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# Design charts-for the subassemblage method of designing unbraced multi-story frames, November 1966

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Plastic Design of Multi-Story Frames

**DESIGN CHARTS FOR  
THE SUBASSEMBLAGE METHOD  
OF DESIGNING  
UNBRACED MULTI-STORY FRAMES**

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by  
**J. Hartley Daniels  
Le-Wu Lu**

**Fritz Engineering Laboratory Report No. 273.54**

RESEARCH INSTITUTE OF ENGINEERING



SYNOPSIS

This report presents 78 design charts which are to be used in conjunction with an earlier report describing the Sway Subassemblage method of designing unbraced multi-story frames.<sup>1</sup> These charts relate the lateral load versus sway deflection behavior of restrained columns permitted to sway. They have been prepared to cover the range of axial load ratios  $P/P_y$  from 0.30 to 0.90, and the range of slenderness ratios  $h/r$  from 20 to 30.

The composition of the design charts is briefly discussed. The scope of application of the charts is also given along with two worked examples to illustrate their use in solving restrained column problems similar to those encountered in the design of multi-story frames.



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## 1. INTRODUCTION

A new semi-graphical method of designing columns and girders in rigidly jointed, unbraced multi-story frames was presented in Reference 1. The design method discussed there, starts with columns and girders which have been selected initially from a previous preliminary design of the story under consideration. Then, through application of an analysis procedure which makes use of the charts presented in this report, the lateral load versus sway deflection behavior of that story can be determined.

The design charts give the lateral load versus sway deflection relationships for restrained columns which are permitted to sway, and which are loaded by a constant axial load  $P$  and a gradually increasing lateral load  $Q$ . In addition to the above loads, the columns are also subjected to a bending moment  $M_n$  defined by

$$M_n = - \left[ Q \frac{h}{2} + P \frac{\Delta}{2} \right] \quad (1)$$

in which  $h/2$  represents the column height and  $\Delta/2$  the sway deflection. The reasons for choosing such a moment is discussed in Reference 1.

Each chart is prepared for a given axial load ratio  $P/P_y$  and a given slenderness ratio  $h/r$  for the restrained column. The lateral load  $Q$  is nondimensionalized as  $Qh/2M_{pc}$  with  $M_{pc}$  being the plastic



moment of the column corresponding to the given  $P/P_y$  ratio.  $M_{pc}$  values for commonly used rolled wide-flange column sections in both A36 and A441 steels are listed in Part II of Reference 3. Each chart contains two sets of curves which define the relationship between  $Qh/2M_{pc}$  and  $\Delta/h$  for certain values of the restraining function. The derivation of these curves and the procedure for preparing the charts, is presented in Appendix I of Reference I. Use is also made of the moment-rotation curves given in Part III of Reference 3 for the preparation of the charts. The application of these charts to the design of columns in unbraced frames, is explained in Reference I and also in lectures 18 and 19 of Reference 2.

## 2. SCOPE OF APPLICATION OF THE CHARTS AND EXAMPLES

The charts presented in this report give a precise in-plane load-deflection curve for any initially straight steel wide-flange column which is pinned at one end and elastically restrained at the other end, while free to sway. In addition, the column must be adequately braced against out of plane movement, and subjected to the loading condition outlined in Section I of this report. The load-deflection curves take into account residual stresses in the column, plastification of the column, reduction of plastic moment due to axial force in the column and the effect of column axial force,  $P$ , acting through the sidesway displacement,  $\Delta$ , commonly referred to as the  $P - \Delta$  moment. Although the columns in an unbraced multi-story frame do not comply exactly with the above requirements, they can be reduced to this case by making certain simplifying assumptions which are stated in Reference I. Thus the charts given in this report can be used to give reasonably accurate designs of such frames.

The following two examples will illustrate the use of these charts in solving restrained column problems similar to those encountered in the design of multi-story frames.

Example I: A 14W264 column, 6 ft. and 9 in. long,\* is framed at its top to a 30W116 beam having a clear span of 22 ft. and 9 in. The lower end of the column and the far end of the beam are both pinned. The column and beam are braced so that only bending in the planes of their

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\* Clear column height to underside of beam.

webs is permitted. The column is subjected to a constant axial force  $P = 1960$  kips and a varying lateral force  $Q$ . In addition, a moment  $M_n$  is applied at the top of the column which is related to the forces  $P$  and  $Q$  according to Eq. 1. Both the column and the beam are made of A36 steel.\* Determine the maximum lateral force that can be resisted by the column, and the corresponding sway and joint rotation.

For the 14W264 section,  $P_y = 2795$  kips and  $r_x = 6.74$  in. Since  $h/2 = 81$  in.,  $h/r_x = 24$ .  $\frac{P}{P_y} = \frac{1960}{2795} = 0.7$ ,  $M_{pc} = 551$  kip-ft. \*\*

For the 30W116 section,  $Z = 377.6$  in.<sup>3</sup>  $M_p = 1132$  kip-ft, and  $I_x = 4919.1$  in.<sup>4</sup>

Note that the plastic moment of the beam is larger than two times that of the column. A plastic hinge will therefore form at the top of the column. The restraining function  $M_r$ , (Ref. 1) is given by

$$\begin{aligned} M_r &= \frac{3EI_x}{L M_{pc}} \theta M_{pc} = \frac{(3) (29500) (4919.1)}{(12) (23.4) (12) (551)} \theta M_{pc} \\ &= 234 \theta M_{pc} \end{aligned} \quad (2)$$

where  $L$  is the clear girder span plus half the column depth. (Ref. 1)

By entering Chart 51 ( $P = 0.70 P_y$ ;  $h = 24 r$ ) and locating the curve for  $M_r = 234 \theta M_{pc}$ , the peak value of  $Qh/2M_{pc}$  is found to be 0.680. Thus the maximum lateral force  $Q_{max}$  which can be resisted by the column is

$$Q_{max} = \frac{(0.680) (551)}{6.75} = 55.6 \text{ kips} \quad (3)$$

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\* For A441 Steel see p. 50 REF. 1

\*\* Part II. REF. 3

The sway deflection index  $\Delta/h$  corresponding to  $Q_{\max}$  is also found from Chart 51 to be equal to 0.0115. Thus the sway deflection at  $Q_{\max}$  is

$$\Delta = (0.0115) (81) = 0.932 \text{ inches.} \quad (4)$$

Referring again to Chart 51, the value of  $M'_r$  corresponding to  $Q_{\max}$  is  $1.94 M_{pc}$ . The joint rotation corresponding to  $Q_{\max}$  is determined by equating  $M_r$  and  $M'_r$

$$\theta = \frac{1.94 M_{pc}}{234 M_{pc}} = 0.0083 \text{ radians.}^* \quad (5)$$

Example 2: Determine the maximum lateral force that can be resisted by the column of Example I, and the corresponding sway and joint rotation, if the beam is reduced in size to a 24W76.

For the 14W264 section,  $P_y = 2795$  kips,  $r_x = 6.74$  in.  
and  $h/4 = 24$ .

$$\frac{P}{P_y} = 0.7, \quad M_{pc} = 551 \text{ kip-ft.}$$

For the 24W76 section  $Z = 200.1 \text{ in.}^3$

$$M_p = 600 \text{ kip-ft. and } I_x = 2096 \text{ in.}^4$$

Note that two times the plastic moment of the column is larger than that of the beam. A plastic hinge will therefore form in the beam at the column face.

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\*Note that the numerical value of  $\theta$  will always be equal to or less than the numerical value of  $\Delta/h$  for positive values of  $M_r$ .

The restraining function  $M_r$  is calculated as shown in Example I.

$$M_r = 100 \theta M_{pc} \quad (6)$$

Assuming an idealized elastic-plastic moment-rotation relationship for the beam, the joint rotation  $\theta$  corresponding to  $M_p$  at the column face is given by

$$\begin{aligned} \theta &= \frac{M_p \ell}{3EI} = \frac{(12) (600) (12) (22.75)}{(3) (29500) (2096)} \\ &= 0.0106 \text{ rad.} \end{aligned} \quad (7)$$

where  $\ell$  is the clear girder span. The maximum value of the restraining function  $M'_r$  is obtained by substituting Eq. 7 into Eq. 6.

$$M'_r = 1.06 M_{pc} \quad (8)$$

Entering Chart 51 ( $P = 0.70 P_y$  ;  $h = 24 r$ ) and locating the curve for  $M_r = 100 \theta M_{pc}$  we find that its intersection with the curve  $M'_r = 1.06 M_{pc}$  gives a value of  $Qh/2M_{pc}$  equal to 0.240. Thus the maximum lateral force  $Q_{max}$  which can be resisted by the column is

$$Q_{max} = \frac{(0.240) (551)}{6.75} = 19.6 \text{ kips} \quad (9)$$

The sway deflection index corresponding to  $Q_{max}$  is 0.0115. Thus the sway deflection at  $Q_{max}$  is

$$\Delta = (0.0115) (81) = 0.932 \text{ inches} \quad (10)$$

### 3. ACKNOWLEDGMENTS

This study is part of a general investigation "Plastic Design of Multi-Story Frames" currently being carried out at Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University. Professor L. S. Beedle is acting Head of the Civil Engineering Department and Director of the Laboratory. This investigation is sponsored jointly by the Welding Research Council, and the Department of the Navy, with funds furnished by the American Institute of Steel Construction, American Iron and Steel Institute, Naval Ships Systems Command and Naval Facilities of Engineering Command. Technical guidance is provided by the Lehigh Project Subcommittee of the Structural Steel Committee of the Welding Research Council. Dr. T. R. Higgins is Chairman of the Lehigh Project Subcommittee.

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Miss S. D. Gubich and Mr. J. M. Gera Jr. prepared the final charts and Mrs. D. Eversley typed the manuscript.

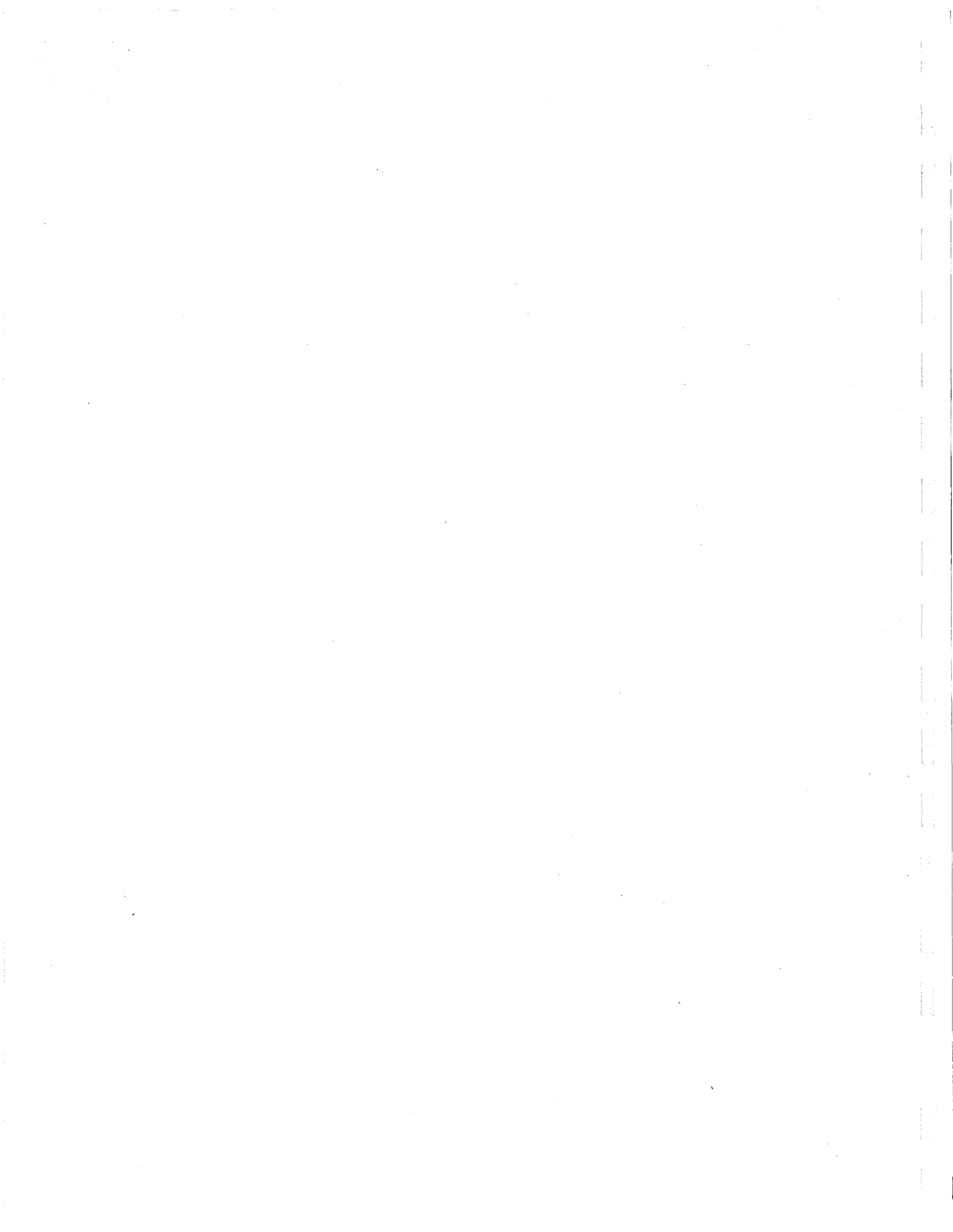
4. NOTATION

A	=	area of member
E	=	elastic modulus
I	=	strong axis moment of inertia
L	=	center to center span length of girder
$M_n$	=	moment at joint or end of member
$M_p$	=	plastic moment
$M_{pc}$	=	reduced plastic moment
$M_r$	=	restraining moment or restraining function
$M_r'$	=	maximum restraining moment
P	=	applied axial force
$P_y$	=	axial yield force = $A \times \sigma_y$
P- $\Delta$	=	secondary overturning moment
Q	=	shear resistance of column
Z	=	plastic modulus
h	=	story height
h/r	=	slenderness ratio
$\ell$	=	clear span length of girder
r	=	radius of gyration
$\sigma_y$	=	static yield stress of material
$\Delta$	=	joint translation or sway
$\Delta/h$	=	deflection index or chord rotation
$\theta$	=	joint rotation

5. REFERENCES

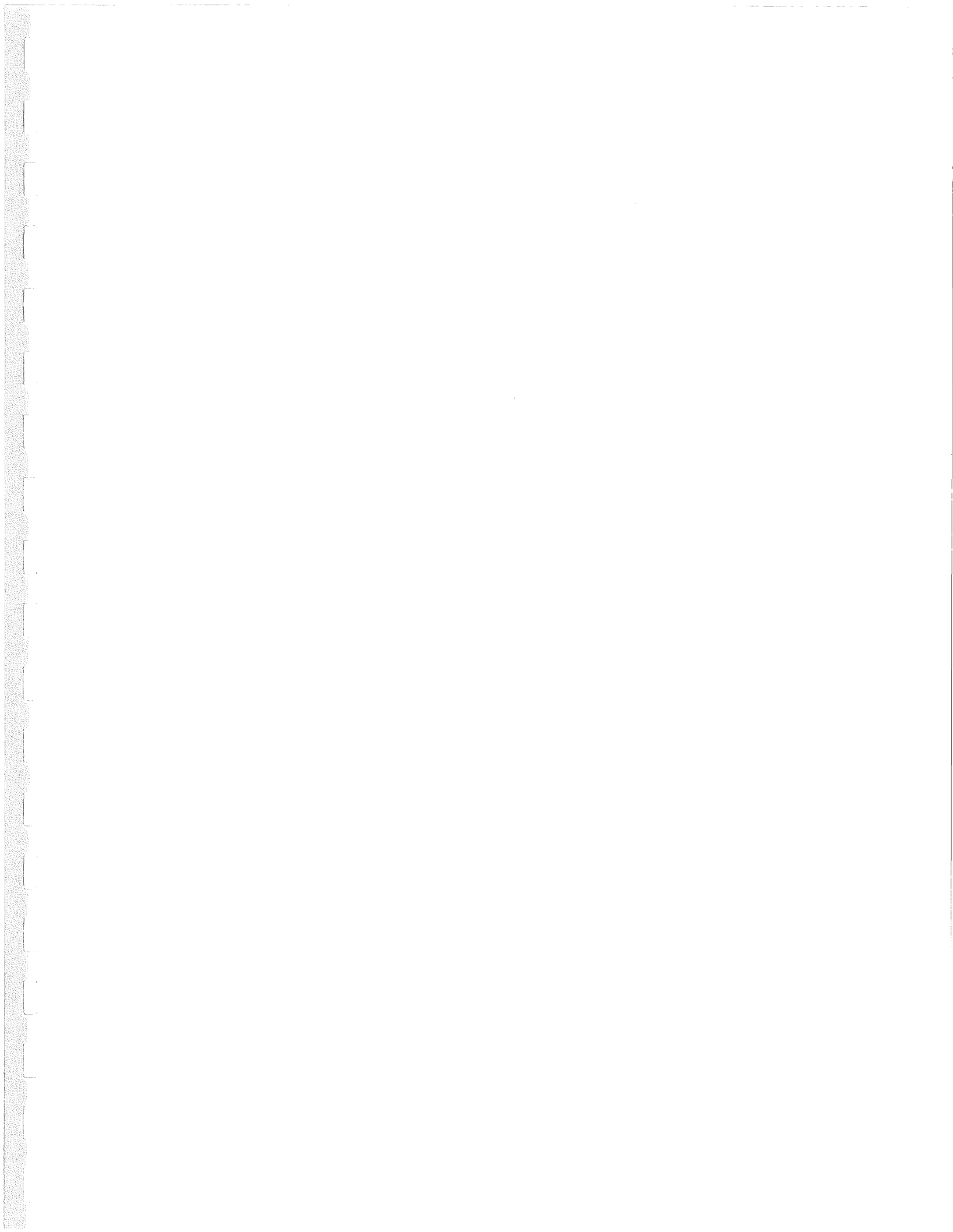
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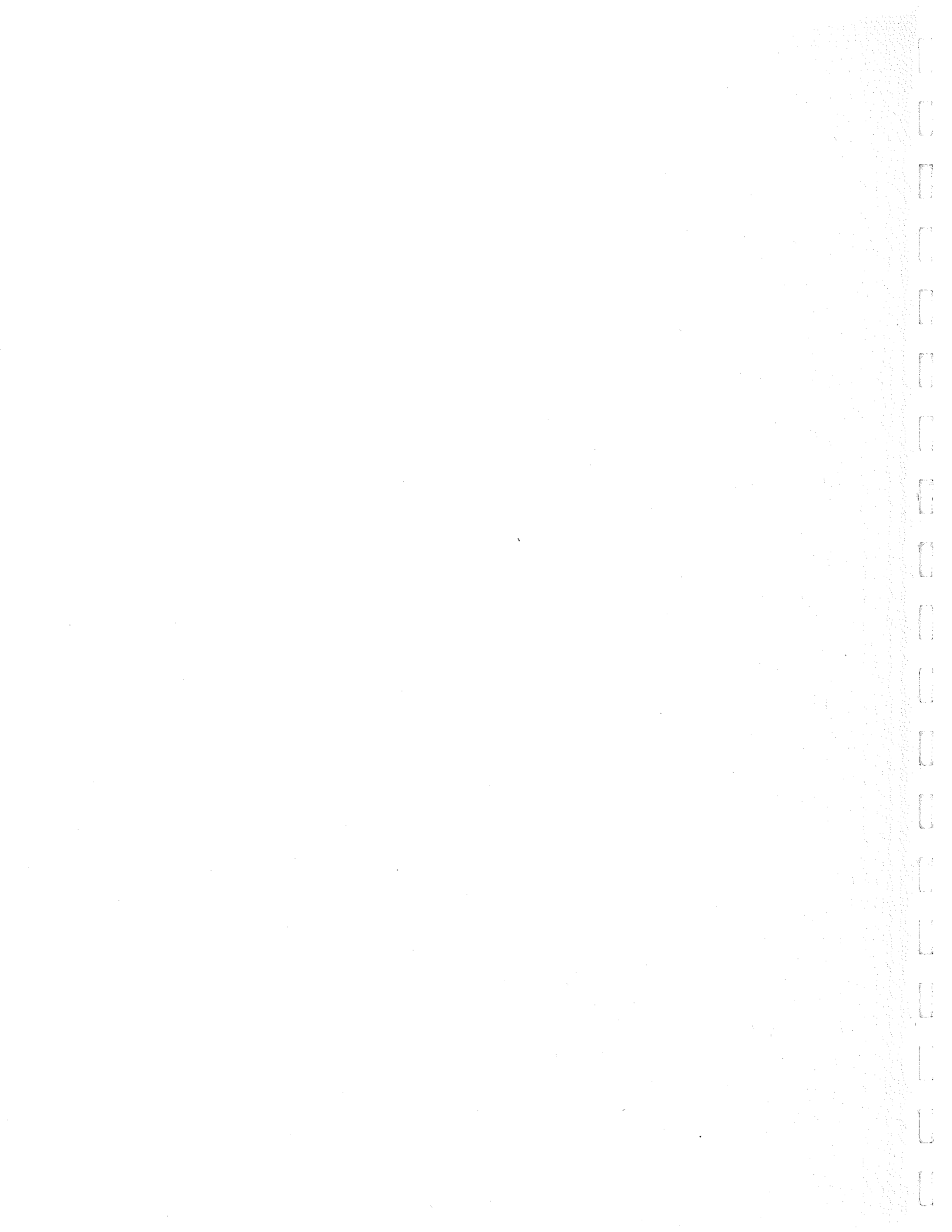




6. CHARTS









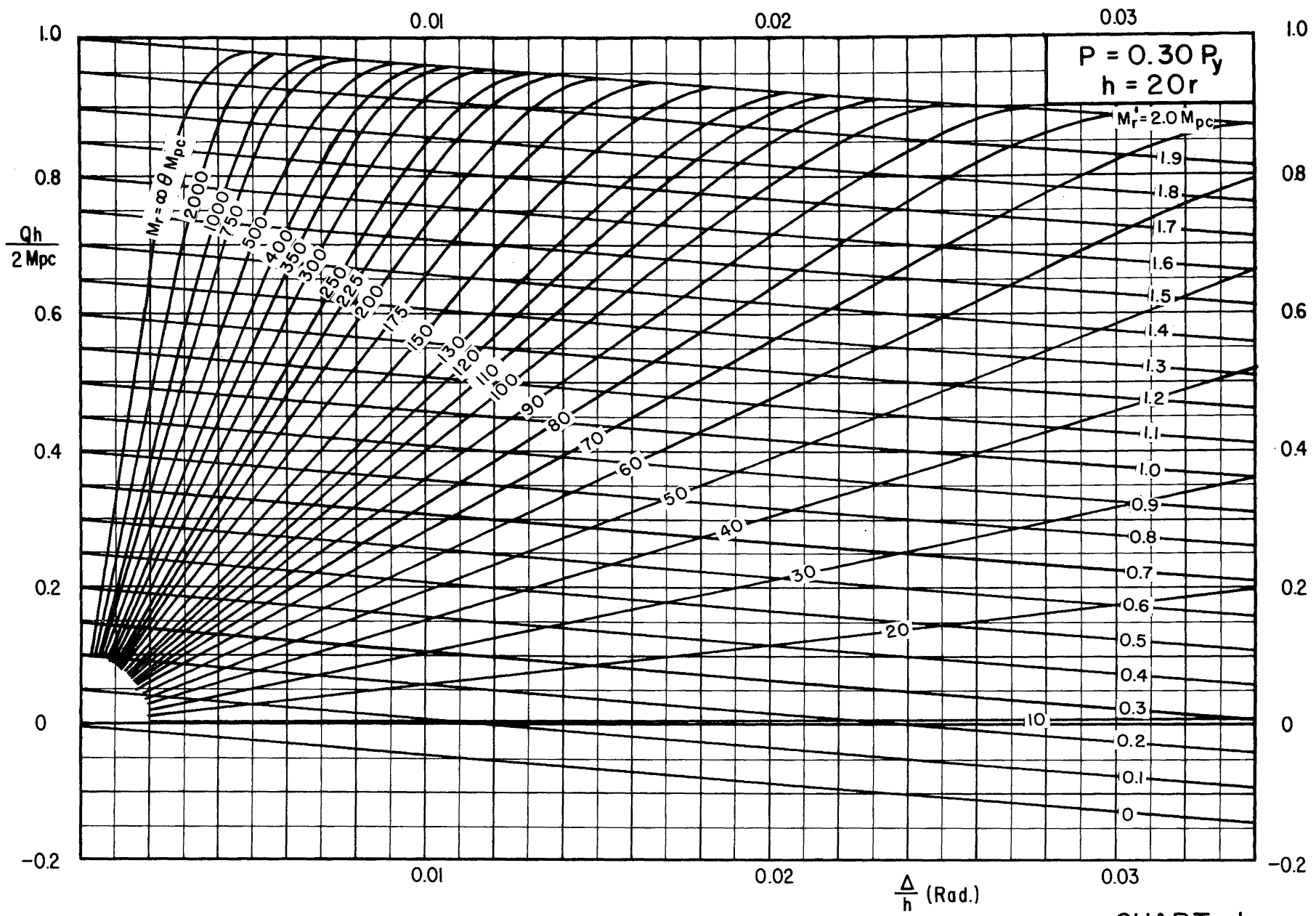


CHART I

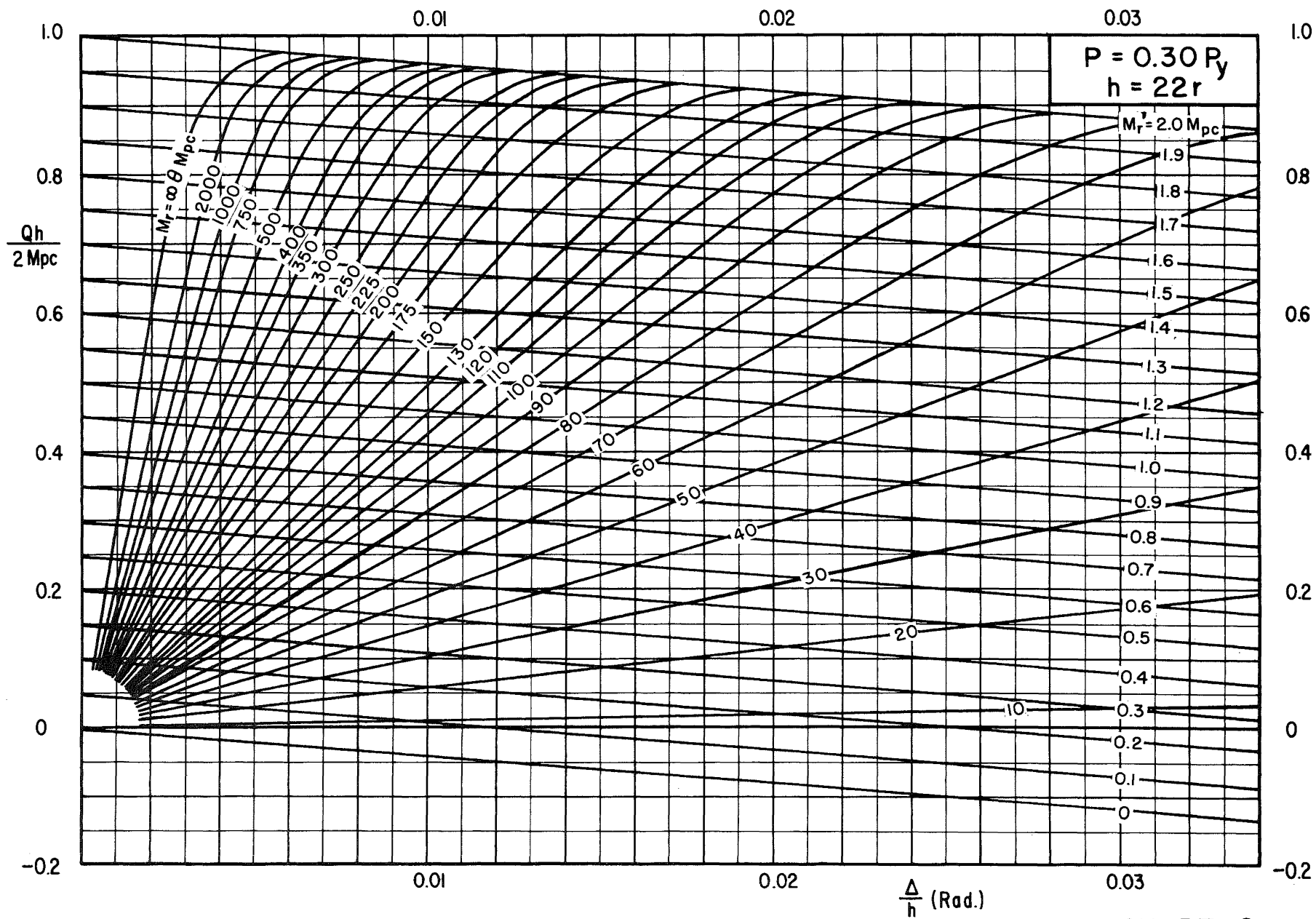
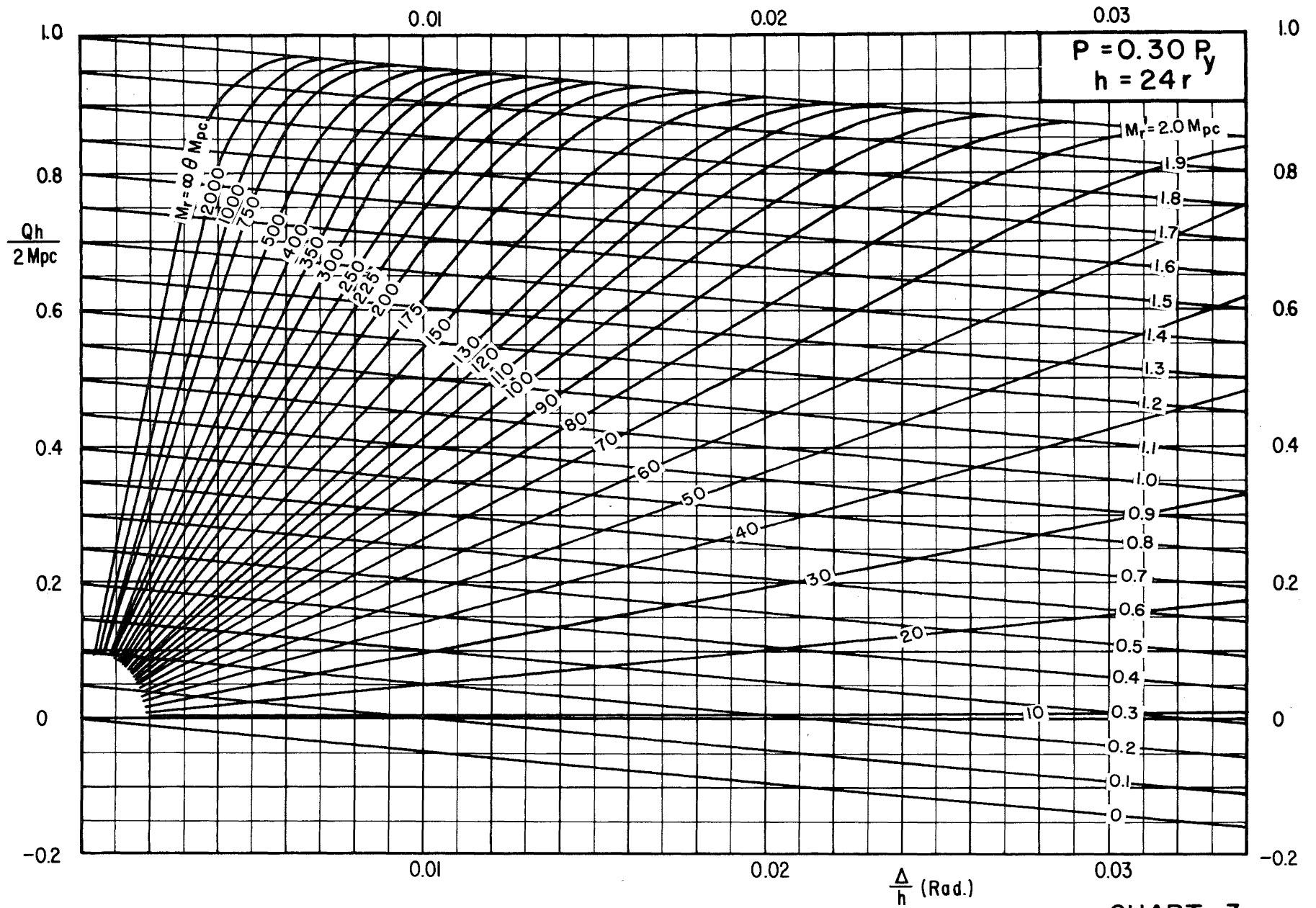


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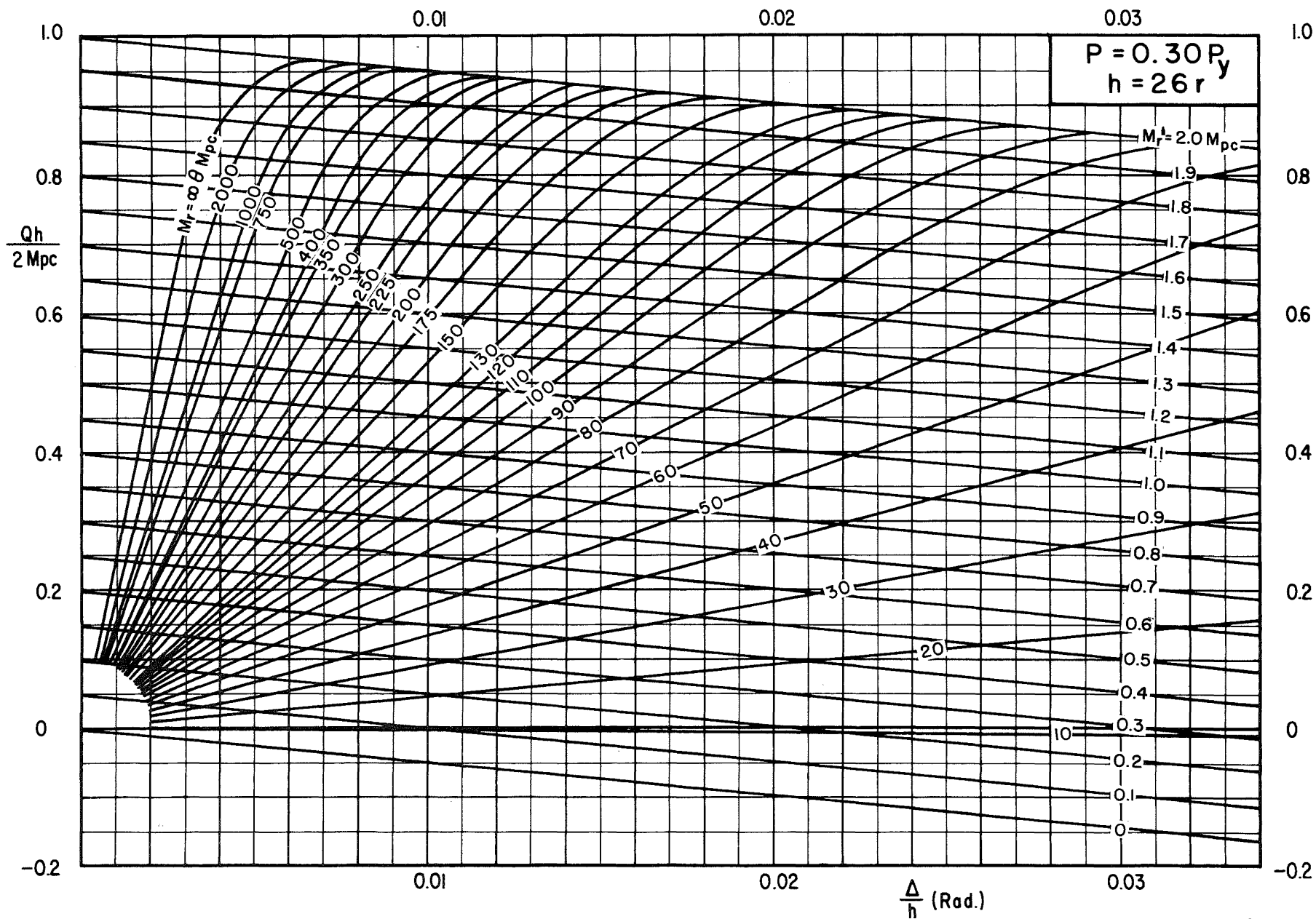


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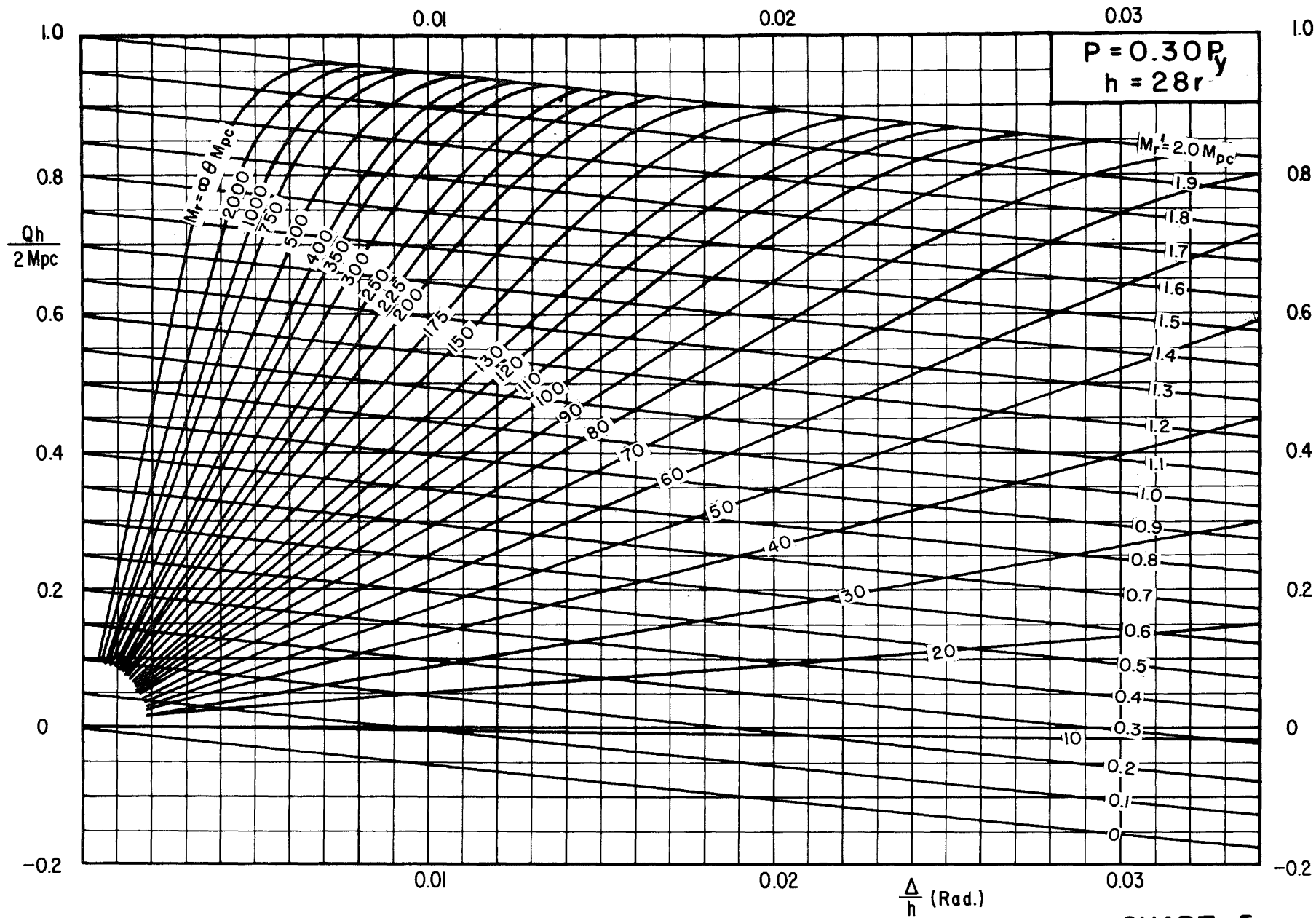


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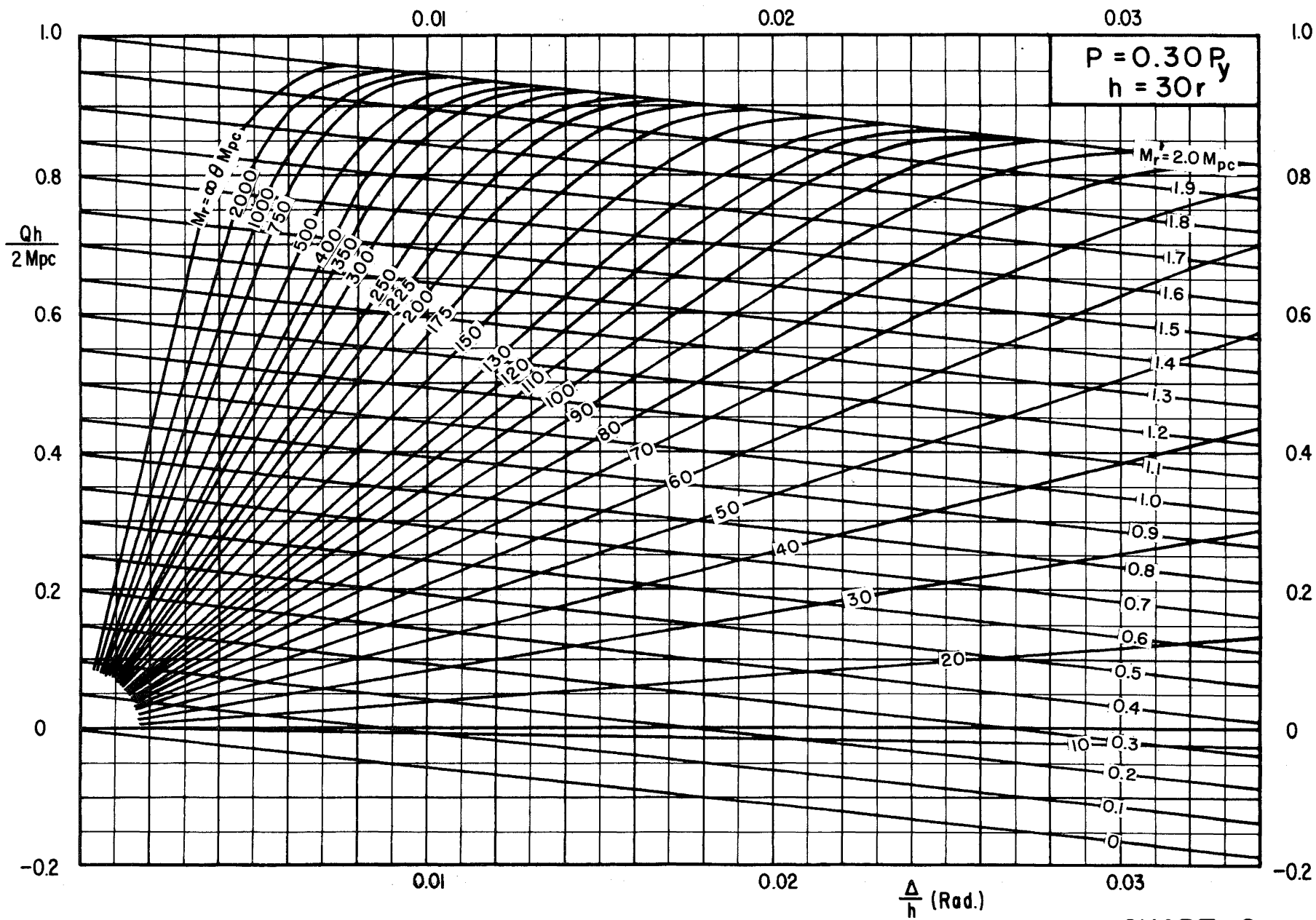
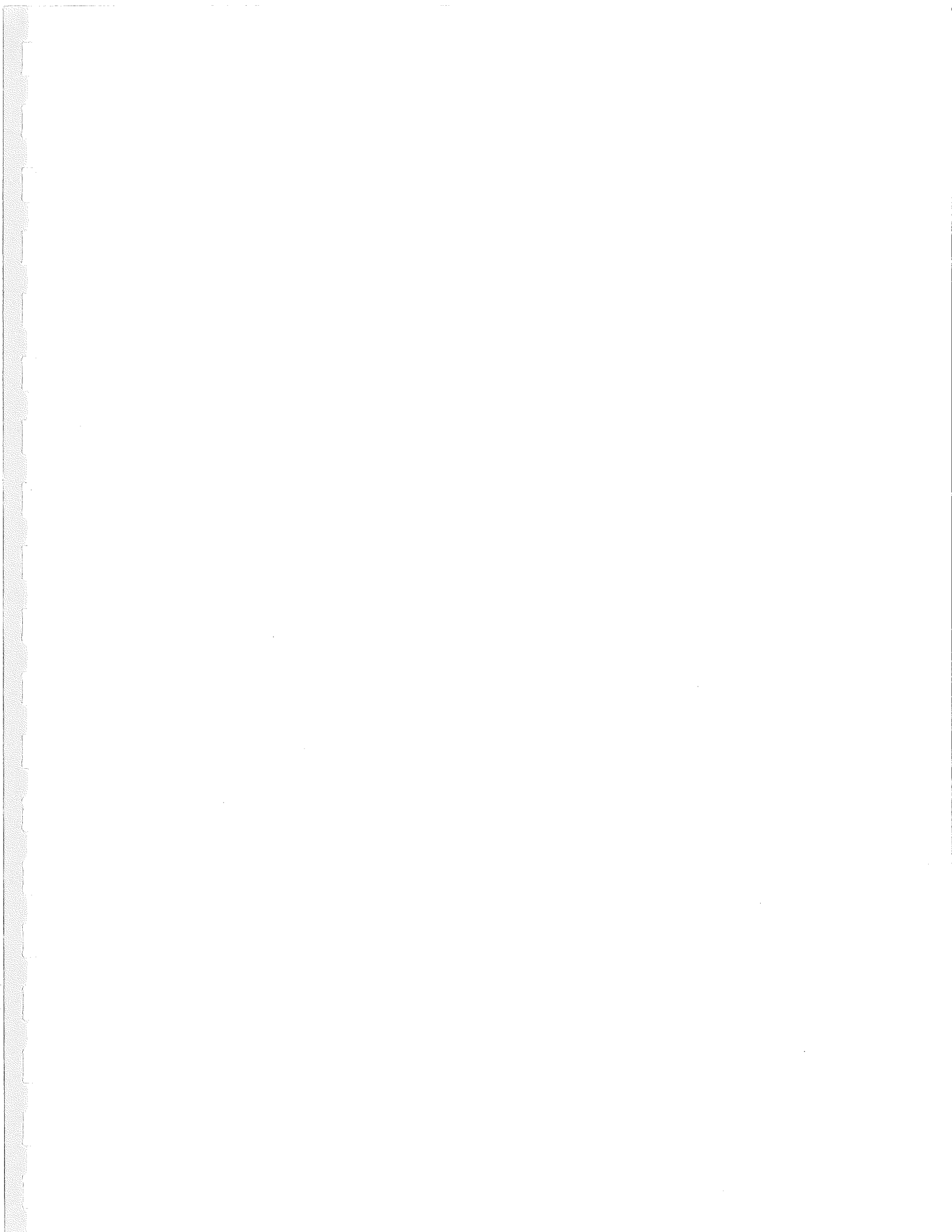


