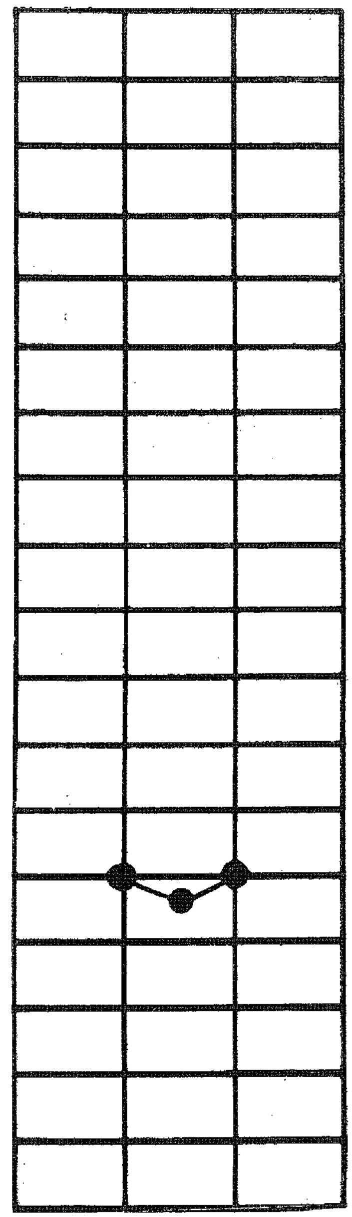
1965

Plastic design of multi-story frames: guest lectures, Lehigh University, Summer Conference, 1965 (65-32)

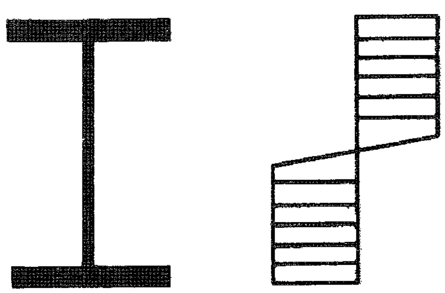
L. Tall

GUEST LECTURES



The
1965 Summer Conference
on

PLASTIC DESIGN OF MULTI-STORY FRAMES



LEHIGH UNIVERSITY
Centennial Anniversary 1965-66

Lehigh University
Department of Civil Engineering
Fritz Engineering Laboratory

1965 Summer Conference

P L A S T I C D E S I G N

O F

M U L T I--S T O R Y F R A M E S

Guest Lectures by:

G. V. Berg

H. J. Degenkolb

J. Heyman

I. Hooper

M. R. Horne

B. G. Johnston

T. Kawai

K. L. Majid

E. P. Popov

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Bethlehem, Pennsylvania

Fritz Engineering Laboratory Report No. 273.46

PREFACE

This series of papers comprise the lectures presented by guest speakers at the 1965 Summer Conference on the Plastic Design of Multi-Story Frames, held at Lehigh University. In effect, this book constitutes part of the proceedings of the conference.

The conference lectures presented the results of research conducted at Lehigh---it was therefore fitting to balance this viewpoint with the results of research and practice conducted elsewhere, both in this country and abroad. Thus, the guest speakers came from all parts of the globe.

The main part of the conference consisted of lectures given by members of the civil engineering department staff at Lehigh University---the lectures were supplemented by a series of laboratory demonstrations to illustrate the principles. These lectures are presented in the books, Plastic Design of Multi-Story Frames, Lecture Notes and Design Aids, published by Lehigh University.

The financial support for the conference was given by the American Iron and Steel Institute, the National Science Foundation, and Lehigh University.

The guest lecturers were:

Glen V. Berg, University of Michigan

Roy W. Clough, University of California, Berkeley

Henry J. Degenkolb, Consulting Engineer, San Francisco

Yuzura Fujita, University of Tokyo, Japan

Jacques Heyman, University of Cambridge, England
Ira Hooper, Consulting Engineer, New York
Michael R. Horne, University of Manchester, England
Bruce G. Johnston, University of Michigan
Tadahiko Kawai, University of Tokyo, Japan
Robert L. Ketter, State University of New York at Buffalo
K. I. Majid, University of Cambridge, England
Nathan M. Newmark, University of Illinois
Egor P. Popov, University of California, Berkeley
John W. Roderick, University of Sydney, Australia
Bruno Thurlimann, Federal Institute of Technology, Switzerland
Udo Vogel, Stuttgart Institute of Technology, Germany
Minoru Wakabayashi, Kyoto University, Japan

The preparation for publication of this series of papers was co-ordinated by Lambert Tall and assisted by Richard K. McFalls.

July 1966
Bethlehem, Pennsylvania

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OPTIMUM DESIGN OF STRUCTURES

by

Bruno Thurlimann
Swiss Fed. Inst. of Tech.

1. Introduction

From an engineering point of view a structure must (1) fulfill its intended use, (2) exhibit the prescribed safety margins (3) respect certain architectural requirements, and (4) be built at a minimum cost. In the past engineers have tried to achieve these objectives through knowledge, experience and intuition. The following is an attempt to put structural design on a more rational basis.

2. Previous Attempts at Solution

Using Elastic Design an explicit analysis of a statically determinate system is possible. For statically indeterminate systems, however, an analysis by trial and error is the only way to arrive at an economical solution. For the forces and stresses in a structure are dependent on the stiffness of the members.

With the introduction of Plastic Design direct solutions have become possible. Using a linear relationship between ultimate load and cross sectional resistances methods of minimum weight design have been introduced (e.g. (1), (2)).

In this paper the structural design problem is put in such a form that Linear Programming Techniques can be applied to its solution. These techniques have been developed and widely used in the field of Operations Research.

3. Simple Example

A simple example is chosen first such that the essentials of the problem are not unnecessarily obscured. Shown in Fig. 1 is a top view of a grid system with two axis of symmetry. The resistance values W (or in this simple example the plastic moments M_p) are W_1, W_2 for the longitudinal and W_3 for the transverse girders as indicated in the figure. Three possib'le loading cases will be investigated, namely (a) uniform load represented by concentrated loads P perpendicular to the plane of the grid at each joint 1 to 6, (b) one concentrated load 3 P at joint 1 or joint 2, and (c) one concentrated load at joint 3 or the other symmetrically located joints 4, 5 or 6.

Assuming no torsional stiffness for the individual girders the system is two times statically indeterminate for vertical loading. To obtain a statically determinate base system hinges are introduced in the beams (3-5) at joint 1 and (4-6) at joint 2. The corresponding redundants are R_7 and R_8 respectively. The moments in the longitudinal girders will be labeled M_1 to M_6 acting at the joints 1 to 6.

The problem consists in finding the relative sizes of the beams such that the design becomes an optimum. Before developing the solution explanations concerning the meaning "optimum" are necessary.

4. Cost Function

Under "optimum" a minimum of the total cost shall be understood. A relationship between cost and resistance value W of a member follows from the following consideration. Fig. 2 shows a wide flange steel section of area A and yield stress σ_y. The resistance value or in this case the plastic moment M_p is:

$$W = M_p = \frac{1}{2} Ay\sigma_y \tag{1}$$

Introducing the parameter

$$\rho = \frac{A_w}{A_F} = \frac{A - 2A_F}{A_F} \quad ; \qquad A = A_F\,(2+\rho) \tag{2}$$

and the plastic section modulus

$$Z = \frac{M_p}{\sigma_y} = A_F h(1 + \frac{\rho}{4})$$

the internal distance y between the resultant compression and tension forces becomes

$$y = 2h\,\frac{1 + \rho/4}{2 + \rho} = h\,\frac{4 + \rho}{4 + 2\rho} \tag{3}$$

with the extreme values

for $A_W = 0$, $\rho = 0$ $\qquad y = h$

and for $A_F = 0$, $\rho \to \infty$ $\qquad y = h/2$.

Taking the simplest possible assumption that the cost is equal to the weight of the section the cost per unit length f is

$$f = c_1 A$$

with c_1 a proportionality factor having the dimensions $\$/ft^3$. Replacing A by its relationship from (1)

$$f = c_1 A = \frac{2c_1}{y\sigma_y}\,W \tag{4}$$

If for a given steel structure the ration $2c_1/\sigma_y$ is a constant a relative cost function for the entire structure can be written as follows

$$F = \Sigma \frac{l_i}{y_i} W_i \qquad (5)$$

where l_i and y_i are the length and internal distance of the member i having constant cross section and a resistance value W_i .

For reinforced concrete sections a similar expression can be derived. In Fig. 3 a rectangular cross section is shown. Assuming a rectangular compressive stress block the plastic Moment M_p follows to:

$$M_p = A_s \sigma_y h \; (1 - \xi/2) \qquad (6)$$

The dimensionless distance of the neutral axis is

$$\xi = \frac{\sigma_y}{\beta} \mu \qquad (7)$$

where: σ_y = yield stress of the reinforcement
A_s = Aera of the reinforcement
β = strength of concrete
μ = A_s/bh = percentage of reinforcement .

For underreinforced sections the value of the term$(1 - \xi/2)$ is practically constant and equals about 0.9. Hence

$$W = M_p \cong 0.9 \; A_s \sigma_y h \quad .$$

A simple cost function can be derived if it is assumed that the concrete section is given and the only free variable is the amount of reinforcement. Following the derivation for the steel section

$$f = c_1 A_s$$

or
$$f = \frac{c_i}{0.9 \; \sigma_y h} W$$

and for a constant yield stress of all reinforcements in a structure the cost function

$$F = \Sigma \frac{l_i}{h_i} W_i \quad . \qquad (8)$$

It is obvious that the cost functions (5) and (8) may need considerable refinement if practical problems are to be solved. Such refinements should consider the influence of different yield stress levels, the fabricating costs of connections, stiffeners etc. Furthermore, it may be necessary to approximate the actual cost function by a linear relationship in the pertinent range of application as shown in Fig. 4.

5. Plastic Analysis of Structure

Having defined a cost function the relationship between loads, resistance values and cost must be established. Using Simple Plastic Theory the Lower Bound Theorem requires that a state of stress in equilibrium with the applied loads is found which does nowhere violate the plasticity conditions.

For a n-times statically indeterminate frame structure a general equilibrium state can be uniquely defined as follows. The moment at a location i is

$$M_i = M_{io} + \sum_{k=1}^{n} m_{io} R_k \tag{9}$$

where: M_{io} = moment at i in the statically determinate base system due to the external loads - inhomogeneous equilibrium state

m_{ik} = moment at i due to the action of the redundant $R_k = 1$ - homogeneous equilibrium state .

For a particular loading condition the general equilibrium state is hence made up of one inhomogeneous state due to the external loads and n independent homogeneous states due to the action of the n redundants R_k .

Taking the example of Fig. 1 with the two redundant moments R_7 and R_8 acting at joints 1 and 2 of the transverse girders (3-5) and (4-6) respectively, Fig. 5 shows the homogeneous equilibrium state due to $R_7 = 1$. In Fig. 6 the inhomogeneous equilibrium state due to the loading cases (b) is given. The general equilibrium state is shown in the form of a matrix in Table 1 with the two independent variables R_7 and R_8.

According to the Lower Bound Theorem all the moments M_i should not violate the plasticity condition, hence must be bound between the positive plastic resistance value W_{Pi} and the negative plastic resistance value W_{Ni}

$$-W_{Ni} \leq M_i \leq W_{Pi} \quad . \tag{10}$$

For a steel section the positive and negative resistance values are identical $W_{Ni} = W_{Pi} = W_i$. Hence the plasticity condition for the moment M_1 can be written

$$-W_1 \leq M_1 \leq W_1 \quad . \tag{11}$$

For the application of the Linear Program it is necessary to introduce non-negative variables

$$X_1 = W_1 - M_1 \geq 0 \tag{12a}$$

$$Y_1 = W_1 + M_1 \geq 0 \tag{12b}$$

with the relationship

$$Y_1 = 2W_1 - M_1 \tag{13}$$

as illustrated in Fig. 7. Hence the two inequalities of Eq. (11) are replaced by two equations (12a), (12b) and the limitations $X_1 \geq 0$ and $Y_1 \geq 0$.

The redundants which are the independent variables are similarly treated

$$- W_3 \leq R_7 \leq W_3 \tag{14}$$

and $\quad \bar{X}_7 = W_3 - R_7 \geq 0 \qquad (15a)$

$$\bar{Y}_7 = W_3 + R_7 \geq 0 \tag{15b}$$

with $\quad \bar{Y}_7 = 2W_3 - \bar{X}_7 \qquad (16)$

Similar expressions are developed for all dependent variables M_i and independent variables R_k. The resulting system of equations represents a lower bound solution. The problem consists in varying the independent variables $\bar{X}_k$ in such a way that the cost function (5)

$$F = \Sigma \frac{l_i}{y_i} W_i$$

becomes a minimum. If the internal distance y_i (distance between the tension and compression resultants, Fig. 2) is constant, the cost function can be simplified to

$$F = \Sigma l_i W_i \tag{17}$$

as only the relative value of the function is of importance.

6. Design Restrictions

In practical applications restrictions may be imposed on the design. For instance the optimum solution may indicate that a member should have W = 0, i.e. should not be present at all. However, architectural requirements indicate the necessity of such a member. Such design restrictions can be classified into three different categories:

a) Relative maximum of a resistance value, e.g.

$$W_2 \leq W_1$$

and on introducing a non-negative auxiliary variable

$$Z = W_1 - W_2 \geq 0 \quad . \tag{19}$$

b) Absolute maximum of a resistance value, e.g.

$$W_3 \leq 0.3\ Pl$$

or $$Z = 0.3\ Pl - W_3 \geq 0 \quad . \tag{20}$$

c) Relative depth of girders, e.g.

$$h_1 = h \ ; \quad h_2 = 4/3\ h \ ; \quad h_3 = 1/2\ h \tag{21}$$

Restriction (c) leads only to a change in the cost function (5), influencing the value of y_i.

It is quite obvious that such restrictions will influence the solution of the Linear Program.

7. Linear Program

The problem is now put into the form of a Linear Program. First the redundants R_k (independent variables), expressed in $\overline{X}$, $\overline{Y}$ according to eq. (15) and the moments M_i, expressed in Y, Y according to eq. (12) are introduced into the equilibrium equation, Table 1. Taking e.g. the moment M_1 due to loading case (a)

$$M_1 = -\ 8/3\ R_7 - 4/3\ R_8 + 2\ Pl$$

the substitution gives

$$(W_1 - X_1) = -\ 8/3\ (W_3 - \overline{X}_7) - 4/3\ (W_3 - \overline{X}_8) + 2\ Pl \tag{18}$$

and

$$X_1 = -\ 8/3\ \overline{X}_7 - 4/3\ \overline{X}_8 + W_1 + 4\ W_3 - 2\ Pl \quad . \tag{19}$$

Similar substitutions in the other equilibrium equations of Table 1 lead to the equations represented in form of a Linear Program in Table 2. Some of the checks on the plasticity condition (e.g. negative moments in the longitudinal girders) are obviously irrelevant and hence deleted.

The cost function (17) leads to

$$F = \Sigma l_i W_i = l(6W_1 + 12W_2 + 2\cdot 2W_3) \quad .$$

As only the relative value is of interest in finding its minimum, it can be written as

$$\overline{F} = 6W_1 + 12W_2 + 4W_3 \qquad (20)$$

as shown in Table 2.

Finally three design restrictions are introduced:

a) Relative restriction

$W_1 \geqq W_2$, keeping h = constant

or $Z_1 = W_1 - W_2 \geqq 0$

b) Absolute restriction

$W_3 \leqq 0.3Pl$, keeping h = constant

$Z_2 = 0.3Pl - W_3 \geqq 0$

c) Variation in girder depth:

Long. Girders W_2 : $h_2 = h$

Long. Girder W_1 : $h_1 = 2/3\ h$

Trans. Girders W_3 : $h_3 = 4/3\ h$

Change in the cost Function, eq. (20):

$$\overline{F} = (6W_1 \frac{h}{h_1} + 12W_2 \cdot \frac{h}{h_2} + 4W_3 \cdot \frac{h}{h_3})$$

$$= 9W_1 + 12W_2 + 3W_3 \qquad (21)$$

The Linear Program, Table 2, can be solved for instance with the "Simplex Algorithm". A corresponding program has been written (3) for the CDC-1604 computer. In the following only the results will be discussed.

8. Discussion of Results

The standard solution with no design restrictions leads to the following results:

$$\begin{aligned} \text{Resistance Values :}\quad & W_1 = 2.0\ Pl \\ & W_2 = 3.0\ Pl \\ & W_3 = 0.5\ Pl \\ \text{Cost Function :}\quad & F = 6W_1 + 12W_2 + 4W_3 = 50 \quad . \end{aligned} \tag{22}$$

The moment diagrams corresponding to the 3 different loading cases are shown in Fig. 8. It should be pointed out that they are statically admissible equilibrium states, i.e. they do not violate the plasticity conditions. However, they do not necessarily correspond to the actual distribution of the internal forces and external reactions in a real case. The actual distribution differs from the shown one by appropriate support movements, hence by a superposition of a homogeneous equilibrium state.

The cases with design restrictions give the following results:

a) Relative Restriction : $W_1 \geq W_2$; $h = \text{const}$

$$\begin{aligned} \text{Resistance Values :}\quad & W_1 = 2.67\ Pl \\ & W_2 = 2.67\ Pl \\ & W_3 = 0.67\ Pl \\ \text{Cost Function :}\quad & F = 6W_1 + 12W_2 + 4W_3 = 50{,}67 \end{aligned} \tag{23}$$

b) Absolute Restriction : $W_3 = 0.3Pl$; $h = const$

Resistance Values : $W_1 = 2.8\ Pl$

$$W_2 = 3.4\ Pl \tag{24}$$

$W_3 = 0.3\ Pl$

Cost Function : $F = 6W_1 + 12W_2 + 4W_3 = 58.8$

c) Variation in girder depth:

Resistance Values : $W_1 = 0\ \ Pl$

$$W_2 = 4.0\ Pl \tag{25}$$

$W_3 = 1.5\ Pl$

Cost Function : $F = 9W_1 + 12W_2 + 3W_3 = 52.5$ *) .

These results show that restrictions may influence the solution in an appreciable manner.

9. Matrix-Formulation

In the following a general formulation in matrix notation is given. The moment at a section i of a frame structure is:

$$m = Ar + b \tag{26}$$

where m is the moment vector, r the redundant vector, b the load vector and A the matrix for the unit actions of the redundants, hence

$$m = \begin{bmatrix} M_1 \\ \cdot \\ \cdot \\ \cdot \\ M_n \end{bmatrix} ; \qquad r = \begin{bmatrix} R_1 \\ \cdot \\ \cdot \\ \cdot \\ R_k \end{bmatrix} ; \qquad b = \begin{bmatrix} B_1 \\ \cdot \\ \cdot \\ \cdot \\ B_n \end{bmatrix}$$

and $$A = \begin{bmatrix} a_{11} & \cdots\cdots\cdots & a_{1k} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{n1} & & a_{nk} \end{bmatrix}$$

---

*) Based on assumption that girder 2 has same depth as in a) and b). In case of constant depth the W-values of c) would give $F = 6W_1 + 12W_2 + 4W_3 = 54$.

The plasticity condition follows to

Moments : $\quad - w_N \leq m \leq w_P$ (27)

In terms of the non-negative variables x and y :

$$x = w_P - m \geq 0$$

$$y = w_N + m \geq 0 \qquad (28)$$

with $\quad y = w_P + w_N - x$

Redundants: $- \overline{w}_N = r \leq \overline{w}_P$ (29)

In terms of the non-negative independent variables $\overline{x}$ and $\overline{y}$:

$$\overline{x} = \overline{w}_P - r \geq 0$$

$$\overline{y} = \overline{w}_N + r \geq 0 \qquad (30)$$

with $\quad \overline{y} = \overline{w}_P + w_N - \overline{x}$

Substituting the m - and r - vectors in eq. (26) by their expressions from eq. (28) and (30) gives:

$$x = A\overline{x} - A\overline{w}_P + w_P - b$$

$$y = -A\overline{x} + A\overline{w}_P + w_N + b$$

and $\quad \overline{y} = - \overline{x} + \overline{w}_P + \overline{w}_N$.

For each loading case a Linear Program represented in Table 3 can be written. The design restrictions constitute a vector

$$z = \begin{bmatrix} Z_1 \\ \cdot \\ \cdot \\ \cdot \\ Z_s \end{bmatrix}$$

where the components $Z_i = Z_i \ (w_P; \ \overline{w}_P; \ w_N; \ \overline{w}_N; \ b)$ are linear functions of the resistance values and/or the loading parameter. The cost function finally is a linear function of the resistance values, hence

$$F = F \ (w_P; \ \overline{w}_P; \ w_N; \ \overline{w}_N)$$

The special form of this program is quite apparent in Table 3. For its solution a special computer program has been written in connection with the paper (3) as mentioned previously.

10. Final Remark

The method presented in this paper is based on the Lower Bound Theorem of Plastic Analysis. An Upper Bound Approach using kinematically admissible velocity fields can be used as well. It is believed, however, that in an actual application the first approach will be simpler as the number of independent variables is equal to the number of redundants. In the kinematic approach the determination of the independent mechanism appears to be more cumbersome.

11. List of References

(1) J. Foulkes : Minimum Weight Design of Structural Frames, Proc. Roy. Soc. (London) A 223, p. 482 (1954).

(2) J. Heyman, W. Prager : Automatic Minimum Weight Design of Steel Frames, Journal Franklin Inst. 266, p. 339 (1958).

(3) Anderheggen, E. : Optimale Bemessung von Stabtragwerken, PhD-Dissertation, Swiss Federal Inst. of Technology (1965).

	Redundants		Loadings		
	R_7	R_8	(a) Pl	(b) Pl	(c) Pl
M_1 =	$-\ 8/3$	$-\ 4/3$	2	4	0
M_2	$-\ 4/3$	$-\ 8/3$	2	2	0
M_3	4/3	2/3	2	0	4
M_4	2/3	4/3	2	0	2
M_5	4/3	2/3	2	0	0
M_6	2/3	4/3	2	0	0
$M_7 = R_7 =$	1	0	0	0	0
$M_8 = R_8 =$	0	1	0	0	0

Table 1 : Equilibrium Matrix : $m = Ar + b$

	$\overline{X}_7(a)$	$\overline{X}_8(a)$	$\overline{X}_7(b)$	$\overline{X}_8(b)$	$\overline{X}_7(c)$	$\overline{X}_8(c)$	W_1	W_2	W_3	Pl
$X_1(a) =$	-8/3	-4/3					1		4	-2
$X_3(a) =$	4/3	2/3						1	-2	-2
$\overline{Y}_7(a) =$	-1								2	
$X_1(b) =$			-8/3	-4/3			1		4	-4
$X_3(b) =$			4/3	2/3				1	-2	
$\overline{Y}_7(b) =$			-1						2	
$X_1(c) =$					-8/3	-4/3	1		4	
$X_3(c) =$					4/3	2/3		1	-2	-4
$\overline{Y}_7(c) =$					-1				2	
$F =$							6	12	4	
$Z_1 =$							1	-1		
$Z_2 =$									-1	0.3
$F_{Mod} =$							9	12	3	

Table 2 : Linear Program

	$\bar{X}_1 \cdots \bar{X}_k$	$W_{P1} \cdots W_{Pn}$	$\bar{W}_{P1} \cdots W_{Pk}$	$W_{N1} \cdots W_{Nn}$	$W_{N1} \cdots W_{Nk}$	
$X_1 =$	$a_{11} \cdots a_{1k}$	1	$-a_{11} \cdots -a_{1k}$			$-B_1$
$\vdots$	$\vdots \quad \vdots$	$\ddots$	$\vdots \quad \vdots$			$\vdots$
$X_n =$	$a_{n1} \cdots a_{nk}$	1	$-a_{n1} \cdots -a_{nk}$			$-B_n$
$Y_1 =$	$-a_{11} \cdots -a_{1k}$		$a_{11} \cdots a_{1k}$	1		B_1
$\vdots$	$\vdots \quad \vdots$		$\vdots \quad \vdots$	$\ddots$		$\vdots$
$Y_n =$	$-a_{n1} \cdots -a_{nk}$		$a_{n1} \cdots a_{nk}$	1		B_n
$\bar{Y}_1 =$	-1		1		1	
$\vdots$	$\ddots$		$\ddots$		$\ddots$	
$\bar{Y}_k =$	-1		1		1	
$F =$		c_1 - - - - - -	- - - - - -	- - - - - - -	- - - - c_m	
$Z_1 =$		d_{11} - - - - - -	- - - - - -	- - - - - - -	- - - - d_{1m}	D_1
$\vdots$		$\vdots$			$\vdots$	$\vdots$
$Z_s =$		d_{s1} - - - - - -	- - - - - -	- - - - - - -	- - - - d_{sm}	D_s

Table 3 : General Linear Program in Matrix Form

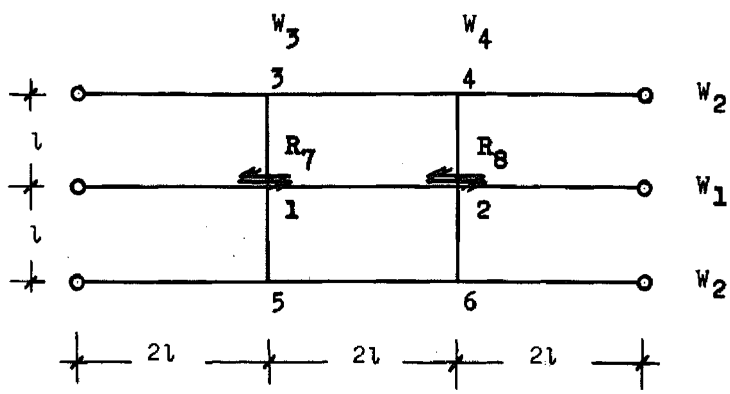


Fig. 1

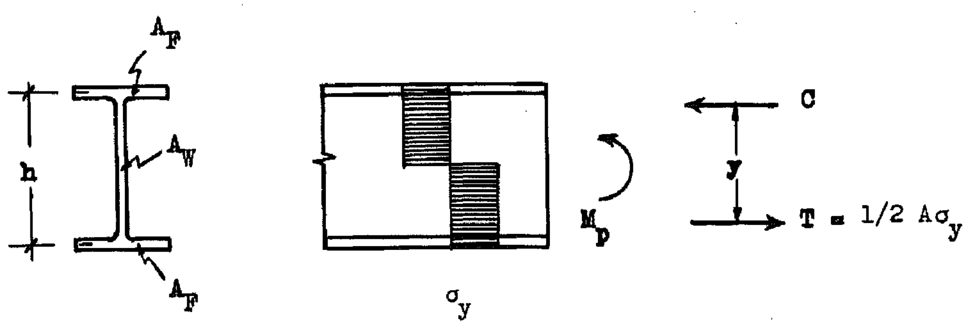
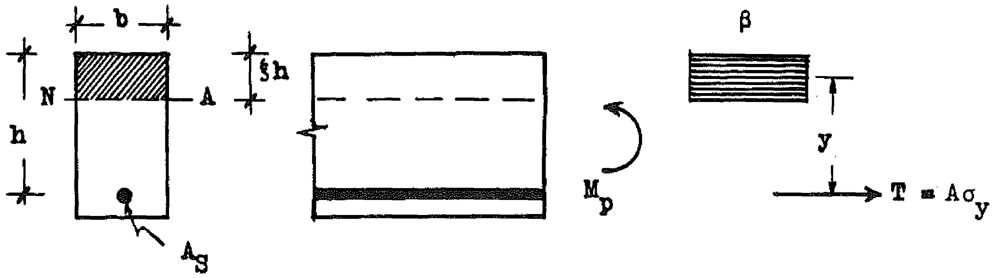
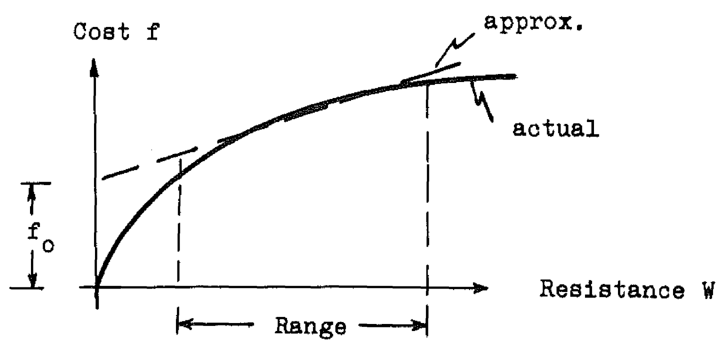


Fig. 2



b
N
A
h
ξh
A_S
M_p
β
y
$T = A\sigma_y$

Fig. 3



Cost f
approx.
actual
f_o
Resistance W
Range

Fig. 4

Action $R_7 = 1$:

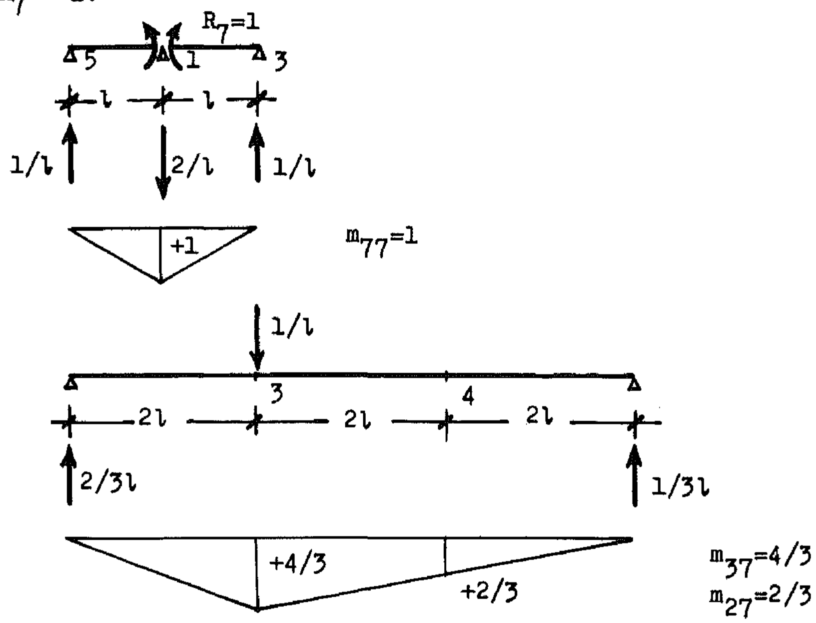


Fig. 5

Case (b)

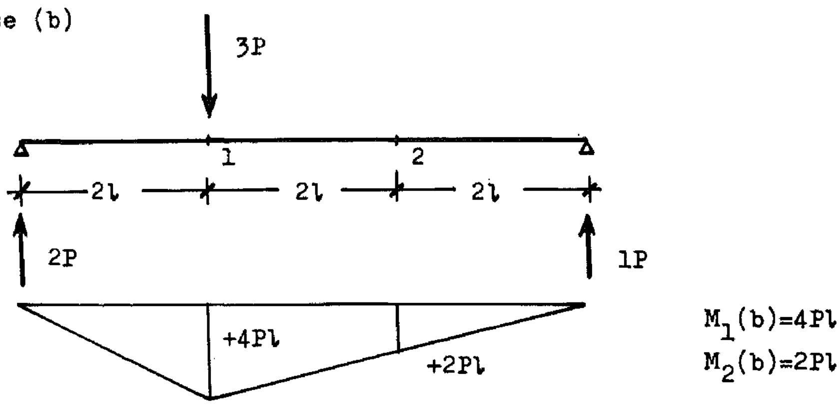


Fig. 6

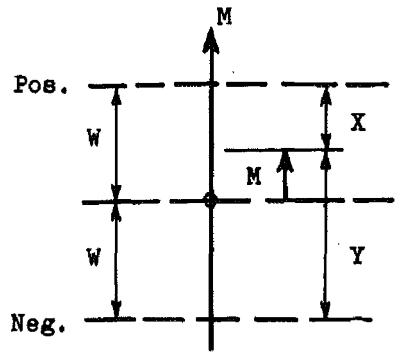

M
Pos.
W
X
M
W
Y
Neg.

Fig. 7

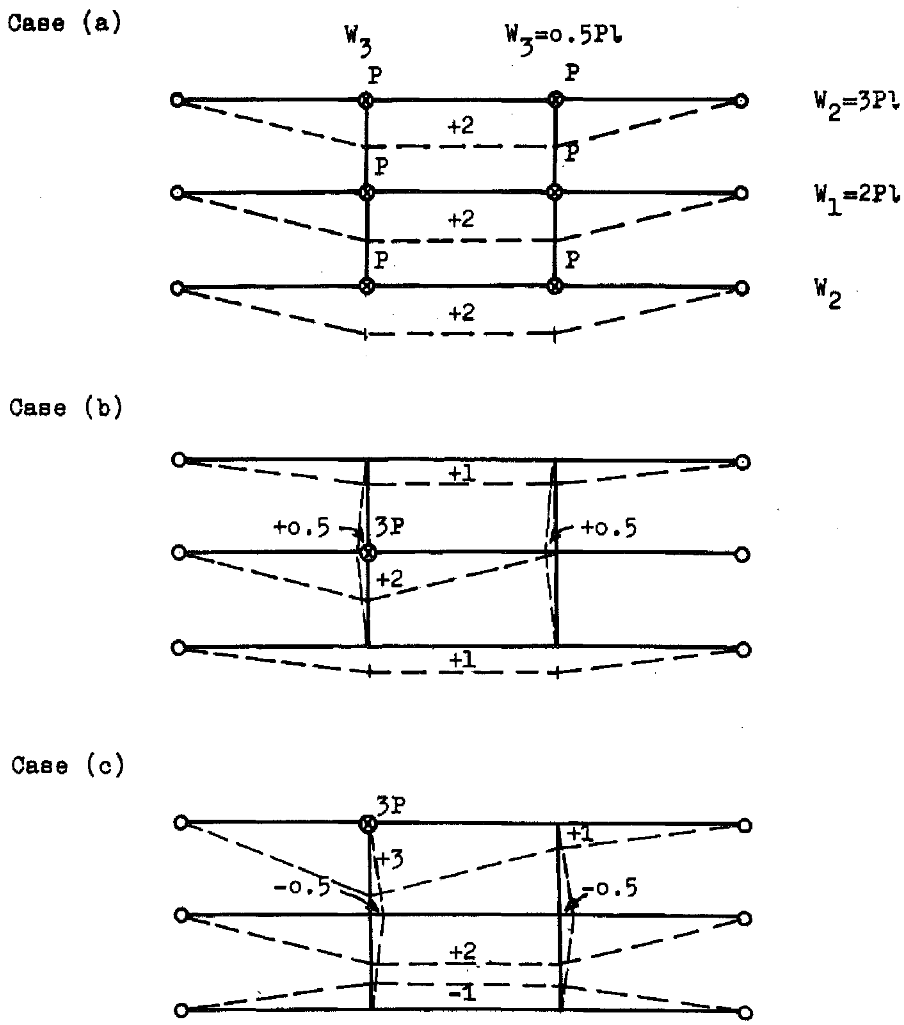

Case (a)
W_3
W_3=o.5Pl
P
P
W_2=3Pl
+2
P
P
W_1=2Pl
+2
P
P
W_2
+2
Case (b)
+1
+o.5
3P
+o.5
+2
+1
Case (c)
3P
+1
+3
-o.5
-o.5
+2
-1

Fig. 8

PLASTIC ANALYSIS AND MINIMUM WEIGHT DESIGN OF MULTI-STORY PLANE FRAMES

by

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A new approach to minimum weight design of multi-story plane portal frames is proposed based on the mechanism method. Development of a computer program and analysis of several test frames are also made in comparison with J. Heyman's method.

The outline of the proposed method is indicated as follows:

(a) FORMULATION OF ALL THE POSSIBLE COLLAPSE MODES OF A GIVEN FRAME

All the possible collapse modes and the corresponding equations of virtual work of a given frame can be automatically formulated through the simple combination of basic data for mechanisms of two elementary structural aggregates. Therefore the ultimate load can be easily determined by computing the smallest limit load under given M_p values.

(b) SCREENING OF THE RESTRAINING INEQUALITIES OBTAINED IN STEP (a)

The number of restraining inequalities obtained in (a) which may increase so fast with the structural complexity can be effectively reduced by computing the limit load of each mechanism for appropriately assumed M_p values and selecting a number of important inequalities which may be operative in the final solution of minimum weight design through comparison and arrangement of these collapse loads in the order.

(c) DETERMINATION OF LOWER BOUND SOLUTION BY LINEAR PROGRAMMING UNDER THE RESTRICTED NUMBER OF INEQUALITIES OBTAINED IN STEP (b)

A lower bound solution of a given problem can be determined by the well established "Dual Simplex Method" in Linear Programming under the restricted number of inequalities obtained in step (b).

These three steps of calculation can be combined, connected and looped into an unified process of automatic computation of which modern electronic digital computers are capable.

It is emphasized that the size of the problems to be solved will not be practically influenced by the scale of the digital computer to be used.

(I) METHOD OF FORMULATION OF ALL THE POSSIBLE COLLAPSE MODES OF A GIVEN FRAME

TWO ELEMENTARY STRUCTURAL AGGREGATES AND SIX JOINTS WHICH CONTROL COLLAPSE MODES OF MULTI-STORY PLANE PORTAL FRAMES

All the possible collapse modes and the corresponding equations of virtual work of a given frame of any structural complexity can be automatically formulated through simple combination of basic data for mechanisms of two elementary structural aggregates as shown in Fig. 2 (a). Furthermore, the mechanisms of these two structural aggregates can be also constructed by simple combination of those of six elementary joints shown in Fig. 2 (b).

POSSIBLE MECHANISMS OF 6 JOINTS

Mechanisms of these six joints are shown in the Tables 1.1, 1.2, 1.3, and 1.4.

COLLAPSE MODES OF ROOF AND INTERMEDIATE FLOOR SYSTEMS

With reference to these tables, combined mechanisms of two elementary structural aggregates can be easily formulated for an arbitrary number of spans. The combined mechanisms thus formulated for one span, two spans and three spans are shown in the Tables 2, 3.1 - 3.3 and 4.1 - 4.4 respectively.

NUMBER OF ALL THE POSSIBLE COMBINED MECHANISMS OTHER THAN BEAM MECHANISMS

The number of all possible combined mechanisms of a given portal frame (m storys, n bays) in which local beam mechanisms are excluded can be determined by the following formula: (see Fig. 3)

$$L\ (m,n) = M\ (n)\ (N_1\ (n))^{m-1} +$$

$$\sum_{p=1}^{m-1}\{M\ (n)\ N_3\ (n)+(m-p-1)\ N_2\ (n)\ N_3\ (n) + N_2\ (n)\}\ (N_1\ (n))^{p-1}$$

where

$L\ (m,n)$ = Total number of combined mechanisms excluding local beam mechanisms

$M\ (n)$ = Number of mechanisms of the roof system of n spans

$N_1\ (n)$ = Number of mechanisms of type 1 for the intermediate floor system of n spans

$N_2\ (n)$ = Number of mechanisms of type 2 for the intermediate floor system of n spans

$N_3\ (n)$ = Number of mechanisms of type 3 for the intermediate floor system of n spans

FORMULATION OF THE EQUATIONS OF VIRTUAL WORK $W_{in} = W_{ex}$ DURING THE COLLAPSE OF ASSUMED MODE (see Fig. 4)

W_{in}

Internal work done (W_{in}) can be easily obtained by the summing up of internal energy absorbed by the plastic hinges of each floor system as given by Tables 2, 3 and 4.

$W_{ex} = W_{ex}^{H} + W_{ex}^{V}$

W_{ex}^{H} : external work done due to horizontal loads.

W_{ex}^{V} : external work done due to vertical loads.

W_{ex}^{H} can be easily computed by knowing the type of collapse mechanism. For example;

(a) Complete collapse

$$W_{ex}^{H} = PL\theta\sum_{i=1}^{m} h_i\ l_i$$

where

$$l_i = \frac{1}{L}\sum_{k=1}^{i} L_k$$

nondimensional vertical distance of the load from the ground.

(b) Collapse of the frame between p and q stories $(1 \leqq p \leqq q \leqq m)$

$$W_{ex}^{H} = PL\theta \left(\sum_{i=p+1}^{q} h_i\ l_i + l_q \sum_{j=q+1}^{m} h_j\right)$$

On the other hand, W_{ex}^{V} can be determined by only summing up numerical values given in the basic data for two structural aggregates.

(II) MINIMUM WEIGHT DESIGN OF PORTAL FRAMES AND ITS COMPUTER PROGRAMMING

Using the method of formulation of all the possible mechanisms of a given frame developed in Part I, Trial and Error method of minimum weight design can be proposed as follows;

1. Assume the M_p value of members appropriately.

2. Calculate the limit loads corresponding to all the possible collapse modes and select a limited number of mechanisms which give unsafe critical loads for the M_p values assumed in 1.

3. Determine new M_p values of the members which may give the minimum weight of the frame under the restricted number of restraining inequalities obtained in 2 by the Linear Programming.

4. Check whether there exist any unsafe collapse mode for M_p values determined in the last stage and if so, return to the step 3 until no unsafe collapse load is found.

The present method of solution can be automatically carried out even on digital computers of comparatively small scale and the flow chart of calculation is given in the Fig. 5-1.

The method for plastic analysis of multi-story frames is essentially the same as that of minimum weight design and its flow chart is also given in Fig. 5-2.

(III) NUMERICAL EXAMPLES

To check the principle of the present method, 7 test problems are solved by using a domestic computer of medium size, "OKITAC 5090C" at the Institute of Industrial Science, University of Tokyo. The computer language is of ALGOL-60 Type, and the library subroutine of the "Dual Simplex Method" is employed for Linear Programming and a problem of plastic analysis of 7 story-2 bay frame is also solved. Problems and their solutions are given in Appendices. Time required to obtain the final solution and number of unknown M_p s, of inequalities operative in the final solution and of all the possible combined mechanisms are also shown in the Table 5.

(IV) CONCLUSION

The results of studies on the seven test problems are summarized as follows;

1. The proposed method will be powerful especially in the case of portal frames with loading and geometrical regularity. This is the reason that the basic data for mechanisms of intermediate floor systems to be stored in a computer will be small. Therefore, it is believed that minimum weight design of frames of considerable complexity can be made on existing computers by increasing the number of computing

cycles. Taking for example, in the case of a large computer like IBM 7090, at least 30 story - 5 bay portal frames may be designed by this method.

2. Application of the present method will be by no means restricted to the cases of such regular frames. Extension of the method to the more general cases can be made only by furnishing the necessary basic data for mechanisms of two structural aggregates. The size of problems to be solved, however, will be much restricted.

3. Choice of initial M_p values will be very important for computing time as in Prof. Heyman's method and in the luckiest cases, it may give the final solution. In the present method lower bound initial M_p values of good approximation can be determined without any intuition.

4. Comparative study on the computing time with that of the others can be hardly made, since it depends upon many parameters such as problems, computers to be used, computer languages and so on. The author, however, believes that it may be comparable with that of Prof. Heyman's method. Also, some improvement will be expected by the method of selection of restraining inequalities which could reduce computing time. The present method is considered the more effective in

the design of frames of higher numbers of stories compared with other methods.

5. In the present method, the condition of a weight compatible mechanism will not be necessary since all the possible collapse modes are always considered and all the restraining inequalities operative in the final solution can be automatically found at the last stage of calculation.

