1962

Ultimate strength tables for beam columns, 1962, Reprint No. 197 (62-3)

J. Prasad

T. V. Galambos

LEHIGH UNIVERSITY INSTITUTE OF RESEARCH

Welded Continuous Frames and Their Components

ULTIMATE STRENGTH TABLES FOR BEAM COLUMNS

by

T. V. Galambos

J. Prasad

January 1962

Fritz Engineering Laboratory Report No. 287.3

LEHIGH UNIVERSITY
BETHLEHEM, PENNSYLVANIA

287

January 25, 1962

DEPARTMENT OF CIVIL ENGINEERING
FRITZ ENGINEERING LABORATORY

MEMBERS OF LEHIGH PROJECT SUBCOMMITTEE (WRC)

Higgins, T.R.	Fox, G.	Marshall, R.W.
Amirikian, A.	Grover, L.	Newmark, N.M.
Beedle, L.S.	Jameson, W.H.	Pisetzner, E.
Crowley, J.M.	Johnston, B.G.	Stuchell, R.M.
Dill, F.H.	Kavanagh, T.C.	Vasta, J.
Epstein, S.	Ketter, R.L.	

RE: Fritz Lab Report No. 287.3
"Ultimate Strength Tables for Beam-Columns" by T.V. Galambos and J. Prasad

Gentlemen:

The enclosed report presents information on the strength of beam-columns in a tabular form for easy use of the designer. The information has been presented previously in several reports as interaction curves. The extent and scale of these curves has never been large enough for effective use, and this report is written to fill in the need for more precise values.

We wish to submit this report for publication as a WRC Bulletin, and we solicit your approval and your comments on the enclosed postcard before March 1, 1962.

Sincerely yours,

Theodore V. Galambos

Theodore V. Galambos

George C. Driscoll Jr.

George C. Driscoll, Jr.

TVG/gem

Enclosures

cc: Messrs. K. H. Koopman
C. F. Larson

Welded Continuous Frames and Their Components

ULTIMATE STRENGTH TABLES FOR BEAM-COLUMNS

by

T. V. Galambos

J. Prasad

This work has been carried out as part of an investigation sponsored jointly by the Welding Research Council and the Department of the Navy with funds furnished by the following:

American Institute of Steel Construction
American Iron and Steel Institute
Institute of Research, Lehigh University
Column Research Council (advisory)
Office of Naval Research (Contract Nonr 610(03))
Bureau of Ships
Bureau of Yards and Docks

Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pennsylvania

December, 1961

Fritz Engineering Laboratory Report No. 287.3

SYNOPSIS

In this report tables are presented for the ultimate strength of as-rolled wide-flange beam-columns bent by end-moments about their major axis. Failure is assumed to take place by excessive bending in the plane of the applied moments. The tables give the critical combinations of length, end-moments and axial force at which failure occurs.

INTRODUCTION

Beam-columns, that is, members which are subjected to an axial force and to bending moments, always fail by inelastic instability.[1] If the bending moments act in one of the principal planes of the cross section, failure may occur due to one of the following instability phenomena:

1. Lateral-torsional buckling
2. Local buckling
3. Excessive bending in the plane of the applied moments.

Lateral-torsional buckling and local buckling will not influence the ultimate strength of as-rolled steel wide-flange beam-columns bent about their major (or strong) axis if the member is adequately braced and if certain width-thickness ratios are not exceeded.[2] For these members failure will thus be due to excessive bending in the plane of the applied moments.

Because this type of failure takes place in the inelastic range, the ultimate strength cannot be computed by an "exact" mathematical procedure. The solution is obtained by numerical integration of the elastic-plastic moment-curvature relationship.[1][3] Interaction curves for various cases of loading are available[1][2][3] and approximate equations for three extreme loading cases are published.[1][2][4]

In applying the available information to a given design problem one is faced with two difficulties: (1) The published interaction curves are drawn at too small a scale and with not enough curves to be of any use in the final design, even though they serve their purpose well in preliminary design. (2) The approximate equations have a limited scope of application and they apply only to extreme cases. The ultimate strength tables of this report are presented in order to permit a more precise analysis than that which is possible with the currently available information.

The tables were prepared from cross-curves constructed from enlarged versions of the originals of the curves reproduced in Refs. 1, 2, and 3. The tables give only the strength of beam-columns. Where additional conditions of rotation requirements must be met[2] further checks are required to assure adequate rotation capacity (Art. 7.4, Ref. 2).