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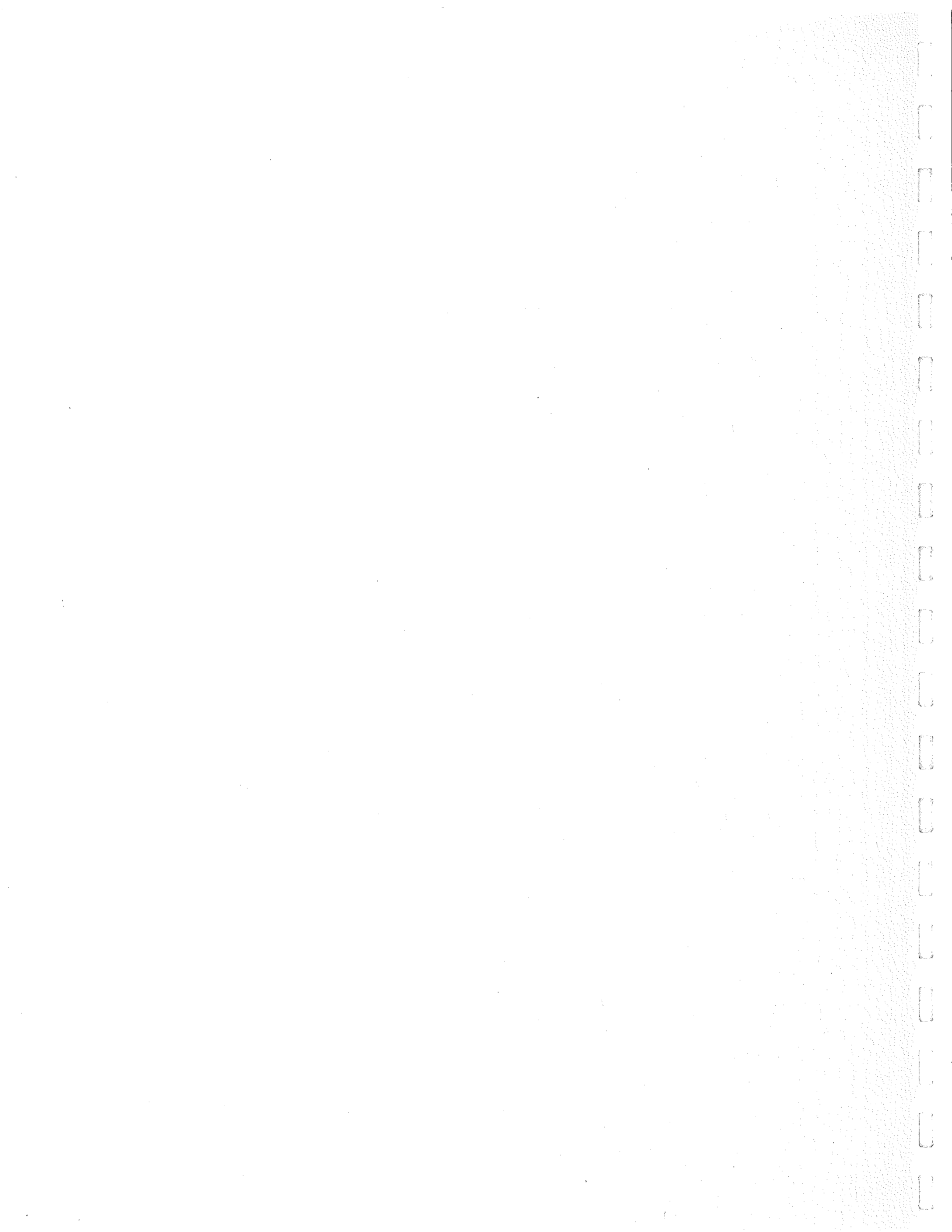






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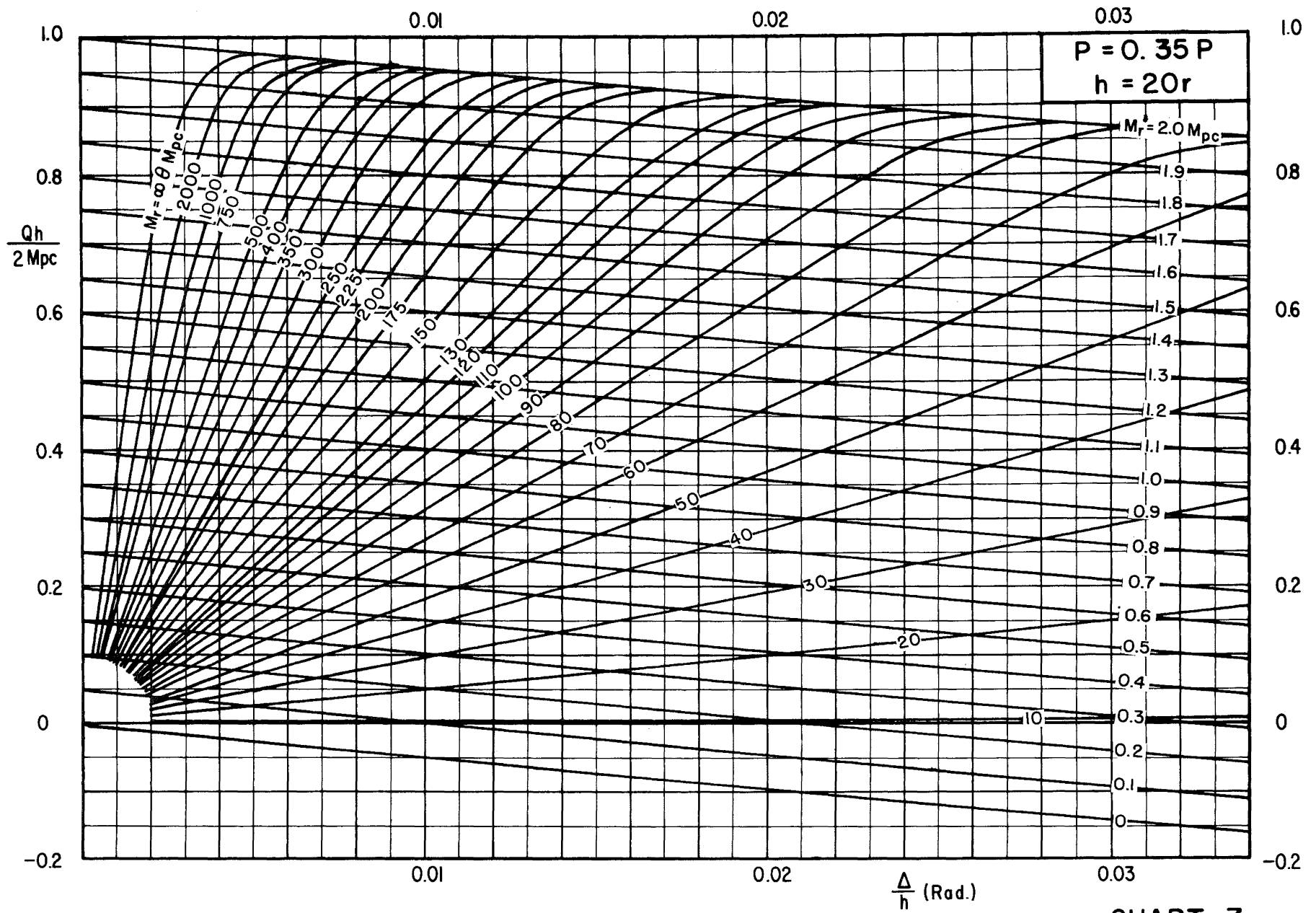


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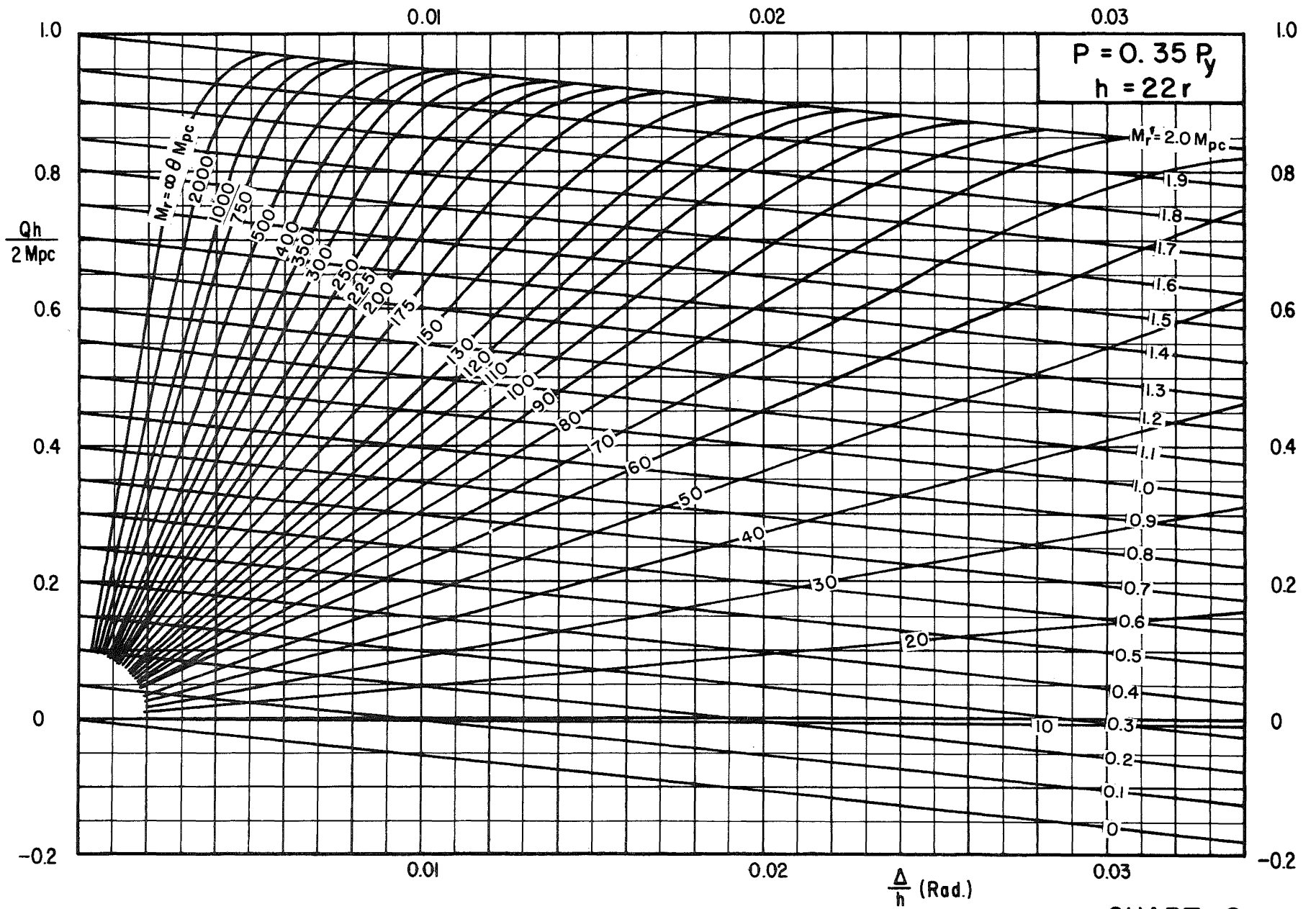


CHART 8

CHART 8

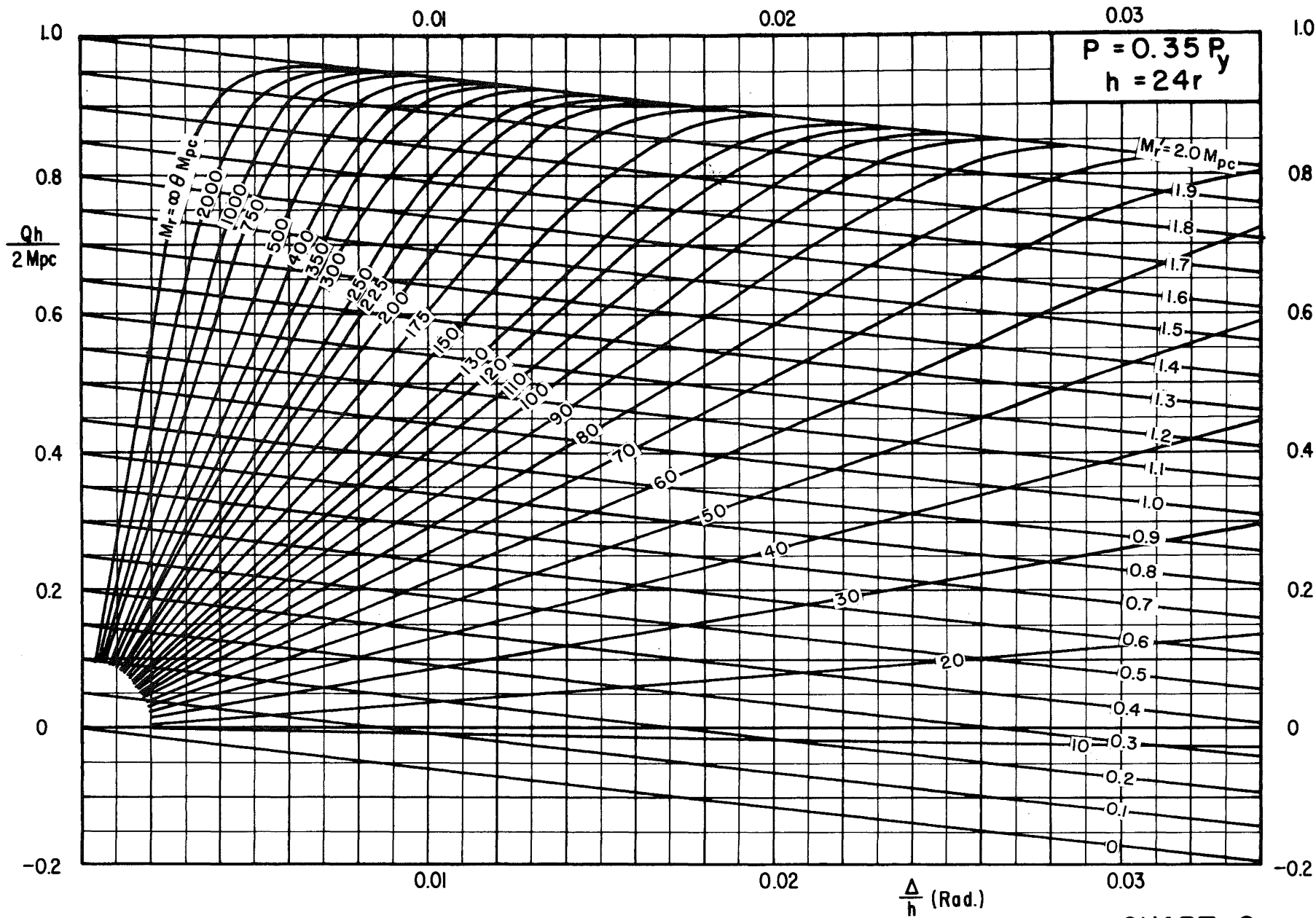


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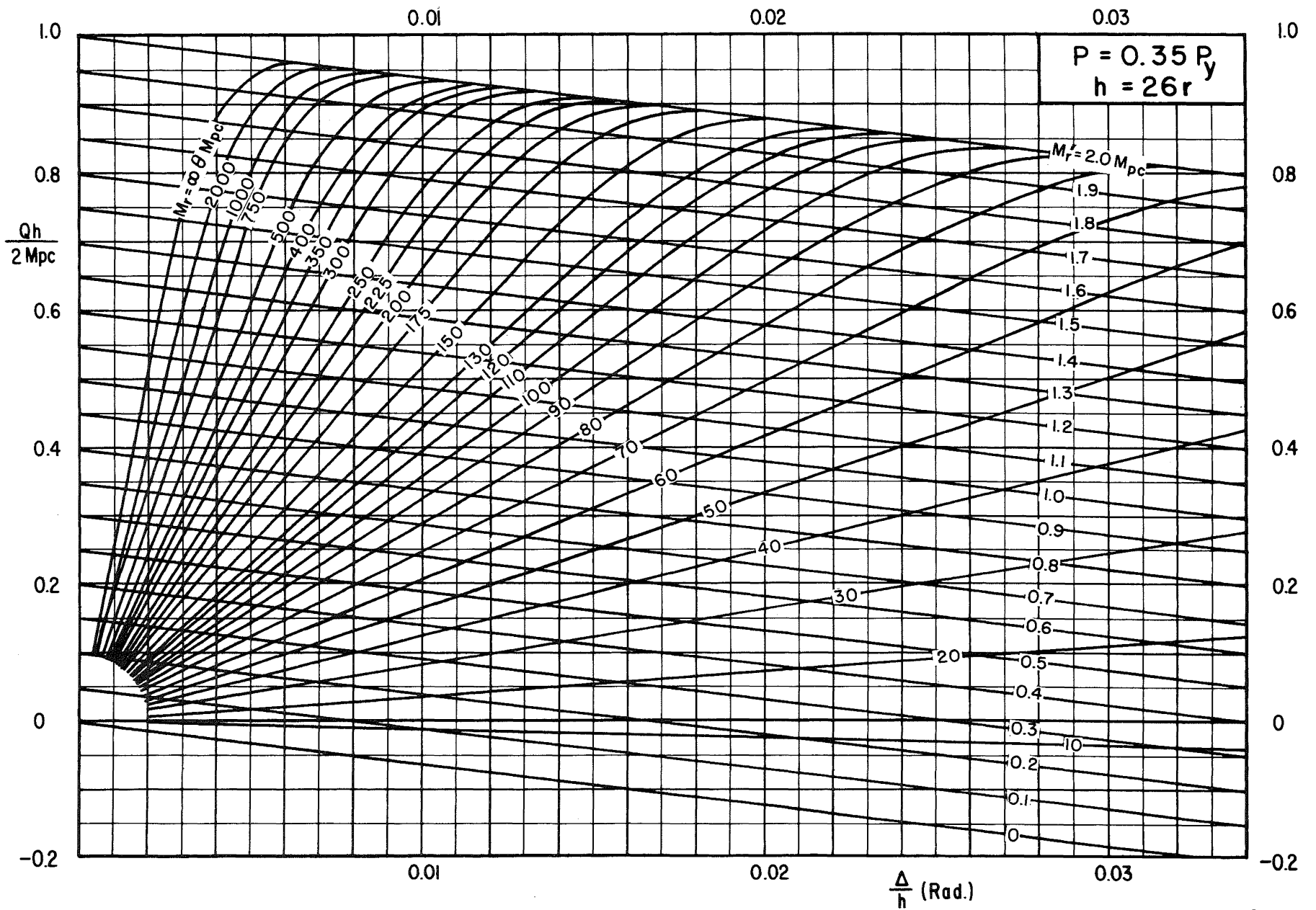


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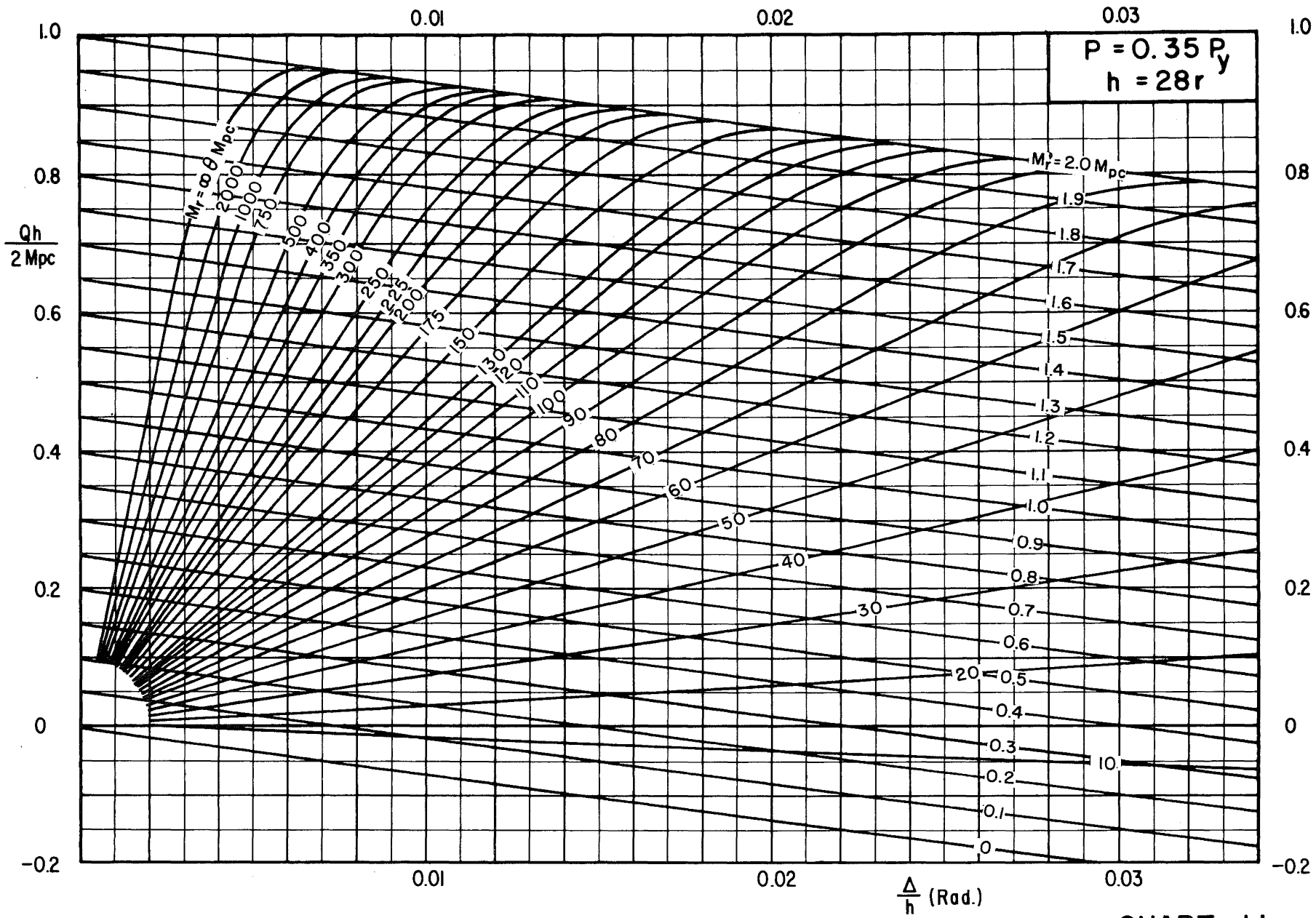


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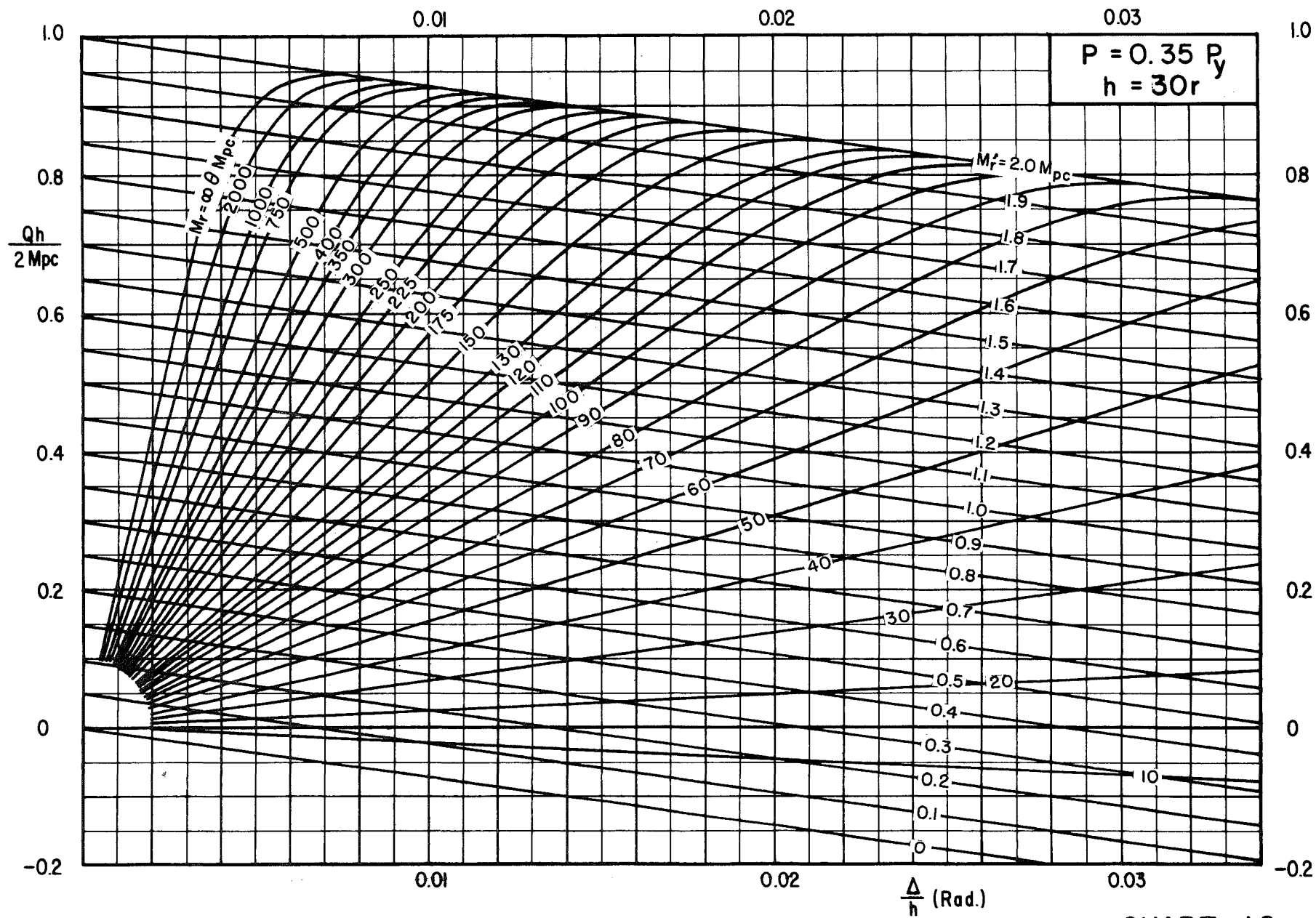
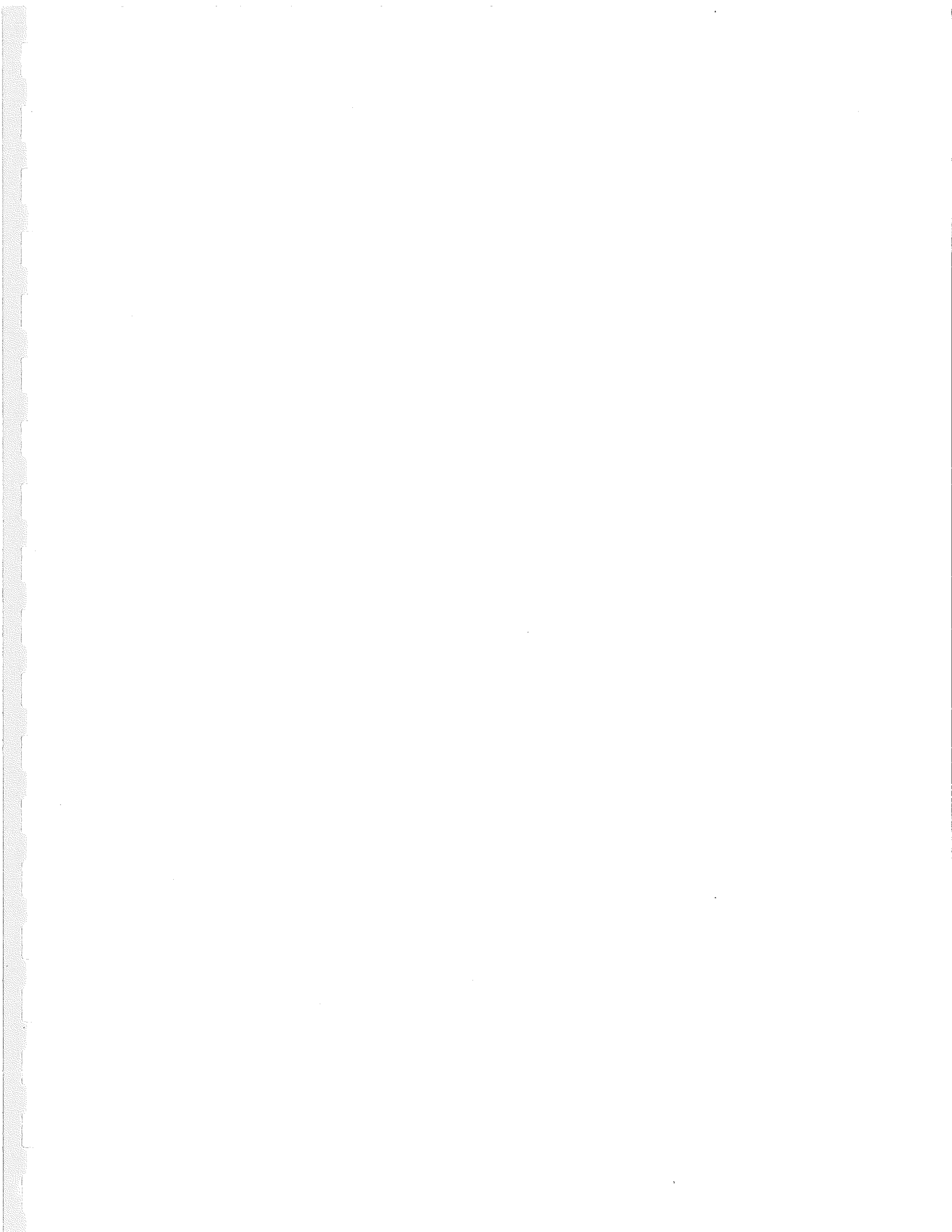
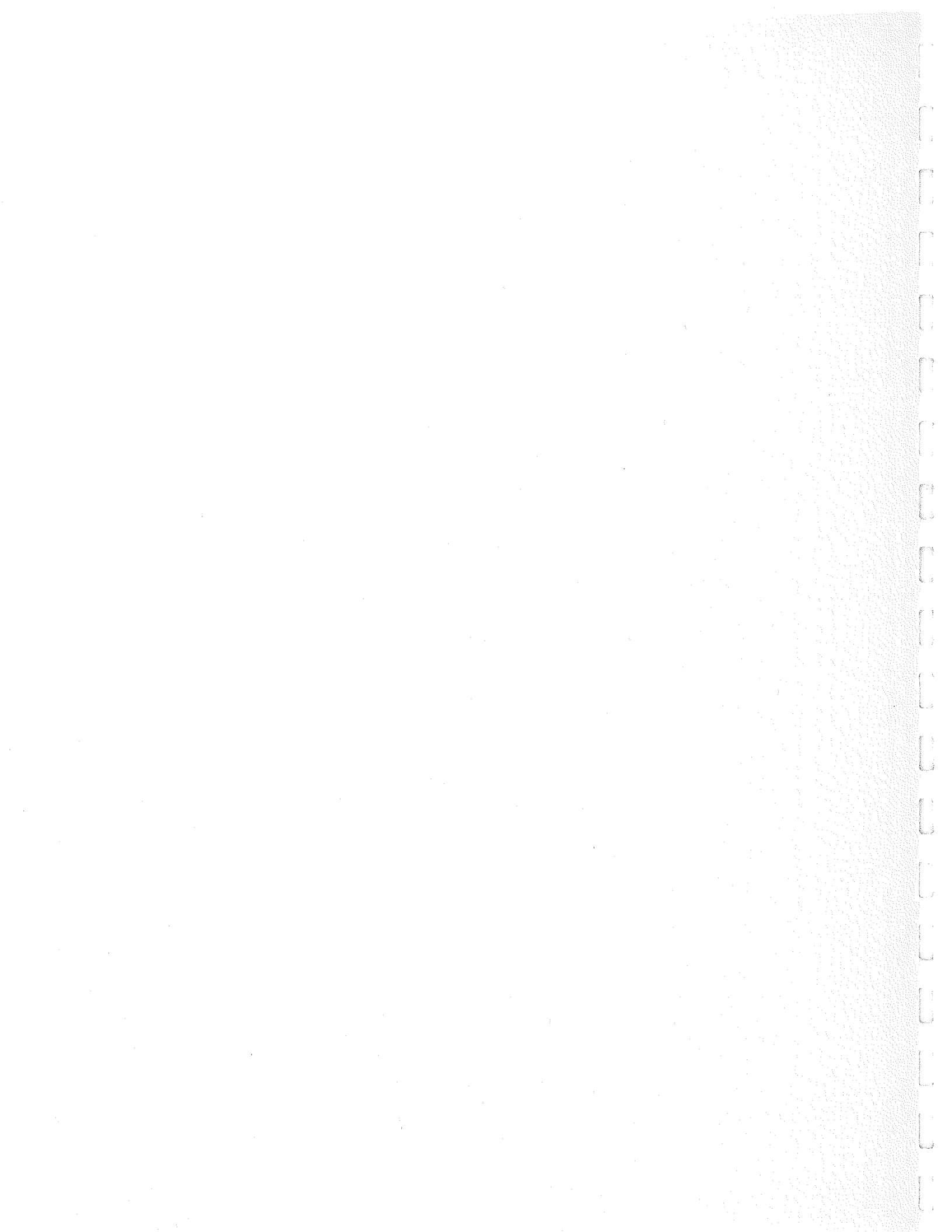


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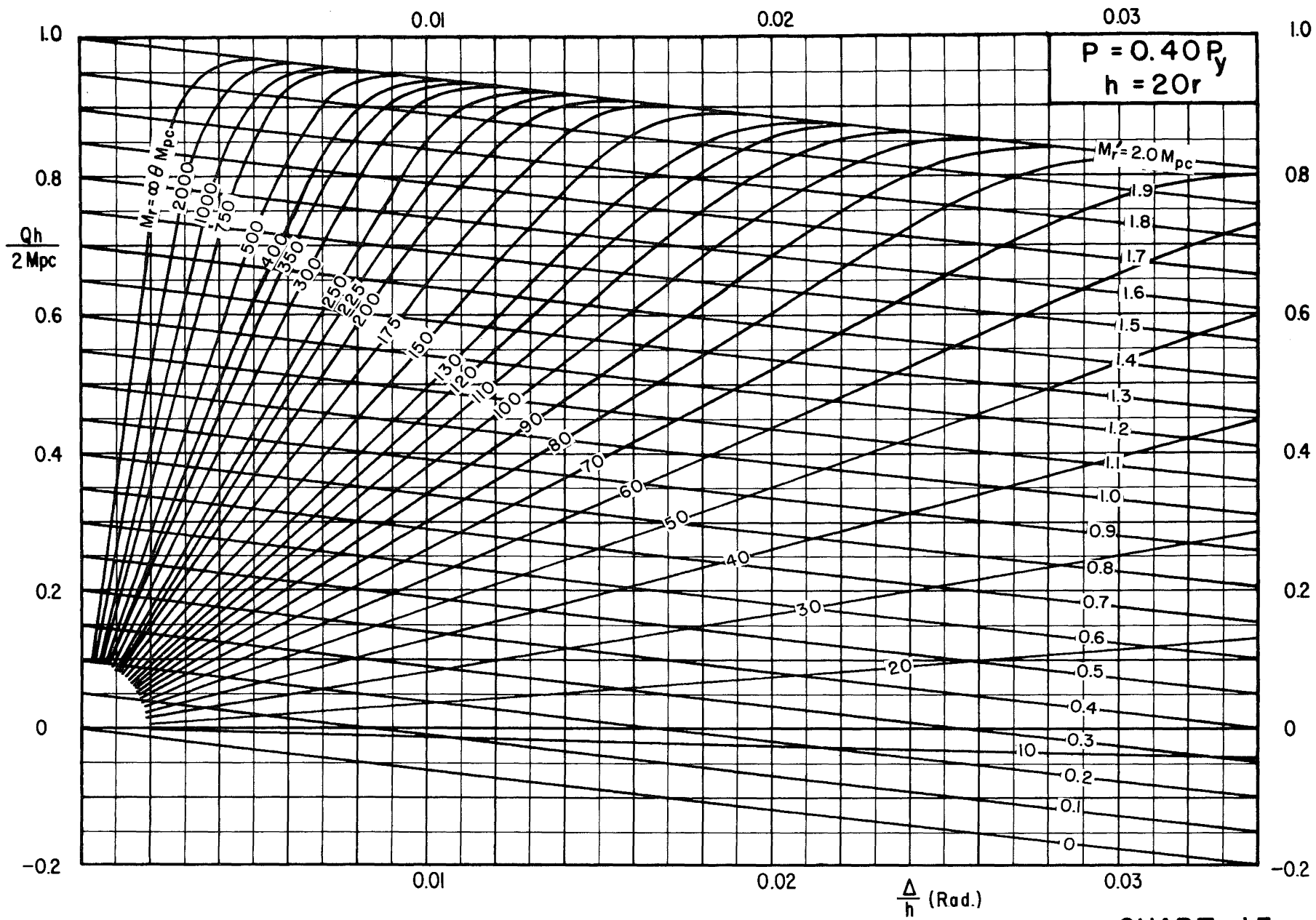


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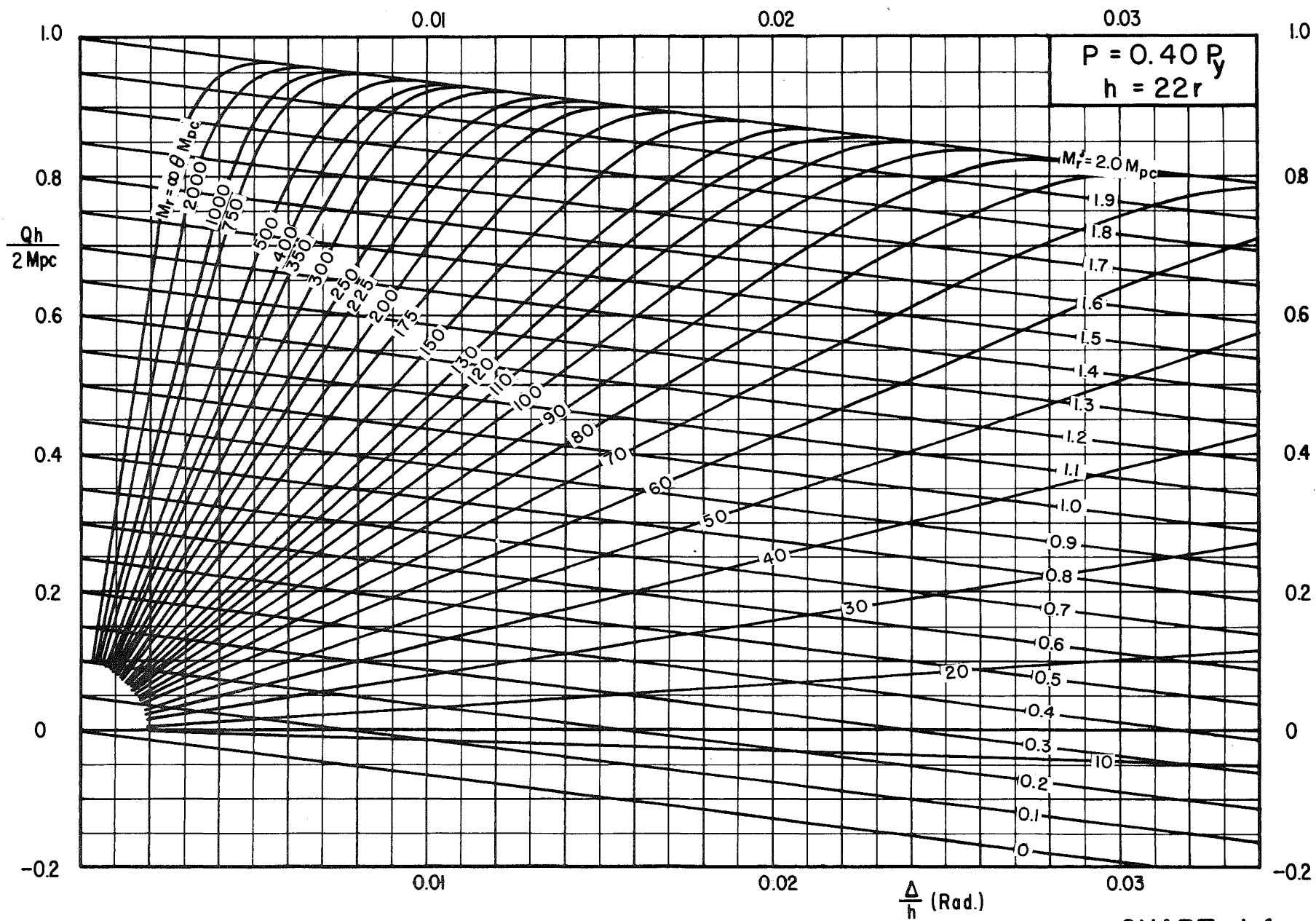


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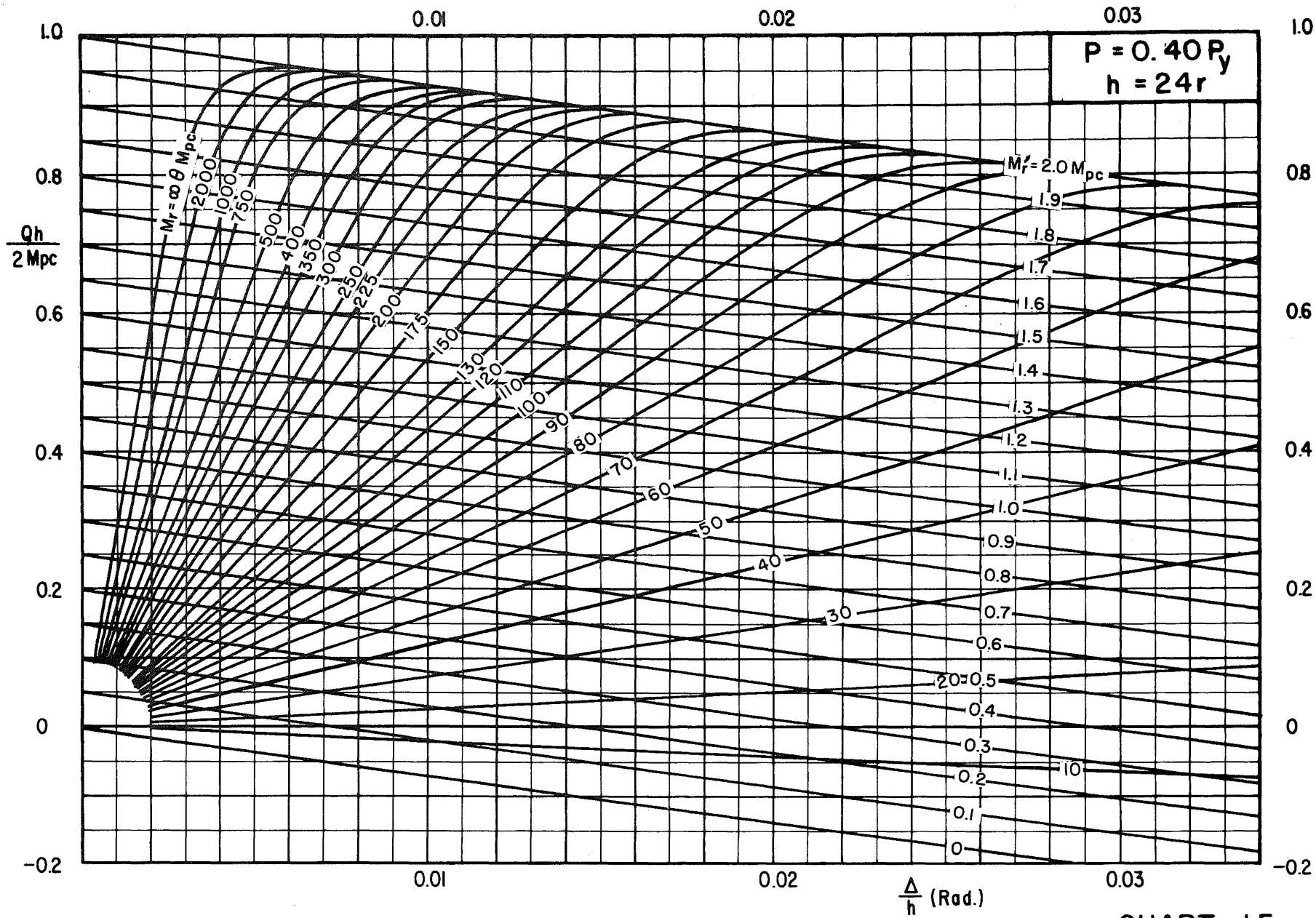


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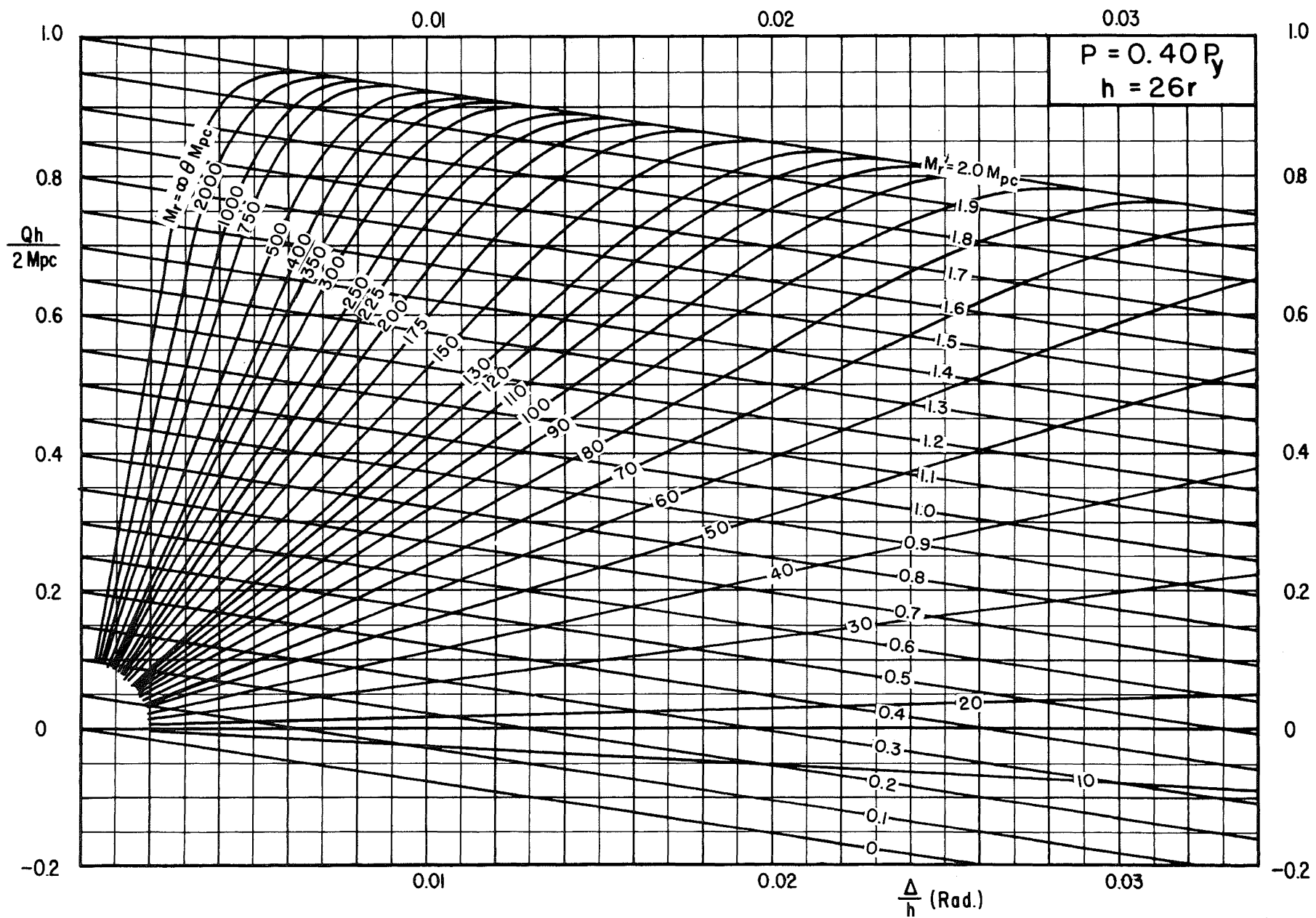