6. Plastic analysis of existing modern building frames will be effectively made by using a large computer like IBM 7090. The size of the problems to be analyzed can be roughly determined by the following formula;

$$m\ (n + 1) \leq \frac{1}{12}\ (\text{Computer capacity - Programm storage})$$

in the case of IBM 7090 and analysis of regular frames

$$m\ (n + 1) \leqq 2500$$

where m: number of story n: number of span

7. From the results of studies on 7 test problems the following discussion on the minimum weight design can be made;

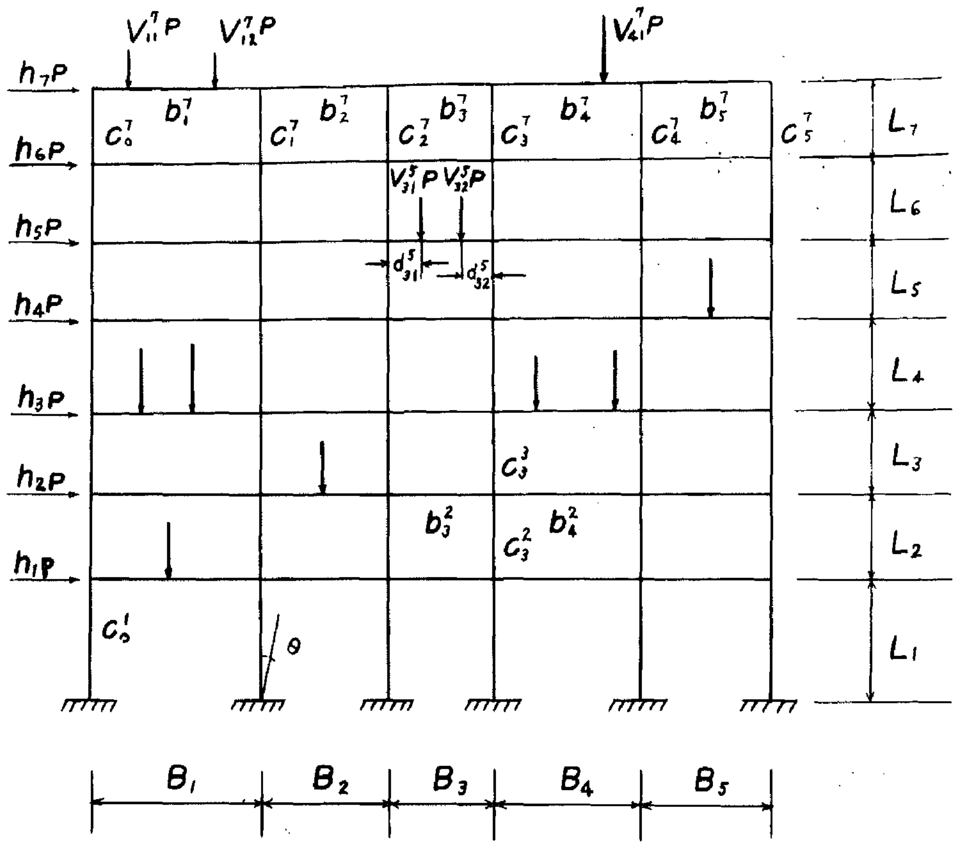
   a) Approximation of distributed load by concentrated loads at two-points may reduce about 10% of the weight of a given frame to compare with the case of one point loading when the vertical loads are considerably larger than horizontal loads, however, in some cases no weight reduction can be experienced.

b) In some types of problems, the number of restraining inequalities in the final solution may be much greater than the number of unknown M_p's.

c) It may be almost impossible to find out some general rules on the nature of minimum weight design of multi-story frames since the design will be much dependent on the magnitude and ratio of the loads as well as the geometrical dimensions of the given frames even in the cases of regular frames.

8. Automatic construction of basic data for two elementary structural aggregates could also be made on digital computers so that the computer programming of fully automatic calculation may be developed. Because of the capacity limitation of existing computers, however, such programming may not always be practical. The author believes that construction of basic data must be done separately from the analysis and design of frames. Furthermore, single-purpose computing programs must be developed for the collapse load analysis as well as for the minimum weight design of frames. In the near future,

he expects the development of more practical programming for analysis and design of multi-story frames on large computers in the machine codes.



θ : Reference angle of hinge rotation at some member during an assumed collapse

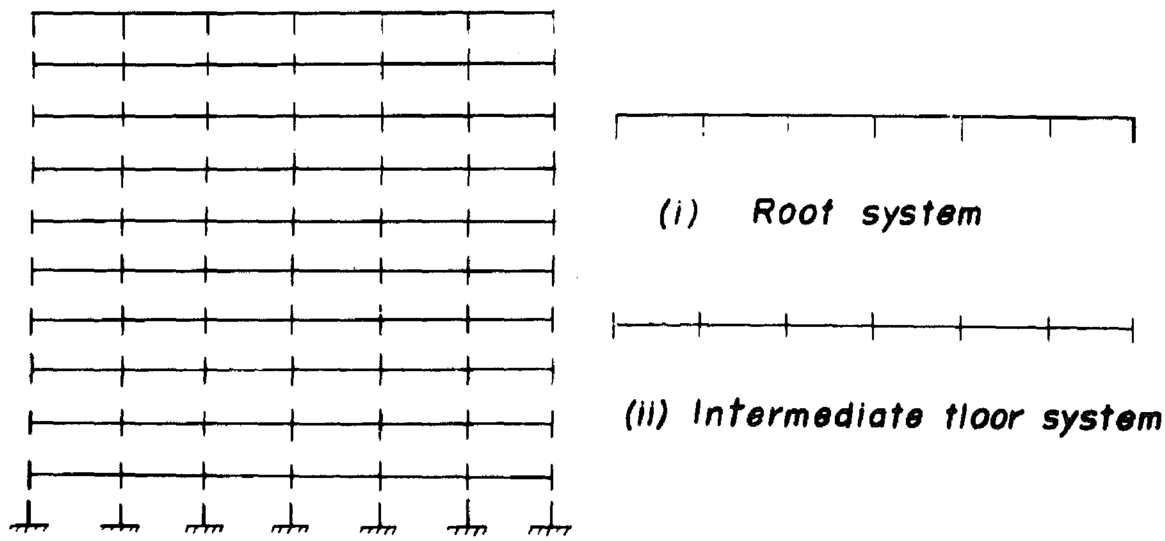
$\varphi_{j1}^i, \varphi_{j2}^i$: Angle of hinge rotation of some beam member due to vertical loads

b_j^i , c_j^i : Nondimensional M_p values of beam and column members

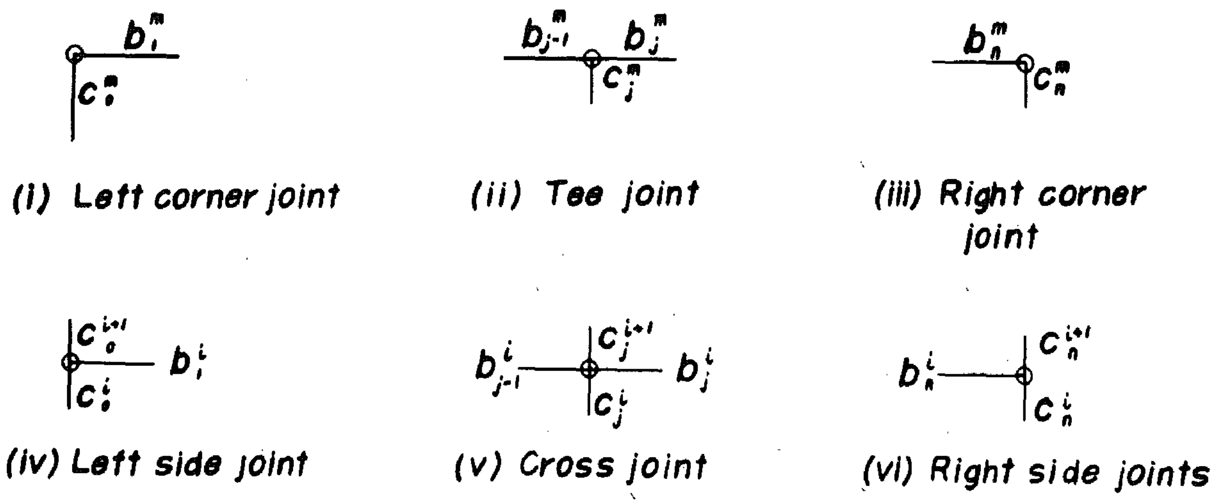
h^i : Load factor of the horizontal load acting on $(i+1)$ floor system

ν_{j1}^i, ν_{j2}^i : Load factors of the vertical loads acting on the j th beam of $(i+1)$ th floor system

Fig. 1. PLANE PORTAL FRAME OF M STORYS & N BAYS AND NOTATIONS TO BE EMPLOYED ($M = 7$, $N = 5$)

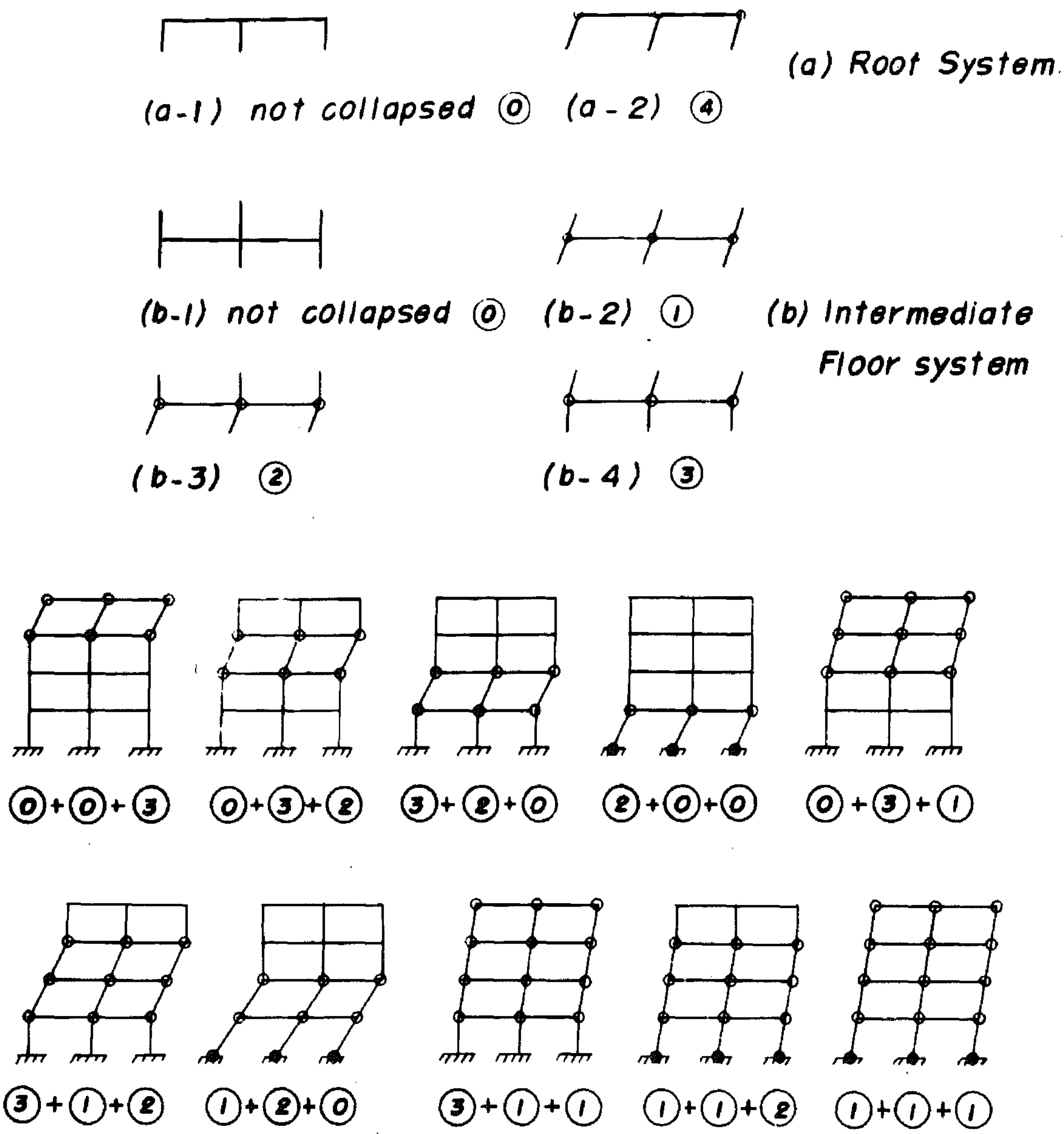


(a) Two structural aggregates



(b) Six joints

Fig. 2 TWO ELEMENTARY STRUCTURAL AGGREGATES AND SIX JOINTS WHICH CONTROL COLLAPSE MODES OF MULTI-STORY PLANE PORTAL FRAMES

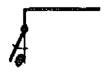
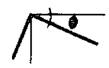


(the collapse mode ④ is only connected to the collapse mode ① or ③ of the 3rd floor system)

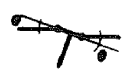
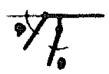
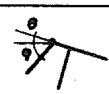
Fig.3 ALL THE POSSIBLE TYPES OF COLLAPSE MODE OF 4 STORYS - 2 BAYS PORTAL FRAME AS CONSTRUCTED BY COMBINATION OF MECHANISMS OF TWO ELEMENTARY STRUCTURAL AGGREGATES

(Only panel type considered and joint mechanisms neglected)

(i) Left corner joint

Mechanism Combination	Joint Mechanism	$W_{in}/M_p\theta$	Plastic moment Relation
O + P		b_1^m	$b_1^m \leq c_0^m$
		c_0^m	$b_1^m \geq c_0^n$
O + B		0	

(ii) Tee joints

Type of Joint Collapse	Mechanism Combination	Joint Mechanism	$W_{in}/M_p\theta$	Plastic moment Relation
	P + P		c_j^m	$c_j^m \leq b_{j-1}^m + b_j^m$
			$b_{j-1}^m + b_j^m$	$c_j^m \geq b_{j-1}^m + b_j^m$
	B + P		$b_{j-1}^m \lambda + c_j^m$	$c_j^m \leq b_{j-1}^m + b_j^m$
			$b_{j-1}^m (1+\lambda) + b_j^m$	$c_j^m \geq b_{j-1}^m + b_j^m$
	P + B		$b_j^m + c_j^m$	$b_j^m + c_j^m \leq b_{j-1}^m$
			b_{j-1}^m	$b_j^m + c_j^m \geq b_{j-1}^m$
	B + B		$b_{j-1}^m (1+\lambda)$	

(iii) Right corner joint

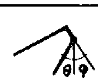
Mechanism Combination	Joint Mechanism	$W_{in}/M_p\theta$	Plastic moment Relation
P + O		b_n^m	$b_n^m \leq c_n^m$
		c_n^m	$b_n^m \geq c_n^m$
B + O		$b_n^m (1+\lambda)$	$b_n^m \leq c_n^m$
		$c_n^m (1+\lambda)$	$b_n^m \geq c_n^m$

Table 1.1 POSSIBLE MECHANISMS OF SIX JOINTS

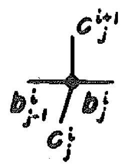
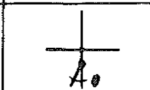
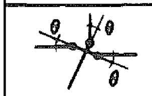
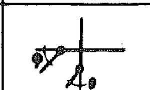
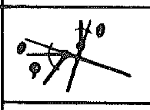
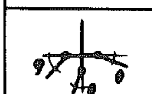
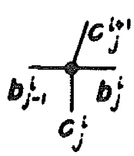
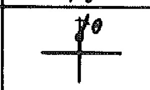
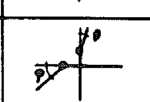
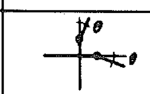
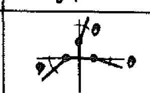
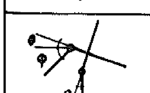
(iv) Left side joint

Type of Joint Collapse	Mechanism Combination	Joint Mechanism	$W_{in}/M_p\theta$	Plastic moment Relation
	O + P		C_0^i	$C_0^i \leq b_1^i + C_0^{i+1}$
			$b_1^i + C_0^{i+1}$	$C_0^i \geq b_1^i + C_0^{i+1}$
	O + B		C_0^{i+1}	
	O + P		b_1^i	$b_1^i \leq C_0^i + C_0^{i+1}$
			$C_0^i + C_0^{i+1}$	$b_1^i \geq C_0^i + C_0^{i+1}$
	O + B		0	
	O + P		C_0^{i+1}	
	O + B		$b_1^i + C_0^{i+1}$	$C_0^i \geq b_1^i + C_0^{i+1}$
			C_0^i	$C_0^i \leq b_1^i + C_0^{i+1}$

(v) Cross joint

Type of Joint Collapse	Mechanism Combination	Joint Mechanism	$W_{in}/M_p\theta$	Plastic moment Relation
C_j^{i+1}, b_{j-1}^i, b_j^i, C_j^i	P + P		$C_j^i + C_j^{i+1}$	$b_{j-1}^i + b_j^i \geq C_j^i + C_j^{i+1}$
			$b_{j-1}^i + b_j^i$	$b_{j-1}^i + b_j^i \leq C_j^i + C_j^{i+1}$
	B + P		$b_{j-1}^i\lambda + C_j^i + C_j^{i+1}$	$b_{j-1}^i + b_j^i \geq C_j^i + C_j^{i+1}$
			$b_{j-1}^i(1+\lambda) + b_j^i$	$b_{j-1}^i + b_j^i \leq C_j^i + C_j^{i+1}$
	P + B		b_{j-1}^i	
	B + B		$b_{j-1}^i(1+\lambda)$	

Table 1.2 POSSIBLE MECHANISM OF SIX JOINTS

Type of Joint Collapse	Mechanism Combination	Joint Mechanism	$W_{in}/M_p\theta$	Plastic moment Relation
	P + P		c_j^i	$b_{j-1}^i + b_j^i \geq c_j^i - c_j^{i+1}$
			$b_{j-1}^i + b_j^i + c_j^{i+1}$	$b_{j-1}^i + b_j^i \leq c_j^i - c_j^{i+1}$
	B + P		$b_{j-1}^i \lambda + c_j^i$	$b_{j-1}^i + b_j^i \geq c_j^i - c_j^{i+1}$
			$b_{j-1}^i (1+\lambda) + b_j^i + c_j^{i+1}$	$b_{j-1}^i + b_j^i \leq c_j^i - c_j^{i+1}$
	P + B		$b_{j-1}^i + c_j^{i+1}$	$b_{j-1}^i + c_j^{i+1} \leq b_j^i + c_j^i$
			$b_j^i + c_j^i$	$b_{j-1}^i + c_j^{i+1} \geq b_j^i + c_j^i$
	B + B		$b_{j-1}^i (1+\lambda) + c_j^{i+1}$	$b_{j-1}^i + c_j^{i+1} \leq b_j^i + c_j^i$
			$b_{j-1}^i \lambda + b_j^i + c_j^i$	$b_{j-1}^i + c_j^{i+1} \geq b_j^i + c_j^i$
	P + P		c_j^{i+1}	
	B + P		$b_{j-1}^i \lambda + c_j^{i+1}$	
	P + B		$b_j^i + c_j^{i+1}$	$b_j^i + c_j^{i+1} \leq b_{j-1}^i + c_j^i$
			$b_{j-1}^i + c_j^i$	$b_{j-1}^i + c_j^{i+1} \geq b_{j-1}^i + c_j^i$
	B + B		$b_{j-1}^i \lambda + b_j^i + c_j^{i+1}$	$b_j^i + c_j^{i+1} \leq b_{j-1}^i + c_j^i$
			$b_{j-1}^i (1+\lambda) + c_j^i$	$b_j^i + c_j^{i+1} \geq b_{j-1}^i + c_j^i$

(Table 1·3)

(vi) Right side joint

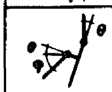
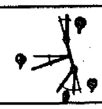
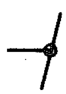
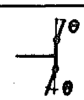
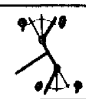
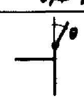
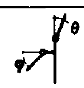
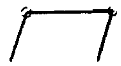
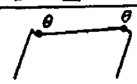
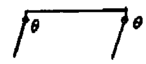
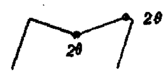
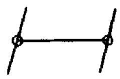
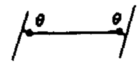
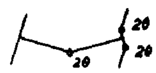
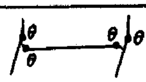
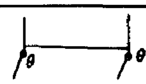
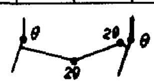
Type of Joint Collapse	Mechanism Combination	Joint Mechanism	$W_{in}/M_p\theta$	Plastic moment Relation
	P+O		c_n^i	$c_n^i \le b_n^i + c_n^{i+1}$
			$b_n^i + c_n^{i+1}$	$c_n^i \ge b_n^i + c_n^{i+1}$
	B+O		$b_n^i\lambda + c_n^i$	$c_n^i \le b_n^i + c_n^{i+1}$
			$b_n^i(1+\lambda)+c_n^{i+1}$	$c_n^i \ge b_n^i + c_n^{i+1}$
			$c_n^i(1+\lambda)+c_n^{i+1}\lambda$	$b_n^i \ge c_n^i + c_n^{i+1}$
	P+O		b_n^i	$b_n^i \le c_n^i + c_n^{i+1}$
			$c_n^i + c_n^{i+1}$	$b_n^i \ge c_n^i + c_n^{i+1}$
	B+O		$b_n^i(1+\lambda)$	$b_n^i \le c_n^i + c_n^{i+1}$
			$(c_n^i+c_n^{i+1})(1+\lambda)$	$b_n^i \ge c_n^i + c_n^{i+1}$
	P+O		c_n^{i+1}	
	B+O		$b_n^i\lambda + c_n^{i+1}$	$b_n^i \le c_n^i + c_n^{i+1}$
			$c_n^i\lambda + c_n^{i+1}(1+\lambda)$	$b_n^i \ge c_n^i + c_n^{i+1}$

Table 1.4 POSSIBLE MECHANISMS OF SIX JOINTS

Type of Collapse	Mech.	$b^m \leq c^m$			$b^m \geq c^m$		
	P						
		2	0	0	0	2	0
	B						
		4	0	0.5	2	2	0.5

Type of Collapse	Mech.	$b^i \leq c^i + c^{i+1}$				$b^i \geq c^i + c^{i+1}$			
	P								
		2	0	0	0				
	B								
		4	0	0	0.5	2	2	2	0.5

Type of Collapse	Mech.	$b^i \leq c^i - c^{i+1}$				$c^i - c^{i+1} \leq b^i \leq c^i + c^{i+1}$				$b^i \geq c^i + c^{i+1}$			
	P									do.			
		2	0	2	0	0	2	0	0				
	B												
		4	0	2	0.5	3	1	1	0.5	2	2	2	0.5

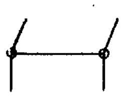
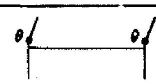
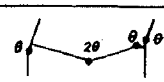
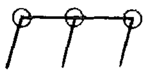
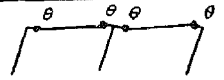
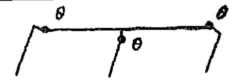
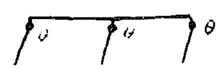
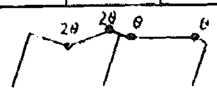
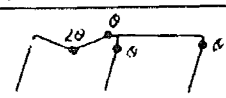
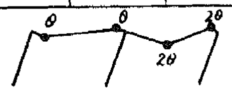
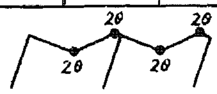
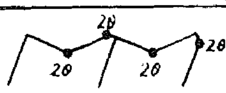
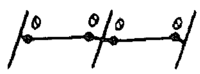
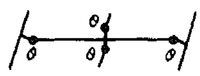
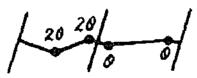
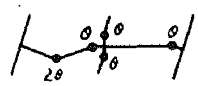
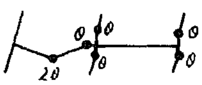
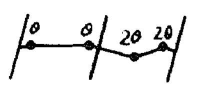
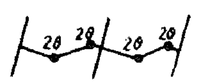
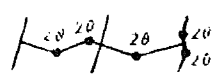
Type of Collapse	Mech.	$b^i \leq c^i - c^{i+1}$				$c^i - c^{i+1} \leq b^i \leq c^i + c^{i+1}$				$b^i \geq c^i + c^{i+1}$			
	P					do.				do.			
		0	0	2	0								
	B												
						3	1	1	0.5	2	2	2	0,5

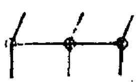
Table 2. COLLAPSE MODES OF ROOF & FLOOR SYSTEMS
(1 span, a single concentrated load at mid span)

Type of Collapse	Mech. Comb.	$b^m \leqq c^m/2$	$c^m/2 \leqq b^m \leqq c^m$	$b^m \geqq c^m$
	P - P	 4 0 0	 2 1 0	 0 3 0
	B - P	 6 0 0.5	 4 1 0.5	 3 2 0.5
	P - B	 6 0 0.5	do.	
	B - B	 8 0 1	do.	 6 2 1

Type of Collapse	Mech. Comb.	$b^i \leqq (c^i+c^{i+1})/2$	$(c^i+c^{i+1})/2 \leqq b^i \leqq c^i+c^{i+1}$	$b^i \geqq c^i+c^{i+1}$
	P - P	 4 0 0 0	 2 1 1 0	
	B - P	 6 0 0 0.5	 4 1 1 0.5	 3 2 2 0.5
	P - B	 6 0 0 0.5	do.	
	B - B	 8 0 0 1	do.	 6 2 2 1

(TABLE 3.1)

Type of Collapse	Mech. Comb.	$b^i \le (c^i - c^{i+1})/2$	$(c^i - c^{i+1})/2 \le b^i \le c^i + c^{i+1}$	$c^i - c^{i+1} \le b^i \le c^i + c^{i+1}$	$b^i \ge c^i + c^{i+1}$	$c^i - c^{i+1} \le b^i \le c^i + c^{i+1}$, $c^i \le c^{i+1}$	$b^i \ge c^i + c^{i+1}$, $c^i \le c^{i+1}$
	P - P	4 0 3 0	2 1 2 0	0 3 0 0	do.	do.	do.
	B - P	6 0 3 0.5	4 1 2 0.5	3 2 1 0.5	do.	do.	do.
	P - B	6 0 3 0.5	do.				
	B - B	8 0 3 1	do.	7 1 2 1	6 2 3 1		6 3 2 1

		$b^i \le c^i - c^{i+1}$	$c^i - c^{i+1} \le b^i \le c^i + c^{i+1}$	$b^i \ge c^i + c^{i+1}$	$c^i - c^{i+1} \le b^i \le c^i + c^{i+1}$, $c^i \le c^{i+1}$	$b^i \ge c^i + c^{i+1}$, $c^i \le c^{i+1}$
	P - P	0 0 3 0	do.	do.	do.	do.
	B - P		3 1 2 0.5	do.	do.	do.
	P - B					
	B - B			6 2 3 1	7 2 1 1	6 3 2 1

(Table 3.2)

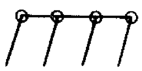
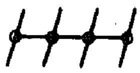
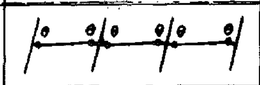
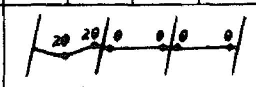
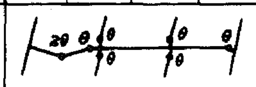
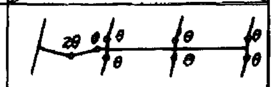
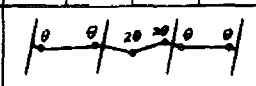
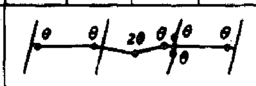
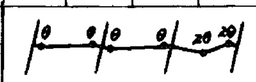
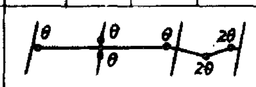
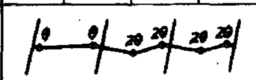
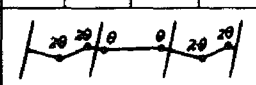
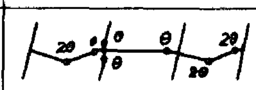
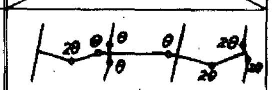
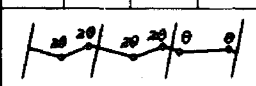
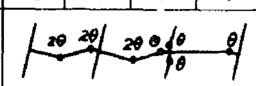
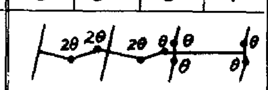
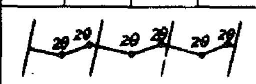
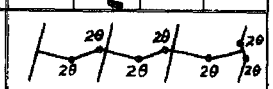
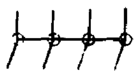
Type of Collapse	Mech. Comb.	$b^m \leq c^m/2$	$c^m/2 \leq b^m \leq c^m$	$b^m \geq c^m$
	P-P-P	6 0 0	2 2 0	0 4 0
	B-P-P	8 0 0.5	4 2 0.5	3 3 0.5
	P-B-P	8 0 0.5	6 1 0.5	
	P-P-B	8 0 0.5	6 1 0.5	
	P-B-B	10 0 1	do.	
	B-P-B	10 0 1	8 1 1	6 3 1
	B-B-P	10 0 1	8 1 1	7 2 1
	B-B-B	12 0 1.5	do.	10 2 1.5

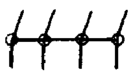
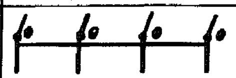
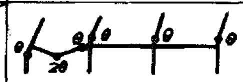
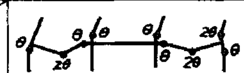
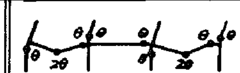
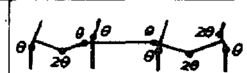
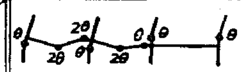
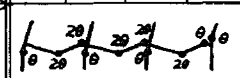
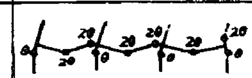
Table4.1 COLLAPSE MODES OF ROOF & FLOOR SYSTEMS
(3 spans, a single concentrated load at each mid span)

Type of Collapse	Mech. Comb.	$b^i \le (c^i+c^{i+1})/2$				$(c^i+c^{i+1})/2 \le b^i \le c^i+c^{i+1}$				$b^i \ge c^i + c^{i+1}$			
	P-P-P												
		6	0	0	0	2	2	2	0				
	B-P-P												
		8	0	0	0.5	4	2	2	0.5	3	3	3	0.5
	P-B-P												
		8	0	0	0.5	6	1	1	0.5				
	P-P-B												
		8	0	0	0.5	6	1	1	0.5				
	P-B-B					do.							
		10	0	0	1								
	B-P-B												
		10	0	0	1	8	1	1	1	6	3	3	1
	B-B-P												
		10	0	0	1	8	1	1	1	7	2	2	1
	B-B-B					do.							
		12	0	0	1.5					10	2	2	1.5

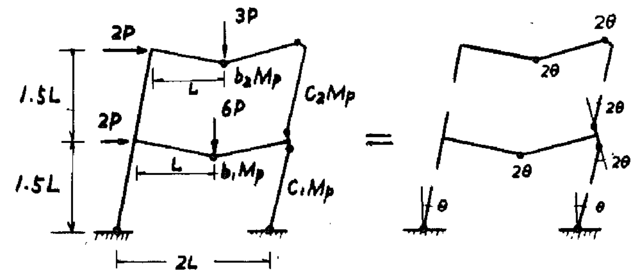
(Table 4.2)

Type of Collapse	Mech. Comb.	$b^i \le (c^i - c^{i+1})/2$	$(c^i - c^{i+1})/2 \le b^i \le c^i - c^{i+1}$	$c^i - c^{i+1} \le b^i \le c^i + c^{i+1}$	$b^i \ge c^i + c^{i+1}$	$c^i - c^{i+1} \le b^i \le c^i + c^{i+1}$, $c^i \le c^{i+1}$	$b^i \ge c^i + c^{i+1}$, $c^i \le c^{i+1}$
	P-P-P	6 0 4 0	2 2 2 0	0 4 0 0	do.	do.	do.
	B-P-P	8 0 4 0.5	4 2 2 0.5	3 3 1 0.5	do.	do.	do.
	P-B-P	8 0 4 0.5	6 1 3 0.5				
	P-P-B	8 0 4 0,5	6 1 3 0.5				
	P-B-B	10 0 4 1	do.				
	B-P-B	10 0 4 1	8 1 3 1	7 2 2 1	6 3 3 1		6 4 2 1
	B-B-P	10 0 4 1	8 1 3 1	7 2 2 1	do.		
	B-B-B	12 0 4 1.5	do.	11 1 3 1.5	10 2 4 1,5		

(Table 4.3)

Type of Collapse	Mech. Comb.	$b^i \le c^i - c^{i+1}$	$c^i - c^{i+1} \le b^i \le c^i + c^{i+1}$	$b^i \ge c^i + c^{i+1}$	$c^i - c^{i+1} \le b^i \le c^i + c^{i+1}$ $c^i \le c^{i+1}$	$b^i \ge c^i + c^{i+1}$ $c^i \le c^{i+1}$
	P-P-P	 0 | 0 | 4 | 0	do.	do.	do.	do.
	B-P-P		 3 | 1 | 3 | 0.5	do	do.	do.
	P-B-P					
	P-P-B					
	P-B-B					
	B-P-B			 6 | 2 | 4 | 1	 7 | 2 | 2 | 1	 6 | 3 | 3 | 1
	B-B-P				 7 | 2 | 2 | 1	do.
	B-B-B				 11 | 3 | 1 | 1.5	 10 | 4 | 2 | 1.5

(Table 4.4)



$W_{in}/M_p\theta$	$W_{ex}^{V}/PL\theta$
$4b_2$	6
$2b_1+2c_1+2c_2$	3
$2c_1$	

$$W_{in} = (2b_1+4b_2+4c_1+2c_2)\,M_p\theta$$

$$W_{ex}^{H} = 2p(1.5L)\theta + 2p(3L)\theta = 9pL\theta$$

$$W_{ex}^{V} = 9pL\theta$$

$$W_{in} = W_{ex}:$$

$$2b_1+4b_2+4c_1+2c_2 = 18K \qquad \left(K = \frac{PL}{M_p}\right)$$

Fig. 4 FORMULATION OF THE EQUATION OF VIRTUAL WORK CORRESPONDING TO AN ASSUMED COLLAPSE MODE

(2 Story - 1 Bay Portal Frame)

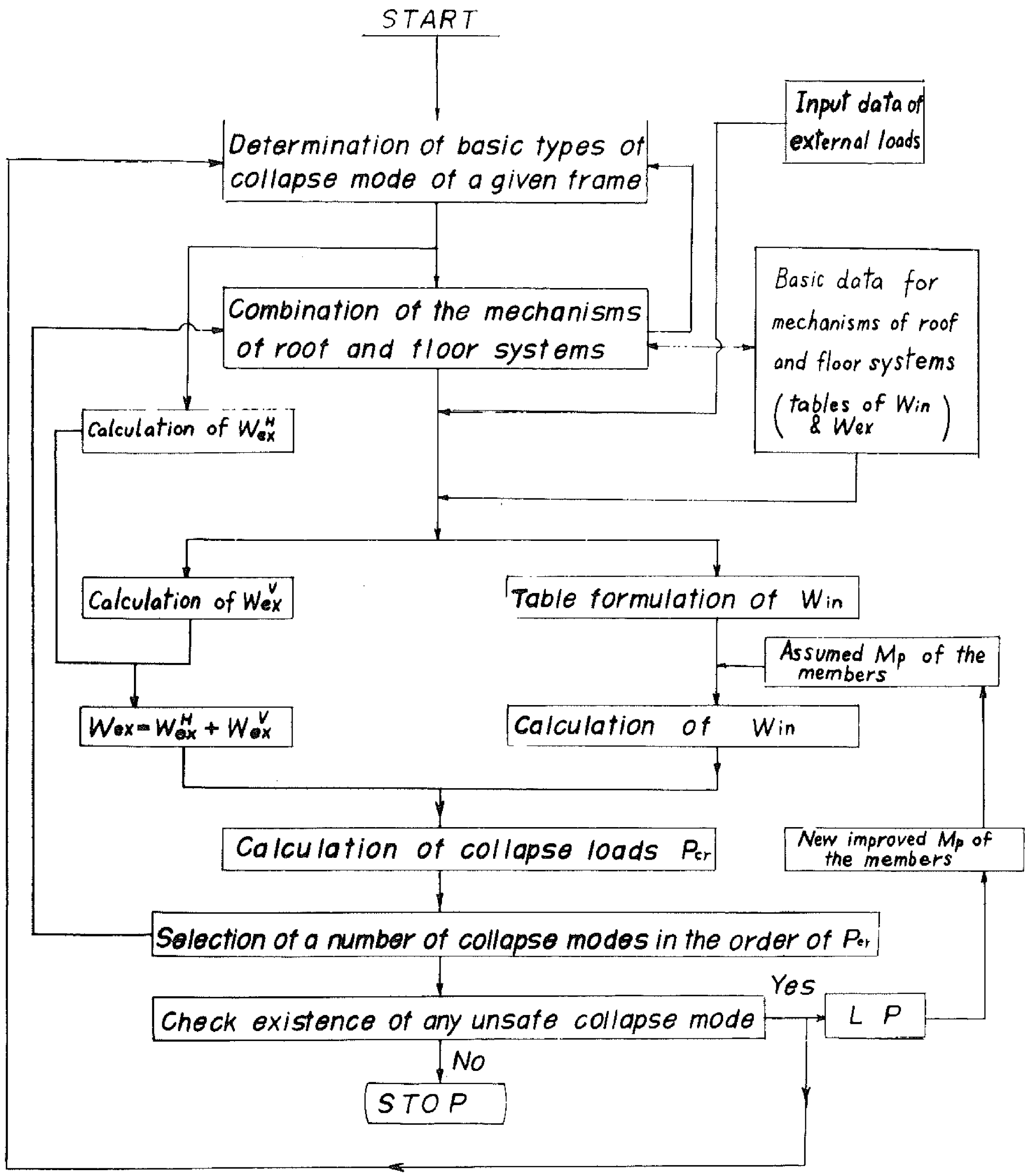


Fig. 5·1 FlOW CHART OF THE PROPOSED METHOD OF MINIMUM WEIGHT DESIGN

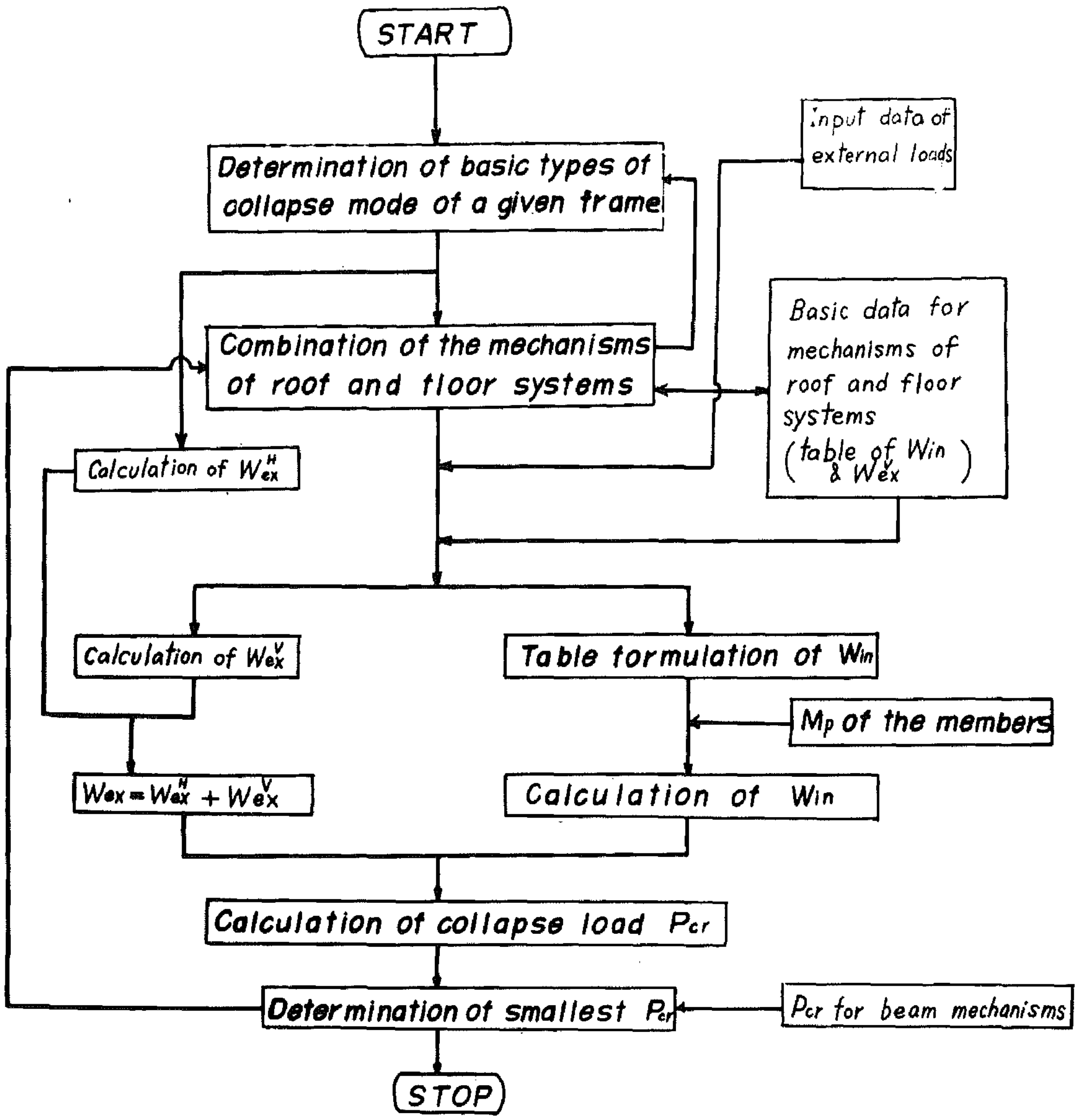


Fig. 5·2 FLOW CHART OF THE PROPOSED METHOD ON DETERMINATION OF A COLLAPSE LOAD

TIMINING by OKITAC-5090

PROBLEM	TIME (sec)
1	100
2	240
3	450
4	4600
5	3400
6	7600
7	3300

SCIENTIFIC MIX COMPARISON OF COMPUTER SPEED

OKITAC : IBM 704 = 20 : 1

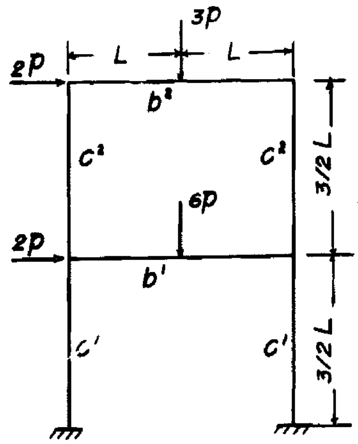
IBM 704 : IBM 7090 = 5.93 : 1

(cf) This program is written in ALGOL 60.

