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Minimum weight design of frames using sway subassemblage theory, December 1970

Hiroshi Yoshida

J. Hartley Daniels

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MINIMUM WEIGHT DESIGN OF FRAMES USING SWAY SUBASSEMBLAGE THEORY

by

Hiroshi Yoshida

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This work has been carried out as part of an investigation sponsored by the American Iron and Steel Institute.

Fritz Engineering Laboratory

Lehigh University

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ABSTRACT

This report considers the theoretical development of a minimum weight design procedure for unbraced multi-story frames which are subjected to combined gravity and wind loads.

The lateral load versus sway deflection of unbraced frames which are designed by the moment balancing method are discussed first. Then the load-deflection behavior of a 3-bay 10-story frame which was designed by the moment balancing method is analyzed using the sway subassemblage method[1,2] and a second-order elastic-plastic method of analysis.[3] The behavior of this unbraced frame under working load values of the combined loads is then discussed. It is shown that the sway deflection of the frame under the working loads is somewhat larger than usually considered practical. Furthermore, it is shown that unbraced multi-story frames designed by the moment balancing method may not in general achieve acceptable sway deflections at working load.

The minimum weight design method using sway subassemblage theory is then described. This method determines the minimum weight of beams and columns in an unbraced multi-story frame considering the following design constraints.

1. A specified maximum sway deflection of a story under combined working loads.
2. No plastic hinges at the working load level of the combined loads.

Once the minimum weight design of the frame has been achieved, the sway subassemblage method of analysis is then used to determine if the frame has the required capacity under factored combined loads. A computer program written in Fortran IV for the minimum weight design of an unbraced frame subject to the above constraints is used in the analyses of the example frame.

1. INTRODUCTION

An unbraced multi-story frame should be designed to meet the following five conditions:

1. Frame buckling does not occur before attainment of the factored gravity load,
2. Frame instability does not occur before attainment of the factored combined gravity and wind loads,
3. No plastic hinges occur under either the working gravity loads or the working combined wind and gravity loads,
4. The sway deflection of each story of the frame under the working combined loads should be restricted to a maximum value, and
5. A minimum weight design with respect to the beams and columns should be achieved.

In general, an unbraced multi-story frame can be designed by trial and error procedures which involves the following three steps;[4]

1. The preliminary design; the selection of tentative beam and column sizes.
2. The analysis; the determination of the adequacy of members selected in step (1) based on strength and stiffness.
3. The revision; the revision of one or more members based on the results of the analysis or on other factors such as minimum weight or economy.

For the preliminary design, the moment balancing method of analysis can be used. However only an estimate of the P-Δ effects is included at this point. The sway subassemblage method of analysis has been developed to check the adequacy of the preliminary design based on frame strength and stiffness.[1,2] The P-Δ effect can be determined from such an analysis[6] and can be compared with that assumed in the preliminary design by moment balancing method. Based on the results of the analysis, a revision of the preliminary design can be made. A subsequent analysis is then required.

However, there has been no rational basis developed to date on which to make the required revision of the preliminary design and at the same time meet all the previous design conditions.

This report presents a method of designing unbraced multi-story frames for the combined gravity and wind load condition which will meet these design conditions. It utilizes both the moment balancing and the sway subassemblage methods previously developed.[1,5] In addition it develops a minimum weight design procedure which is based on the basic assumptions of the sway subassemblage method.[1]

The nature of problem of designing frames for minimum weight has been clarified considerably by the work of J. Foulkes.[7] This work has been extended by further investigations.[8,9] It was assumed in Ref 7 that;

1. The full plastic moments of the members are unaffected by shear force and axial thrust,
2. An infinite range of sections is available, and

3. The curve which represents the relation between the weight per unit length and the full plastic moment of the section can be replaced by a straight line.

Messrs. Moshe F. Rubinstein and John Karagozian[10] discuss the preliminary design of an unbraced frame on a minimum weight basis using the following assumptions:

1. Plastic hinges form only in the beams.
2. A linear variation of member sizes with story height is assumed.
3. The contributions of the beams and columns to the flexibility of a building frame are separated and a conservative ratio between those contributions is established.

They conclude that it is more efficient to provide increased stiffness of the beams in the exterior bays of an unbraced frame.

T. M. Murray also treats the optimum design of unbraced frames. However, this work does not consider either the effect of P-Δ moments or the sway limitation at the working loads.

In the minimum weight method of design to be developed in this report, the following conditions are assumed for the frame and loading (in addition to the assumptions on which the sway subassemblage method of analysis are based).

1. The full plastic moments of the members are reduced by the axial thrusts,

2. Only those shapes listed in the AISC Manual of Steel Construction[12] are available,
3. The effect of P-Δ moments in the behavior of the frame are considered,
4. The members selected are adequate for the factored gravity load condition,
5. A working load sway limitation under the combined loads is considered,
6. No plastic hinges occur under the working loads, and
7. A minimum weight design of the frame at the working combined loads is achieved.

Since the minimum weight design procedure does not consider frame strength and stiffness under the factored combined gravity and wind loads, the minimum weight design is then checked using the sway subassemblage method of analysis. If the frame does not achieve the required capacity under the factored combined loads, another minimum weight design can be performed. To achieve increased factored load capacity of the story under combined loads, the minimum weight design can be repeated using either of the following criterion:

1. A smaller working load sway limitation is specified, or
2. The same working load sway limitation is retained but the formation of plastic hinges is delayed to a specified level of loads greater than the working load level.

To illustrate the design procedure developed in this report, Frame B of Ref. 6, will be used.

2. PRELIMINARY DESIGN OF FRAME B BY MOMENT BALANCING METHOD

The load deflection behavior of Frame B as designed in Ref. 6 will be examined under both working and design ultimate combined loads using the sway subassemblage method of analysis. The dimensions and loading for Frame B are shown in Fig. 1. The member sizes determined by the moment balancing method are shown in Fig. 2.[6] The axial thrusts in the columns under working and design ultimate combined loads must be estimated before calculating the load deflection behavior of a story. In the sway subassemblage method of analysis, these axial thrusts can be assumed to remain constant during application of the wind load. Axial thrusts due to gravity loads can be based on the tributary column area.[13] Axial thrusts due to the wind load however can also be estimated under the desired load level. Several methods for estimating the axial thrusts in the columns either at the working or the design ultimate level of the wind load will be discussed in this report.

Approximate methods of analysis are available for elastic frames, such as the cantilever method.[14] A modified elastic solution for the frame will be used in this report to determine the approximate value of the axial thrusts in the columns under working wind loads.

Using the assumptions of the sway subassemblage methods of analysis, a one-story assemblage at level n is isolated from an unbraced multi-story frame as shown in Fig. 3. The axial thrusts in the columns can be determined by the slope-deflection method of analysis under the following assumptions.

1. The total horizontal shear forces in the columns above and below level n are the same, and
2. The sway deflections for each column are the same.

Table 1 shows the axial thrusts in the columns due to working wind load as determined by this method. Figure 4 shows the lateral load versus sway-deflection behavior at working loads for levels 4, 6, 8 and 10 in Frame B using the sway subassemblage computer program.[15] The vertical axis shows the applied lateral shear force non-dimensionalized by the working index Δ/h of the story where Δ is the sway deflection and h is story height.

Under the design ultimate load level, the axial thrusts in the columns can be determined by assuming the following distribution of bending moments in a one-story assemblage.

1. The bending moments at the leeward ends of the beams are at the full plastic moment,
2. The bending moments at the windward ends of the beams or within the spans, whichever is applicable (this depends on the magnitude of beam loading) are at the full plastic moment, and
3. At each joint the sum of the bending moments in the beams is equal to or less than ΣM_{pc} for the columns.

Table 2 shows the axial thrusts in columns due to the design ultimate wind loads by this method. Figure 5 shows the lateral-load versus sway-deflection behavior of levels 4, 6, 8 and 10 of Frame B for the design ultimate combined loads.

The following observations can be made from Figs. 4 and 5.

1. The sway deflections under the working combined loads are probably too large for practical designs.
2. Plastic hinges form considerably before the attainment of working loads in levels 4 and 6.
3. The strengths of levels 4 and 6 are considerably below the desired design ultimate load level of the combined loads (L.F. = 1.3).

In Figs. 6 and 7, curve 1 shows the lateral-load versus sway-deflection behavior for constant gravity load (L.F. = 1.3) as calculated by the sway subassemblage method of analysis. Curve 2 shows the same behavior for proportionally increasing gravity load with the lateral load calculated in a step-by-step method for gradually increasing gravity load using the sway subassemblage method of analysis. Curve 3 was obtained by an "exact" second-order elastic-plastic analysis.[3] For level 6 the degree of approximation using the sway subassemblage method is not too large. Therefore, the strength of level 6 under the constant gravity load (L.F. = 1.3) as shown by curve 1 should be fairly accurate. It can be seen that considerably lower strength was obtained at level 6 when gravity loads were held constant at their design ultimate values.

A similar comparison was made for level 8 as shown in Fig. 7. In this case curves 2 and 3 indicate close agreement between the "exact" and the sway subassemblage methods. For level 8, then, curve 1 which was obtained for a constant gravity load (L.F. = 1.3) should be very accurate. However, considerably larger strength under non-proportional load is available at level 8. It can be noted however from Fig. 7 that under the design ultimate values of combined gravity and lateral loads, all three curves are in close agreement.

3. MINIMUM WEIGHT DESIGN OF FRAMES

3.1 Shear Distribution Factors for a Sway Subassemblage

Fig. 3 shows the loading condition for a one story assemblage isolated from an unbraced multi-story frame. The axial forces in the columns are determined as discussed in Chapter 2. The total shear force due to lateral loading can be calculated from the loading condition. However, the distribution of shear force to each column must be determined. The one story assemblage shown in Fig. 3 can be divided into four sway subassemblages[1] as shown in Fig. 8. Figure 9 shows a typical interior sway subassemblage. The restraining coefficients $K_{i-1,i}$ and K_{ji} in Fig. 9 can be approximately expressed by Eqs. 54 and 56 in Ref. 1.

The relationship between the horizontal shear force, $\lambda_1 Q_n$ and deflection index $\rho = \Delta/h$ in Fig. 9 can be expressed by

$$\lambda_1 Q_n = \frac{\left(\frac{I_{i-1,i}}{L_{i-1,i}}\xi_{i-1} + \frac{I_{ij}}{L_{ij}}\xi_{i+1}\right)\left(4\frac{EI_i}{h}U_i - P_i h\right) - U_i P_i h \frac{I_i}{h}}{\left(\frac{I_{i-1,i}}{L_{i-1,i}}\xi_{i-1} + \frac{I_{ij}}{L_{ij}}\xi_{i+1} + \frac{I_i}{h}U_i\right) h}\,\rho \qquad (1)$$

where $\xi_{i-1} = \dfrac{3 - K_{i-1,i}}{4 - K_{i-1,i}}$

$\xi_{i+1} \quad \dfrac{3 - K_{ji}}{4 - K_{ji}}$

$U_i \quad \dfrac{1}{C_i}(C_i^2 - S_i^2)$

$$C_i = \frac{c_i}{c_i^2 - s_i^2}$$

$$S_i = \frac{s_i}{c_i^2 - s_i^2}$$

$$c_i = \frac{1}{\phi^2}\ (1 - \phi \coth \phi)$$

$$s_i = \frac{1}{\phi^2}\ (\frac{\phi}{\sin \phi} - 1)$$

$$\phi = \frac{h}{2}\sqrt{\frac{P_i}{EI_i}}$$

The derivation of Eq. 1 is given in Appendix I.