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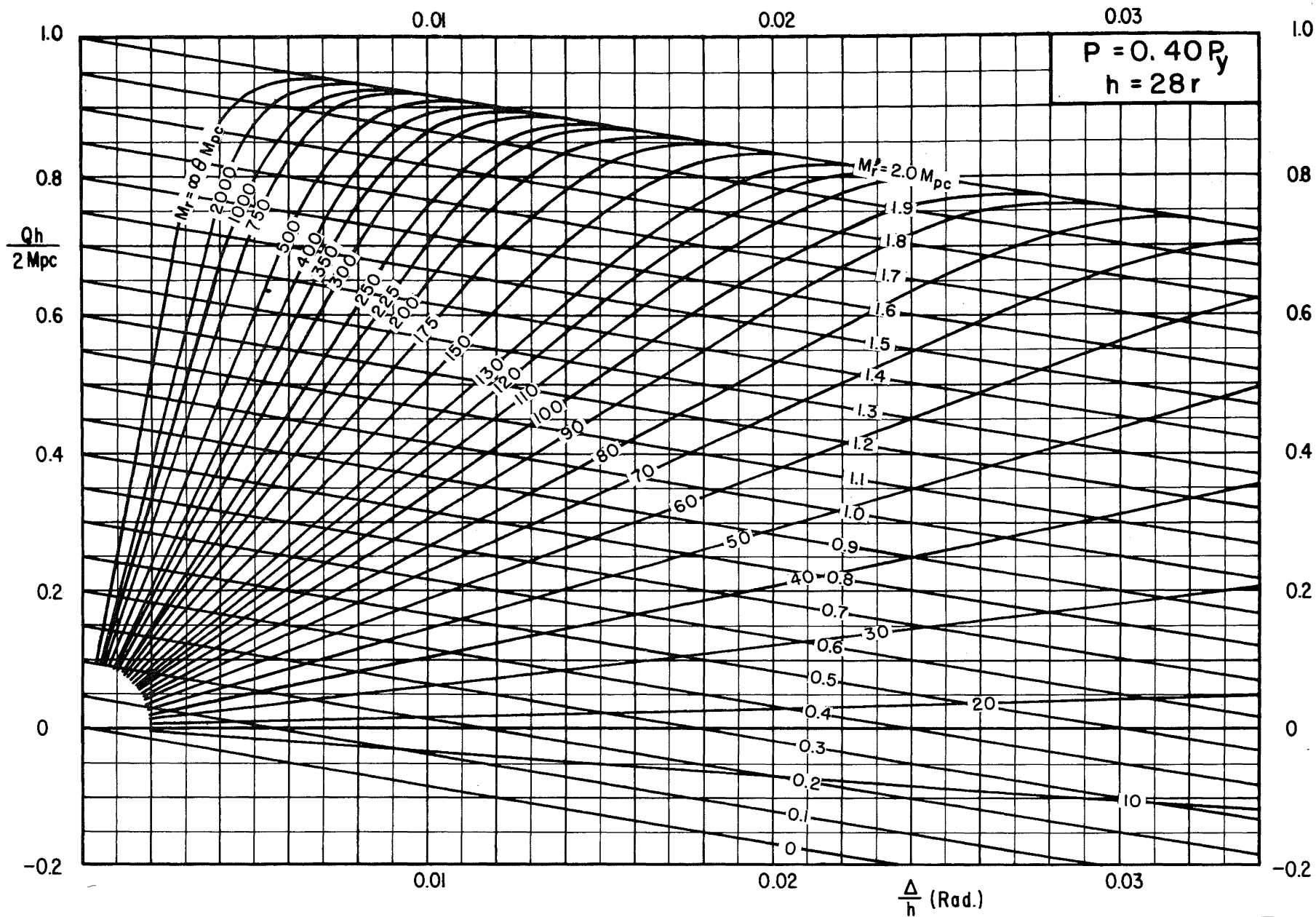


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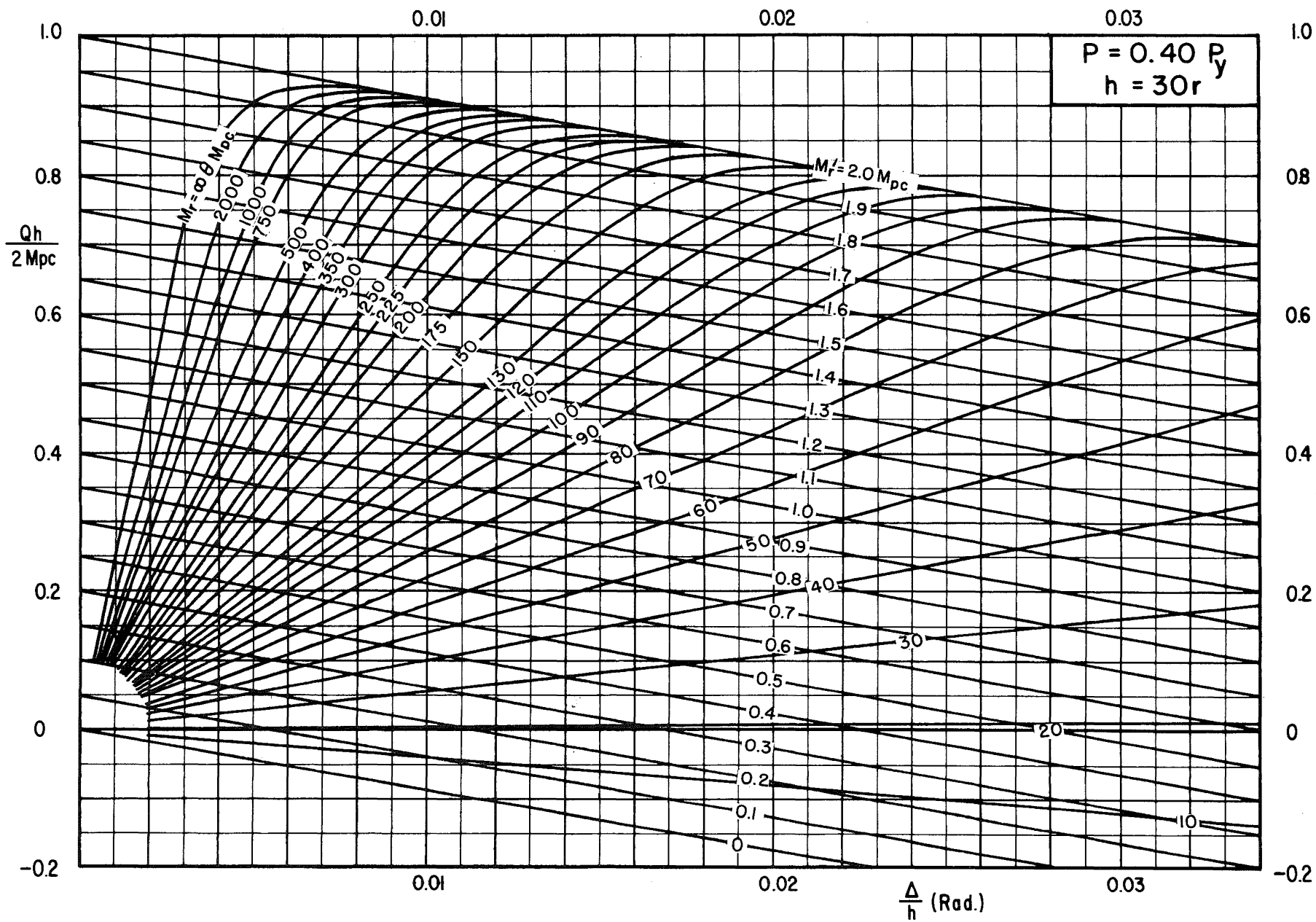


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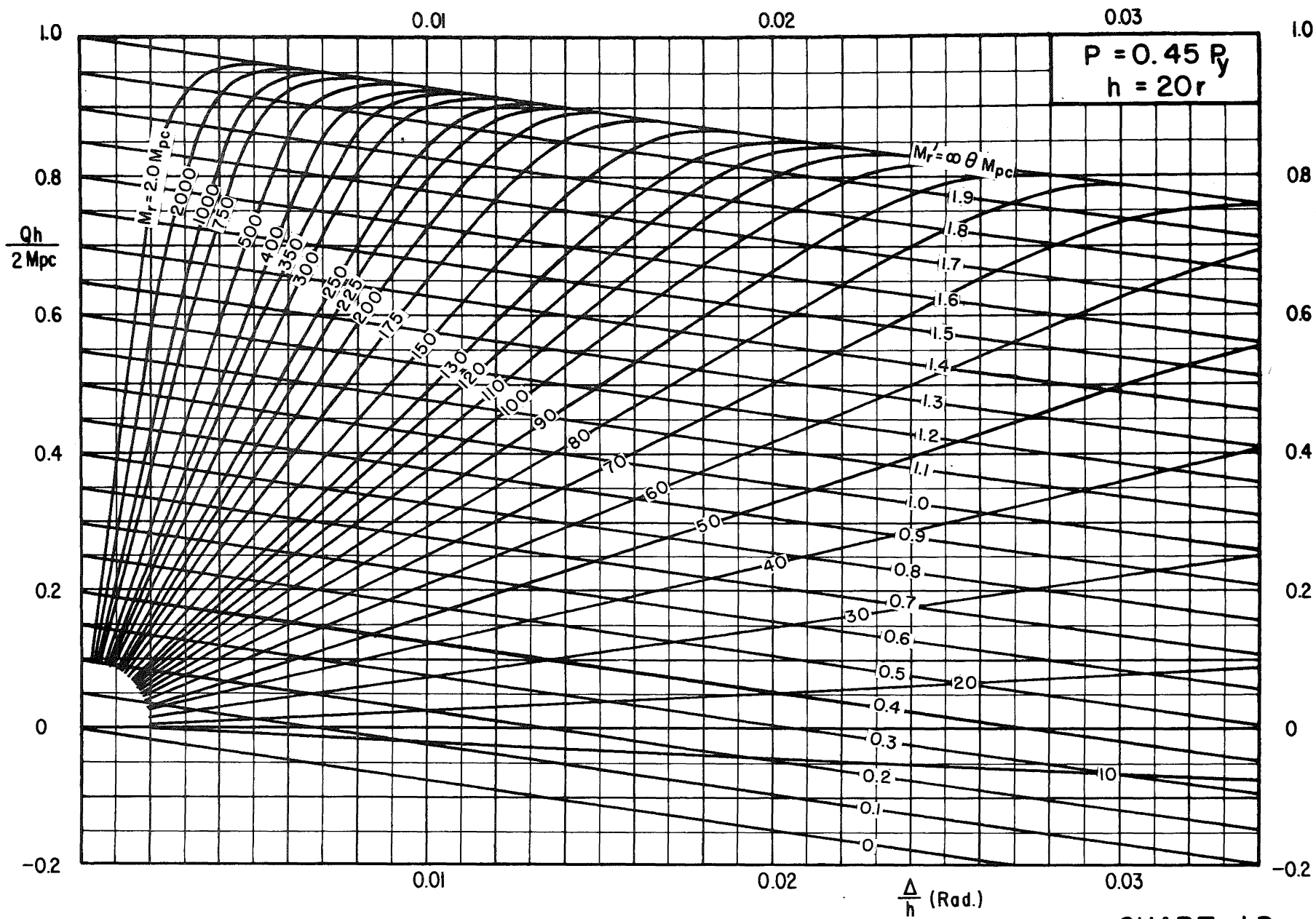


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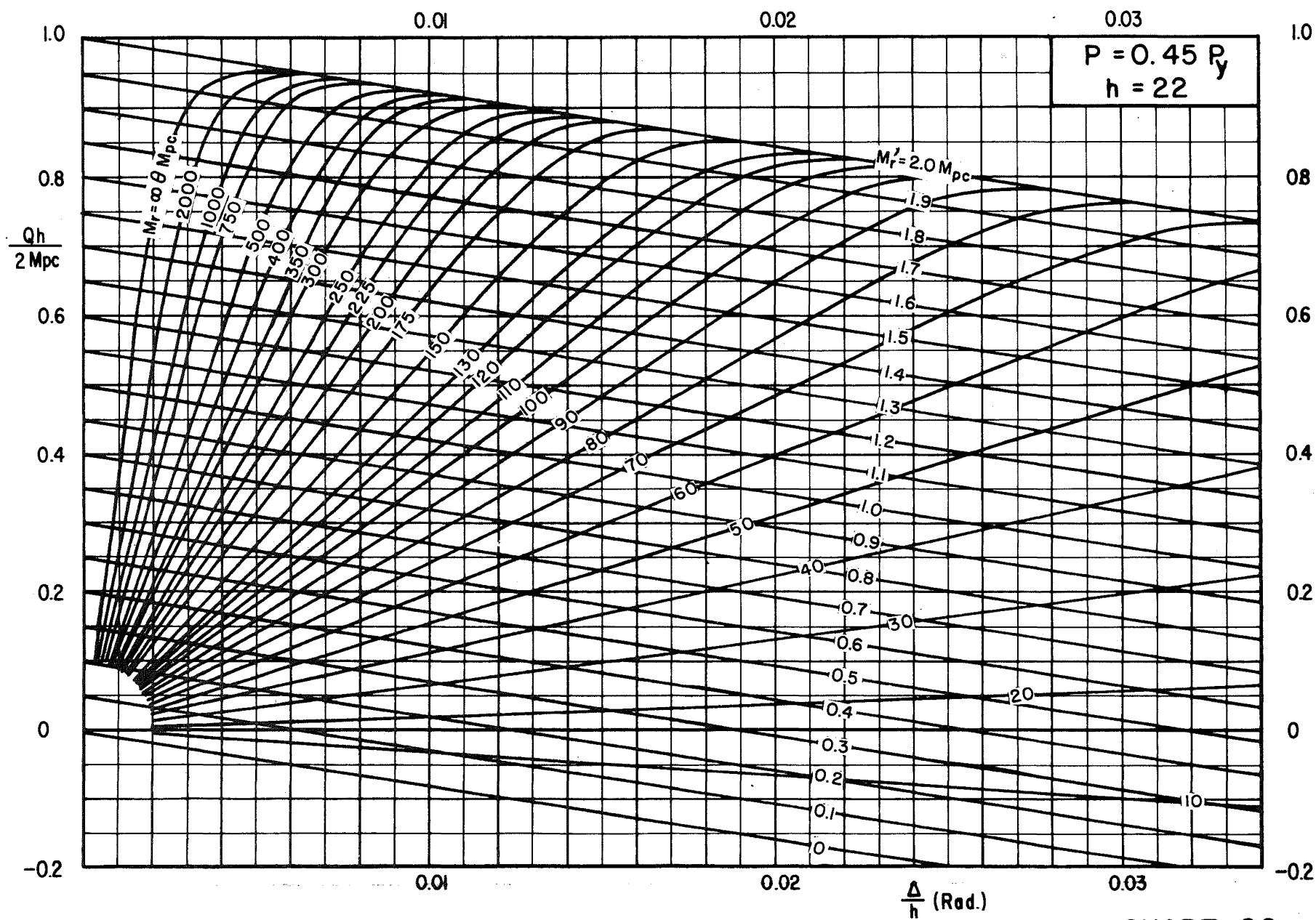


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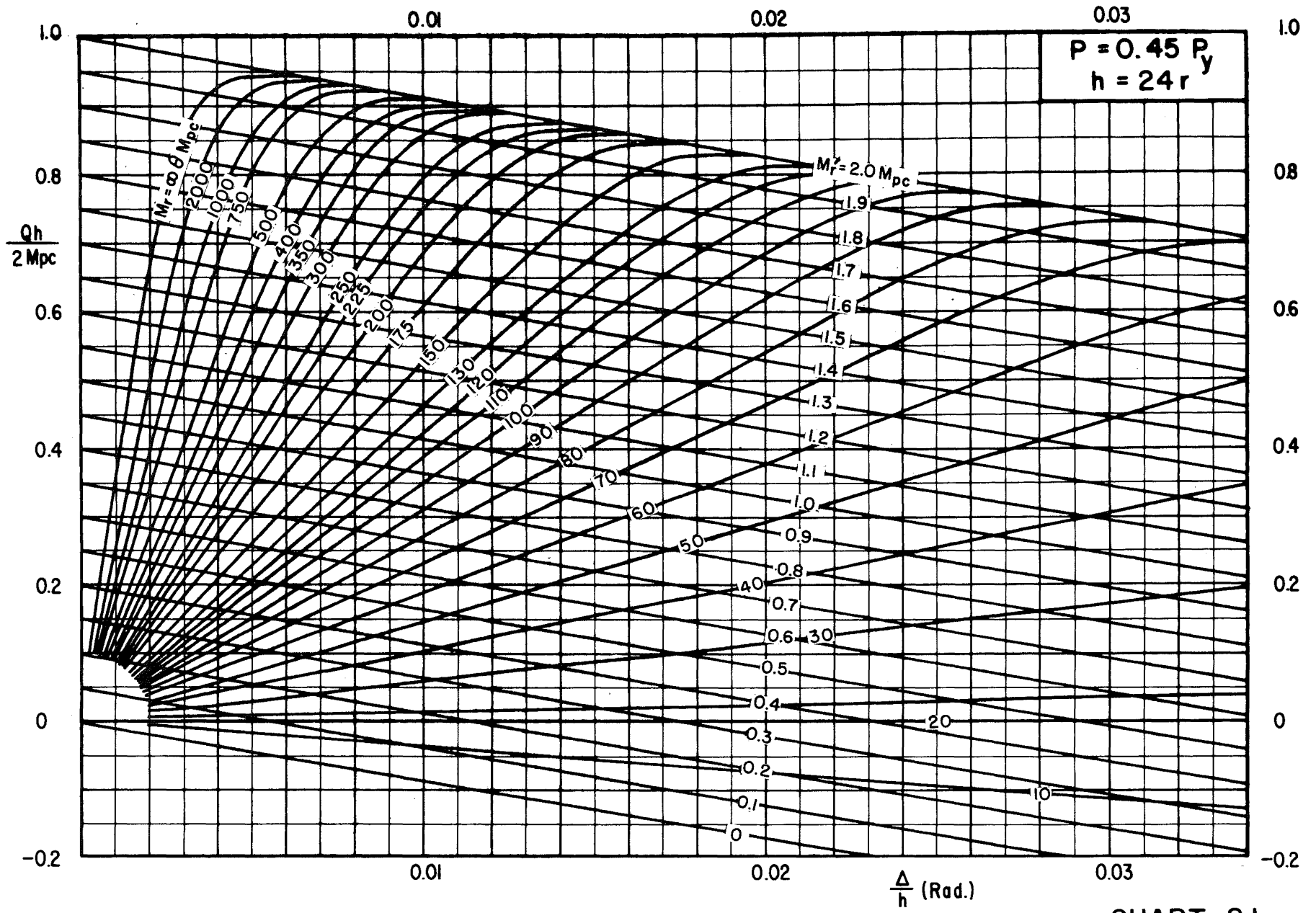


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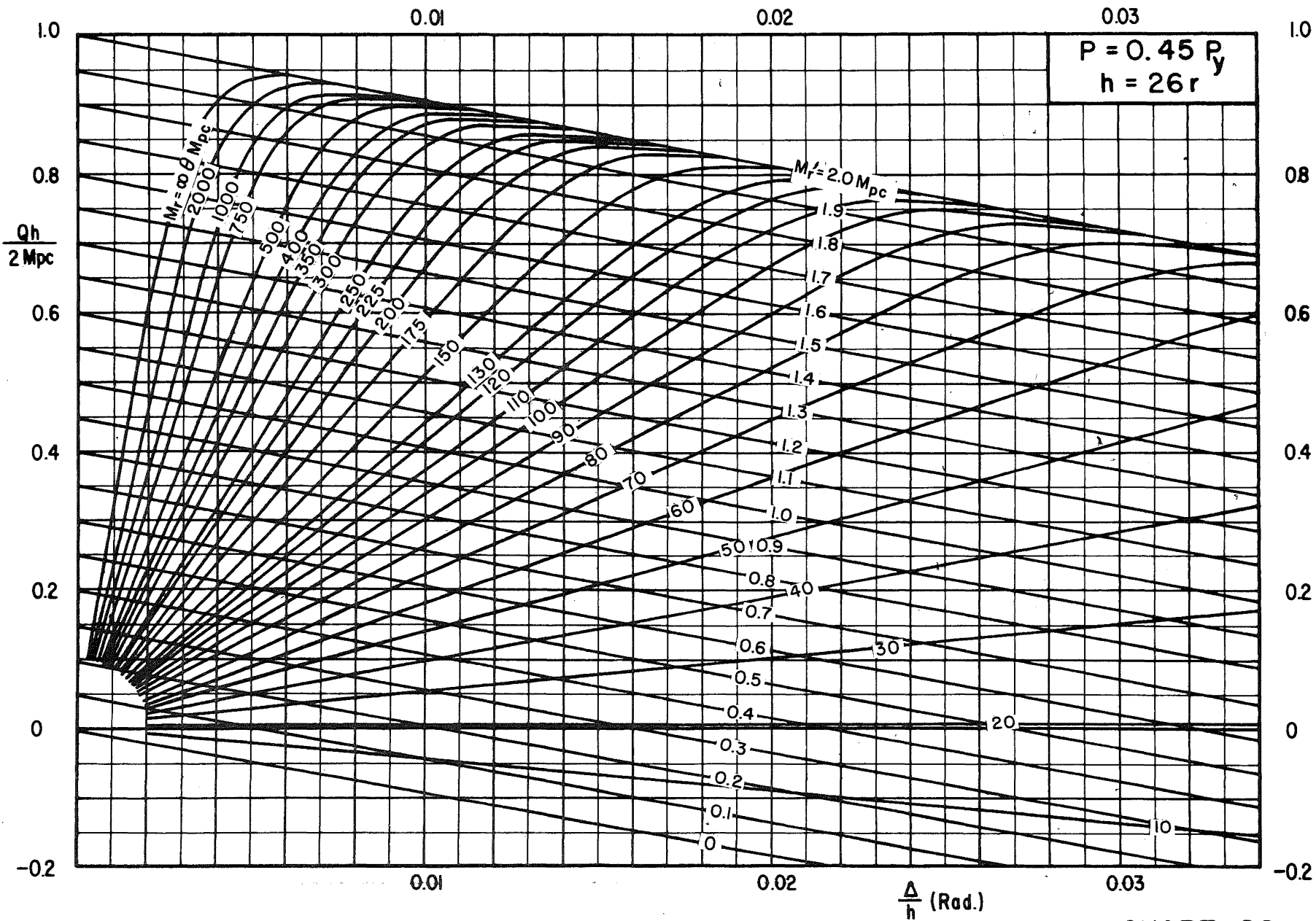


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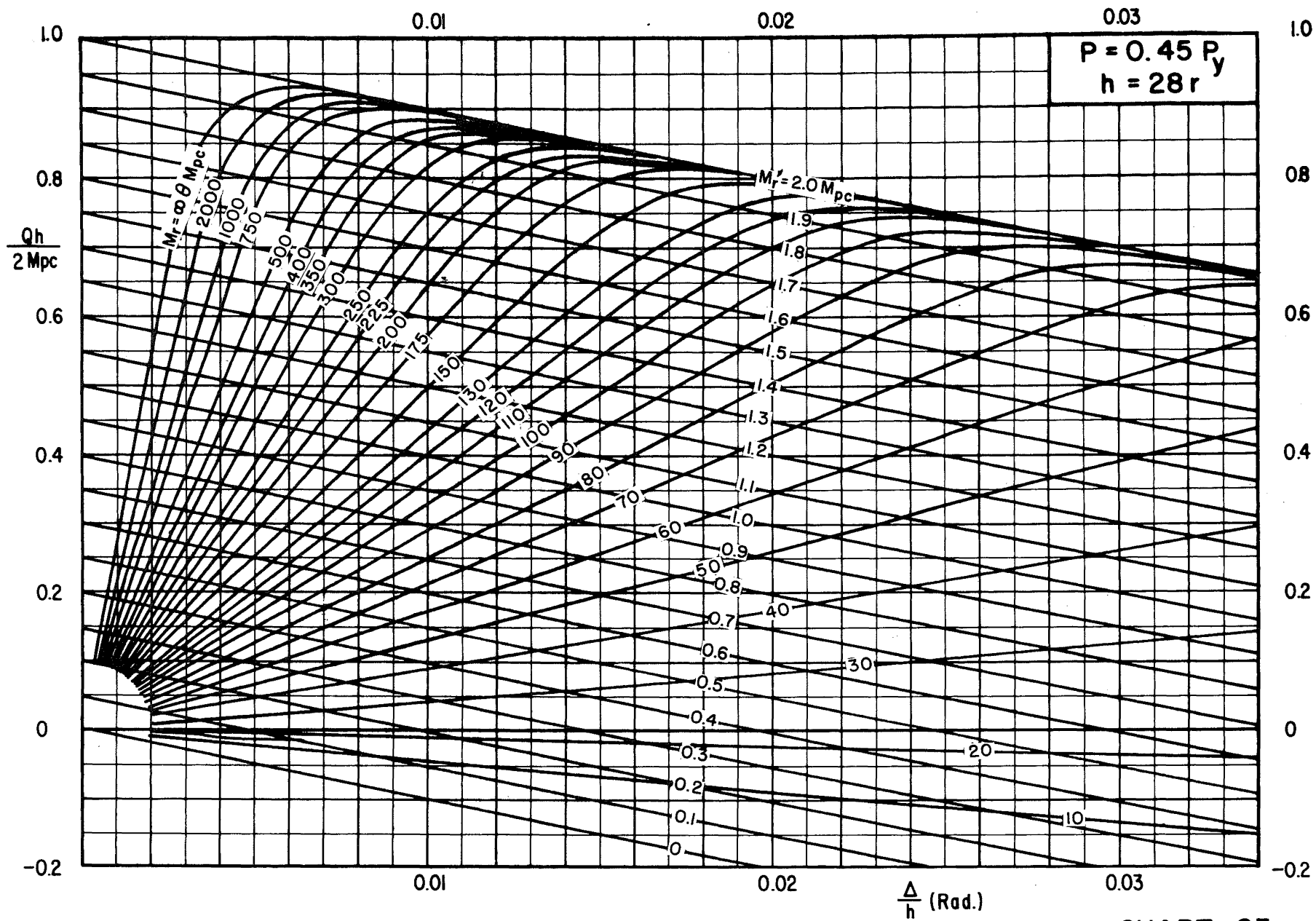


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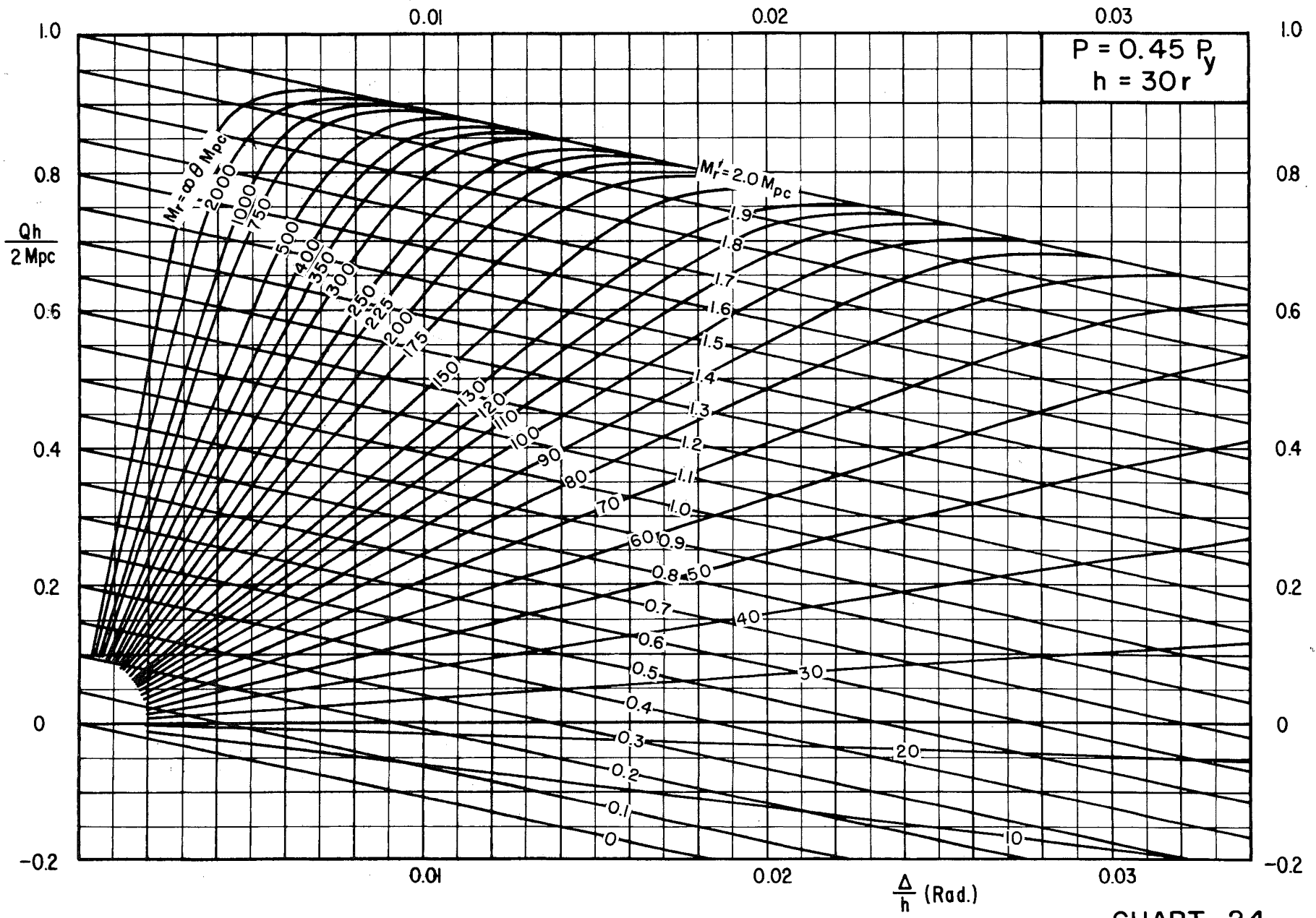
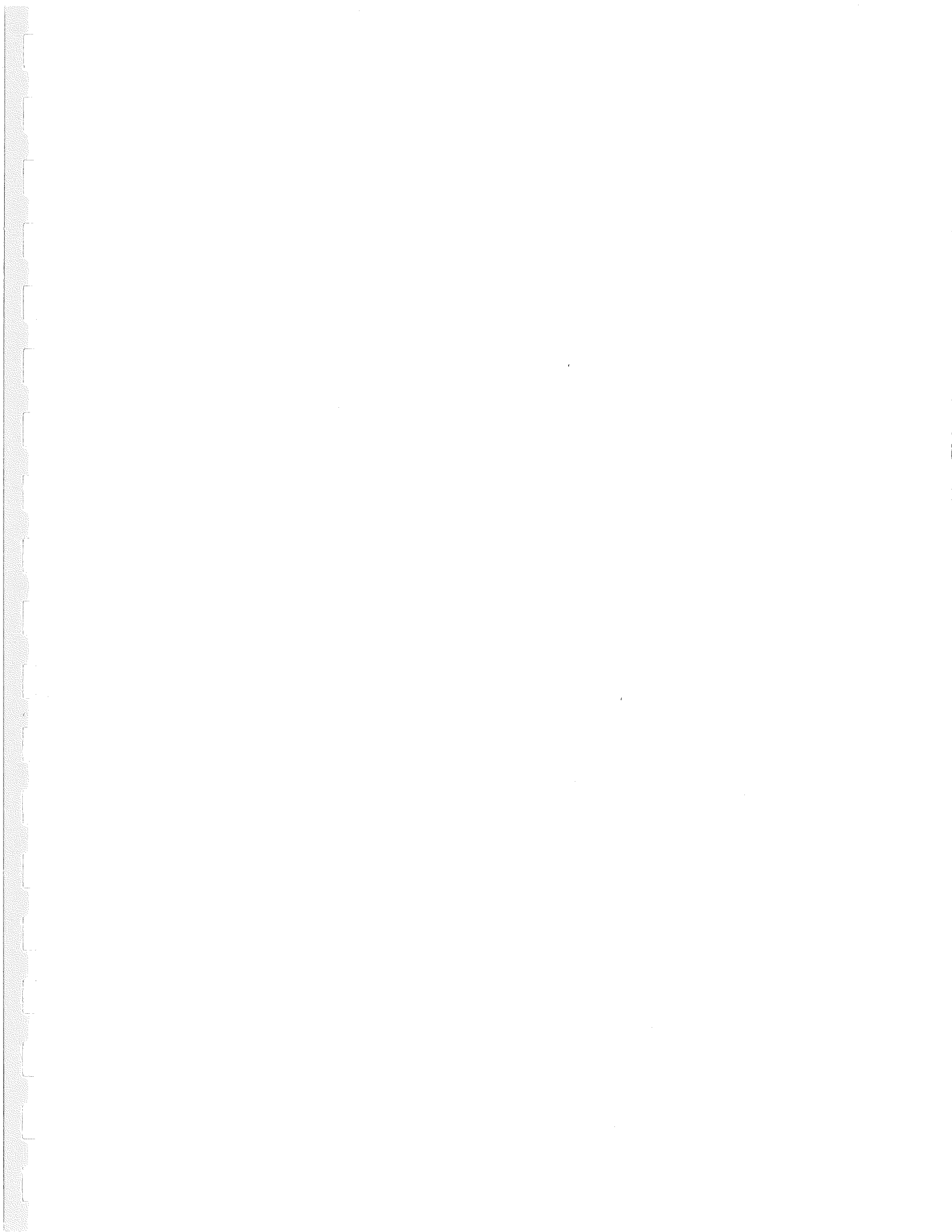
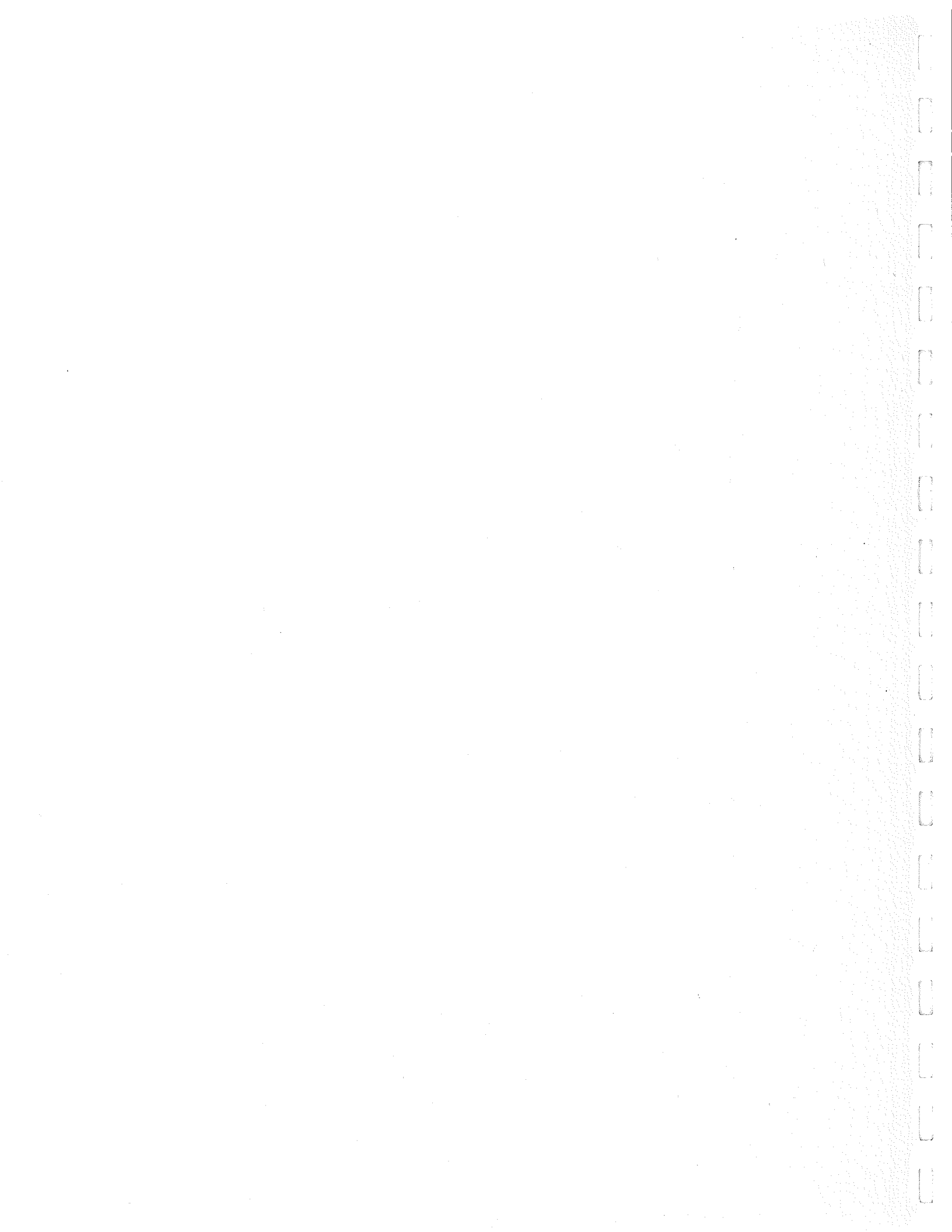


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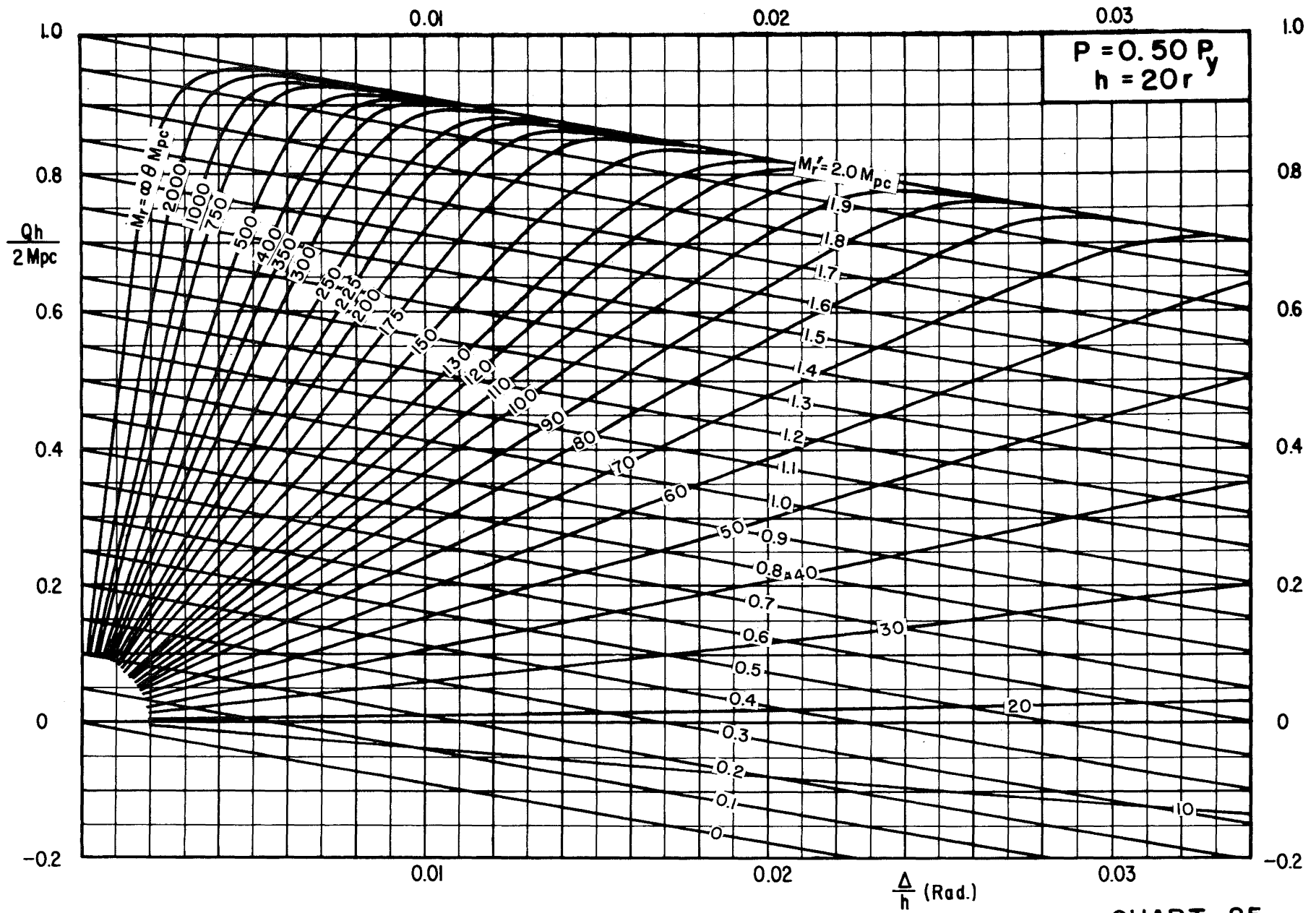


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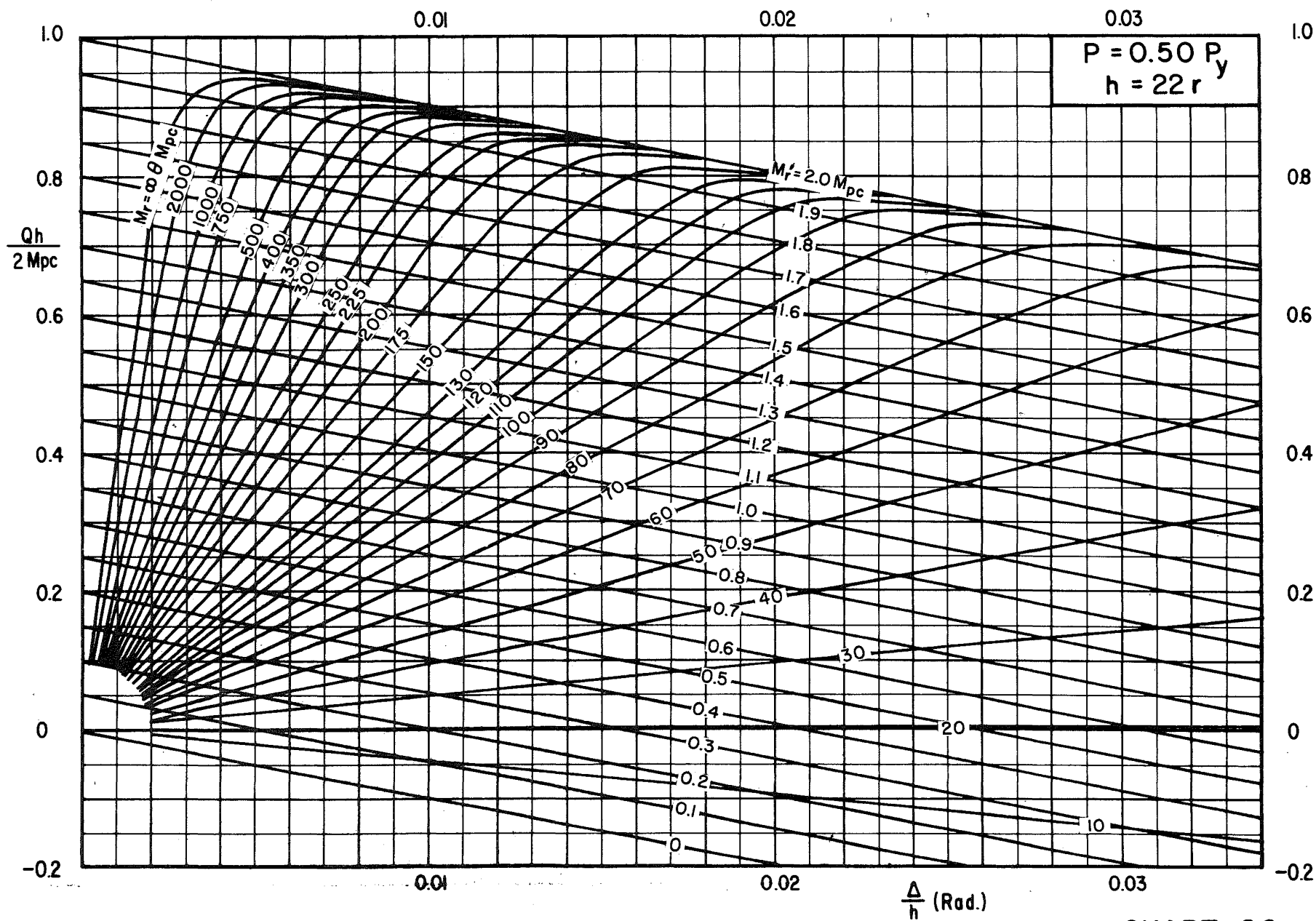


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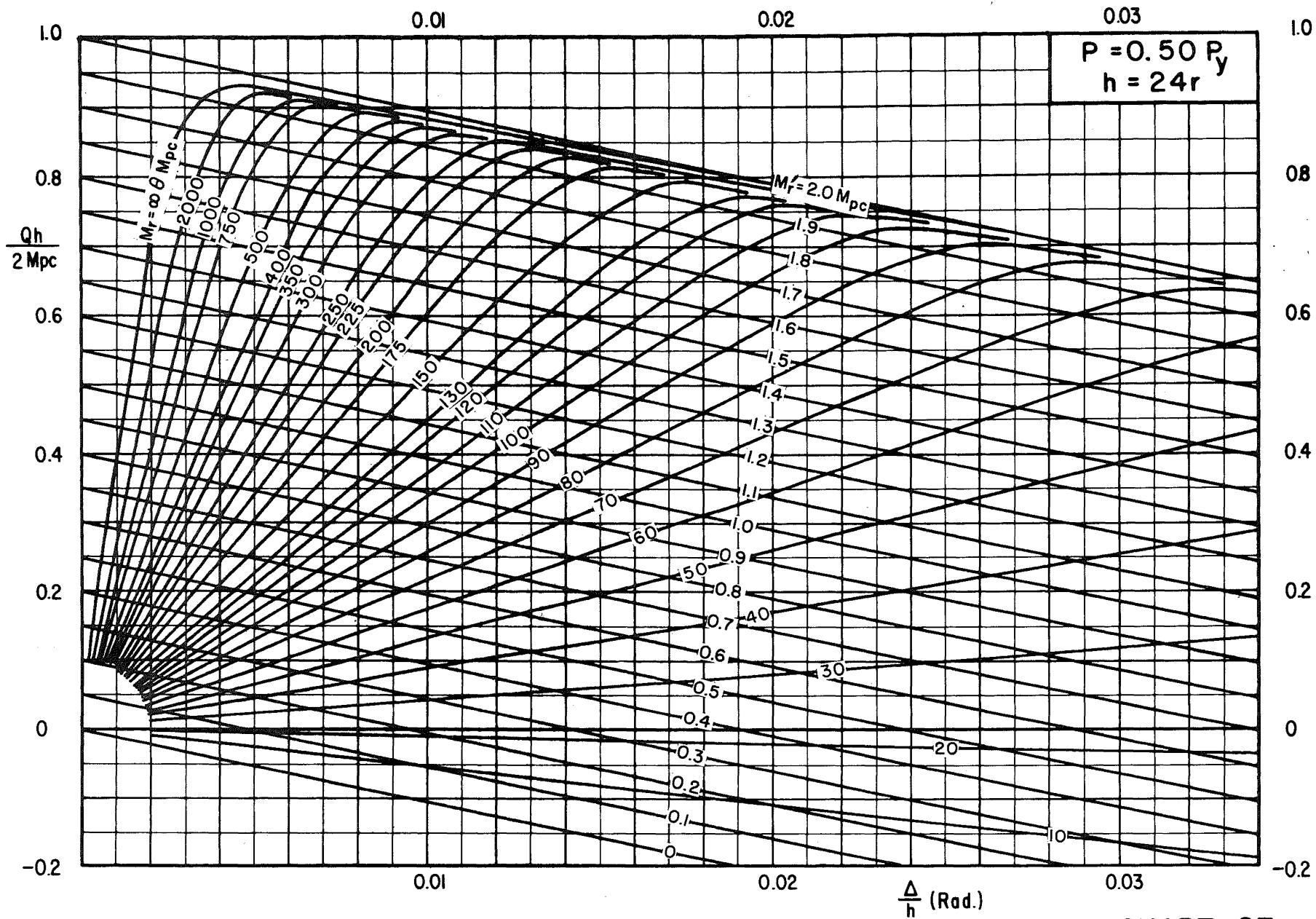