PROBLEM	No. OF COLLAPSE MODE	No. OF INEQUALITIES OPERATIVE IN FINAL S.L.	No. OF MEMBERS
1	29	6	4
2	68	6	4
3	106	6	4
4	3355	6	6
5	404	27	8
6	2208	27	8
7	784	24	4

Table 5. RESULTS OF STUDY ON 7 TEST PROBLEMS

PROBLEM - 1



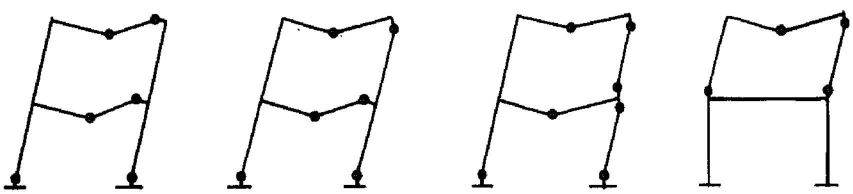
$b' = 8/3 = 2.\dot{6}$

$b^2 = 1$

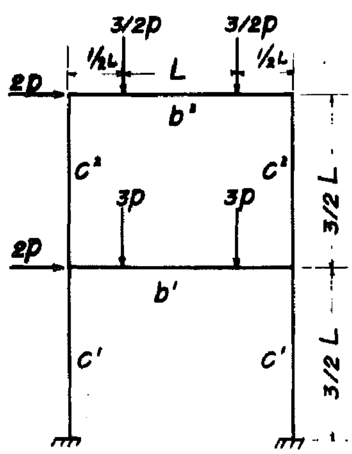
$c' = 5/3 = 1.\dot{6}$

$c^2 = 1$

$W_f / M_p L = 46/3 = 15.\dot{3}$



PROBLEM - 2



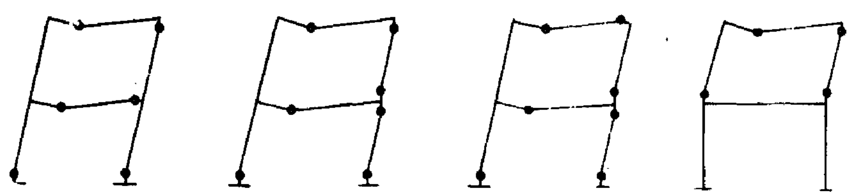
$b' = 120/49 = 2.448979591$

$b^2 = 6/7 = 0.8571428571$

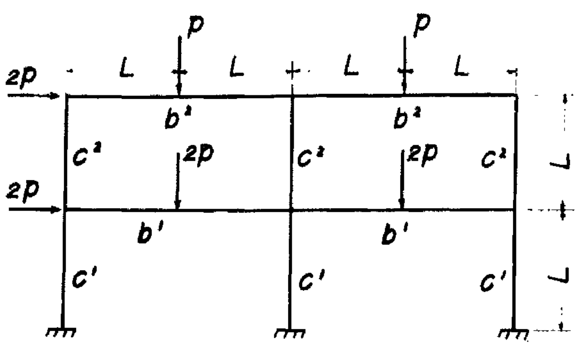
$c' = 78/49 = 1.591836734$

$c^2 = 6/7 = 0.8571428571$

$W_f / M_p L = 684/49 = 13.959183673$



PROBLEM - 3



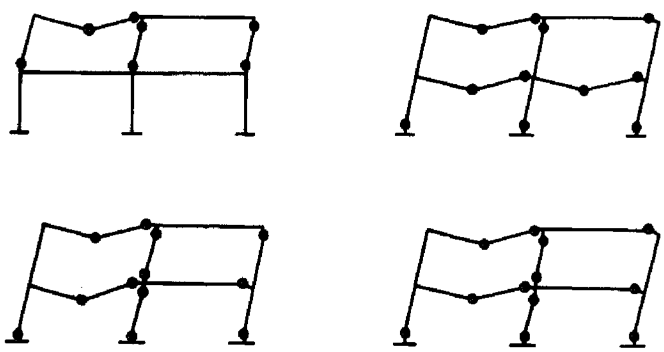
$b' = 0.8125$

$b^2 = 0.375$

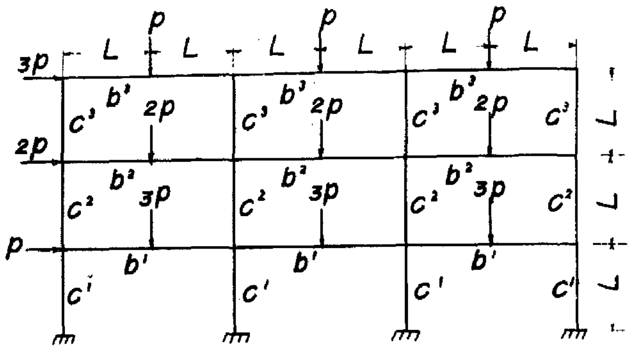
$c' = 0.875$

$c^2 = 0.375$

$W_f / M_p L = 8.5$

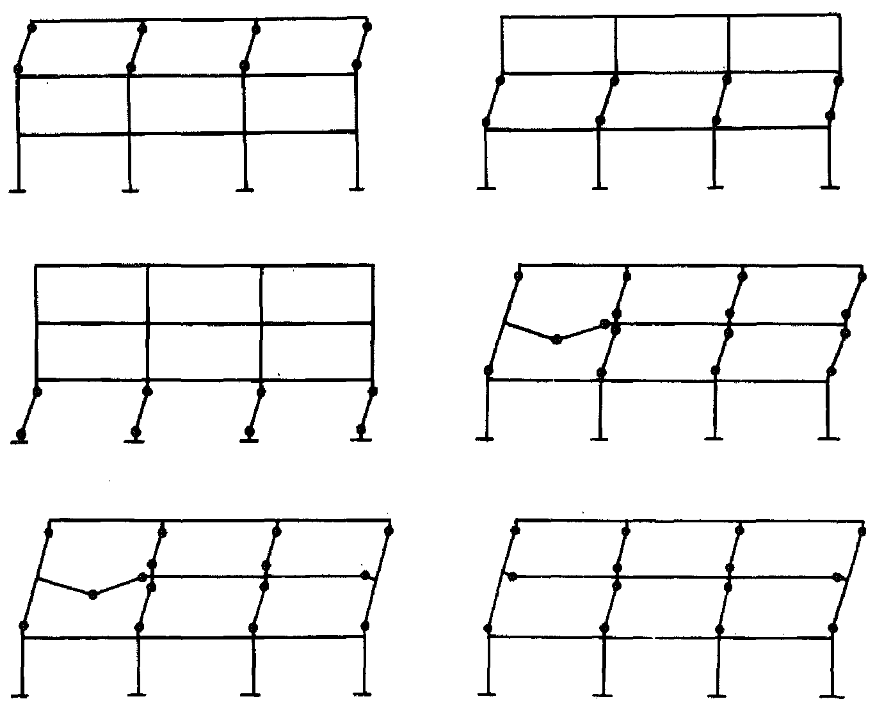


PROBLEM - 4

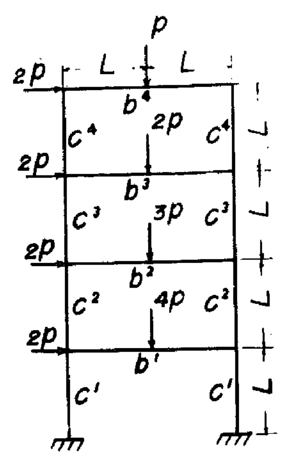


b' = 1.5
b² = 1.0
b³ = 0.5
c' = 0.75
c² = 0.625
c³ = 0.375

$W_f / M_p L = 25.0$

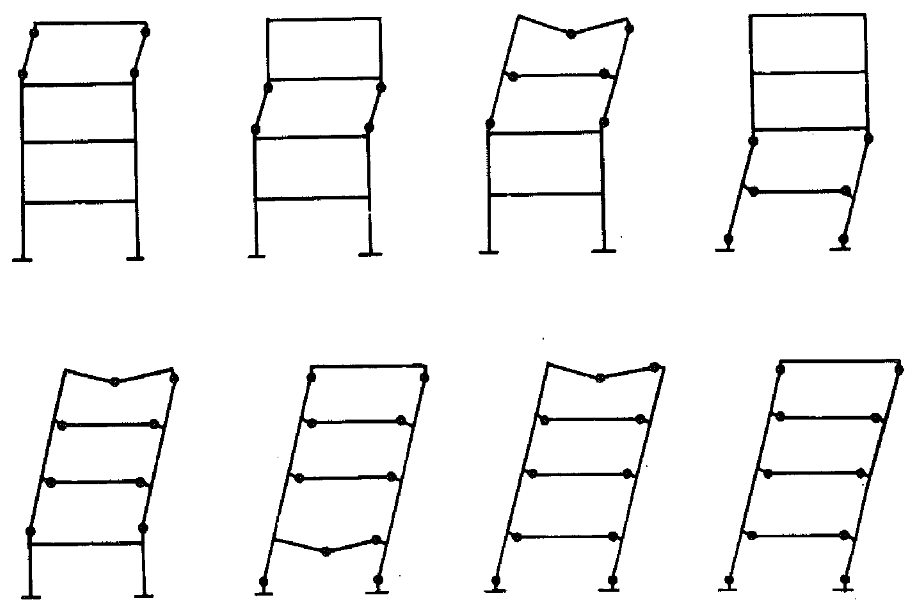


PROBLEM - 5

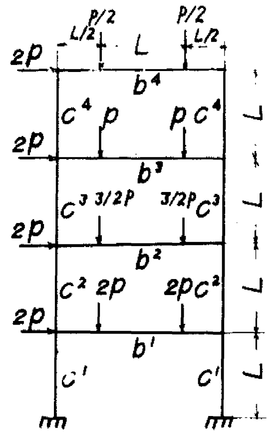


$b^1 = 2.0$
$b^2 = 2.5$
$b^3 = 1.5$
$b^4 = 0.5$
$c^1 = 3.5$
$c^2 = 1.5$
$c^3 = 1.0$
$c^4 = 0.5$

$W_f / M_p L = 26.0$

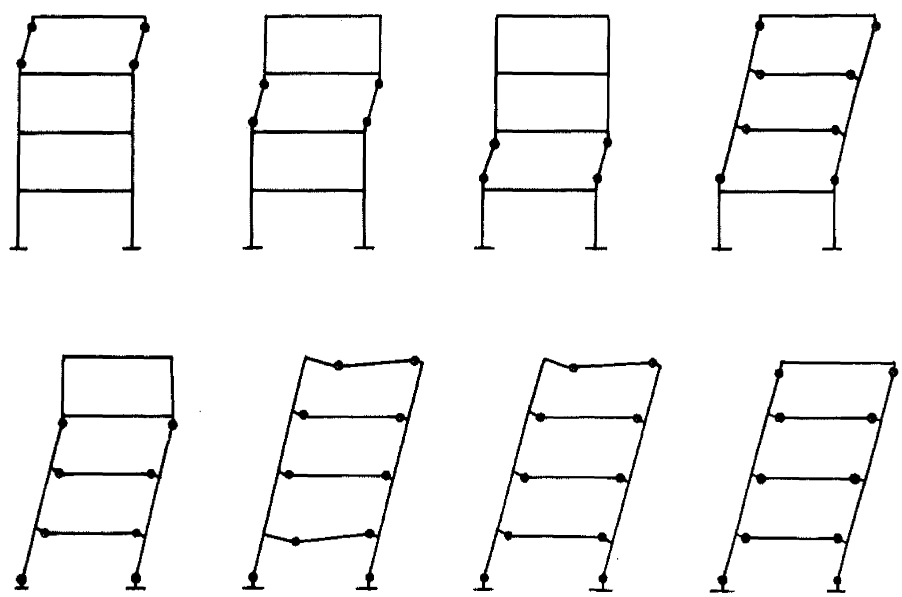


PROBLEM - 6

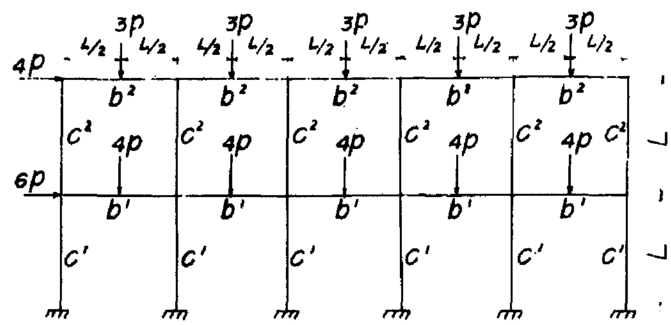


$b^1 = 2.0$
$b^2 = 2.5$
$b^3 = 1.5$
$b^4 = 0.5$
$c^1 = 3.5$
$c^2 = 1.5$
$c^3 = 1.0$
$c^4 = 0.5$

$W_f / M_p L = 26.0$



PROBLEM - 7



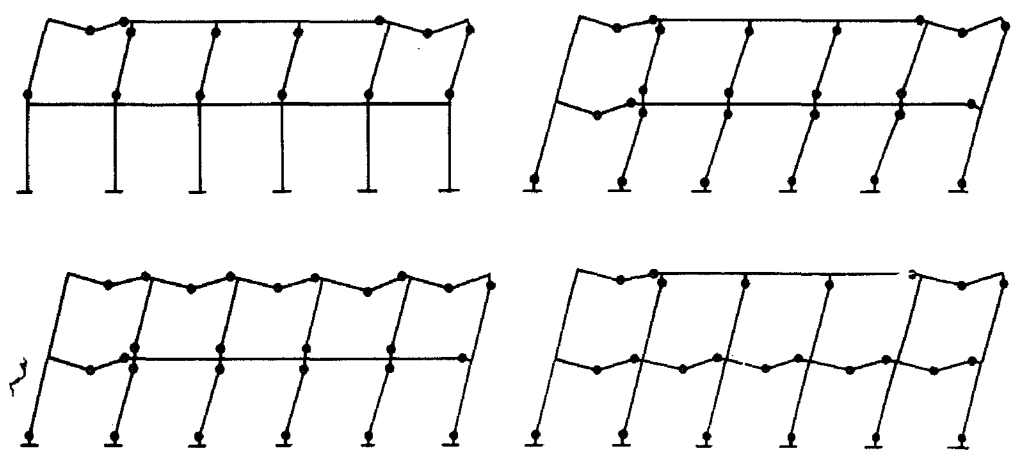
$b^1 = 457/550 = 0.8\dot{3}0\dot{9}$

$b^2 = 0.47$

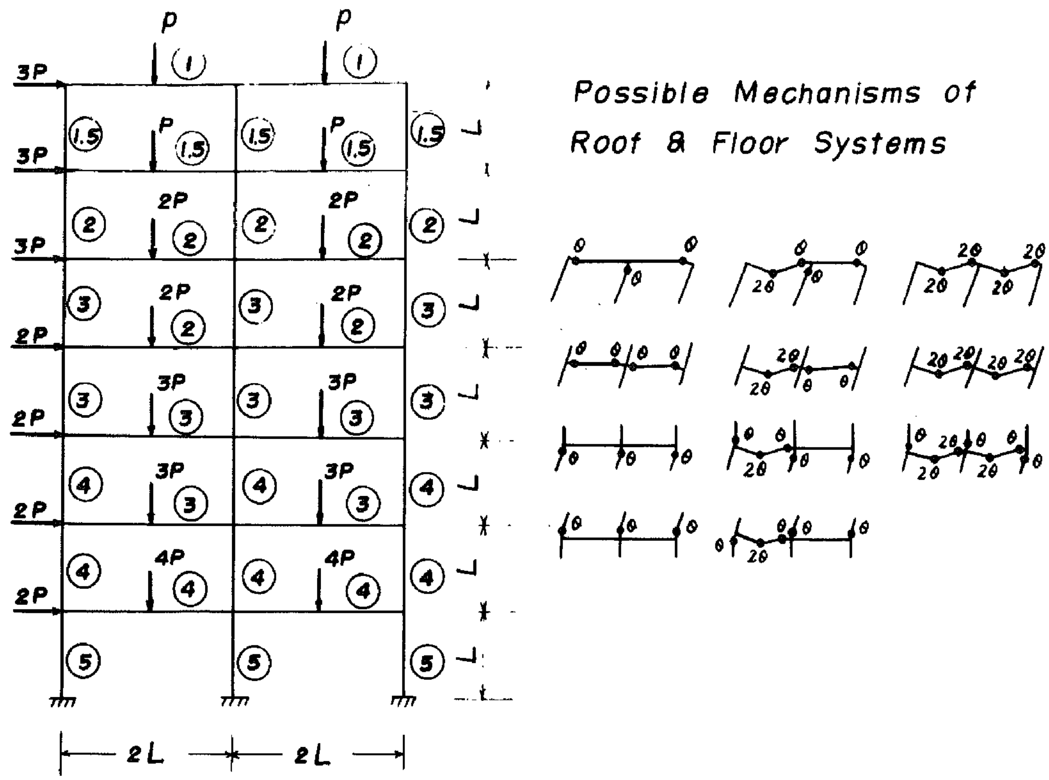
$c^1 = 519/550 = 0.9\dot{4}\dot{3}6$

$c^2 = 0.38$

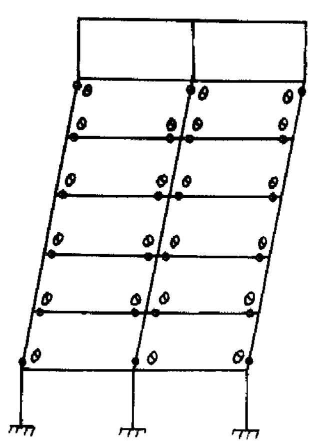
$W_f / M_pL = 15891/1100 = 14.44\dot{6}\dot{3}$



Problem - 8 Plastic Analysis of a 7 Story - 2 Bay Frame



Ultimate Load & Corresponding Collapse Mode



$$P_{ult} = \frac{29}{27}\ \frac{M_P}{L} = 1.074\ \frac{M_P}{L}$$

number of collapse modes = 6543

timing 330 min.

PLASTIC DESIGN OF BRACED MULTI-STORY FRAMES

by

Jacques Heyman

Cambridge University

I am most honoured to have been asked to talk here; I am particularly pleased because there has been for a long time a great deal of cooperation between my university at Cambridge and Lehigh. Much information has been exchanged, and it is perhaps not surprising that many of the design rules that we have proposed independently show a common ancestry. I do not wish, however, to go too deeply into the history of the development of plastic design methods; this morning I would like to discuss, very briefly, a report of a Joint Committee of the Institution of Structural Engineers and the Institute of Welding in England, entitled "Fully Rigid Multi-Storey Welded Steel Frames". This report is really a sketch for a design code for such structures, for which wind loading is not taken by structural action of the main members. The Report does not discuss how bracing against wind should be provided; it assumes that wind loads are accounted for by shear walls, cross-bracing, or by some other means.

For example, we have put up in England a six-storey building (Fig. 6) in which the end (gable) frames are built in rigidly to existing structures. In this design, therefore, we ignored the wind altogether, and designed for gravity loading only.

This report, which does not as yet have official recognition, is the only plastic design code existing in England; although plastic design has been going on for some 17 years, this is the first occasion on which anything has been codified. There are many reasons, I think, for this state of affairs. We could say that we did not wish to put down in black and white design rules that we know were somewhat empirical, that might hamper future progress, that might stop practicing engineers from making their own recommendations and modifications, and so on.

Such reasons are, I think, just rationalizations of our belief that design rules could not be formulated in black and white. Well, we have been triumphantly proved wrong by Lehigh by the production of these magnificent volumes that we have all received; and before this, Lehigh had produced the book on "Structural Steel Design", which is, among other things, a manual for the plastic design of certain classes of structures. The three new volumes now go a very long way to enable the practicing engineer to design multi-storey frames.

In 1948 we permitted plastic design in England by inserting a clause in the new edition of British Standard 449. The clause was merely permissive, and gave no help in how to use plastic theory. In fact (as I suppose is usual in such matters), one building at least had already been built by 1948.

Professor Roderick among others, had beaten the gun and designed a structure for the British Welding Research Association at Abington; this fatigue laboratory was, as far as I know, the first building in England of a pitched portal type to be designed by plastic theory. The BWRA has given much support to the development of plastic theory in England, and it is appropriate that this first building should have been put up at their research station.

In fact, we have a habit of building, when we can, plastic structures in our own backyard. Immediately after the war, we started a large re-development programme at the Engineering Laboratories at Cambridge University, and the first big six-storey block was built soon after the war ended, unfortunately too soon for us to have been in a position to use plastic theory. However, this building has been successively extended over the years and the center wing, completed in 1957, was designed according to plastic theory by Professor Horne. As far as I know this is the first such multi-storey frame to be designed in England. It is an all-welded structure, and we prepared alternative designs so that we could compare a conventional bolted design with the finally built plastic design.

It turned out that the plastic design saved some 28% of material. The price of the plastic design was some 28% more than the corresponding elastic design, so that the final cost was exactly the same. One reason for this is that, eight years ago, England was not used to the idea of site welding, which we demanded for the completion of this structure. The contractors who tendered for the building loaded their tenders against the welded structure. This situation has changed radically, as we will see later on.

The next plastic design we did at Cambridge was of the north wing of the same laboratory building; this was a plastic composite design. We felt we were ready at this time to design a multi-storey building using the floor slab as part of the structure, that is, allowing a composite plastic hinge to form at mid-span of the beams. If I can make a small digression here on our state of research on composite design, there are many interesting lines that can be pursued. One line that we are <u>not</u> pursuing is the question of effective width of slab that can be taken into account. This is not a live question from the point of view of research because the full plastic moment that can be developed at the cross section is almost independent of the width of concrete that is assumed. Indeed, the problem is to try and use to full advantage whatever concrete width is available. One can use so little of the slab normally because the steel is insufficient in area, and a common design procedure is to strengthen the steel beam at center span by use of a cover plate to increase the area of steel in tension, and thus try and throw more of the concrete slab in compression. The tendency is towards an enormous central moment developed by the slab and the reinforced steel work, and a small moment at the ends of the beam, so that the bending moment diagram looks almost like a simply supported beam.

Despite this tendency towards simply-supported design, it does seem certain that the most economical design would take place in the way that I have described, with very heavy moments at the center of a beam, and small moments at the ends. In particular, small moments are induced in the columns.

Still digressing, one of the live questions at Cambridge concerns questions of continuous beams, where the composite action is complicated; in particular much more must be known about the transverse reinforcement required in regions of hogging bending moment. One needs to know more also about the sort of spacing and type of connectors that can be used. We did not know enough about this topic when we designed the north wing at Cambridge, and so we put in too many connectors, in order to be safe.

There is also the question of columns which is a very live question. And there is a whole host of constructional problems yet to be solved. For example, how and where should beams be propped while continuous floor slabs are being cast? Returning now to the North wing extension to the existing building, it was not an arbitrary choice to use a composite design. In fact, we made four separate designs in this case; a conventional elastic design, a conventional elastic composite design, a straightforward plastic design, and the plastic composite design. We had to match up with the existing building and this means that we had to match the storey heights. In addition, the new wing was to incorporate a drafting office of a clear span of about fifty feet. The depth of plain steel beam required to span fifty feet to carry the floor loads that we specified for this building was too great. The elastic composite design was much better in this respect, and the plastic design was about the same as the elastic composite design. The plastic composite design, besides being the most economical, both in material and in cost, also gave us a very practical design from the point of view of headroom and other design requirements for the building.

The saving was about 20% in material, and this time we achieved just about that saving in cost. This was seven years after the previous building that I have just described. In fact, I think the saving is not so important as the fact that this was really the only possible design that we could make.

The completion of this wing just about fills up the site that we have available at Cambridge. The site was first opened in 1930, and a series of single-storey north light portal frames was put up as the first laboratories together with some two-storey buildings. The next move to get the increased space that we require was to replace the 1930 buildings by a four-storey building. In common with all the new buildings on the site, we have specified a superimposed load of 200 pounds per square foot. This is enormous, of course, but it means that we can use any equipment anywhere in the building. However, it does make the steel work very heavy and it explains to some extent my previous remarks about the problem of headroom with the composite design.

The first stage of this rebuilding program has just started, and the first block has been designed as far as possible in accordance with the recommendations of the Joint Committee. In codifying the rules for design certain assumptions have been made which are not altogether obeyed in the design for the new wing. For example, half of this report approximately is taken up with design charts for columns, and these charts have been prepared for steel of BS 15 which corresponds approximately to A-36 steel.

We have actually built the structure in BS 968 steel, which corresponds as I understand, to about A-440 or A-441 steel, and design in this higher grade steel is not covered by this booklet. That is, numerical tables given here are not applicable to the higher grade steel, but the design philosophy incorporated in the booklet is of course applicable to the design that we have made, and we followed that design philosophy as far as possible.

This design philosophy is, first of all, that all beams shall be designed plastically. This of course leads to very great simplifications compared with the elastic design of a continuous frame. As Professor Driscoll showed in a previous lecture, each beam can be designed separately, almost independently of its neighbors. Very occasionally one has to go back and check on the initial design of a beam (see Fig. 2 below).

Secondly, the Code specifies that under the factored collapse loads applied to the beams the columns shall be designed elastically. That is, they shall be checked to remain just elastic and stable at collapse of the beams. This is a fairly reasonable design assumption, and one to which many people are tending. There is very little reserve of strength left in a column once yield has been exceeded, and little material is being wasted by adopting this plastic beam/elastic column approach.

In addition, the Code specifies how one is to determine the worst design conditions in a column. Here a limited substitute frame is proposed which stems from the substitute frame proposed by the Steel Structures Research Committee between 1929 and 1936, when they produced their three reports. It is really the same substitute frame proposed by Lehigh.

While talking about the Steel Structures Research Committee we can perhaps recall that that Committee made tests on real structures which were going up in the early '30's in London; a railway terminal, a block of flats, etc. As is well known, the bending moments they observed really bore virtually no relation to the bending moments assumed in design. That is to say, the elastic bending moments assumed by the conventional designer were just not observed in practice in this series of tests done on full-scale structures. And of course, this has been corroborated by many other workers since.

The fact remains of course that an elastic design carried out in accordance with the provisions of standard building codes does not normally fall down; that is, the elastic distribution of stress calculated, or determined in one way or another by the designer, seems to provide a good basis for the design of steel structures, and I think one of the most important contributions of plastic theory is to prove that this must be so for a ductile structure. The elastic distribution of bending moments is only one of an infinite number of possible equilibrium distributions of bending moments, and Professor Beedle showed us that if we present the structure with a reasonable equilibrium distribution of bending moments, and base a design of the structure on that distribution, then we will always have a safe design.

What the Joint Committee proposes therefore is to present the designer with a reasonable equilibrium distribution of bending moments for a multi-storey frame, the bending moments for the beams being the plastic distribution and the bending moments for the columns being determined by means of a substitute frame.

The Committee's work can perhaps best be discussed with reference to the design that we carried out for the latest extension at the Engineering Laboratories. This extension is called "Inglis A" in memory of Sir Charles Inglis, who was the professor before Sir John Baker. As you see (Fig. 1) this is a four-bay, four-storey structure; the spans are basically forty feet. The loads here, 71.2 live load, 79.3 dead load, are in long tons.

This slide (Fig. 2) illustrates the one case where there was some slight adjustment to be made in the beam design. Most of the beams were completely straightforward. Here at the top right hand corner of the frame the beam had a stronger section than the combined columns. The full plastic moment at the center of the beam was 7,400 units. Allowing for shear at the end of the beam the full plastic moment was reduced to 6,950, and the bending moment diagram can then be completed to give a moment of 5,550 at the right hand. Now the full plastic moment available at collapse from the two columns is the sum of 3,330 and 2,640, that is 5,970, compared with the 5,550 required. The design is therefore safe.

This is the limited substitute frame (Fig. 3) proposed by the Joint Committee. The far ends of all members are assumed to be fixed, except that a pinned footing can be allowed for. The live load will induce a collapse mechanism in each of the loaded beams, since they have been designed in that way. These beams will have zero stiffness, and in any elastic distribution of bending moments, these members will not participate.

Figure 4 is a page from the Lehigh book on structural steel design, and shows various alternatives for substitute frames. Notice, looking at the center picture, that the two beams which are collapsing have been taken out of the picture and replaced effectively by an M_p acting there, and the dead load acts on the other two beams. Springs are shown at the far ends of these members and at top left is the substitute frame that the Joint Committee proposes. The alternative substitute frames show all ends pinned, the columns fixed etc.

To show how the Joint Committee's proposals work, Fig. 5 gives one particular column with the calculations laid out. These calculations are really very simple. One can make a conventional moment distribution, using the stiffnesses shown in the figure, and distributing the out-of-balance moments of (6830 - 4800) = 2030. Alternatively, the Joint Committee give simple tables of the functions α and β for different values of $K/\Sigma K$. A one step distribution can then be made, taking α times the out-of-balance moment at one end of the column and subtracting β times the out-of-balance moment at the other, winding up with the moments of 630 and 510 shown.

These values, together with the known axial load in the column length, are the checking values for the design. Notice that the checkerboard loading has produced almost uniform single curvature, and this is the worst design case for any column.

The main building in Fig. 6 was the first building just after the war, designed by conventional means. The wing in the foreground is the composite structure of which I spoke first and which was completed and opened last year.

The new building is shown in Fig. 7 with the steel work nearly erected. Fig. 8 gives another view of the steel work from a viewpoint almost at right angles. It shows the protective hoods to enable welding to be carried on in bad weather. Fig. 9

shows one of these welding hoods being used.

A completed main beam joint is seen in Fig. 10 showing a full strength butt weld to the web and a down-hand butt weld for the flanges. The erection cleat has been knocked off at this stage, and the four holes remain in the structure with no harmful effect.

Fig. 11 shows a minor axis beam connection. The minor axis beams were in fact bolted rather than welded and this slide shows the shear stiffening that we adopted for the major axis beams framing into end columns. The cover plates make for an effective minor axis detail while providing the necessary shear reinforcement to the web of the column. Fig. 12 gives a view of the completed steel work.

In all these three designs that we carried out in our own back yard we wanted to try out our latest developments, but in fact each one of them, with the possible exception of the first, has shown economies; not only economies in money and material but also economies in design time. Indeed before I came to Lehigh for this conference, I didn't know of any simple way at all to design a multi-storey structure, other than the Joint Committee's method. This to me is the important thing about the development of plastic theory, the way in which it saves design time, the way in which it enables the designer to obtain an understanding of structural behavior that he can't get by elastic design methods, and the way in which, in the future, it will enable the plastic designer to tackle a whole new range of structures, which are just beyond possibility at the moment.

Discussion

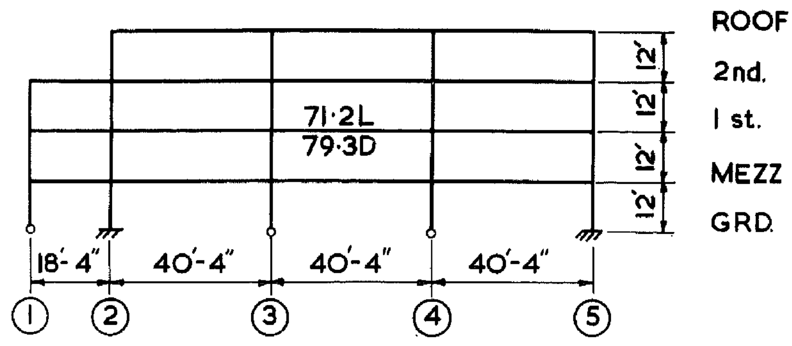
"Has the Joint Committee presented evidence to support the following assumptions: (1) Elastic columns at ultimate load (which was one of the design assumptions). For example, have elastic plastic analyses been made to show that the columns designed by Joint Committee recommendations do in fact remain elastic at ultimate load? And secondly, what about the fixed end boundary conditions for reduced frame. Has the influence of other boundary conditions been studied and evaluated to show that the fixed boundary is safe but not excessively so?"

I don't think I can answer these questions individually. A blanket answer would be that since the publication of the final report of the Steel Structures Research Committee in 1936, work has been continuous, both in England and abroad. In England, which is the work I know best, for example, the Building Research Station carried on work along these lines and for the last 10 or 15 years Dr. Wood has been extremely active and a great deal of the Joint Committee's report must be put to his credit. He has made continuing investigations on the problems about which questions have been asked just now; the answer is that these questions have been studied, and as far as we know the assumptions being made are both conservative and also realistic in the sense that we're not wasting too much material. In addition to the work of the Building Research Association and Dr. Wood, Professor Horne has also been interested specifically in the column problem, and has come to similar conclusions working along different lines from Wood.

A second question on this paper is, "Will you explain how wind load can be resisted without causing lateral sway deflection of the frame? What limits on sway induced by wind are considered by the Joint Committee, and why are these limits appropriate?"

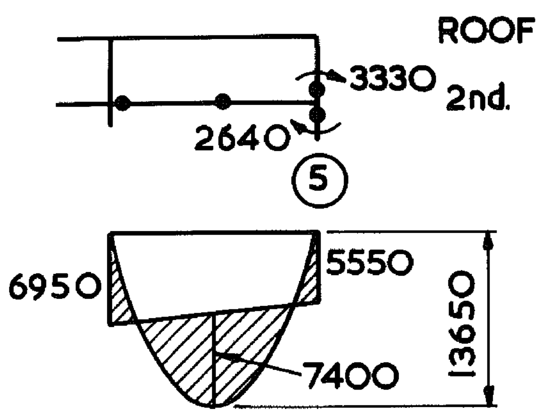
No limits are specified in the Joint Committee report. It is a relatively simple matter, as indeed we learned last week, to get an estimate of deflections for a braced frame. Having got that estimate of deflections it can be taken into account in the design.

I have been asked to say something about load factors. In England, it's been common to use a factor of 1.75 for plastic design. This compares with the old factor of 1.85 here which has now been reduced to 1.70. In the U. S., I think, a one-third allowance is made for wind stress, our elastic allowance is 25%, and this brings us both out with 1.4, as the load factor to be taken for combined gravity plus wind loading. Now the Joint Committee first of all cuts out wind, and secondly believes that it has a very rational method. For this type of building only, designed in accordance with the Committee's recommendations, it suggests that the load factor to be used should be 1.5 against gravity loading.



TYPICAL FRAME INGLIS "A"

Fig. 1



DESIGN OF SECOND FLOOR BEAM

Fig. 2

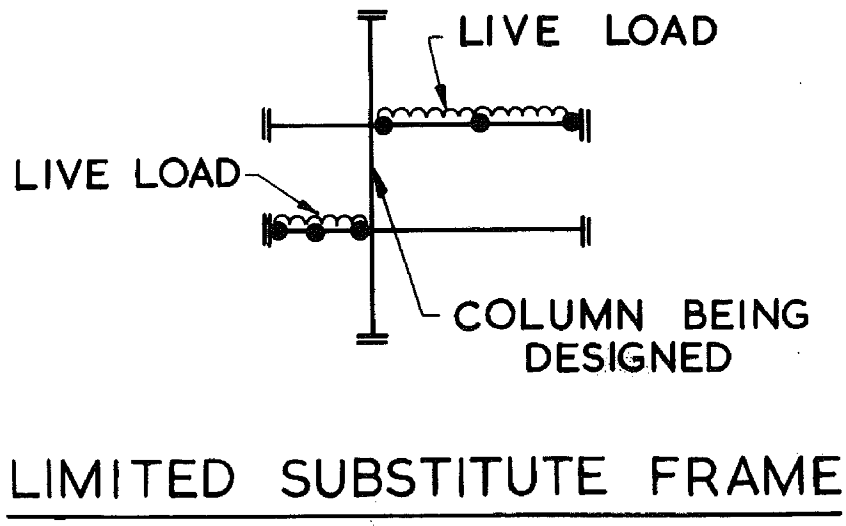

LIVE LOAD
LIVE LOAD
COLUMN BEING DESIGNED
LIMITED SUBSTITUTE FRAME

Fig. 3

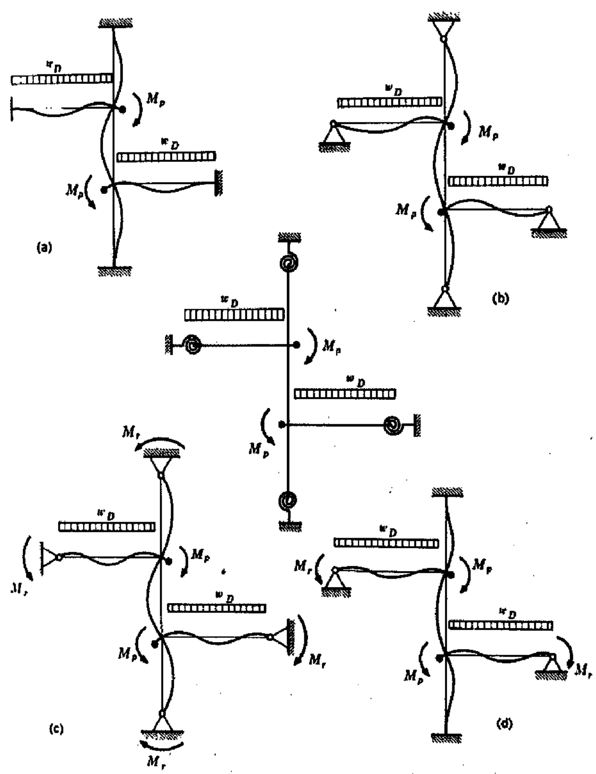
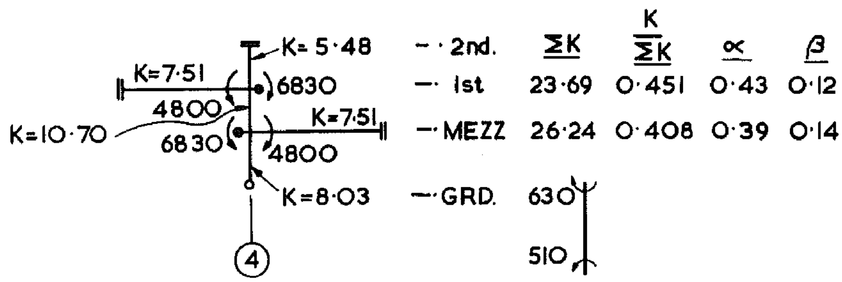

(a)
(b)
(c)
(d)

Fig. 4



DESIGN OF COLUMN 4, MEZZ / 1st FLOOR

Fig. 5

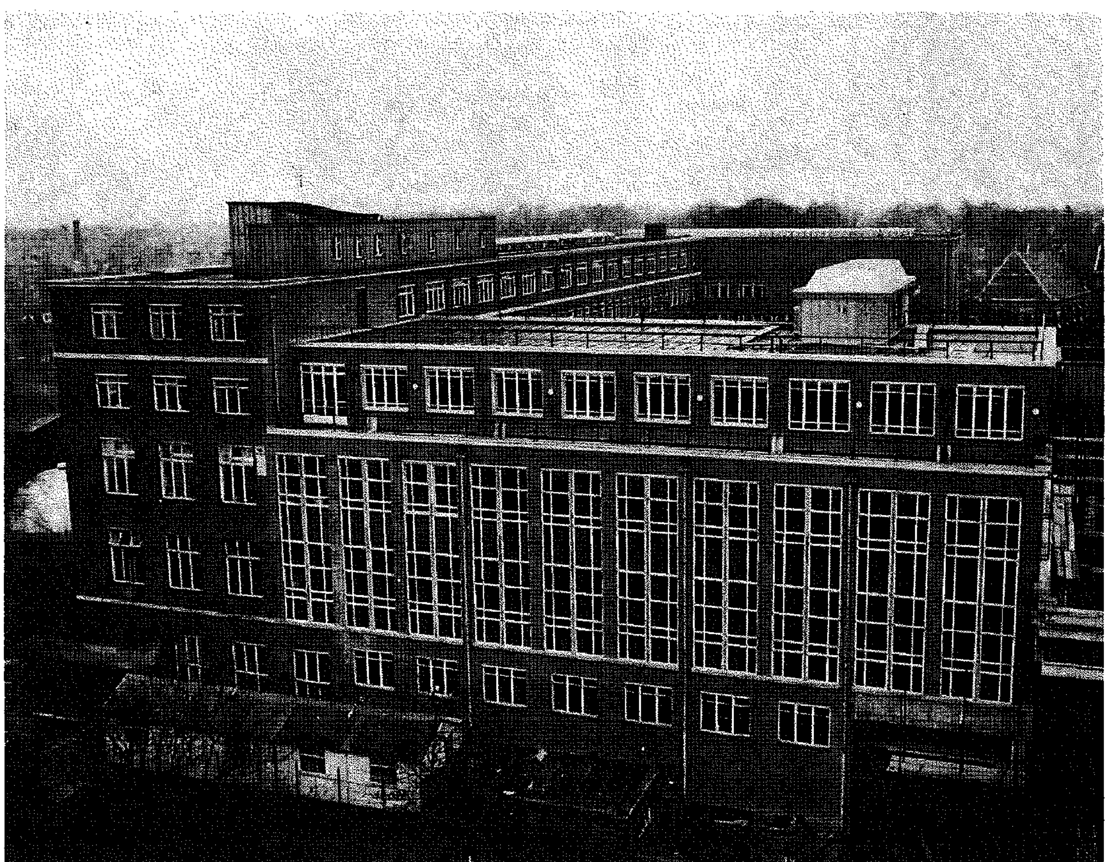


Fig. 6

Fig. 7

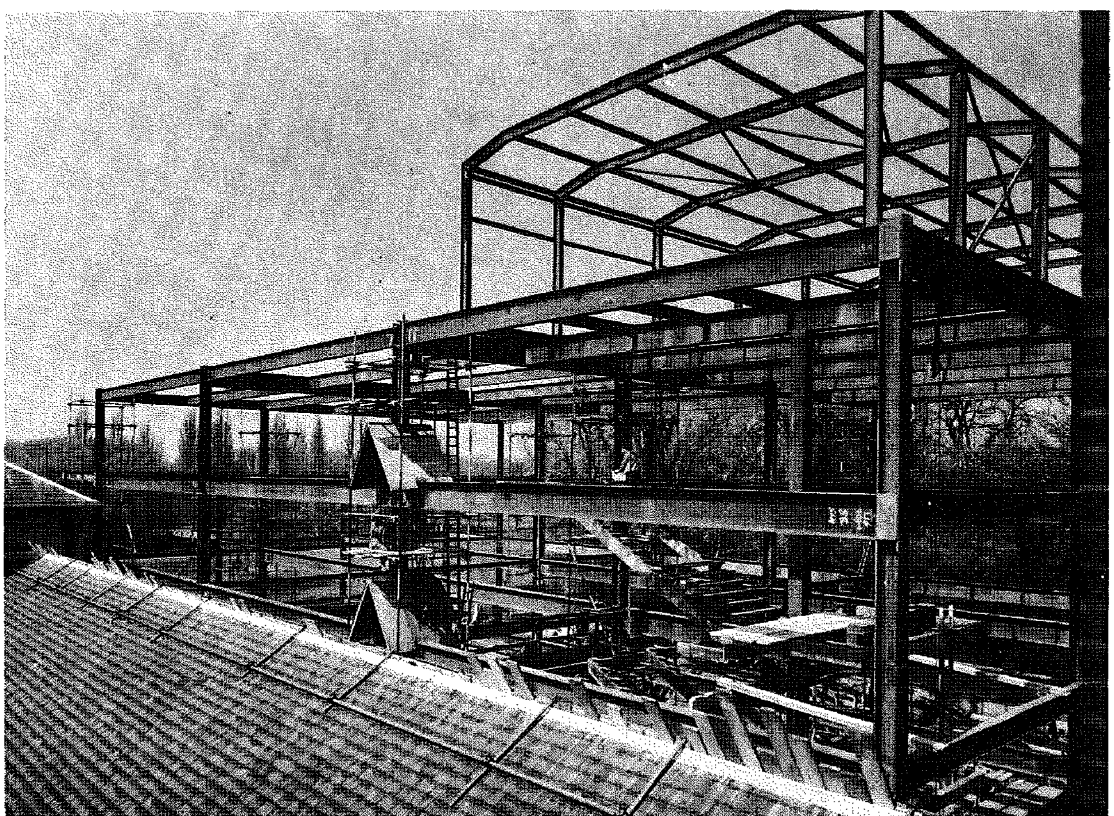


Fig. 8

Fig. 9

Fig. 10

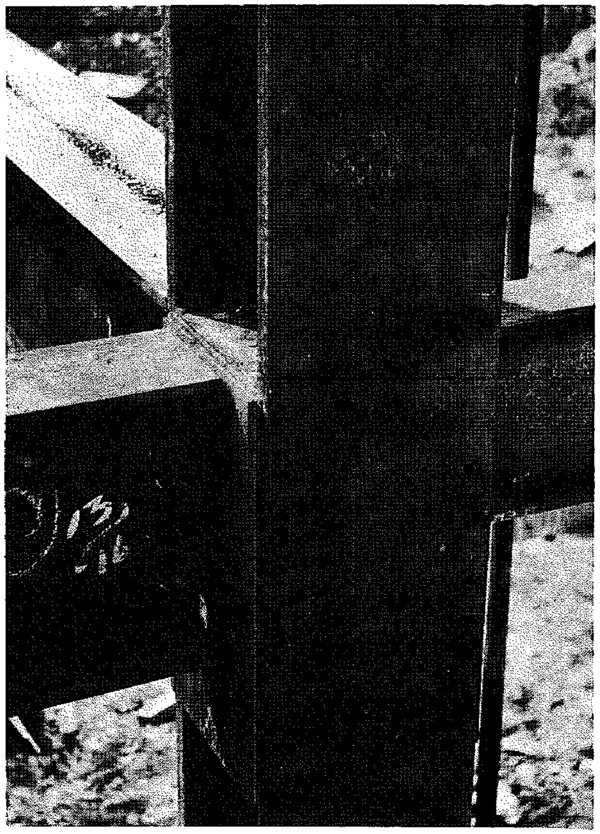


Fig. 11

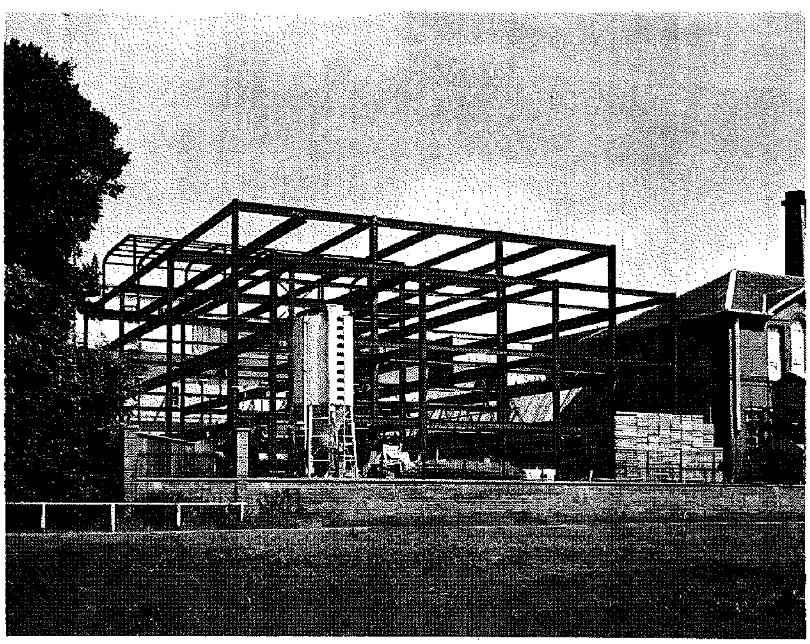


Fig. 12

THE DESIGN OF SWAY FRAMES IN BRITAIN

by

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SYNOPSIS

The design of multi-storey sway frames is complicated by the incidence of frame instability, and rigid-plastic methods of design must be modified to allow for this. In a method proposed by Heyman, an assumed pattern of plastic hinges is used to derive suitable sections, a degree of conservatism being introduced by designing the columns elastically, thus allowing for frame instability for frames within some unspecified limits. Holmes and Gandhi have calculated some special stability functions which allow for the effect of storey drift on frame moments, plastic hinges being confined in their method to the beams. More recently, a computer program has been developed which performs an elastic-plastic design by an iterative process, use being made of a routine which calculates accurately the elastic-plastic failure load of any given plane frame. It is found that the computer method is much more economic than the hand methods of Heyman and Holmes and Gandhi, and more versatile in that it deals with irregular frames.