The sway deflections of each sway subassemblage under the applied horizontal shear force Q_n are assumed to be equal. Also the sum of the column shears for each sway subassemblage is equal to the total applied shear force Q_n. Using these relations, the horizontal shear distribution factor λ_i can be determined as follows:

$$\lambda_i = \frac{\dfrac{\left(\dfrac{I_{i-1,i}}{L_{i-1,i}}\ \xi_{i-1} + \dfrac{I_{ij}}{L_{ij}}\ \xi_{i+1}\right)\left(4\ \dfrac{EI_i}{h}\ U_i - P_i h\right) - U_i\ P_i h\ \dfrac{I_i}{h}}{\left(\dfrac{I_{i-1,i}}{L_{i-1,i}}\ \xi_{i-1} + \dfrac{I_{ij}}{L_{ij}}\ \xi_{i+1} + \dfrac{I_i}{h}\ U_i\right) h}}{\displaystyle\sum_{i+1}^{n} \dfrac{\left(\dfrac{I_{i-1,i}}{L_{i-1,i}}\ \xi_{i-1} + \dfrac{I_{ij}}{L_{ij}}\ \xi_{i+1}\right)\left(4\ \dfrac{EI_i}{h}\ U_i - P_i h\right) - U_i\ P_i h\ \dfrac{I_i}{h}}{\left(\dfrac{I_{i-1,i}}{L_{i-1,i}}\ \xi_{i-1} + \dfrac{I_{ij}}{L_{ij}}\ \xi_{i+1} + \dfrac{I_i}{h}\ U_i\right) h}} \tag{2}$$

3.2 The Relationship Between Moment Inertia of Beam and Column for a Constant Sway

Based on Eq. 1, the compatibility condition for the interior sway subassemblage shown in Fig. 9 for the given horizontal shear force and restricted working load sway ρ is as follows:

$$\lambda_i \left(\frac{I_{i-1,i}}{L_{i-1,i}} \xi_{i-1} + \frac{I_{ij}}{L_{ij}} \xi_{i+1} + \frac{I_i}{h} U_i \right) h\, Q_n = \left(\frac{I_{i-1,i}}{L_{i-1,i}} \xi_{i-1} + \frac{I_{ij}}{L_{ij}} \xi_{i+1} \right)$$

$$x \left(4 \frac{EI_i}{h} U_i - P_i h \right) \rho - U_i P_i h \frac{I_i}{H} \rho \tag{3}$$

Expressing Eq. 2 in terms of the moment of inertia of column I_i which is required for maintaining constant sway P gives

$$\frac{I_i}{h} = \frac{\left(\lambda_i h\, Q_n + \rho\, P_i h \right) \left(\frac{I_{i-1,i}}{L_{i-1,i}} \xi_{i-1} + \frac{I_{ij}}{L_{ij}} \xi_{i+1} \right)}{4EU_i \rho \left(\frac{I_{i-1,i}}{L_{i-1,i}} \xi_{i-1} + \frac{I_{ij}}{L_{ij}} \xi_{i+1} \right) - U_i P_i h\, \rho - \lambda_i U_i h\, Q_n} \tag{4}$$

where $\frac{I_{i-1,i}}{L_{i-1,i}} \xi_{i-1} + \frac{I_{ij}}{L_{ij}} \xi_{i+1} > \frac{1}{4E\rho} (P_i h\, P + \lambda_i h\, Q)$

Also expressing Eq. 2 in terms of the moment of inertia of the beams which is required for maintaining constant sway P gives

$$\frac{I_{ij}}{L_{ij}} = \frac{4 \frac{EI_i}{h} U_i \rho \frac{I_{i-1,i}}{L_{i-1,i}} \xi_{i-1} - (\lambda_i h\, Q + P_i h \rho) \frac{I_{i-1,i}}{L_{i-1,i}} \xi_{i-1} - (P_i \rho + \lambda\, Q_n) U_i h \frac{I_i}{h}}{(\lambda_i h\, Q_n + P_i h\, \rho - 4 \frac{EI_i}{h} U_i\, \rho)\; \xi_{i+1}} \tag{5}$$

where $\frac{I_i}{h} > \frac{1}{4EU_i \rho} (\lambda_i Q_n + P_i \rho)\, h$

If the moment of inertia of either the beam or the column in the sway subassemblage shown in Fig. 9 is known, the moment of inertia of the other member which maintains the constant sway condition can be found from Eqs. 4 or 5.

3.3 The Minimum Weight Design Process

The three-step design process for an unbraced multi-story frame has been described previously in Chapter 1. In this article, the following optimum design procedure will be described in accordance with those steps.

1. A frame which is designed by the moment balancing method is taken as the preliminary design.[6]
2. The axial thrusts in the columns due to the working loads are calculated using the method described in Chapter 2.
3. The bending moments in the beams and columns are then calculated under the working combined loads.
4. The distribution factors λ_i are calculated by Eq. 1 for each column.
5. Each one-story assemblage is then divided into sway subassemblages.
6. The beam and the column for the windward sway subassemblage is first optimized with respect to weight using Eqs. 4 and 5.
7. The plastic moment condition for the beam and column determined in step (6) is then checked using the bending moments calculated in step (3).
8. The combination of beam and column which gives a minimum weight and satisfies the plastic moment condition is then selected as the first trial members for the windward sway subassemblage.
9. For the first interior sway subassemblage, the column and leeward beam are then optimized with respect to weight. The windward beam which was previously chosen in step (8) is held constant.

10. All interior sway subassemblages are optimized in the same way preceeding from the windward to the leeward side of the one story assemblage.
11. The column in the leeward sway subassemblage remains. This column is determined by Eq. 4 and the plastic moment condition.

After all members of a one story assemblage are determined, the calculation must be repeated from step (3) to (11) using new value of λ_1 until convergence is obtained. The previous procedure is carried out for wind from both directions such that all members chosen satisfy the 4 conditions listed in Chapter 1 for the minimum weight design of the frame. The final members obtained are then used when the story is checked for its capacity under the design ultimate value of the combined loads.

4. DESIGN EXAMPLE AND RESULTS

Using a computer program based on the work described in Chapter 3, the minimum weight design of Frame B was obtained for assumed working load sway limitations of $\rho_L = \Delta_L/h = 0.001$, 0.0015, 0.002, 0.0025, 0.003 and 0.004. The resulting designs of Frame B for $\rho_L = 0.002$, 0.0025, 0.003 and 0.004 are shown in Figs. 10 to 13. The weights of the one-story assemblages at levels 2, 4, 6, 8 and 10 for each sway limitation are plotted in Fig. 14.

As shown in Fig. 14 the weights of one-story assemblages at levels 2 and 4 do not change appreciably for the ranges of sway limitation $\rho_L = 0.0015$ to 0.004 and 0.0025 to 0.004, respectively. This means that the member sizes at levels 2 and 4 are likely controlled by the plastic moment condition under the design ultimate gravity loads alone. At levels 6, 8 and 10, the weight of each one-story assemblage increases gradually as the working load sway limitation decreases from 0.004 to 0.0025 and then increases sharply for sway limitations less than about 0.0025. Figure 14 also shows that for the same working load sway limitation, the minimum weight frame at levels 6, 8 and 10 is up to 5.0 percent lighter than the frame obtained by the moment balancing method (Fig. 2). For a sway limitation of about 0.002 at working loads, however, the minimum weight frame is somewhat heavier than the frame designed by moment balancing as would be expected.

Although Fig. 14 indicates that the gravity load condition controls the design of levels 2 and 4, the sway subassemblage method is not expected to yield accurate solutions in the upper stories of a frame.[6] Therefore, the minimum weight solutions at levels 2 and 4 are somewhat questionable.

The actual deflection indexes (Δ/h) of each level under the working load must prove to be equal to the given sway limitation of each level for a minimum weight frame. Figure 15 shows the lateral-load versus sway-deflection behavior at level 8 of the minimum weight frames under the working loads. The deflection indexes for $\rho_L = 0.001$ to $\rho_L = 0.003$ are very close to those specified. The deflection index for $\rho_L = 0.004$ is much less than the specified working load sway limitation because the member sizes of this one-story assemblage were determined by the plastic moment condition and not by sway limitations. Figure 15 also shows that in this case a plastic hinge forms just after the attainment of the working load level of the lateral load. Figures 16 and 17 show the lateral-load versus sway-deflection behavior under the design ultimate combined loads for levels 6 and 8 of three minimum weight frames. In levels 6 and 8, all one-story assemblages corresponding to sway limitations of 0.001 to 0.003 have sufficient strength under the design ultimate combined loads. However, the strengths of levels 6 and 8 for a sway limitation $\rho_L = 0.004$ are less than the design ultimate load level (L.F. = 1.3).

From the view point of strength and economy, the sway limitation of $0.0025 \sim 0.003$ is available as far as Frame B is concerned.

5. CONCLUSIONS

A design method and an associated computer program has been developed for the minimum weight design of unbraced multi-story frames. The theoretical basis of the method is the sway subassemblage method of analysis. The computer program is limited to rigid plane-frames of up to thirty stories and five bays. Uniformly distributed beam loads and equal story heights are assumed.

The calculation results were compared to Frame B from the Lehigh Summer Conference notes for weight, strength and stiffness.

The weights of one-story assemblages in levels 8 and 10 decrease 3.5% and 5% against Frame B, respectively, for a deflection index at working load of about 0.003.

Further improvements of the program would be: 1) to use mixed yield stress levels for beams and columns; 2) to restrict member sizes for the convenience of construction; and 3) to extend the program to apply to any arbitrary gravity loads on the beams as well as to uniformly distributed loads.

The frame designed by this computer program must be checked with respect to strength and stiffness using either the sway subassemblage method of analysis or any other second-order elastic-plastic method of analysis.

6. NOMENCLATURE

E	=	Modulus of elasticity;
h	=	story height;
I	=	moment of inertia;
K	=	restraint coefficient;
L	=	span length;
P	=	axial force;
Q	=	lateral load;
Q_{WL}	=	working level of lateral load;
λ	=	shear distribution factor;
Δ	=	sway deflection of one story assemblage;
Δ_L	=	working load sway limitations;
ρ	=	Δ/h (deflection index);
ρ_L	=	Δ_L/h.