THE SCOPE OF APPLICATION OF THE TABLES

The tables presented herein are only a convenient arrangement of already published information. They represent no new work, and the values are subject to the same limitations as those set forth in the original reports.[1][3] The tables are thus valid for as-rolled wide-flange ASTM-A7 steel sections having a maximum compressive residual stress at the flange tips of 0.3 times the yield stress and which are subjected to end-moments about the major axis. It is assumed that failure is due to excessive bending in the plane of the applied moments, and that lateral-torsional buckling and local buckling will not influence the ultimate strength.

The tables list the maximum end-moment which a beam-column of a given size and length can support under a given axial force and for a given ratio of the end-moments. All values are given non-dimensionally and therefore the tables are applicable for all wide-flange shapes. The pertinent non-dimensional ratios are as follows:

L/r = Slenderness ratio

P/P_y = Axial load ratio

β = End-moment ratio

M_o/M_p = Maximum moment ratio

where

L = Length of the beam-column

r = Major radius of gyration

P = Axial load to be supported

$P_y = A\sigma_y$ = Yield load

A = Cross-sectional area

σ_y = Yield stress (33ksi for A7 steel)

β = Ratio of the smaller to the larger end-moment. When the two moments cause single curvature deformation, the value of β is positive; when they cause double curvature deformation, the value of β is negative. Because β is always a ratio of a smaller to a larger number, $-1.00 \leqslant \beta \leqslant +1.00$.

M_o = Larger of the two end-moments.

$M_p = Z\sigma_y$ = Plastic moment of the section.

Z = Plastic modulus about the major axis.

The ultimate moment values are given for slenderness ratios from 0 to 120, in increments of 10; for axial load ratios from 0 to P_{cr}/P_y (where P_{cr} is the tangent-modulus load for the corresponding axially loaded column) in increments of 0.05; and for end-moment ratios β from -1.00 to +1.00 in increments of 0.2. The spacing of the points is sufficiently close such that linear interpolation can be used for intermediate points.

The use of the tables can be illustrated by the following typical example:

Given:

From the simple plastic analysis of a rigid frame it is found that a particular member of length L = 180 in. is subjected to the following forces:

$P = 125^k$;

$M_o = 376$ in-kips;

$\beta M_o = 126$ in-kips therefore $\beta = 0.335$

Solution:

1st trial: 8WF24

$P/P_y = 0.536$; $M_o/M_p = 0.493$; $L/r = 52.6$

From the tables (for $\beta = 0.4$, $P/P_y = 0.55$; $L/r = 60$)* it is found that $(M_o/M_p)_{max} = 0.40 < 0.493$ and therefore the 8WF24 section is inadequate.

2nd trial: 8WF28

$P/P_y = 0.459$; $M_o/M_p = 0.420$; $L/r = 52.2$

From the tables (for $\beta = 0.4$; $P/P_y = 0.50$; $L/r = 60$) it is found that $(M_o/M_p)_{max} = 0.47 > 0.420$ and therefore the 8WF28 section is adequate.

ACKNOWLEDGEMENT

This study is part of a general investigation "Welded Continuous Frames and Their Components" currently being carried out at the Fritz Engineering Laboratory of the Civil Engineering Department of Lehigh University under the general direction of Lynn S. Beedle. The investigation is sponsored jointly by the Welding Research Council and the Department of the Navy, with funds furnished by the American Institute of Steel Construction, the American Iron and Steel Institute, Lehigh University Institute of Research, the Office of Naval Research, the

*Since interpolation is not used, the more severe values of β, P/P_y, and L/r are chosen.

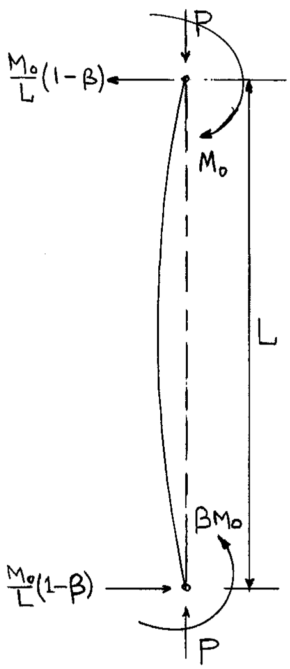
Bureau of Ships and the Bureau of Yards and Docks. The Column Research Council acts in an advisory capacity.