1. INTRODUCTION

The use of plastic theory in the design of sway frames is elementary provided there is no danger of frame instability reducing the failure load significantly below the rigid-plastic collapse value. When this danger is not present (often satisfied in frames of only a few storeys), structures may be designed merely by assuming a suitable distribution of plastic hinges at collapse. If desired, the theoretical minimum weight structure[1,2] may be derived, using either hand or computer methods[3,4,5].

The incidence of frame instability[6] is however of potential importance in all sway frames, and must be allowed for either implicitly or explicitly in design methods intended for general use with multi-storey frames. In some frames, strain-hardening is sufficient to compensate for the theoretical reduction which would occur in the collapse load of a purely elastic-plastic frame due to instability[7,8]. In general, however, except in single storey frames, some systematic means of allowing for frame instability must be introduced, and the present paper describes procedures developed in Britain for dealing with this problem.

Design codes for building frames in Britain require two loading conditions to be investigated, namely dead and superimposed vertical loads with and without wind loading. The load factor for "vertical loading" (that is, dead and superimposed vertical loads) currently specified is $\lambda_1 = 1.75$, while that for "combined loading" (that is, vertical loading plus wind loads) is $\lambda_2 = 1.40$. For a range of frames up to a certain number of storeys (dependent on loading intensities, number of bays and storey height to beam span ratio), vertical loading will be more critical for the design of the beams than combined loading. These limits have been studied with reference to plastic design methods[9,10]. As an approximate rule[9], vertical loading is found to be more critical in frames with number of storeys less than $0.105 \frac{w}{p} \frac{qL^2}{H^2}$ where H is the mean storey height of the building in feet, L the mean beam span, q the number of bays, p the mean intensity of total wind pressure on the vertical projected area of the building and w is the working value of the intensity of total vertical loading per floor.

In frames of height less than the critical number of storeys n, a beam of span L carrying a total uniformly distributed vertical load W will require a full plastic moment not less than $\lambda_1 \frac{WL}{16}$. This is therefore a lower limit on the size of a beam, no matter how many storeys there may be in the structure. The design methods to be described differ in the way in which beam sections influenced by wind loading are specified, and in the way in which columns are designed.

2. DESIGN METHOD PROPOSED BY HEYMAN

In this method, Heyman[10,11] assumed the distribution of plastic hinges for combined loading at load factor λ_2 as shown in Fig. 1. His method applies to regular frames only, consisting of q equal bays of span L, each carrying a vertical load (working value) of W. To simulate the effect of a uniformly distributed load, the total beam load W is divided into three concentrated loads, $\frac{W}{4}$ applied at each end and $\frac{W}{2}$ at mid-span. Each storey has a height of H, and the wind load acting at each floor level is Q ($\frac{Q}{2}$ on roof beams).

The hinge moments of the beams in the r th storey from the top are B_r (roof beams B_1), C_r is the hinge moment of an internal column ($r = 1$ corresponding to the uppermost columns) and C_r^* is the hinge moment of an external column. The pattern of hinges assumed by Heyman was arrived at as suitable for design purposes after examining a number of alternatives. Heyman gives the following results for B_r, C_r and C_r^* in a multi-storey multi-bay frame.

$$\left.\begin{array}{l}
\text{For } r \neq 1:- \\
B_r \not< \lambda_1 \frac{WL}{12}, \quad B_r \not< \lambda_2 \left\{ \frac{WL}{12} + \frac{QH}{6q}(r-1) \right\}, \\
C_r + C_{r-1} \not< X_r \text{ where } X_r = \lambda_2 \frac{QH}{q}(r-1), \\
C_r^* = \tfrac{1}{2} C_r . \\
\text{For } r = 1:- \\
B_1 \not< \lambda_2 \left\{ \frac{WL}{12} + \frac{QH}{24q} \right\}, \\
C_1 \not< C_1^*, \quad C_1^* \not< B_1 .
\end{array}\right] \qquad (1)$$

A single bay frame must be treated as a special case (see references 10 and 11).

Heyman proposes the use of the beam hinge moments given above to design the beams according to full plastic moment values. For the columns, because of the possibility of instability effects, Heyman suggests that the column hinge values C_r and C_r^* should be regarded as limiting <u>elastic</u> values, so that the columns are designed just to remain elastic under the terminal moments C_r or C_r^* in the presence of the appropriate axial load. This facilitates the checking of any possibility of failure of a column length due to instability by using a method developed by Horne[12,13].

Although the columns are designed by Heyman's formulae (1) to remain elastic, it could not be assumed, without making some analytical check, that they would in fact remain elastic at the factored combined loads, since the values of C_r and C_r^* are calculated on the assumption that hinges exist in the columns at collapse. Heyman therefore proposes a check analysis to ensure that the columns remain elastic at the working load level, and that the sections should be increased in those cases where the yield stress is exceeded. He suggests a simple approximate method of calculating the column moments.[11] He also suggests an approximate method for checking sway deflexions at working loads[11] to ensure that these are not excessive.

Heyman's design method has the advantage of directness and simplicity. Frame instability is not considered, but there is a degree of conservatism in the design procedure introduced by the elastic design of the columns. For this reason, Heyman's method is safe for a range of multi-storey frames, although the limits of this range have not in any way been investigated. By checking additional moments introduced by sway at working loads, some idea may be gained of the likely importance of frame stability, and with this safeguard, Heyman's method appears to be a valid one. Comparisons of particular cases with designs derived by the computer method described below indicate that Heyman's method may be excessively conservative for frames up to five or six storeys. Heyman's method suffers finally from the disadvantage that it applies only to regular frames.

3. DESIGN METHOD PROPOSED BY HOLMES AND GANDHI

Holmes and Gandhi[14] propose a design procedure for regular frames in which, at collapse under combined loading, plastic hinges are assumed to be confined to the beams. Down to a certain number of storeys from the top (Zone I) the beam section is controlled by vertical loading with hinges forming as in Fig. 2(a). In Zone II lying below Zone I, the beam section is controlled by combined loading with hinges in the pattern shown in Fig. 2(b), and for tall frames a third pattern of hinges as shown in Fig. 2(c) may control the beam sections in Zone III lying below Zone II.

Holmes and Gandhi take $\lambda_1 = 1.75$ (load factor for vertical loading) and $\lambda_2 = 1.40$ (load factor for combined loading). Denoting beam span by l and storey height by h, vertical load per beam by W and storey shear per bay by H, beam full plastic moment by B, internal column moment by C_I and external column moment by C_E, the ranges of the three zones and the design formulae for each zone are as follows.

<u>Zone I</u> Applicable when

$$0 < A(mHh)_{av}/Wl \leqslant \frac{1}{16} .$$

$$B = 1.75\ Wl/16,$$

$$C_E \geqslant B/2,$$

$$C_I \geqslant 1.4\ A_c mHh/2.$$

<u>Zone II</u> Applicable when

$$\frac{1}{16} < A(mHh)_{av}/Wl \leqslant \frac{1}{4} .$$

$$B = 1.4\ \frac{Wl}{16} + 1.4A\ \frac{(mHh)_{av}}{4} ,$$

$C_E \geqslant B/2,$

$C_I \geqslant 1.4A_c mHh/2.$

Zone III Applicable when,

$A(mHh)_{av}/Wl > \frac{1}{4}.$

$B = 1.4A(mHh)_{av}/2,$

$C_E \geqslant B/2,$

$C_I \geqslant 1.4A_c mHh/2.$

The coefficients m are the stability function defined by Merchant[6,15], and dependent on the ratio of the factored axial thrust P in the appropriate column length to the Euler critical load P_E for that member. The quantities $(mHh)_{av}$ are the average values of (mHh) in the storeys above and below the beam under consideration. The coefficients A and A_c are special stability functions introduced by Holmes and Gandhi to allow for the effect of flexural deformations in the beams and columns on the moments in the frame. These stability functions depend not only on $\frac{P}{P_E}$, but also on the zone in which the members lie, and the stiffness ratios $\frac{K_U}{K_L}$ and $\frac{K_L}{K_B}$ where K_U, K_L and K_B are the elastic stiffnesses (second moment of area divided by length) of the upper and lower column lengths and beam respectively. In the design procedure, therefore, it is necessary to make an estimate of these stiffness ratios, use the formulae to calculate the design moments, and iterate if the initial guess at a stiffness ratio was not sufficiently accurate.

The method of Holmes and Gandhi is a major step forward in the development of a design (as opposed to merely analytical) procedure for multi-storey frames with explicit allowance for frame instability effects. It suffers however from two major defects. Firstly, it applies only to highly regular frames, and although Holmes and Gandhi suggest a procedure for dealing with frames with varying beam spans, this modification appears to lead to excessively conservative results. Secondly, no guarantee exists that at collapse, plastic hinges would form in anything like the pattern assumed. In the presence of instability effects, the uniqueness theorem of plasticity ceases to be valid, and the failure load of a frame designed by the method of Holmes and Gandhi could theoretically be either above or below the design load. From comparisons made with designs produced by computer, it appears that, because of various approximations made by Holmes and Gandhi on the safe side, their method appears to be conservative. Their procedure is at present the most thorough hand method available in Britain.

4. DEVELOPMENT OF COMPUTER PROGRAM FOR DESIGN OF MULTI-STOREY FRAMES

Program for Analysis of Elastic-Plastic Failure Loads

The suitability of approximate design methods such as those described above can only be assessed by carrying out numerous collapse analyses of frames so designed. The testing of the efficiency of the design procedures also requires ideally a method of producing highly efficient designs by any method, however involved, so that relative economies may be assessed.

The "exact" calculation of the failure loads of multi-storey frames, allowing for frame instability, has received much attention in Britain. The principles of elastic-plastic behaviour of frames have been very thoroughly explored[6,16,17]. Hand calculation of failure loads is excessively laborious, but digital computer methods[3,18,19] are now available. Of these methods, that of Jennings and Majid[19] appears to be the most efficient for steel frames. It is a displacement method in which the effect on stiffness of axial loads is allowed for by the introduction of stability functions, while the effect of plastic hinges is allowed for by systematically modifying the stiffness matrix as each hinge forms. The flow diagram for the calculation is shown in Fig. 3. The program automatically follows the formation of plastic hinges as the load is increased, and ceases the calculation when the determinant of the stiffness matrix becomes negative. This stage represents the attainment of the collapse load.

Design Criteria

The frame is required to sustain vertical loading up to a load factor of at least λ_1 and combined loading up to a load factor λ_2 where $\lambda_2 < \lambda_1$. In addition, certain restrictions are placed on the stages at which plastic hinges are allowed to form in individual members. Basing analysis on a unit form factor (where the form or shape factor is the ratio of full plastic moment to moment at first yield in the extreme fibres), it is stipulated that:-

1) no plastic hinge shall form in a beam at a load factor λ less than unity for either vertical or horizontal loading,
2) no plastic hinge shall form in a column at a load factor less than λ_1 for vertical loading, nor less than λ_2 for

combined loading.

Since the columns are designed to remain elastic up to load factor λ_1 or λ_2, their suitability with respect to lateral stability may be checked manually[12,13] after the design is complete.

It is appreciated that other design criteria - particularly those relating to permissible deflexions at working loads - may also have to be considered, and it is assumed that such criteria will be separately investigated as required.

Design Procedure

The flow diagram for the complete computer program is summarised in Fig. 4. An initial choice of sections is made according to some suitable formulae. It has been found that the final design does not depend on the initial choice in all frames so far considered, but the more reasonable the choice, the fewer iterations required. Heyman's formulae may be used, but a closer estimate ("modified Heyman formulae") is derived by replacing $\frac{WL}{12}$ by $\frac{WL}{16}$ and X_r and X_r^* by $\frac{X_r}{1.46}$ and $\frac{X_r^*}{1.46}$ in equation (1). This gives lower beam moments but higher column moments than in Heyman's formulae.

The preliminary design is then analysed by an elastic-plastic method of analysis (Method B) which is less time-consuming and slightly less accurate than Method A (Fig. 3). This enables tests to be applied in the program to check which design criteria are not being satisfied. The sections of the various members are then modified to satisfy the design criteria, the approximation being made that the load factor at which a hinge first forms will be changed in simple proportion to the full plastic moment of the member. Although this is necessarily a crude approximation, it causes an automatic modification of the section in the correct sense, and is found to give a rapidly convergent iterative procedure. A final check on the suitability of all sections is made by performing an accurate analysis by Method A (Fig. 3).

5. EXAMPLES OF AUTOMATICALLY DESIGNED FRAMES

Four Storey, Single Bay Frame

This frame, shown with working values of loads in tons in Fig. 5(a), was used by Heyman[11] to illustrate his method. The sections chosen by Heyman for $\lambda_1 = 1.75$ and $\lambda_2 = 1.40$ are shown in columns 2 and 3 of Table 1, the members being numbered according to the system shown in Fig. 5(a). Starting with modified Heyman formulae, three iterations of the automatic design procedure gave a satisfactory and economical design with the sections shown in columns 4 and 5 of Table 1. The order of hinge formation, with the load factors at which hinges form, is shown for combined loading in Fig. 5(b). It will be seen that no beam hinge forms below $\lambda = 1.00$ and no column hinge below $\lambda = 1.40$. Final collapse occurs when $\lambda = 1.50$, the number of hinges being 10. This compares with a simple plastic collapse load factor of 1.60 with 11 hinges. The load-deflexion curve for the horizontal sway of the top storey is shown in Fig. 6.

Comparisons with Heyman's design are of interest. Heyman's frame has a weight of 5.64 tons, and elastic-plastic analysis shows it to have a load factor at failure for combined loading of 1.76. The automatically computed design has a weight of 4.91 tons. Hence for this frame, Heyman's procedure is conservative.

Eight-Storey, Single-Bay Frame Resting on Flexible Foundations

Although in the foregoing four-storey frame, stability effects are sufficiently important (6%) to necessitate taking them into account, they do not dominate the design. The present eight-storey frame is chosen to test whether the design procedure can be used successfully to proportion the members of a frame in which frame stability effects are dominant.

The dimensions and working loads are illustrated in Fig. 7(a). The modified Heyman formulae give the sections shown in columns 2 and 3 of Table 2. An elastic-plastic analysis of this structure under combined loading (Fig. 7(b)) shows a completely inadequate load factor at collapse of 0.86, the large discrepancy compared with the required value of 1.40 being due to frame instability. A single modification of the members by the standard routine gives the sections shown in columns 4 and 5 of Table 2, the order of hinge formation being shown in Fig. 8(a). The design criteria are satisfied, the collapse load factor being 1.43. A check on behaviour under vertical loading (Fig. 8(b)) shows entirely elastic behaviour up to a load factor of 2.49 so that, as might be expected, this system of loading exercises no control over the design.

It is not suggested that the design obtained for this highly artificial frame would be acceptable in practice. At working load under combined loading, the top sway deflexion is 6.8 inches, or 1/153 of the height, and is certainly excessive. The example shows conclusively however that the method is capable of dealing with frames in which overall stability is of prime importance.

Irregular Frame

The ability of the method to deal with highly irregular frames may be illustrated by reference to the frame in Fig. 9. No approximate formulae are available for a preliminary design, but this may be obtained quite readily by postulating any system of internal forces capable of sustaining the applied loads. It is necessary to adopt suitable minimum sections for the beams to enable them to support the vertical loads, and the performance of the structure must be checked by computer for wind blowing from either direction. For this frame, a satisfactory design was obtained after four trials. The order of hinge formation with wind blowing from the left is shown in Fig. 10, final failure occurring at a load factor of 1.54. With wind from the right, the failure load factor is 1.58, while for vertical loading, failure occurs at a load factor of 1.77. The final sections are shown in columns 2 and 3 of Table 3.

6. CONCLUSIONS

The derivation of a computer method of design for multi-storey frames based on elastic-plastic behaviour is a useful step forward. It provides a means of assessing the validity and efficiency of manual methods of design that have been suggested. It shows that, for frames of a few storeys, the methods of both Heyman and of Holmes and Gandhi are conservative, and it is possible that the computer method may aid the development of more refined manual procedures.

The possibility remains, however, that a computer method may remain the only truly reliable procedure for multi-storey sway frames. That the methods of Heyman and Holmes and Gandhi have proved conservative for frames of a few storeys is no guarantee that they may not be unconservative for more extensive frames. Moreover, no manual design procedure is yet available for irregular frames, whereas the computer method is fully versatile. The further refinement of the computer method is therefore well worthwhile, and steps are being taken to incorporate an automatic check on the lateral-torsional stability of column lengths. This may enable plastic hinges to be allowed in the columns below load factors of λ_1 and λ_2, thus leading to greater economy in design.

An interesting point arises in relation to the convergence of the iterative design procedure. It is well known that, in redundant structures, iterative elastic design procedures based on maximum permissible elastic stresses may not be convergent. No such problems arise in simple plastic design, and this certainty of convergence is an attractive feature of plastic design methods. Multi-storey frames are not plastic hinge mechanisms at the point of collapse, but the lack of any convergence troubles in the automatic design procedure shows that the degree of plasticity is sufficient to prevent the divergent iterative process characteristic of some elastic structures.

Although the computer design method described in this paper results in a structure capable of sustaining the applied loads at load factors which are above the minimum without being excessive, there is no guarantee that the resulting design is the minimum weight structure. The problem of the absolute minimum weight design of structures subject to elastic-plastic instability is proposed as the subject of further study in the Civil Engineering Department of the University of Manchester.

ACKNOWLEDGEMENTS

The work described in this paper is part of a general study into computer methods for the analysis and design of structures in the elastic and elastic-plastic range, partially supported by a grant from the Science Research Council, and carried out in the Engineering Department, Faculty of Science of the University of Manchester.

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TABLE 1

Member		Heyman's Design		Sections by Computer	
		Section	S	Section	S
1		2	3	4	5
Beams	2	18x7½x60 U.B.	122.8	16x7x40 U.B.	72.7
	5	18x7½x60	122.8	16x7x40	72.7
	8	18x7½x55	111.7	16x7x40	72.7
	11	18x7½x55	111.7	16x7x40	72.7
Columns	1,3	10x10x60 U.C.	75.0	12x12x79 U.C.	119.2
	4,6	10x10x60	75.0	10x10x60	75.0
	7,9	10x10x60	75.0	10x10x60	75.0
	10,12	10x10x60	75.0	10x10x60	75.0
Total Weight		5.64 tons*		4.91 tons*	

*long ton = 2240 lbs

Summary of sections for example 1

Four storey single bay frame (Fig. 5(a))

TABLE 2

Member		First Trial		Second Trial	
		Section	S	Section	S
	1	2	3	4	5
Beams	1,5	36x12x170 U.B.	667	36x16½x230 U.B.	942.5
	3	12x6½x27 U.B.	38.0	16x7x36 U.B.	63.8
	7	10x5¾x29	34.6	14x6¾x34	54.5
	10	10x5¾x25	29.6	14x6x34	54.5
	13	10x5¾x21	24.1	14x6¾x30	47.1
	16	10x5¾x21	24.1	12x6½x27	38.0
	19	8x5¼x17	15.8	10x5¾x21	24.1
	22	8x5¼x17	15.8	8x5¼x20	19.1
	25	8x5¼x17	15.8	8x5¼x17	15.8
Columns	2,4	12x12x106 U.C.	163.5	14x16x158 U.C.	286.1
	6,8	10x10x60	75.0	14x14½x87	151.4
	9,11	10x10x49	60.3	12x12x79	119.2
	12,14	10x10x49	60.3	10x10x49	60.3
	15,17	8x8x35	34.7	8x8x35	34.7
	18,20	8x8x31	30.4	8x8x31	30.4
	21,23	6x6x20	15.1	8x8x31	30.4
	24,26	6x6x15.7	11.2	6x6x15.7	11.2
Total wt.		6.18 tons*		8.35 tons*	

*long ton = 2240 lbs

Summary of sections for example 2

Eight storey single bay frame resting on flexible foundations (Fig. 7(a))

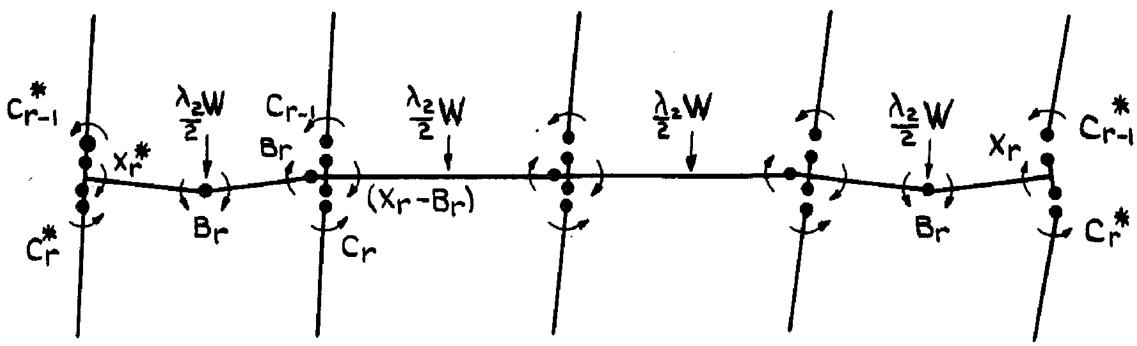
TABLE 3

Member		Sections by Computer	
		Section	S
	1	2	3
Beams	2	10 x 5¾ x 21	24.1
	5	8 x 5¼ x 17	15.8
	8	12 x 6½ x 27	38.0
	11	18 x 7½ x 55	111.7
	13	10 x 5¾ x 25	29.6
	17	12 x 6½ x 27	38.0
	20	8 x 5¼ x 20	19.1
Columns	1	6 x 6 x 20	15.1
	3	8 x 8 x 31	30.4
	4	6 x 6 x 20	15.1
	6	8 x 8 x 31	30.4
	7	8 x 8 x 31	30.4
	9	8 x 8 x 40	39.8
	10	12 x 12 x 65	97.0
	12	10 x 10 x 49	60.3
	14	6 x 6 x 20	15.1
	15	8 x 8 x 40	39.8
	16	8 x 8 x 31	30.4
	18	8 x 8 x 31	30.4
	19	6 x 6 x 25	18.9
	21	6 x 6 x 25	18.9
Total wt.		5.08 tons*	

*long ton = 2240 lbs

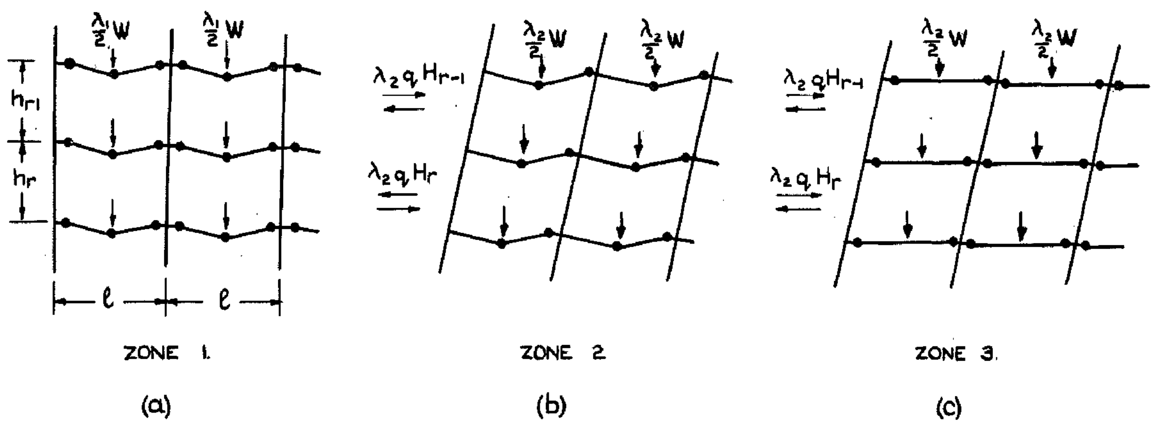
Summary of sections for example 3

Irregular frame (Fig. 9)



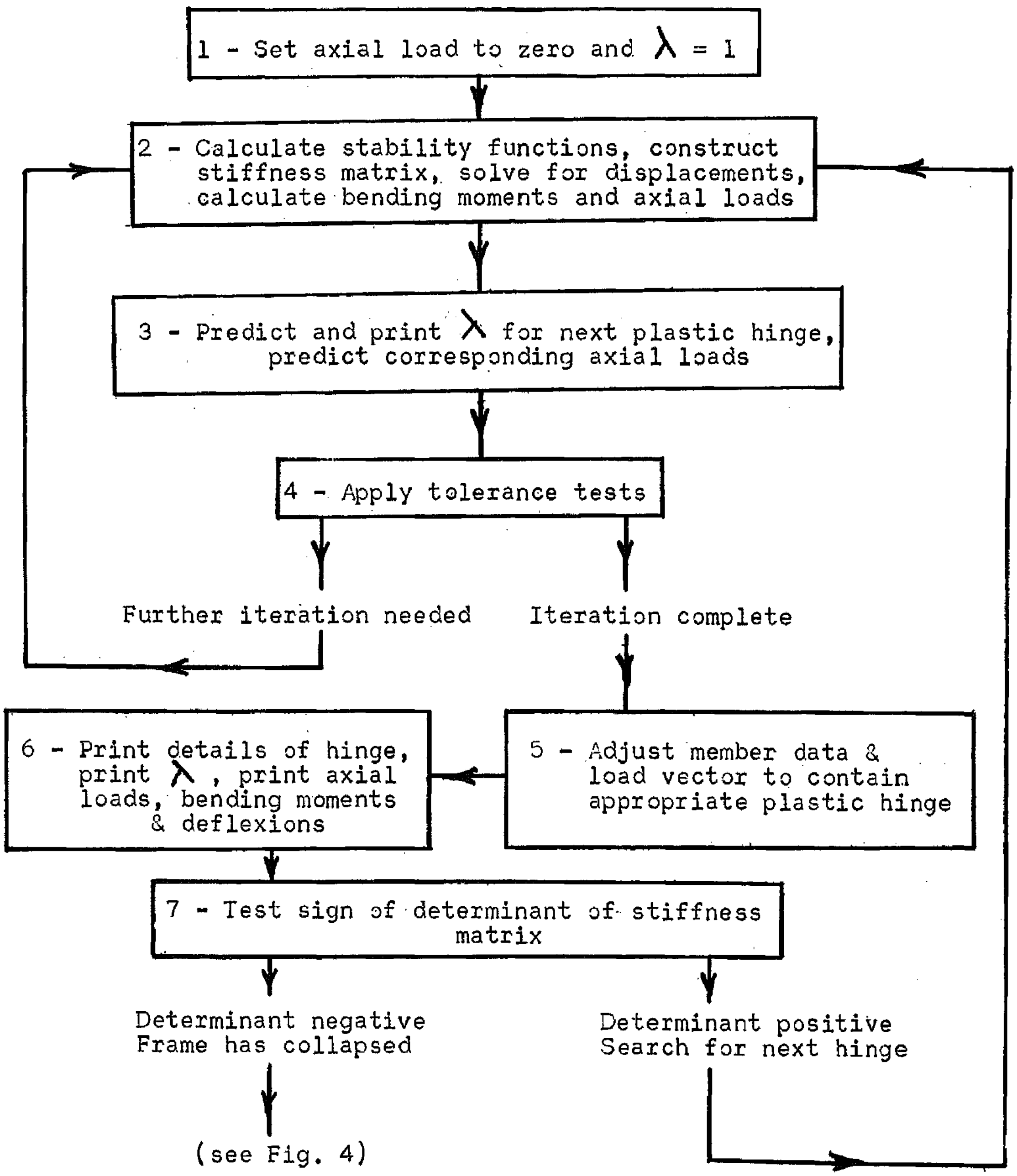
DISTRIBUTION OF HINGES IN HEYMAN'S METHOD
AT r^{th} BEAM FROM TOP.

FIG. I.



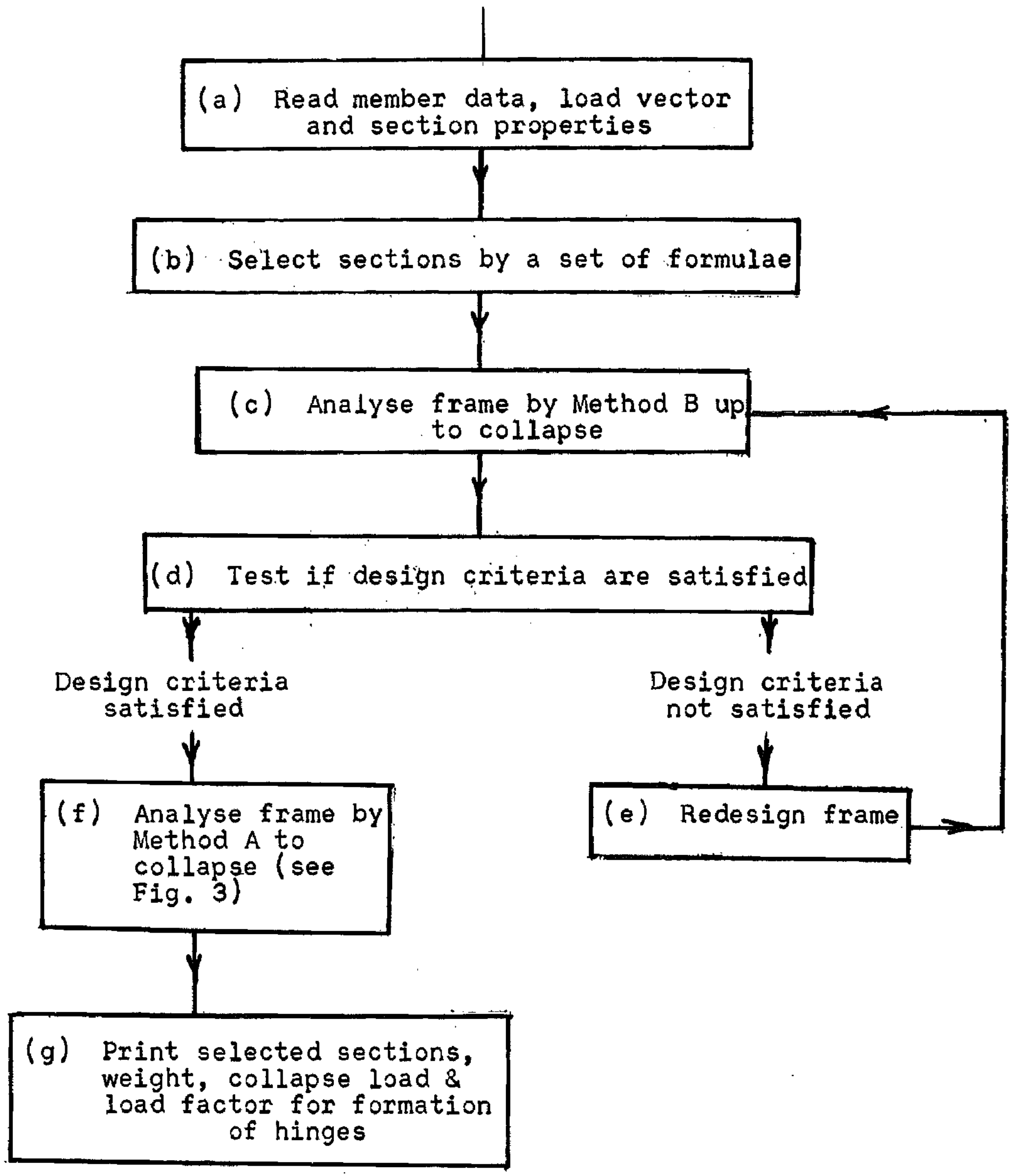
DISTRIBUTIONS OF HINGES IN METHOD OF HOLMES AND GANDHI.

FIG. 2.



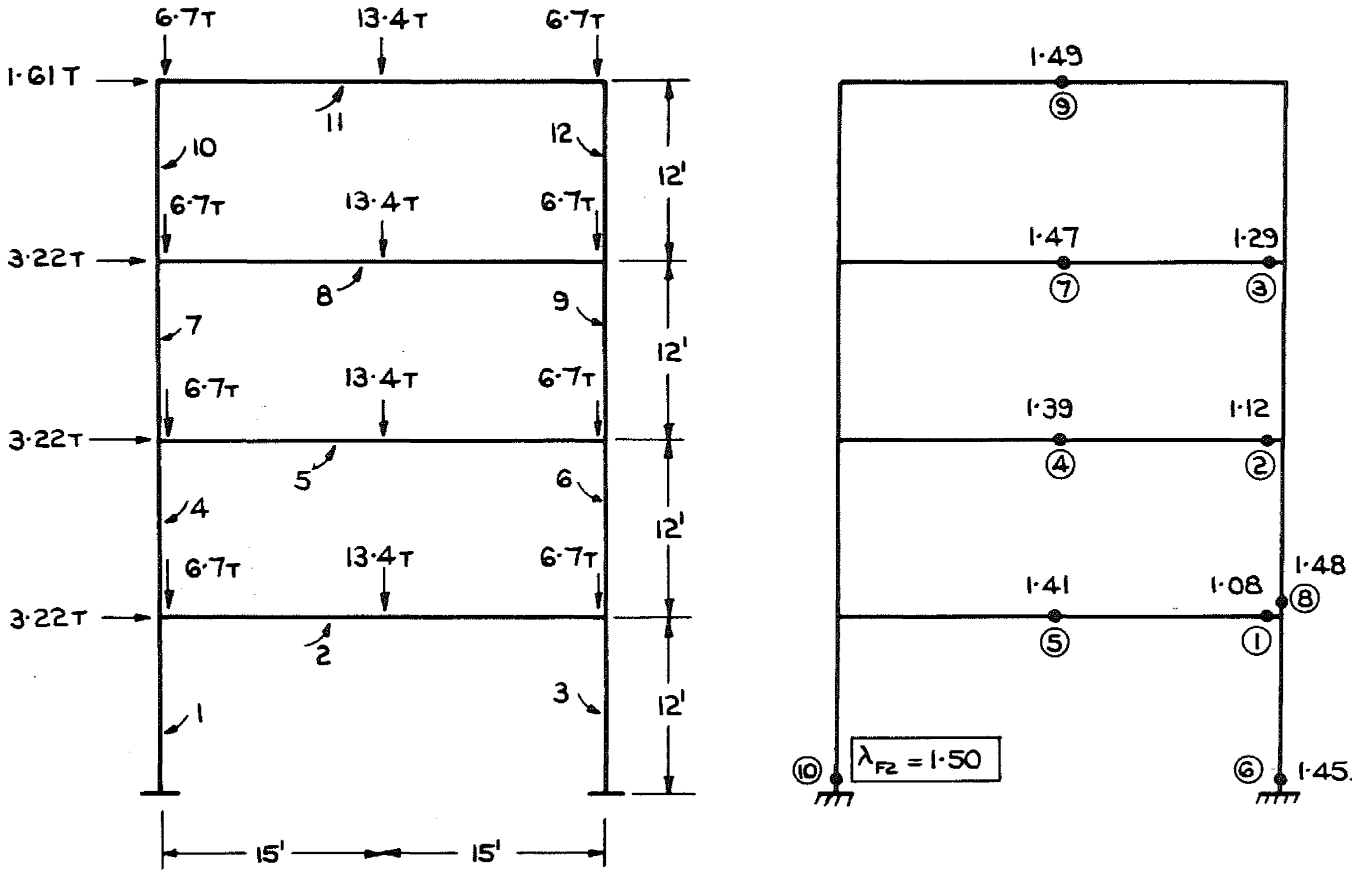
Flow diagram for elastic-plastic analysis up to collapse. Method A

FIG. 3



General flow diagram

FIG. 4



DIMENSIONS, LOADS (IN TONS) AND NUMBERING OF MEMBERS.

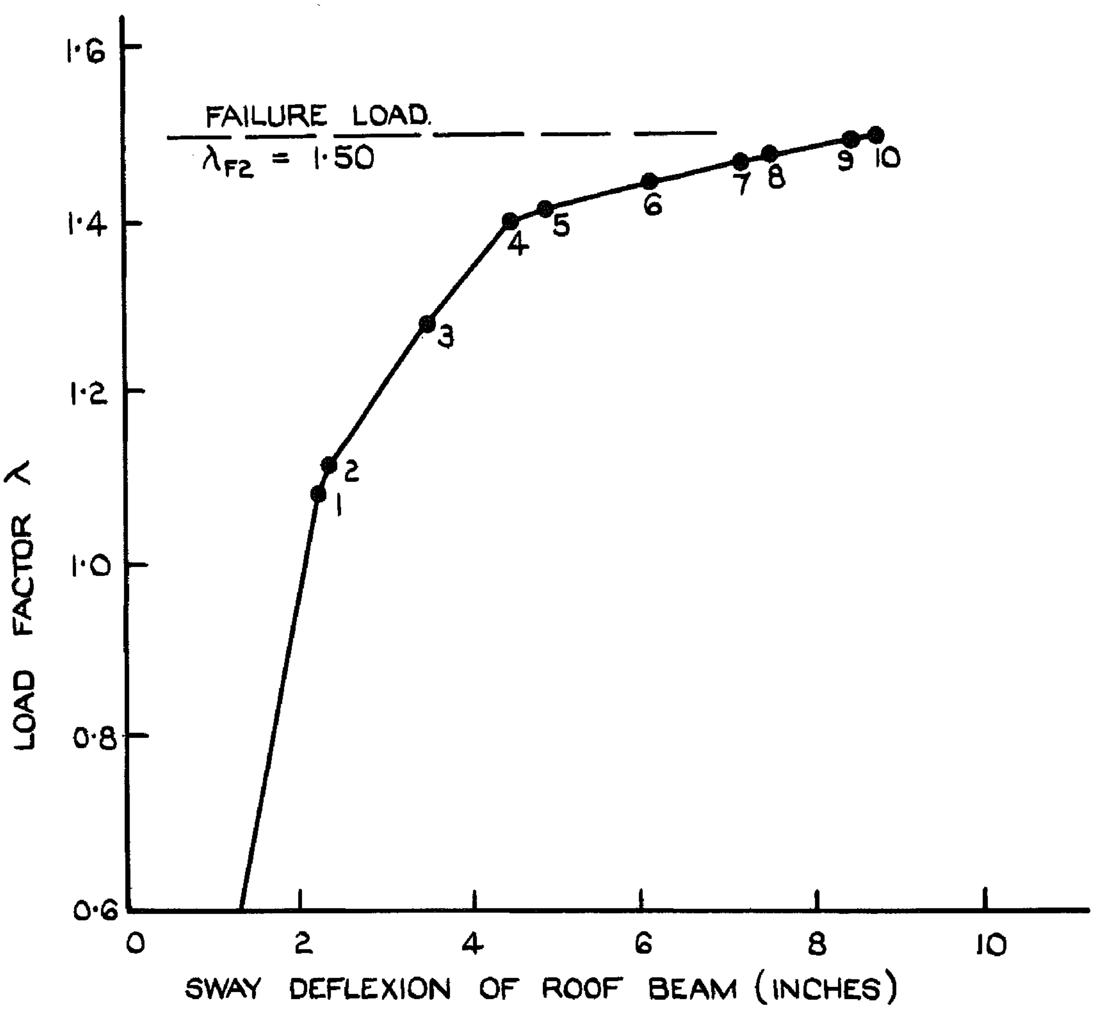
(a)

ORDER OF HINGE FORMATION AND LOAD FACTORS AT WHICH THEY FORM. FINAL DESIGN. (COMBINED LOADING).

(b)

DESIGN EXAMPLE I.

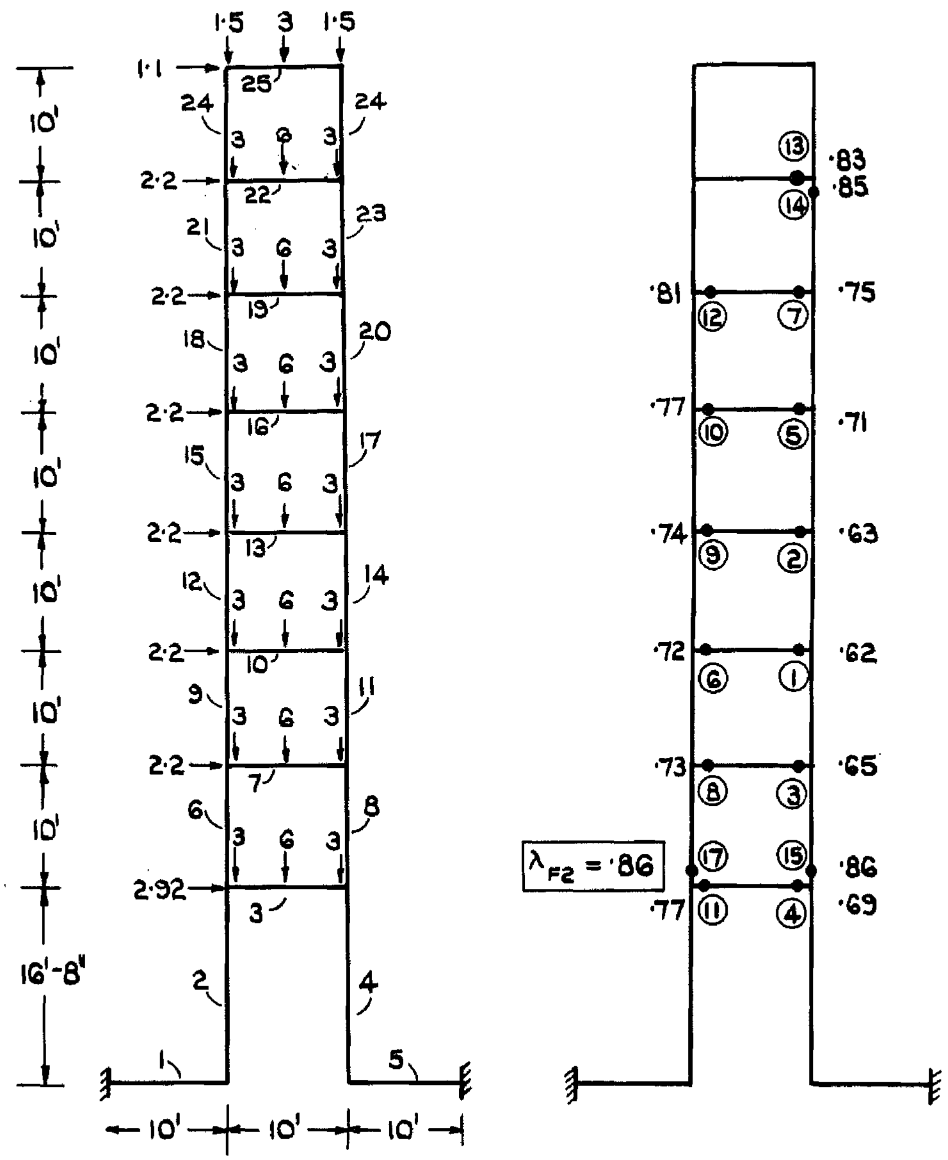
FIG. 5.



LOAD - DEFLEXION CURVE - FINAL DESIGN
COMBINED LOADING.

DESIGN EXAMPLE 1.

FIG. 6.



DIMENSIONS, LOADS (IN TONS) AND NUMBERING OF MEMBERS.