7. APPENDIX I

DERIVATION OF THE RELATIONSHIP BETWEEN THE HORIZONTAL SHEAR FORCE AND SWAY DEFLECTION (Eq. 3.1)

Reference will be made to Fig. 9 throughout the derivation of Equation 1.

Using the slope deflection equations, equilibrium of moments at joint i is given by

$$4E\ \frac{I_{i-1,i}}{L_{i-1,i}}\ \xi_{i-1} + \frac{I_{ij}}{L_{ij}}\ \xi_{i+1} + \frac{I_i}{h}\ U_i\ \ \theta_i - 4E\ \frac{I_i}{h}\ \rho\ \ U_i = 0$$

where

$$\xi_{i-1} = \frac{3 - K_{i-1,i}}{4 - K_{i-1,i}}$$

$$\xi_{i+1} = \frac{3 - K_{ij}}{4 - K_{ij}}$$

$$U_i = \frac{1}{C_i}\ (C_i^{\ 2} - S_i^{\ 2})$$

$$C_i = \frac{c_i}{c_i^{\ 2} - s_i^{\ 2}}$$

$$S_i = \frac{s_i}{c_i^{\ 2} - s_i^{\ 2}}$$

$$c_i = \frac{1}{\phi^2}\ (1 - \phi \coth \phi)$$

$$s_i = \frac{1}{\phi^2}\ (\frac{\phi}{\sin \phi} - 1)$$

$$\phi = \frac{h}{2}\sqrt{\frac{P_i}{EI_i}}$$

The equilibrium shear force in the sway subassemblage is given by

$$\frac{h}{2}\lambda Q_n + \frac{1}{2} P\Delta = -2E \frac{I_i}{h} U_i (\theta_i - P)$$

Eliminating θ_i, Eq. 1 is obtained.

8. TABLES AND FIGURES

TABLE 1

Axial Forces in Columns due to the Working Wind Loads

(kips)

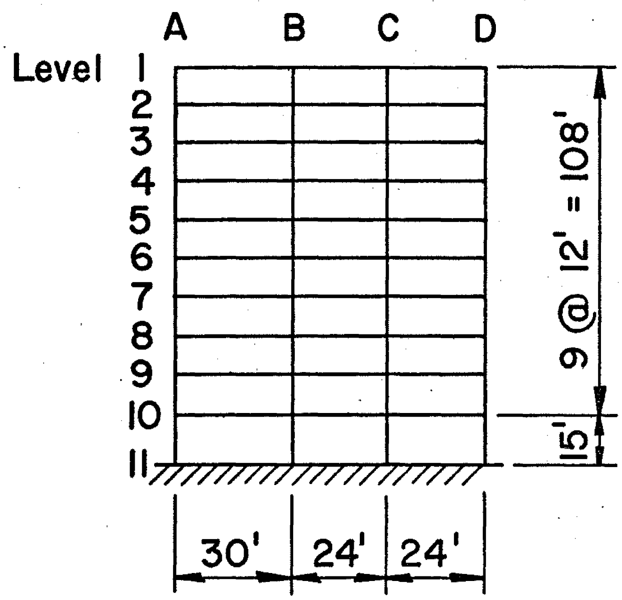
Level	A	B	C	D
1	-0.31	0.18	-0.07	0.20
2	-2.20	1.22	-0.33	1.31
3	-4.82	1.95	-0.46	3.33
4	-8.50	2.97	-0.65	6.17
5	-12.49	3.08	-0.84	10.25
6	-17.38	3.22	-1.08	15.24
7	-23.16	3.35	-1.27	21.08
8	-28.34	1.07	-1.46	28.73
9	-34.18	-1.44	-1.96	37.58
10	-40.76	-4.27	-2.54	47.57

TABLE 2

Axial Forces in Columns due to the Design Ultimate Wind Loads

(kips)

Level	A	B	C	D
1	-8.92	6.61	0	2.31
2	-17.75	9.93	0	7.82
3	-26.64	12.66	0	13.98
4	-35.53	15.39	0	20.14
5	-48.17	13.95	0	34.22
6	-60.86	12.51	0	48.30
7	-73.45	11.07	0	62.38
8	-89.74	-1.62	0	91.36
9	-110.42	-13.90	0	124.31
10	-131.09	-20.18	0	157.26



Bent Spacing = 24'

Working Loads:

Roof W_L = 30 psf

W_D = 60 psf

Floor W_L = 80 psf

W_D = 80 psf

Exterior Wall

W_D = 45 psf

Wind 20 psf

Fig. 1 Frame B: Geometry and Loading

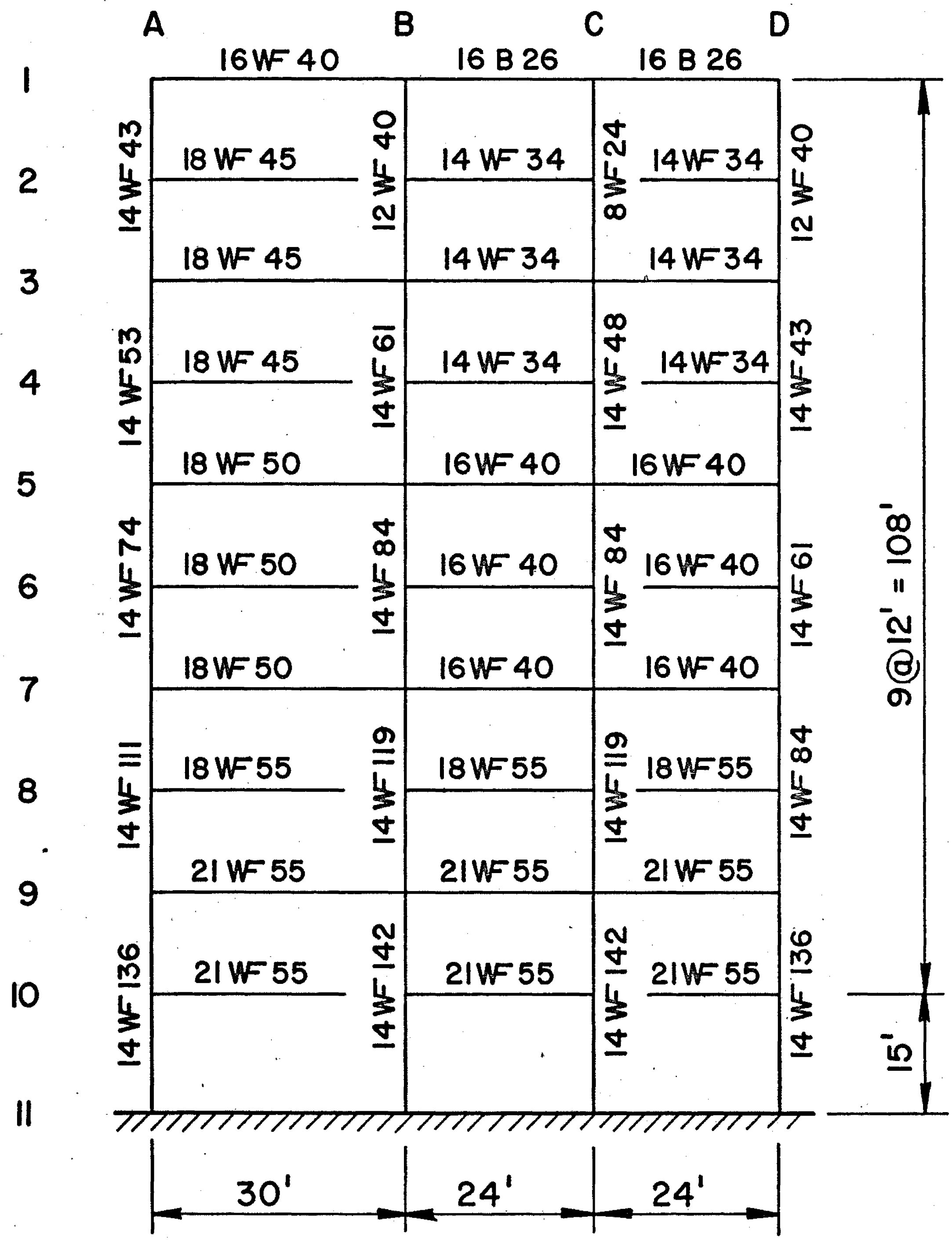


Fig. 2 Frame B: Member Sizes Required by Moment Balancing Method (Ref. 6)