Data for constructing the curves for β = +0.5, -0.5 and -1.0 were supplied by Dr. R. L. Ketter, whose cooperation is greatly appreciated.

ULTIMATE STRENGTH TABLES FOR BEAM-COLUMNS

Scope of the Tables:

As-rolled steel wide-flange sections of ASTM A-7 steel* bent about their strong axis and meeting the bracing rules and width-thickness limitations set forth in Chapter 6 and 7 of Ref. 2.



$-1.00 \leqslant \beta \leqslant +1.00$

in increments of 0.2

$0 \leqslant L/r \leqslant 120$

in increments of 10

$0 \leqslant P/P_y \leqslant P_{cr}/P_y$

in increments of 0.2

---

*For use with other steels (up to the 50 ksi high-strength steels), these tables may be used if the slenderness ratio is adjusted as follows[1]:

$$\left(\frac{L}{r}\right)_{\text{adjusted}} = \left(\frac{L}{r}\right)_{\text{actual}} \sqrt{\frac{\sigma_y \text{ actual}}{33}}$$

ULTIMATE M_o/M_p VALUES FOR β = -1.0

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.98
0.10	0.97	0.97	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
0.15	0.95	0.95	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.93	0.93
0.20	0.91	0.91	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90
0.25	0.86	0.86	0.86	0.86	0.86	0.85	0.85	0.85	0.85	0.84	0.84	0.83	0.83
0.30	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.80	0.79	0.78	0.76
0.35	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.73	0.73	0.72	0.70
0.40	0.70	0.70	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.67	0.67	0.66	0.63
0.45	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.63	0.63	0.62	0.61	0.59	0.58
0.50	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.58	0.58	0.56	0.54	0.53	0.51
0.55	0.53	0.53	0.52	0.52	0.52	0.52	0.52	0.52	0.51	0.50	0.49	0.46	0.43
0.60	0.47	0.47	0.47	0.47	0.46	0.46	0.46	0.46	0.46	0.44	0.42	0.37	0.24
0.65	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.40	0.40	0.38	0.37	0.32	--
0.70	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.34	0.33	0.31	0.27	0.06	--
0.75	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.29	0.28	0.19	0.03	--	--
0.80	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.22	0.20	--	--	--	--
0.85	0.18	0.18	0.18	0.18	0.17	0.14	0.10	--	--	--	--	--	--
0.90	0.12	0.12	0.12	0.12	0.10	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

ULTIMATE M_o/M_p VALUES FOR $\beta = -0.8$

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.98
0.10	0.97	0.97	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
0.15	0.95	0.95	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.93	0.93
0.20	0.91	0.90	0.90	0.90	0.90	0.90	0.90	0.89	0.89	0.89	0.89	0.88	0.88
0.25	0.86	0.86	0.86	0.86	0.86	0.85	0.85	0.85	0.85	0.84	0.83	0.82	0.81
0.30	0.81	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.79	0.77	0.75	0.74
0.35	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.73	0.71	0.67	0.62
0.40	0.70	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.68	0.65	0.59	0.49
0.45	0.64	0.64	0.64	0.64	0.64	0.63	0.63	0.62	0.62	0.61	0.58	0.51	0.42
0.50	0.59	0.58	0.58	0.58	0.58	0.58	0.58	0.57	0.56	0.53	0.50	0.42	0.29
0.55	0.53	0.52	0.52	0.52	0.52	0.52	0.52	0.51	0.50	0.49	0.45	0.34	0.23
0.60	0.47	0.47	0.47	0.46	0.46	0.45	0.45	0.45	0.44	0.39	0.31	0.25	0.09
0.65	0.41	0.41	0.41	0.41	0.41	0.40	0.40	0.40	0.37	0.32	0.25	0.14	--
0.70	0.35	0.35	0.35	0.35	0.35	0.35	0.34	0.33	0.33	0.25	0.14	0.03	--
0.75	0.30	0.30	0.30	0.29	0.29	0.28	0.28	0.25	0.20	0.14	0.02	--	--
0.80	0.24	0.24	0.24	0.23	0.23	0.22	0.21	0.18	0.16	--	--	--	--
0.85	0.18	0.18	0.18	0.17	0.16	0.13	0.08	--	--	--	--	--	--
0.90	0.12	0.12	0.12	0.11	0.06	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