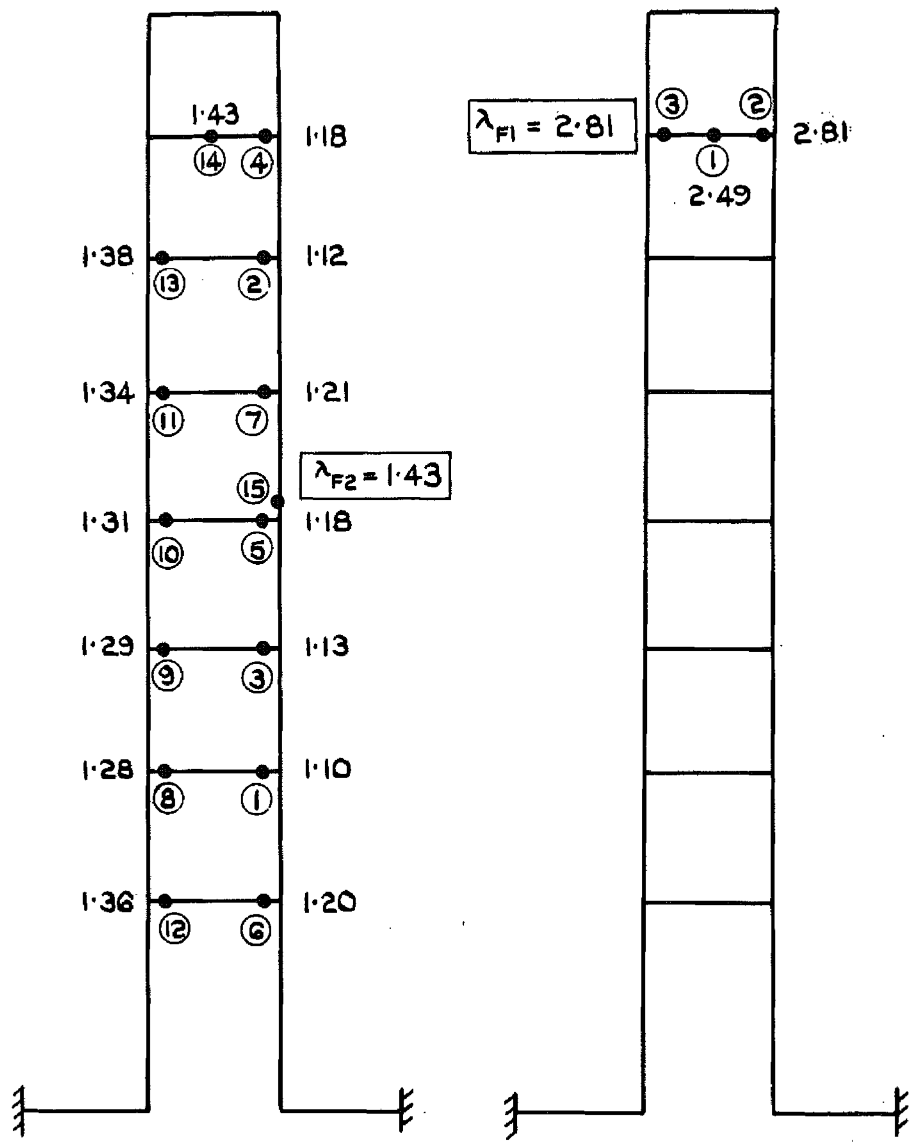
(a)

ORDER OF HINGE FORMATION AND LOAD FACTORS AT WHICH THEY FORM, FIRST TRIAL. (COMBINED LOADING).

(b)

DESIGN EXAMPLE 2.

FIG. 7



COMBINED LOADING

(a)

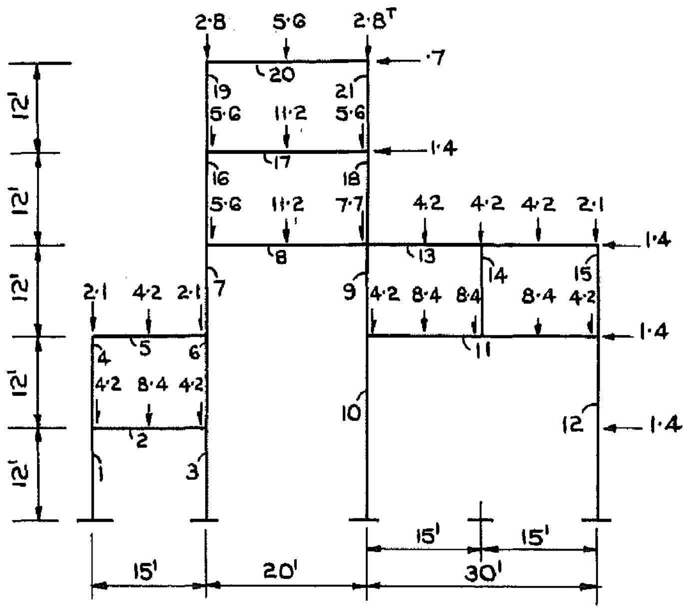
VERTICAL LOADING.

(b)

ORDERS OF HINGE FORMATION AND LOAD FACTORS AT WHICH HINGES FORM. SECOND TRIAL.

DESIGN EXAMPLE 2

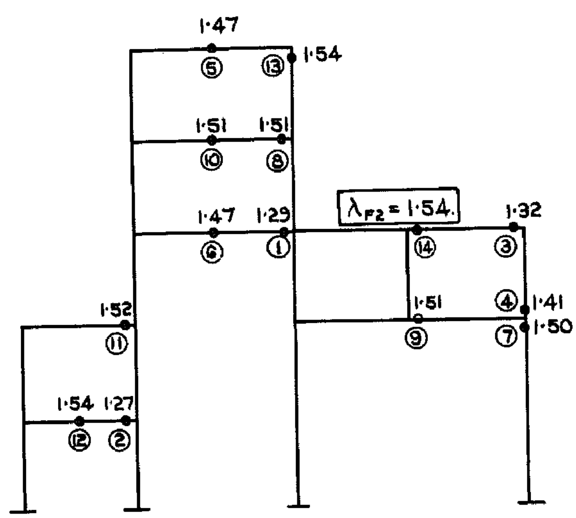
FIG. 8



DIMENSIONS LOADS (IN TONS) & NUMBERING OF MEMBERS.

DESIGN EXAMPLE 3.

FIG. 9.



FINAL DESIGN (COMBINED LOADING WITH WIND FROM LEFT)

DESIGN EXAMPLE 3.

FIG. 10.

THE INFLUENCE OF DEFORMATIONS ON THE ULTIMATE LOAD OF RIGID STEEL FRAMES

By Udo Vogel[1]

SYNOPSIS

An extension due to some "second order effects" of the simple plastic theory is developped. A set of equations for the unknown terms, one of which is the critical load factor P_{cr} is established. These equations may be solved numerically by a process of iteration. Theoretical and experimental investigations were made to check the proposed approximate theory.

INTRODUCTION

In the "Simple plastic theory" for the design of rigid steel frames the following important assumptions are made:

1. The deformations of the structure are so small, that their influence on the equations of equilibrium may be neglected.

2. Neither a single member nor the whole structure may show any effect of instability in the plane of the framework before performing a yield mechanism.

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However, it is known, that in many cases of structures with great slenderness ratio or great axial loads the deformations even in the elastic range are so large that they must be taken into account, because they may increase the bending moments considerably. Furthermore we know from the behavior of the eccentrically loaded column, that in the range of partial plastification, the equilibrium may become instable without bifurcation, if by increasing load the resistance of the internal stresses may not increase to the same extent as the external forces.

The ultimate load then lies at the maximum of the load-deformation-curve (see Fig.1)[2,3]

FIG.1.- LOAD-DEFORMATION-CURVE OF A BEAM-COLUMN

The reason for this behavior is that the structure suffers a loss of stiffness by increasing of the axial forces and by plastification, so that the deformations increase more rapidly than the external forces.

This behavior is found also in statically indeterminate systems, which are subjected to compression and bending, such as continuous

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[2] Jezek,K.: "Die Festigkeit von Druckstäben aus Stahl" Springer-Verlag Wien 1937

[3] Chwalla,E.: "Theorie des außermittig gedrückten Stabes aus Baustahl" DER STAHLBAU 7 (1934),S.161/65,173/76, 18o/84

beams with axial and transverse loads[4] or frames[5,6]. It is called a "Problem of instability without bifurcation of equilibrium" or "frame instability". To resolve such problems the "theory of inelastic instability" must be used.

Now as the ultimate load of a structure is defined as the load, under which increasing deformations take place without increasing loads, the "simple plastic theory" represents a "theory of inelastic instability" too.

But compared with the more exact theory in the simple plastic theory some simplyfying assumptions are made. These assumptions are:

a) the idealized stress-strain-relationship for the ductile material,

b) the localisation of yielding at the plastic hinges,

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4 Chwalla,E.: Außermittig gedrückte Baustahlstäbe mit elastisch eingespannten Enden und verschieden großen Angriffshebeln" DER STAHLBAU lo (1937), S.49/52

5 Oxfort,J.: "Über die Begrenzung der Traglast eines statisch unbestimmten biegesteifen Stabwerkes aus Baustahl durch das Instabilwerden des Gleichgewichts" DER STAHLBAU 3o (1961),S.33/46

6 Vogel,U.: "Über die Traglast biegesteifer Stahlstabwerke" DER STAHLBAU 32 (1963), S.1o6/113

c) the neglect of deformations,

d) the neglect of the reducing influence of axial forces on the bending-stiffness of the members.

These simplifying assumptions are suitable and verified by tests for most frameworks of steel, the members of which are subjected dominantly to bending moments.

But there are adverse cases in which members have great slenderness-ratio or are subjected to high compression, for instance the columns in the lower stories of multi-story frames. Here the application of the simple plastic theory may lead to a design on the unsafe side. Therefore a more accurate theory is necessary. But the application of the exact theory of inelastic instability, well known from the papers of Kårmån, Chwalla, Ježek and other authors, is too tedious for frameworks with more than two or three members.

Therefore an approximate theory has been developped, which includes some of the analytical advantages of the simple plastic theory, but which on the other hand takes into account the influence of deformations and axial thrust, which may not be neglected, if the problem of inelastic instability will be considered correctly.

APPROXIMATE THEORY OF INELASTIC INSTABILITY

Besides the postulation of an idealized elastic-plastic material, as it is known in the simple plastic theory, the following assumptions in the proposed approximate theory of inelastic instability are made:

a) The ultimate load is reached, when a failure mechanism is created by a sufficient number of plastic hinges.

b) For the calculation of deformations the spread of plastic zones in the neighbourhood of plastic hinges is neglected.

On the other hand the following "second-order-effects" are taken into account:

a) The influence of deformations on the equilibrium conditions,

b) the influence of axial thrust on the plastic moment capacity and on the bending-stiffness of the members.

Therefore the theory may be called "second order theory of plasticity".[7]

This method will now be developped in short:

At first a yield mechanism of a frame as shown in FIG.2 is considered just in the moment, when the ultimate load is attained.

FIG.2.- LOADS AND DEFORMATIONS

The joints and plastic hinges of the frames will have some (unknown) displacements δ_K as result of the elastic and plastic deformations which have taken place during the increasing of the external loads Q_j.

FIG.3.- VIRTUAL DISPLACEMENTS

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7 Vogel,U.: "Die Traglastberechnung stählerner Rahmentragwerke nach der Plastizitätstheorie II.Ordnung" Stahlbau-Verlag Köln 1965

If now the structure moves by a virtual displacement as it is shown in FIG.3, it follows from the "principle of virtual works" for the case of equilibrium:

$$\sum_{j=1}^{r} Q_j \cdot v_j = \sum_{i=1}^{n} M_{Pci} \cdot \vartheta_i \tag{1a}$$

With regard to the assumption of proportional loading, the external loads Q_j are defined by a mutual factor P_{cr}, the value of which is to be calculated.

Since the virtual displacements v_j depend on the geometry of the frame and therefore on the displacements δ_K, and the plastic moments M_{PCi} depend on the axial-forces N_i, equation (1a) may be written as:

$$P_{cr} = F(\delta_K, N_i) \qquad \begin{array}{l} K = 1,2 \;........\; m \\ i = 1,2 \;........\; n \end{array} \tag{1b}$$

This equation contains with the load factor P_{cr}

$$z = 1 + n + m$$

unknown terms, wherein:

n = number of unknown axial-forces = number of plastic hinges

m = number of unknown displacements δ_K of the failure-mechanism, just in the moment, when the last plastic hinge has formed, but not yet rotated.

Now, for the n unknown N_i, with well-known methods of statics n independent equations may be found in the form:

$$N_i = \Phi_i(P_{cr}\,,\ \delta_K\,,\ M_{PCi})$$

Since $M_{PCi} = M_{Pi} \cdot f_{(Ni)}$ is a function of N_i the n equations for N_i may be written as:

$$N_i = \Phi_i\,(P_{cr}\,;\ \delta_1, \delta_2 \ldots \delta_m\,;\ N_1, N_2 \ldots N_n) \qquad i = 1,2 \ldots\ldots n \qquad (2)$$

Now, m further expressions must be found for the m unknown displacements δ_K.

Therefore the slope-deflection equations for a member of the frame will be established.

FIG.4.- LOADS AND DEFORMATIONS OF A BEAM-COLUMN

With the signs of FIG.4, from the differential equation of a beam subjected to transverse and normal forces follows:

$$\varphi_{ab} = \frac{\delta_{ab}}{l_{ab}} + \frac{l_{ab}}{EI_{ab}}\left[\alpha'_{ab}(M_{ab} - M^E_{ab}) - \beta'_{ab}\,(M_{ba} - M^E_{ba})\right] \qquad (3a)$$

$$\varphi_{ba} = \frac{\delta_{ab}}{l_{ab}} + \frac{l_{ab}}{EI_{ab}}\left[\alpha'_{ab}(M_{ba} - M^E_{ba}) - \beta'_{ab}\,(M_{ab} - M^E_{ab})\right] \qquad (3b)$$

wherein

M^E_{ab} and M^E_{ba} = end-moments of the clamped beam, owing to the transverse loads, calculated with regard to the influence of deformations and axial-forces (second order theory).

$$\alpha'_{ab} = \frac{\sin \varepsilon_{ab} - \varepsilon_{ab} \cdot \cos \varepsilon_{ab}}{\varepsilon_{ab}^2 \cdot \sin \varepsilon_{ab}} \tag{4a}$$

$$\beta'_{ab} = \frac{\varepsilon_{ab} - \sin \varepsilon_{ab}}{\varepsilon_{ab}^2 \cdot \sin \varepsilon_{ab}} \tag{4b}$$

which are tabulated in some handbooks as functions

$$\text{of} \quad \varepsilon_{ab} = l_{ab} \cdot \sqrt{\frac{N_{ab}}{E I_{ab}}} \tag{4c}$$

(Neglecting the influence of deformations and normal-forces, it is known that $\alpha' = \frac{1}{3}$ and $\beta' = \frac{1}{6}$)

To get m equations for the m unknown $\delta_{ab,K} = \delta_K$ in the moment of failure, there are m equations of continuity available, which may be obtained by equalizing the endslopes of beams or columns meeting at the same rigid knee and at the point of the last hinge. Thereby the endslopes φ are eliminated and a system of linear equations is obtained, which may be written as:

$$\begin{aligned}
\alpha_{11} \cdot \delta_1 + \alpha_{12} \cdot \delta_2 \qquad & = a_{10} = \Psi_1 (P_{cr}, N_i) \\
\alpha_{21} \cdot \delta_1 + \alpha_{22} \cdot \delta_2 \qquad & = a_{20} = \Psi_2 (P_{cr}, N_i) \\
+ \alpha_{32} \cdot \delta_2 + \alpha_{33} \cdot \delta_3 & = a_{30} = \Psi_3 (P_{cr}, N_i) \\
\vdots \qquad & \\
+ \alpha_{m,m-1} \cdot \delta_{m-1} + \alpha_{mm} \cdot \delta_m & = a_{m0} = \Psi_m (P_{cr}, N_i)
\end{aligned} \tag{5}$$

with the solution:

$$\delta_K = \frac{\Delta_K}{\Delta} = \Psi_K (P_{cr}, N_i) \qquad K = 1,2 \, \ldots\ldots\ldots \, m \tag{6}$$

(Often the value of α_{12} does not exist, then the equations (5) may be solved easily step by step).

The equations (1b), (2) and (6) represent a system of $z = 1 + n + m$ equations for the z unknown terms, the one of which is the critical-load-factor P_{cr}. The disadvantage of these equations is that they may not be resolved explicitly for the unknown terms, which are included in a transcendent and non-linear form. Therefore a programm of iteration for the numerical calculation has been developped, which is explained in a systematic sheme as follows:

<u>1. step:</u> (adequate to the "simple plastic theory")

$N_i = 0 \;,\; \delta_K = 0 \longrightarrow P_{cr}^{(1)}$	from equation		(1b)
$N_i = 0 \;,\; \delta_K = 0$ and $P_{cr}^{(1)} \longrightarrow N_i^{(1)}$	"	"	(2)
$N_i = N_i^{(1)}$ and $P_{cr}^{(1)} \longrightarrow \delta_K^{(1)}$	"	"	(6)

<u>2. step:</u>

$N_i = N_i^{(1)},\; \delta_K = \delta_K^{(1)} \longrightarrow P_{cr}^{(2)}$	"	"	(1b)
$N_i = N_i^{(1)},\; \delta_K = \delta_K^{(1)}$ and $P_{cr}^{(2)} \longrightarrow N_i^{(2)}$	"	"	(2)
$N_i = N_i^{(2)}$ and $P_{cr}^{(2)} \longrightarrow \delta_K^{(2)}$	"	"	(6)

check : $P_{cr}^{(1)} - P_{cr}^{(2)} \leqslant \Delta P$ (of arbitrary value): finish

$P_{cr}^{(1)} - P_{cr}^{(2)} > \Delta P$ (" " "): next step

.
.
.

<u>n. (last) step:</u>

$N_i = N_i^{(n-1)}, \; \delta_K = \delta_K^{(n-1)} \longrightarrow P_{cr}^{(n)}$ from equ. (1b)

$N_i = N_i^{(n-1)}, \; \delta_K = \delta_K^{(n-1)}$ and $P_{cr}^{(n)} \longrightarrow N_i^{(n)}$ " (2)

$N_i = N^{(n)}$ and $P_{cr}^{(n)} \longrightarrow \delta_K^{(n)}$ " (6)

$$\text{check : } P_{cr}^{(n)} - P_{cr}^{(n-1)} \leqslant \Delta P \longrightarrow \text{finish}$$

As in the simple plastic theory some controls of the calculation are necessary, so as : control of equilibrium,
control that the plastic moment condition is not be violated,
control of the last plastic hinge (control of deformation) .

Now the question is, how the actual behavior of structures verifies the outlined approximate theory:

VERIFICATION OF THE APPROXIMATE THEORY OF INELASTIC INSTABILITY:

At the Technische Hochschule Stuttgart, Germany, both theoretical and experemental investigations were made in order to check the developped approximate theory.

The results of some of these investigations will be demonstrated in a few pictures:

Theoretical investigations

FIG.5.- ULTIMATE LOAD OF THE ECCENTRICALLY LOADED COLUMN WITH I-SECTION

FIG.5 shows the results of Jezeks more exact theory of inelastic instability compared with the results of the proposed approximate theory for an eccentrically loaded column with wide flange shape.[7]

The agreement of the approximate theory with the exact theory is sufficient for practical cases.

FIG.6.- LOAD-DEFORMATION-CURVE OF A SIMPLE FRAME

FIG.6 shows the load-deformation-relationship for a simple frame, the members of which have rectangular cross-sections. The results of the approximate theory (solid line) are compared with Oxforts [5,6] exact theory (dashed line).

The approximate value of the ultimate load with $P_{cr} = 0,456 \cdot b \cdot h$ at the point of the last plastic hinge, lies near the exact value of $P_{cr} = 0,488 \cdot b \cdot h$.

On the other hand the result of the simple plastic theory is $P_{cr} = 0,833 \cdot b \cdot h$, which is far on the unsafe side. Thereby the reducing influence of the axial thrust on the plastic moment capacity is taken into account, but the influence of deformations, which decreases the load carrying capacity rapidly, is neglected.

At this point it is necessary to state, that in some cases of statically indeterminate systems, the assumption that the ultimate load is reached, when the last plastic hinge has been formed, leads to a value of the ultimate load, which is lower than the maximum of the load-deformation-curve. So the approximate theory sometimes may show results, which are too far "on the safe side", though the influence of spread of plastic zones is neglected for calculating the deformations.

In such cases it would be desirable to calculate the maximum of the load-deformation-curve as an approach for the ultimate load, but - due to the second order effects - this is much more difficult than calculating the load belonging to the last plastic hinge. Furthermore the load at the last plastic hinge is independent on the sequence of the load application. But this is not true for the peak of this curve.

Experimantal investigations

FIG.7.- RESULTS OF MODEL-TESTS

FIG.7 shows the results of tests on model-columns with rectangular cross-section, which were subjected to constant end-slopes and increasing axial loads.[8] Tests were made for different

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[8] Pelikan,W. and Vogel,U.: "Die Tragfähigkeit von Stahlstützen in Geschoßbauten mit Betondecken", DER STAHLBAU 33 (1964) S. 161/167

ratios of slenderness, and the figures show, that the test results are in very good agreement with the predicted theoretical curves of the proposed approximate theory.

FIG.8.- TEST ARRANGEMENT FOR A FULL SCALE TEST

FIG.9.- FOTOGRAPH OF THE TEST SPECIMES IN THE HYDRAULIC TESTING MACHINE

FIG.8 and 9 show the arrangement for a full scale ultimate load test of a framework, consisting of a steel column, rigidly connected with a concrete beam.[8] The column is clamped at the foot. The load acts eccentrically to the column axis.

In FIG.10 the result of the full scale test is shown.

FIG.10.- LOAD-DEFORMATION-CURVE OF THE FRAME

Whereas the predicted value of the ultimate load by the approximate theory was $P_{cr} = 59,5$ t, the load reached in the test was $P_{test} = 58,8$ t, which is only 1,2% per cent below the predicted value. It has to be noted, that the frame was designed so that the column had to break down by forming three plastic hinges at the top, at the foot and at the point of maximum strength between top and foot.

All these theoretical and experimental investigations demonstrate the qualification of the proposed approximate theory of inelastic instability, which is recommended for cases with great axial thrust and not prevented side-sway.

SUMMARY

In this paper it is shown, that an extension of the simple plastic theory for rigid steel frames is necessary in many cases, in which the influence of deformations (and axial thrust) is not negligible for a design "on the safe side". As the exact theory of inelastic instability is too complicated, an approximate theory is proposed, which makes - compared with the exact theory - the following simplifying assumptions:

a) As in the simple plastic theory the plastic hinge-mechanism is declared as the right failure mechanism for the ultimate load,

b) the spread of plastic zones in the neighbourhood of plastic hinges is neglected for the calculation of deformations.

For verifying the theory which is called "Second order theory of plasticity" several analytical and experimental investigations were made, some of the results of which are explained. Good agreement between theory and tests were obtained.

For rigid steel frames with more than two or three members of beams or columns a systematic programm for calculating the ultimate load with regard to the proposed approximate theory has been developped.

ACKNOWLEDGEMENTS

The developed investigations were performed when the author was an assistent of Prof.Dr. Pelikan at the Lehrstuhl für Stahlbau und Holzbau of the Technische Hochschule Stuttgart. The model-tests were made at the Institut für Spannungsoptik und Modellmessung and the full-shape test at the Otto-Graf-Institute of the T.H.Stuttgart.

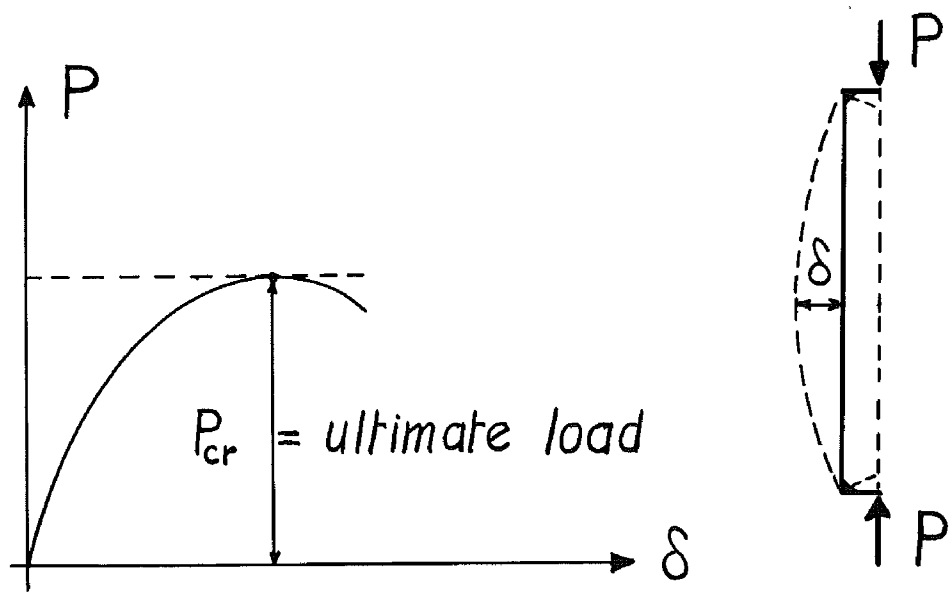

P
P_{cr} = ultimate load
δ
P
δ
P

FIG. 1

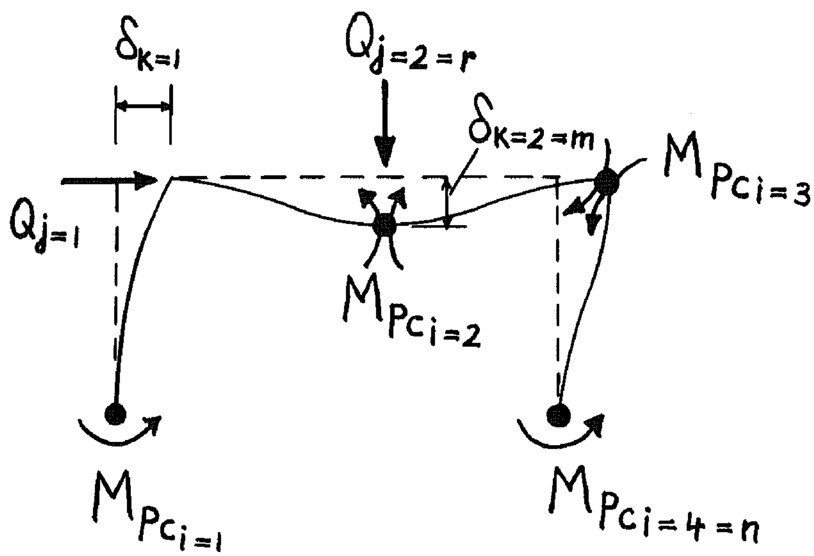

$\delta_{K=1}$
$Q_{j=2=r}$
$\delta_{K=2=m}$
$M_{Pc_{i=3}}$
$Q_{j=1}$
$M_{Pc_{i=2}}$
$M_{Pc_{i=1}}$
$M_{Pc_{i=4=n}}$

FIG. 2

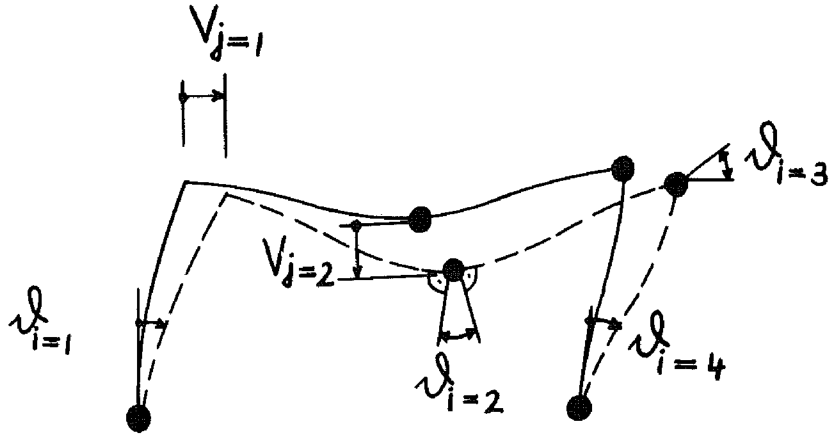

$V_{j=1}$
$\vartheta_{i=3}$
$V_{j=2}$
$\vartheta_{i=1}$
$\vartheta_{i=4}$
$\vartheta_{i=2}$

FIG. 3

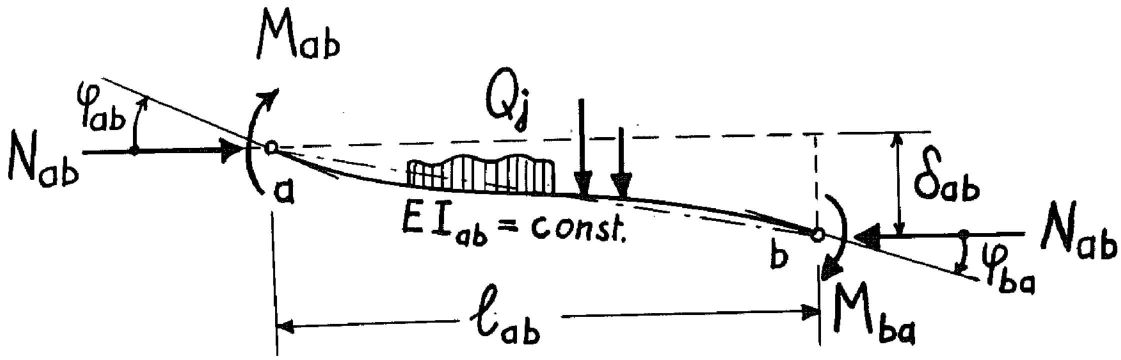
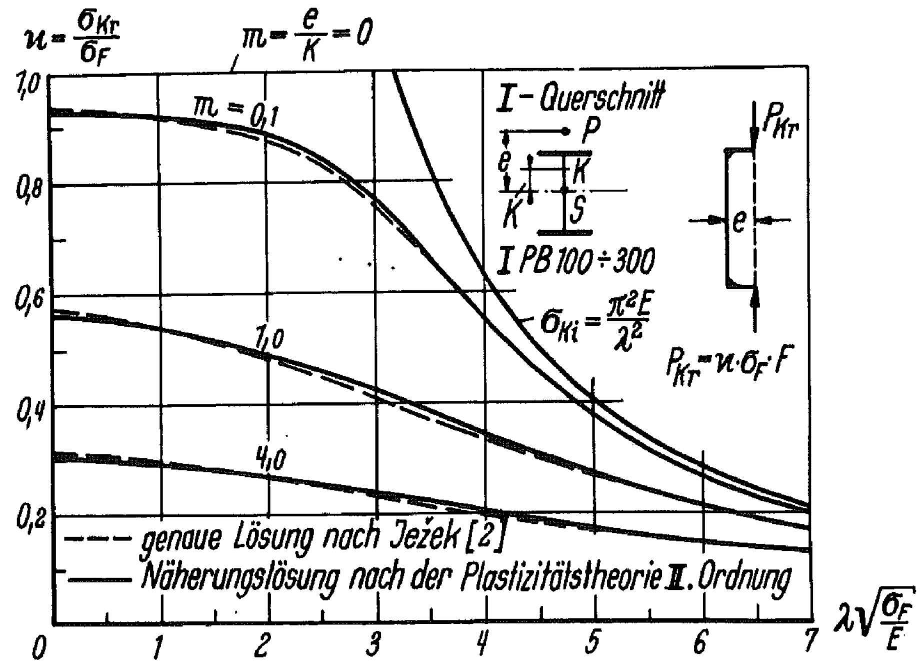

M_{ab}
φ_{ab}
Q_j
N_{ab}
a
δ_{ab}
N_{ab}
$EI_{ab} = const.$
b
φ_{ba}
ℓ_{ab}
M_{ba}

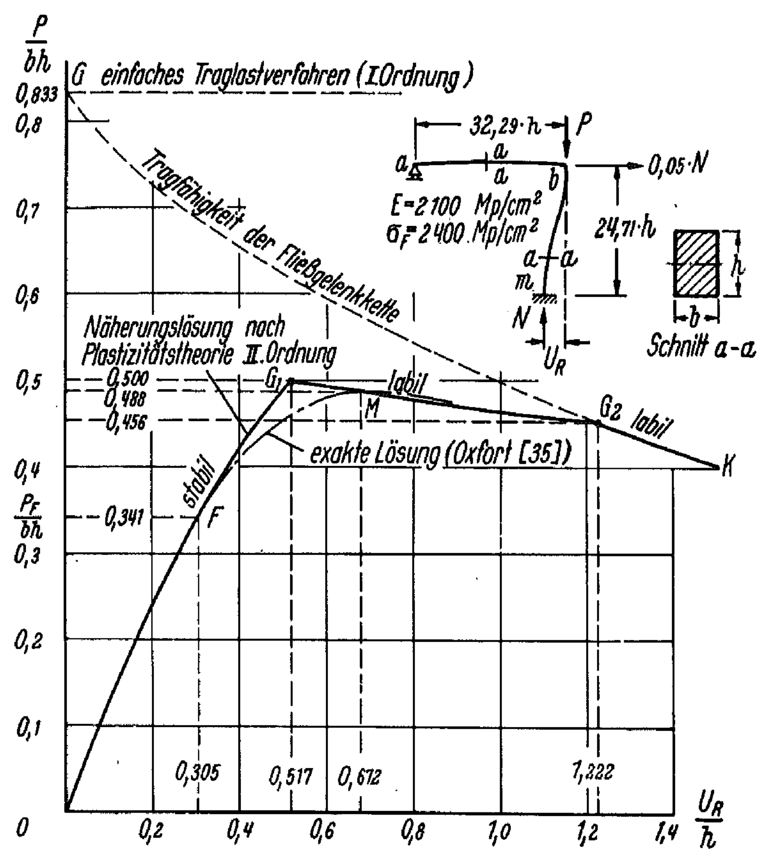
FIG. 4

Fig. 5



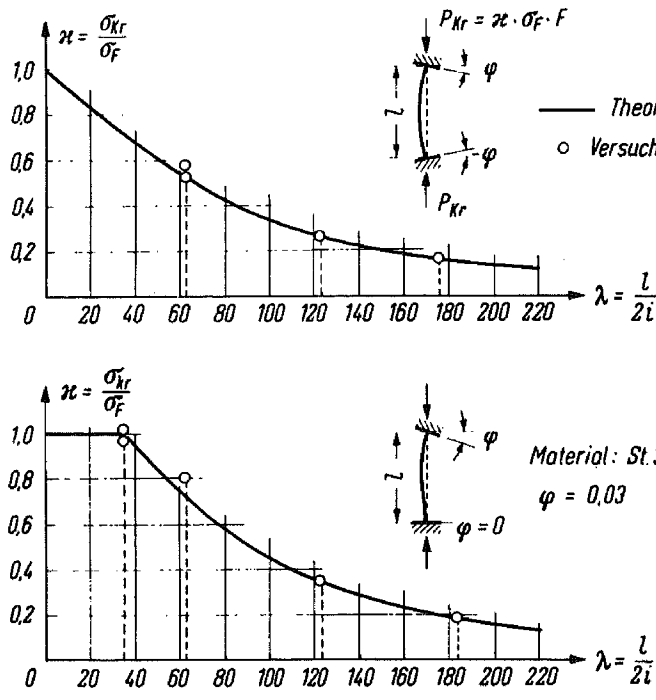
$\varkappa = \frac{\sigma_{Kr}}{\sigma_F}$
$m = \frac{e}{K} = 0$
m = 0,1
1,0
4,0
I - Querschnitt
P
e
K
K
S
I PB 100 ÷ 300
$\sigma_{Ki} = \frac{\pi^2 E}{\lambda^2}$
P_{Kr}
e
$P_{Kr} = \varkappa \cdot \sigma_F \cdot F$
genaue Lösung nach Ježek [2]
Näherungslösung nach der Plastizitätstheorie II. Ordnung
$\lambda \sqrt{\frac{\sigma_F}{E}}$

Fig. 6



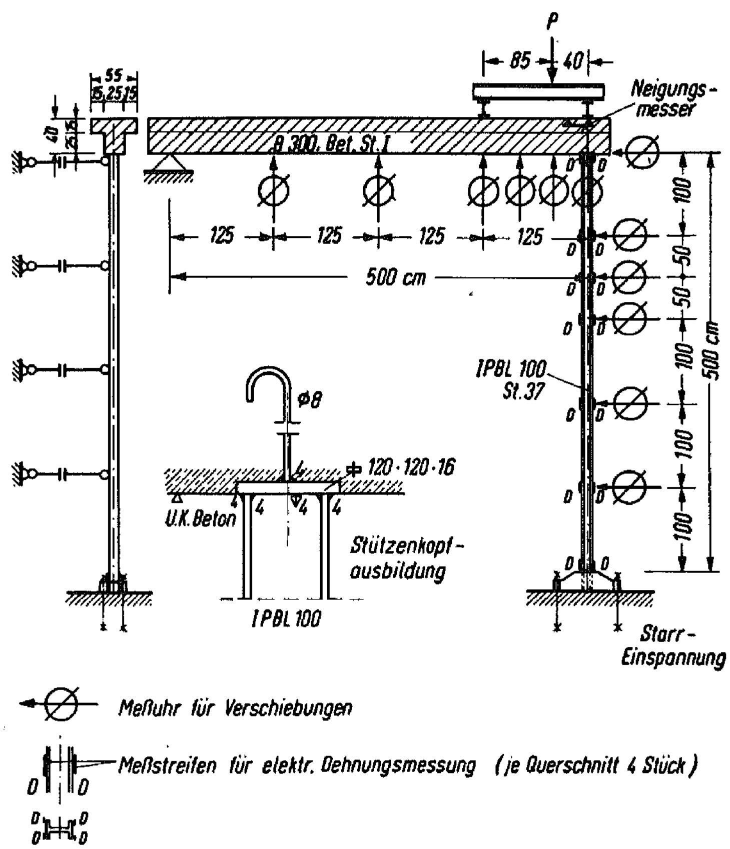
$\frac{P}{bh}$
G einfaches Traglastverfahren (I Ordnung)
0,833
Tragfähigkeit der Fließgelenkkette
32,29·h
P
0,05·N
a
b
E = 2100 Mp/cm²
σ_F = 2400 Mp/cm²
24,71·h
a-a
m
N
U_R
h
b
Schnitt a-a
Näherungslösung nach Plastizitätstheorie II. Ordnung
0,500
0,488
0,456
G_1
M
labil
G_2
labil
K
exakte Lösung (Oxfort [35])
stabil
F
$\frac{P_F}{bh}$
0,341
0,305
0,517
0,672
1,222
$\frac{U_R}{h}$

Fig. 7



$\varkappa = \frac{\sigma_{Kr}}{\sigma_F}$
$P_{Kr} = \varkappa \cdot \sigma_F \cdot F$
Theorie
Versuchswert
$\lambda = \frac{l}{2i}$
Material: St.37
φ = 0,03
φ = 0

Fig. 8



P
Neigungs-messer
B 300, Bet. St. I
500 cm
IPBL 100 St.37
U.K. Beton
Stützenkopf-ausbildung
IPBL 100
120·120·16
Starr-Einspannung
Meßuhr für Verschiebungen
Meßstreifen für elektr. Dehnungsmessung (je Querschnitt 4 Stück)

Fig. 9

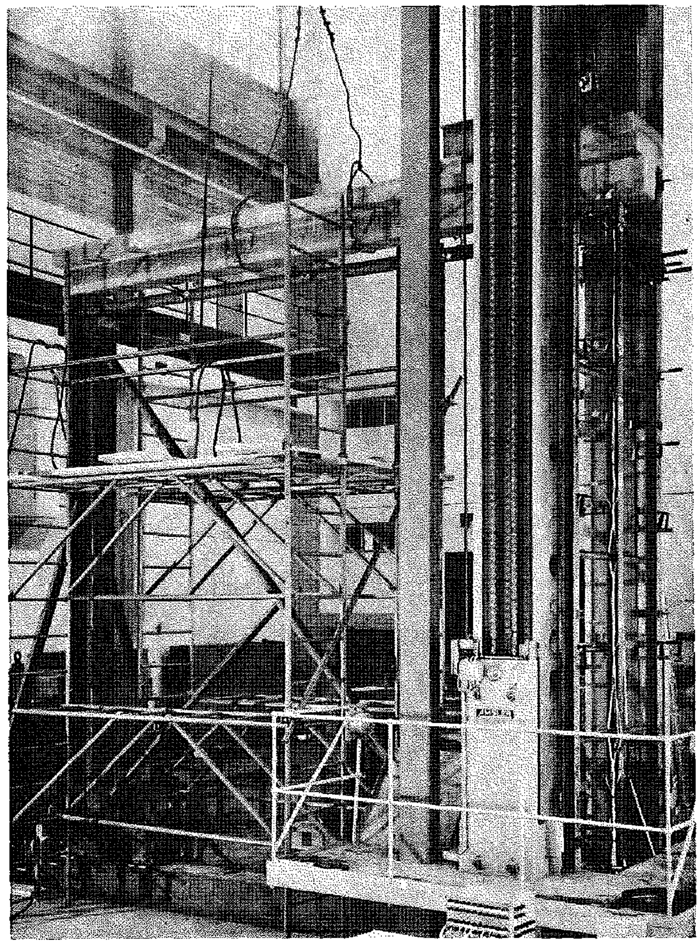
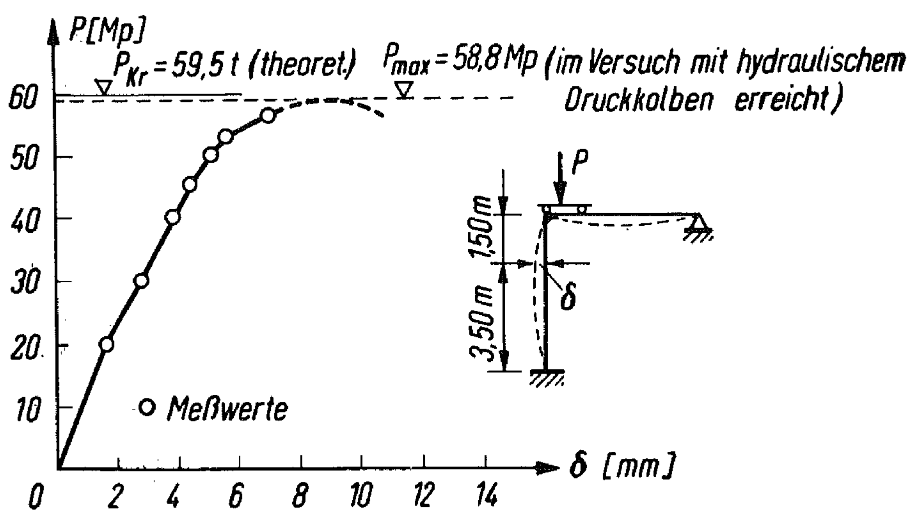


Fig. 10


P[Mp]
P_{Kr} = 59,5 t (theoret.)
P_{max} = 58,8 Mp (im Versuch mit hydraulischem Druckkolben erreicht)
60
50
40
30
20
10
0
2
4
6
8
10
12
14
δ [mm]
P
1,50 m
3,50 m
δ
o Meßwerte

THE RESTORING FORCE CHARACTERISTICS OF MULTI-STORIED FRAMES

BY

MINORU WAKABAYASHI

Professor, Kyoto University

1. Introduction

Much attention has been given to earthquake-resistant design by engineers and researchers in the field of structural engineering. First of all, an outline will be given on the design method of multi-storied frames against earthquakes used in Japan.

Dynamic analysis is utilized in designing tall buildings against earthquake forces, particularly those taller than one hundred feet. The dynamic analysis comprises the following four steps:

1) First, static horizontal forces are assumed for a trial design.
2) The designed building is replaced by a dynamically equivalent system composed of masses and springs as shown in Fig. 1.
3) The response of the system is analyzed with the help of a digital or analogue computer for some strong earthquake excitation recorded in the past.
4) The maximum displacement is calculated. If the relative displacement between consecutive stories goes beyond an allowable value, the design is revised.

Since an essential part of the process lies on the force-displacement relationship in the system, the restoring force characteristics or the relationship between the relative displacement and the horizontal force applied on a story is of main interest. Typical examples of the restoring force characteristics are shown in Fig. 2, where (H) denotes the horizontal force and (U) the relative displacement. These examples give quite different dynamic behaviors of the system. The example (a) may be termed elastic-perfectly plastic, (b) bilinear with postive slope and (c) bilinear with negative slope. Case (b) is, of course, most desirable and (c) is most disadvantageous. The main causes for unfavorable characteristics like (c) are considered to be destabilizing phenomena due to vertical loads, lateral or local buckling and so forth. The influence of the vertical loads is relatively small in a low building, but it may be important in a tall building.

The object of this discussion is twofold. One is to make clear the restoring force characteristics of frames in the presence of vertical loads. The other is to investigate the behavior of bracing under repeated loads; there is no doubt about the advantages of using bracing for earthquake and wind resistant purposes.

2. Effects of Vertical Loads on the Restoring Force Characteristics of a Rigid-Frame

(1) Elastic Buckling of a Multi-Storied Frame

Before discussing the inelastic behavior of rigid frames under vertical and horizontal loads, the general characteristics of the elastic buckling of a frame due to vertical loads applied only to the top of the columns will be reviewed.