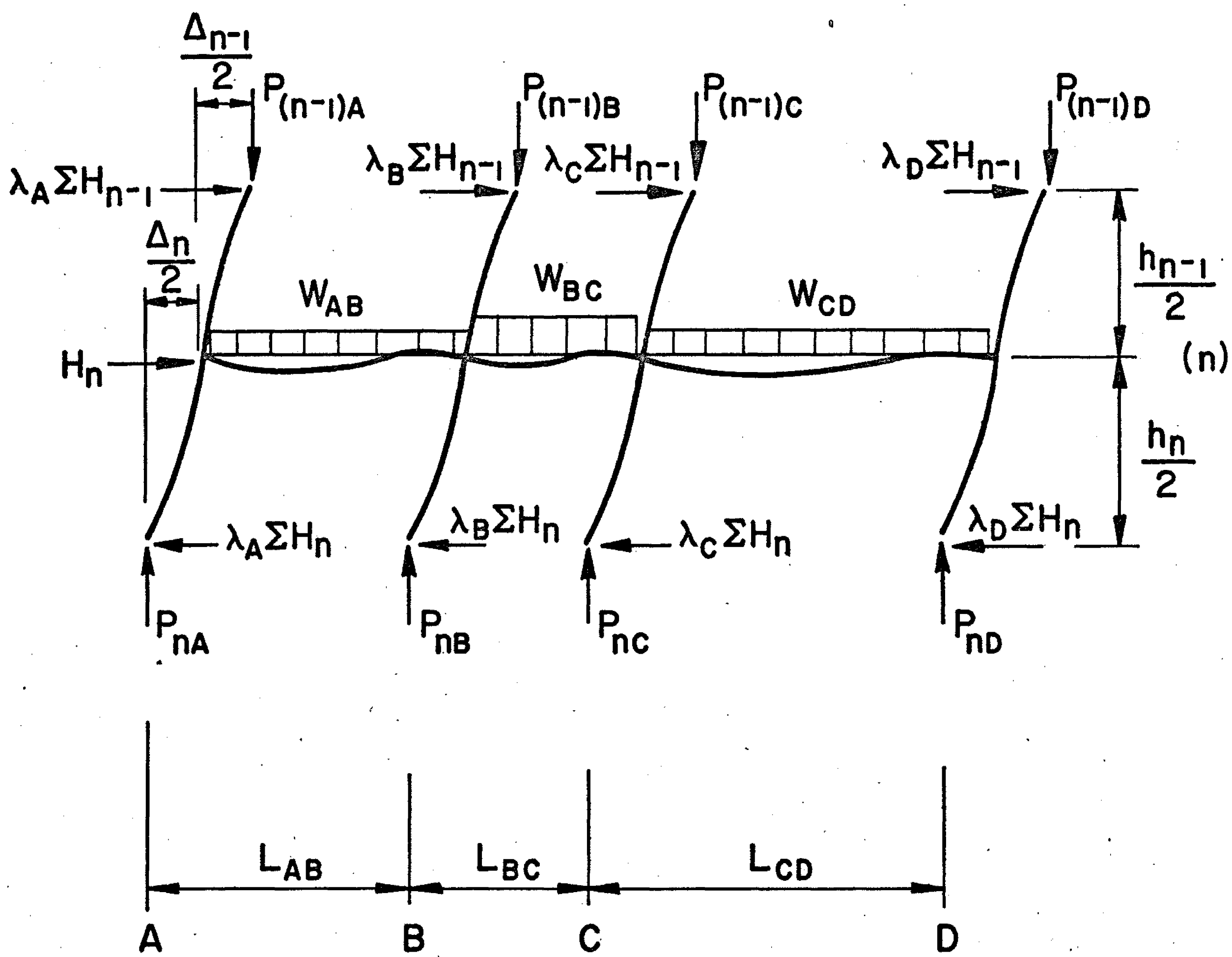


Fig. 3 One-Story Assemblage

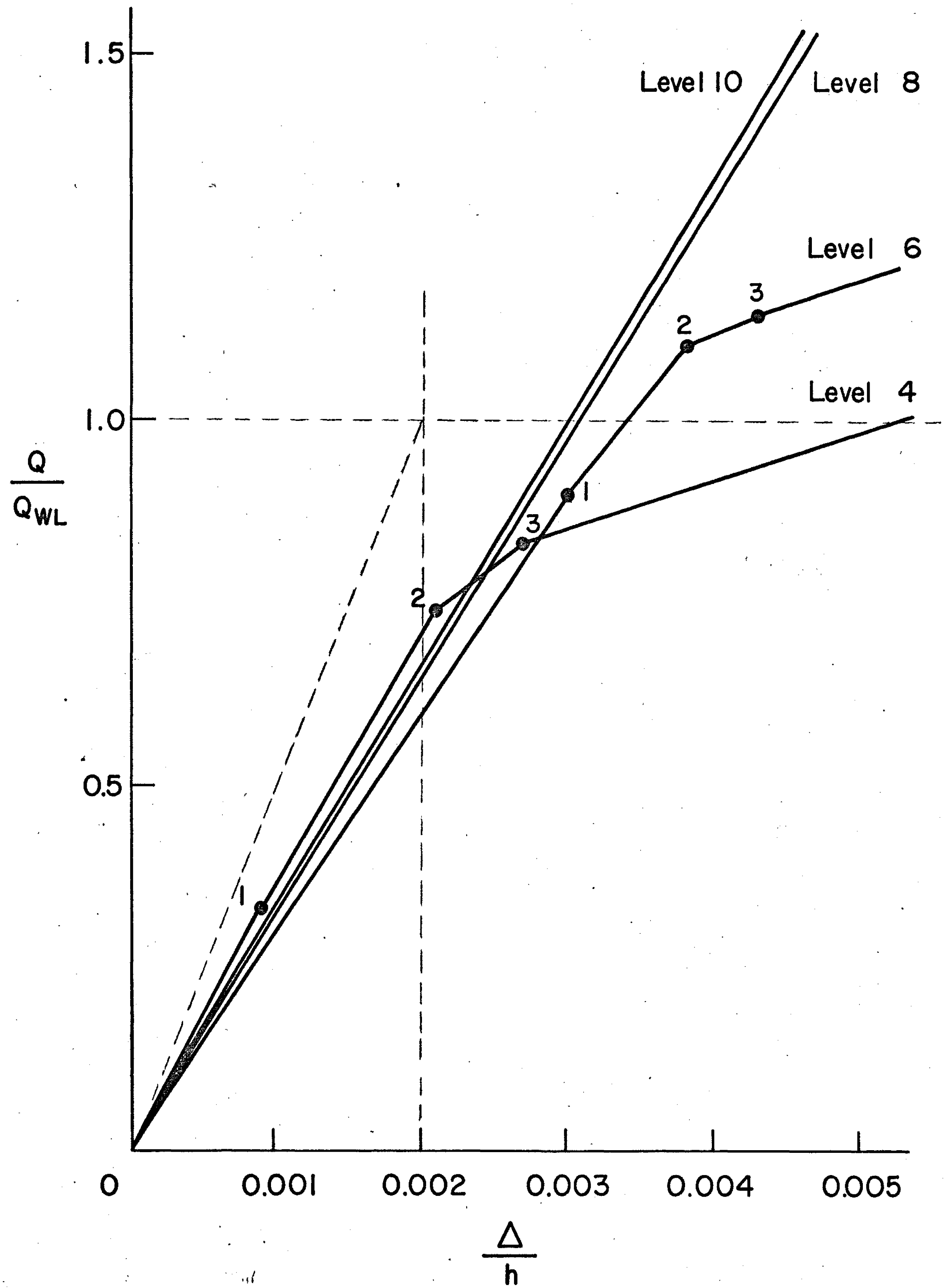


Fig. 4 Lateral-Load Versus Sway-Deflection Under Working Combined Loads

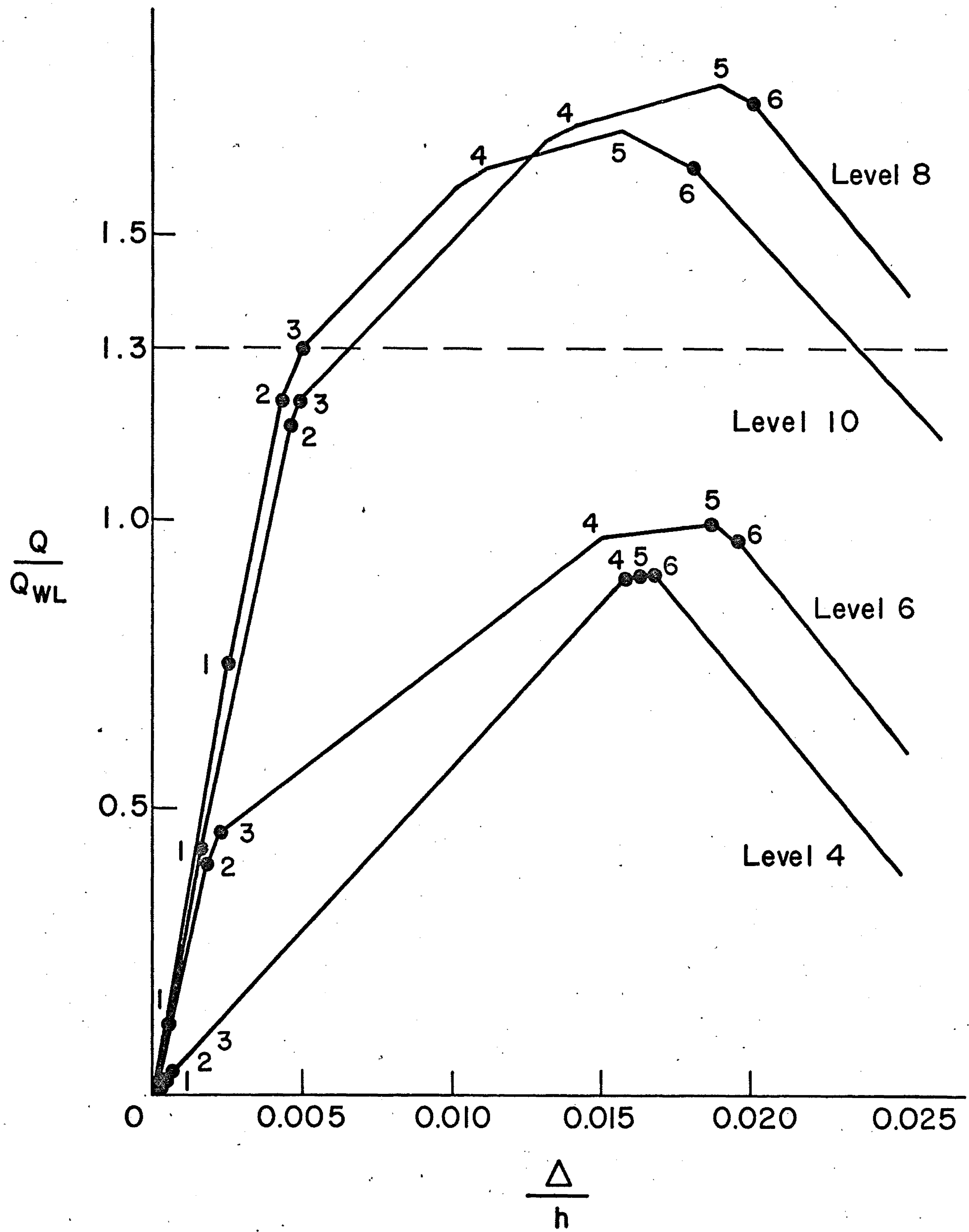


Fig. 5 Lateral-Load Versus Sway-Deflection Under Design Ultimate Loads