ULTIMATE M_o/M_p VALUES FOR β = -0.6

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.98
0.10	0.97	0.97	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
0.15	0.95	0.94	0.94	0.94	0.94	0.94	0.94	0.93	0.93	0.93	0.93	0.92	0.92
0.20	0.91	0.90	0.90	0.90	0.90	0.90	0.90	0.89	0.89	0.89	0.89	0.87	0.86
0.25	0.86	0.86	0.86	0.86	0.86	0.85	0.85	0.85	0.85	0.84	0.82	0.80	0.79
0.30	0.81	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.78	0.75	0.72	0.69
0.35	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.72	0.68	0.61	0.54
0.40	0.70	0.69	0.69	0.69	0.69	0.69	0.69	0.68	0.68	0.66	0.61	0.53	0.41
0.45	0.64	0.64	0.64	0.64	0.64	0.63	0.63	0.62	0.61	0.59	0.55	0.44	0.30
0.50	0.59	0.58	0.58	0.58	0.58	0.58	0.58	0.56	0.54	0.50	0.45	0.33	0.19
0.55	0.53	0.52	0.52	0.52	0.52	0.52	0.52	0.51	0.48	0.44	0.37	0.24	0.11
0.60	0.47	0.47	0.47	0.46	0.46	0.45	0.45	0.44	0.41	0.34	0.24	0.15	0.02
0.65	0.41	0.41	0.41	0.41	0.41	0.40	0.40	0.38	0.34	0.26	0.16	0.05	--
0.70	0.35	0.35	0.35	0.35	0.35	0.34	0.33	0.31	0.29	0.20	0.07	0.01	--
0.75	0.30	0.30	0.30	0.29	0.29	0.28	0.26	0.22	0.16	0.09	0.01	--	--
0.80	0.24	0.24	0.24	0.23	0.23	0.21	0.18	0.15	0.12	--	--	--	--
0.85	0.18	0.18	0.18	0.17	0.15	0.12	0.06	--	--	--	--	--	--
0.90	0.12	0.12	0.12	0.10	0.06	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

ULTIMATE M_o/M_p VALUES FOR $\beta = -0.4$

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.98
0.10	0.97	0.97	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.95	0.95
0.15	0.95	0.94	0.94	0.94	0.94	0.93	0.93	0.93	0.93	0.93	0.93	0.92	0.91
0.20	0.91	0.90	0.90	0.90	0.90	0.90	0.90	0.89	0.89	0.88	0.88	0.86	0.84
0.25	0.86	0.86	0.86	0.86	0.86	0.85	0.85	0.84	0.84	0.83	0.80	0.77	0.75
0.30	0.81	0.80	0.80	0.80	0.80	0.80	0.80	0.79	0.79	0.77	0.73	0.67	0.61
0.35	0.75	0.75	0.75	0.75	0.75	0.74	0.74	0.74	0.74	0.70	0.65	0.56	0.47
0.40	0.70	0.69	0.69	0.69	0.69	0.68	0.68	0.67	0.66	0.63	0.56	0.47	0.35
0.45	0.64	0.64	0.64	0.63	0.63	0.63	0.62	0.61	0.59	0.54	0.47	0.37	0.23
0.50	0.59	0.58	0.58	0.58	0.58	0.57	0.57	0.54	0.50	0.46	0.38	0.26	0.14
0.55	0.53	0.52	0.52	0.52	0.52	0.51	0.51	0.49	0.44	0.35	0.26	0.16	0.05
0.60	0.47	0.47	0.47	0.46	0.45	0.45	0.45	0.42	0.37	0.29	0.19	0.08	0.01
0.65	0.41	0.41	0.41	0.40	0.40	0.39	0.38	0.36	0.31	0.21	0.10	0.03	--
0.70	0.35	0.35	0.35	0.34	0.34	0.33	0.31	0.29	0.22	0.16	0.04	0.01	--
0.75	0.30	0.29	0.29	0.28	0.28	0.27	0.23	0.19	0.12	0.05	0.01	--	--
0.80	0.24	0.24	0.24	0.23	0.22	0.20	0.15	0.12	0.08	--	--	--	--
0.85	0.18	0.18	0.18	0.17	0.15	0.11	0.06	--	--	--	--	--	--
0.90	0.12	0.12	0.12	0.10	0.05	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