Since the complete analysis of a buckling load requires too much rigor when the frame has many stories and bays, some assumptions on the proportions of the frame will be made. Figs. 3 to 5 show how the buckling loads or the effective length of columns change according to the relative stiffness of the beams to the columns, the number of stories and bays, rigidity of the columns in the upper and lower stories, the distribution of the column loads, etc.*

Fig. 3 shows the relationship between the effective length of a column and the relative stiffness of a beam to that of the column in a uniform one bay multi-storied frame for some number of stories. The frame is subjected to two equal vertical loads at the top of the frame. It is seen that the effective length increases as the number of stories increases, but there is little difference in the effective length if the frame has more than five stories. Fig. 4 shows how the effective length decreases as the number of bays increases. The ordinate is proportional to the effective length of a column and, the abscissa is the stiffness ratio of a beam to a column.

In an actual building, the columns do not have the same dimensions for all stories, but the cross-sectional area of columns and the vertical loads are larger at lower stories than upper stories. Fig. 5 shows an example of the effective length in a non-uniform multi-storied frame. Column (1) of the table in the figure is for a uniform frame, and columns (a), (b) and (c) are for non-uniform frames. Little difference is seen in the effective length when the

*Reference: M. Wakabayashi, "The Restoring Force Characteristics of Multi-Storey Frames", Bulletin of the Disaster Prevention Research Institute of Kyoto University, Kyoto, Vol. 14, Part 2, 1965, pp. 29-47 axial loads vary from story to story as compared with a uniform frame subjected to vertical loads at the top of the columns of the top floor. In the calculation of the above example an assumption concerning the cross-sectional areas and the distribution of vertical loads has been made. The assumption is such that each column has the same value of elastic buckling load if each column were simply supported at the both ends. In order to check the validity of this assumption an actual building has been investigated (Table 1). In this example the parameter Z takes approximately a constant value in all columns except the columns at the top floor. Therefore, the above assumption is regarded as reasonable.

(2) Miniature Model Tests of Portal Frames

In order to study the behavior of a frame subjected to constant vertical loads and increasing horizontal force, several tests were made as shown in Figs. 6-9. The miniature model is a rectangular portal frame of rectangular cross-section. A specimen is composed of two similarly made portal frames placed parallel to each other. The frames are rigidly connected by several bars to prevent instability of the frame due to lateral displacement. Each frame is cut from a steel plate. The vertical loads are applied by means of two jacks and the horizontal force is provided by a testing machine. In Fig. 8, rollers at A are to let the specimen end move freely in the direction parallel to the beam portion of the specimen. Tests were made on about thirty specimens of various values of slenderness ratio, relative stiffness and vertical load. Some experimental results are shown in Fig. 10. Horizontal force (H) is taken as the ordinate and the horizontal displacement (U) of the top of the columns as the abscissa. Solid lines show the experimental results and dotted lines the theoretical predictions. The parameter (k) is the ratio of the vertical force to the tangent-modulus buckling load of the frame. As the value of (k) becomes greater, the maximum load decreases and the destabilizing phenomena becomes important. As shown in the figure, the theoretical analysis is based on an elastic-perfectly plastic moment-curvature relationship, and is made for one-half of the frame. The discrepancy between the experiment and the theory is considered to be due mainly to the idealization of the moment-curvature relationship, which does not include the exact transition phase from the elastic to the plastic range. The idealization also neglects the strain-hardening phenomena. In Fig. 11, the test results are compared with Sakamoto's theory.** He approximately takes account of the expansion in the plastic region and of strain-hardening. A general agreement is seen between them.

(3) Miniature Model Tests of Multi-Storied Frames

Some specimens of three and five storied frames have been tested. Fig. 12 shows the specimens. A specimen is machined from a 60mm thick plate stock. Minute care is taken in order to leave no residual stresses. The experimental apparatus is shown in Fig. 13. A specimen end is fixed by a support with the shape of the letter "L". Cylindrical rollers are placed between the support and the

** Reference: J. Sakamoto, "Elastic-Plastic Behavior of Steel Frames (Part II), Transactions of the Architectural Institute of Japan, No. 113, 1965, pp. 7-11 (In Japanese) testing machine. Vertical loads are applied by the testing machine and horizontal force by an oil jack. The rollers serve to eliminate restraints against the horizontal displacement at the specimen end. Fig. 14 shows the shapes of specimens after testing. The various modes of collapse are seen to depend on the proportions of the frame. Fig. 15 shows the distribution of the plastic zones at the instant of collapse. The size and shape of a plastic zone are estimated from the distribution of strains measured by wire strain gages. In analyzing the frame the moment-curvature relationship is assumed to be perfectly plastic, and also the effect of shear in beams on the axial force of columns is assumed to be negligible. An outline of the analysis is shown in Fig. 16. First, an elastic analysis on frame (a) is made, taking the axial forces into account; then the first plastic hinge location will be known. The analysis is repeated assuming the frame is elastic everywhere except at the point of the first plastic hinge as in (c). After repeating similar procedures, the plastic collapse state is found with a mechanism like (d). The final deformation at the state of collapse is determined by superimposing the deformations at each stage. In Fig. 17, the horizontal force (H) is plotted as a function of the horizontal displacement (U). Again, solid curves are for experimental results and dotted curves for the theoretical predictions. The discrepancy may again be due mainly to the assumed elastic-perfectly plastic moment-curvature relationship; the shape factor of the cross-section not being exactly equal to unity, and neglecting strain-hardening in the calculations.

Comparison of the experimental values of the maximum load with Rankine's Formula in Fig. 18 shows reasonably good agreement.

(4) Tests of Portal Frames of Wide-Flanges

In order to examine the behavior of a portal frame of hot-rolled wide flange sections, an experiment was performed as shown in Fig. 19. Two frames are connected with each other by small wide flanges at joints and the middle of each member. Figs. 20 and 21 show the loading system. The method of loading is similar to that in the tests already mentioned. Fig. 22 shows the relationship between the horizontal force and the horizontal displacement at the top of the columns. The vertical loads are as much as 30% of the yield force of the columns. In the analysis an elastic-perfectly plastic moment-curvature relationship is again assumed.

3. Behavior of Frames with Bracing

For earthquake- or wind-resistant design, bracing is often used in framed structures. Some tests have been made on portal frames of wide flanges with bracing to observe the behavior of bracing under static and repeated loadings. Fig. 23 shows the types of bracing and loading included in the tests. The experimental apparatus is shown in Figs. 24 and 25. Fig. 26 is the picture of specimens after test loaded statically in one direction. The bracing has been designed not to deflect outside of the plane of the frame so that all bracing buckles in the plane of the frame.

In K-type frames, the beam is loaded downward by the tensile bracing after the compressive bracing buckles, and the collapse mechanism is such that there is a plastic hinge at the center of the beam. The theoretical relationship between the compressive force and the contraction of the bracing is drawn in Fig. 27. The ends of the bracing are assumed to be so constrained that they are displaced as if rigidly connected with the frame. Figs. 28 to 30 show the load-displacement relationships of the frame. Solid lines are experimental and dotted lines theoretical.

There is some difference noted in the load-displacement relationships according to the type of bracing used. It is also noted that the shape of the curves for braced frames is essentially different from that of a frame without bracing.

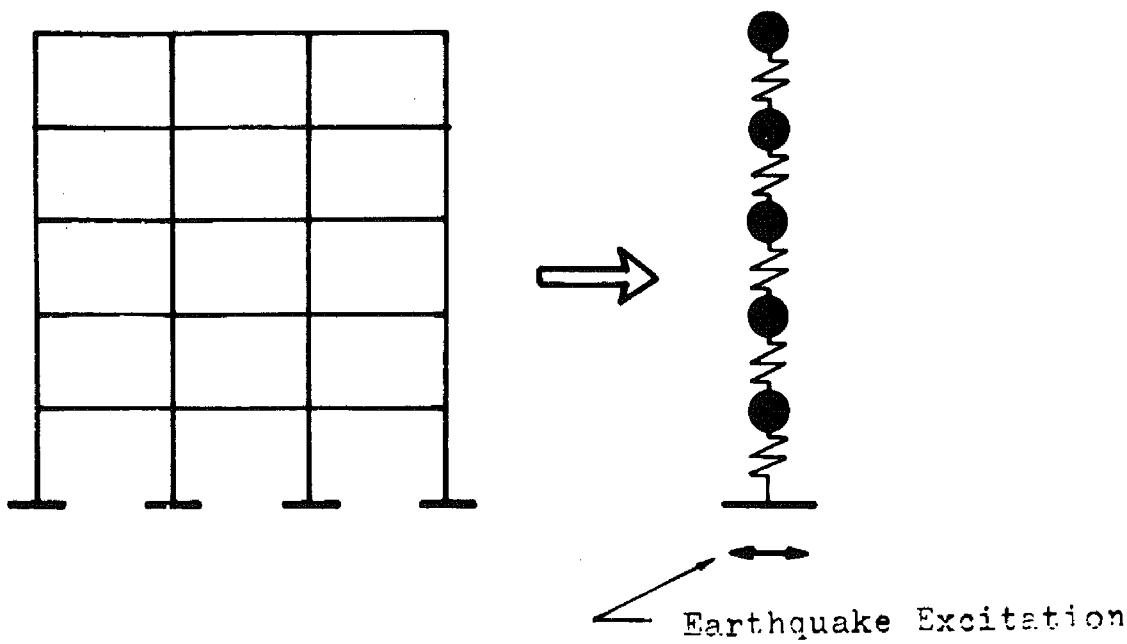
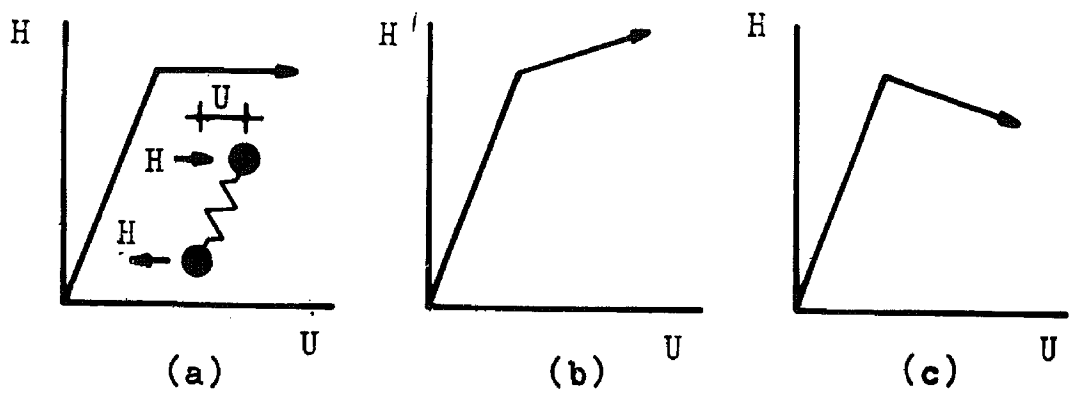


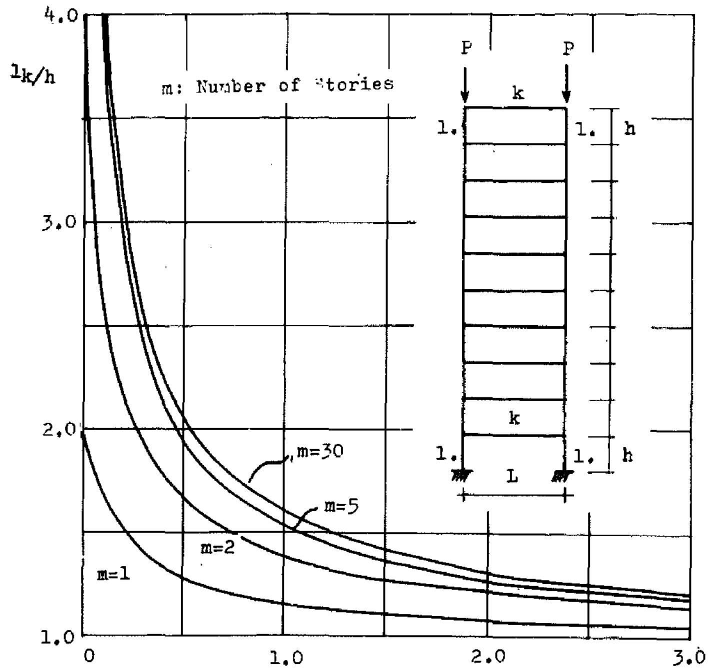
Fig. 1 Replacement of a Frame by a Simple Model



Examples of

Restoring Force Characteristics

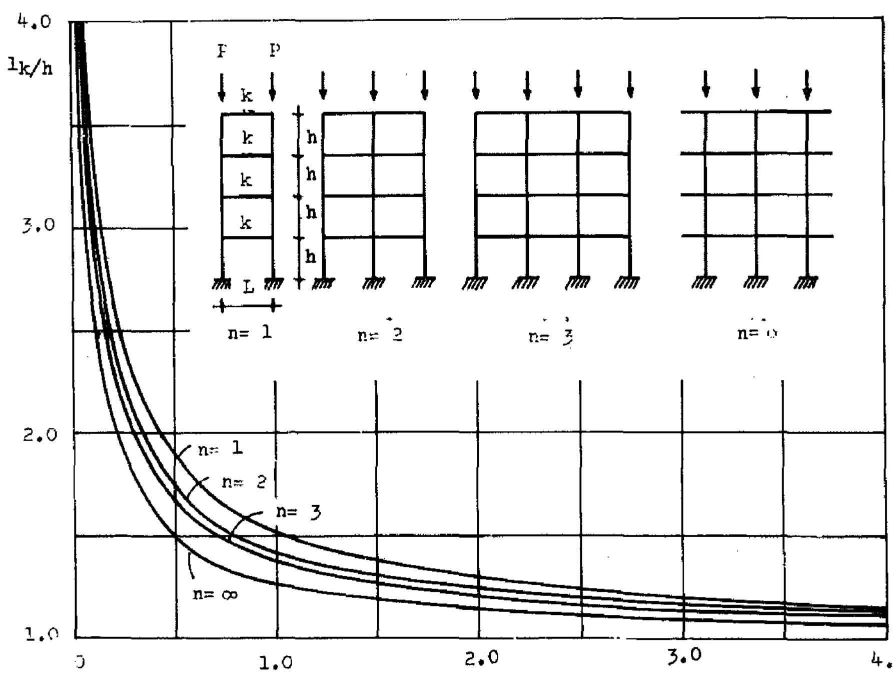
Fig. 2



Buckling Length of a Column
in a Single-Bay Frame

$k = \frac{EI_b}{EI_c} \cdot \frac{h}{L}$

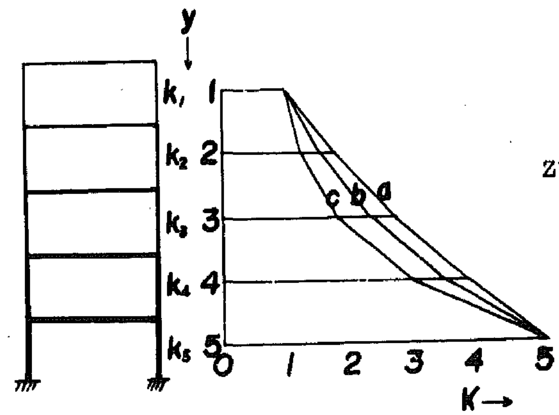
Fig. 3



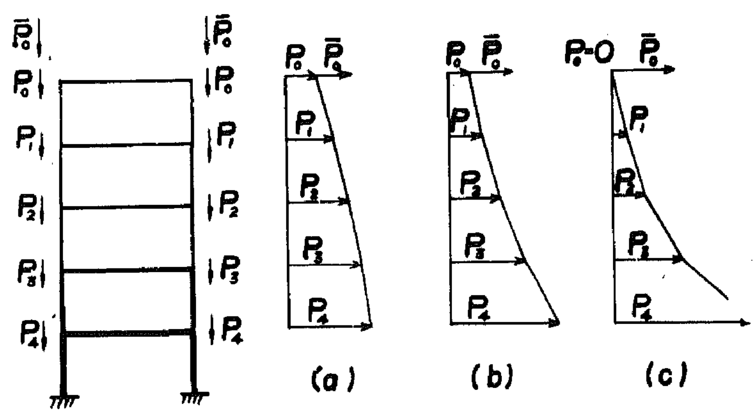
Buckling Length of a Column
in Four-Storied Frames

$k = \frac{EI_b}{EI_c} \cdot \frac{h}{L}$

Fig. 4



$$Z = \sqrt{\frac{Nh^2}{EI}} = \text{const}$$



k_b	k=const.	$k_5 = 5k_1$		
	(1)	(a)	(b)	(c)
	l_k/h	l_k/h	l_k/h	l_k/h
0.5	1.931	1.931	1.919	1.898
1.0	1.537	1.539	1.530	1.522
1.5	1.380	1.380	1.375	1.368

$$k_b = \frac{2 \times (\text{Beam Stiffness})}{k_{n-1} + k_n}$$

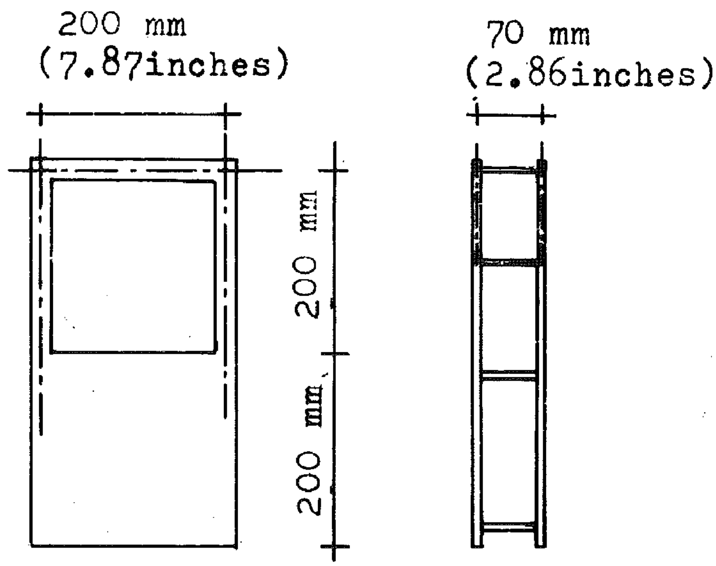
k_n = Column Stiffness of n^{th} Story.

Buckling Length of a Non-uniform Multi-storied Frame

Fig. 5

Story	Axial Load N(tons)	Column Length h(cm)	$Z^2 = \frac{Nh^2}{EI}$	Slenderness ratio
25	35	350.0	8.91 10	29.19
20	193	350.0	32.69	26.19
15	363	350.0	42.49	24.95
10	543	350.0	42.53	23.72
5	734	350.0	37.93	21.78
1	942	412.5	47.28	24.08
B4	1079	500.0	35.82	25.12

Table 1. An Example of The Actual Building: Tokyo Station Project



Model Specimen

Fig. 6

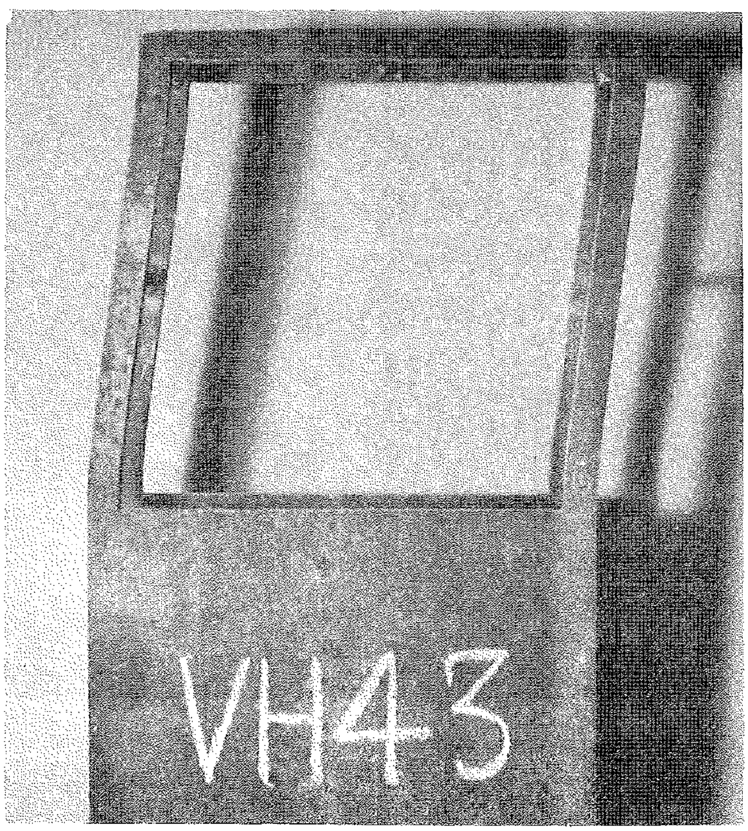
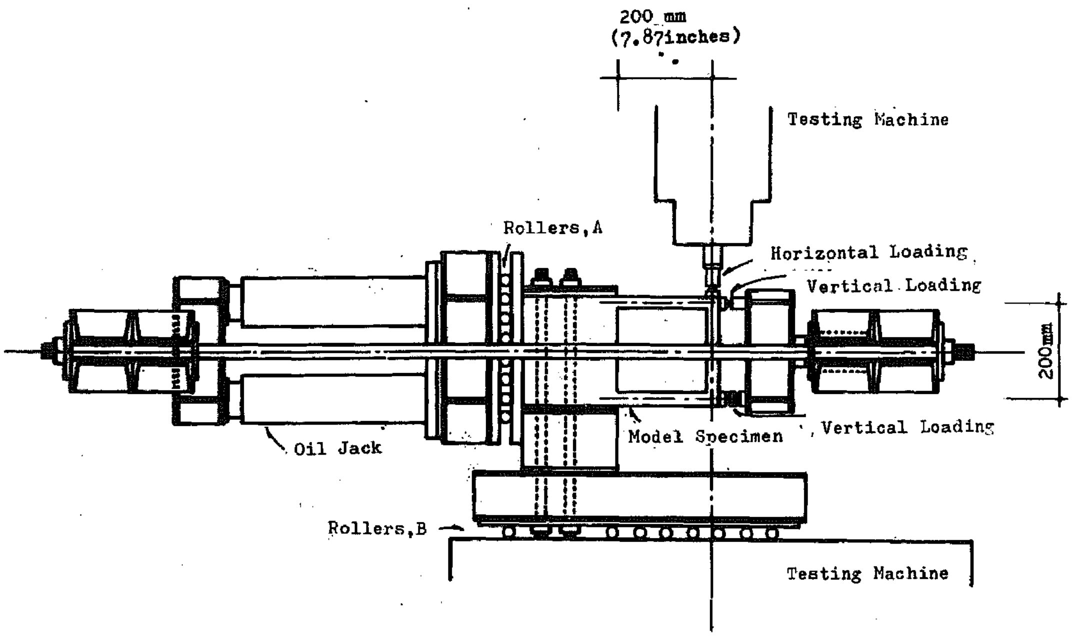


Fig. 7



Experimental Apparatus

Fig. 8

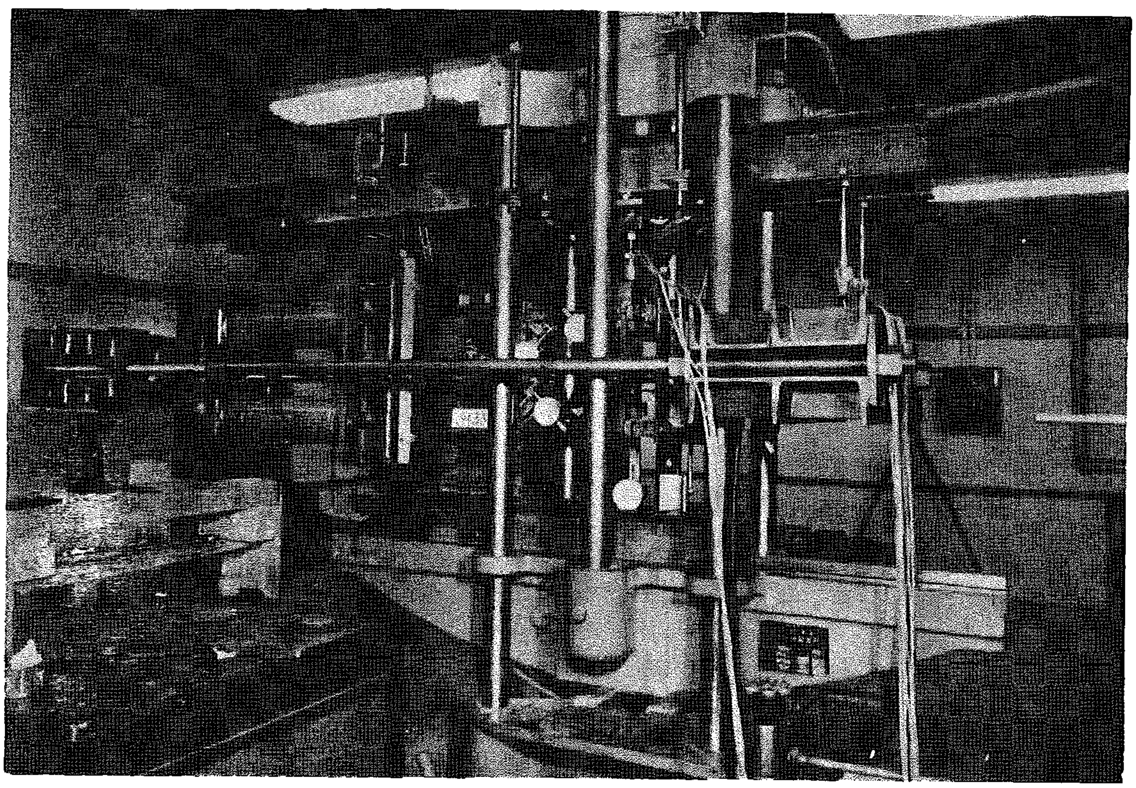
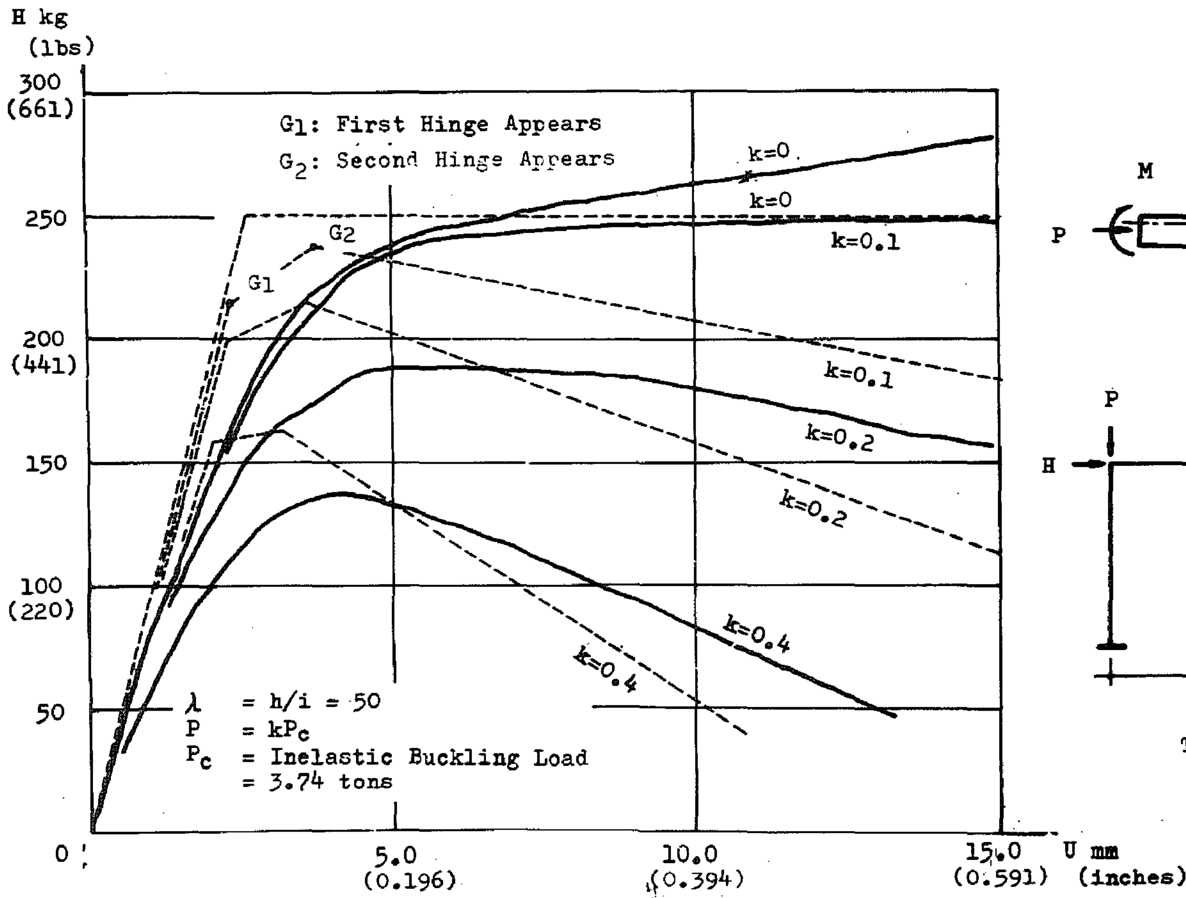
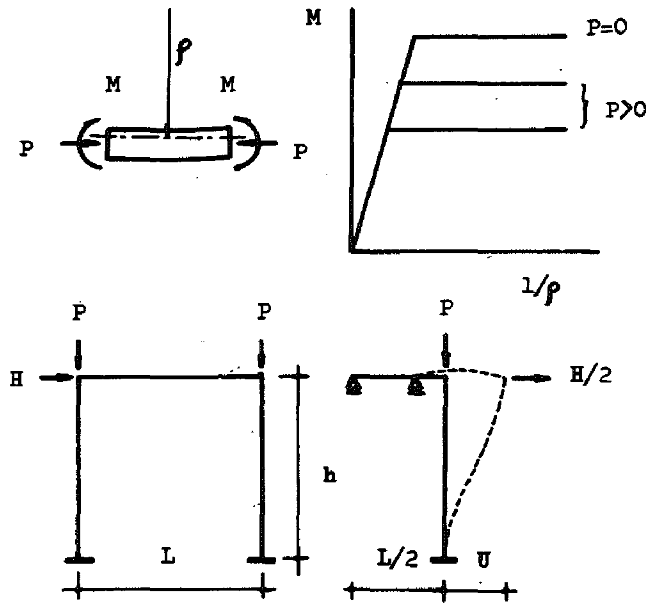


Fig. 9

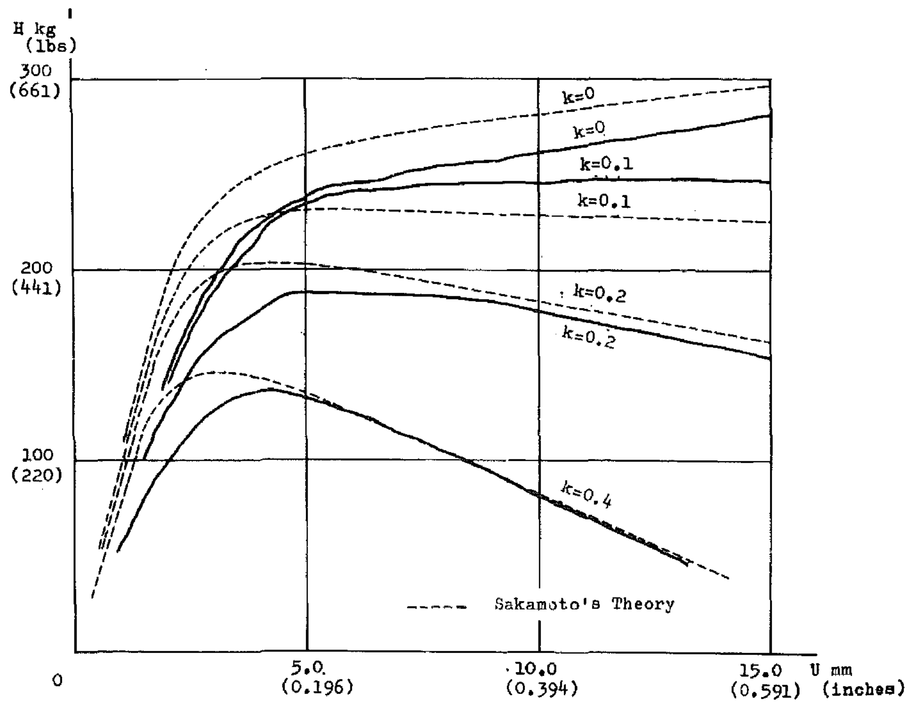


Load-Displacement Relation



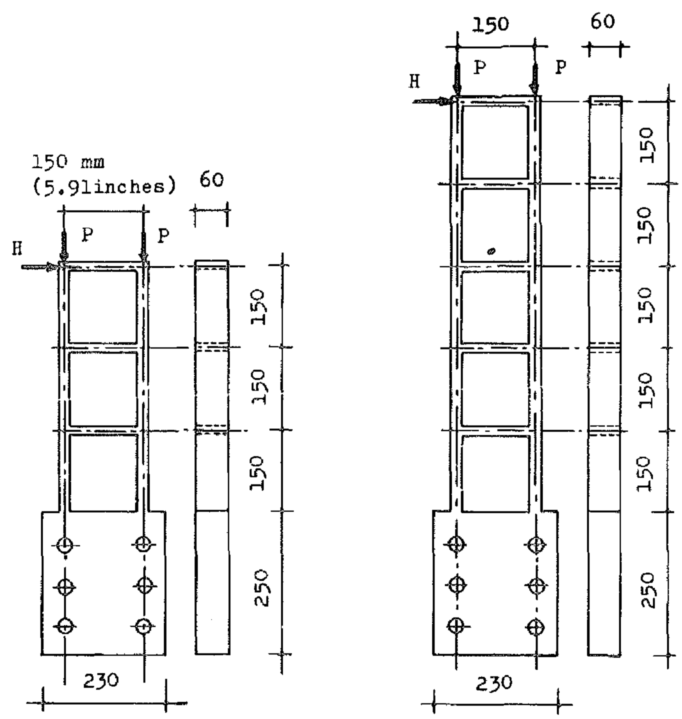
Theoretical Assumption

Fig. 10



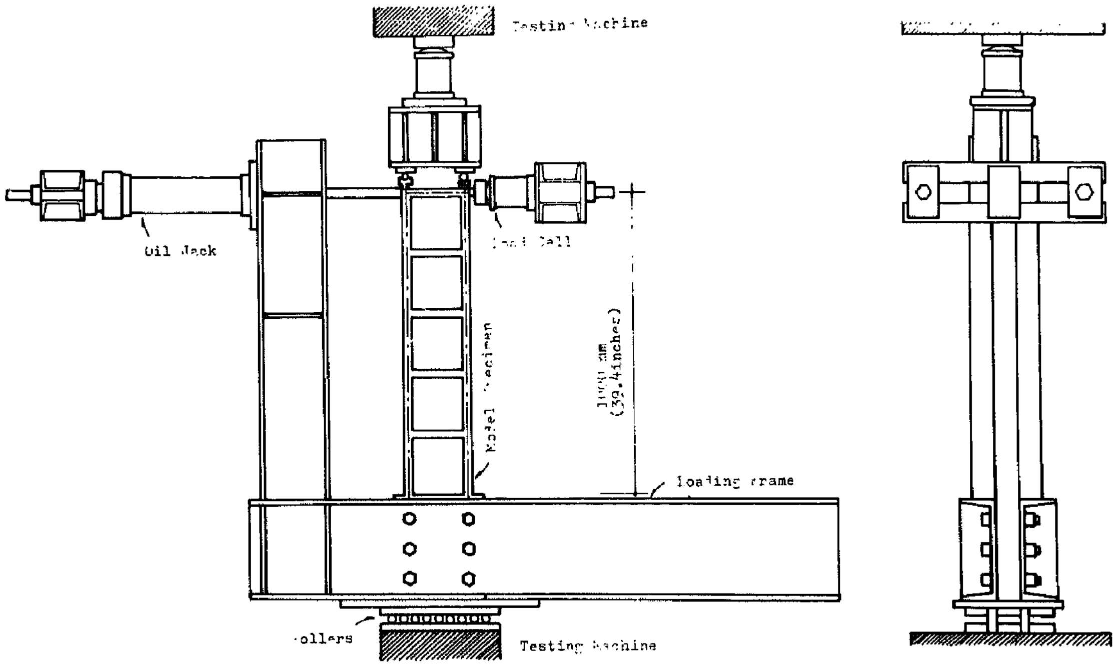
Load-Displacement Relation

Fig. 11



Model Specimens

Fig. 12



Experimental Apparatus

Fig. 13

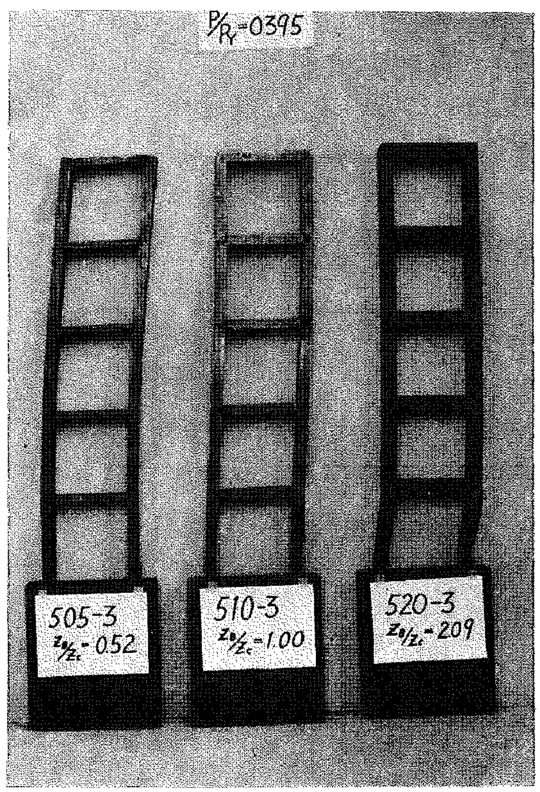
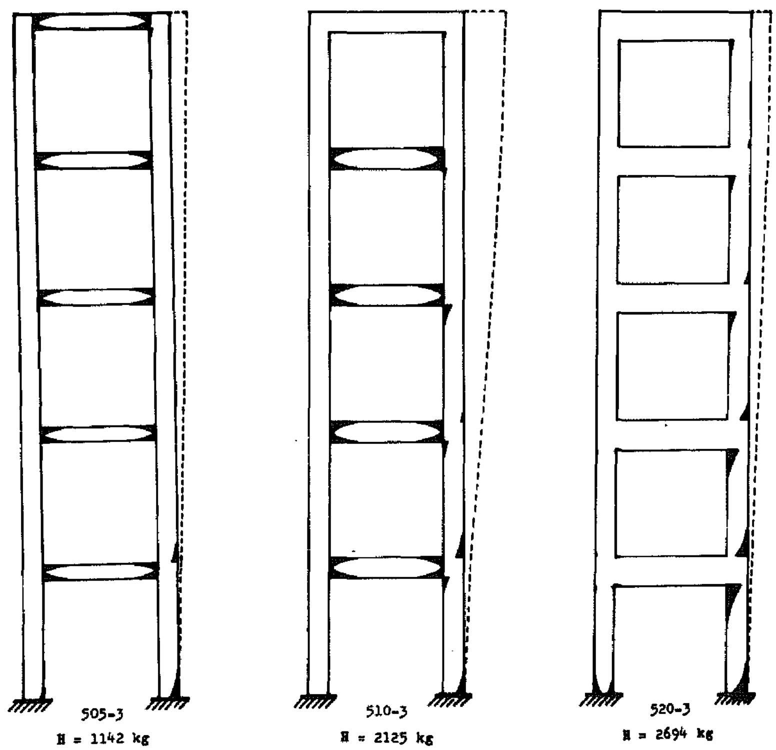
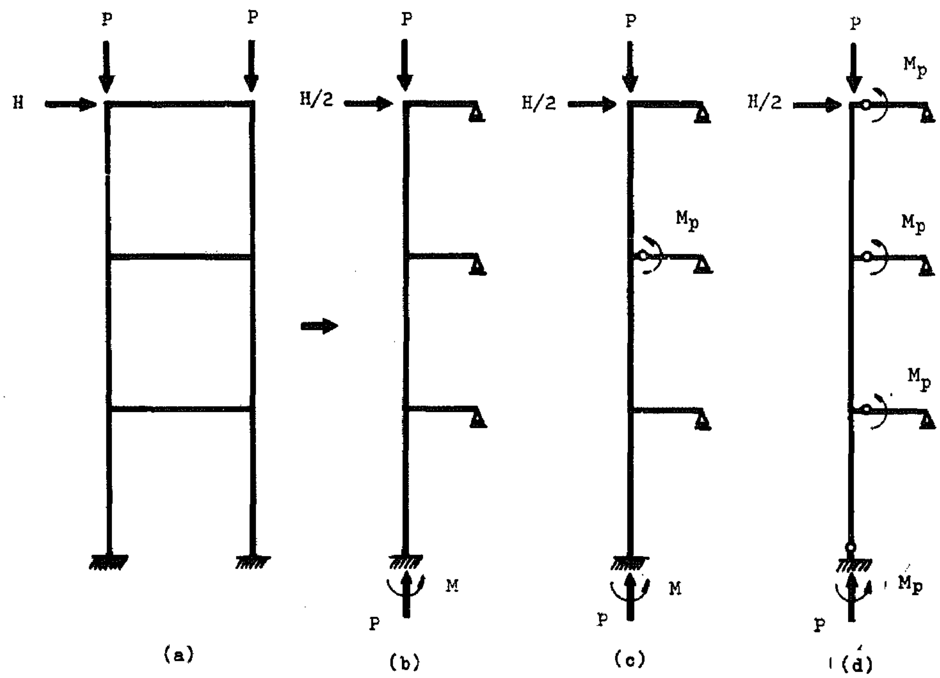