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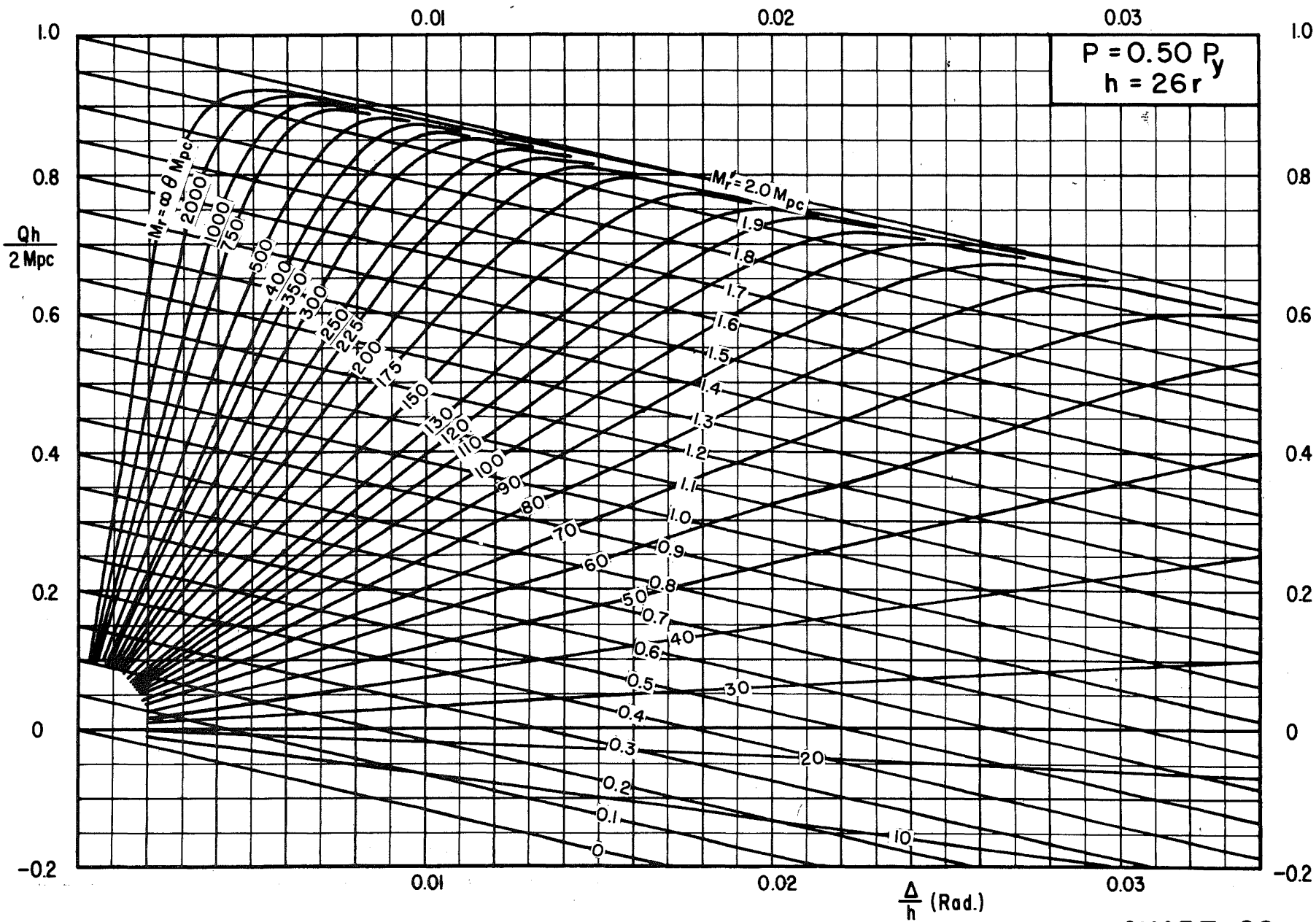


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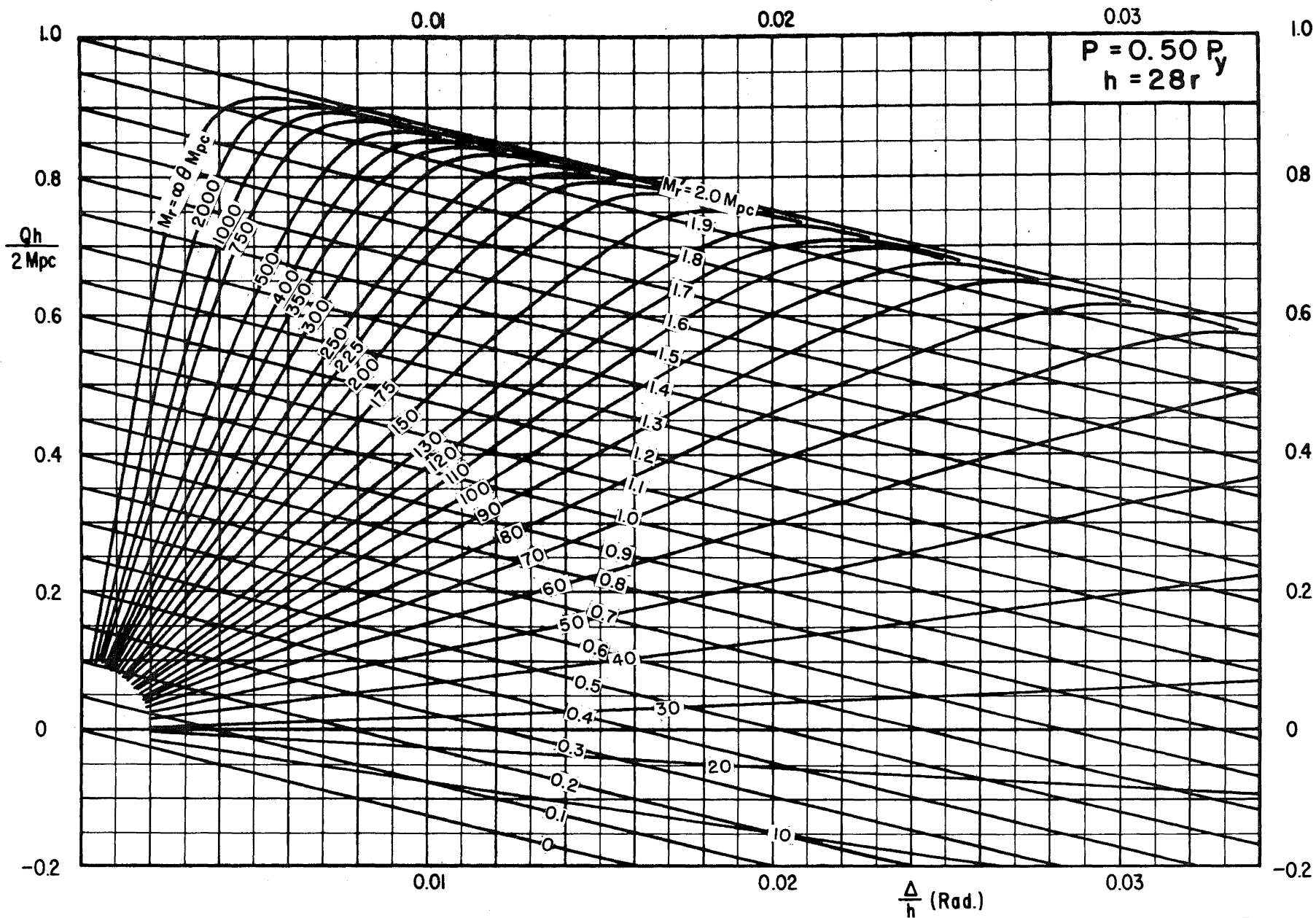


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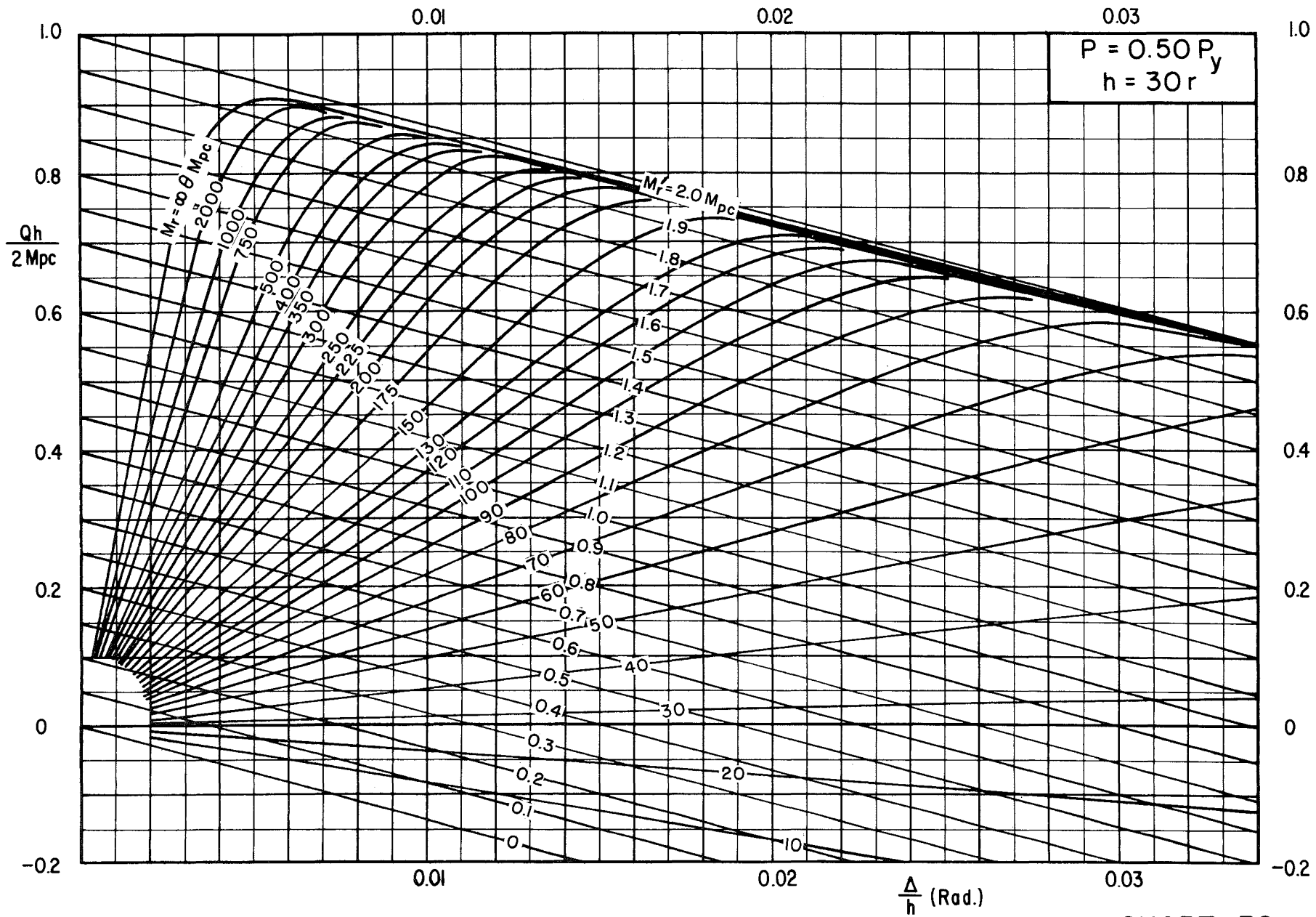
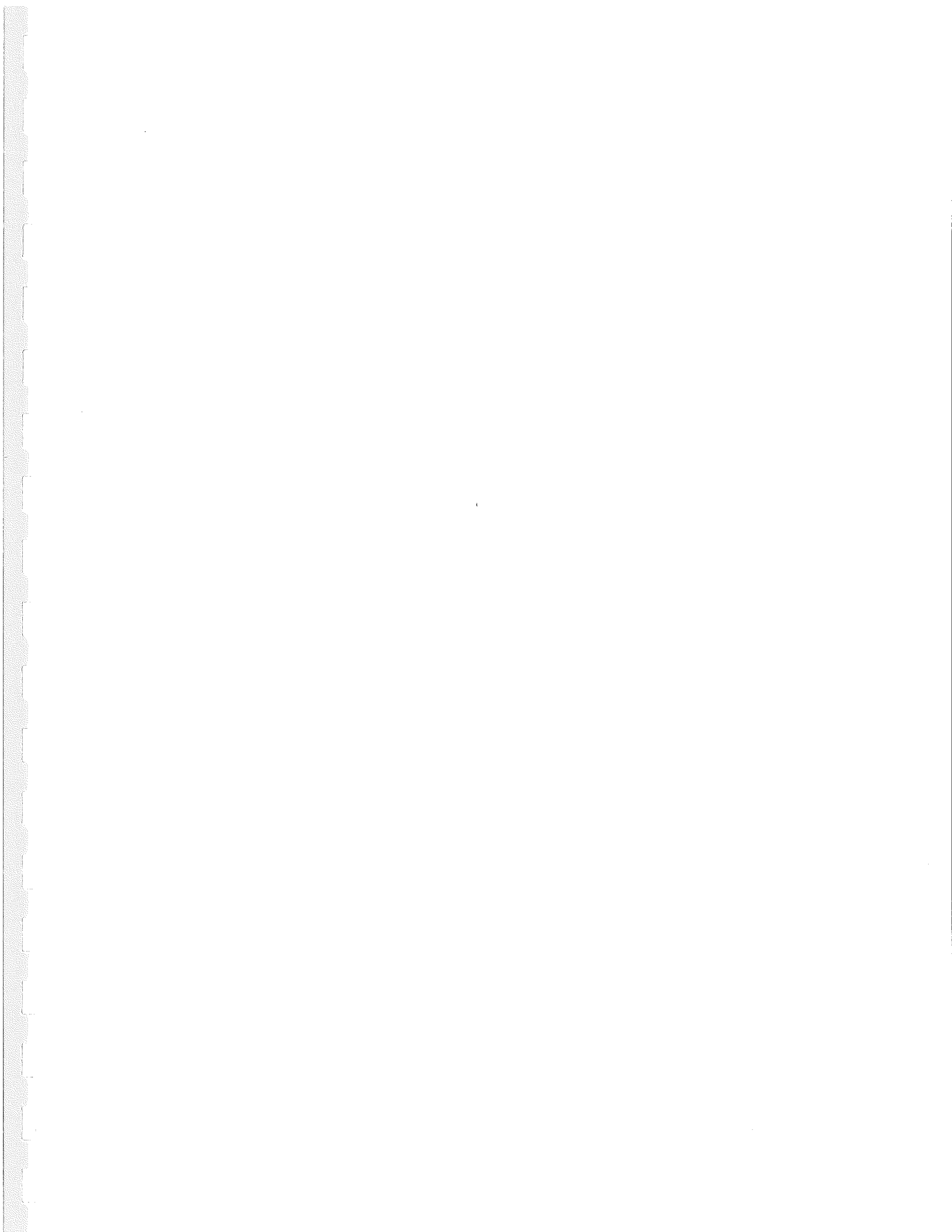


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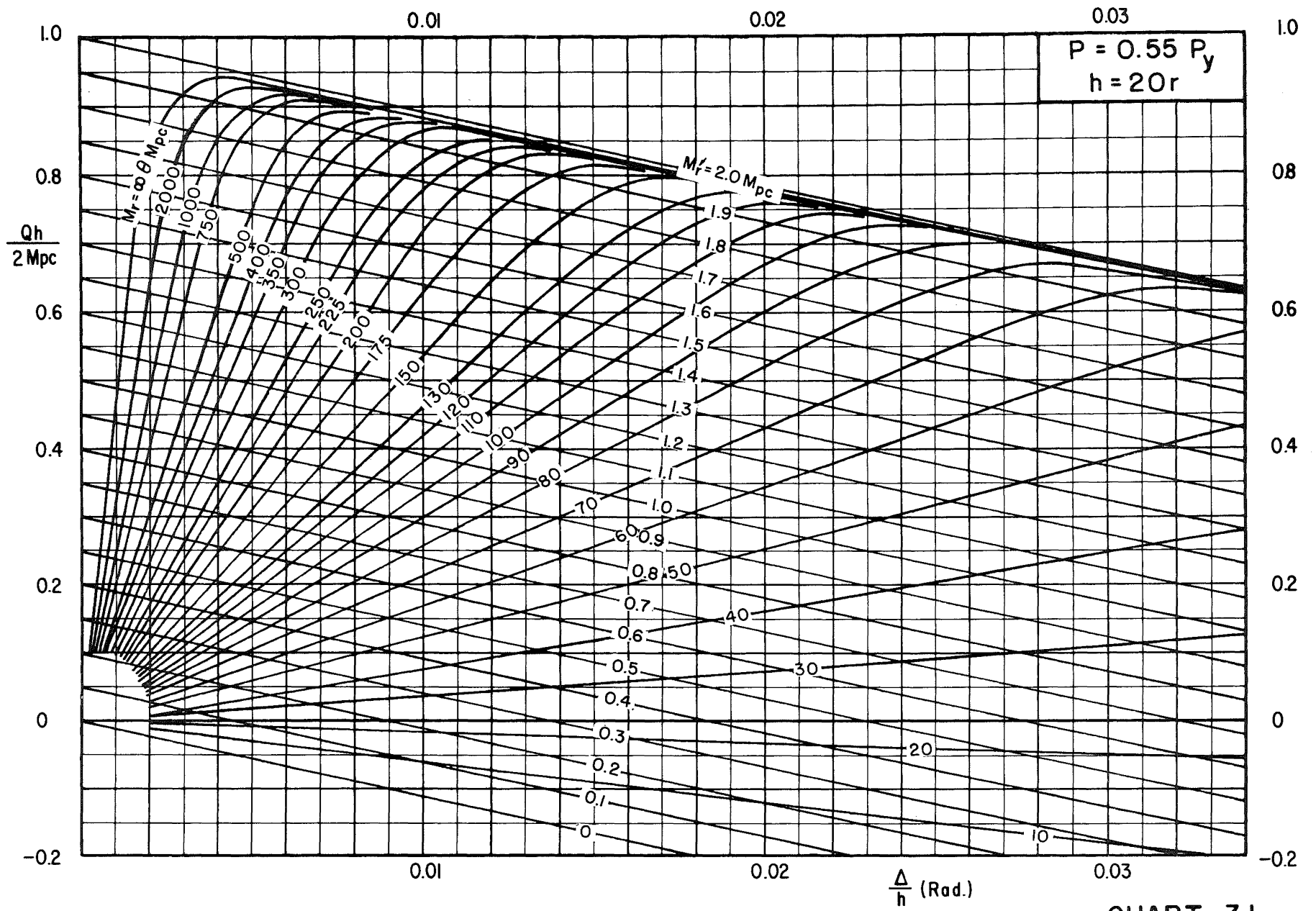












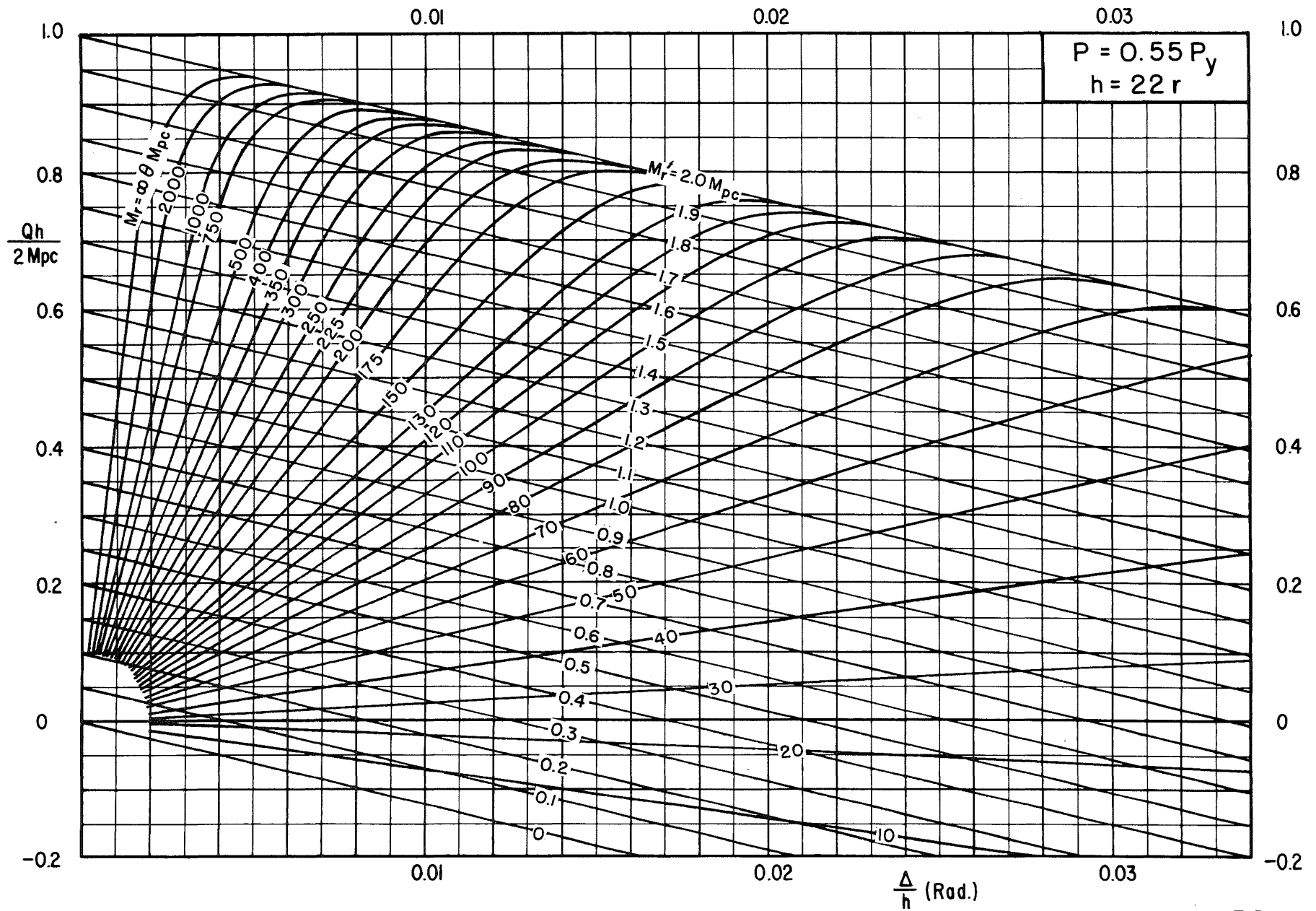


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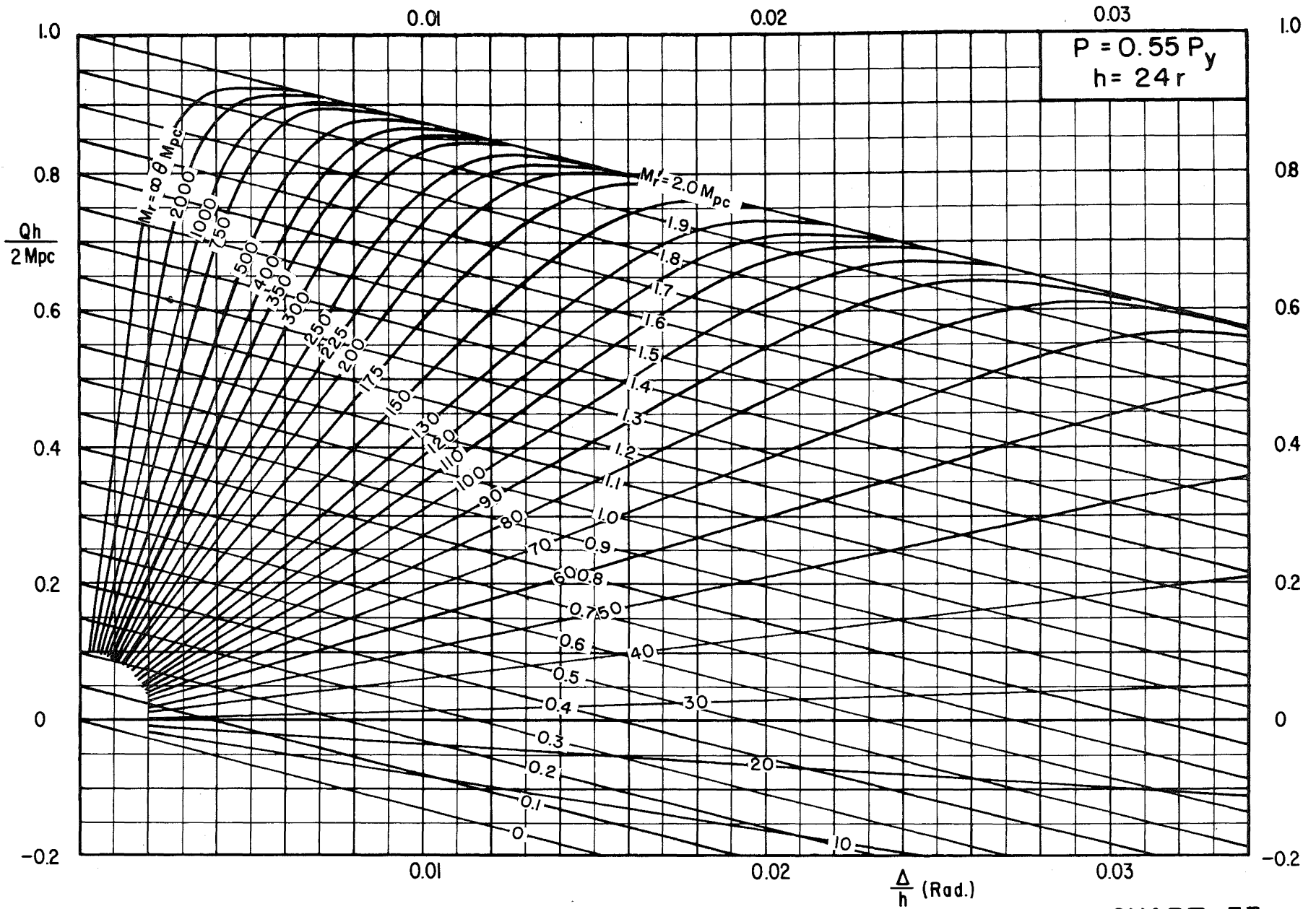


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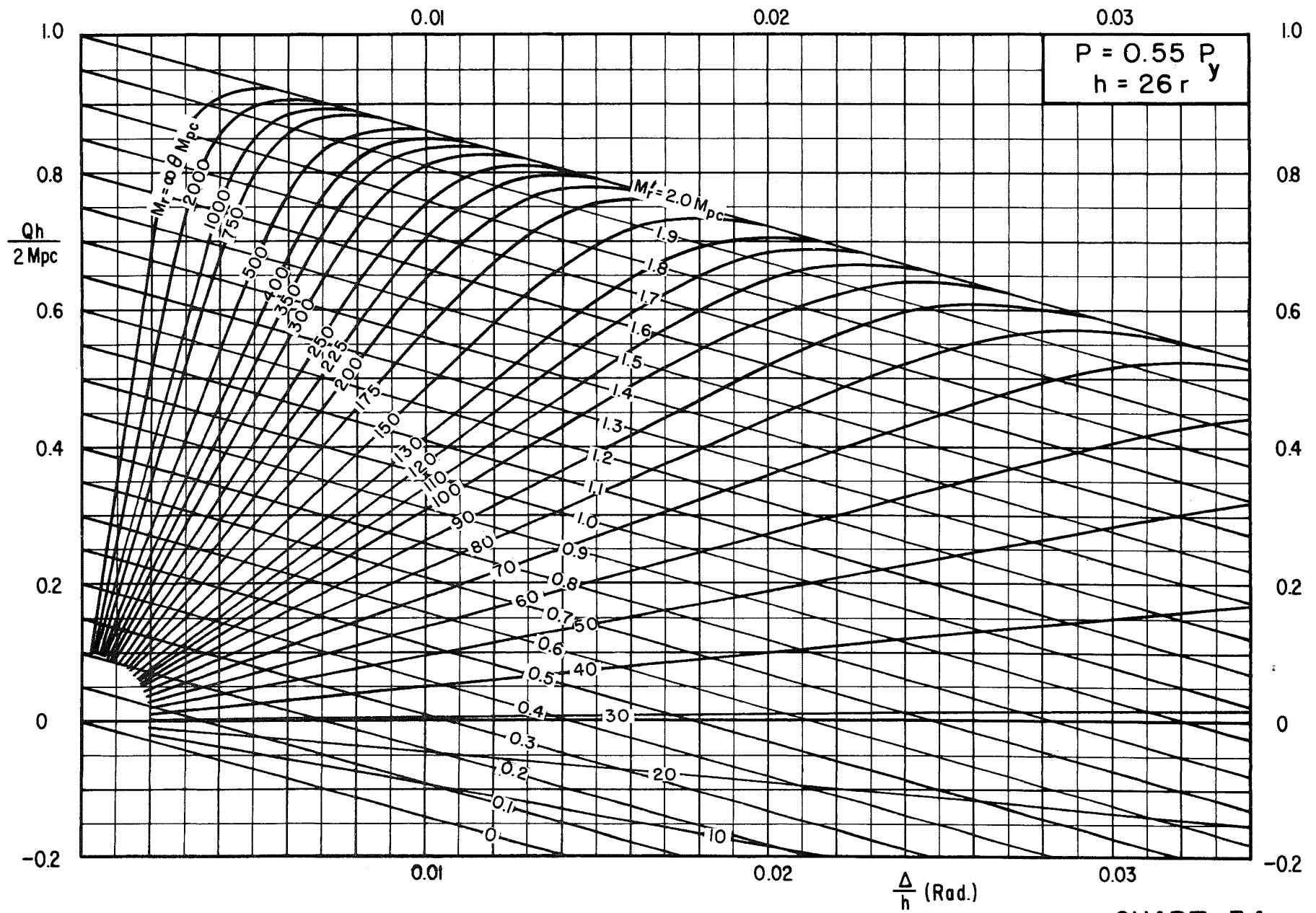


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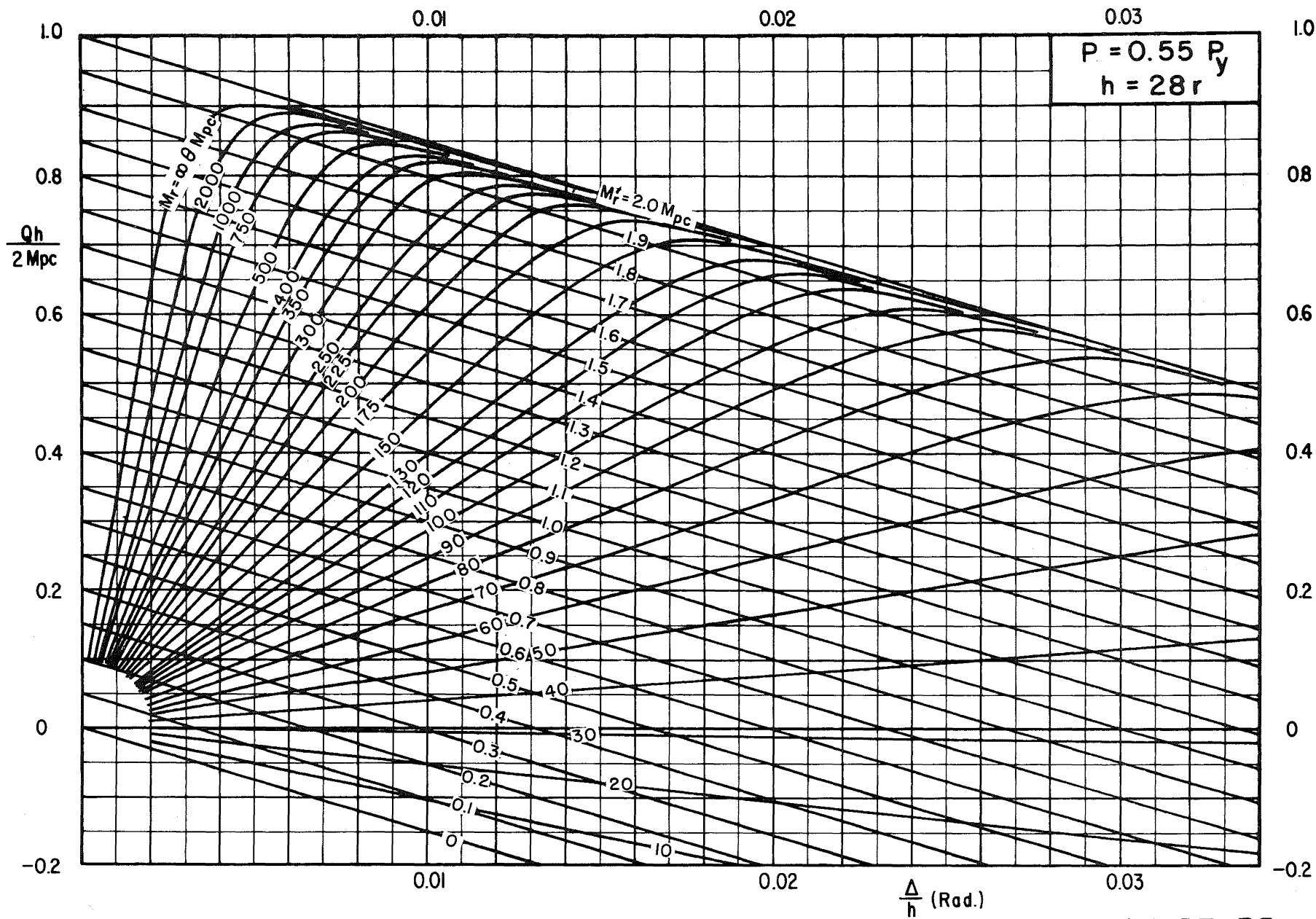


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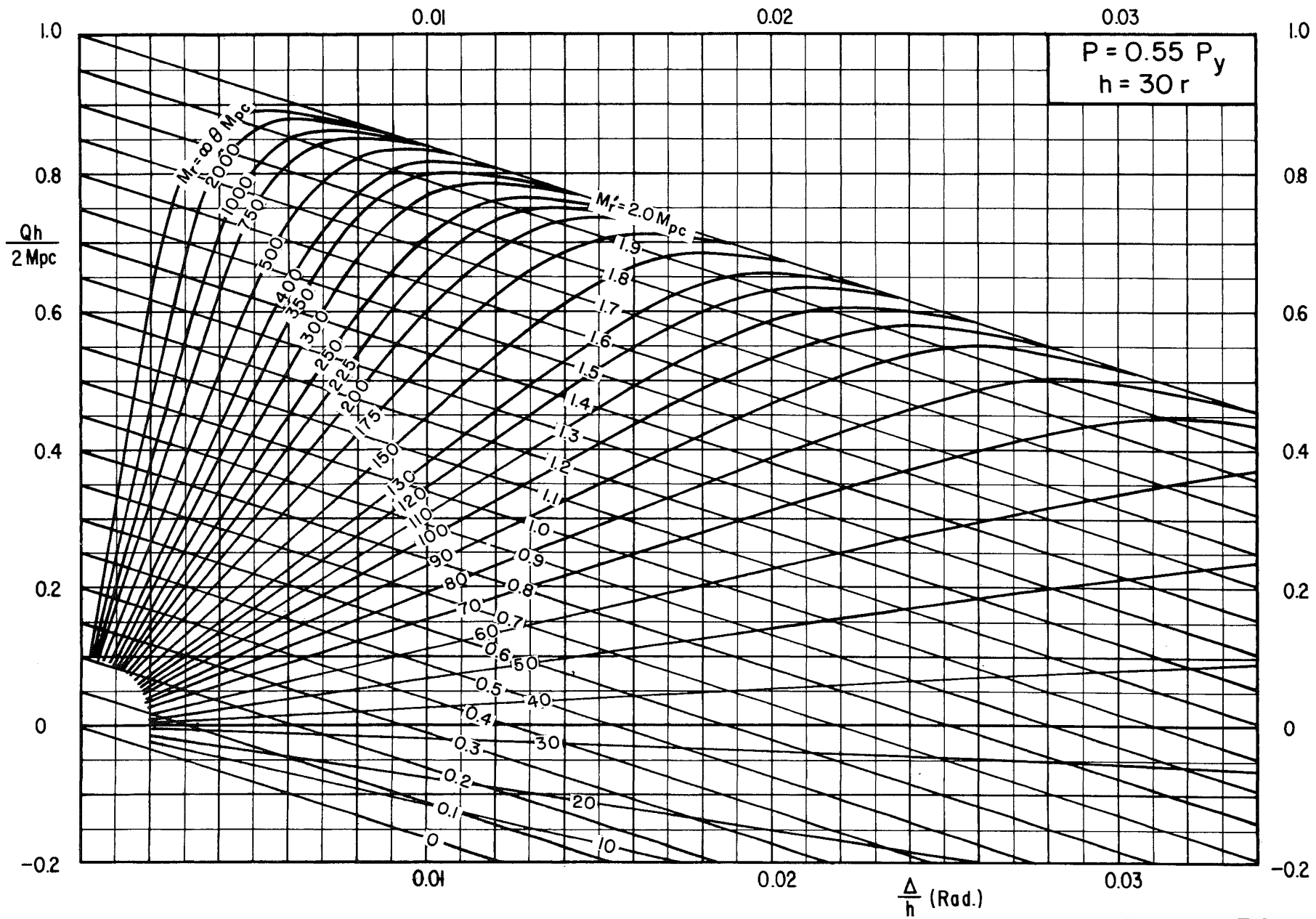
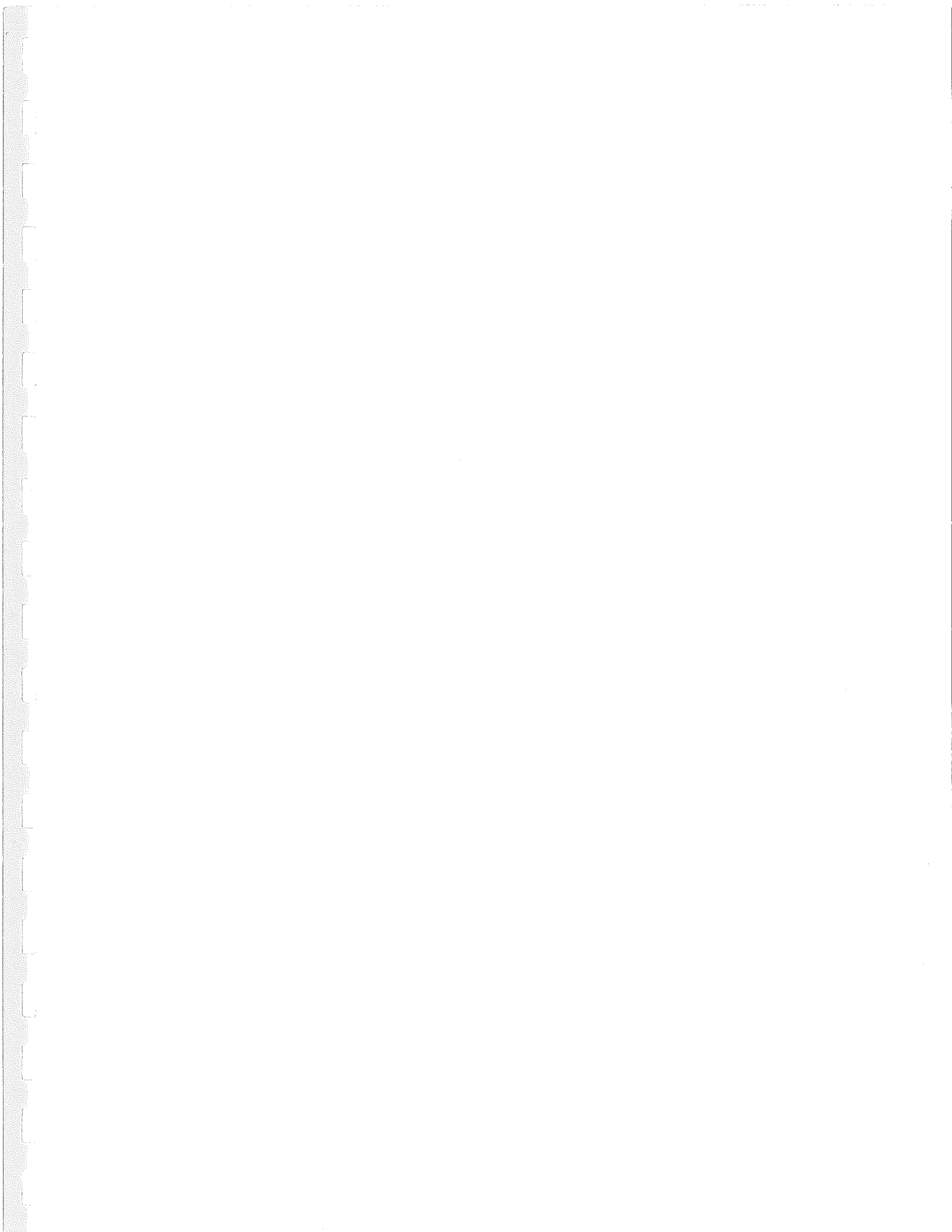
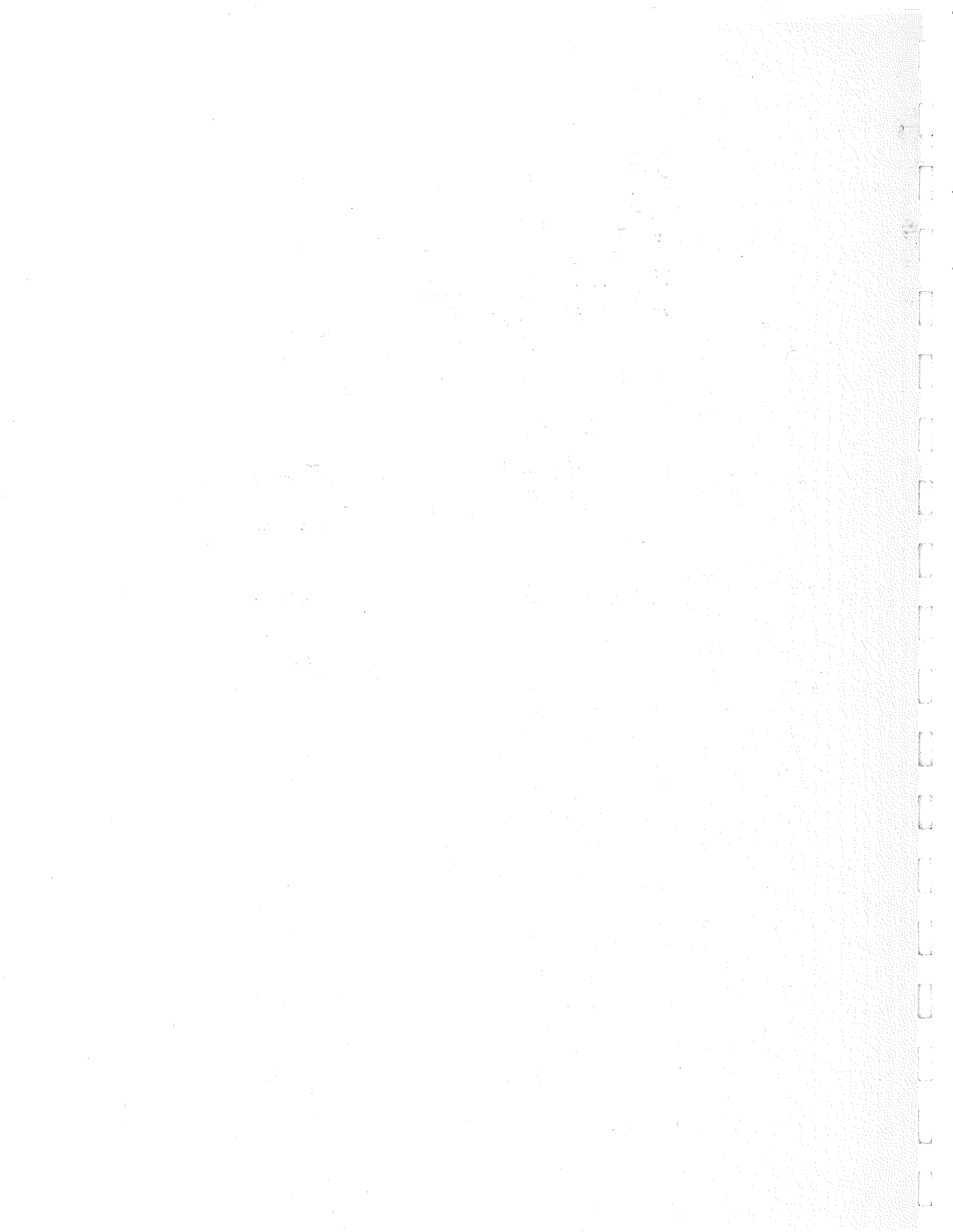


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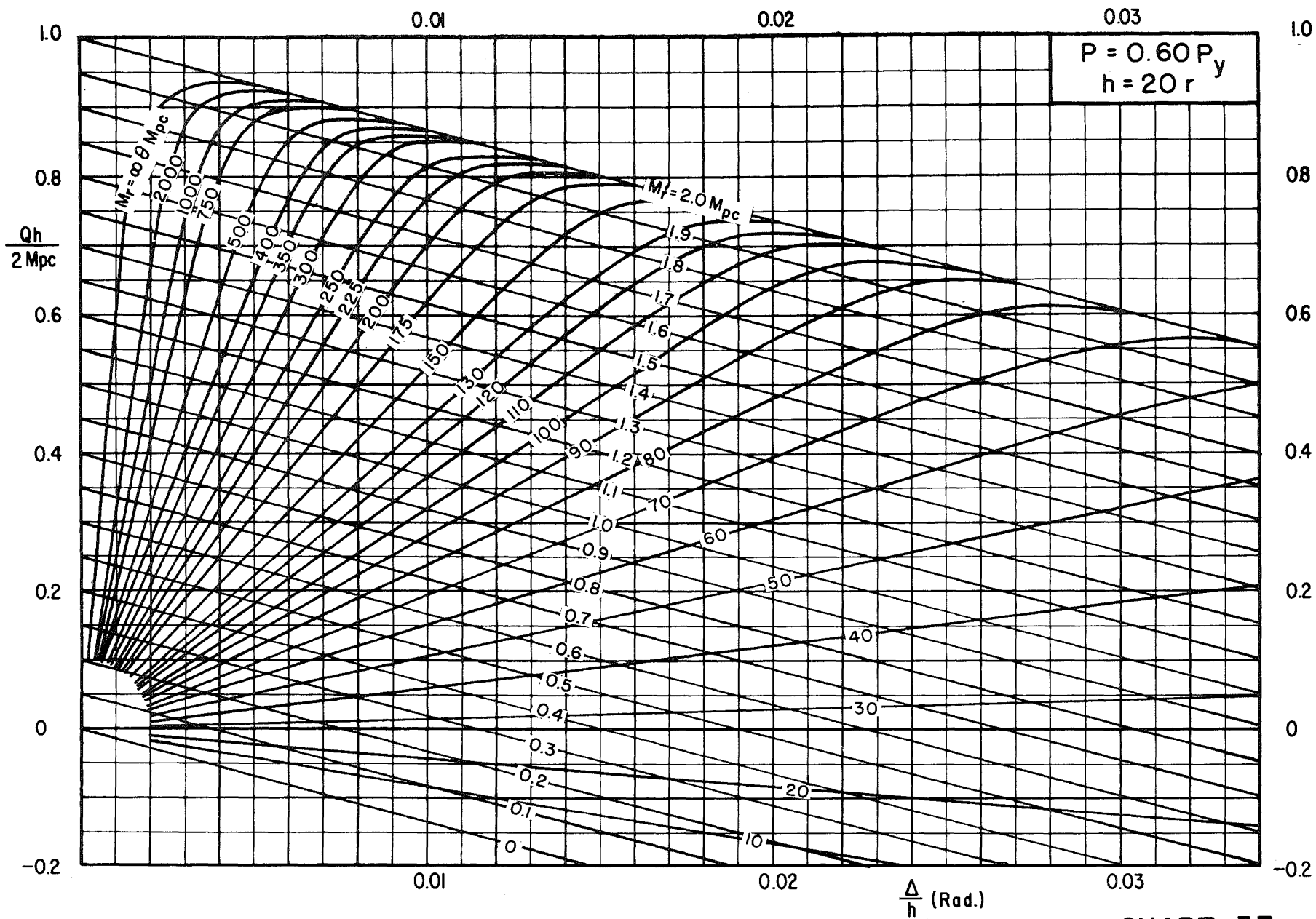