ULTIMATE M_o/M_p VALUES FOR β = -0.2

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.97	0.97
0.10	0.97	0.97	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.95	0.95	0.94	0.93
0.15	0.95	0.94	0.94	0.94	0.94	0.94	0.94	0.93	0.93	0.92	0.92	0.90	0.88
0.20	0.91	0.90	0.90	0.90	0.90	0.90	0.90	0.89	0.89	0.88	0.86	0.84	0.81
0.25	0.86	0.86	0.86	0.85	0.85	0.85	0.85	0.84	0.83	0.81	0.78	0.74	0.69
0.30	0.81	0.80	0.80	0.80	0.80	0.79	0.79	0.78	0.77	0.74	0.69	0.62	0.52
0.35	0.75	0.75	0.75	0.74	0.74	0.73	0.73	0.71	0.70	0.67	0.61	0.51	0.40
0.40	0.70	0.69	0.69	0.69	0.69	0.68	0.67	0.65	0.62	0.58	0.51	0.40	0.29
0.45	0.64	0.64	0.64	0.63	0.62	0.62	0.61	0.58	0.54	0.47	0.39	0.30	0.19
0.50	0.59	0.58	0.58	0.57	0.57	0.56	0.55	0.51	0.46	0.40	0.32	0.20	0.12
0.55	0.53	0.52	0.52	0.51	0.51	0.50	0.49	0.46	0.39	0.27	0.19	0.11	0.04
0.60	0.47	0.47	0.47	0.46	0.45	0.44	0.43	0.38	0.32	0.24	0.14	0.05	0.01
0.65	0.41	0.41	0.41	0.40	0.39	0.37	0.36	0.31	0.27	0.16	0.07	0.03	--
0.70	0.35	0.35	0.35	0.34	0.34	0.31	0.29	0.25	0.17	0.12	0.03	0.01	--
0.75	0.30	0.29	0.29	0.29	0.27	0.25	0.21	0.16	0.10	0.03	0.01	--	--
0.80	0.24	0.24	0.24	0.23	0.21	0.18	0.13	0.09	0.04	--	--	--	--
0.85	0.18	0.18	0.18	0.17	0.14	0.10	0.05	--	--	--	--	--	--
0.90	0.12	0.12	0.12	0.10	0.05	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

ULTIMATE M_o/M_p VALUES FOR β = 0.00

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.96	0.96
0.10	0.97	0.97	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.95	0.94	0.93	0.91
0.15	0.95	0.94	0.94	0.90	0.94	0.94	0.94	0.93	0.93	0.92	0.90	0.88	0.85
0.20	0.91	0.90	0.90	0.85	0.90	0.90	0.90	0.89	0.88	0.86	0.83	0.79	0.75
0.25	0.86	0.86	0.86	0.80	0.85	0.84	0.84	0.82	0.81	0.78	0.73	0.67	0.60
0.30	0.81	0.80	0.80	0.74	0.80	0.79	0.77	0.75	0.73	0.70	0.65	0.57	0.41
0.35	0.75	0.75	0.75	0.68	0.74	0.72	0.71	0.68	0.66	0.62	0.55	0.46	0.34
0.40	0.70	0.69	0.69	0.63	0.68	0.67	0.65	0.62	0.58	0.52	0.45	0.34	0.24
0.45	0.64	0.64	0.64	0.63	0.62	0.61	0.59	0.54	0.49	0.42	0.34	0.25	0.16
0.50	0.59	0.58	0.57	0.56	0.56	0.54	0.52	0.48	0.41	0.34	0.26	0.17	0.10
0.55	0.53	0.52	0.52	0.51	0.50	0.48	0.46	0.42	0.34	0.23	0.15	0.09	0.04
0.60	0.47	0.47	0.47	0.46	0.44	0.42	0.40	0.34	0.27	0.19	0.11	0.05	0.01
0.65	0.41	0.41	0.41	0.40	0.38	0.35	0.34	0.27	0.21	0.13	0.06	0.02	--
0.70	0.35	0.35	0.35	0.34	0.33	0.30	0.27	0.20	0.14	0.09	0.03	0.01	--
0.75	0.30	0.29	0.29	0.29	0.27	0.23	0.19	0.14	0.08	0.03	0.01	--	--
0.80	0.24	0.23	0.23	0.23	0.21	0.17	0.12	0.08	0.03	--	--	--	--
0.85	0.18	0.18	0.18	0.16	0.13	0.09	0.05	--	--	--	--	--	--
0.90	0.12	0.12	0.12	0.10	0.05	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