Fig. 14



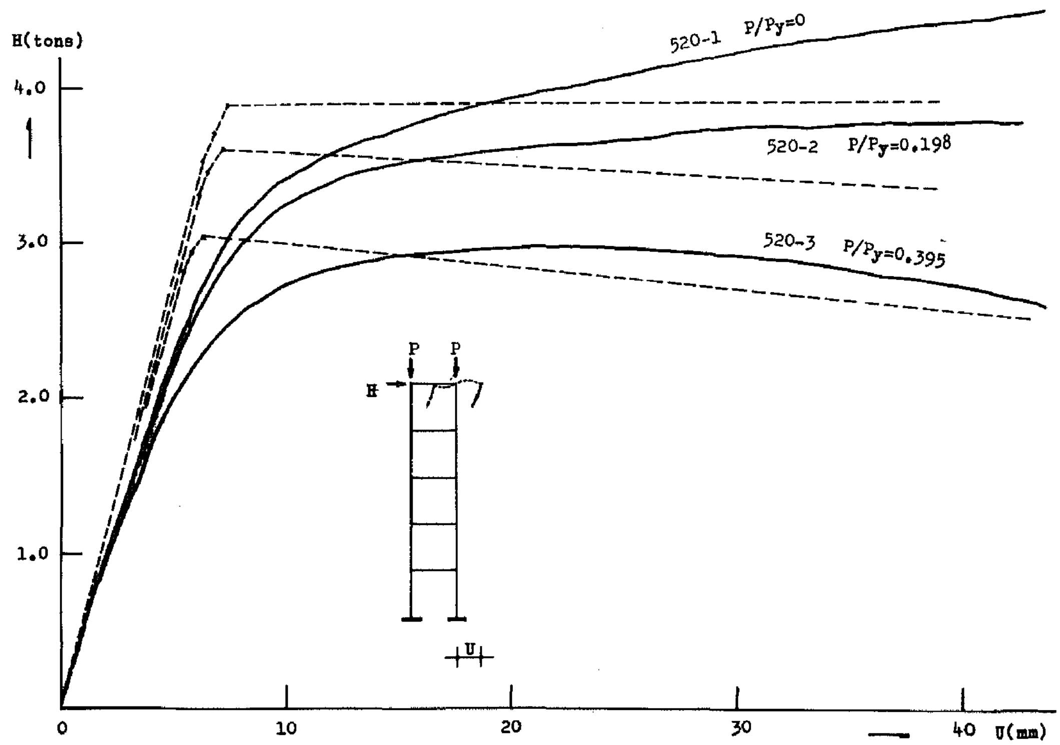
Distribution of Plastic Zones

Fig. 15



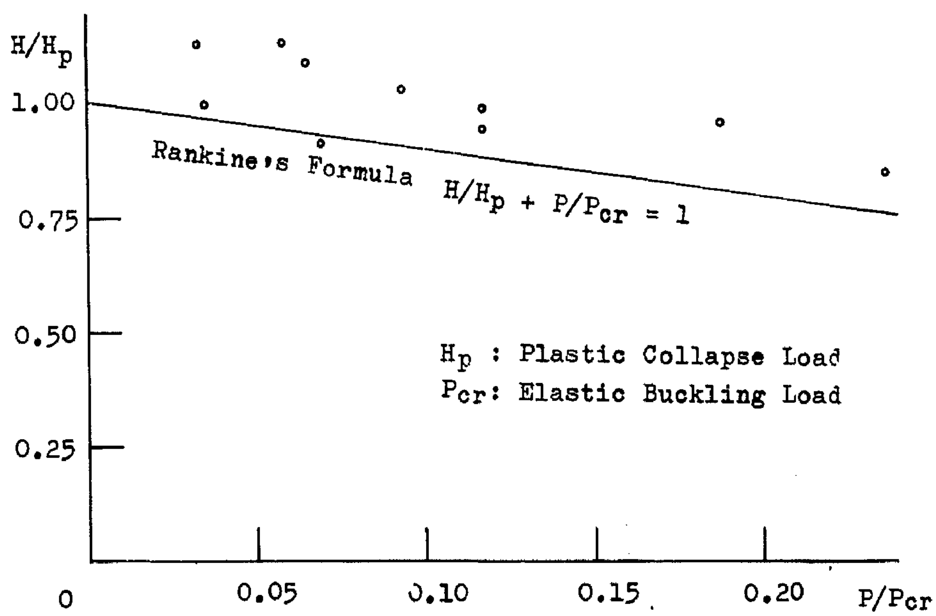
Method of the Analysis

Fig. 16



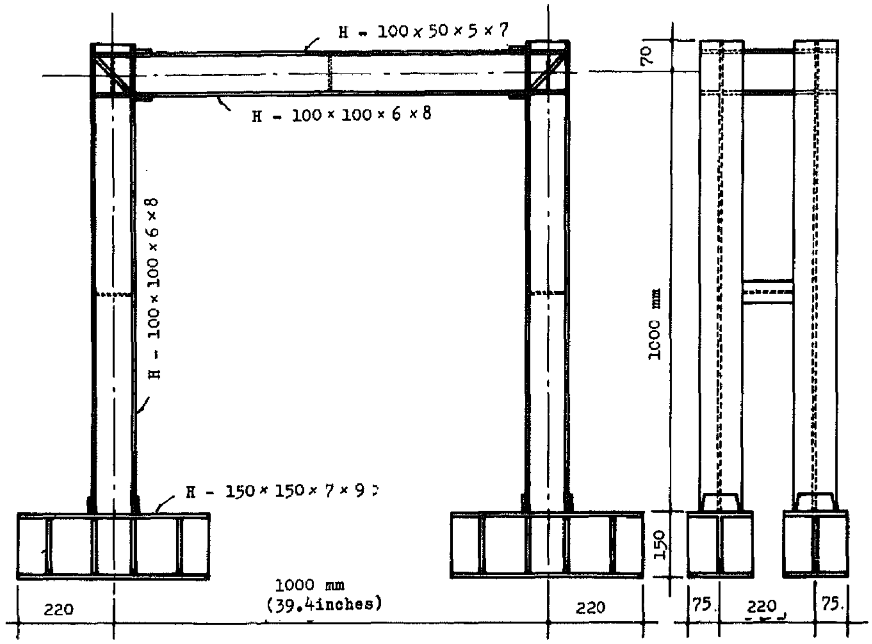
Load-Displacement Relation

Fig. 17



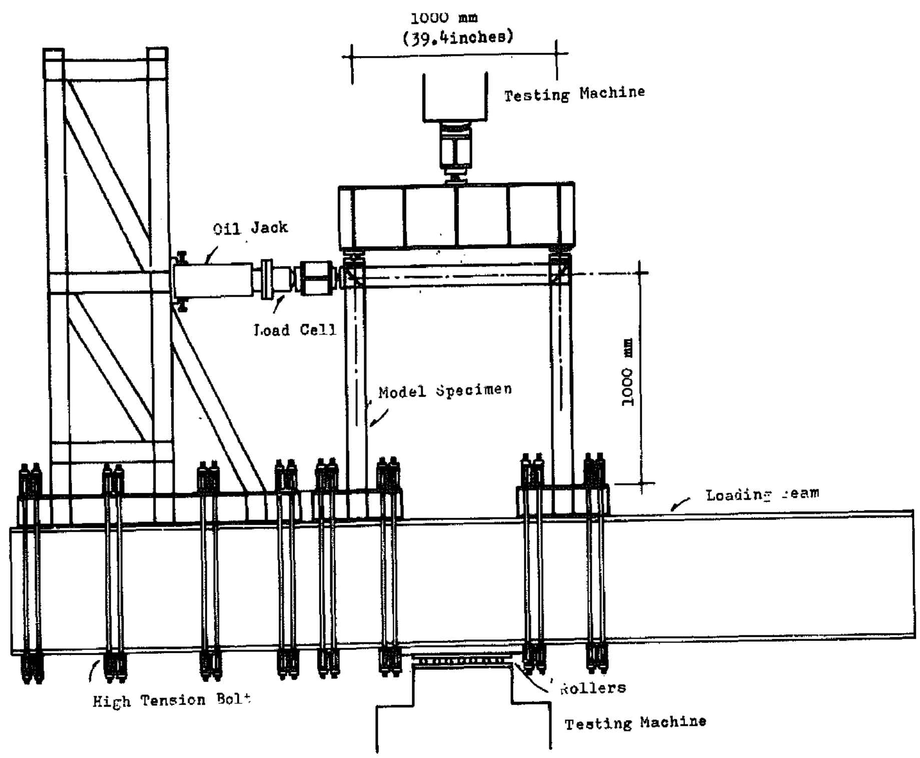
Maximum Loads

Fig. 18



Model Specimen

Fig. 19



Experimental Apparatus

Fig. 20

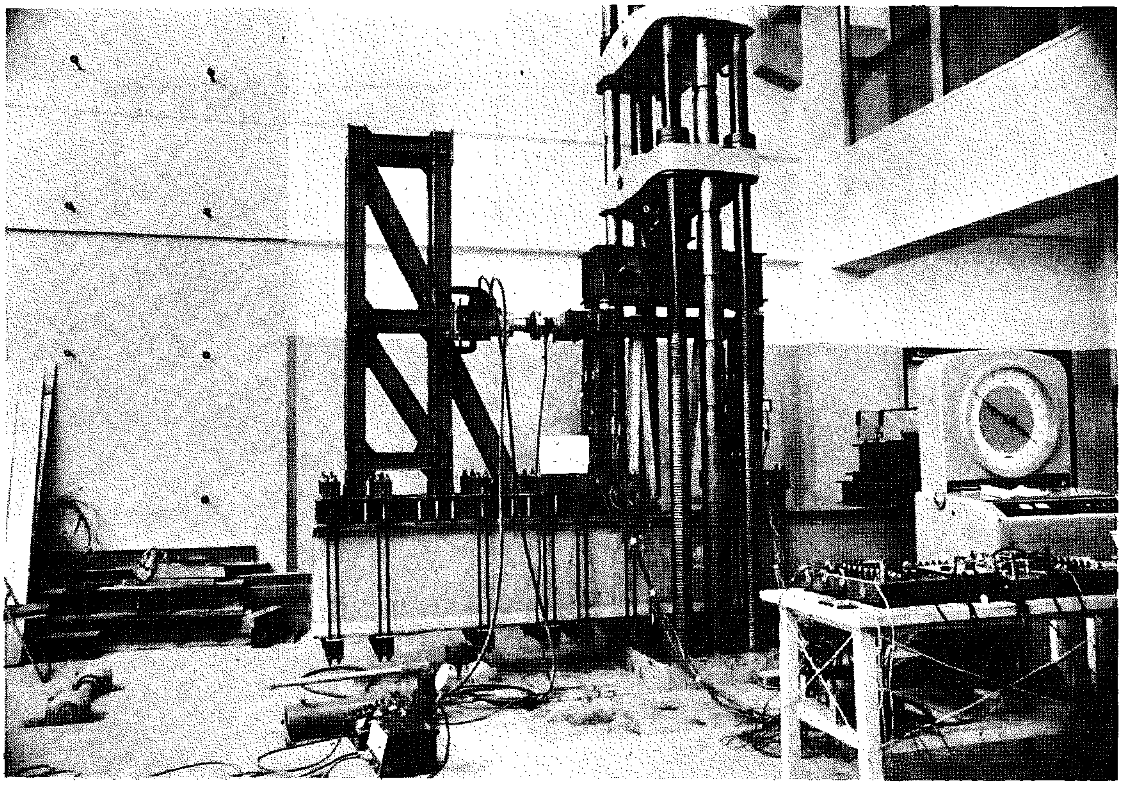


Fig. 21

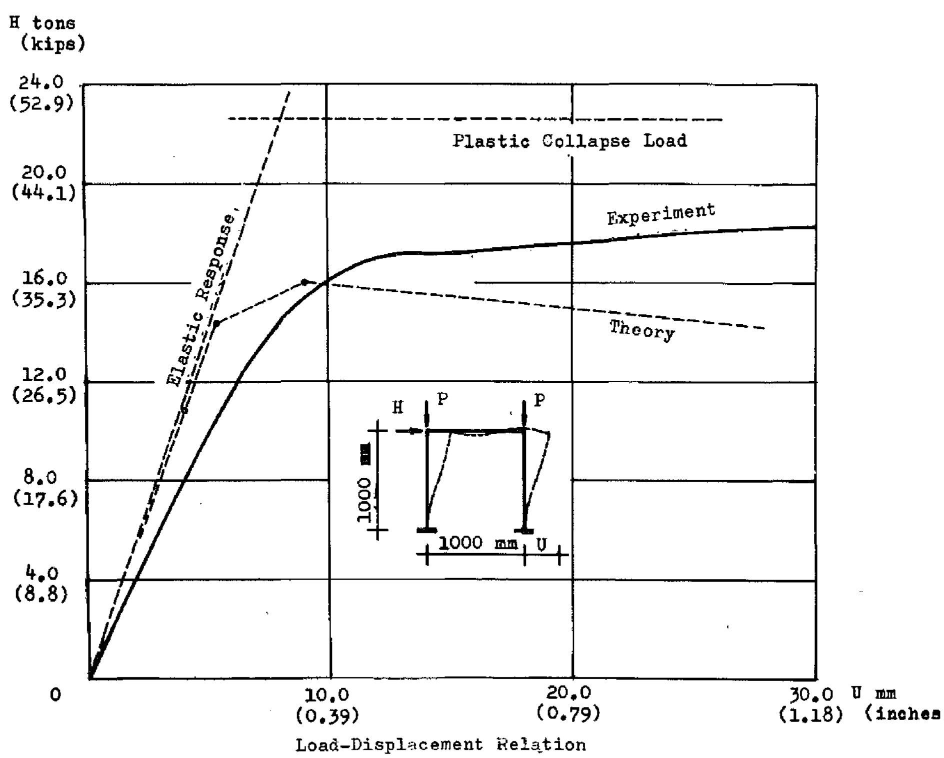
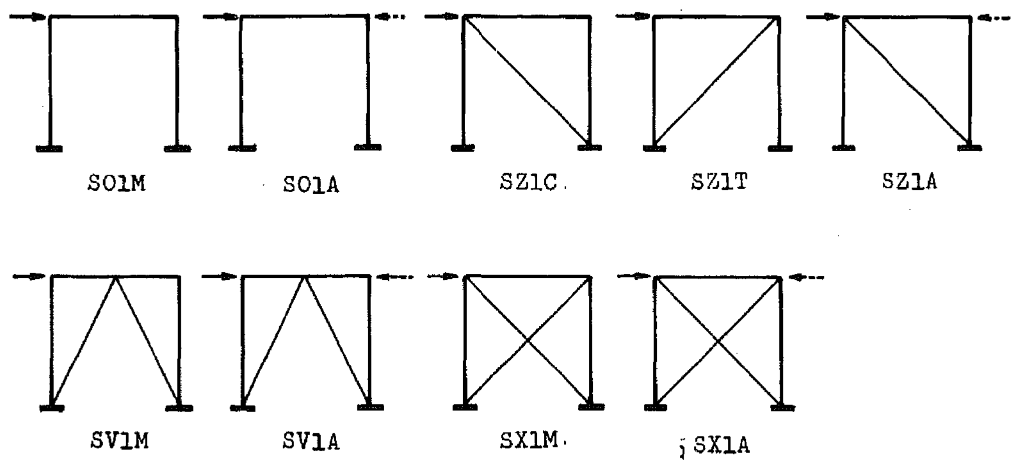
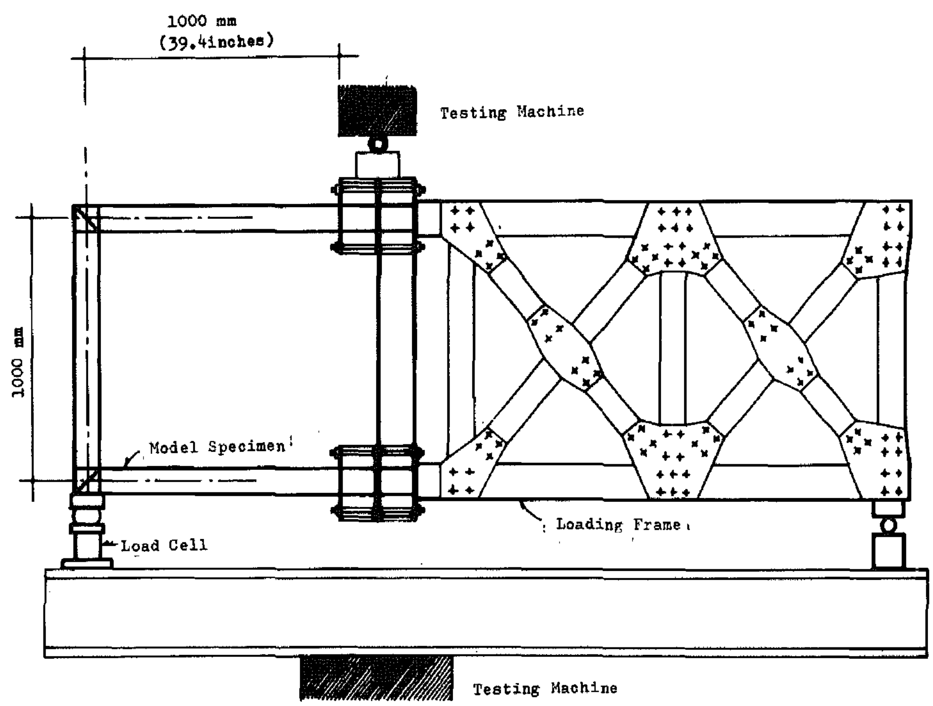

H tons
(kips)
24.0
(52.9)
20.0
(44.1)
16.0
(35.3)
12.0
(26.5)
8.0
(17.6)
4.0
(8.8)
0
10.0
(0.39)
20.0
(0.79)
30.0
(1.18)
U mm
(inches)
Plastic Collapse Load
Experiment
Theory
Elastic Response
H
P
P
1000 mm
1000 mm
U
Load-Displacement Relation

Fig. 22



Tests of Frames with Bracing

Fig. 23



Experimental Apparatus

Fig. 24

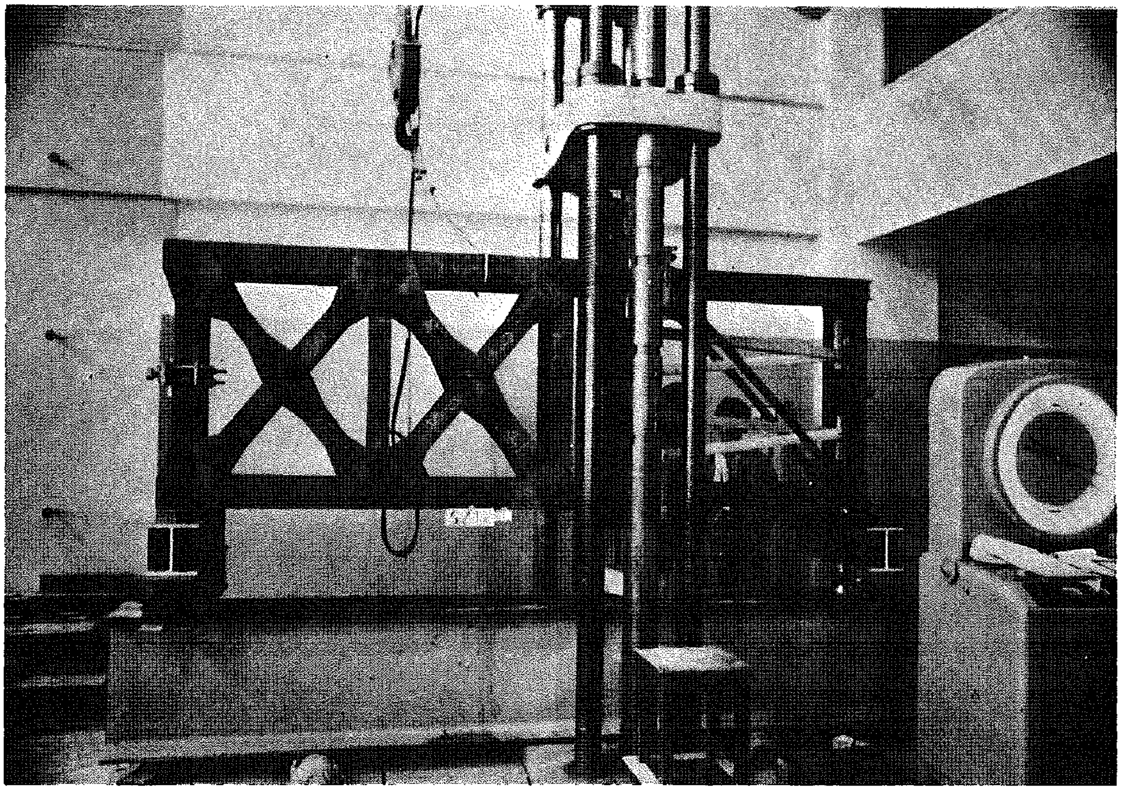


Fig. 25

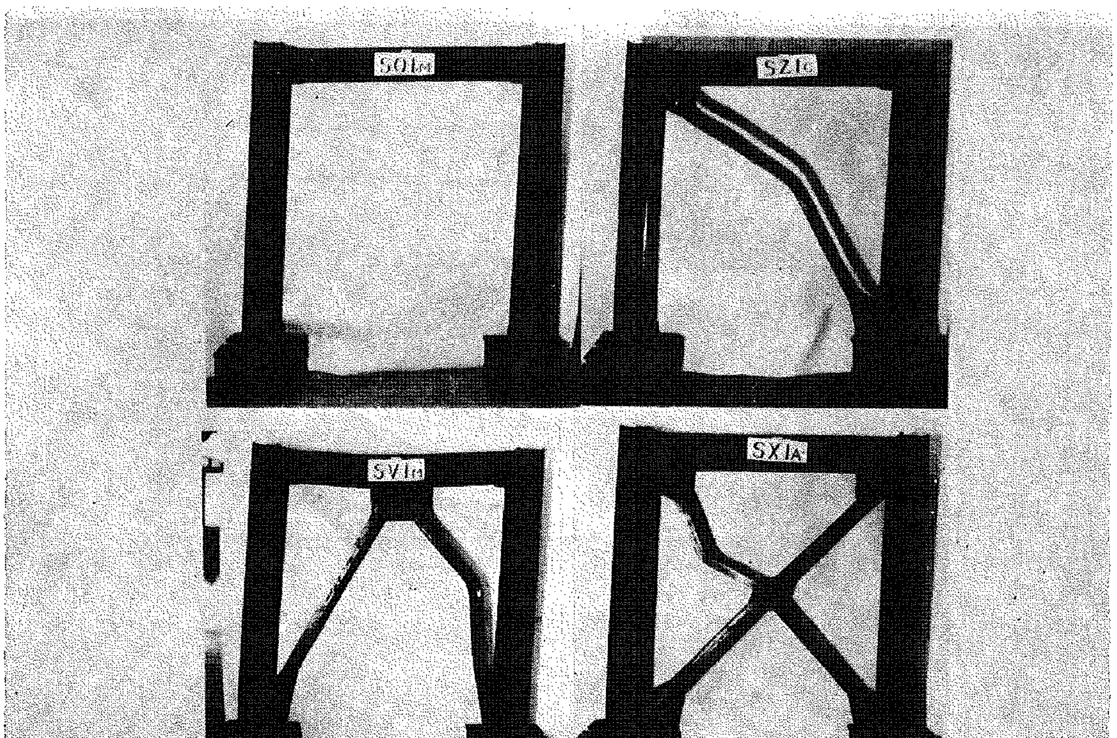
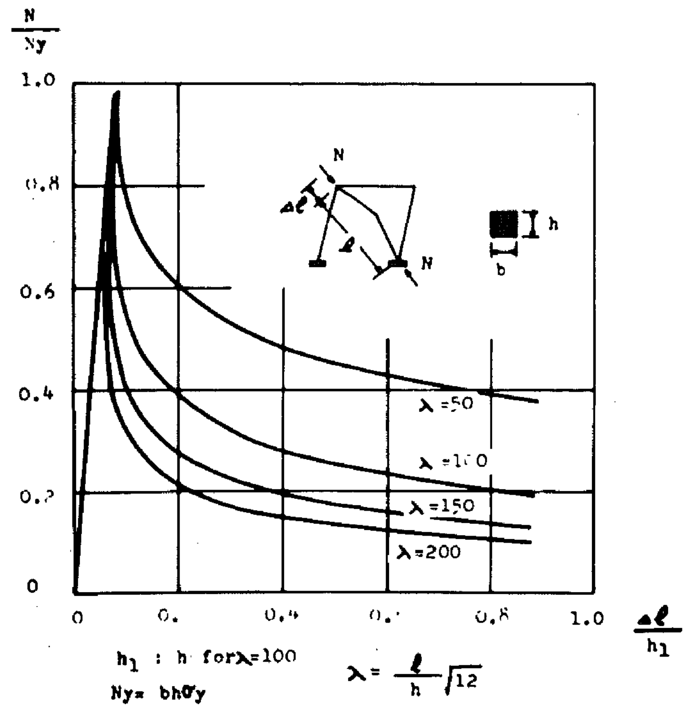
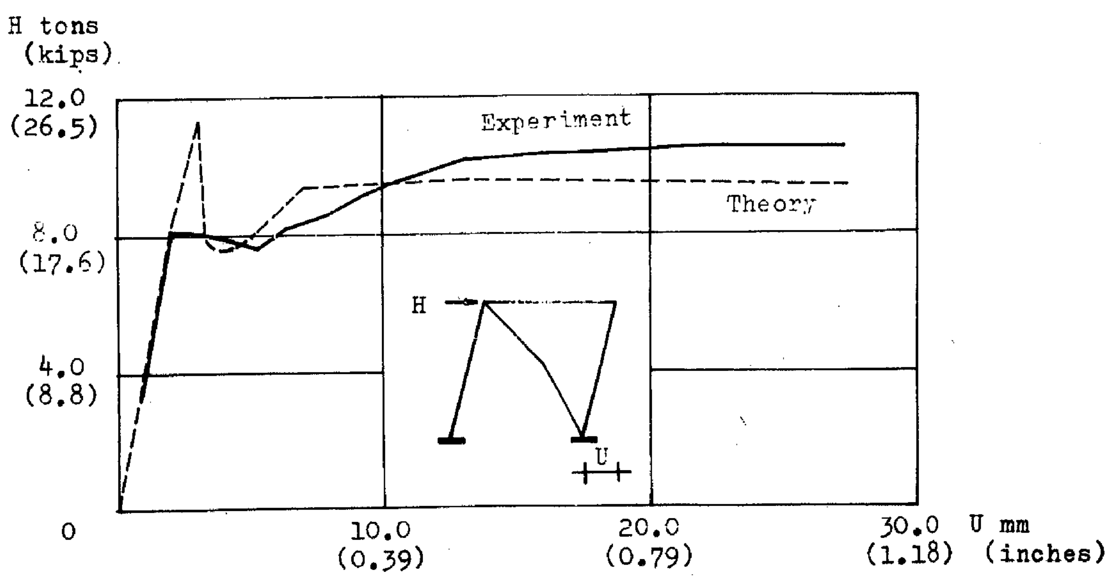


Fig. 26



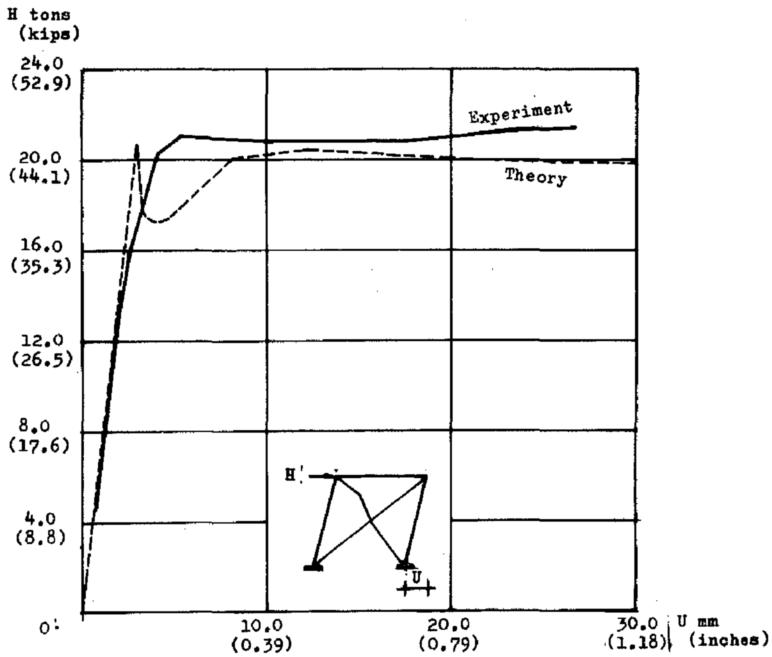
Axial Force - Contraction Curve

Fig. 27



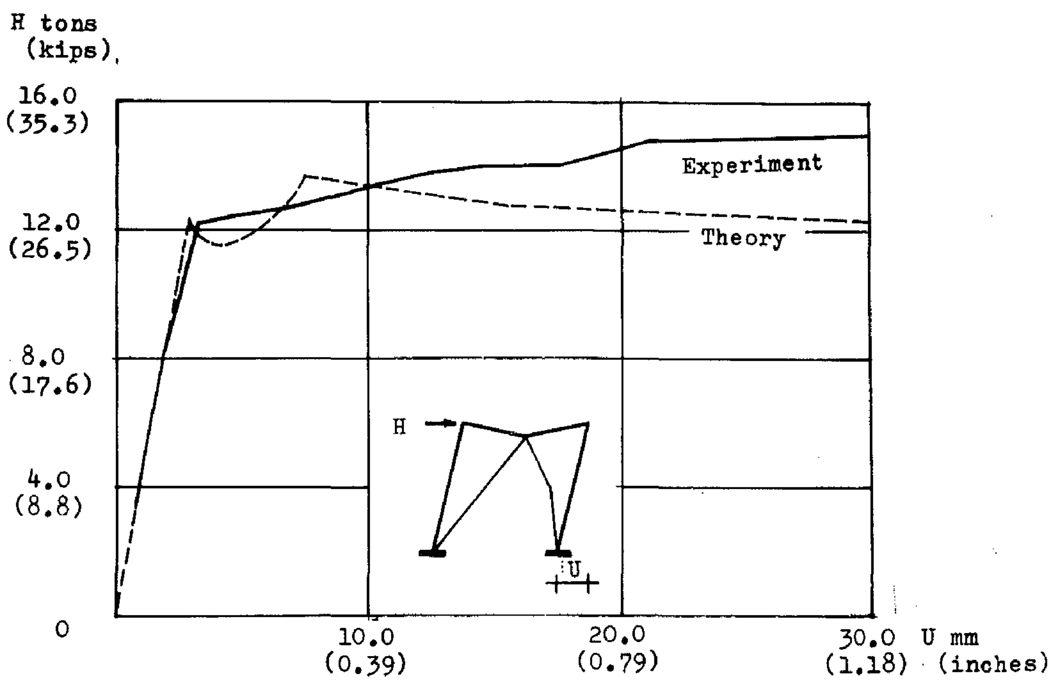
Load-Displacement Relation

Fig. 28



Load-Displacement Relation

Fig. 29



Load-Displacement Relation

Fig. 30

PRACTICAL DESIGN PROBLEMS FOR MULTI-STORY FRAMES

By Ira Hooper, [1] F.ASCE

INTRODUCTION

Traditional methods of multi-story design have been based on simplifying assumptions; they require experience, judgement and some courage for successful application. Another requirement is the strong maintenance of one's convictions, which has led to spirited differences of opinions.

A somewhat similar situation occurred in theology of a few centuries ago, when a popular subject for discourse was the question, "How many angels could dance on the head of a pin?" The question was never finally answered, but after a few hundred years of attention by profound thinkers, it stopped bothering people.

Now with the passage of time and as a result of the 1963 AISC Design Specification, many of the old problems are no longer troublesome, but a brand new set of problems have arisen.

This is not intended to be adverse criticism. The 1963 document is a fine piece of work. It incorporates advances in theory, a better understanding of how structures behave, and the results of many laboratory investigations. It is understandable that such an expansion of the entire field should develope some rough spots. This paper will discuss some of these areas of trouble; the three main topics will be:

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1. - Rigid vs semi-rigid connections.

2. - Design procedures for rigid frames.

3. - Column interaction and effective length.

A short discussion of miscellaneous items will follow the three topics.

RIGID VS. SEMI-RIGID CONNECTIONS

The origin of this debate goes back to the beginnings of iron construction, when simple span beams were used to rest on posts and when the relatively short buildings were strongly braced by heavy masonry walls. The concepts of continuity and frame stability were not necessary.

Then, with growing congestion of urban areas, multi-story buildings were required. The need for wind resistance was recognized and was met by designing vertical trusses between interior columns that cantilevered from the foundations. These trusses also provided frame stability for vertical loads, without the conscious consideration of the designer. Rigid connections and continuity were still not required, and were not used.

The next development was the high-rise building, where wind loads required some rigid connections, especially at the lower floors. The designer was confronted with the need to develope design methods for complicated rigid frames after having spent a lifetime of doing calculations for simple beams and axially loaded columns. In order to avoid the seemingly endless mathematics, he bravely took advantage of plasticity long before plastic design was even thought of. He designed the beams as simple spans for vertical loads, then designed the connections to resist the wind moments. The connection material was arranged to accommodate plastically the strains caused by vertical loads, after which the connection still had the elastic strength to resist the wind moment.

Many great buildings were successfully designed with this ingenious simplification, and generations of engineers were educated in its use. Engineering education used to be rigid and authoritarian, which occasionally led to the confusion of simple man-made rules with the actual workings of nature. We now realize that the simplified method did not consider frame stability; it simply assumed the effective column length was the story height. The masonry walls and partitions very probably acted as sway bracing, but we still do not have practical design methods for this effect.

We are now at the nub of the problem. Does the engineer continue to design semi-rigid connections today for economical use of steel's plasticity, or is he trying to save himself the strain of abandoning familiar procedures to learn new methods? The two main arguments against rigid connections are:

- Columns are punished by rigid frame moments due to vertical loads.
- Rigid connections cost more than the girder weight saved.

In considering the first of these two objections, it should be noted that only the exterior columns at the top of a building are adversely affected by rigid frame moments due to vertical loads. At interior columns, the girder moments tend to balance and cancel out. At lower floors, the wind moments are large and dictate the joint design. With the current trend to higher wind loads, the region where columns are affected adversely by rigid frame moments is small. These small regions can be eliminated by setting the exterior columns inside the building so that exterior cantilevers balance the interior girder moments. Another scheme is to relieve the moment by using simple framing connections at the exterior columns; this scheme requires a wide building with enough interior columns to provide the needed frame stability.

With regard to the cost of rigid connections, the savings in girder weight for full continuity can be as much as 33% under elastic design and up to 50% under plastic design, when compared with a simple span. Such savings more than pay for the cost of welded rigid connections which give additional benefits, such as reduced girder deflections, reduced drift of the entire frame, and reduced effective lengths of columns. The last item is particularly interesting to the designer, because finding the effective length of a column with semi-rigid connections is a long calculation.

For these reasons, rigid connections seem to be indicated for multi-story frames.

Some designers feel the best solution of the rigid frame is not a solution at all. They claim that a round, square or hexagonal, stressed-skin tower, designed similarly to an airplane fuselage will avoid a lot of paper-work, save steel and end up with a stiffer structure.

DESIGN METHODS

In the good old days, a designer could break his building into separate members. Each beam, girder or column was separately designed and was supposed to be unaffected by the sizes of adjacent members. Design consisted of neat, concise, book-keeping that was as therapeutic as knitting. This procedure can still be used, today, for frames with hinged connections, braced against sidesway.

But modern rigid frames without sway bracing no longer permit convenient isolation of each member. Engineering now approaches the complexity of politics or the physics of gravitational fields, where a change anywhere in the system immediately affects every part of the system. The simple cause-effect relationship, so necessary for quick mental grasp, is greatly complicated by reverberating responses, or "feedback", from distantly affected parts. One must now try to deal with a whole process, rather than a series of separate events.

While the process in nature allows all the interacting effects to occur at once, the designer must try to understand the process by taking one thing at a time. Breaking the complex whole into manageable parts begins by separating the analyses for wind loads and for vertical loads.

Wind load analyses are usually either the portal or the cantilever method. While these methods are approximate, they have been shown to be in good agreement with more "exact" methods for building heights up to 25 and 35 stories. Extreme refinement of the wind analysis is not justified, since the load magnitudes can not be established with precision.

The analysis for vertical loads is a series of successive trials. A typical scheme follows:

Step 1 Estimate the member sizes, and the relative stiffnesses.

Step 2 Apply the loads and distribute the moments; find the moment, thrust and shear in each member.

Step 3 Choose member sizes to accommodate the moments, thrusts and shears.

Step 4 Compare with the estimated sizes of Step 1. Adjust the sizes and repeat the whole procedure until adequate convergence occurs.

For Step 1, the original estimate of girder sizes usually assumes uniform loading and full fixity, resulting in end moments of WL/12. The column sizes can be estimated by choosing a section that can carry the sum of the floor loads above with an unbraced length about the major axis of twice the story height.

Each member is designed for the conditions of (1) vertical loads without wind and (2) vertical loads with wind at increased allowable stresses; the larger size governs.

If the wind condition governs, the trial sizes for vertical load without wind must be increased, and new moment distributions are required. The experienced designer will anticipate this time-consuming development and will learn where to estimate the initial member size for wind plus vertical load. The girders will be governed by vertical loading in the upper stories, and by wind in the lower stories; the point of transition depends on the relative values of vertical and horizontal forces. Columns do not follow so clear a pattern, and can even zig-zag from one load condition to the other as they go down the building.

The moment distributions might best be made by electronic computer. A rigorous analysis requires the handling of masses of numbers and many successive trials. If a design office has access to a machine with ample capacity, programs are available to do the work.

If manual methods must be used, it is best to borrow a standard procedure from concrete design for vertical load analysis in which each floor of a rigid bent is isolated with its rigidly connected columns; columns are considered fixed at the floor above and the floor below. Once again it seems unreasonable to require "exact" solutions for frame analysis when the loads can not be precisely assigned.

COLUMN INTERACTION & EFFECTIVE LENGTH

The 1963 AISC Specification establishes a host of variables for columns with bending. It is quite understandable that design engineers have not rushed to adopt the new rules. They now have to work longer to design steel that weighs less, so that their fees are reduced.

With familiarity, the formulas become easier to apply and simple charts have been devised by many offices. Several design aids have been developed, with tables, to reduce the amount of numerical work required.

The most frequent questions asked by designers are:

(1) Which formula will govern interaction?

(2) What does the Commentary mean by the statement: "Where

the design of a building frame is based primarily upon the effect of large side loading or upon a drift limitation, the effective column length may generally be taken as the actual unbraced length."

(3) What can be done to speed the process to find K for effective length?

For slenderness ratios over 80, Fig. 1. shows that Formula (7a) will govern except for a small region of very high moment, when Formula (6) controls.

For slenderness ratios less than 60, it appears that each of the three formulas controls a part of the range. The combined curve is very complicated, but note that the straight line for Formula (6) is a close approximation, generally within 5%.

Some actual column dimensions may be helpful at this time. A 10 ft. long, 12" WF column has an l/r ratio of 40 about the y-axis. In usual construction, the y-axis is braced against sidesway and the x-axis is not. K_y is therefore equal to 1 and K_x must be calculated. If K_x is less than 1.7, the y-axis will govern for slenderness under axial load, since the ratio of the two radii of gyration is usually about 1.7. With K_x of 2.5, the slenderness ratio about the x-axis is approximately 60. These values occur frequently.

For practical purposes, it is suggested that in designing columns with sidesway permitted, it is satisfactory to use Formula (6) when the slenderness ratio is less than 60, and that Formula (7a) will govern when the ratio is above 80.

Fig. 1 gives some understanding of the second question, regarding large side loading and drift. For short, stiff columns with large moments, Formula (7b) tends to govern. Formula (7b) is not affected by column slenderness. Also note that the Formula (7a) curves for low slenderness ratios are close together. In this range, the design is not sensitive to changes in effective length and an arbitrary rule limiting the K-value to 1 can be justified to save needless numerical work. It is reasonable to assume that buildings designed for large side loading or drift would tend to have stiff columns. But, when the slenderness ratio climbs above 40, columns are sensitive to changes in slenderness, and the writer can find no simple reason to disregard the need to evaluate K for effective length. Until the wording of the Commentary is made clear and numerical limits have been set, columns should be designed according to the specification requirements.

As to the third question, the K-values vary little between floors of a multi-story unbraced frame. After the calculations have been done for the top two stories, excellent guesses can be made for the succeeding floors below. For the very first trials at the top floor, without any other guides, a K_x-value of 2 is suggested.

MISCELLANEOUS

At this point, the writer would like to review some practical problems, without going into great detail.

Just as it is the politician's most important duty to get elected, if he is to achieve any good for his constituents, it is the engineer's first concern to stay in business. He will tend to resist any changes in design procedures, since he must pay for the education of his employees. He will absolutely balk at new methods if they greatly increase his design time. He must be allowed time to become familiar with changes and he must be furnished with practical design aids as soon as new methods are published.

The digital computer is generally accepted as the solution to the problem of handling the growing volume of computations. Some excellent programs for complete analyses have been developed, but they require large-capacity machines not available to most engineers. The development by the steel industry of modest programs for small machines would be very helpful. The small programs could be planned as sub-programs for inclusion in larger, future programs.

Refinement of live loads by research is needed. The origins of some occupancy loads have been lost, but the load-values persist by inertia and precedent. It is obvious that 50 PSF for office space is too heavy for desks, chairs and a few persons. It is equally obvious that 50 PSF is too little for modern business computer rooms.

Much attention has been given to wind-pressures, but the final report of the ASCE Task Committee, published in the 1961 Transactions does not present enough definite recommendations for the practising engineer. A clear code covering a wide variety of modern buildings is still needed.

Present design methods do not result in unique solutions; the results are dictated by the original estimates of sizes. It would be interesting to be able to optimize a rigid frame, to see how much weight might be saved; such procedures are now being developed.

This discussion was not intended to be a gloomy inventory of current difficulties in steel design. The new problems are, rather, a healthy sign of vigorous growth. As the older, central problems are absorbed and resolved, new developments occur on the periphery of our growing knowledge. This periphery is the engineer's exciting, risky frontier and he can consider himself fortunate that it will never be finally conquered or settled.

CONCLUSIONS

(1) Rigid connections seem to be indicated for most multi-story structures.

(2) For unbraced frames, column interaction will be governed by AISC Formula (7a) when the slenderness ratio exceeds 80. When the slenderness ratio is less than 60, Formula (6) will give close approximations.

(3) Design engineers need new design aids, programs suitable for small computers, new information on live loads and wind load, and procedures for optimization of design.

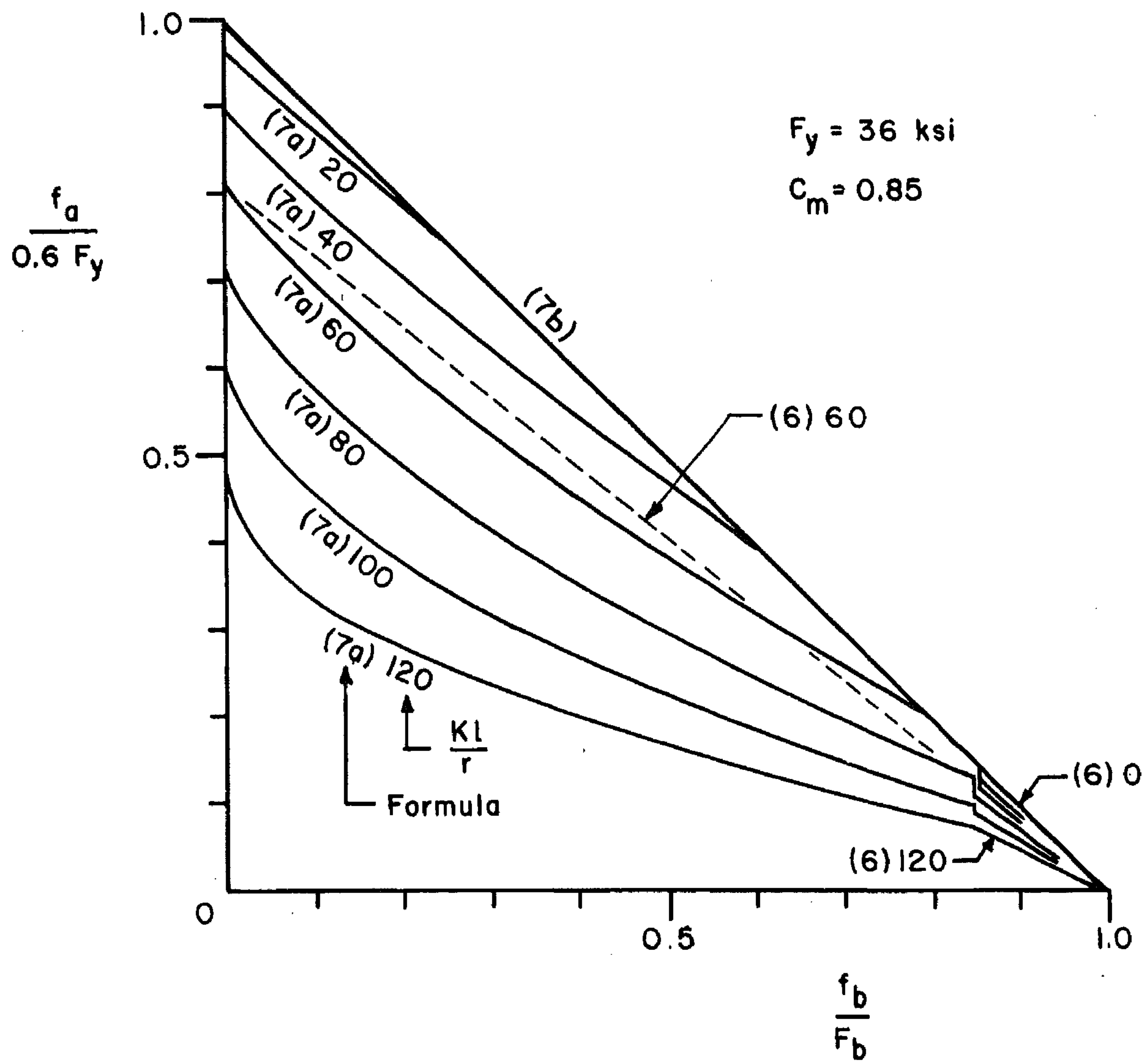

FIG. 1. – COLUMN INTERACTION FORMULAS SIDESWAY NOT PREVENTED

STUDIES IN COMPOSITE CONSTRUCTION

by

J. W. Roderick*

Dr. Roderick thanked the chairman for his kind remarks, and the Conference organisers for having invited him to be one of the guest speakers. As had been stated, Dr. Roderick had for many years been interested in the structural behaviour of the steel skeleton, but in recent times had turned his attention to the integral behaviour of the concrete clad steel frame which forms the effective structure of many present day multi-storey buildings. Currently, the designer is permitted to take some account of the concrete casing in proportioning the steel frame, but the indications are that the allowances are conservative to the point of being uneconomical. Accordingly, the programme of research now being carried out in the School of Civil Engineering of the University of Sydney, is intended to throw some light on the plastic and ultimate load behaviour of the concrete clad steel frame. Obviously the column presents the most interesting problem, but its solution is intimately concerned with the flexural behaviour of the beam. Also, because the New South Wales Department of Main Roads - one of the sponsors of the programme - was interested in the beam problem as it relates to bridge structures, attention had been directed first to the traditional composite beam of rolled steel joist attached to a concrete slab by shear connectors. A number of beams of this type had been examined to study the influence on structural behaviour of different concretes and of the strength of the steel used for the connectors. Also, a series of concrete encased steel

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joists had been tested to collapse as pin-ended columns subjected both to concentric and to eccentric loads.