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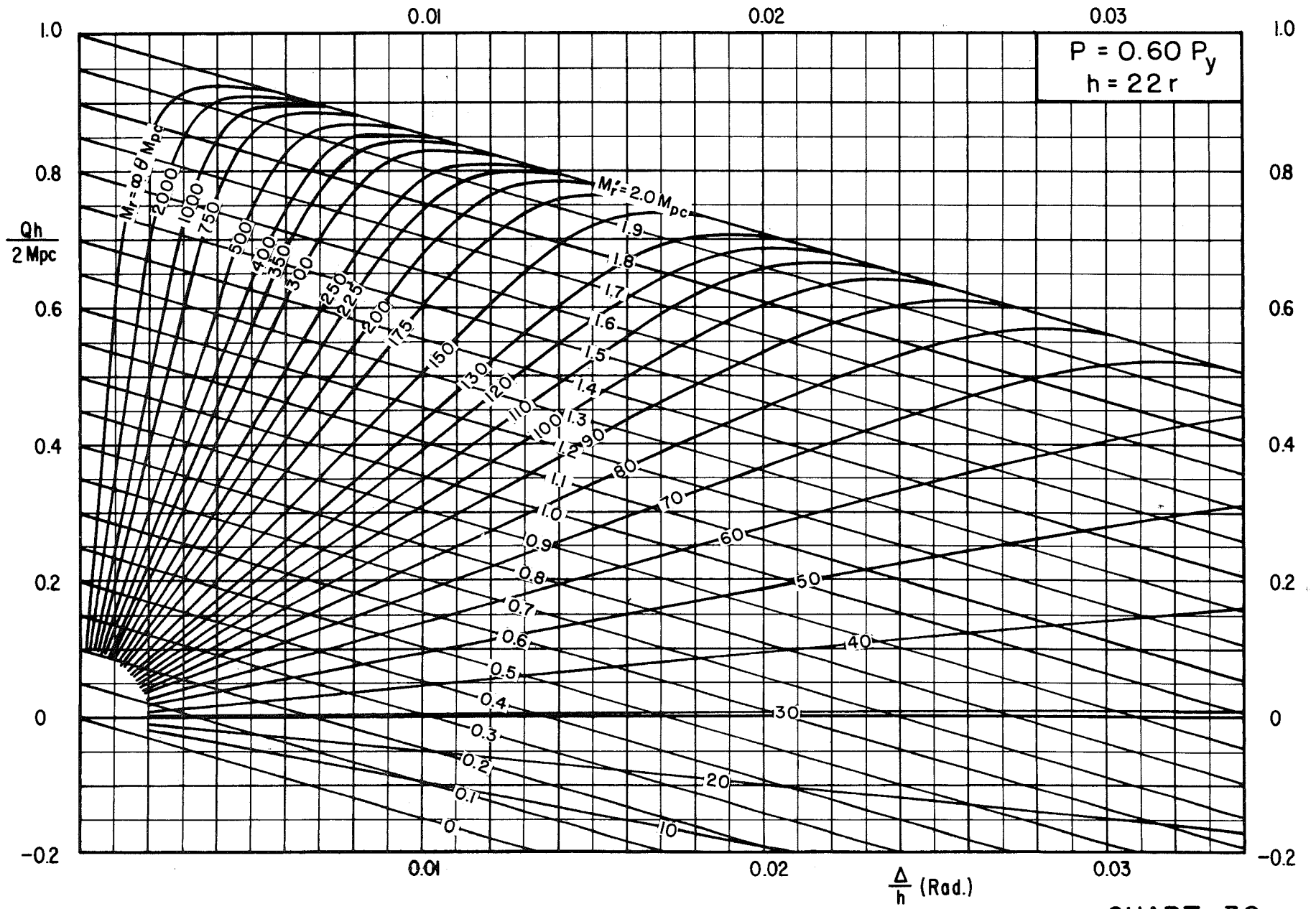


CHART 38

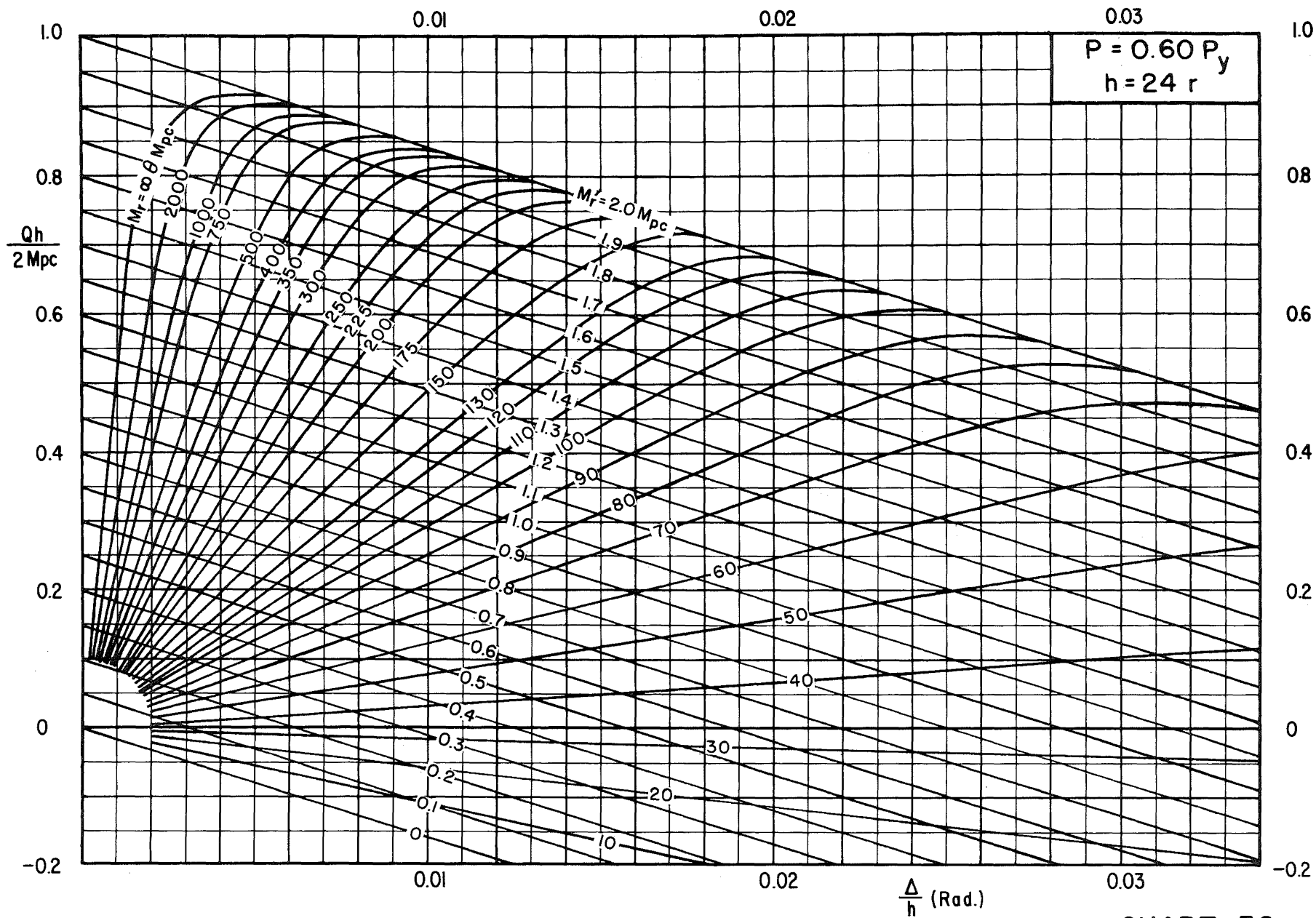


CHART 39

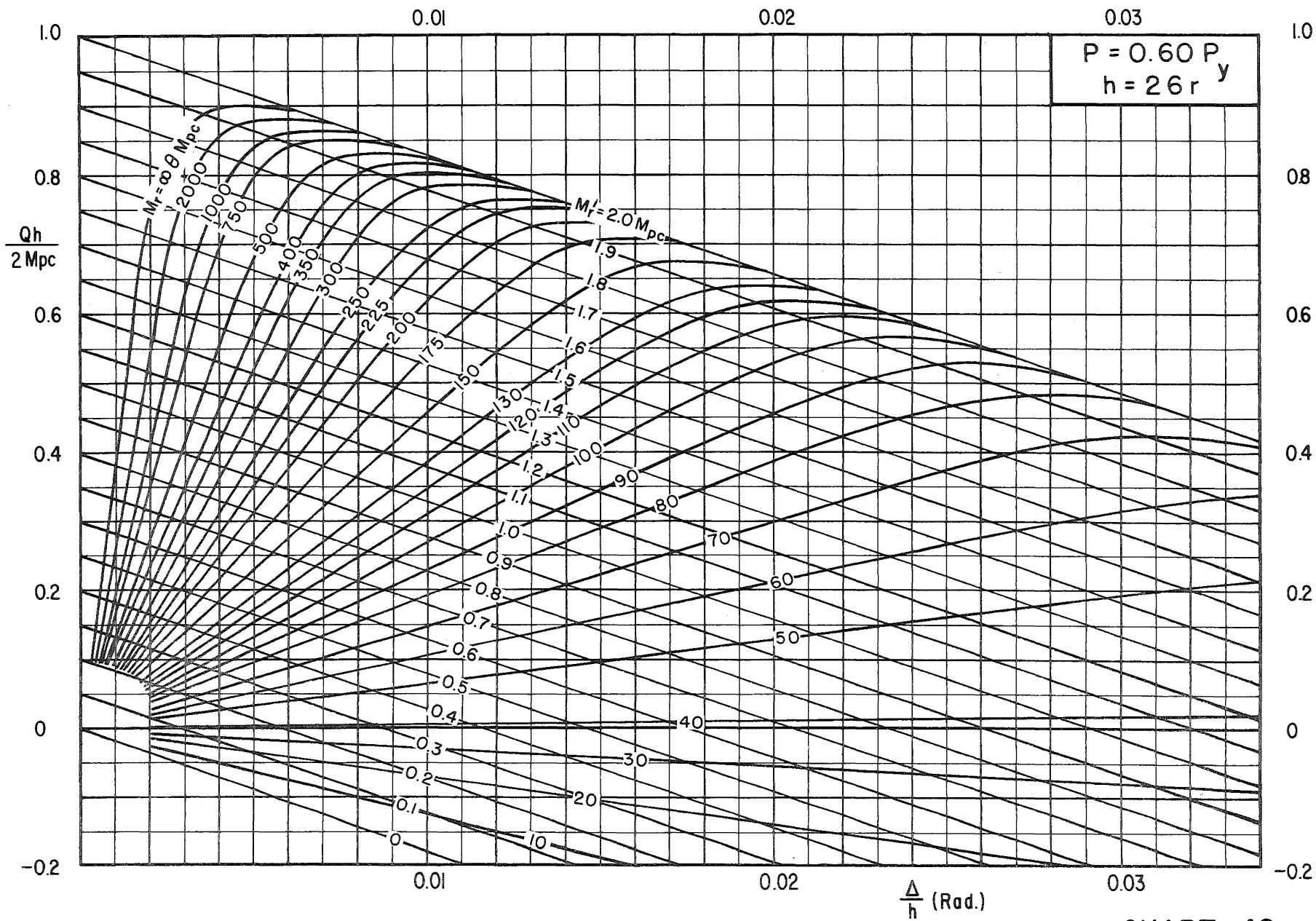


CHART 40

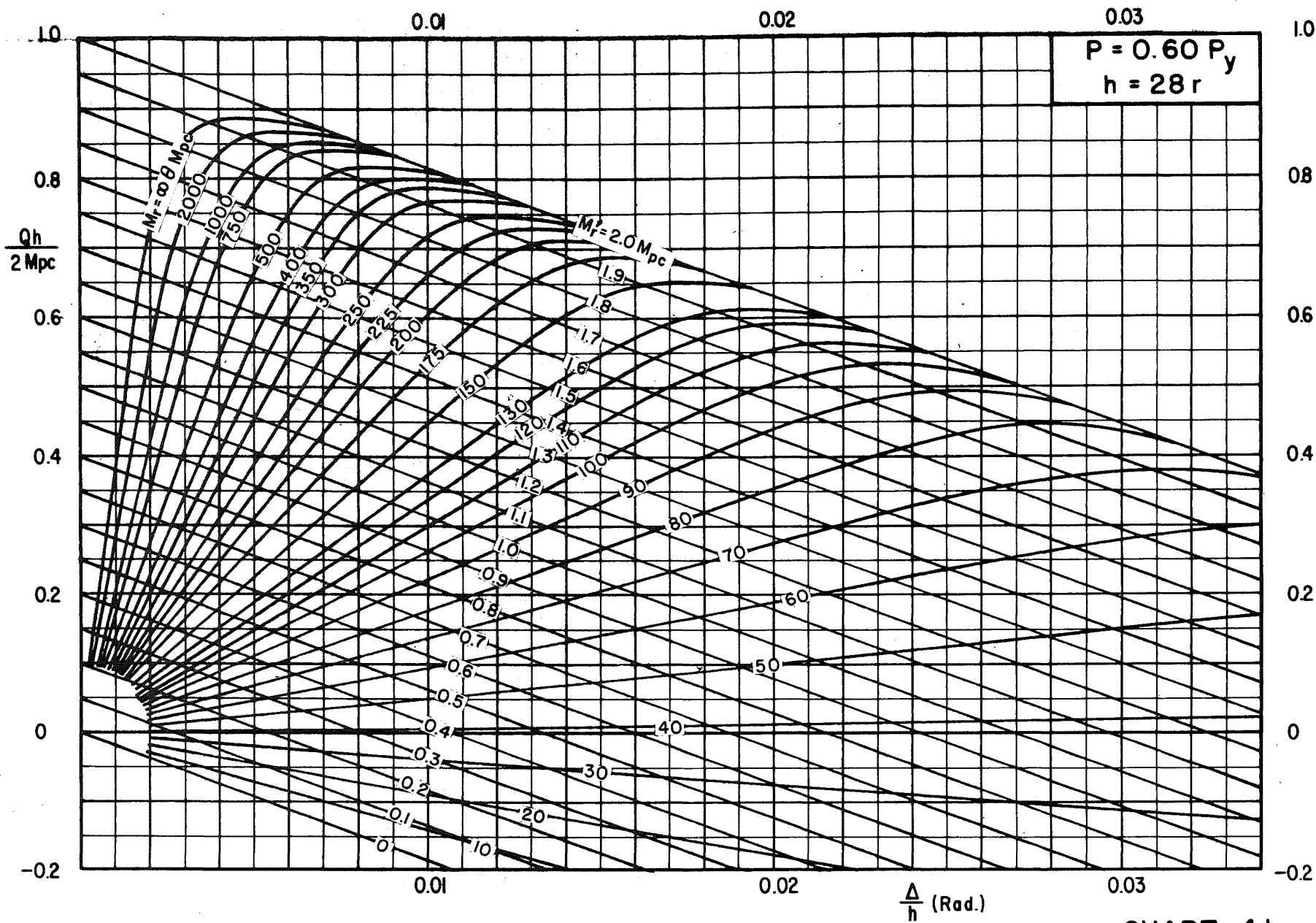


CHART 41

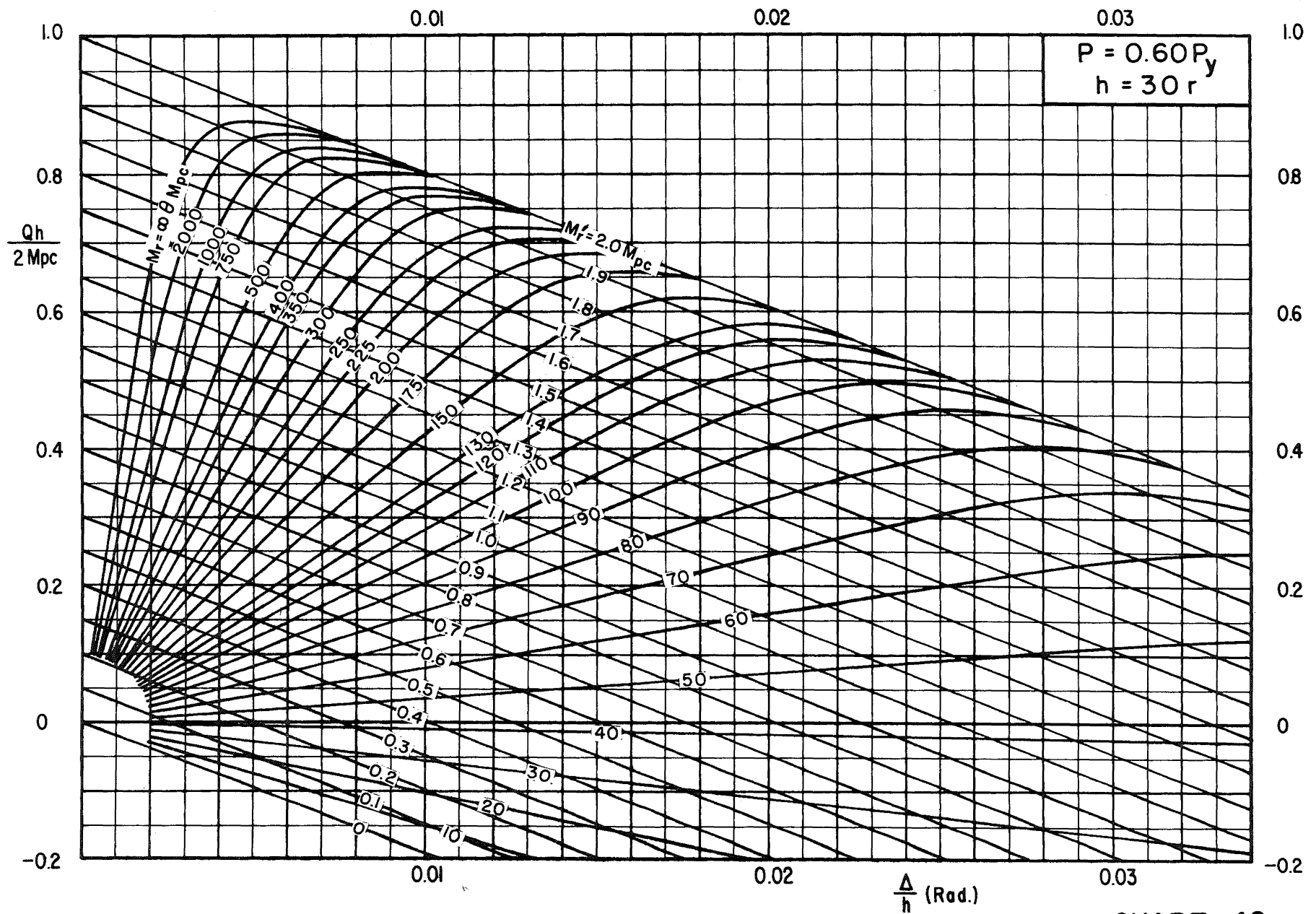
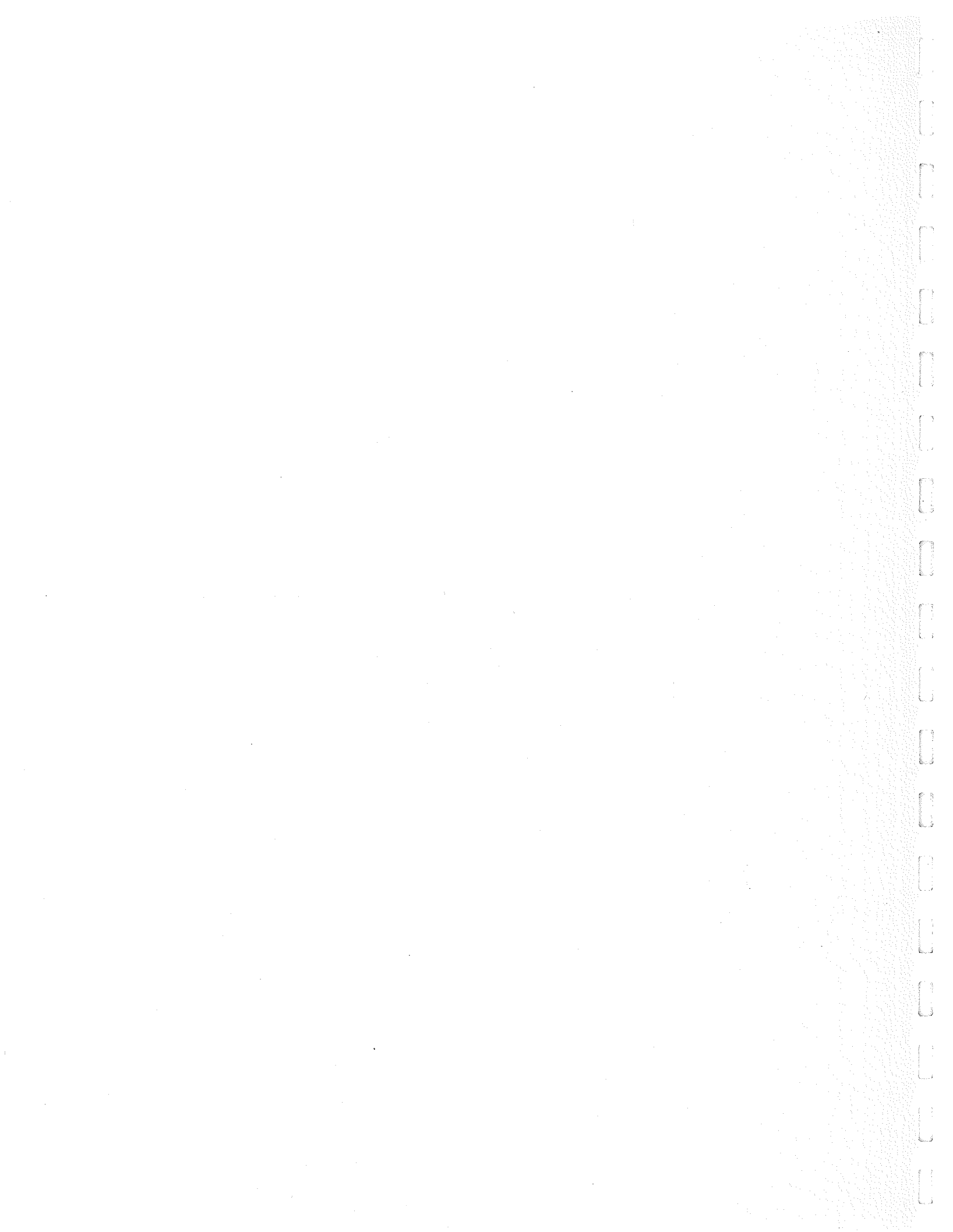