ULTIMATE M_o/M_p VALUES FOR β = +0.20

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.95	0.94
0.10	0.97	0.97	0.96	0.96	0.96	0.96	0.96	0.95	0.95	0.94	0.92	0.90	0.88
0.15	0.95	0.94	0.94	0.94	0.94	0.94	0.94	0.93	0.92	0.89	0.87	0.84	0.80
0.20	0.91	0.90	0.90	0.90	0.90	0.89	0.89	0.87	0.86	0.82	0.78	0.74	0.68
0.25	0.86	0.86	0.86	0.85	0.85	0.84	0.83	0.80	0.77	0.73	0.67	0.60	0.52
0.30	0.81	0.80	0.80	0.80	0.80	0.78	0.75	0.71	0.69	0.63	0.58	0.50	0.36
0.35	0.75	0.75	0.75	0.74	0.73	0.71	0.68	0.65	0.60	0.55	0.48	0.40	0.29
0.40	0.70	0.69	0.69	0.68	0.67	0.65	0.63	0.58	0.53	0.46	0.38	0.29	0.20
0.45	0.64	0.63	0.63	0.62	0.61	0.59	0.56	0.51	0.45	0.37	0.30	0.21	0.14
0.50	0.59	0.58	0.57	0.56	0.55	0.53	0.50	0.45	0.37	0.30	0.22	0.15	0.08
0.55	0.53	0.52	0.52	0.51	0.49	0.47	0.43	0.38	0.31	0.22	0.14	0.09	0.04
0.60	0.47	0.47	0.46	0.45	0.43	0.40	0.36	0.30	0.24	0.16	0.09	0.05	0.01
0.65	0.41	0.41	0.41	0.39	0.37	0.34	0.29	0.23	0.17	0.12	0.06	0.02	--
0.70	0.35	0.35	0.35	0.34	0.32	0.28	0.24	0.17	0.12	0.06	0.02	0.01	--
0.75	0.30	0.29	0.29	0.28	0.26	0.22	0.17	0.12	0.07	0.03	0.01	--	--
0.80	0.24	0.23	0.23	0.22	0.20	0.16	0.11	0.07	0.02	--	--	--	--
0.85	0.18	0.17	0.17	0.15	0.12	0.09	0.04	--	--	--	--	--	--
0.90	0.12	0.11	0.10	0.08	0.04	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

ULTIMATE M_o/M_p VALUES FOR β = +0.4

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.96	0.95	0.94	0.93
0.10	0.97	0.96	0.96	0.96	0.96	0.96	0.96	0.95	0.93	0.92	0.90	0.88	0.85
0.15	0.95	0.94	0.94	0.94	0.94	0.93	0.93	0.91	0.89	0.86	0.83	0.79	0.75
0.20	0.91	0.90	0.90	0.89	0.89	0.88	0.88	0.86	0.83	0.78	0.74	0.68	0.63
0.25	0.86	0.85	0.85	0.84	0.84	0.83	0.80	0.76	0.72	0.68	0.61	0.55	0.47
0.30	0.81	0.80	0.80	0.79	0.79	0.76	0.72	0.68	0.63	0.57	0.51	0.44	0.33
0.35	0.75	0.75	0.75	0.74	0.72	0.59	0.65	0.61	0.55	0.48	0.42	0.30	0.24
0.40	0.70	0.69	0.69	0.68	0.66	0.63	0.60	0.54	0.48	0.41	0.33	0.26	0.17
0.45	0.64	0.63	0.63	0.62	0.60	0.57	0.53	0.47	0.41	0.34	0.27	0.19	0.12
0.50	0.59	0.59	0.57	0.56	0.54	0.51	0.47	0.41	0.33	0.27	0.20	0.14	0.07
0.55	0.53	0.52	0.52	0.50	0.48	0.44	0.40	0.34	0.27	0.20	0.14	0.08	0.04
0.60	0.47	0.46	0.46	0.44	0.42	0.37	0.33	0.28	0.21	0.15	0.09	0.04	0.01
0.65	0.41	0.40	0.40	0.38	0.36	0.33	0.27	0.21	0.15	0.10	0.05	0.02	--
0.70	0.35	0.35	0.35	0.33	0.31	0.27	0.22	0.15	0.10	0.05	0.02	0.01	--
0.75	0.30	0.29	0.28	0.27	0.25	0.21	0.16	0.10	0.06	0.03	0.01	--	--
0.80	0.24	0.23	0.23	0.20	0.18	0.14	0.10	0.06	0.02	--	--	--	--
0.85	0.18	0.17	0.16	0.14	0.11	0.08	0.04	--	--	--	--	--	--
0.90	0.12	0.11	0.09	0.07	0.04	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