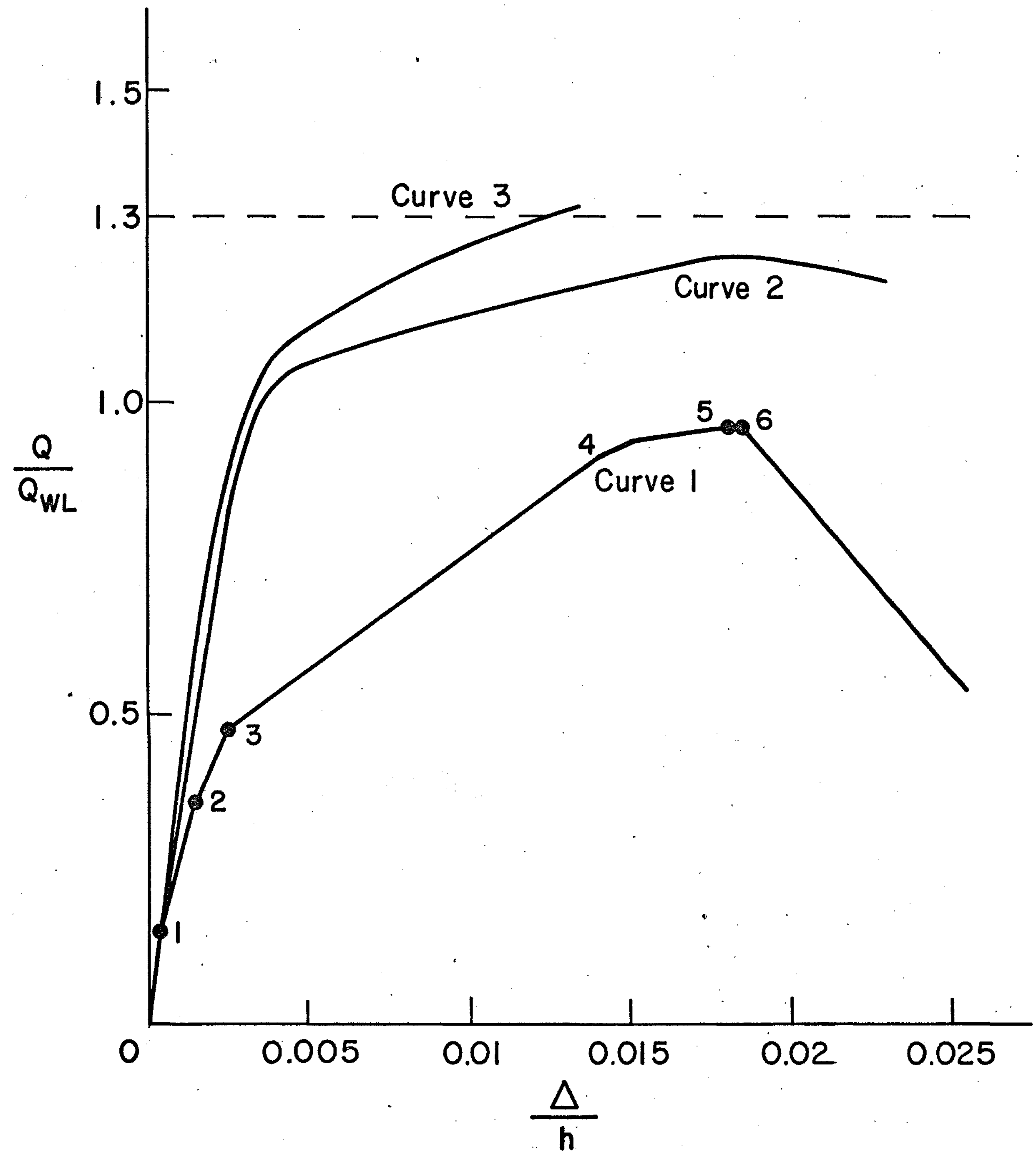


Fig. 6 Lateral-Load Versus Sway-Deflection (Level 6)

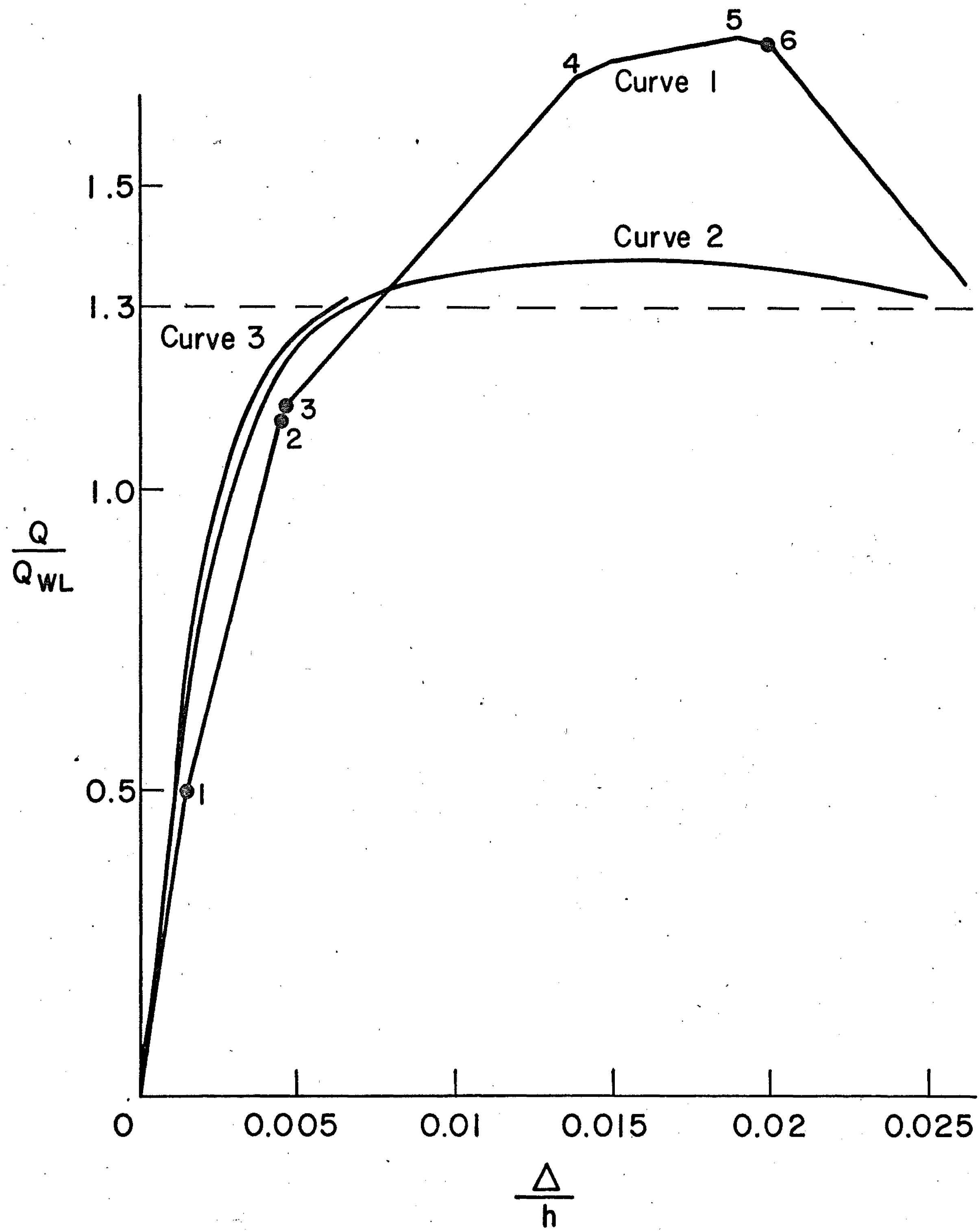


Fig. 7 Lateral-Load Versus Sway-Deflection
(Level 8)

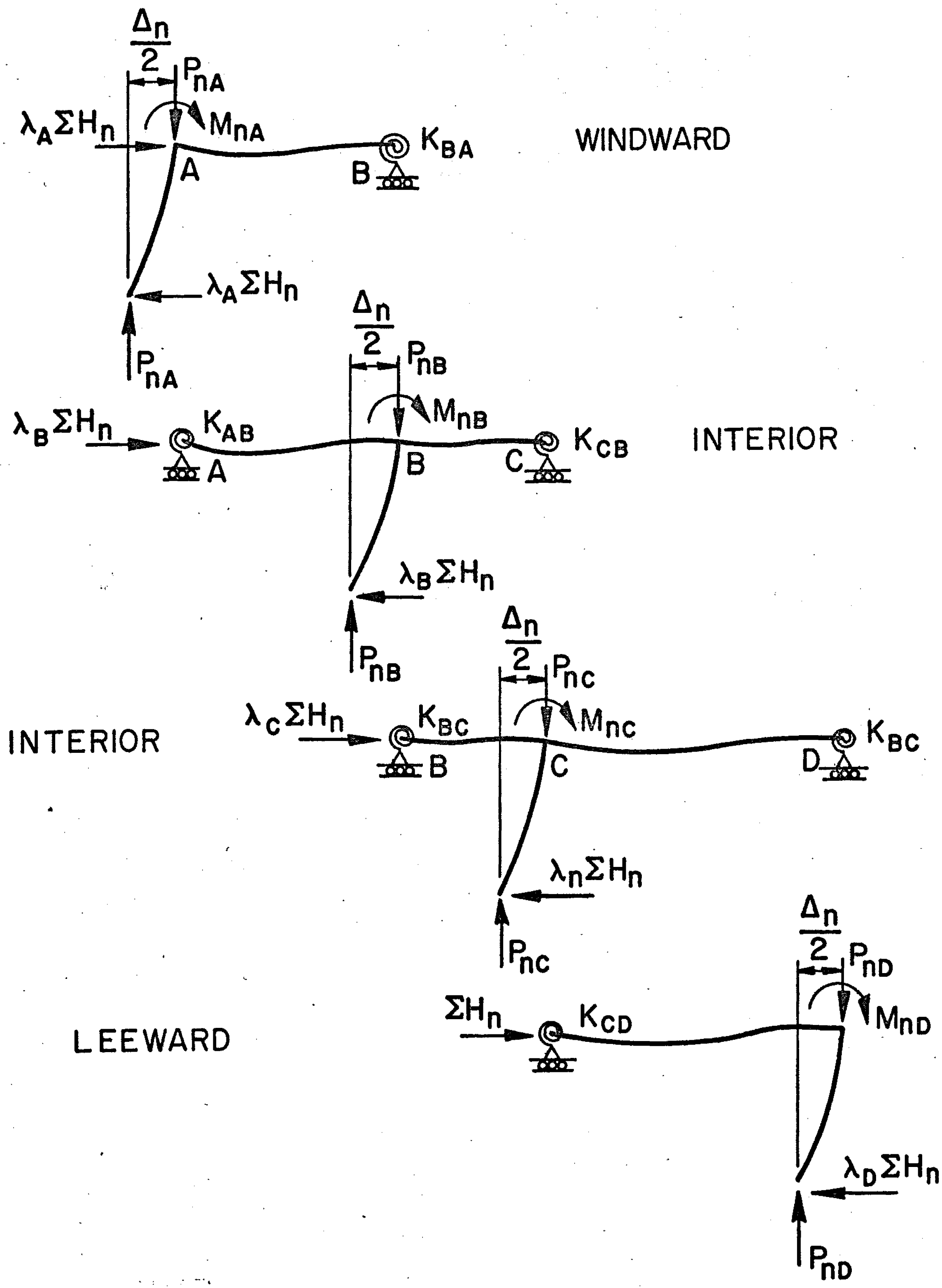
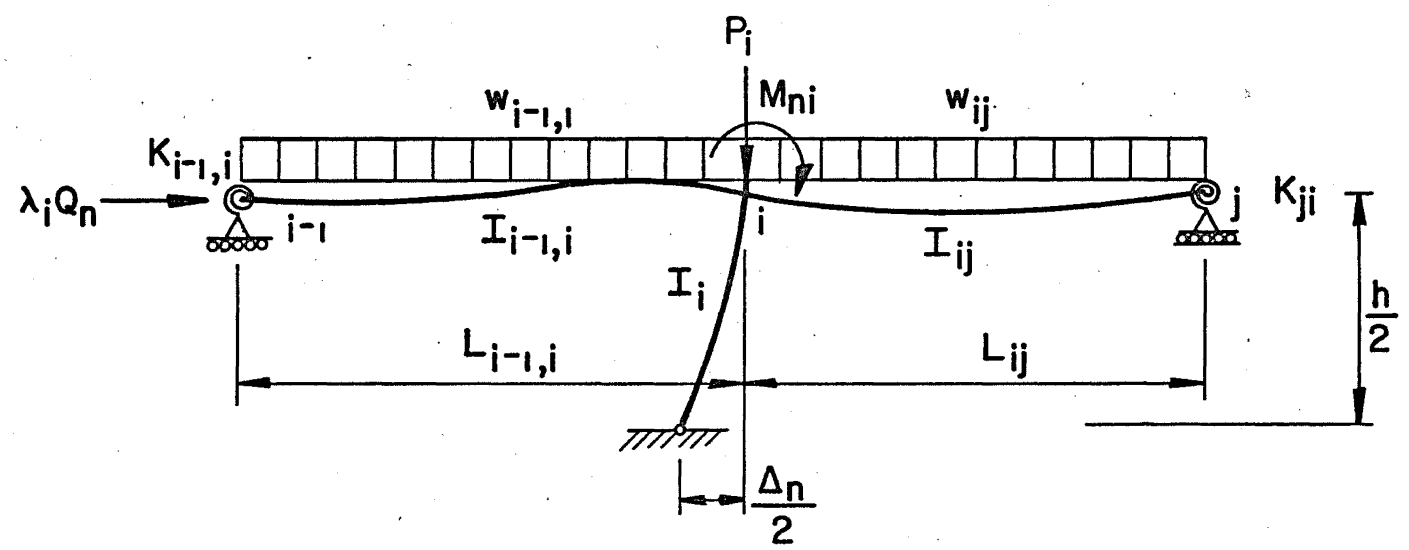