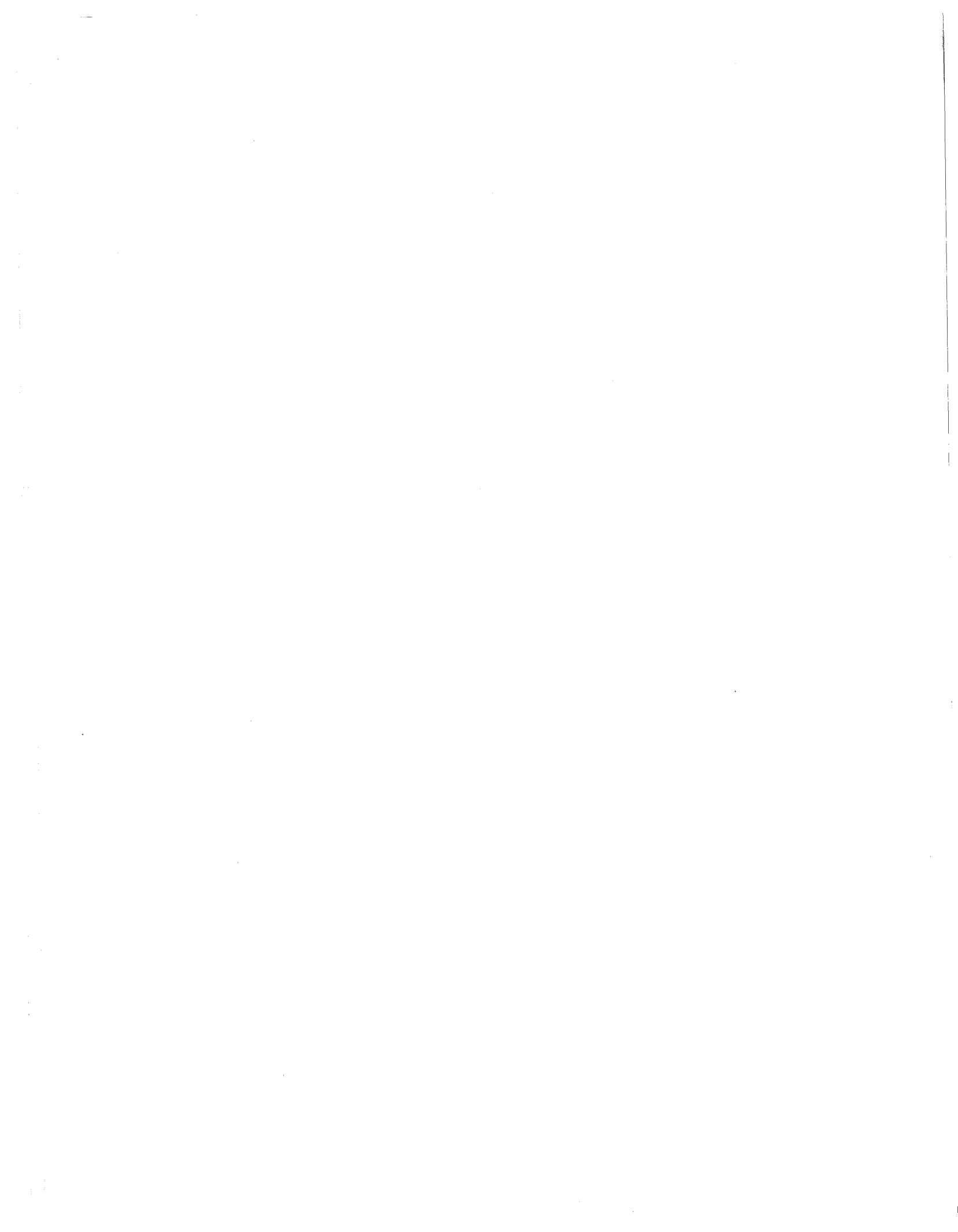
CHART 42











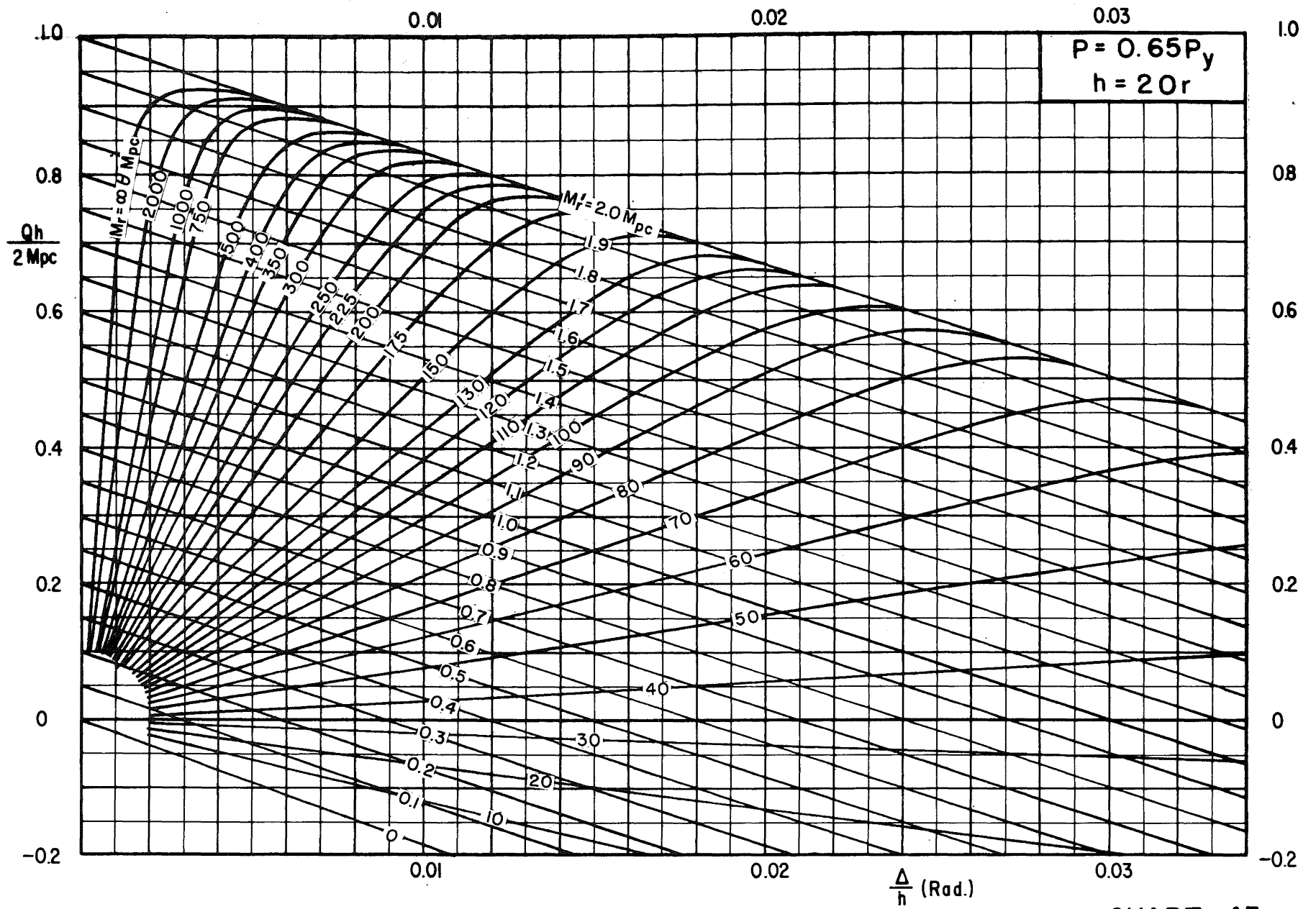


CHART 43

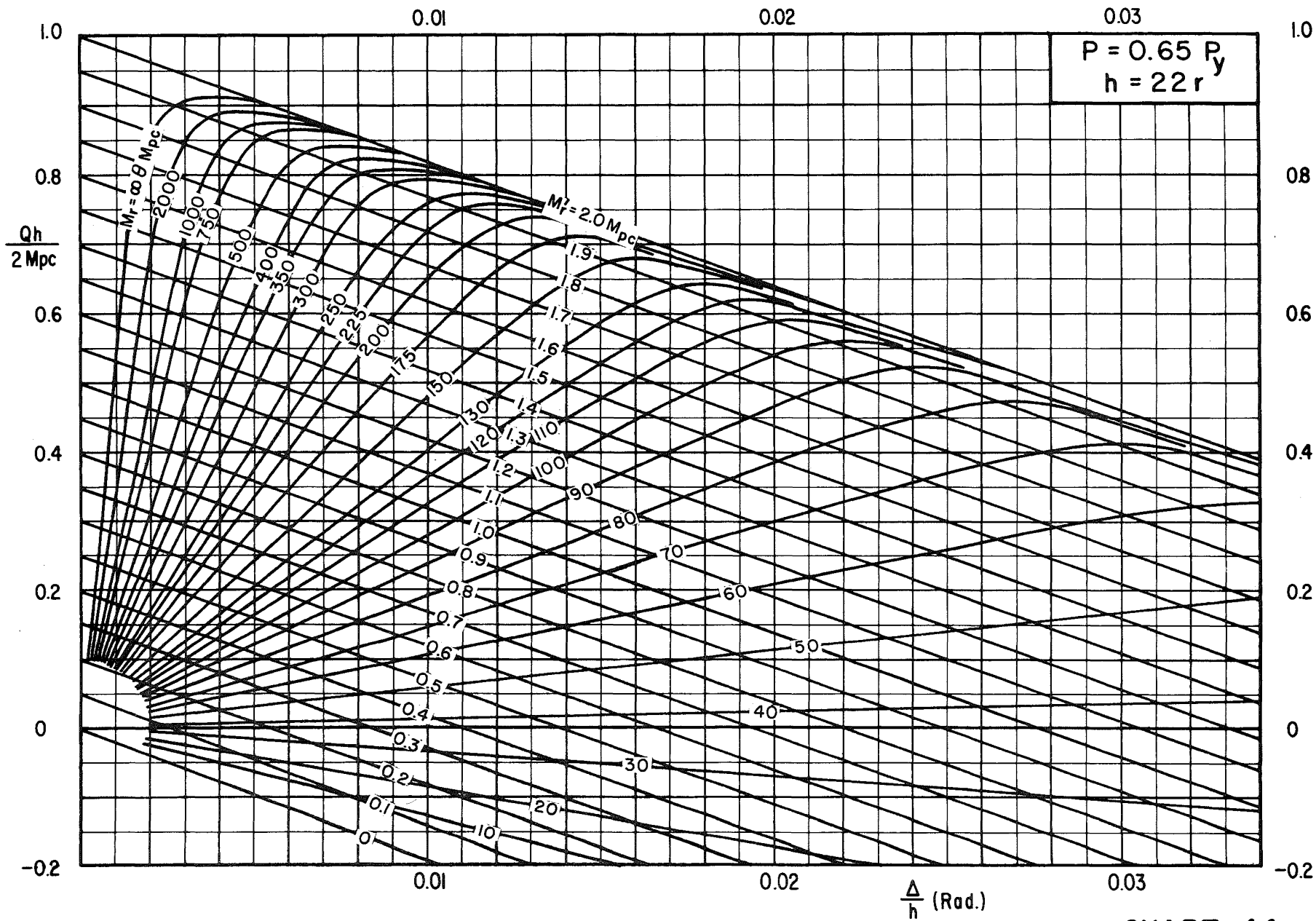


CHART 44

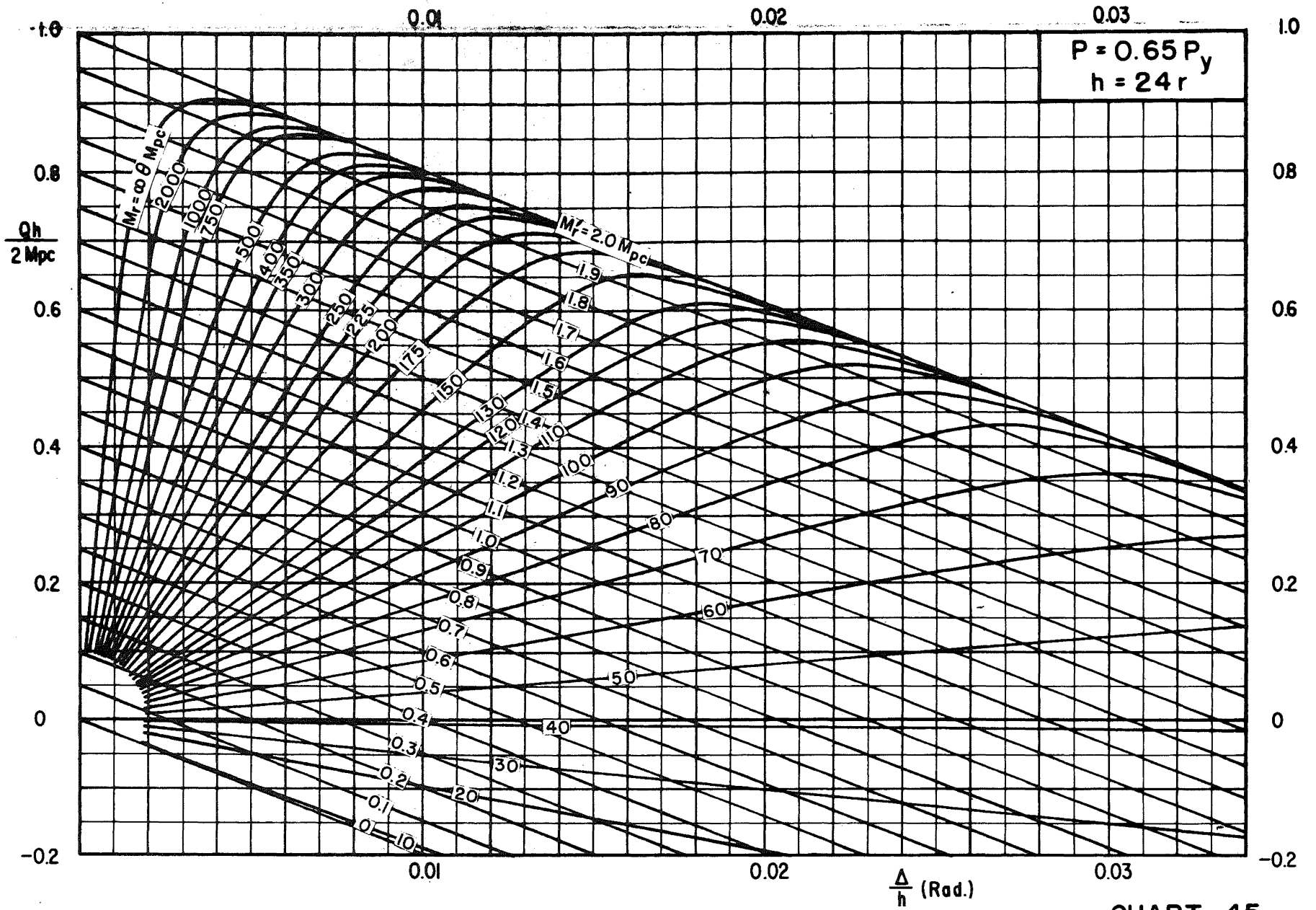


CHART 45

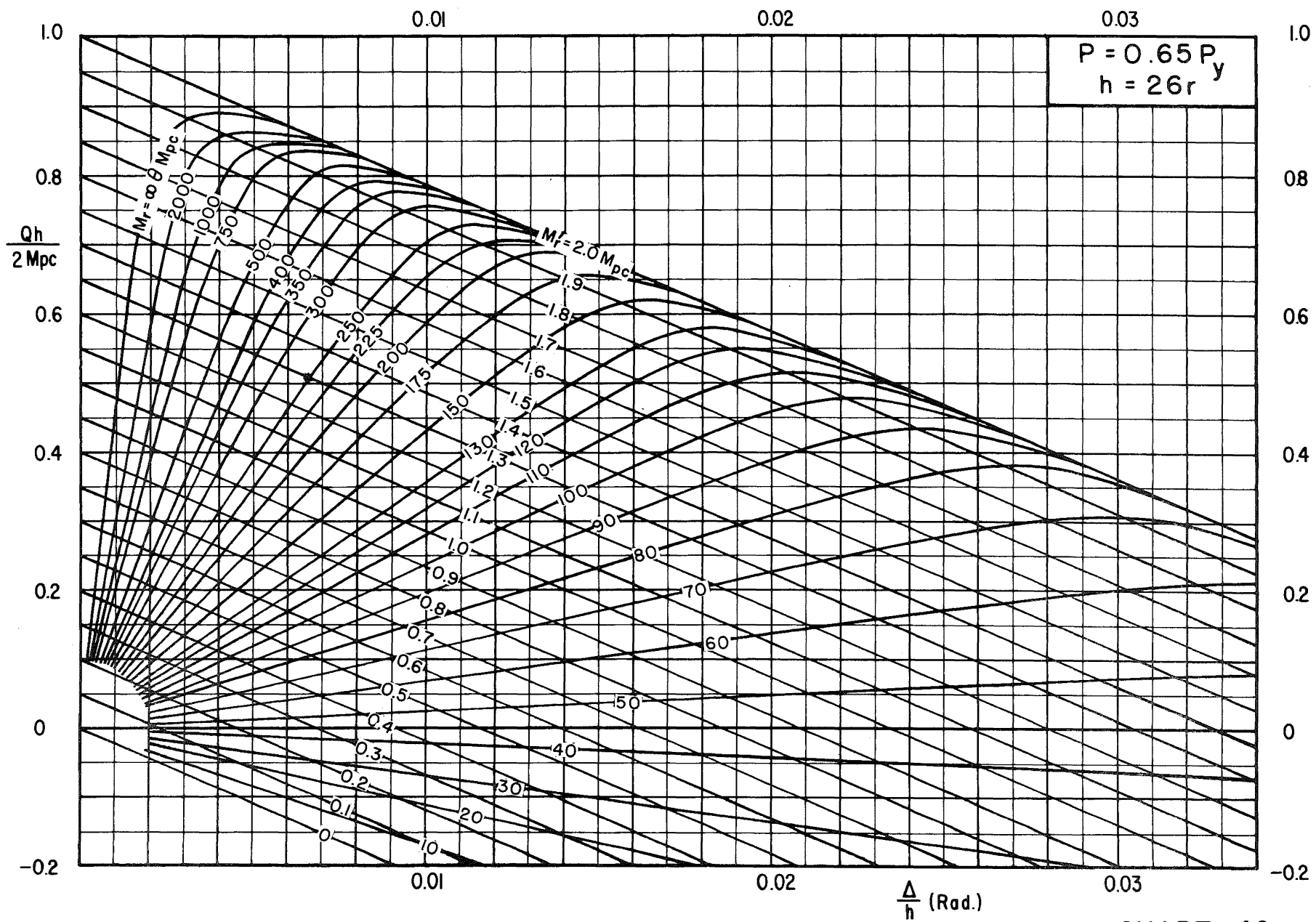


CHART 46



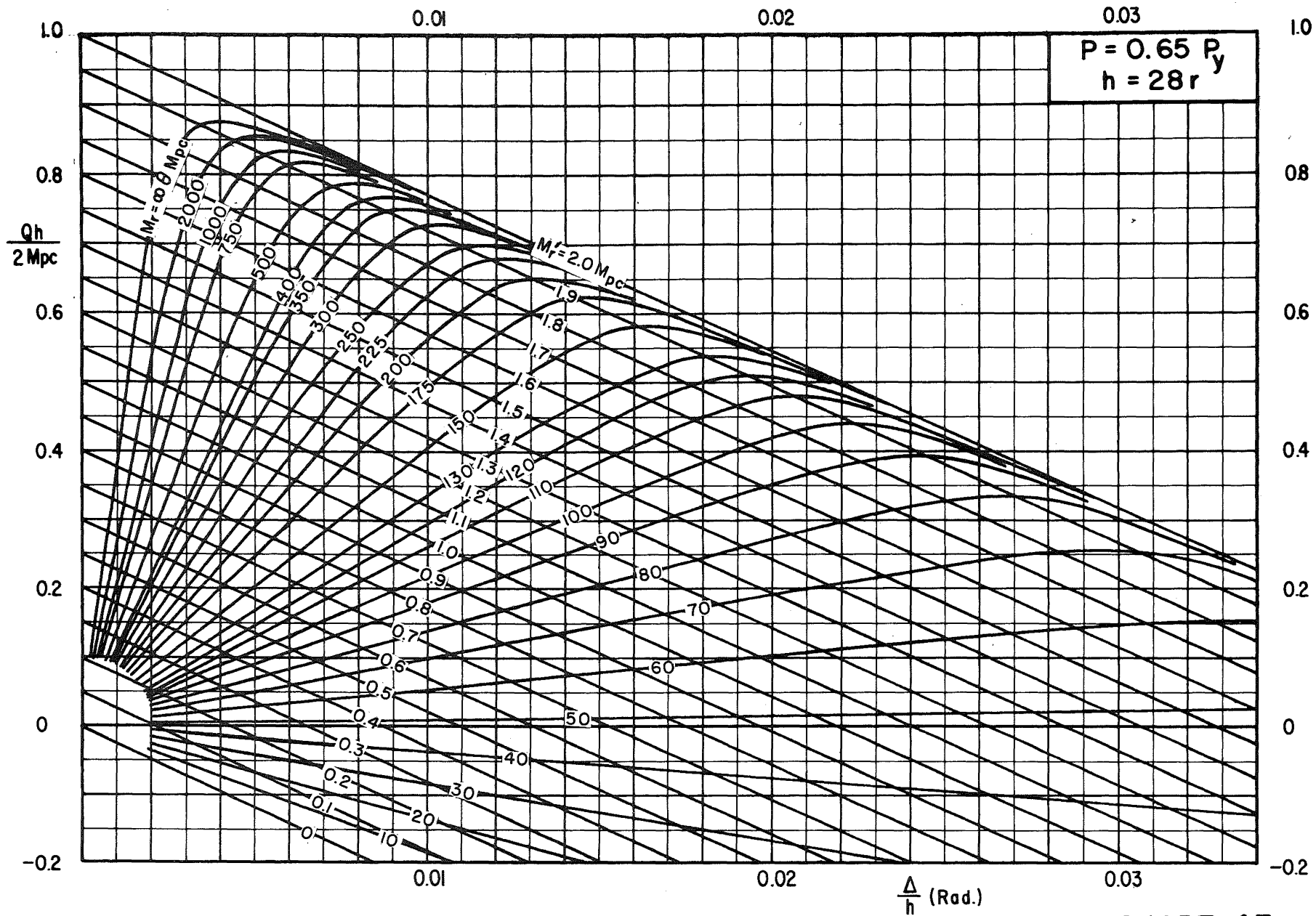


CHART 47

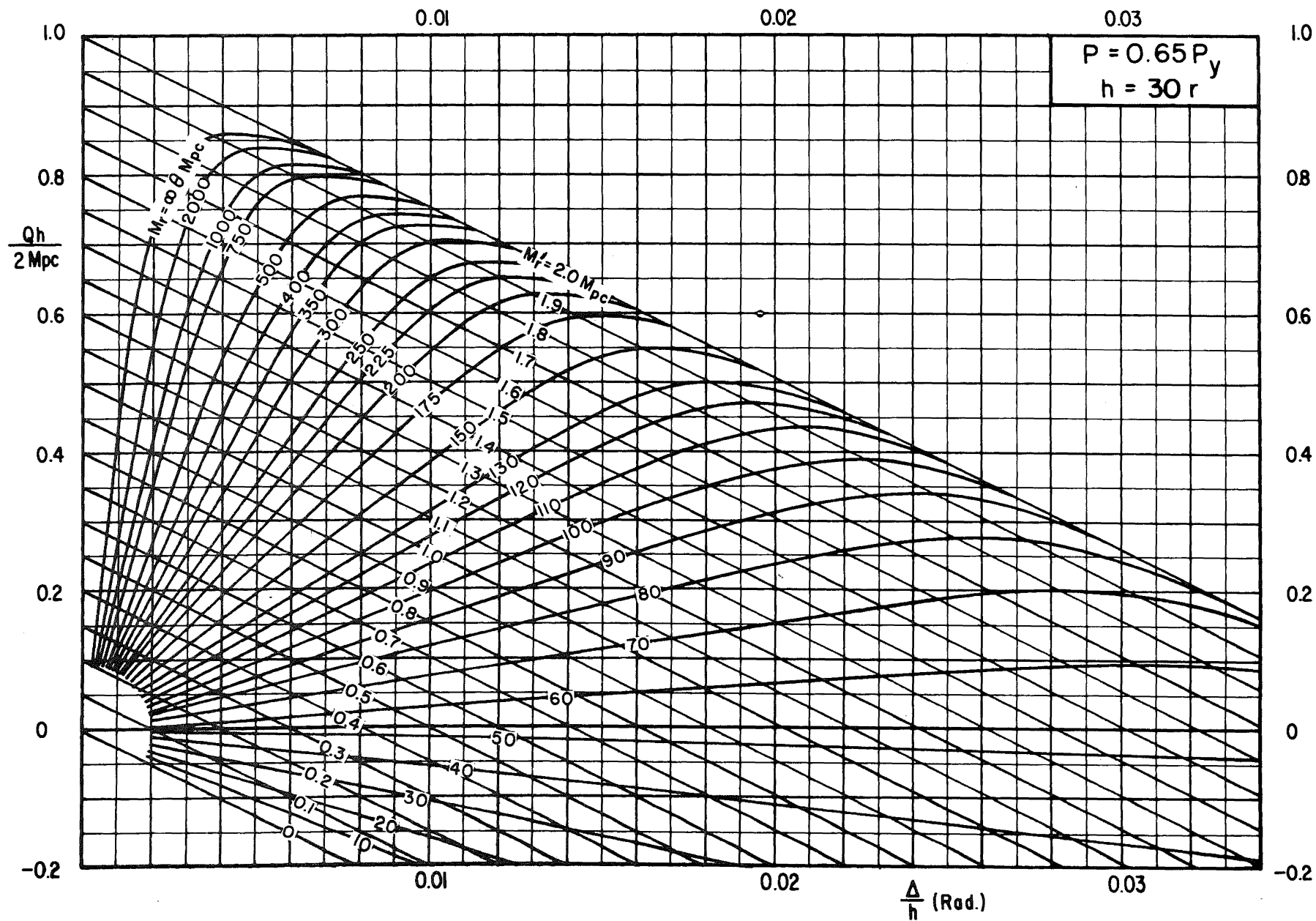
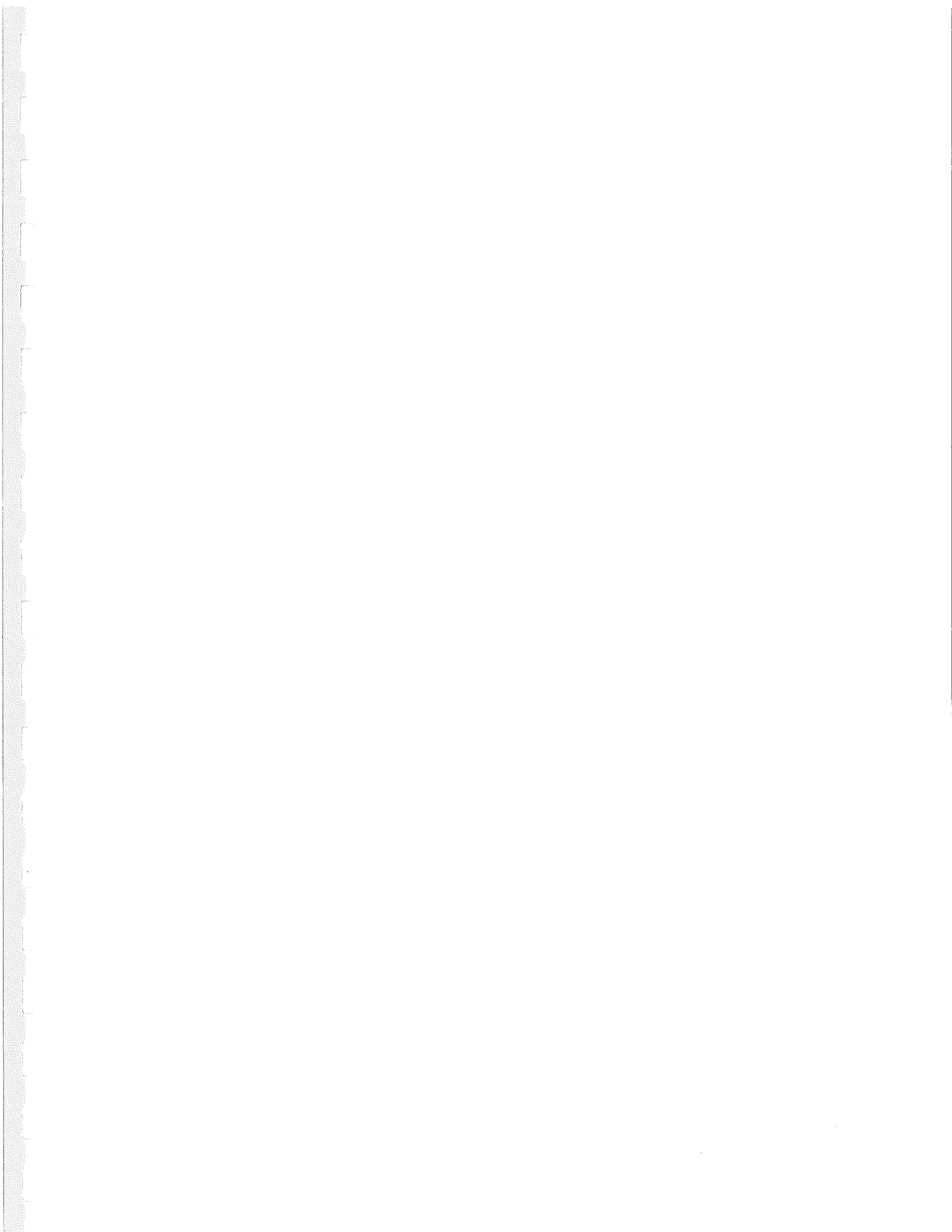
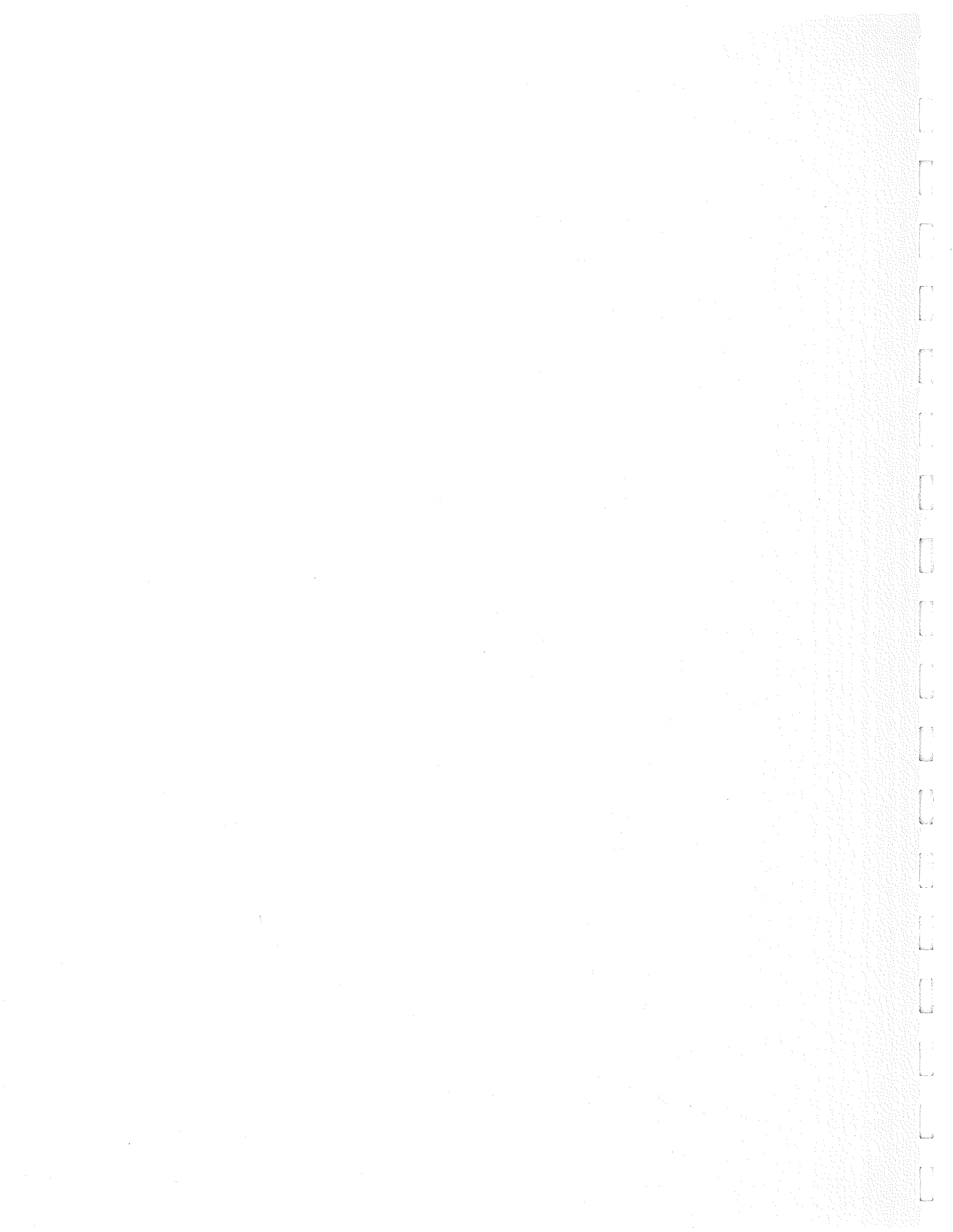


CHART 48









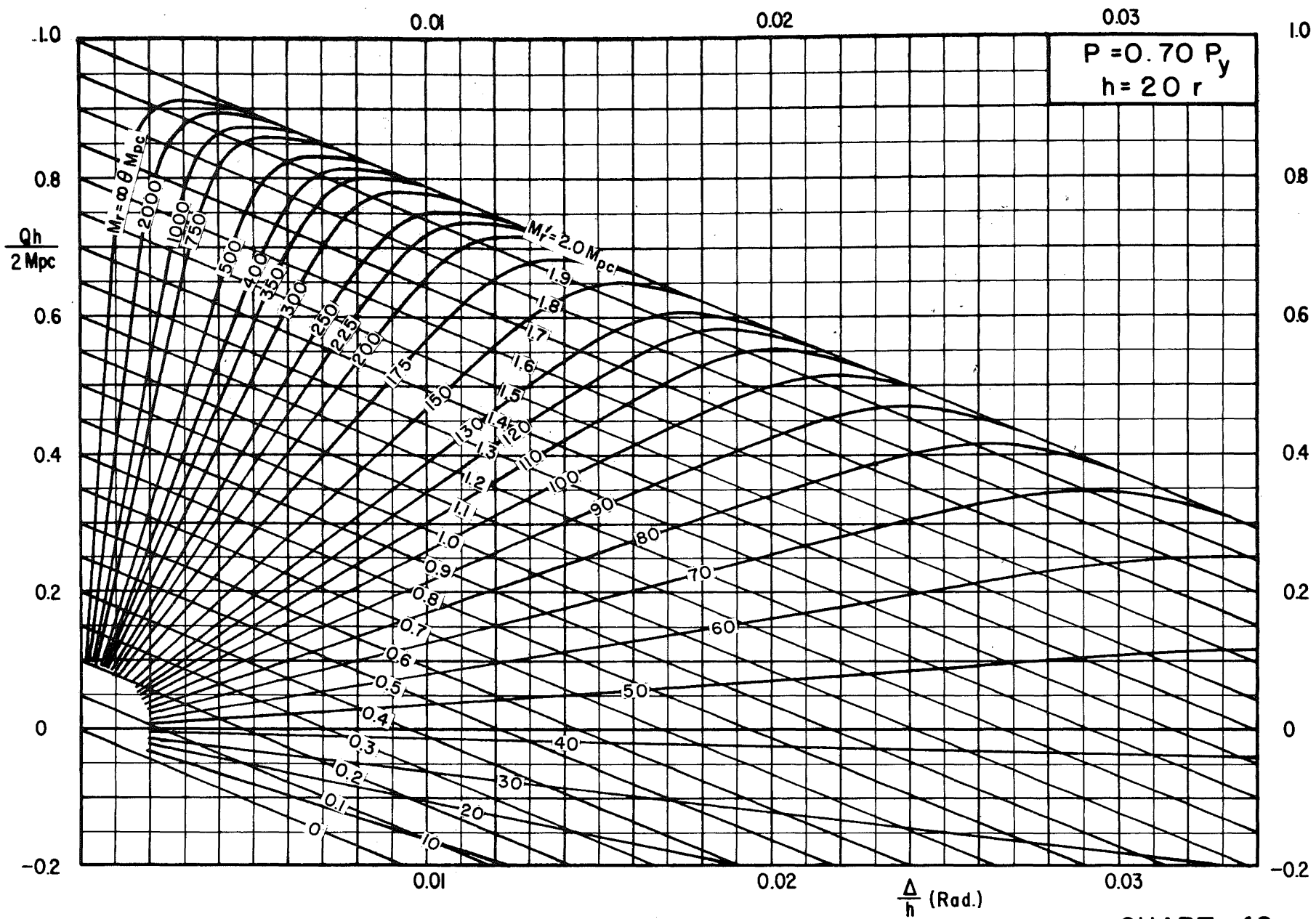


CHART 49

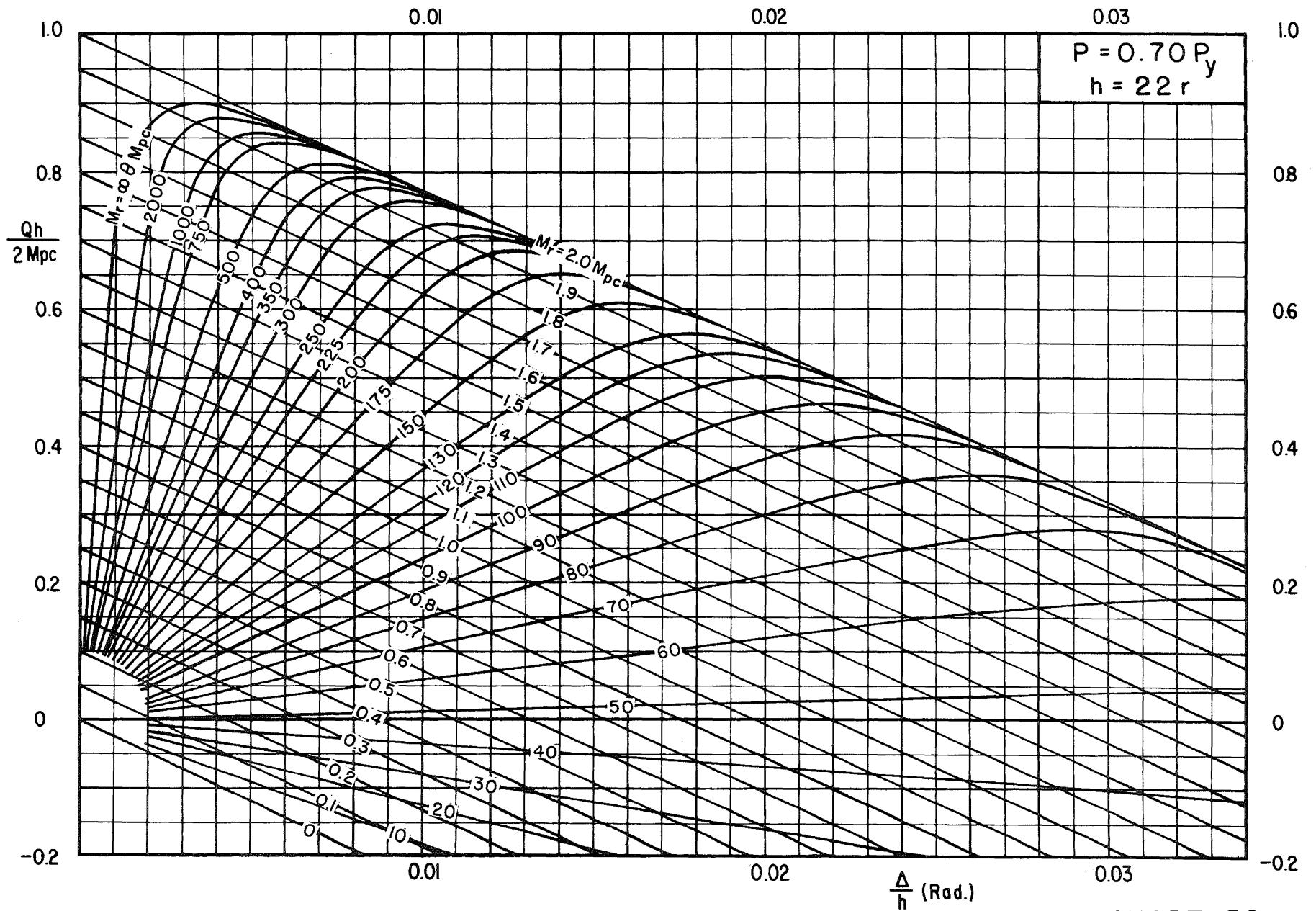


CHART 50



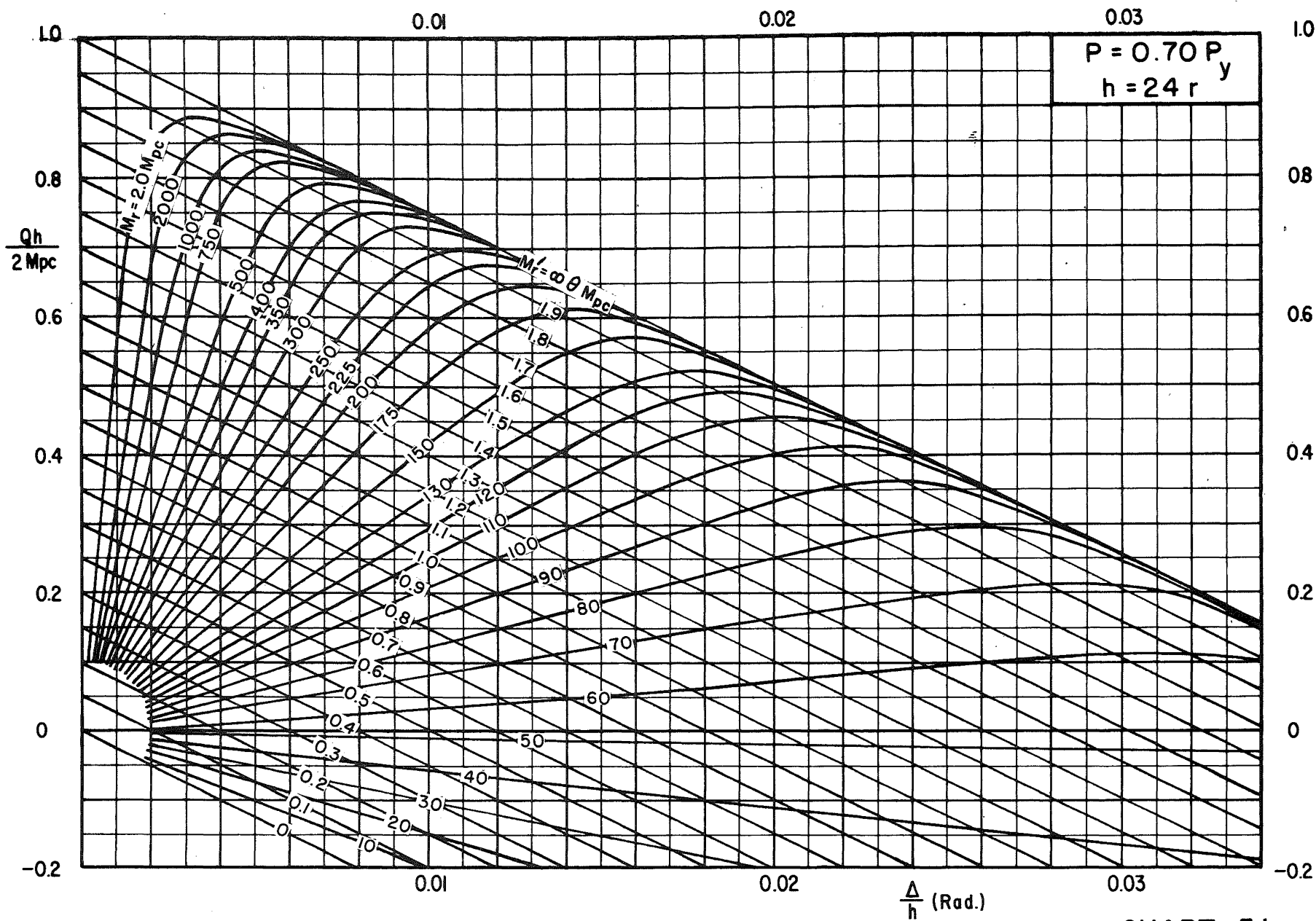


CHART 51

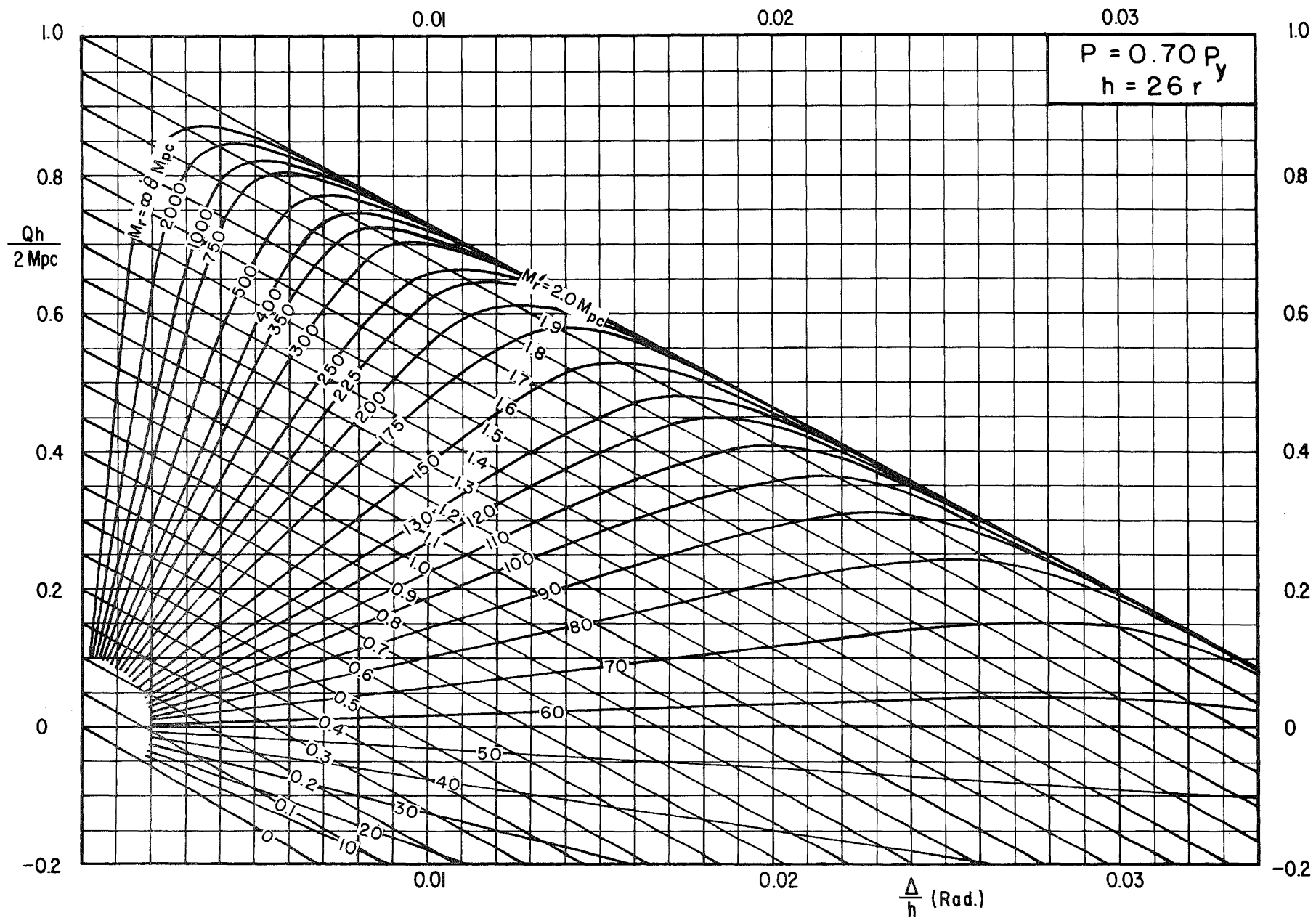


CHART 52

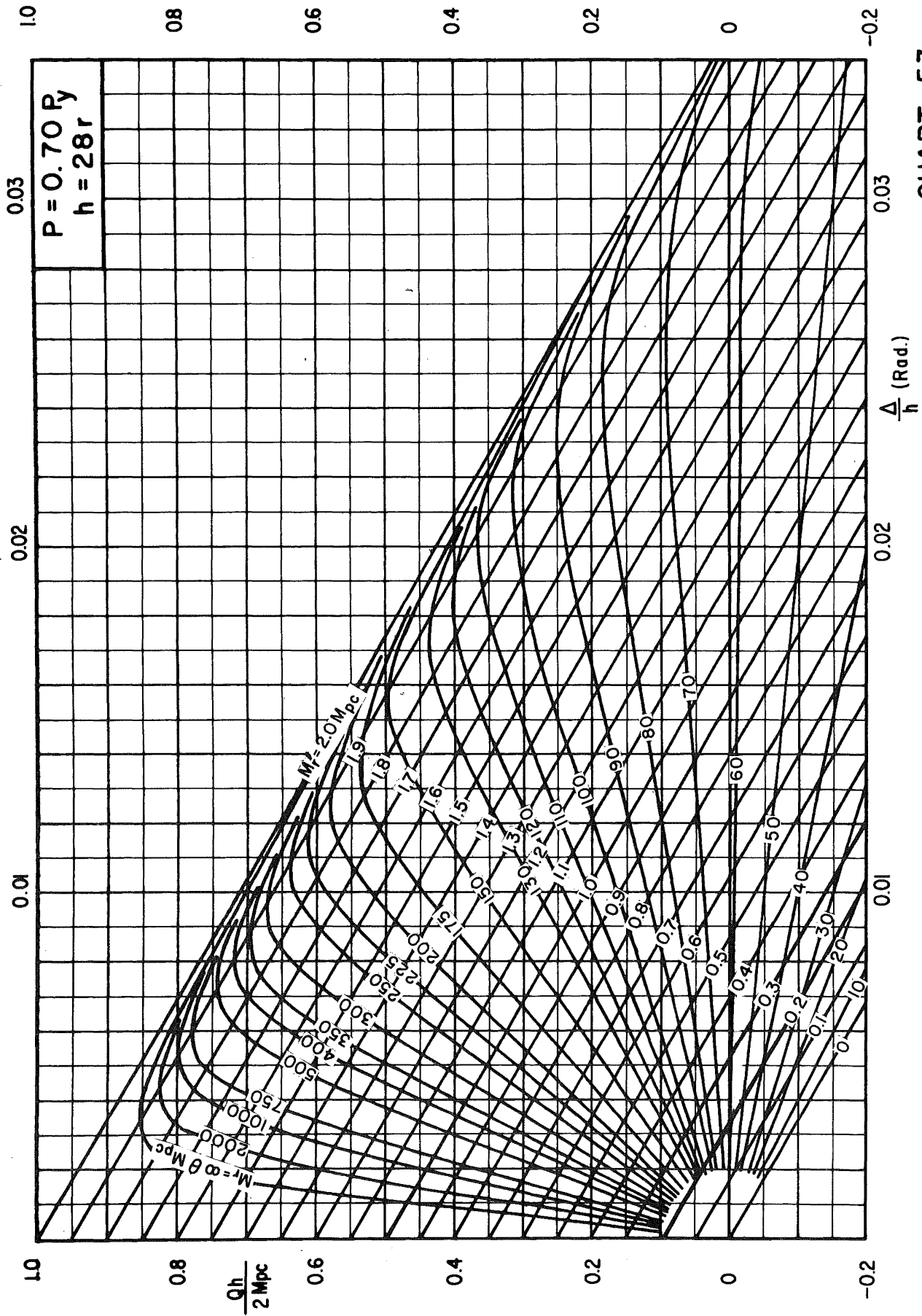


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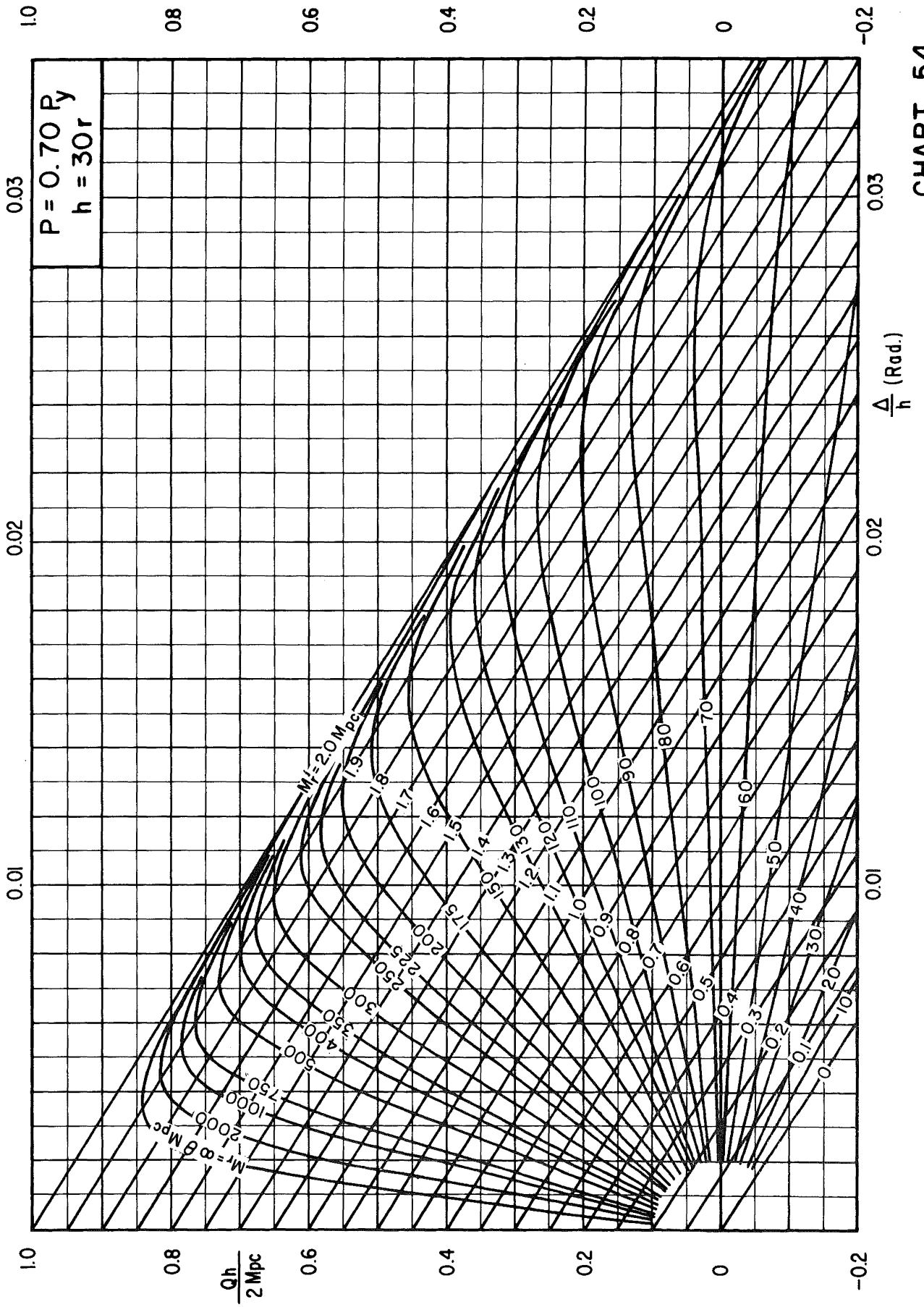
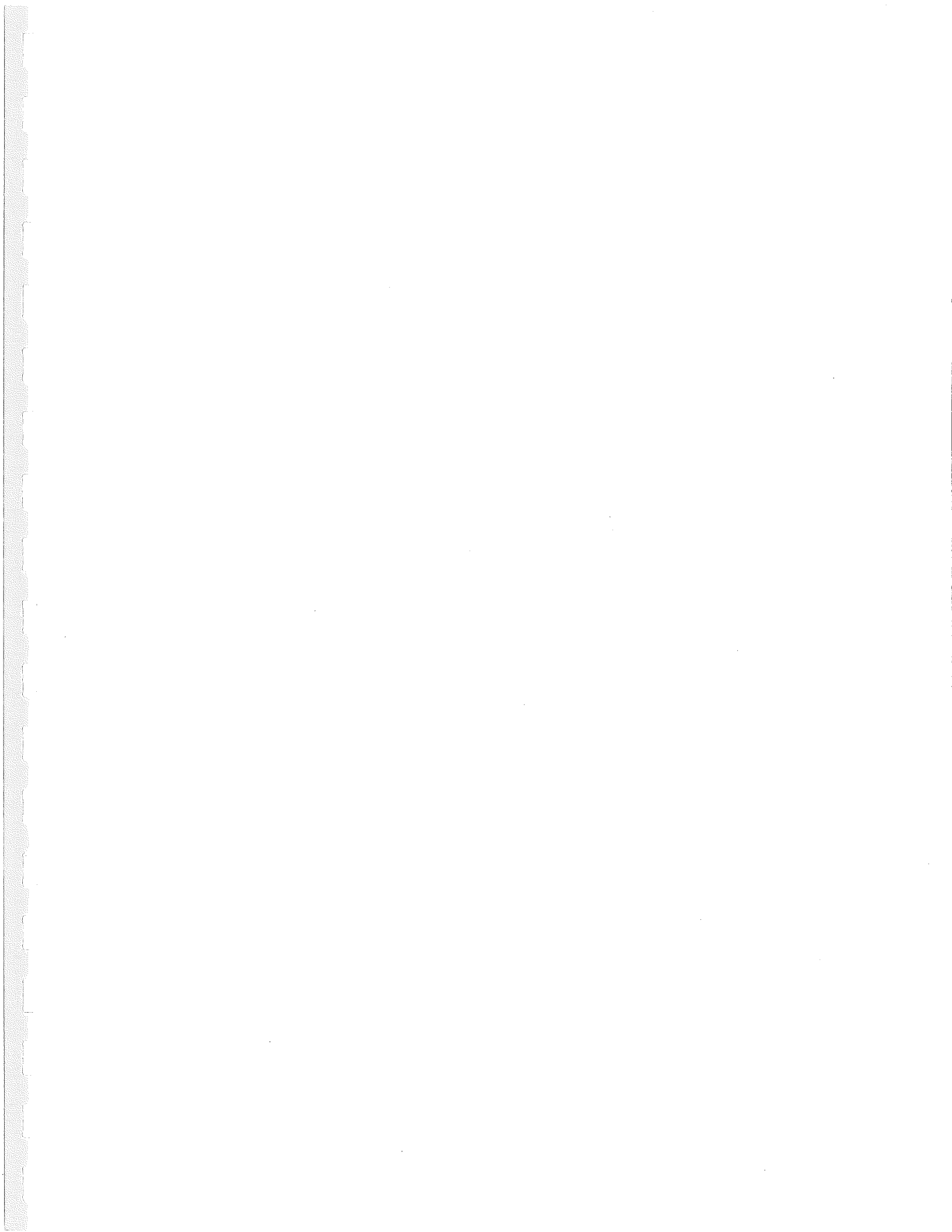
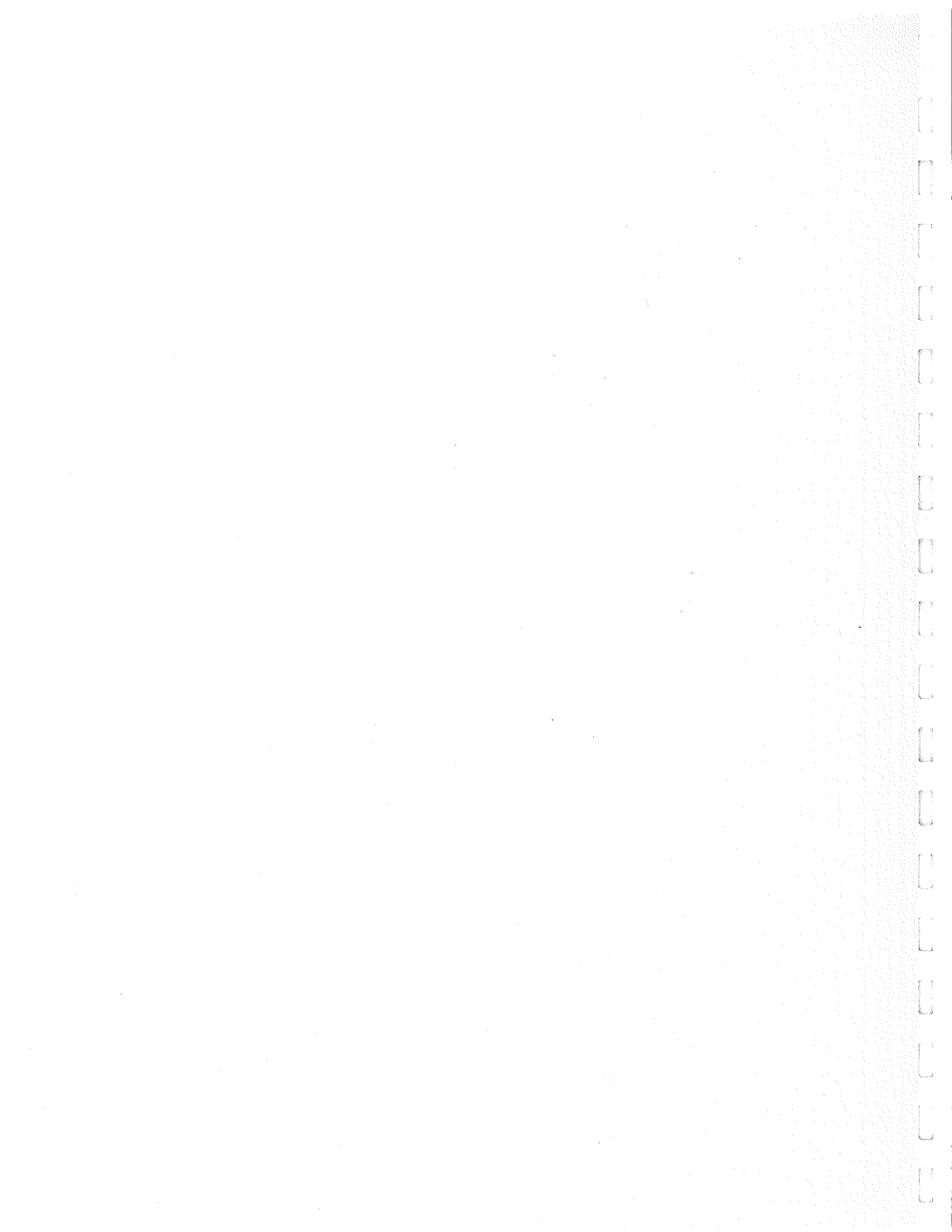