ULTIMATE M_o/M_p VALUES FOR β = +0.6

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.98	0.98	0.98	0.97	0.97	0.96	0.96	0.95	0.94	0.93	0.91
0.10	0.97	0.97	0.96	0.96	0.96	0.95	0.94	0.93	0.91	0.89	0.87	0.84	0.82
0.15	0.94	0.94	0.94	0.93	0.93	0.92	0.91	0.88	0.85	0.82	0.79	0.75	0.70
0.20	0.91	0.90	0.90	0.89	0.89	0.87	0.86	0.82	0.78	0.74	0.69	0.63	0.57
0.25	0.86	0.85	0.85	0.84	0.83	0.80	0.77	0.73	0.68	0.63	0.56	0.50	0.43
0.30	0.81	0.80	0.80	0.79	0.77	0.73	0.69	0.54	0.57	0.52	0.46	0.38	0.31
0.35	0.75	0.74	0.74	0.73	0.71	0.66	0.62	0.57	0.51	0.44	0.37	0.30	0.22
0.40	0.70	0.69	0.68	0.66	0.65	0.61	0.56	0.51	0.44	0.38	0.30	0.23	0.16
0.45	0.64	0.63	0.62	0.60	0.58	0.54	0.50	0.44	0.37	0.31	0.24	0.17	0.11
0.50	0.59	0.58	0.57	0.55	0.53	0.48	0.43	0.38	0.31	0.25	0.18	0.13	0.06
0.55	0.53	0.52	0.51	0.49	0.46	0.42	0.37	0.31	0.25	0.19	0.13	0.08	0.03
0.60	0.47	0.46	0.46	0.43	0.40	0.35	0.30	0.25	0.19	0.13	0.08	0.04	0.01
0.65	0.41	0.40	0.39	0.37	0.35	0.31	0.25	0.19	0.14	0.09	0.05	0.01	--
0.70	0.35	0.34	0.34	0.32	0.29	0.25	0.20	0.13	0.10	0.05	0.02	0.01	--
0.75	0.30	0.29	0.28	0.26	0.23	0.20	0.14	0.09	0.05	0.03	0.01	--	--
0.80	0.24	0.23	0.22	0.19	0.17	0.13	0.09	0.05	0.02	--	--	--	--
0.85	0.18	0.21	0.15	0.12	0.10	0.07	0.03	--	--	--	--	--	--
0.90	0.12	0.10	0.08	0.05	0.03	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

ULTIMATE M_o/M_p VALUES FOR β = +0.8

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.98	0.98	0.97	0.96	0.96	0.95	0.95	0.94	0.93	0.91	0.90
0.10	0.97	0.97	0.96	0.95	0.94	0.93	0.92	0.90	0.88	0.86	0.84	0.81	0.78
0.15	0.95	0.94	0.93	0.92	0.90	0.89	0.87	0.84	0.81	0.78	0.74	0.70	0.65
0.20	0.91	0.90	0.89	0.87	0.86	0.84	0.82	0.77	0.73	0.69	0.64	0.58	0.52
0.25	0.86	0.85	0.84	0.82	0.80	0.76	0.73	0.68	0.64	0.59	0.52	0.47	0.40
0.30	0.81	0.79	0.79	0.76	0.74	0.69	0.66	0.60	0.54	0.48	0.43	0.36	0.29
0.35	0.75	0.74	0.72	0.70	0.68	0.63	0.58	0.53	0.47	0.41	0.34	0.27	0.20
0.40	0.70	0.68	0.67	0.64	0.61	0.57	0.53	0.47	0.40	0.35	0.28	0.22	0.15
0.45	0.64	0.63	0.61	0.58	0.56	0.50	0.46	0.40	0.34	0.28	0.22	0.16	0.10
0.50	0.59	0.57	0.55	0.52	0.50	0.45	0.40	0.34	0.28	0.23	0.17	0.11	0.06
0.55	0.53	0.51	0.49	0.47	0.44	0.39	0.34	0.28	0.23	0.18	0.12	0.08	0.03
0.60	0.47	0.45	0.44	0.42	0.38	0.33	0.28	0.23	0.18	0.12	0.08	0.04	0.01
0.65	0.41	0.40	0.38	0.35	0.33	0.28	0.23	0.18	0.13	0.09	0.05	0.01	--
0.70	0.35	0.34	0.33	0.30	0.27	0.22	0.18	0.12	0.09	0.05	0.02	0.01	--
0.75	0.30	0.28	0.25	0.24	0.21	0.18	0.13	0.09	0.05	0.02	0.01	--	--
0.80	0.24	0.22	0.21	0.18	0.15	0.12	0.08	0.05	0.02	--	--	--	--
0.85	0.18	0.16	0.13	0.11	0.09	0.06	0.03	--	--	--	--	--	--
0.90	0.12	0.10	0.07	0.05	0.03	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