Fig. 8 Sway Subassemblages



$$M_{ni} = \frac{1}{2} h \lambda_i Q_n + \frac{1}{2} \Delta_n P_{ni}$$

Fig. 9 An Interior Sway Subassemblage

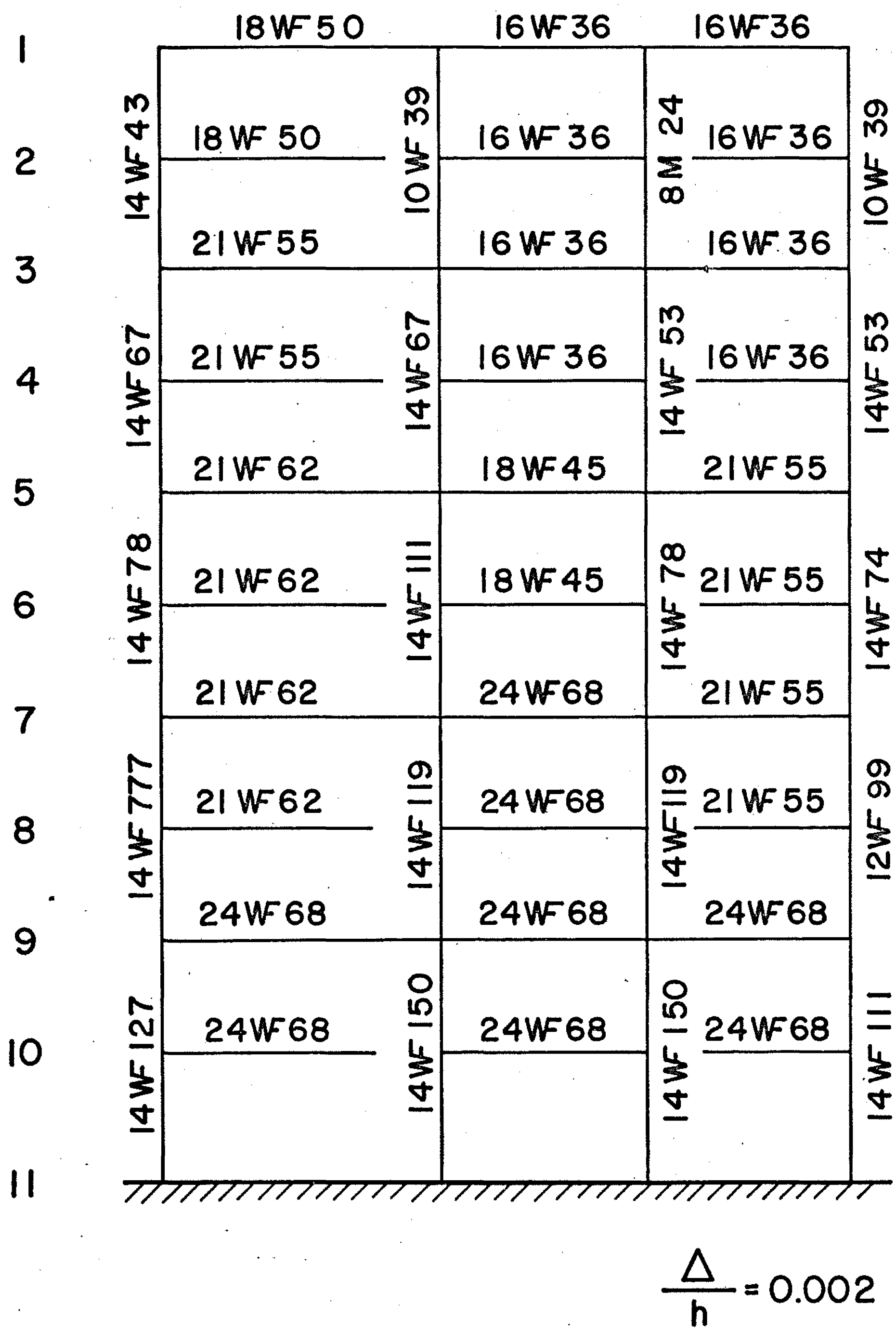


Fig. 10 Minimum Weight Design Frame
(Sway Limitation $\frac{\Delta L}{h} = 0.002$)

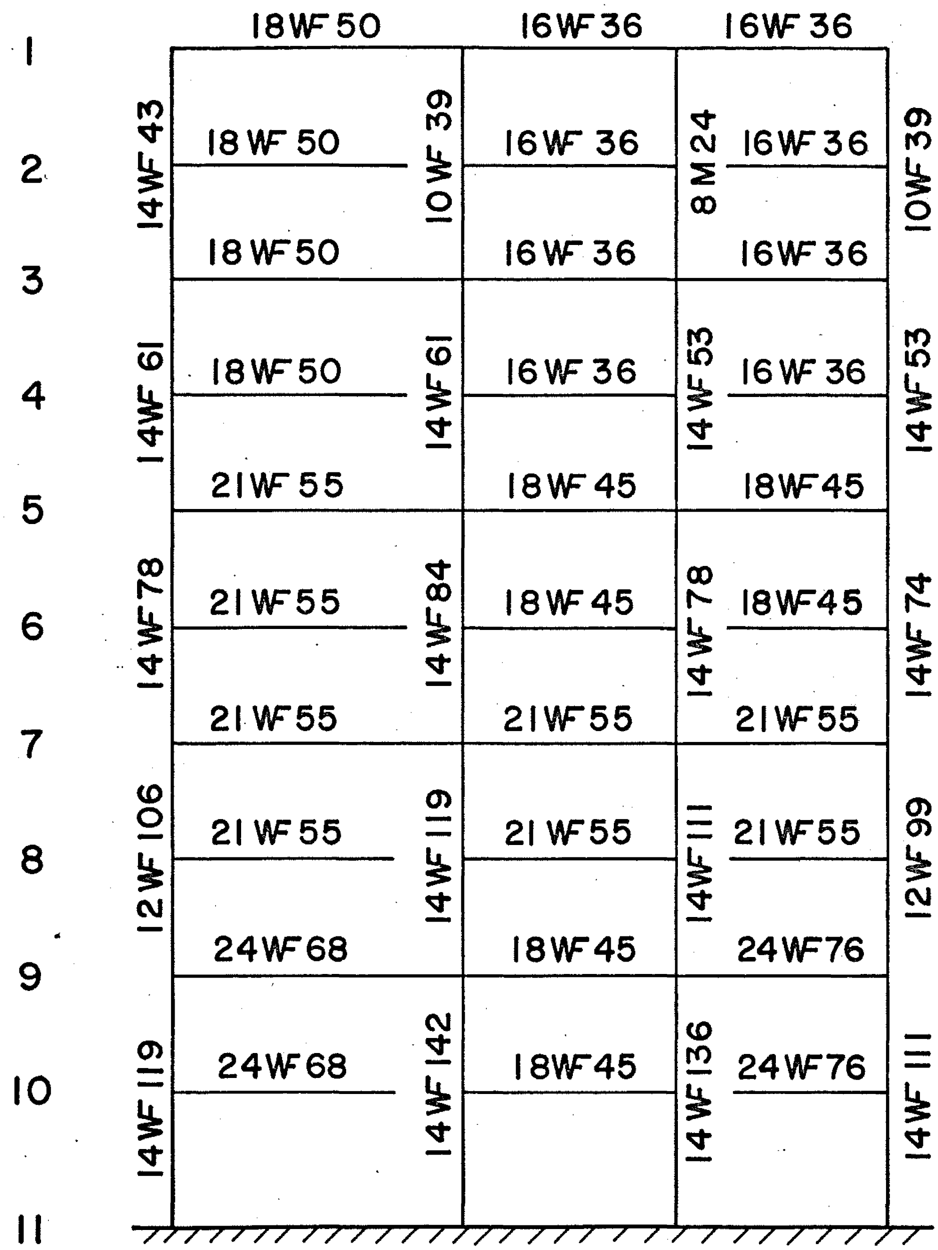


Fig. 11 Minimum Weight Design Frame
(Sway Limitation $\frac{\Delta L}{h}$ = 0.0025)