CHART 54











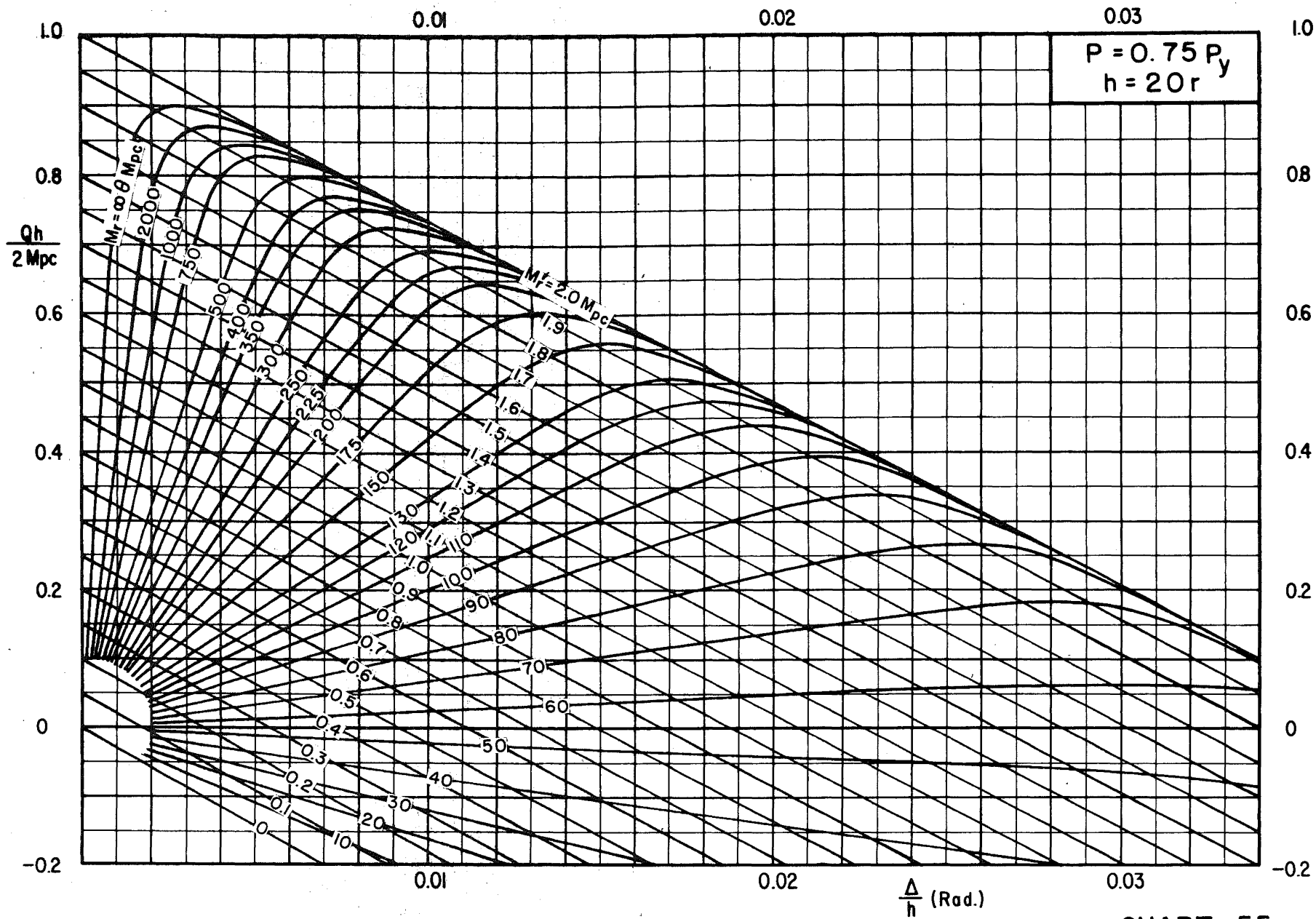


CHART 55

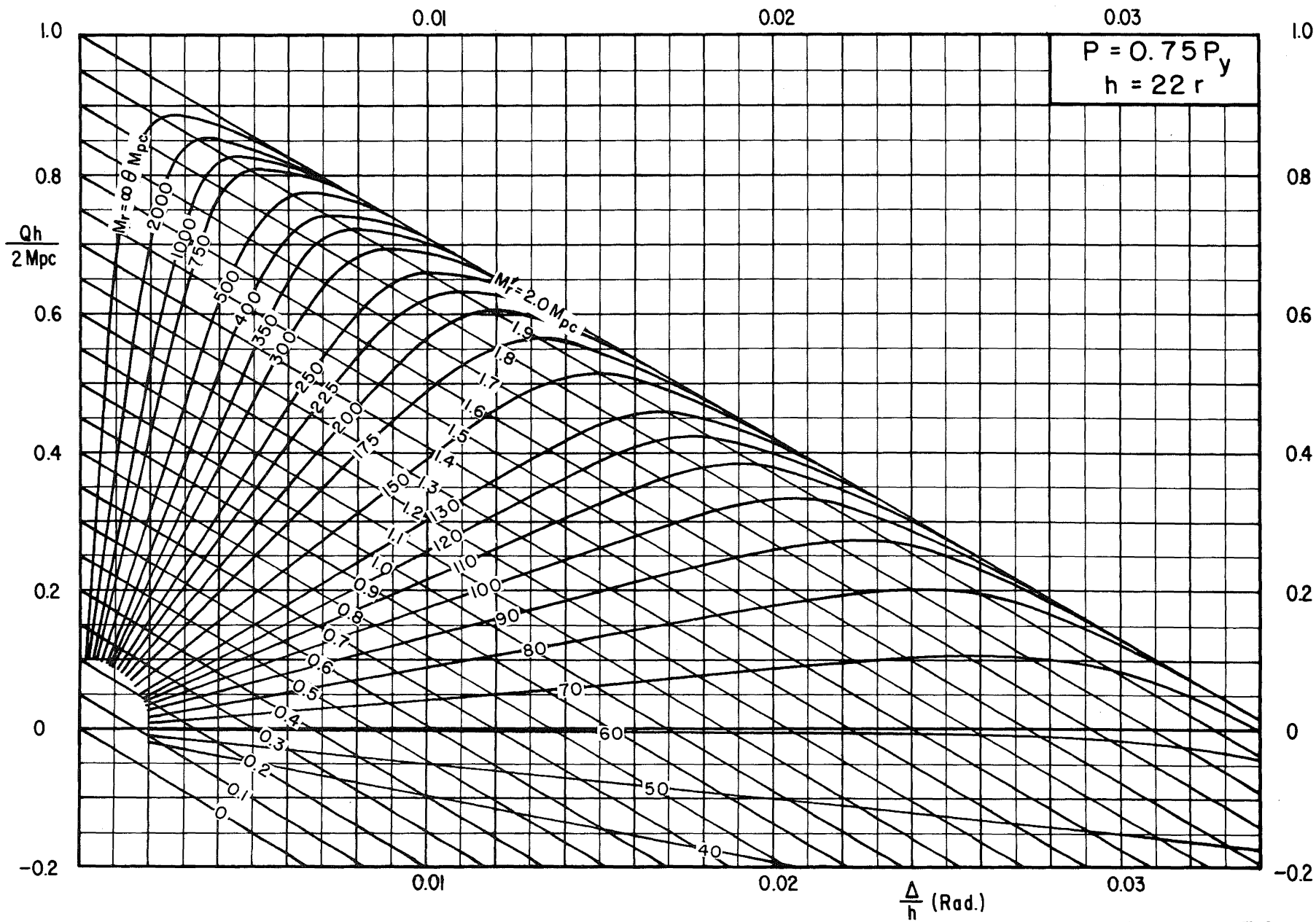


CHART 56

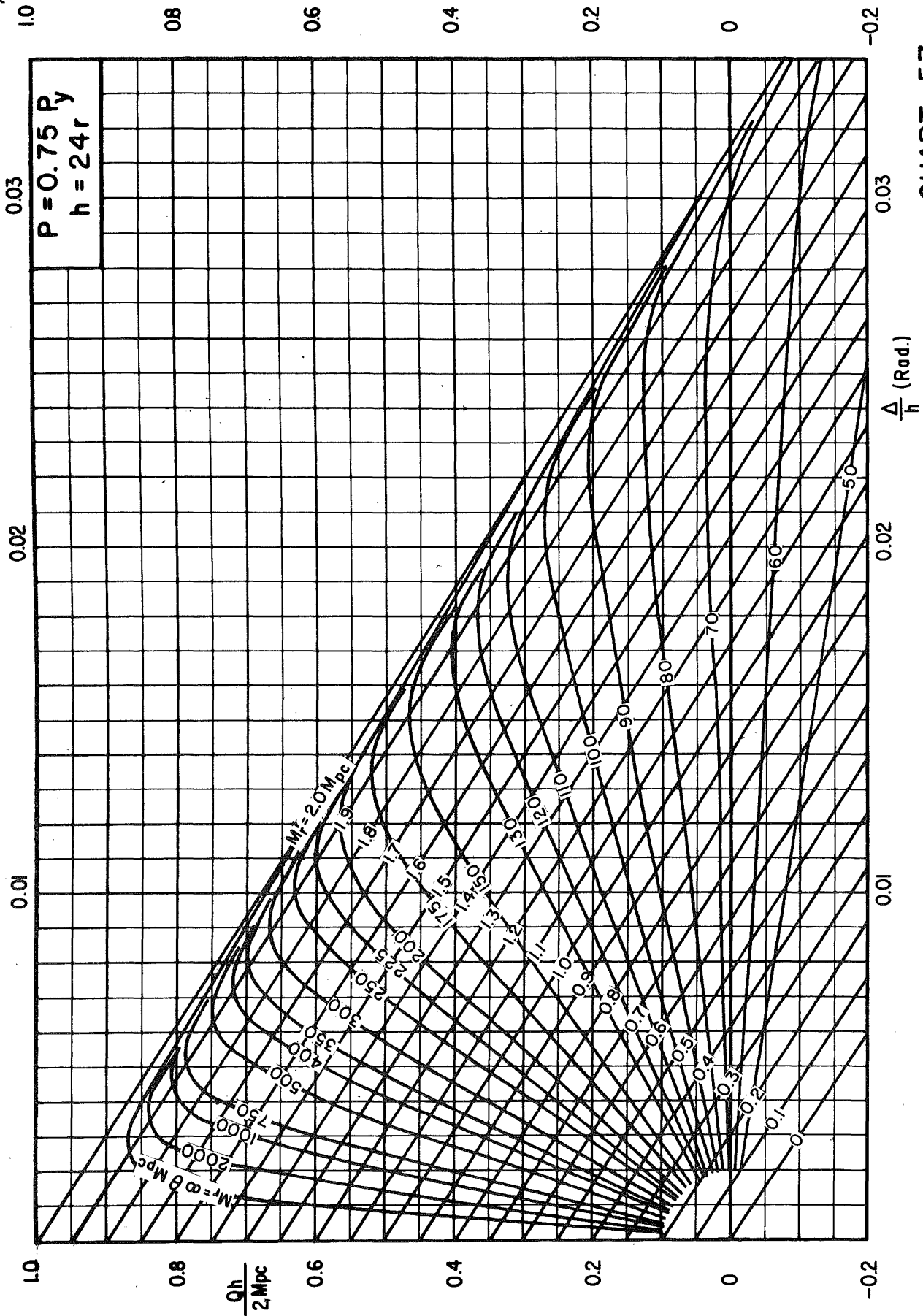


CHART 57

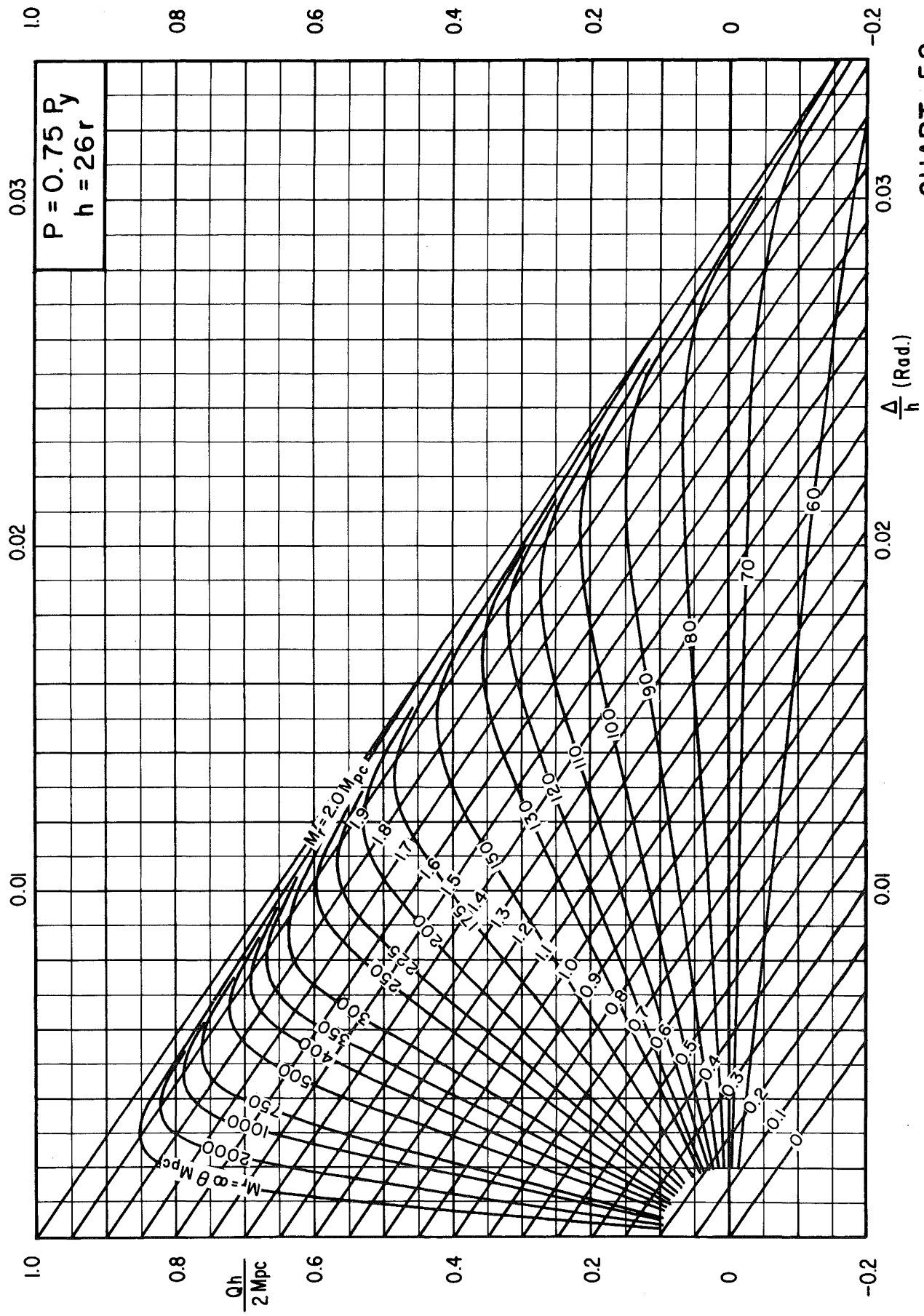


CHART 58

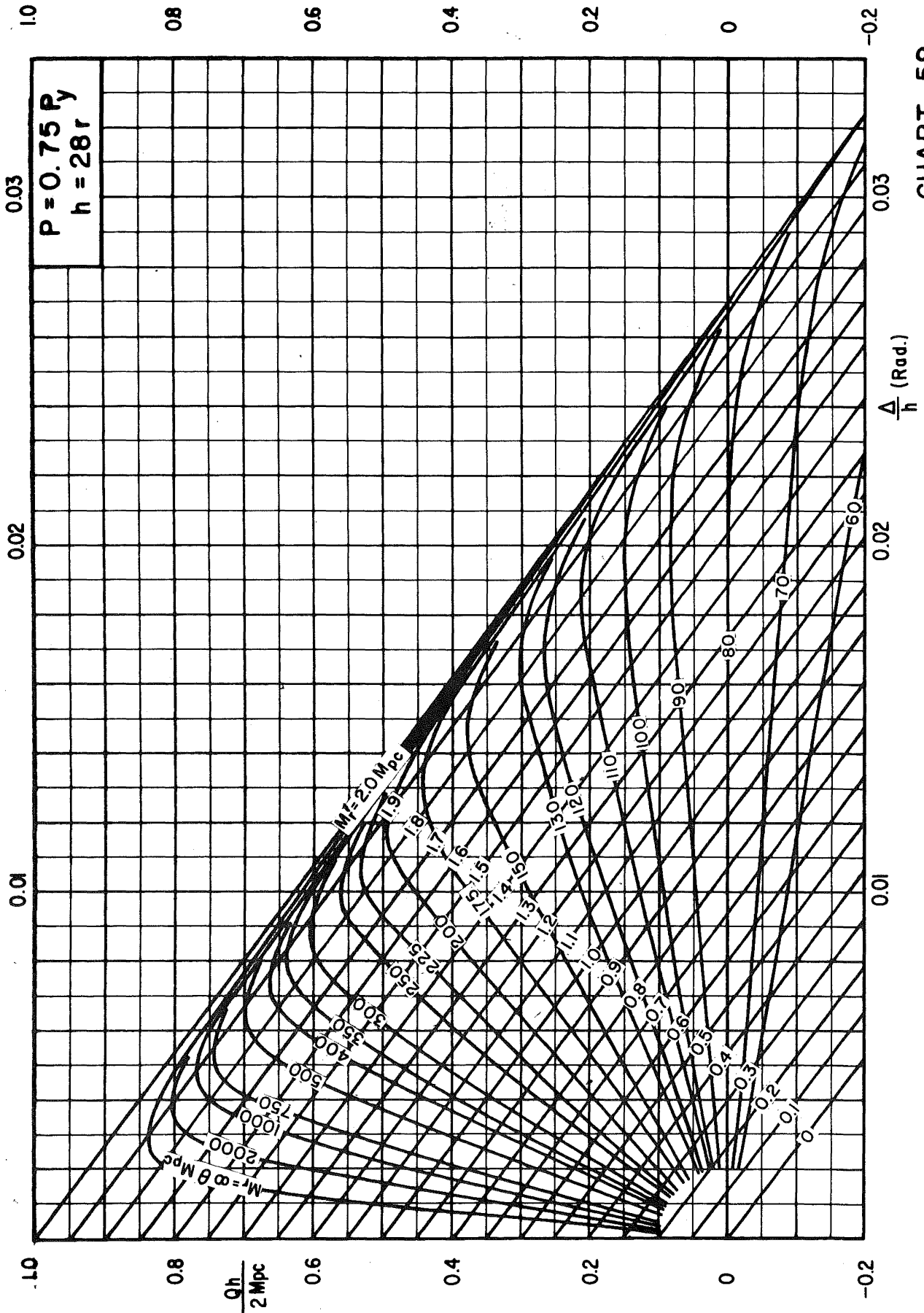


CHART 59

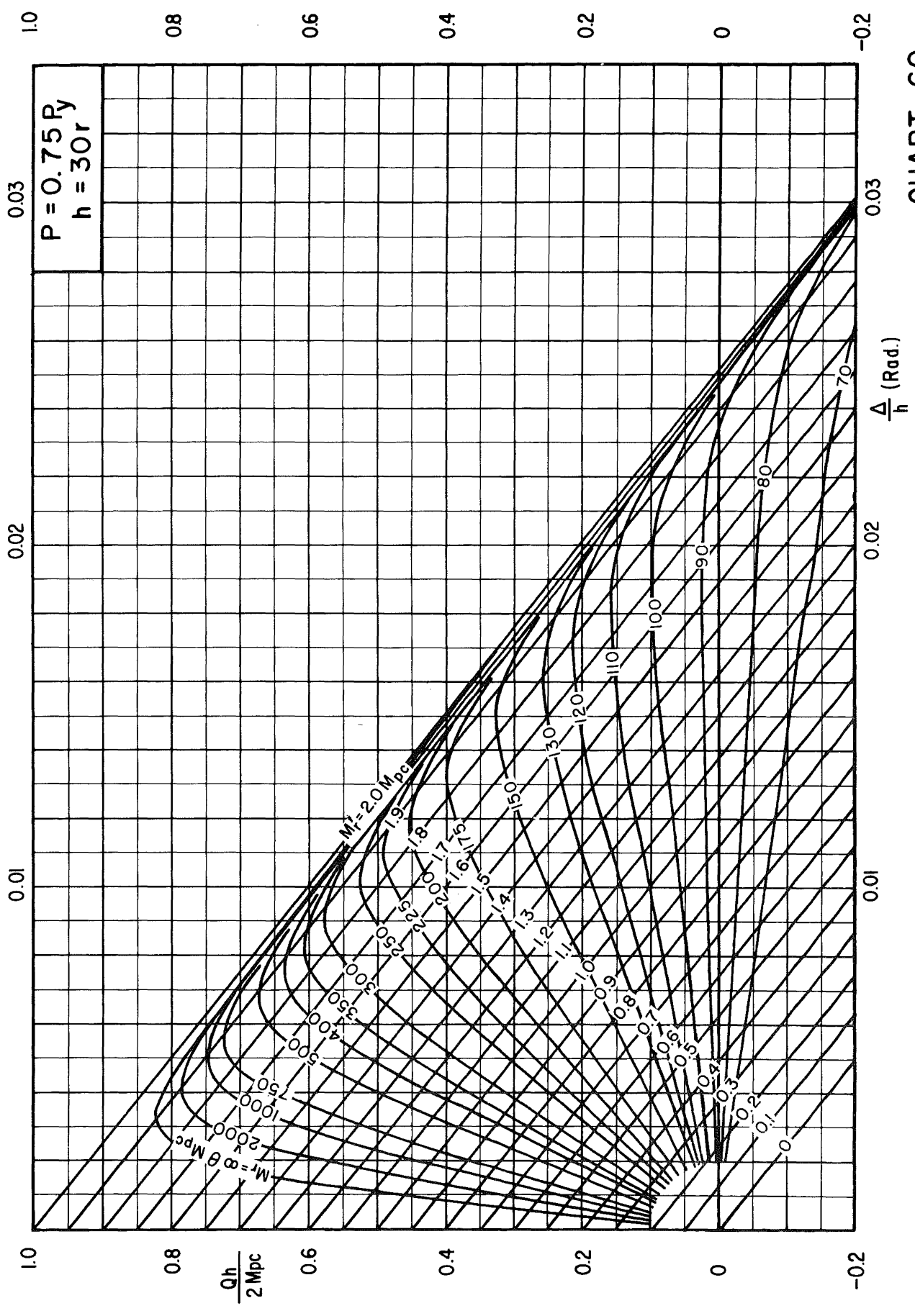
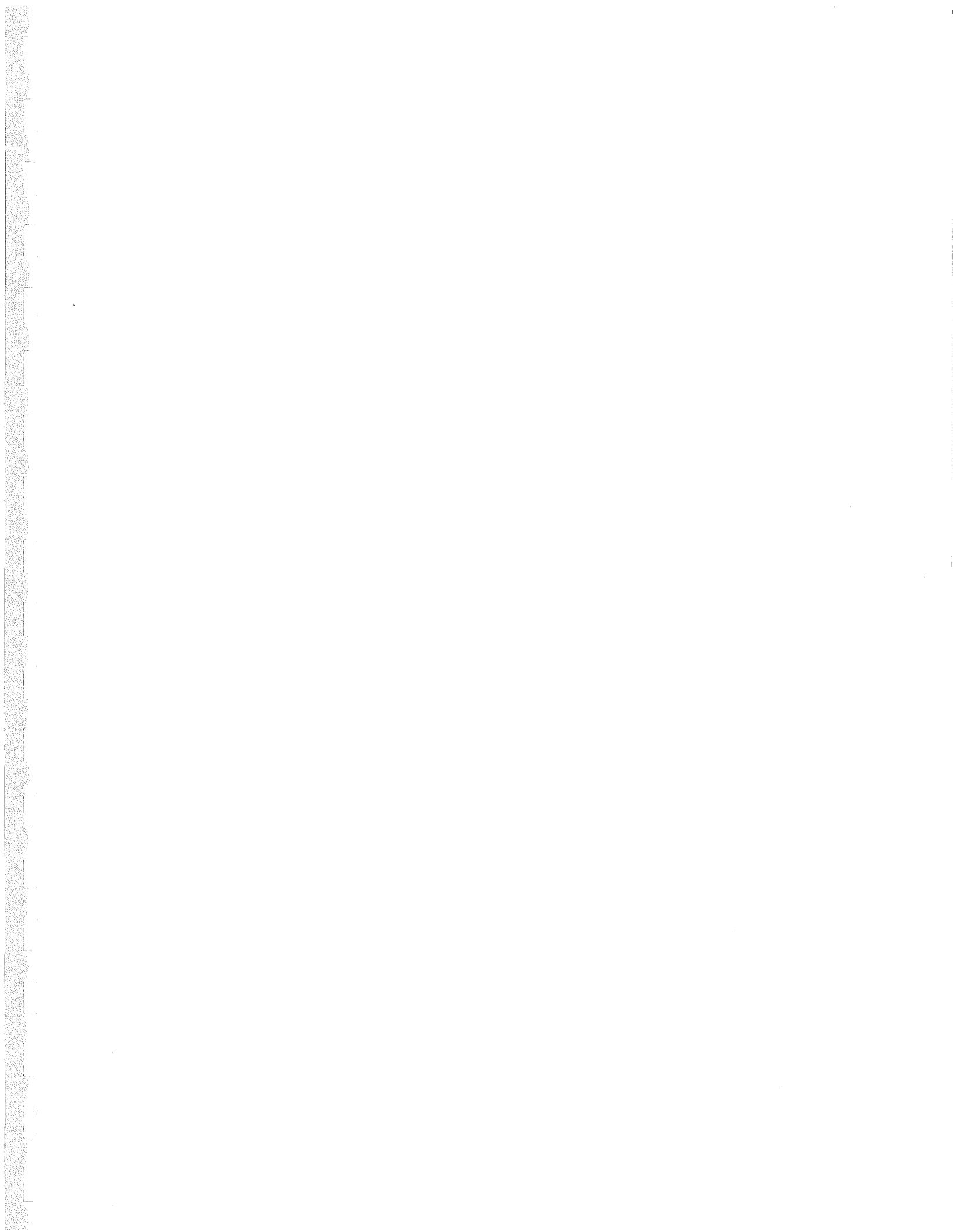
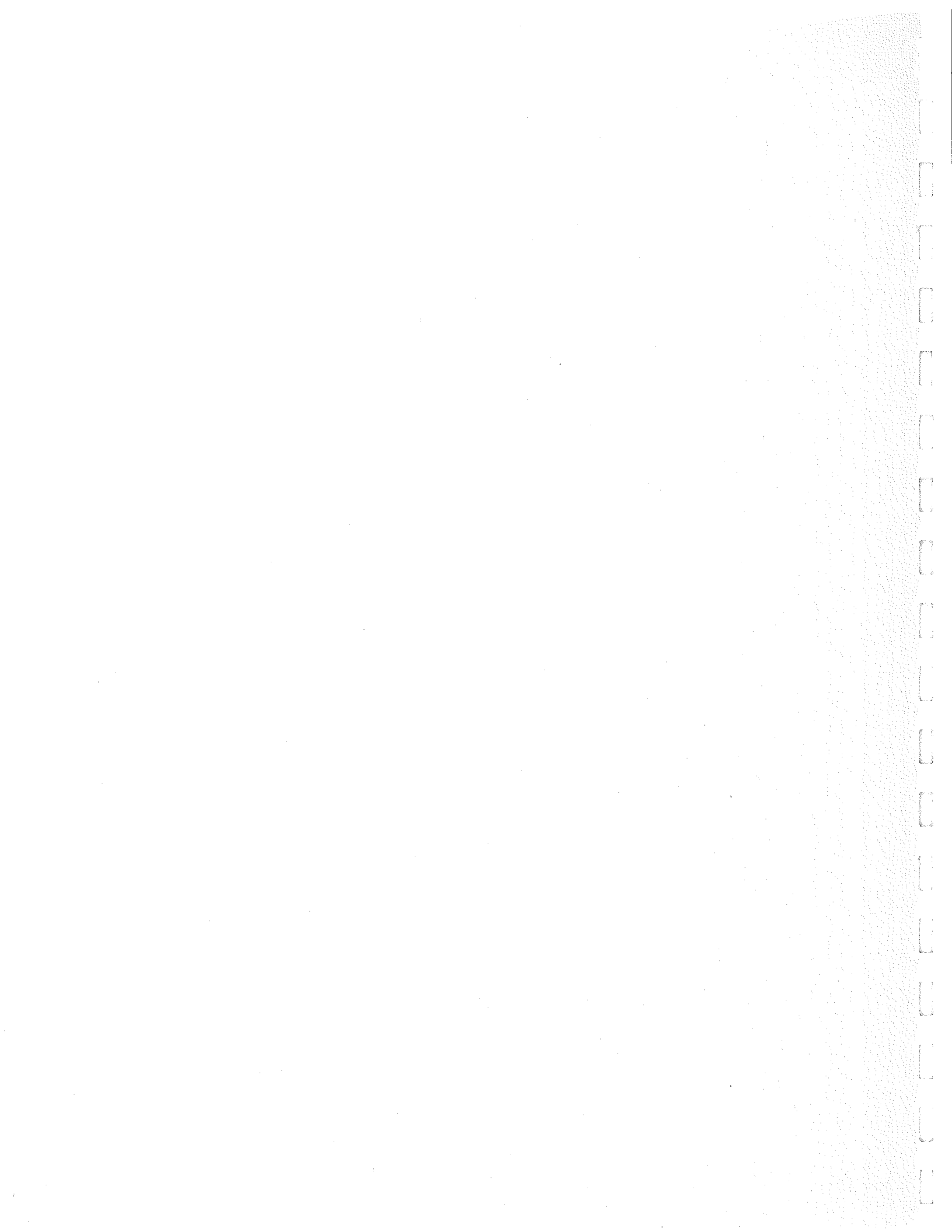