ULTIMATE M_o/M_p VALUES FOR $\beta = +1.0$

P/P_y	L/r=0	L/r=10	L/r=20	L/r=30	L/r=40	L/r=50	L/r=60	L/r=70	L/r=80	L/r=90	L/r=100	L/r=110	L/r=120
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	0.99	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.93	0.92	0.91	0.90	0.89
0.10	0.97	0.96	0.95	0.93	0.92	0.90	0.88	0.87	0.85	0.82	0.80	0.77	0.74
0.15	0.95	0.93	0.91	0.88	0.86	0.84	0.81	0.79	0.77	0.74	0.68	0.64	0.60
0.20	0.91	0.88	0.86	0.84	0.81	0.78	0.75	0.72	0.68	0.63	0.59	0.54	0.48
0.25	0.86	0.84	0.81	0.78	0.75	0.72	0.67	0.64	0.60	0.55	0.50	0.43	0.37
0.30	0.81	0.93	0.76	0.72	0.69	0.65	0.61	0.56	0.52	0.46	0.40	0.34	0.28
0.35	0.75	0.72	0.70	0.66	0.62	0.58	0.54	0.49	0.44	0.38	0.33	0.27	0.20
0.40	0.70	0.67	0.63	0.60	0.57	0.52	0.48	0.43	0.38	0.32	0.27	0.20	0.14
0.45	0.64	0.61	0.59	0.55	0.51	0.47	0.42	0.37	0.32	0.26	0.21	0.15	0.09
0.50	0.59	0.55	0.52	0.49	0.45	0.41	0.36	0.31	0.27	0.22	0.16	0.10	0.06
0.55	0.53	0.50	0.46	0.43	0.40	0.36	0.31	0.26	0.22	0.16	0.12	0.07	0.03
0.60	0.47	0.44	0.42	0.38	0.35	0.31	0.26	0.21	0.17	0.12	0.06	0.04	0.01
0.65	0.41	0.39	0.36	0.32	0.29	0.25	0.21	0.17	0.13	0.08	0.04	0.01	--
0.70	0.35	0.32	0.30	0.27	0.24	0.20	0.16	0.12	0.09	0.05	0.02	0.01	--
0.75	0.30	0.27	0.24	0.22	0.18	0.15	0.11	0.08	0.05	0.02	0.01	--	--
0.80	0.24	0.22	0.19	0.16	0.13	0.09	0.07	0.04	0.02	--	--	--	--
0.85	0.18	0.15	0.13	0.10	0.07	0.05	0.03	--	--	--	--	--	--
0.90	0.12	0.09	0.07	0.05	0.02	--	--	--	--	--	--	--	--
0.95	0.06	0.04	0.02	--	--	--	--	--	--	--	--	--	--
1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

REFERENCES

1. R. L. Ketter, T. V. Galambos
COLUMNS UNDER COMBINED BENDING AND THRUST,
Proceedings of the ASCE, 85 (EM2) (April 1959)

2. WRC - ASCE Committee
COMMENTARY ON PLASTIC DESIGN IN STEEL
ASCE Manual No. 41, (1961)

3. R. L. Ketter
FURTHER STUDIES ON THE STRENGTH OF BEAM-COLUMNS
Proceedings of the ASCE, 87 (ST6) (August 1961)

4. AISC
PLASTIC DESIGN IN STEEL
AISC, New York, 1959