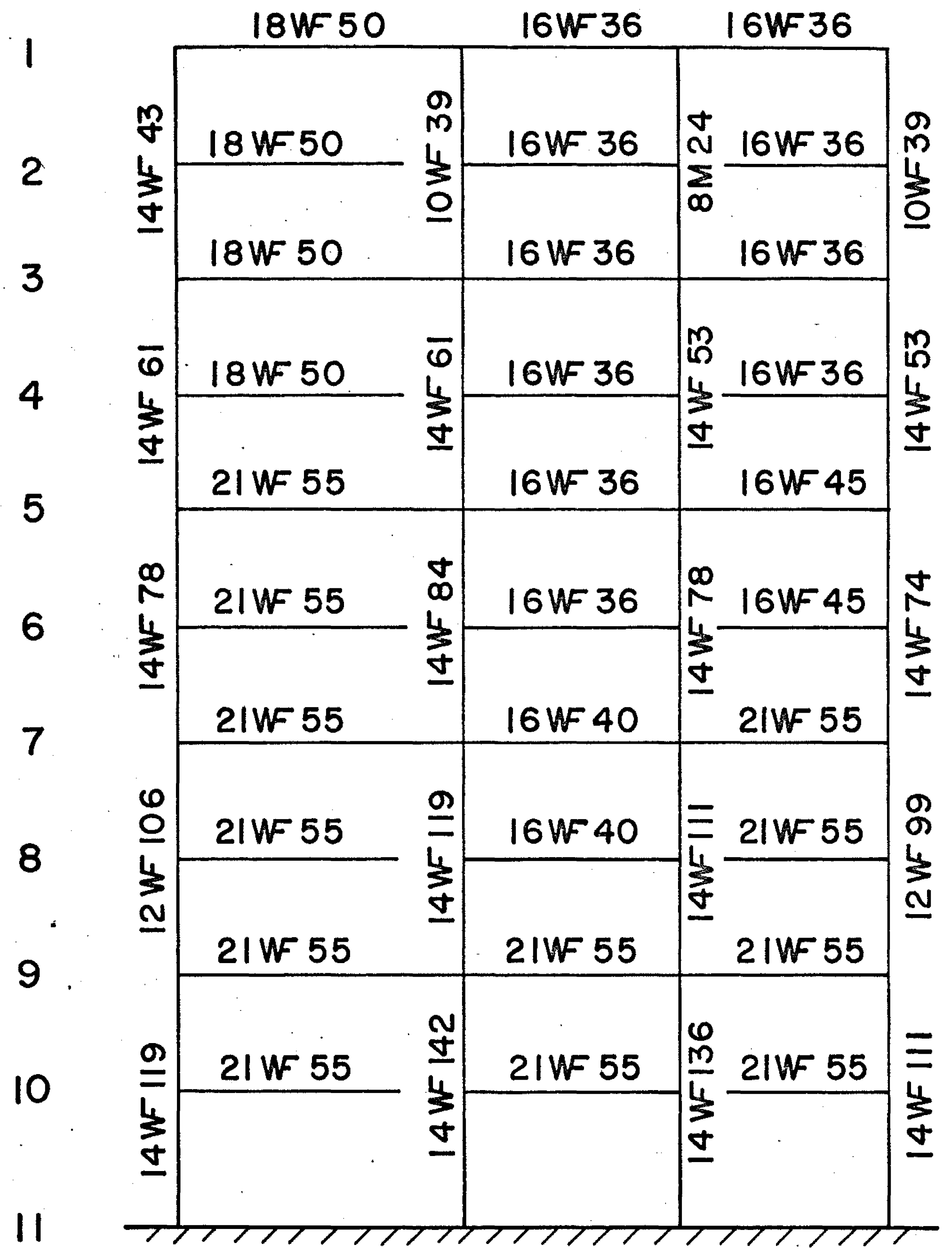


Fig. 12 Minimum Weight Design Frame
(Sway Limitation $\frac{\Delta L}{h} = 0.003$)

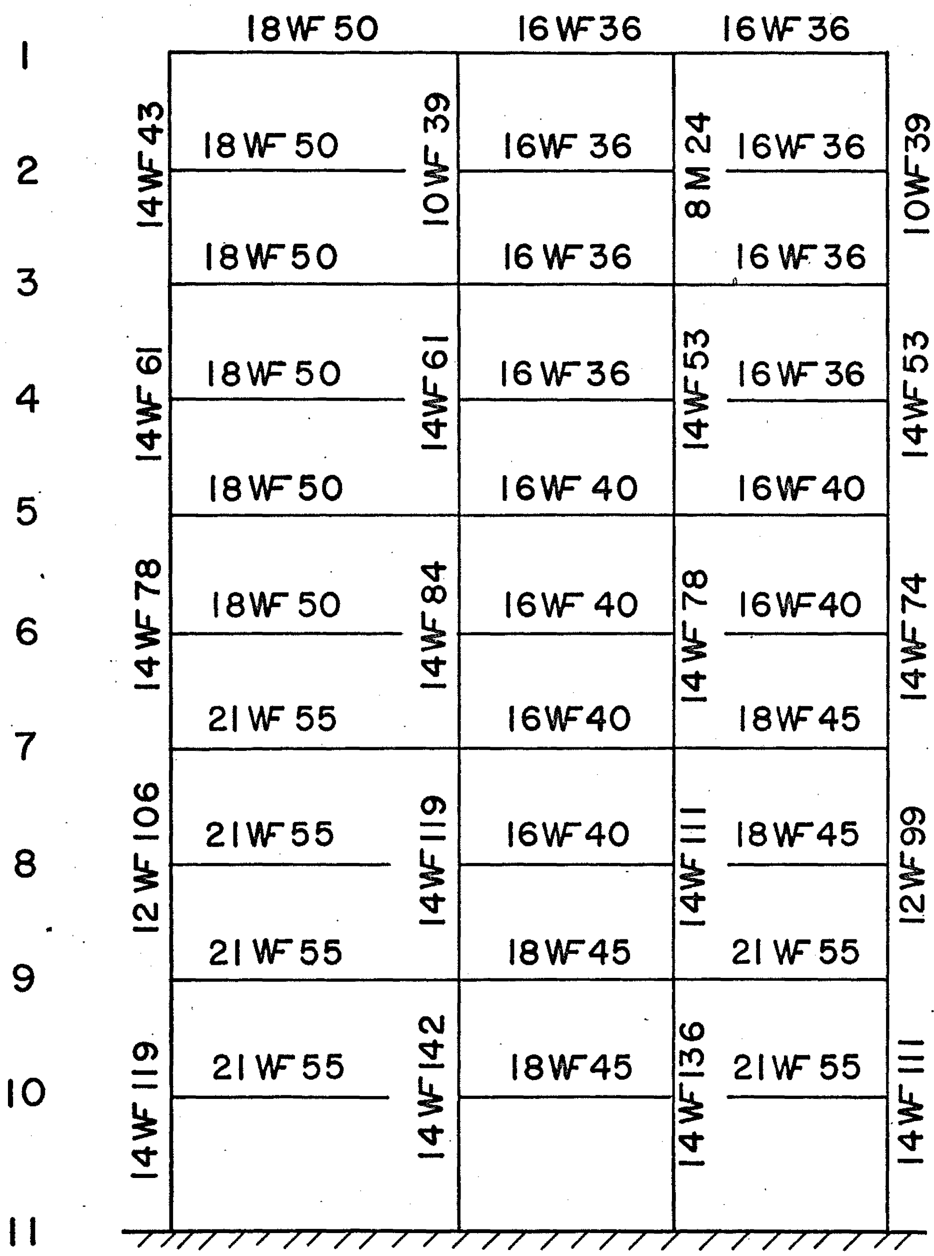


Fig. 13 Minimum Weight Design Frame
(Sway Limitation $\frac{\Delta L}{h} = 0.004$)

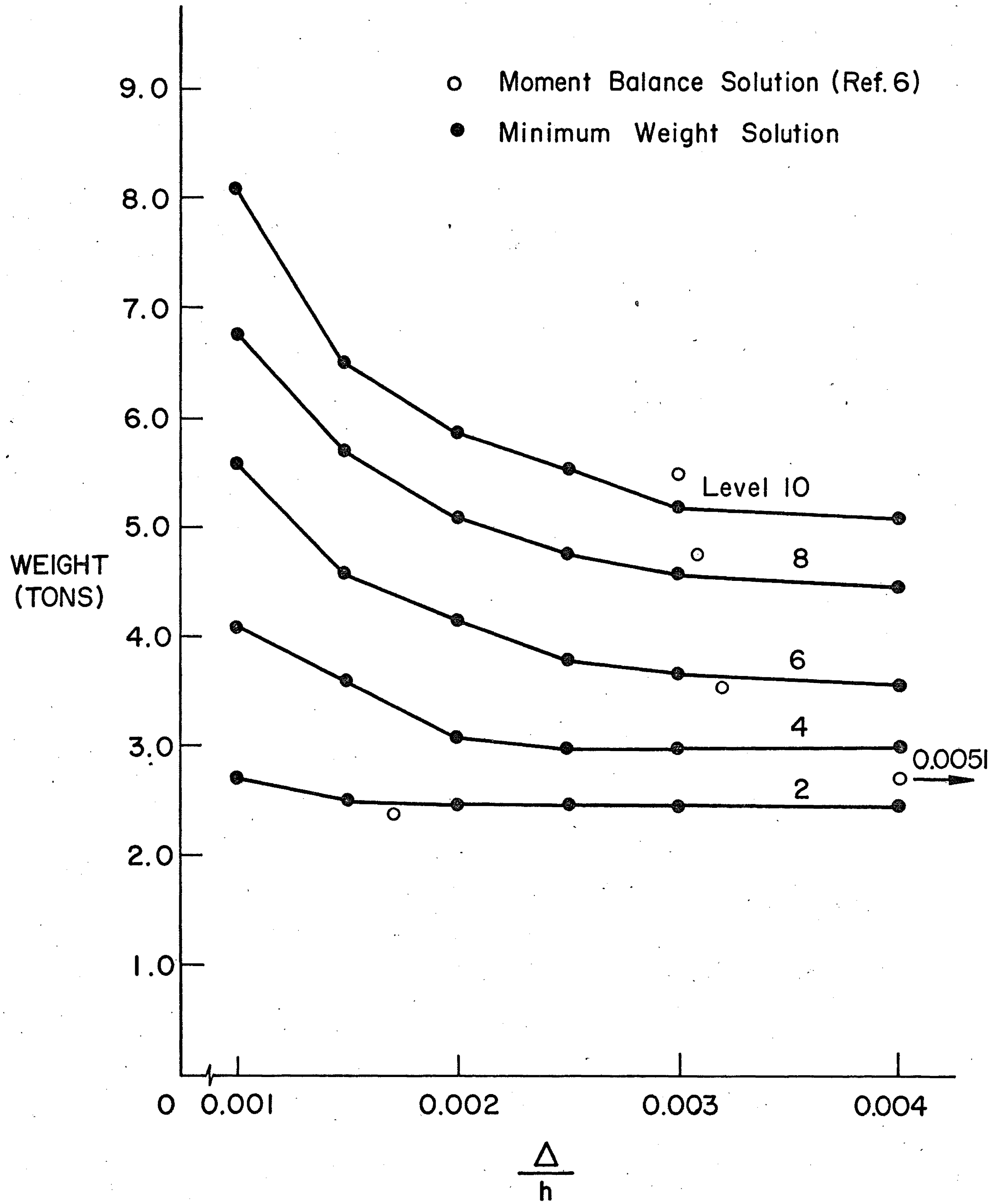


Fig. 14 The Relationship between Weight of One Story Assemblages and Sway Limitations