CHART 60











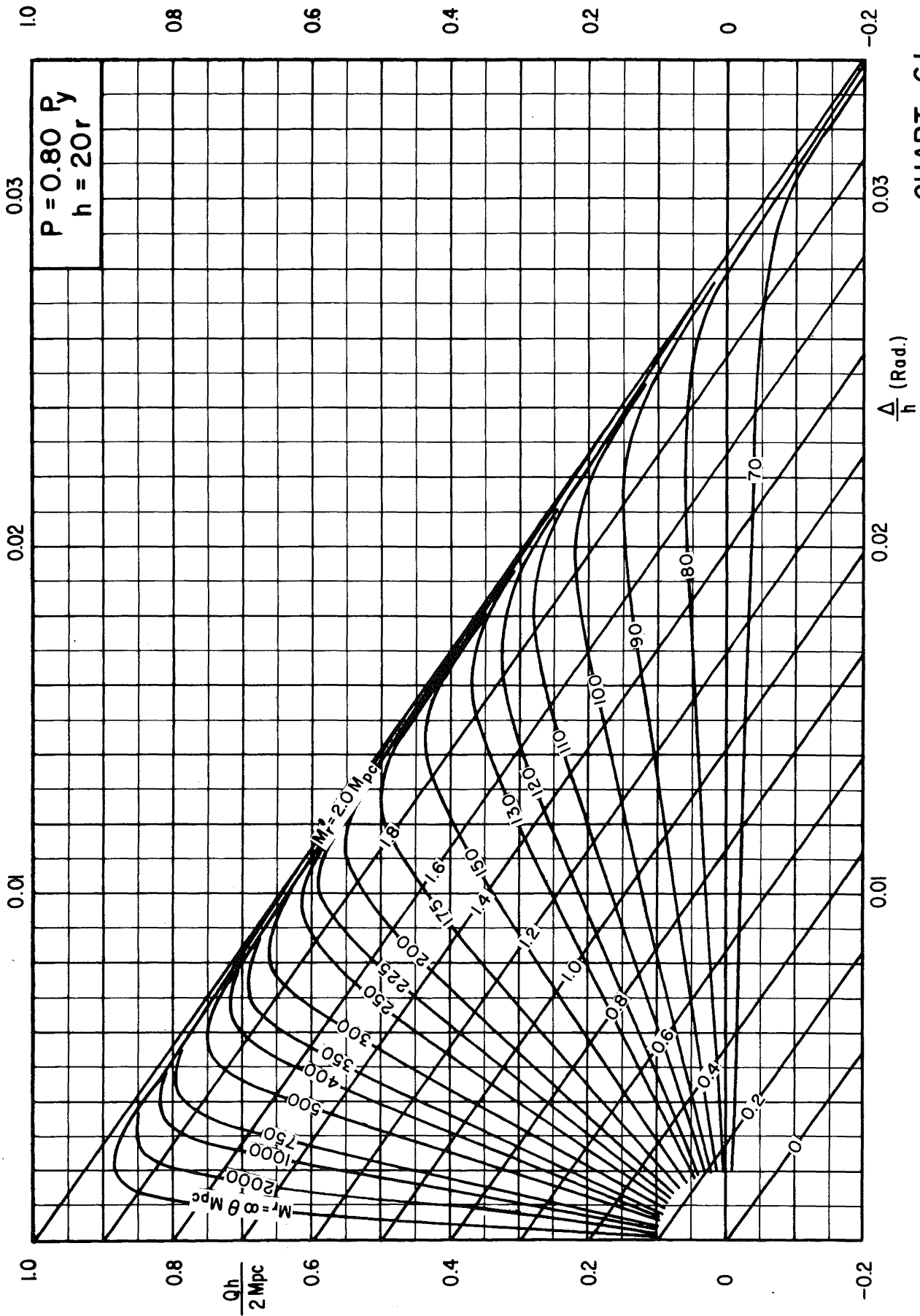


CHART 61

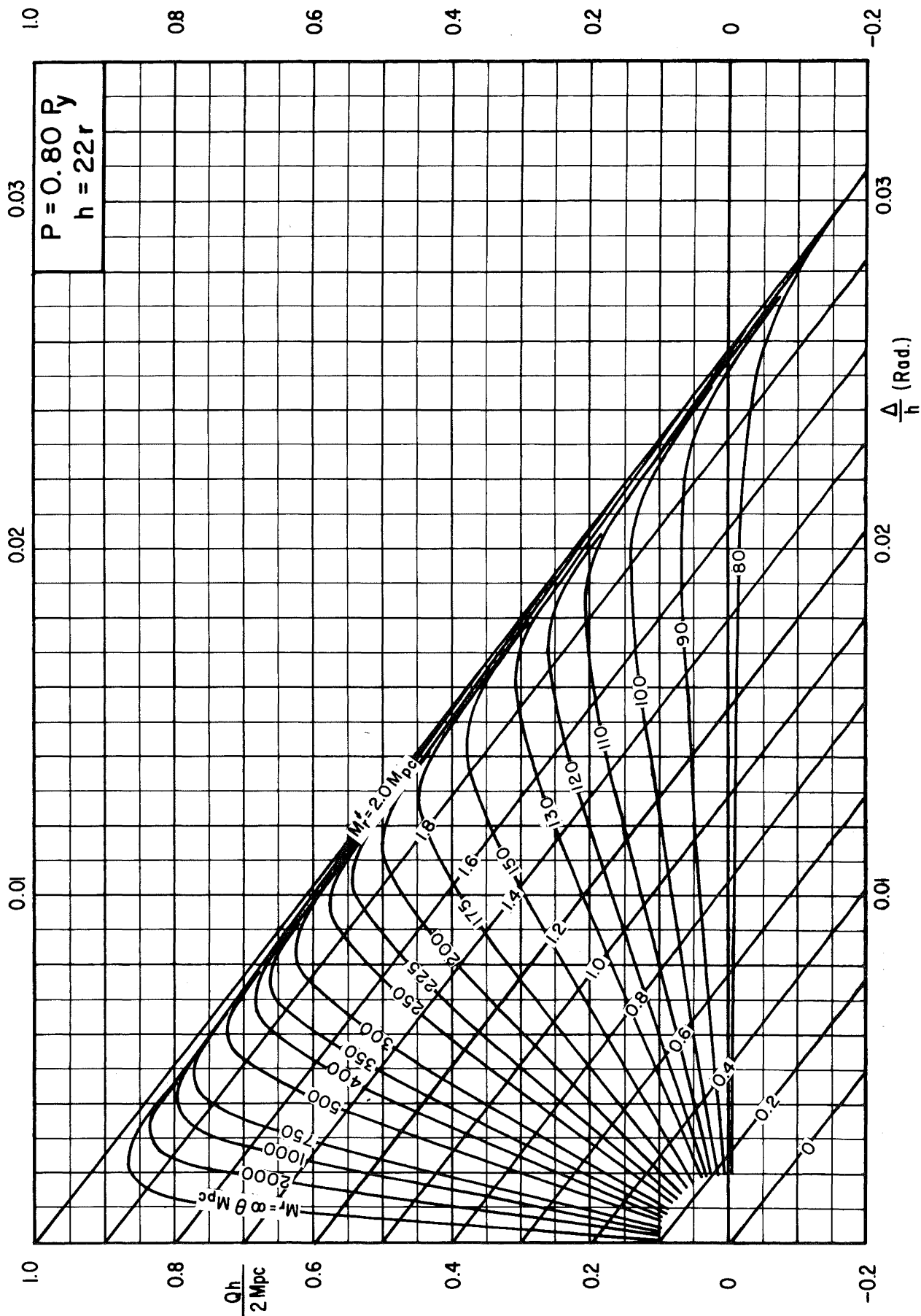


CHART 62

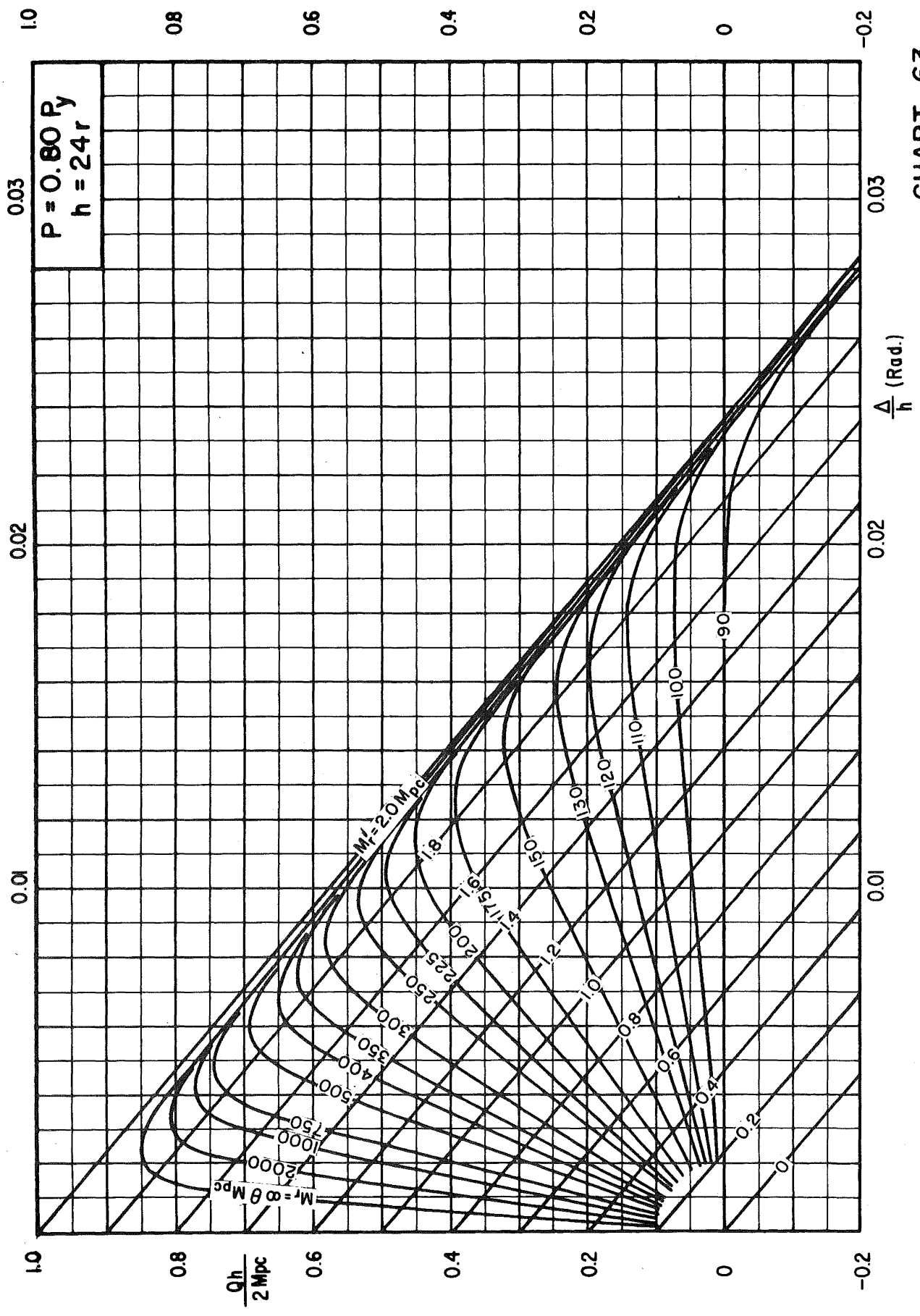


CHART 63

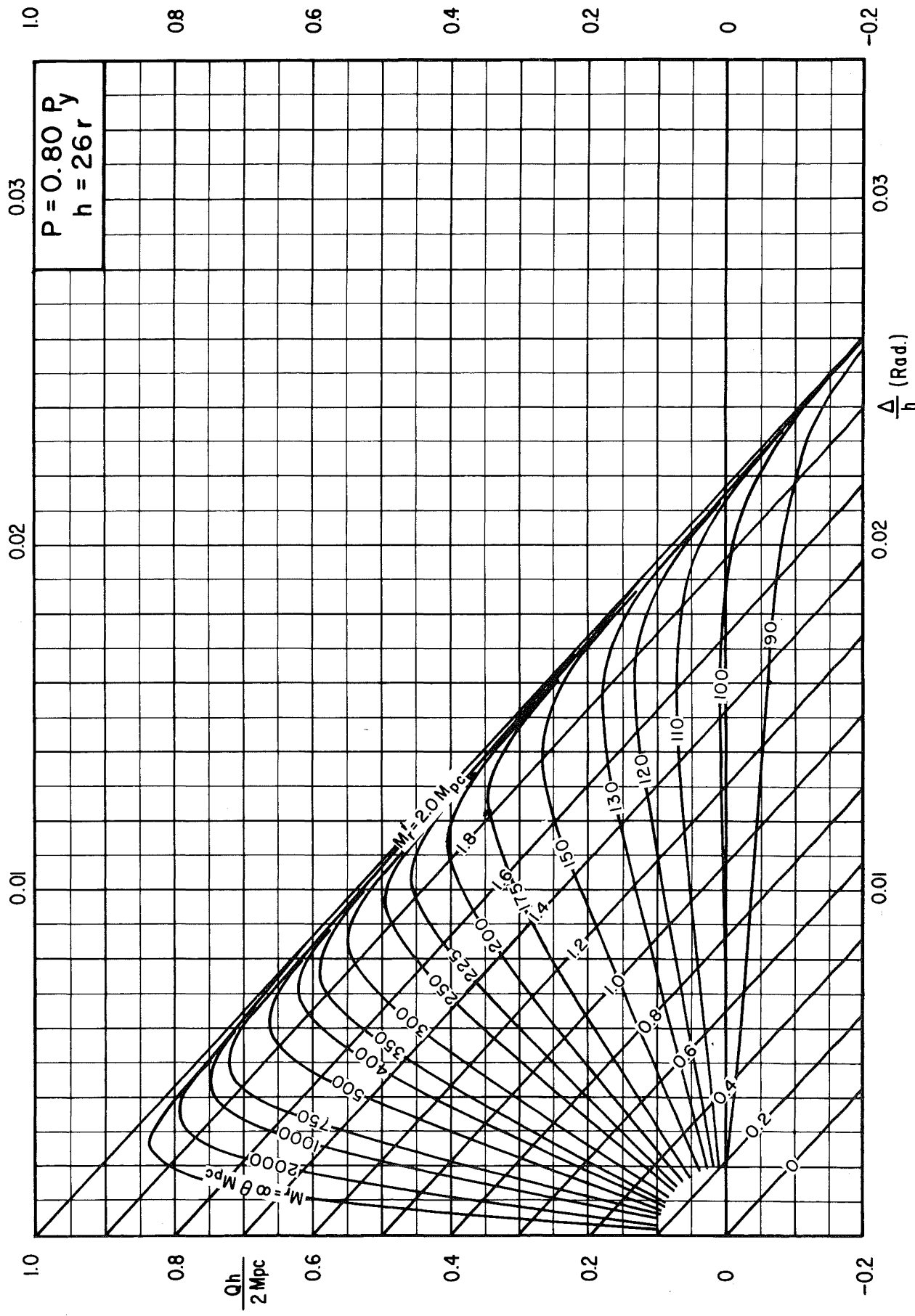


CHART 64

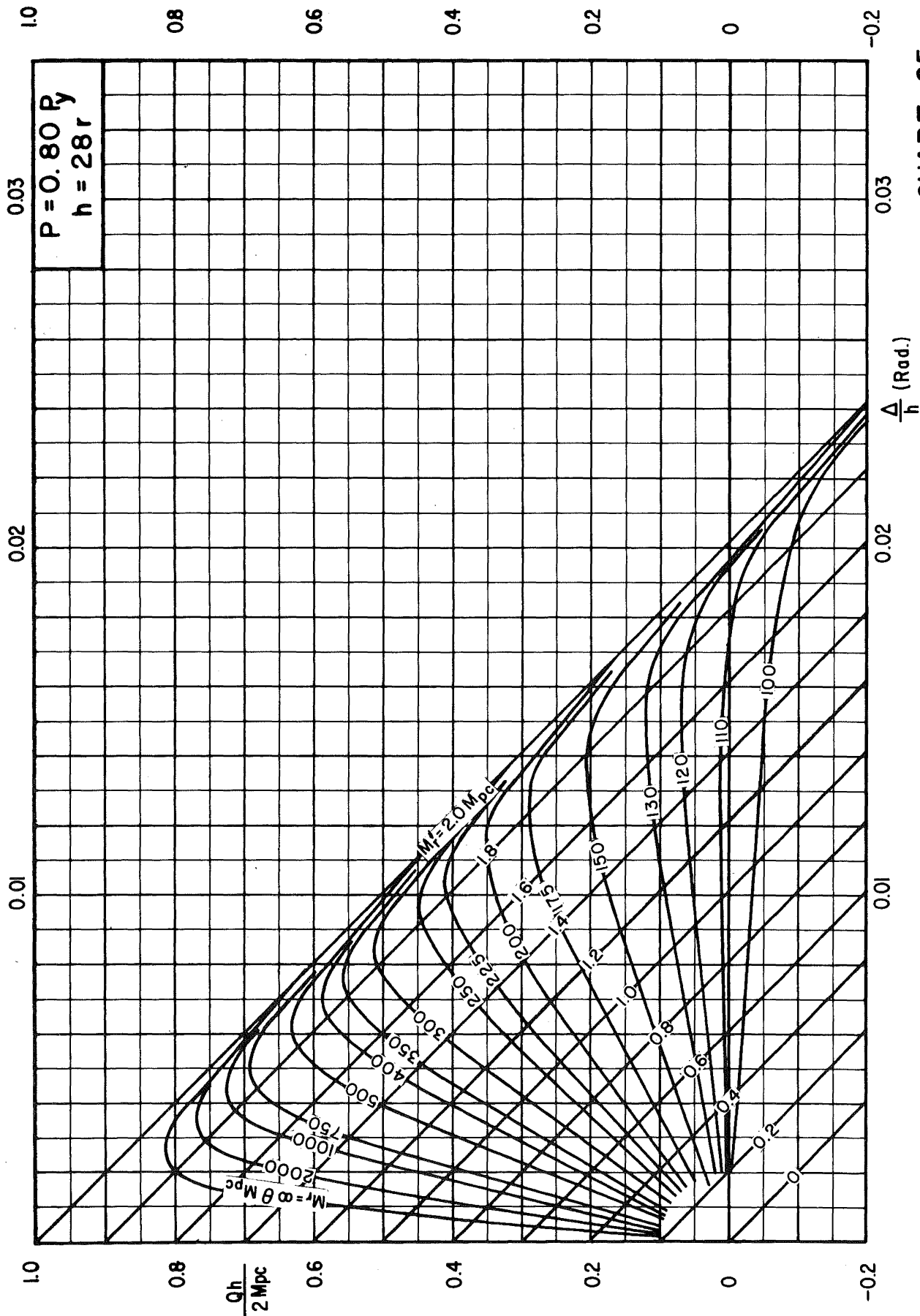


CHART 65

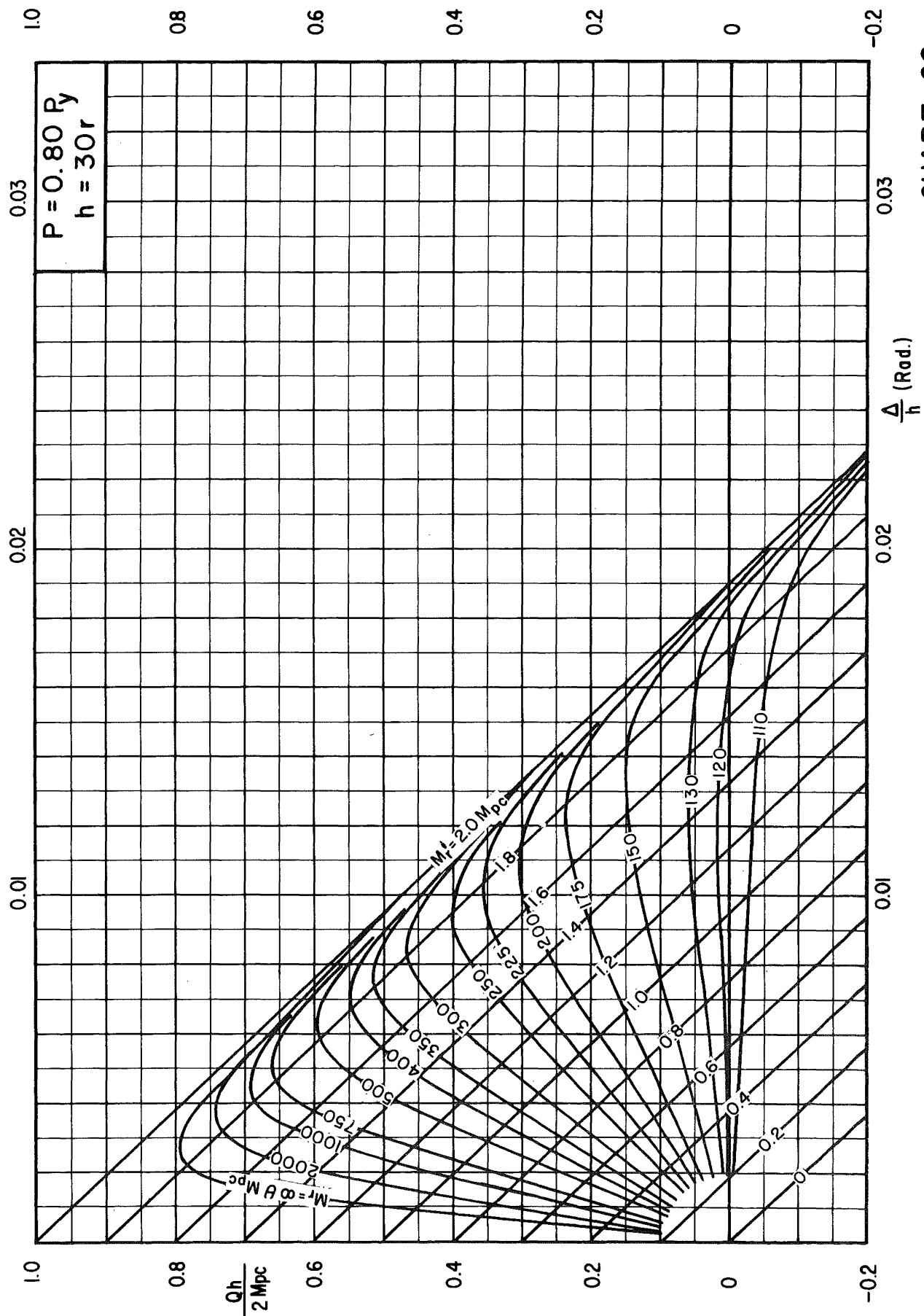


CHART 66











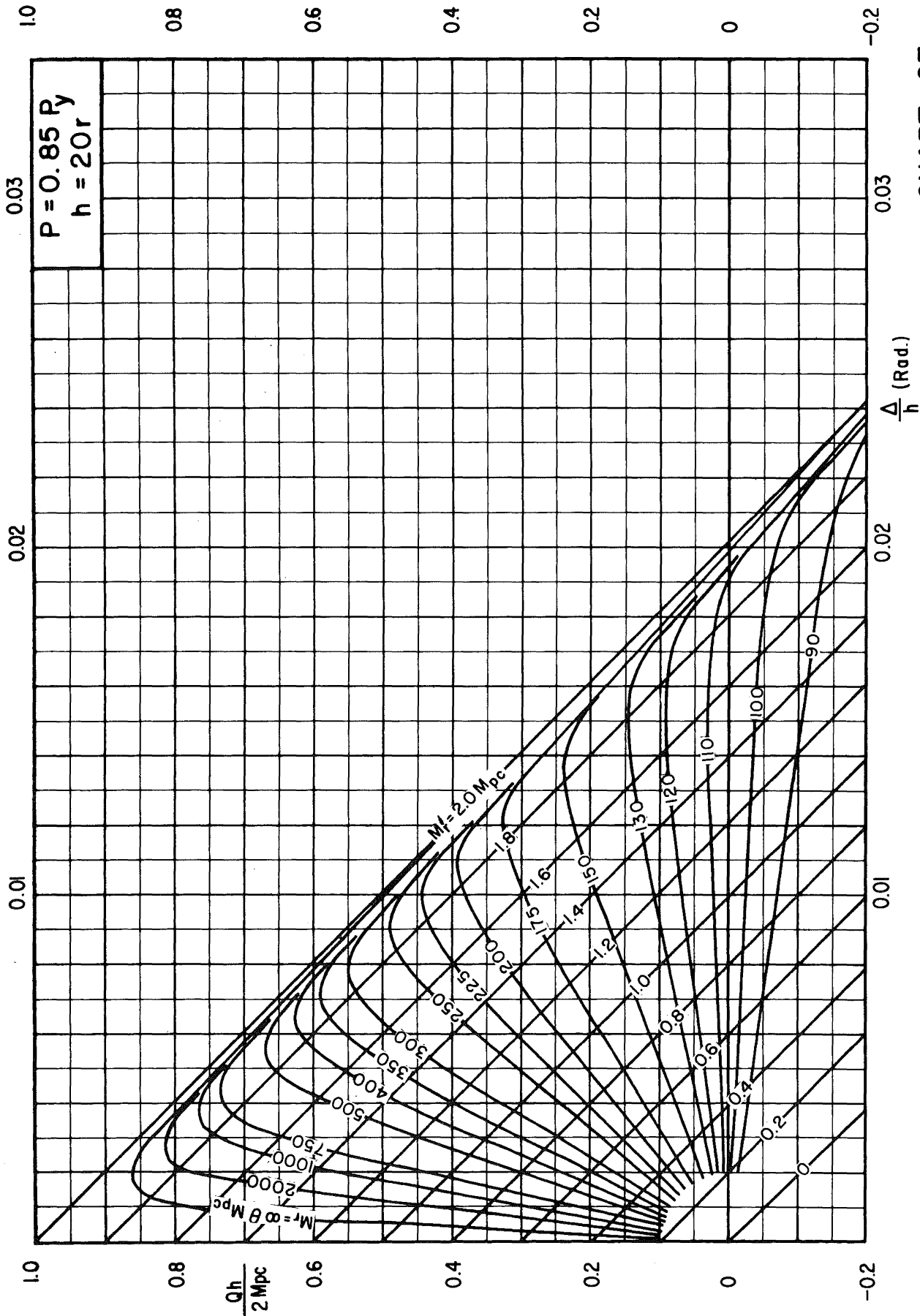


CHART 67

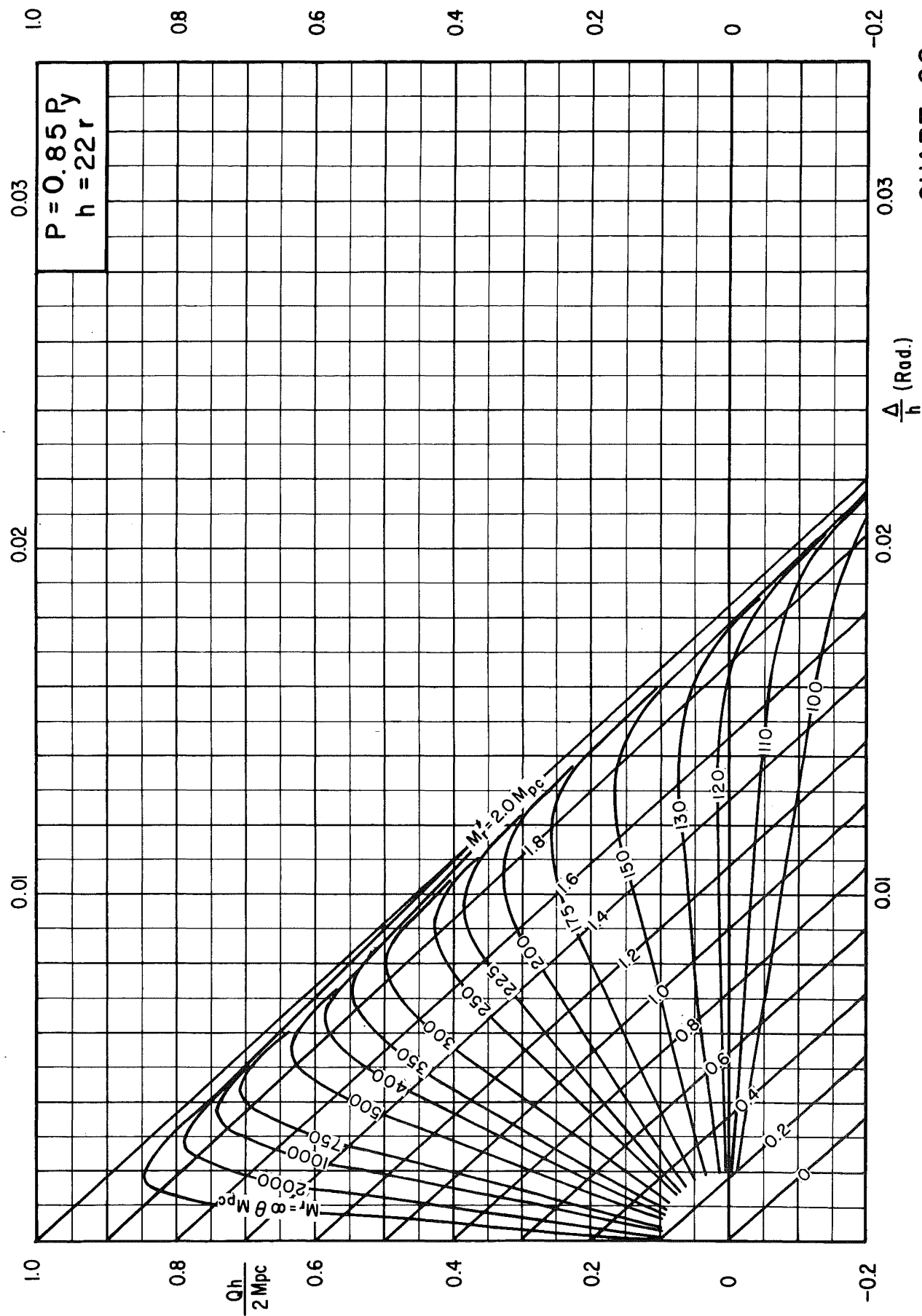


CHART 68

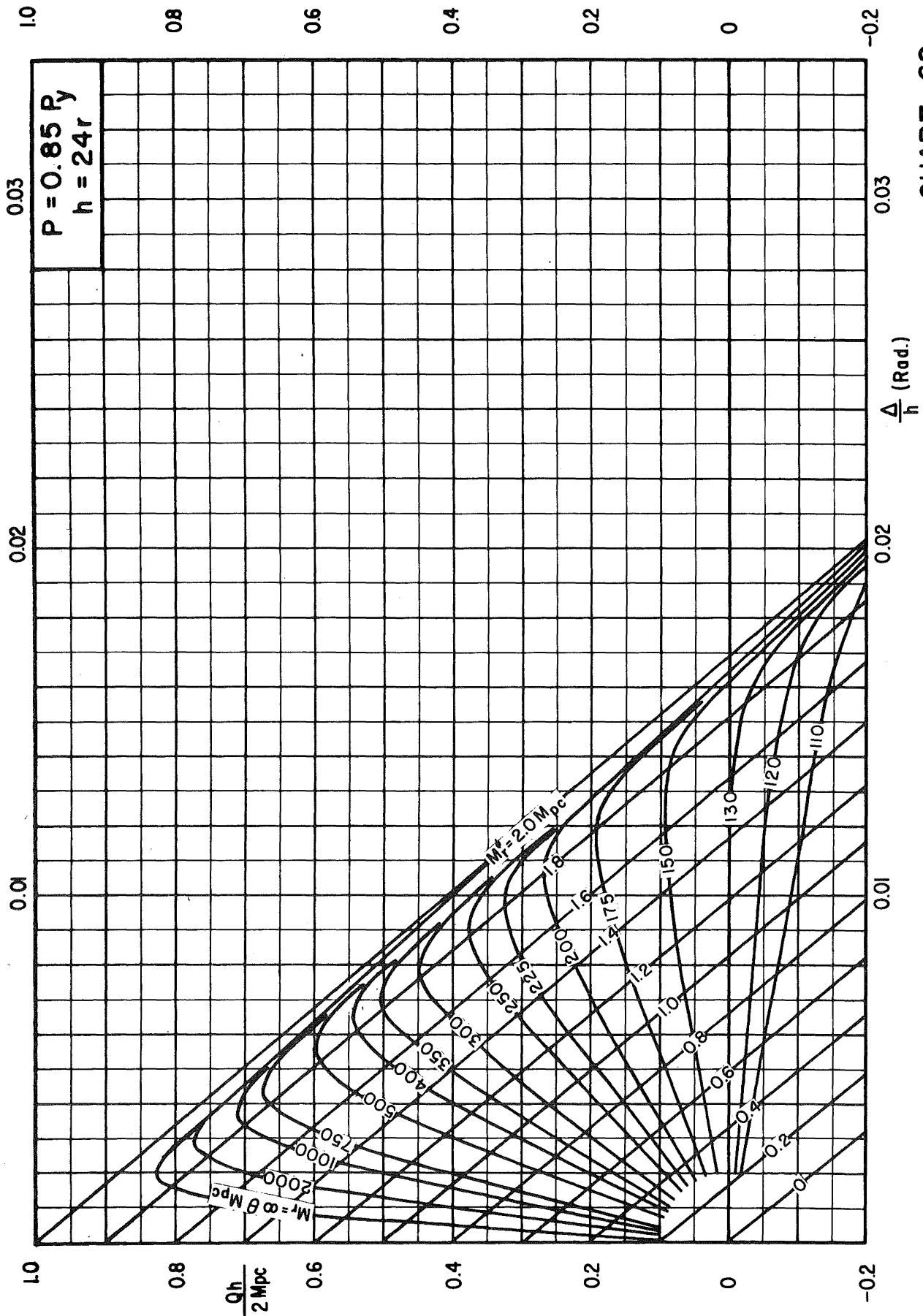


CHART 69

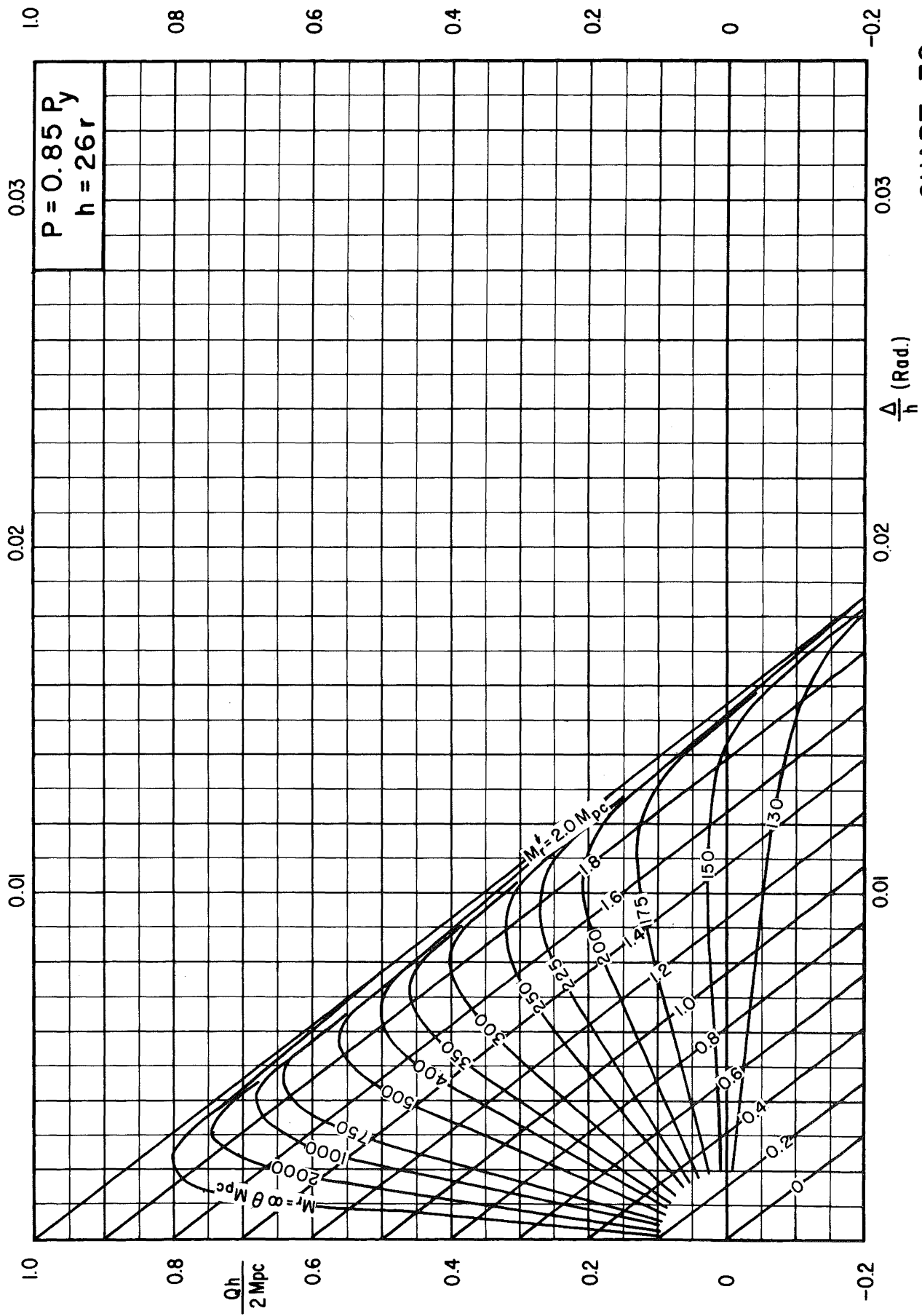


CHART 70



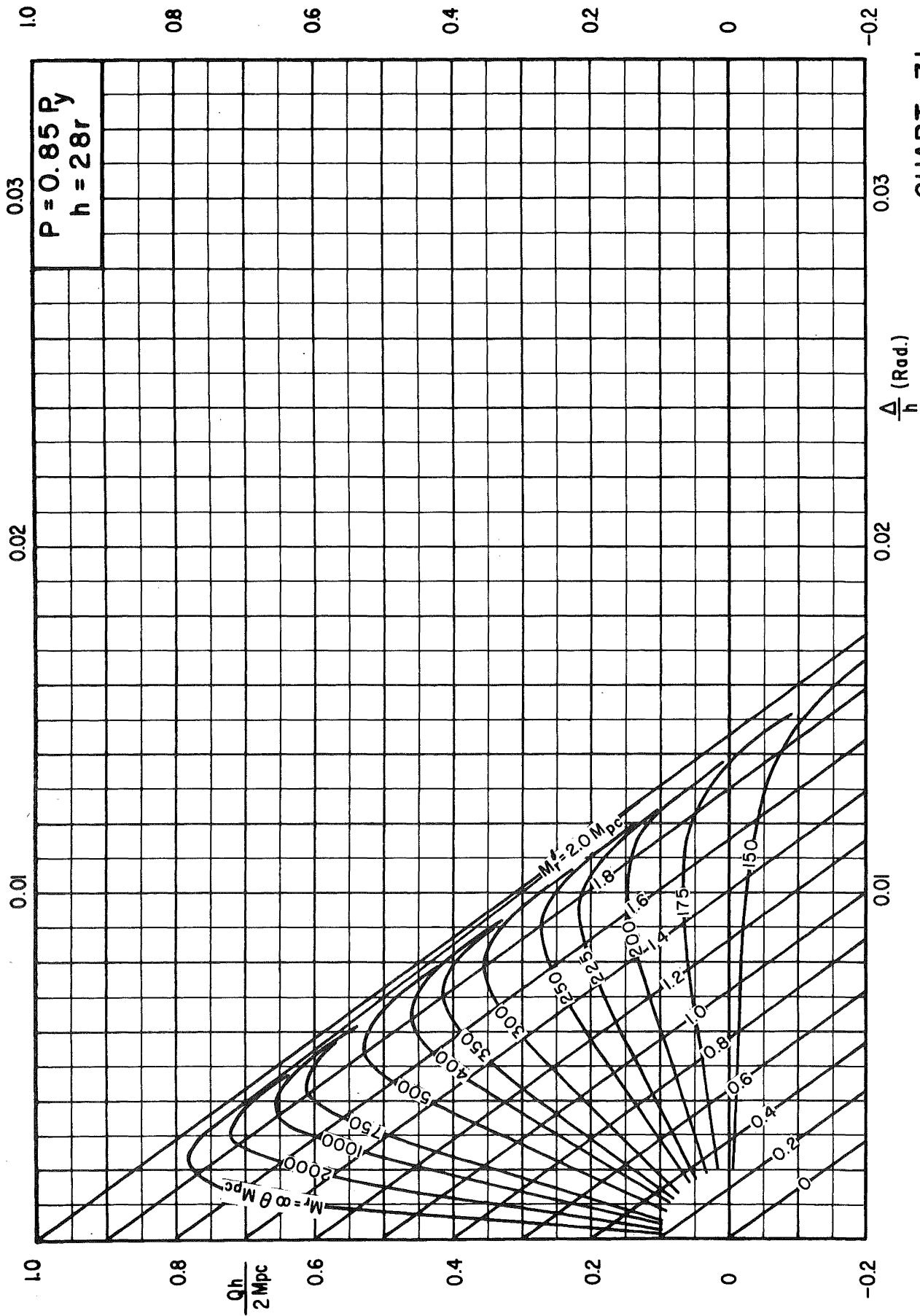


CHART 71

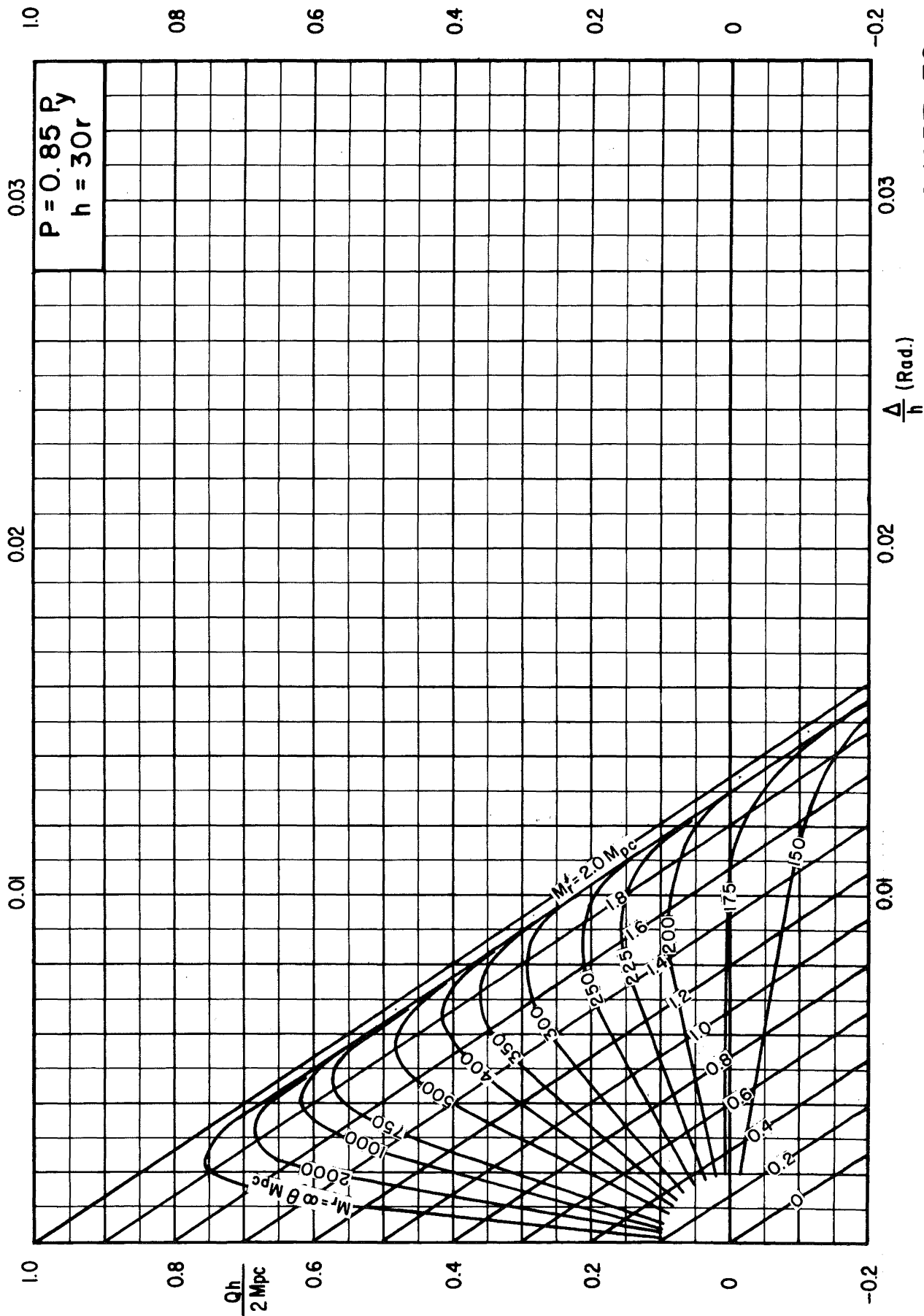
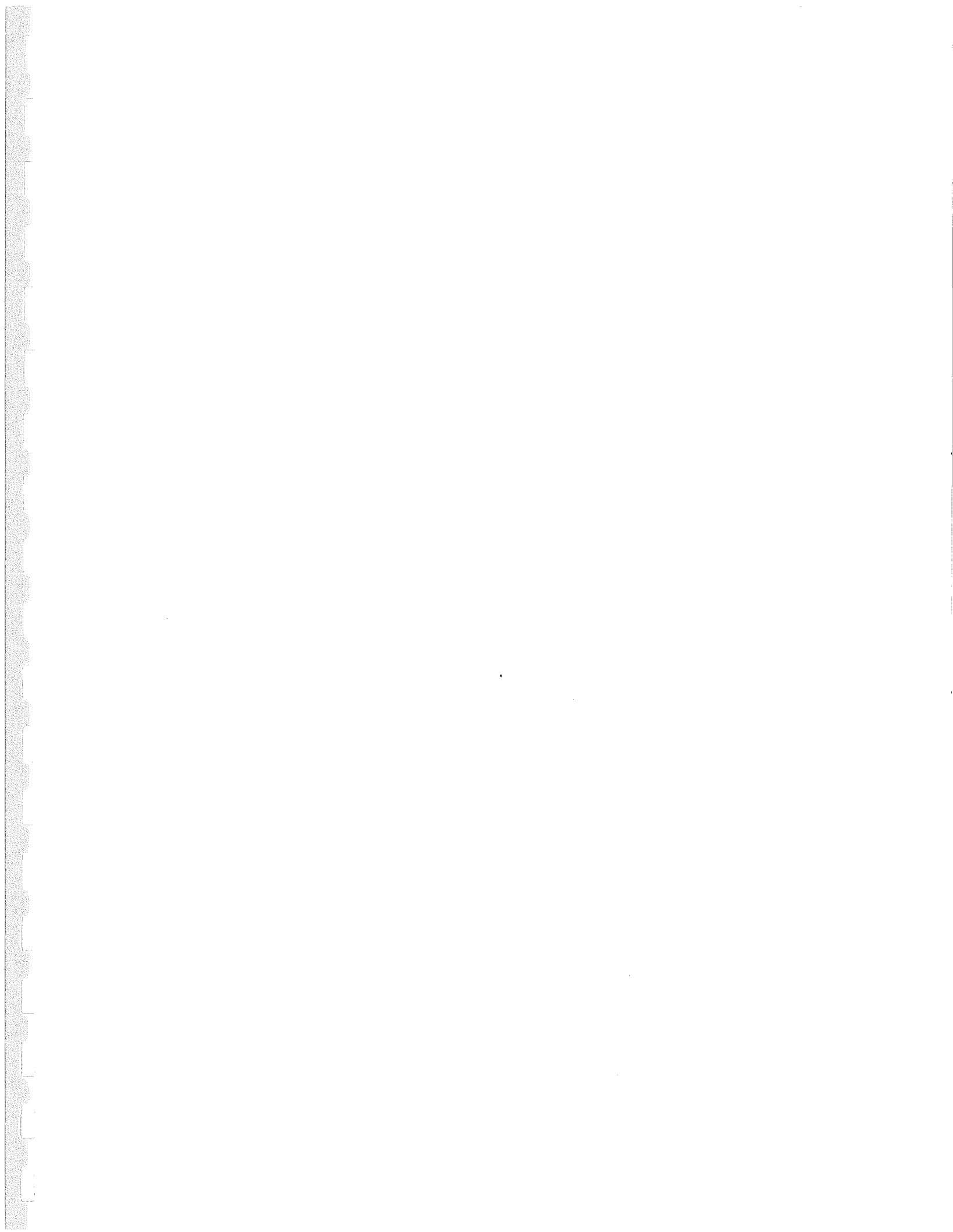
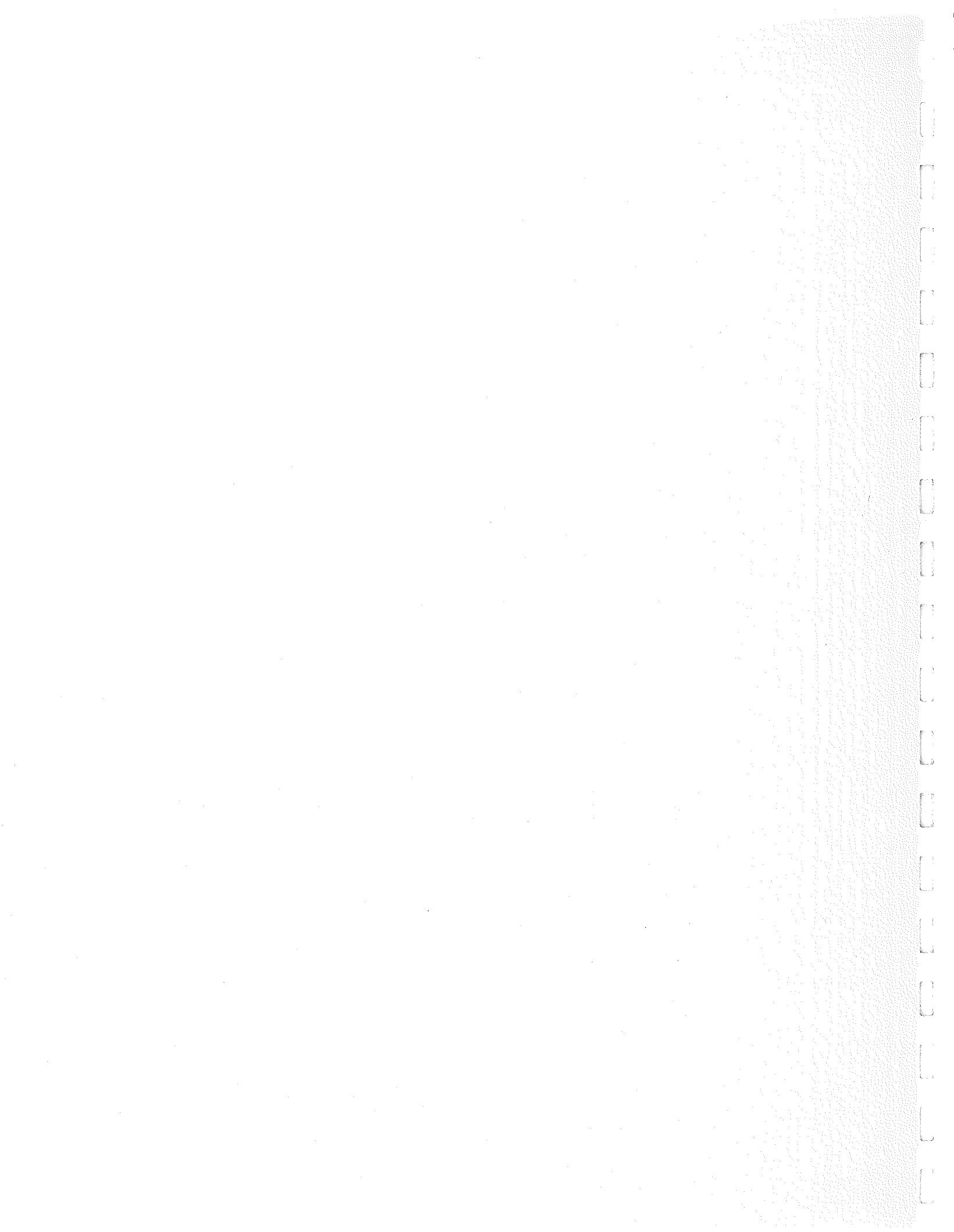


CHART 72









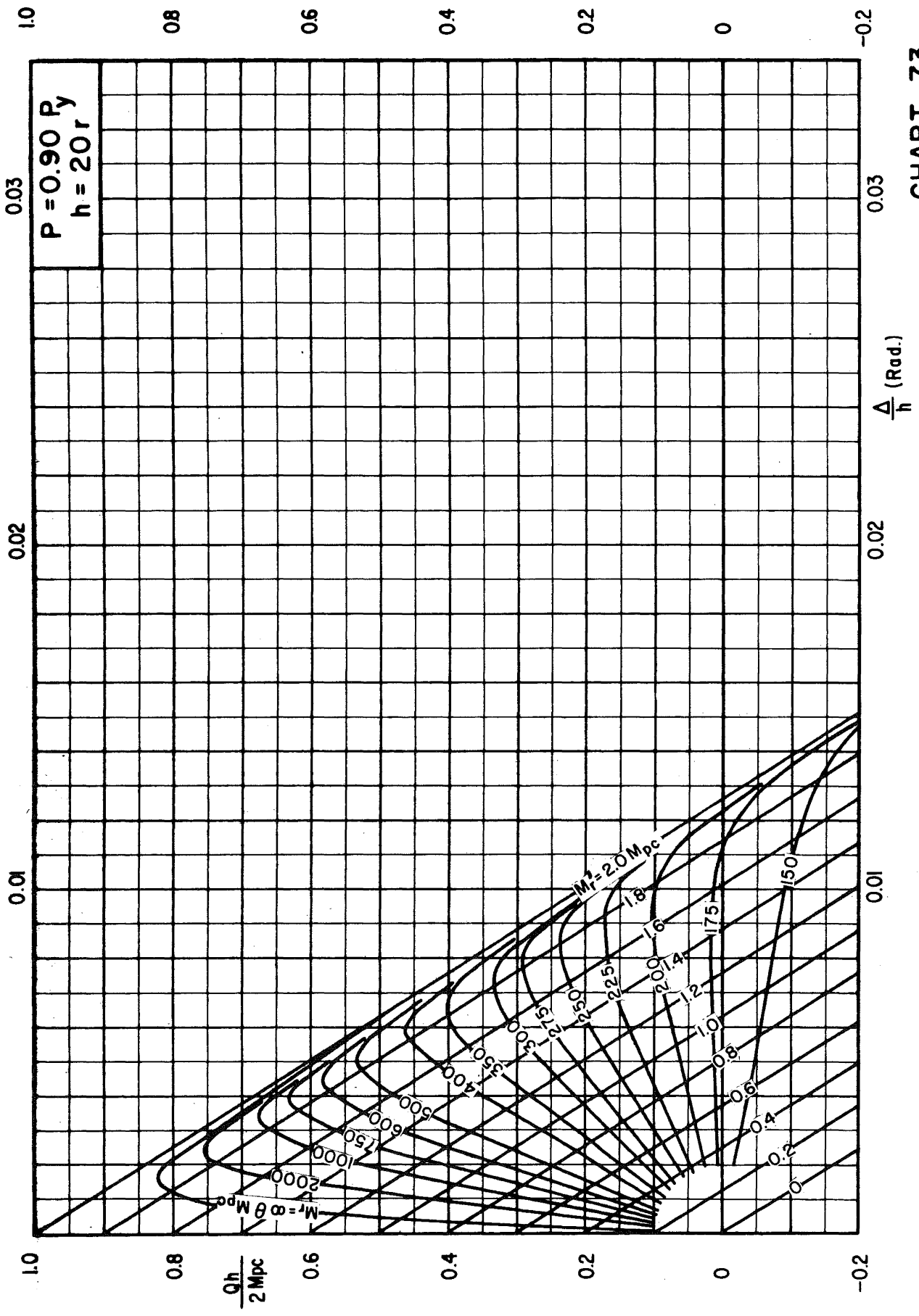


CHART 73

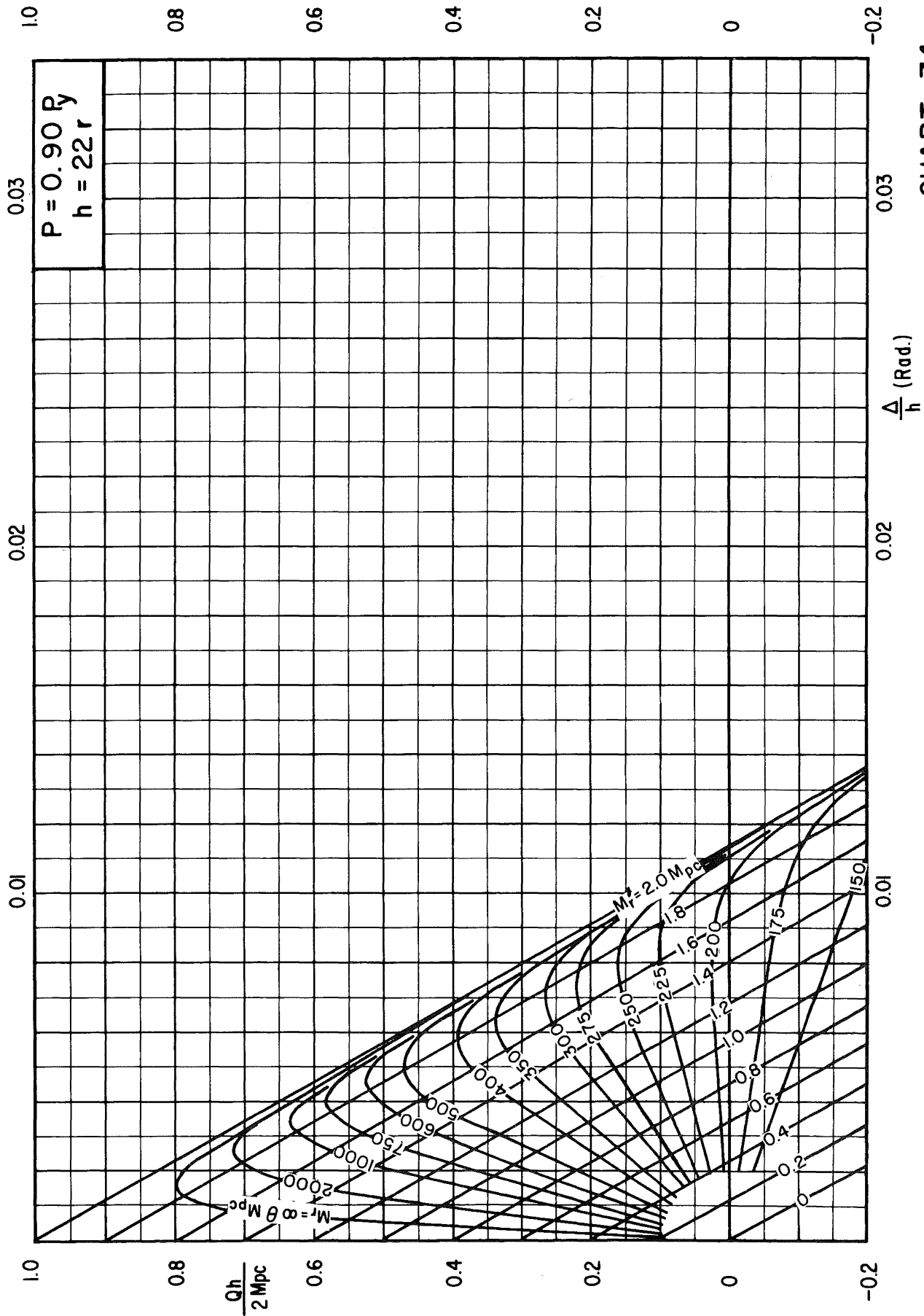


CHART 74



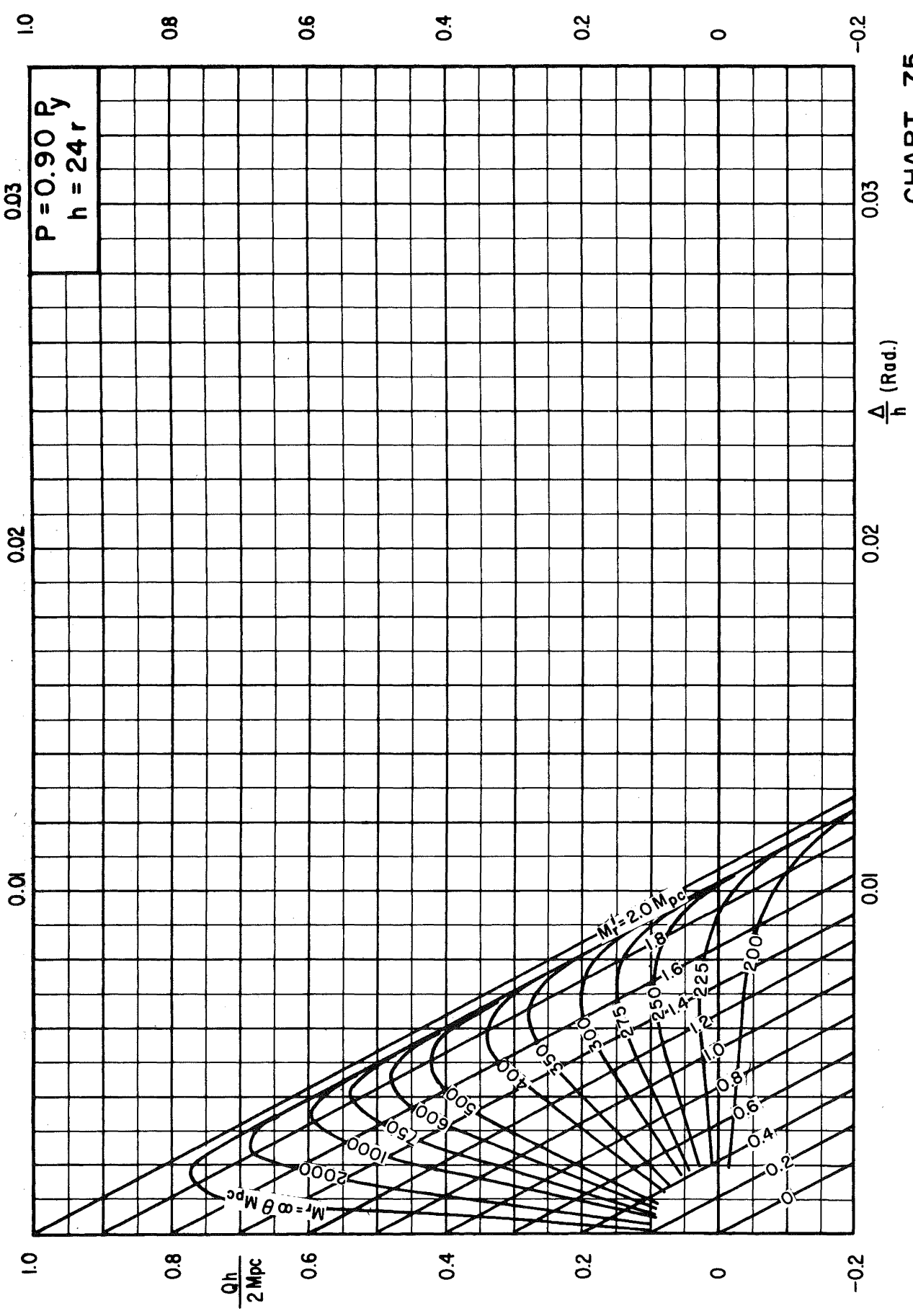


CHART 75

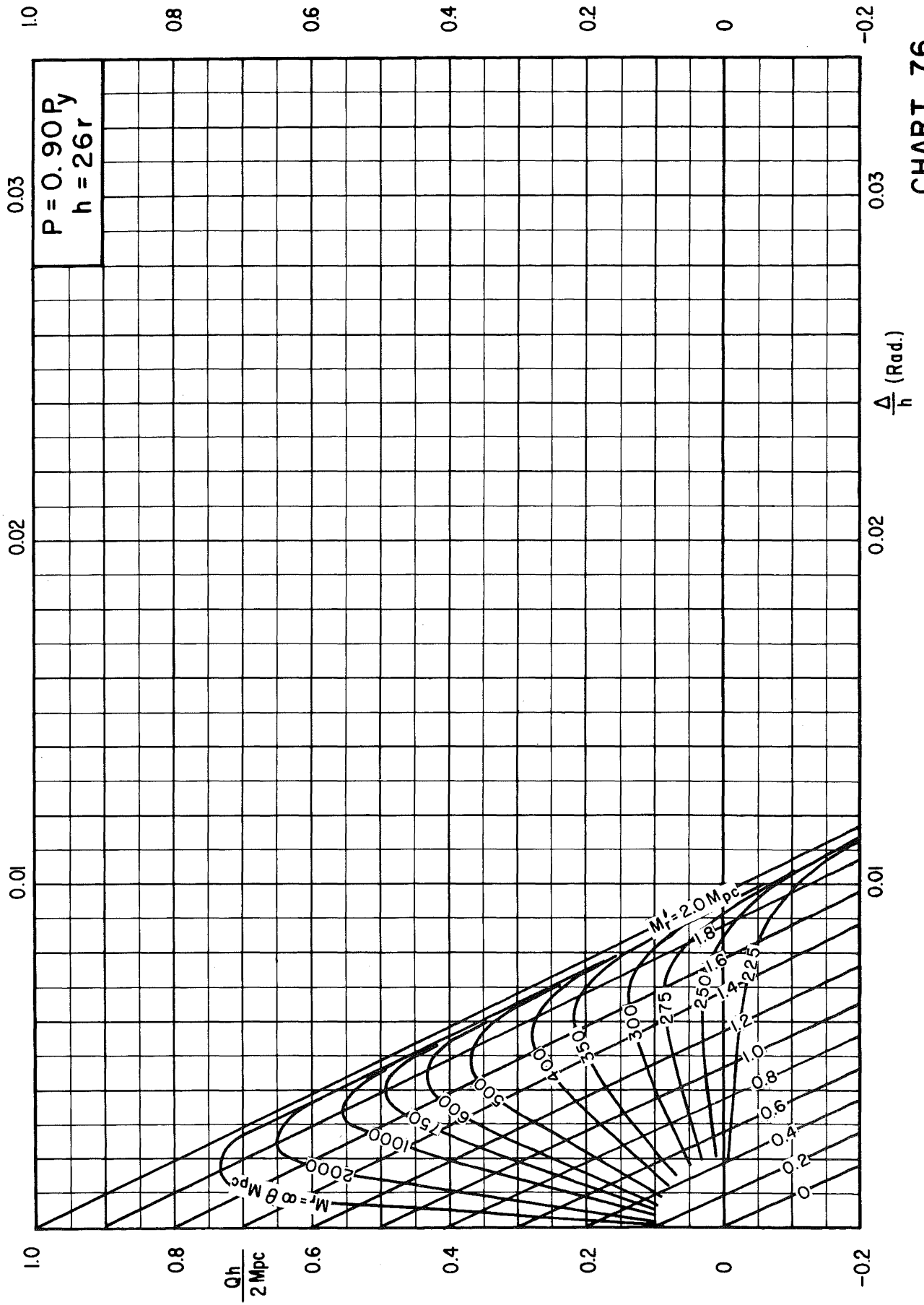


CHART 76

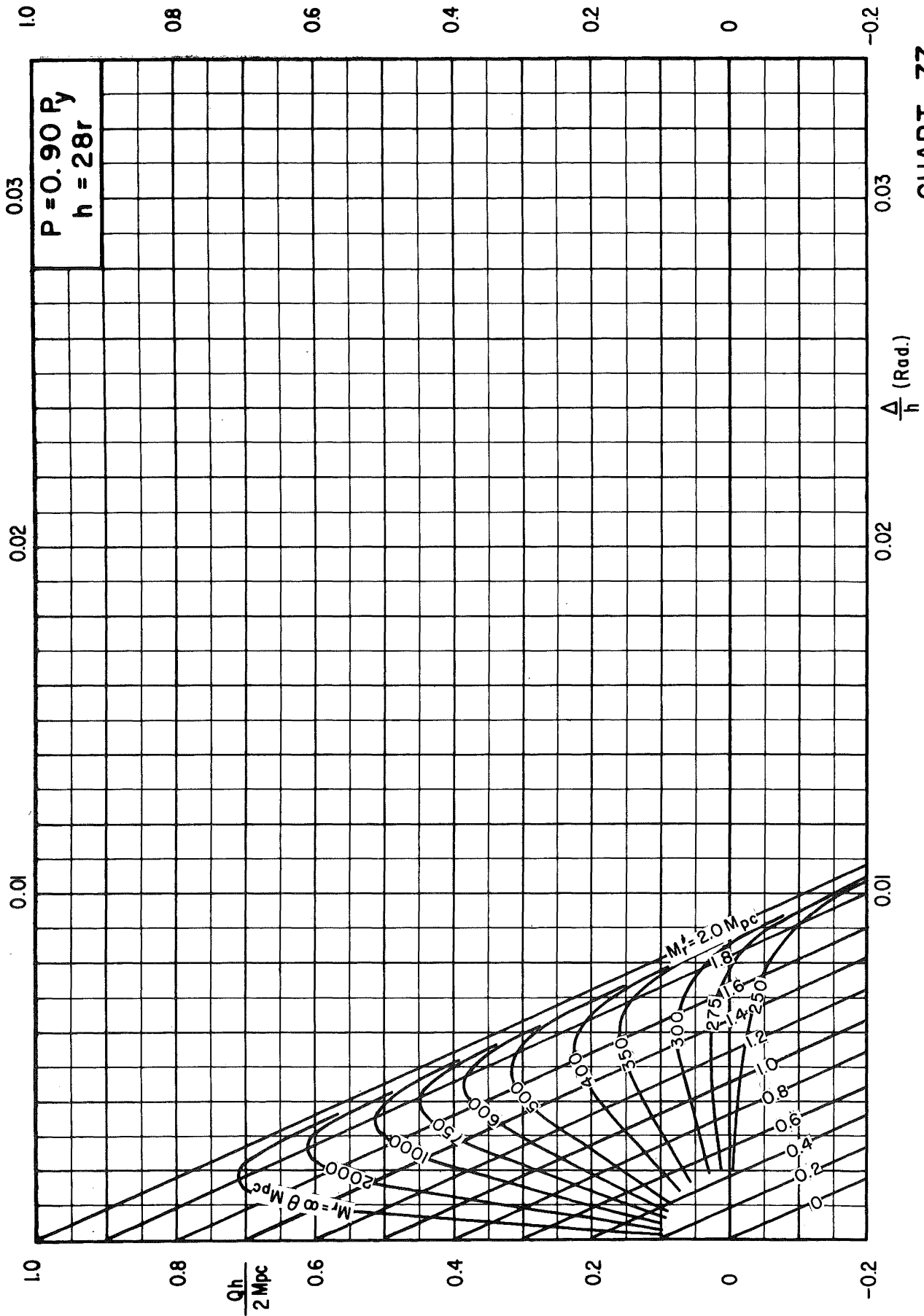


CHART 77

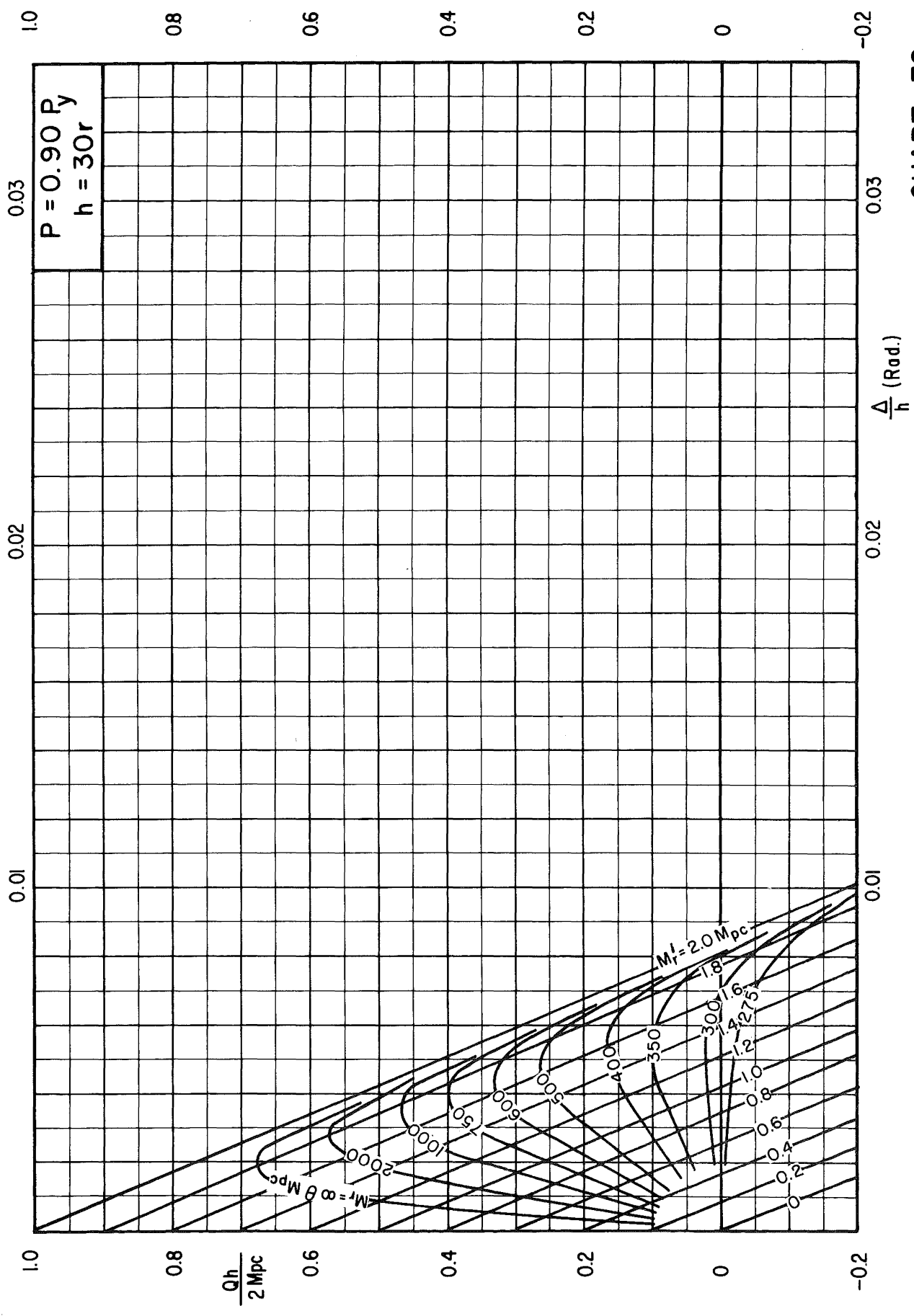


CHART 78