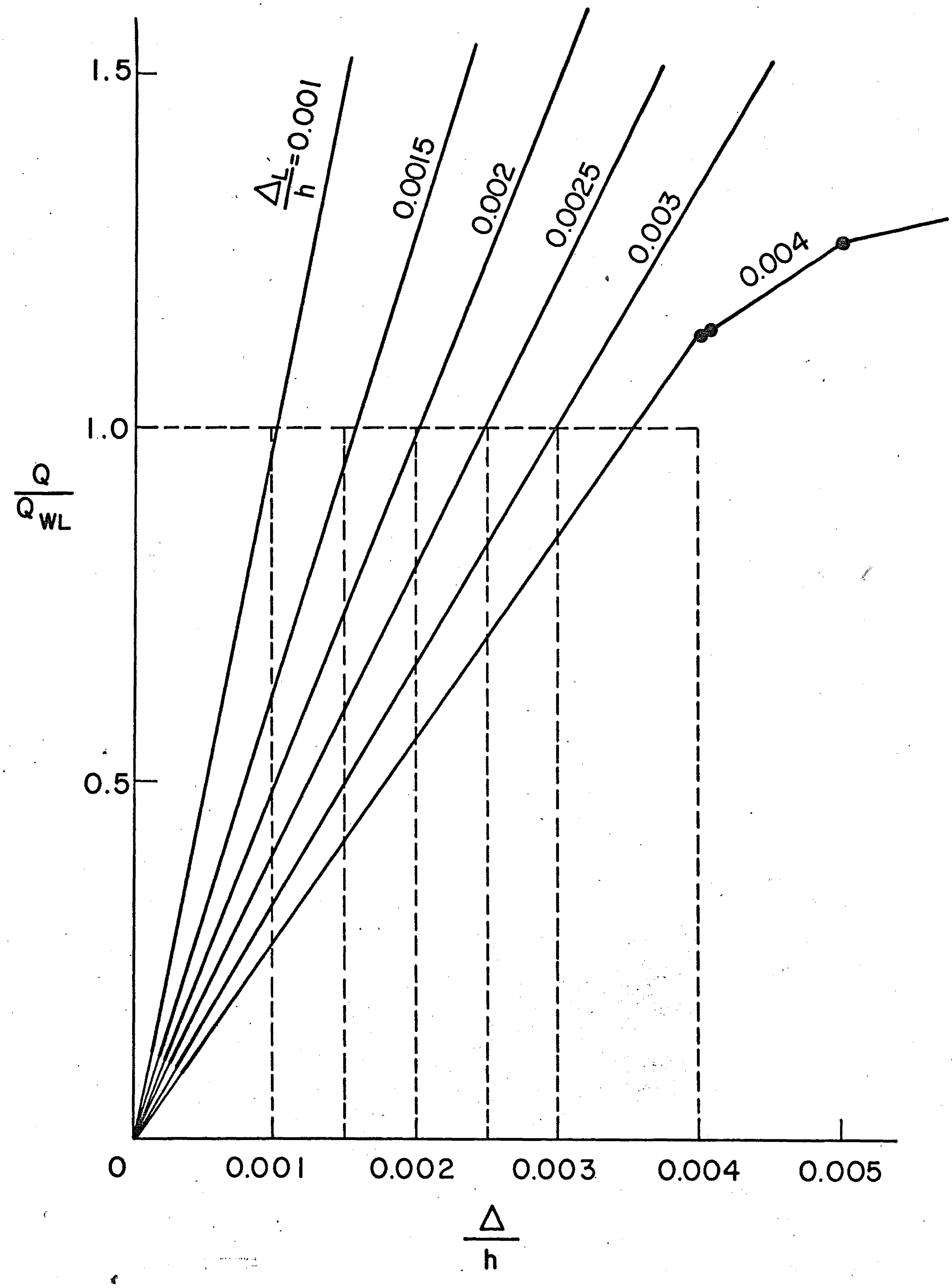


Fig. 15 Lateral Load Versus Sway Deflection
Under the Working Combined Load (Level 8)

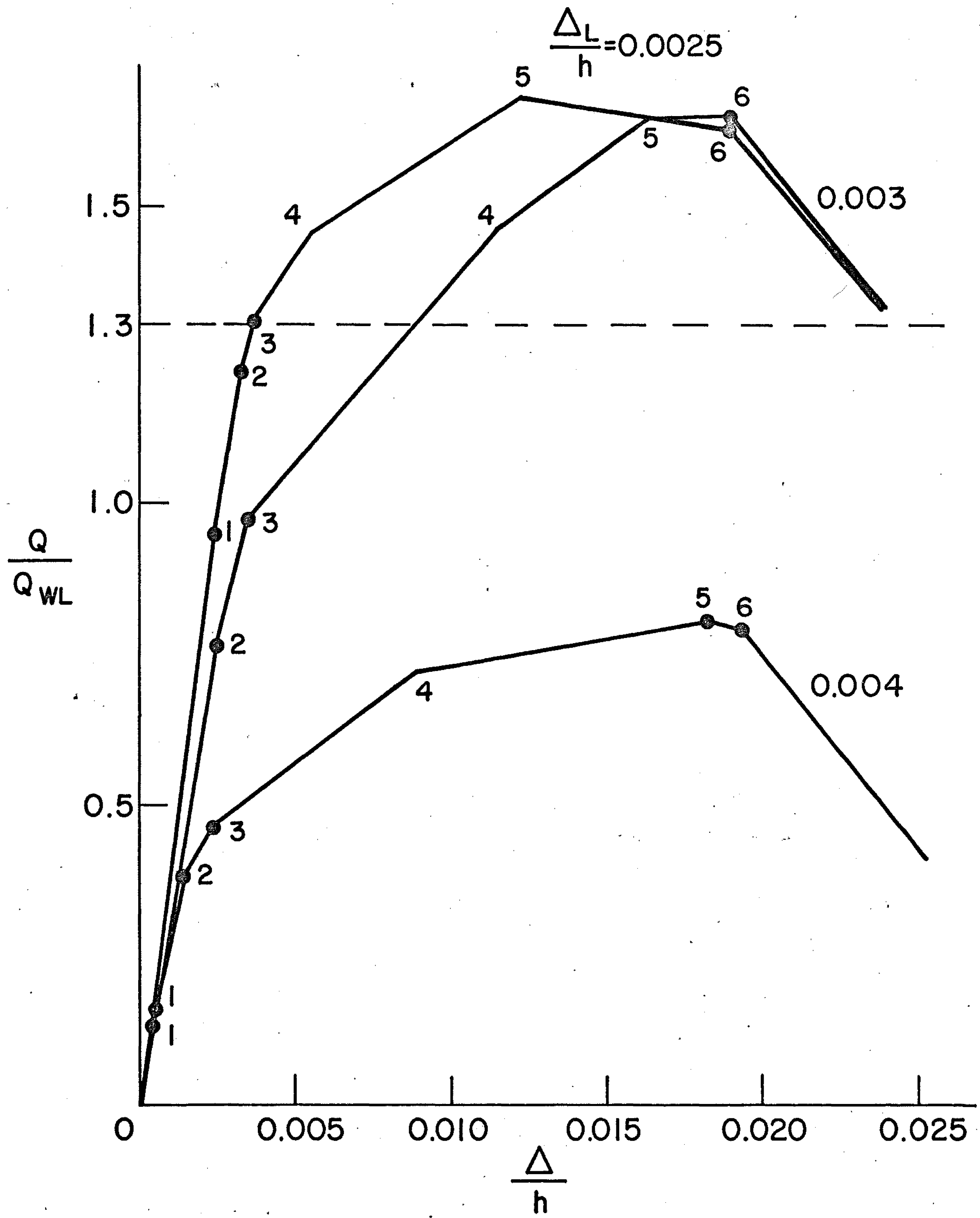


Fig. 16 Lateral-Load Versus Sway-Deflection Under Design Ultimate Combined Load (Level 6)

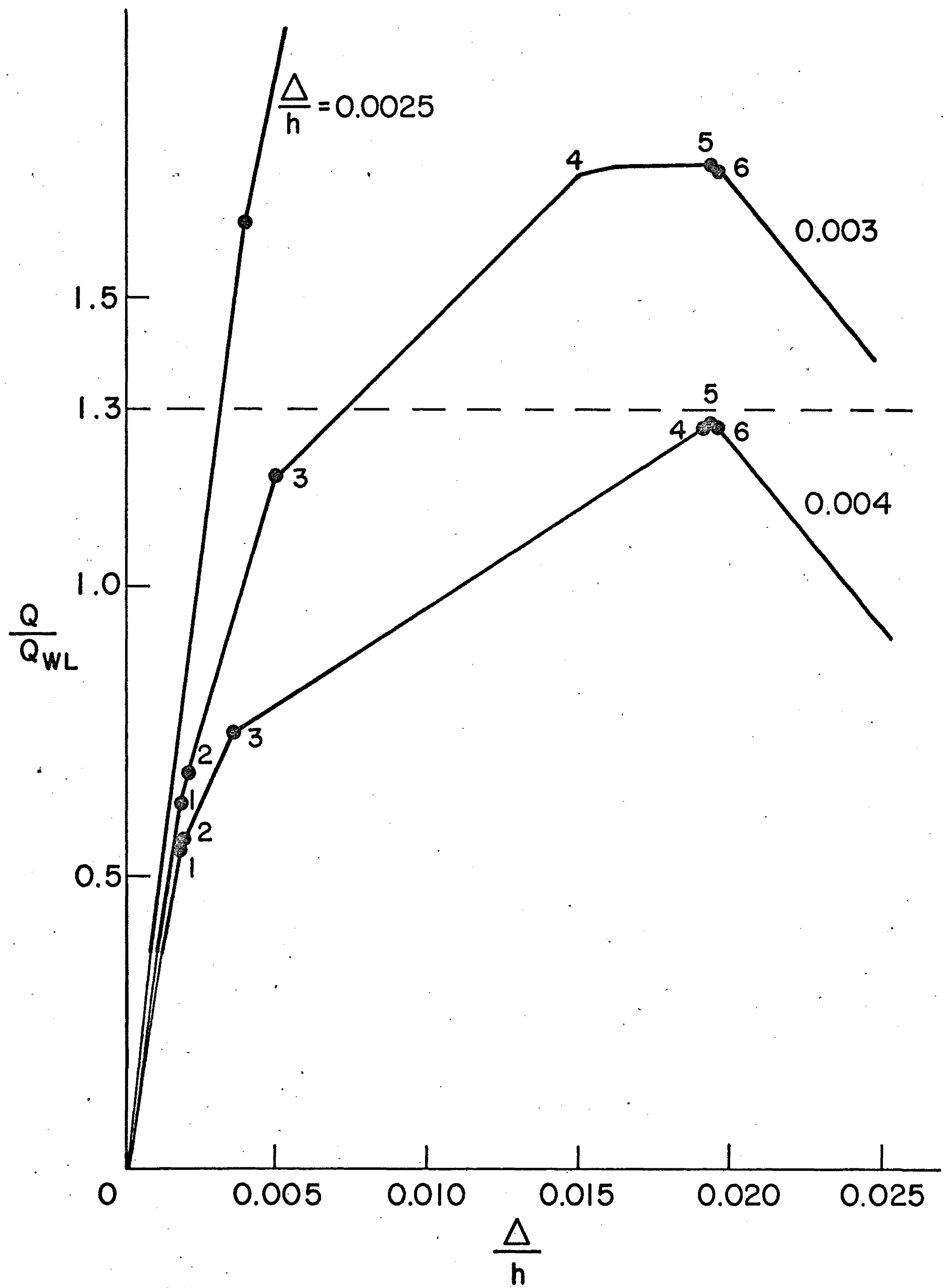


Fig. 17 Lateral-Load Versus Sway-Deflection
Under Design Ultimate Combined Load (Level 8)

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