The speaker was aware that he and his group were late-comers in this field; indeed they had only been seriously engaged on this task for about eighteen months. The present account was a progress report on a certain amount of experimental work and the derivation of analytical procedures to explain the behaviour observed. Attention was however being given to a number of problems which had hitherto not been examined in any great detail.

SIMPLE COMPOSITE BEAMS

In 1961 tests were carried out on a series of small scale beams composed of 6in. x 3in. rolled steel joists and 2½in. thick concrete slabs, to examine the effectiveness of various types of shear connectors. Taking into account the economics of the problem, it had been concluded that the welded stud connector had many advantages. This connector is admittedly among the more flexible, but as tests on small scale beams had revealed, deflections of these members were not particularly sensitive to differences in load-slip characteristics of the connector as determined from push-out tests.

In more recent tests carried out at Sydney on push-out and beam specimens of a size approaching full scale, the opportunity had been taken to examine the influence of using different steels for the studs, and for the slabs, both normal and lightweight concretes. In the latter expanded shale aggregates were used giving a concrete density of about 108 $lb/ft.^3$. From Fig.1(a) it will be seen that the three stud steels had been included in the programme, one having a yield stress of 40 $kips/in.^2$ (M), another with a yield stress of 65 $kips/in.^2$ (J) and the third, a cold worked material with a maximum stress approaching 90 $kips/in.^2$ (K). Studs of all three materials were tested in push-out specimens having slabs of normal concrete and gave the results shown at (b) in Fig.1. The differences in load-slip curves are not so marked as the stress-strain curves for the studs would suggest.

For the studs of steels J and M push-out specimens of the type shown in Fig.2 were made up with slabs of both normal and lightweight concrete having average crushing strengths of 4,350 and 4,600 $lb/in.^2$ respectively. The corresponding elastic modulii were 3.85×10^6 and 2.20×10^6 $lb/in.^2$; here again the large difference in stiffness is not particularly evident in the load-slip curves shown at (c) and (d) of Fig.1. In Fig.2 can be seen a push-out specimen with the cold worked studs (K), broken open after testing to failure. The studs remained comparatively straight and rotated by yielding of the root material in the joist beneath the weld; ultimate failure was brought about by a combination of this rotation and crushing of the concrete. These two phenomena tended to predominate with the result that changes in stud strength did not have a particularly marked effect. However for the studs of lower strengths (J and M), it proved easier to achieve reliable weldments and in subsequent work it was decided to use studs of the lower strengths and to discard the work-hardened material (K).

Details of the three full-scale beams are given in Fig.3. As will be seen from the Table at (c), both of the beams A1 and A2 had slabs of normal concrete: in the former the studs were of a steel with a yield stress of 32.5 $kips/in.^2$; for the studs in the latter beam the corresponding stress was 62.9

kips/in.^{2}. The stress-strain diagrams for these stud steels are shown at (a) of Fig.4. For the third beam (A3) the slabs were of lightweight aggregate and the studs were similar to those used for the beam A1. The stress-strain diagrams for the various concretes are plotted at (b) in Fig.4; the values of the modulii of elasticity were 4.12×10^6, 4.02×10^6 and 2.18×10^6 lb/in.^{2} for beams A1, A2 and A3 respectively. These beams were typical of a number used in a large building in the city of Sydney. The prototypes were designed for uniformly distributed loading on the basis of the A.I.S.C. recommendations making necessary the provision of fourteen 3/4in. diameter x 3in. long stud connectors in the shear length. The beams were however tested under a symmetrical two-point loading giving a bending moment diagram approximating to that for the uniformly distributed loading; and for this arrangement of concentrated loads there were only ten studs in the shear length. This had the effect of producing greater slip and thus of emphasising characteristics of particular interest in the investigation.

Each beam was tested by applying the two-point loading ((a) Fig.3) with hydraulic jacks reacted through portal frames attached to a strong floor as shown in Fig.5. Deflections and strains were recorded in the usual manner by a series of dial and electric resistance strain gauges. The central deflections observed during these tests are plotted against total applied load for each beam in Fig.4(c). Here again, despite the considerable difference in properties of the concretes and stud of the steels, the three curves of deflections are very similar in form and lie remarkably close together.

In an attempt to explain the above behaviour, an analysis has been developed for the composite beam relating load and deflection in the non-linear range right up to the point of collapse. Under elastic conditions, and when the neutral axis lies in the concrete, the resultant bending stress distribution will be that shown at (b) in Fig.6, where the change in value at the interface represents the effect of slip characteristic of the flexible stud connector. After yielding of the steel has occurred and the extreme fibre stress in the concrete has exceeded 85 percent of the cylinder strength, the stress distribution is assumed to be as at (c); and when the section reaches its maximum moment of resistance the corresponding distribution is taken to be that at (d).

In the absence of slip, curvatures and strains at a section subjected to a given bending moment, are uniquely defined by the moment curvature relationship; when slip occurs strains corresponding to a particular curvature are not unique and the problem becomes more complex. To facilitate the solution of the latter case, a finite difference method has been used in which changes in slope (θ) and deflection (δ) have been related to curvatures (ρ) at n discrete positions along the length (l) of a beam by the following expressions which apply irrespective of the state of the material of which the beam is composed:

$$\theta = \frac{l}{n}(\rho_1 + \rho_2 + --- \rho_n) \qquad (1)$$

$$\delta = \frac{l}{2n}\left[\theta + \frac{2l}{n}\left\{\rho_1(n-1) + \rho_2(n-2) + --- \rho_{n-1}\right\}\right] \qquad (2)$$

To evaluate these quantities it is necessary to determine the relationship between curvature and moment of resistance. For a typical case of a partially yielded section as considered in Fig.7, the strain distribution is given by the area AJEFPH, and the shaded area ABCEFDGH corresponds to the stress distribution for the selected condition of yielding. It can be shown that the depth of the axis NN .is given by

$$k_2 D = [\sqrt{\{\alpha(\alpha + 2\beta - 2s) + 4c_1^2\}} - \alpha] \qquad (3)$$

where α is a term relating the geometrical and material properties of slab and joist, β is a purely geometrical term and s is a measure of the slip occurring at the particular section. In addition, the moment of resistance can be expressed in terms of the curvature by the equation

$$\begin{aligned} M = \tfrac{1}{2}E_m\rho[&k_2 D z'_{ct} + 2dz_m + A_m(k_1 D + 2c)(\beta - s - k_2 D) \\ &-2(bz_f + az_w + b_2 z''_f) \\ &- \{2c_1 z''_{ct} + c_1 A''_{ct}(k_2 D - 2c_1)\} \\ &- 2A_f(d - b)(2d_1 - b) \\ &- 2A_w(a + 2b_2)(d - a - 2b) \\ &- 2b_2 A''_f(d + b_2 - 2d_1)] \end{aligned} \qquad (4)$$

in which the terms in A and z are areas and modulii respectively of the variòus components of the composite section. Similar relationships can be determined for different stages of yielding of the cross-section and can be used to obtain a relationship between M and ρ for the solution of equations of the form given by (1) and (2).

As indicated above, when the connectors are assumed to be rigid and the term s can be taken to be zero, the evaluation of deflections right up to collapse is comparatively straightforward. Theoretical load deflection curves determined in this way are given in Figs.8 and 9 for the beams A1 and A3 and are labelled "Complete interaction". These curves are based upon the yield stress of the steel joist and a maximum stress of 85 percent of the cylinder strength for the concrete slab. The ultimate loads calculated in this way are considerably greater than those observed in the tests indicating a significant loss of composite action in both beams. It will however be recalled that as tested under the particular two point loading, they were effectively underdesigned since only ten studs were provided in the shear length.

In work commenced more recently values of the slip s for use in the computations were being obtained from a simple mathematical curve fitted to that determined from push-out tests on the particular type of connector. To evaluate beam deflections taking slip into account, a relaxation method was being developed. Commencing from a determination of the loads on individual connectors for the case of no slip, true values of these loads were obtained by an iterative process in which the connectors were successively relaxed until they attained a state of equilibrium compatible with their stiffnesses. These calculations were being done on a computer using a programme arranged to output information including beam deflections and connector displacements and loads. This method had worked satisfactorily in the range where the steel and concrete were elastic; beyond this point certain problems of convergence arose and these were still being studied.

In the range in which the relationship between load and slip was linear for the connector, deflections could, of course, be evaluated in accordance with Newmark's direct solution; the corresponding results are shown in Figs.8 and 9.

CONCRETE ENCASED COLUMNS

The type of column being studied is shown in Fig.10. All of these were of 4in. x 3in. rolled steel joist having a 2in. concrete cover as indicated at (a); spiral reinforcement was provided at the ends; and each member was tested as a pin-ended column effectively of length 7ft. as shown at (b). Details of eccentricity of loading and of material properties are given for three test columns in the Table at (c). They were loaded in an Amsler machine as shown in Fig.11 and in all cases failure occurred by bending about the minor axis of the joist.

The specimens tested under concentric loading failed catastrophically with considerable spalling of the concrete at the centre section; as the eccentricity was increased in subsequent tests the failure became more gradual and was accompanied by cracking without spalling. This will be evident from the photographs of the centre sections of columns (1) and (3) as shown in Fig.12. Load deflection curves for all three columns are plotted in Fig.13. An analytical solution for the determination of collapse load can be obtained by further use of equations of the type (1) and (2). The imperfections of the member were assumed to be represented by initial sinusoidal curvature of the same order as that of the bare steel column and expressed in terms of the Perry-Robertson constant η . As a result of this curvature any axial load produces bending; commencing from the displacements corresponding to the imperfection it is possible by iteration on a computer using equations similar to (1) and (2), to arrive at the deflected form of the column which is in equilibrium under a given load. All that is necessary to pursue this type of solution into the non-linear range is a knowledge of the relationship between moment and curvature after yielding of the steel and crushing of the concrete commences. It will be found that a stage is eventually reached where, for any increase in axial load, the iteration ceases to be convergent thus indicating that the column has become unstable. No attempt has been made to pursue the calculation beyond this theoretical collapse load to obtain the drooping characteristic of the load deflection curve.

For a general solution, it is necessary to examine a number of different cases involving yielding of the steel and/or crushing of the concrete. Strain distributions for a typical case in which both materials have exceeded the limiting strains (e_y and e_{cy}) are shown at (b) and (c) in Fig.14; the shaded areas at (a) and (d) represent the materials strained beyond these limits. The equations relating moment (M) axial load (P) and curvature (ρ) have the following forms:

$$M = E_c \rho \Big[I_T - \sum A_c (\bar{x}_c - x_1)\, \bar{x}_c - \sum I_c - n \sum A_s (\bar{x}_s - x_2)\, \bar{x}_s - n \sum I_s - \sum A_k (\bar{x}_k - x_3)\, \bar{x}_k - \sum I_k \Big] \qquad (5)$$

$$
\begin{aligned}
P \quad = \quad & A_T E_c \,(e_{cy} - x_1 \rho) \\
& - E_c \rho \Big[\sum A_c \,(\bar{x}_c - x_1) + n \sum A_s \,(\bar{x}_s - x_2) \\
& - \sum A_k \,(\bar{x}_k - x_3) \Big] \qquad (6)
\end{aligned}
$$

where the terms A, $\bar{x}$ and I refer to the areas, positions of centroids of areas, and second moments of areas of the various elastic and non-elastic components of the composite section in Fig.14.

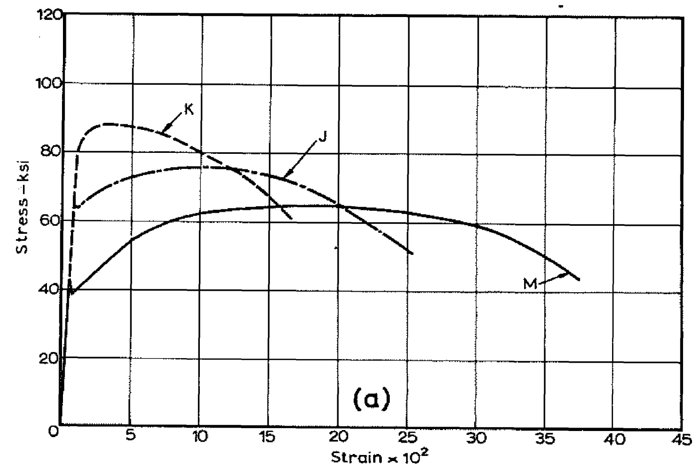
A number of theoretical values of the collapse load were determined for column (1), the member subjected to a concentric axial load. The significant portion of the load deflection curve corresponding to each of these is shown in Fig.15 together with the selected values of the three variables, the modular ratio (n), the degree of imperfection (η) and the limiting stress for the concrete (f_{cy}). It will be seen that as compared with the experimental result, a minimum theoretical curve was obtained for $n = 10$, $\eta = 0.0015 \frac{l}{k}$ and $f_{cy} = 0.85 F_c'$. These values were subsequently used to determine theoretical values of the collapse loads of the other two columns; the values so obtained are compared with experimental results in Fig.16.

Also shown in that figure are two curves based on loads required by the rules of the British Standard 449 The Use of Steel in Buildings. The upper curve represents the working load obtained from a formula assuming the member to act in simple compression and to have a modular ratio of 30. The values given by the lower curve represent the permissible axial load, since the rule when applied to the particular joist section, limits the load to twice the working value for the bare steel member. The results given here would suggest that the rules in the British Standard are particularly conservative for composite members where the strength of the concrete encasement can be guaranteed.

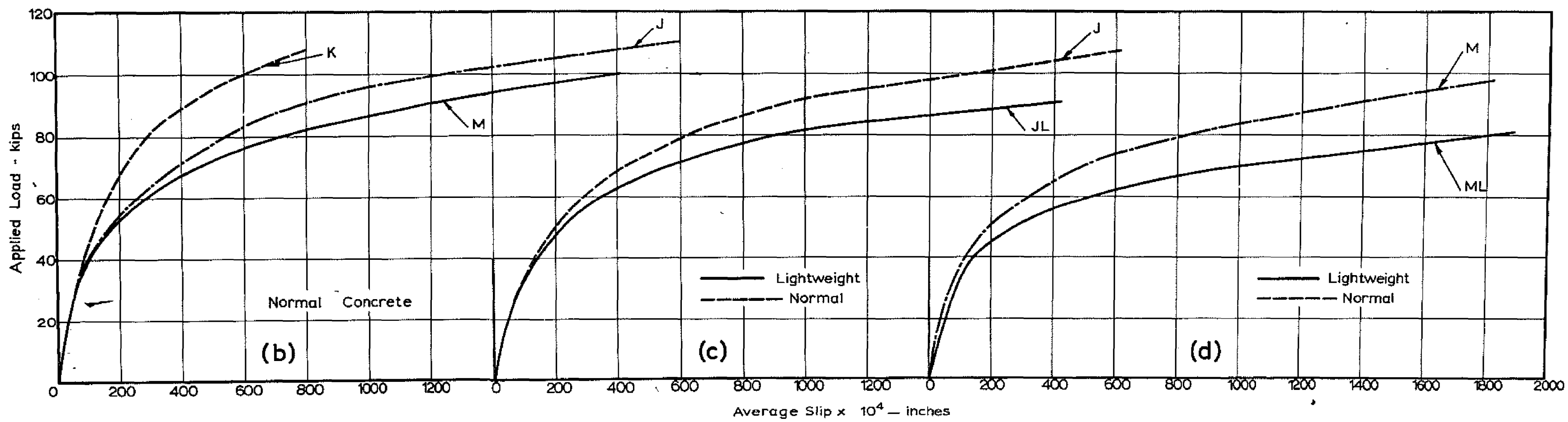
OTHER STUDIES

In addition to the work described above, several other studies have recently been started. In the case of simple beams these include a further examination of the behaviour of studs and of slabs of lightweight concrete; a study is also being made of the effects of dispensing with the shear connectors and encasing the joist in concrete and making it integral with the slab. Preliminary tests are also being carried out on inverted composite beams with shear connectors and various amounts of reinforcement in the slab. The results so obtained should be of value in designing continuous beams for the next stage of the investigation. A start has also been made on a determination of the load deformation characteristics and of the load carrying capacity of studs in push-out specimens when subjected to sustained and to repeated loading.

In conclusion the author wished to acknowledge the support given to the research programme by the New South Wales Department of Main Roads, the Australian Road Research Board and the Australian Institute of Steel Construction. He also wished to express his thanks to his colleagues Dr. N. M. Hawkins, Mr. P. T. Brown, Mr. D. Rogers and Mr. Y. O. Loke for their part in these investigations.



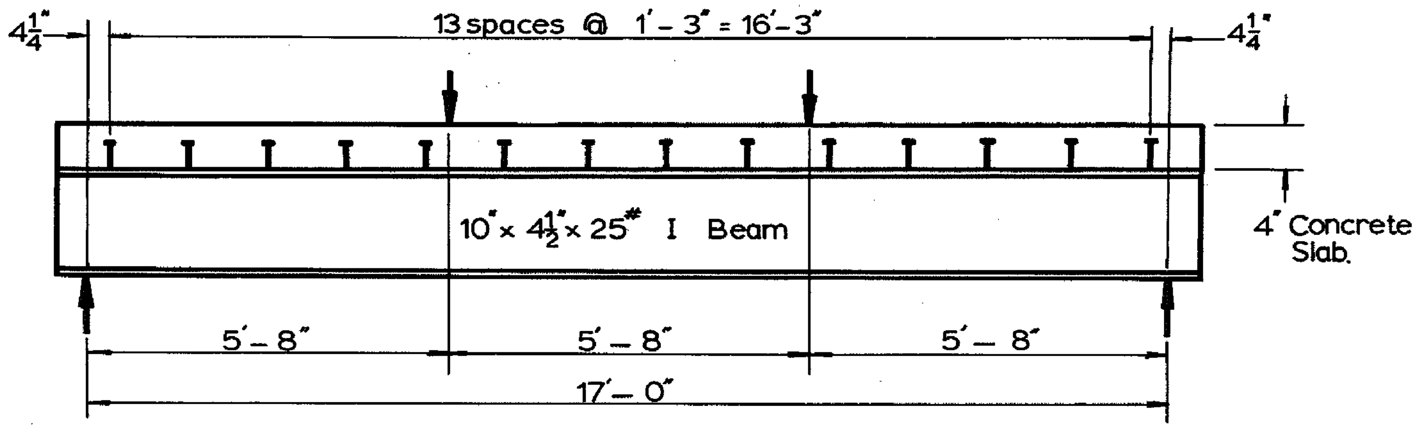
— STUD STEELS —



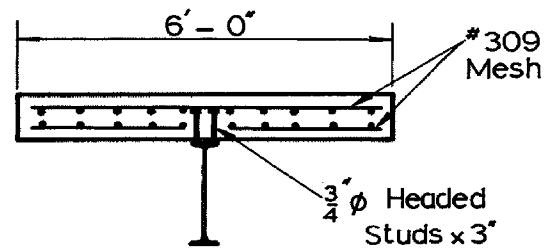
— PUSH - OUT TESTS —

FIG. 1.

FIG. 2. STUDS (K) IN NORMAL CONCRETE.



(a).



(b).

Beam No.	Studs		Concrete		Yield Stress of Joist.
	Yield psi	U. T. S. psi	Strength psi	Type	psi
A1	32 500	65 800	4 040	Normal Weight	43 050
A2	62 900	75 800	4 360	Normal Weight	43 100
A3	32 500	65 800	4 680	Light Weight	42 900

(c).

FIG. 3.

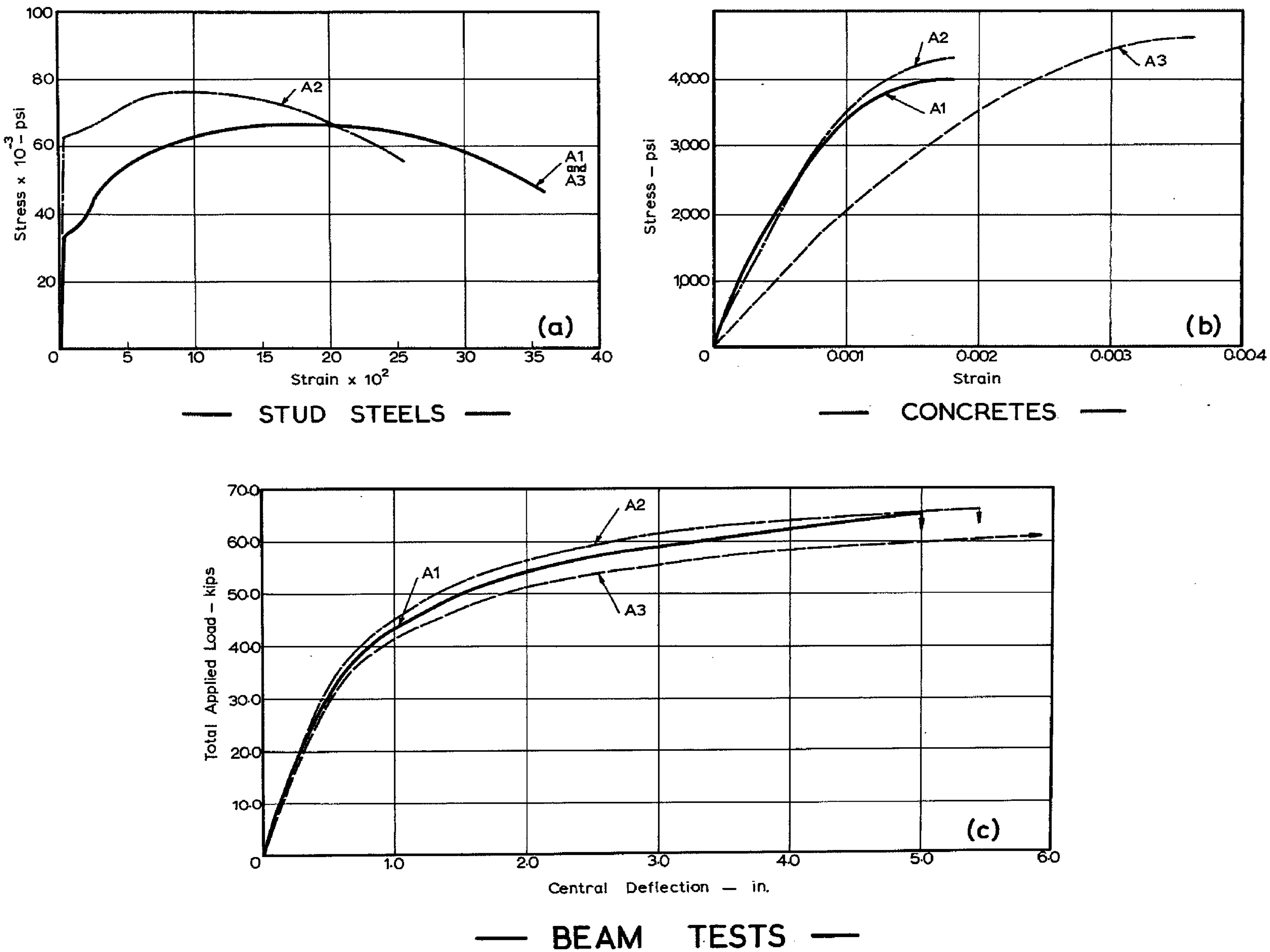

100
80
60
40
20
0
Stress x 10^{-3} – psi
A2
A1 and A3
(a)
5
10
15
20
25
30
35
40
Strain x 10^2
STUD STEELS
4,000
3,000
2,000
1,000
Stress – psi
A2
A1
A3
(b)
0.001
0.002
0.003
0.004
Strain
CONCRETES
70.0
60.0
50.0
40.0
30.0
20.0
10.0
Total Applied Load – kips
A2
A1
A3
(c)
1.0
2.0
3.0
4.0
5.0
6.0
Central Deflection — in.
BEAM TESTS

FIG. 4.

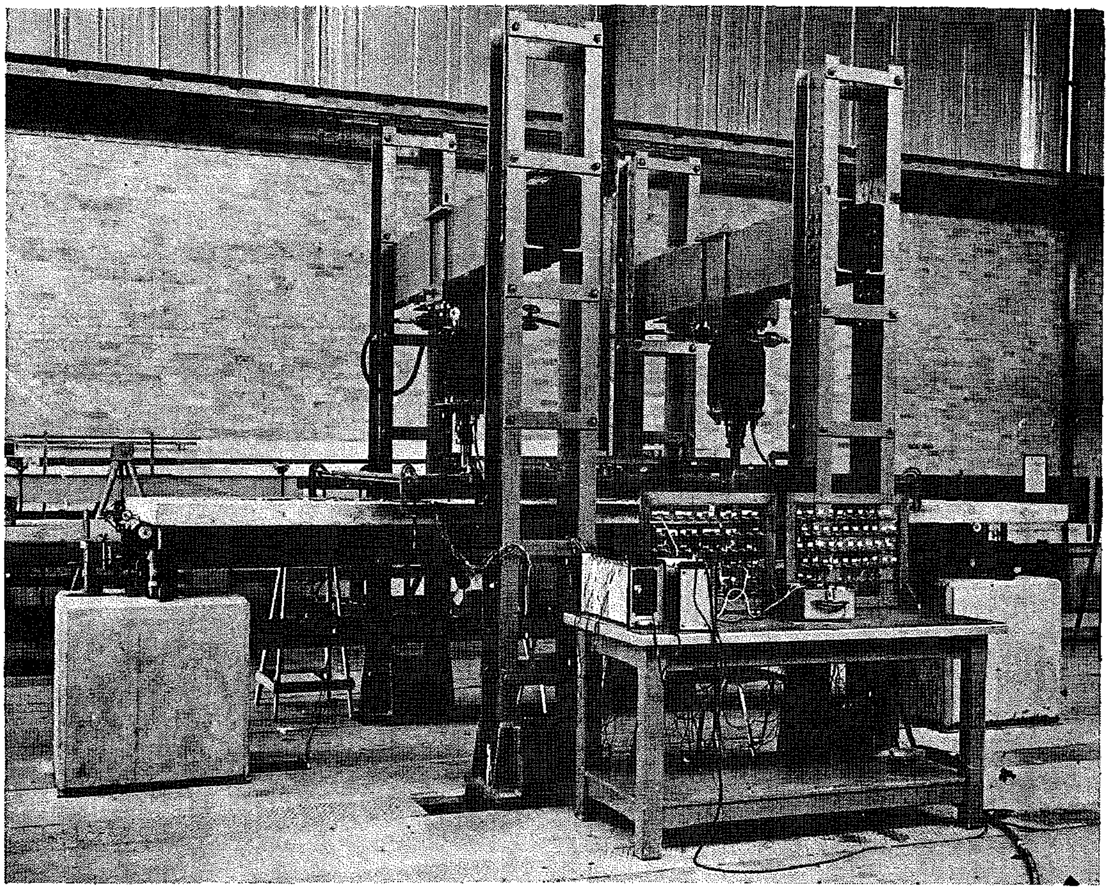


FIG. 5. GENERAL ARRANGEMENT OF LOADING RIG FOR COMPOSITE BEAM TESTS

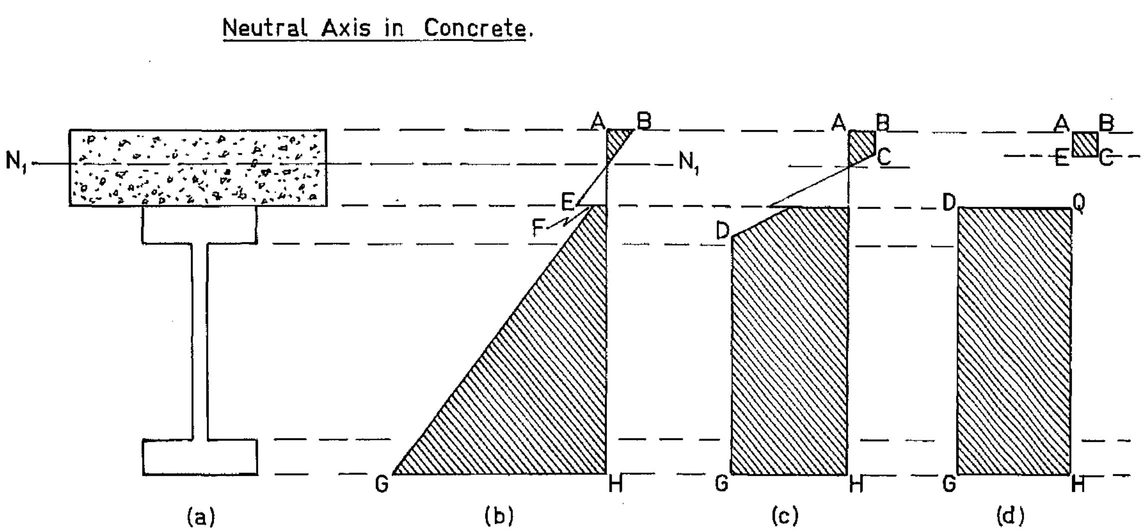
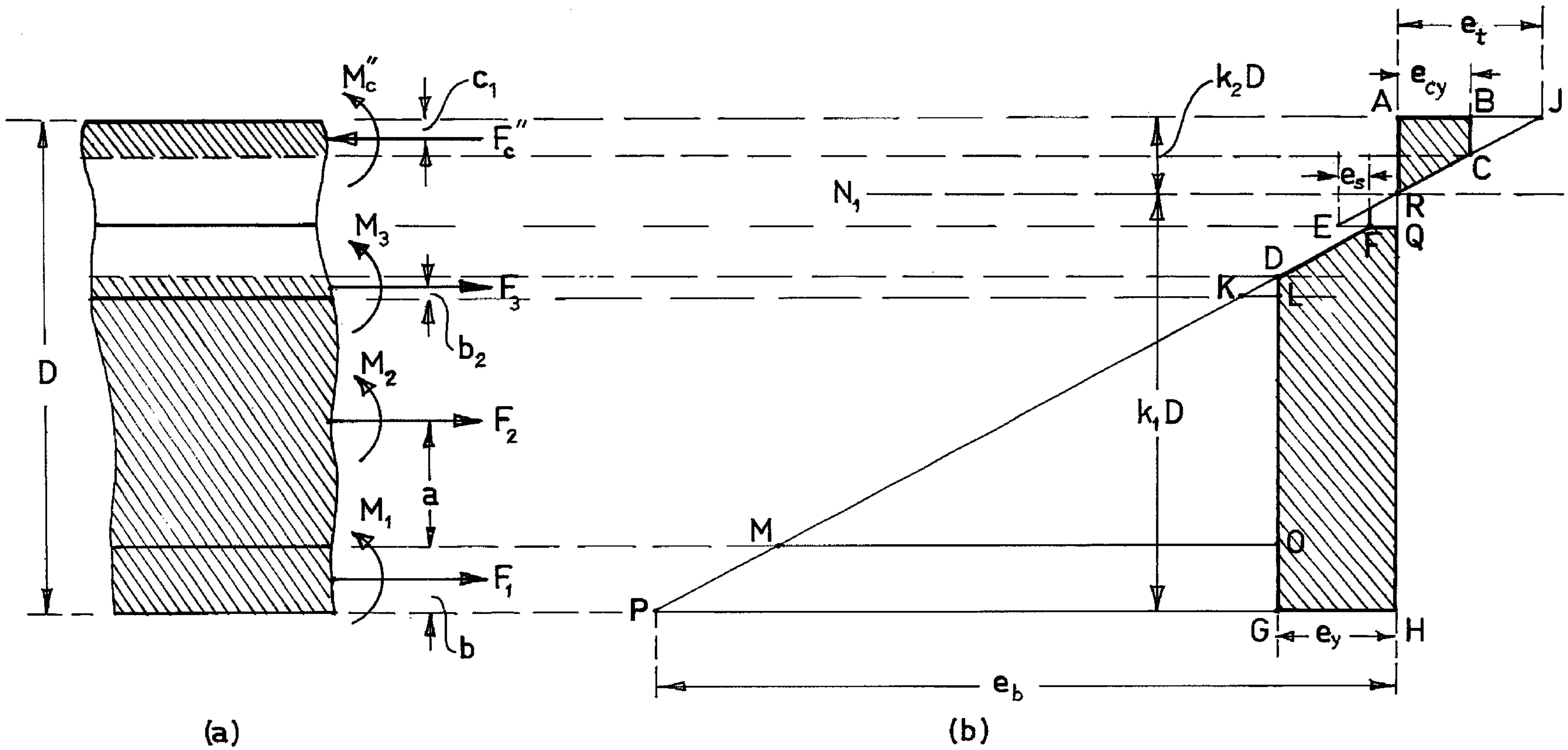
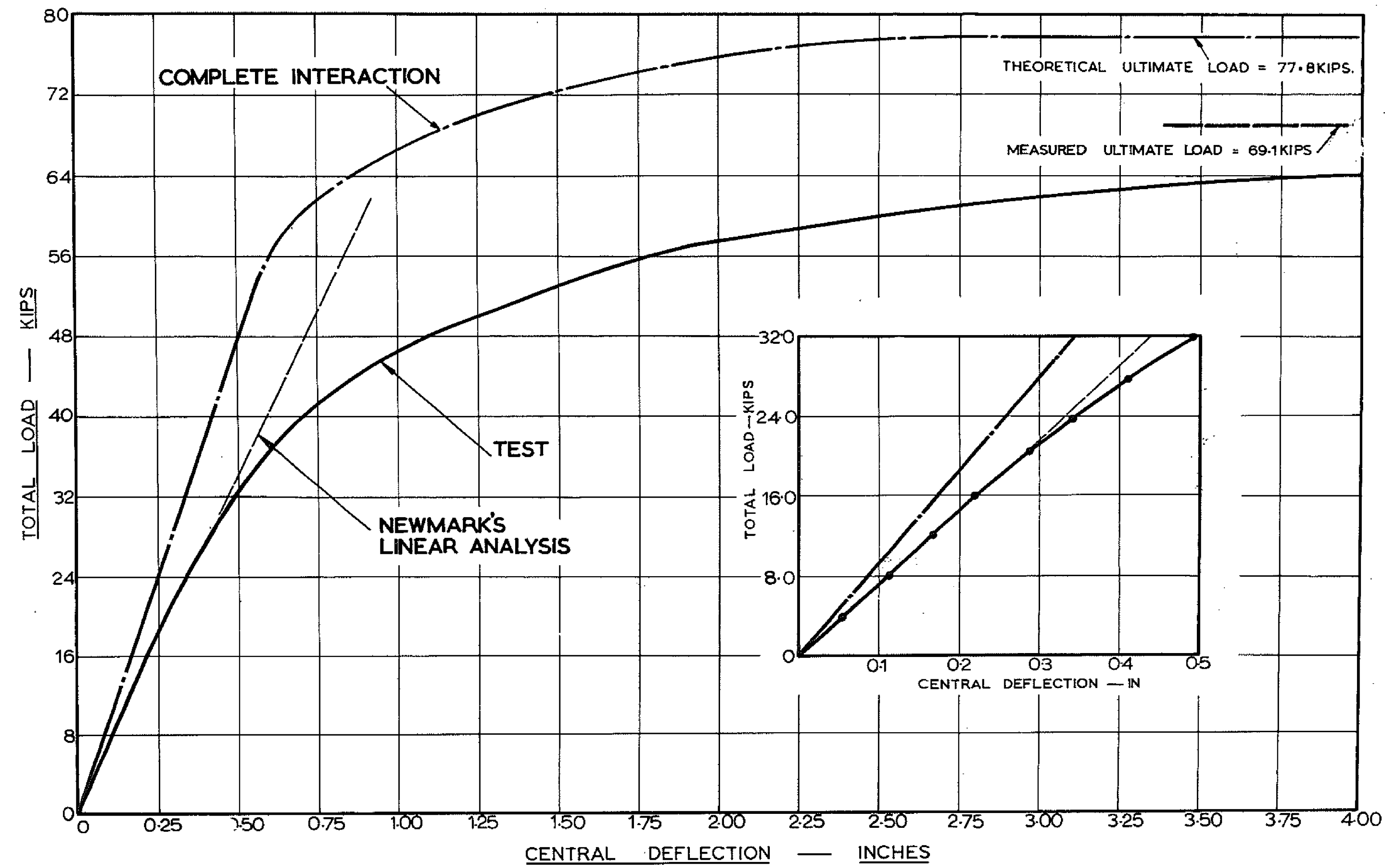


FIG.6.



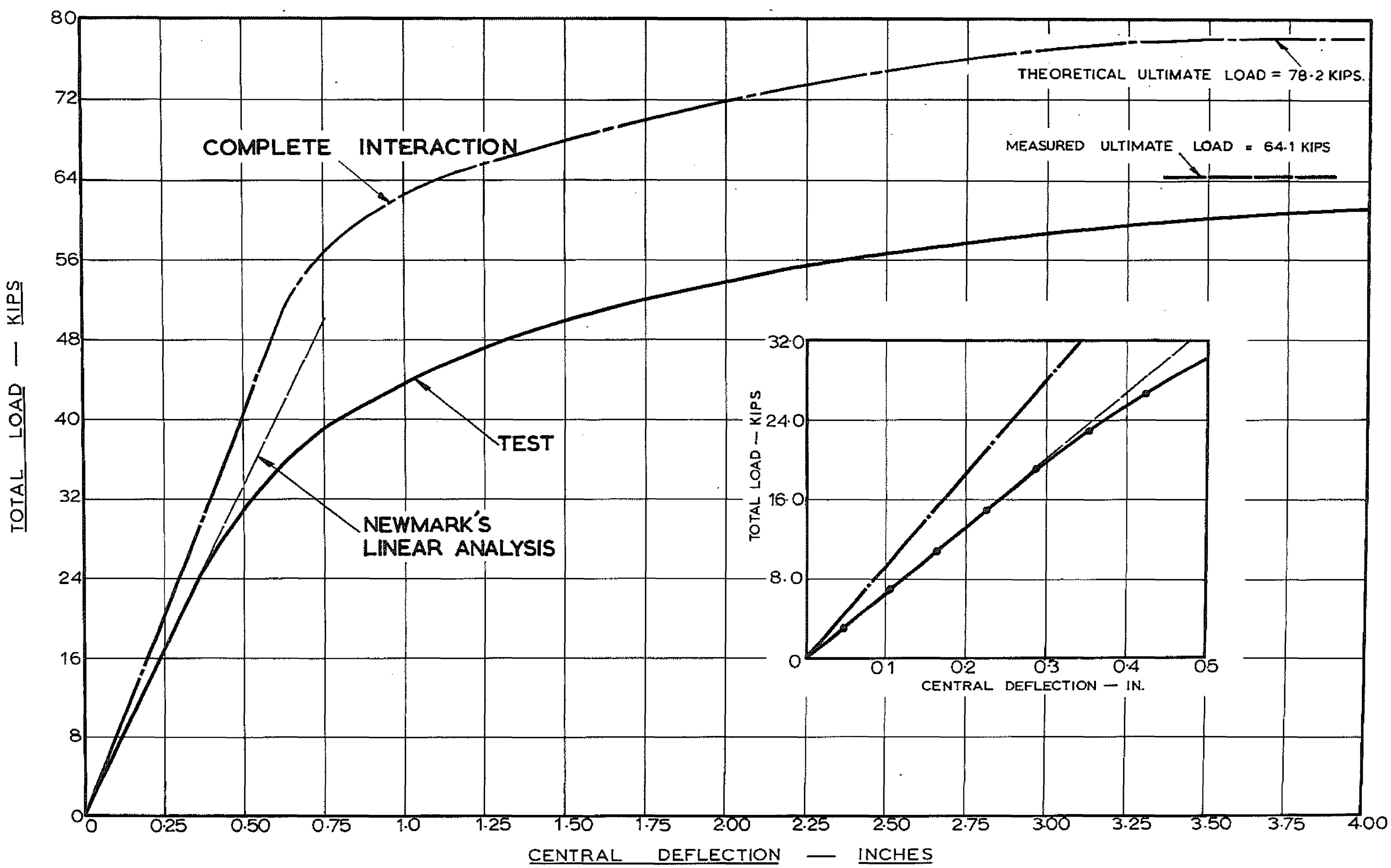
D
M''c
c1
F''c
M3
F3
b2
M2
F2
a
M1
F1
b
k2D
N1
k1D
et
ecy
es
A
B
J
C
R
E
Q
F
D
K
L
M
O
P
G
H
ey
eb
(a)
(b)

FIG.7.


COMPLETE INTERACTION
THEORETICAL ULTIMATE LOAD = 77·8KIPS.
MEASURED ULTIMATE LOAD = 69·1KIPS
TEST
NEWMARK'S
LINEAR ANALYSIS
TOTAL LOAD — KIPS
CENTRAL DEFLECTION — INCHES
80
72
64
56
48
40
32
24
16
8
0
0
0·25
0·50
0·75
1·00
1·25
1·50
1·75
2·00
2·25
2·50
2·75
3·00
3·25
3·50
3·75
4·00
TOTAL LOAD — KIPS
CENTRAL DEFLECTION — IN
32·0
24·0
16·0
8·0
0
0·1
0·2
0·3
0·4
0·5

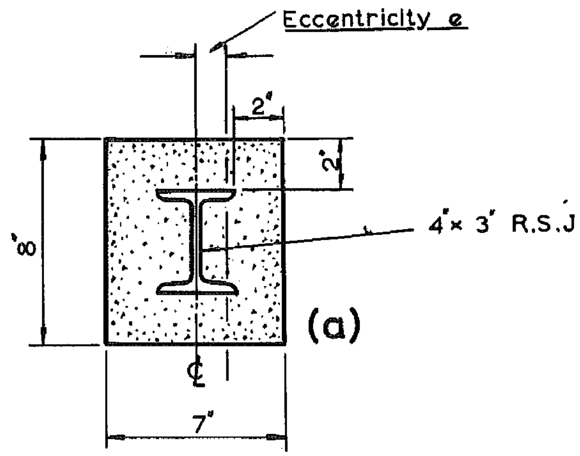
— BEAM A1 —

FIG. 8.

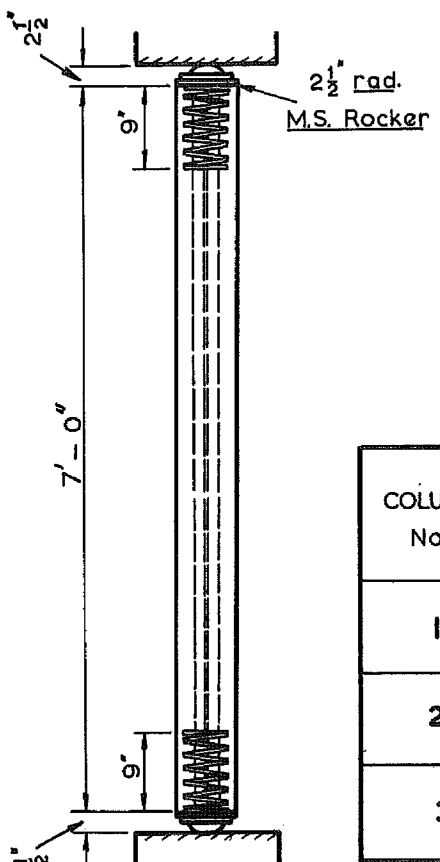

THEORETICAL ULTIMATE LOAD = 78·2 KIPS.
MEASURED ULTIMATE LOAD = 64·1 KIPS
COMPLETE INTERACTION
TEST
NEWMARK'S LINEAR ANALYSIS
TOTAL LOAD — KIPS
CENTRAL DEFLECTION — INCHES
TOTAL LOAD — KIPS
CENTRAL DEFLECTION — IN.

—— BEAM A3 ——

FIG. 9.



(a)



(b)

COLUMN No.	ECCENTRICITY e (in.)	STEEL YIELD STRESS f_y (psi)	CONCRETE STRENGTH F'_c (psi)
1	0	42 400	3 690
2	0.4	42 400	3 910
3	0.8	42 400	4 270

(c)

COLUMN TESTS

FIG. 10.

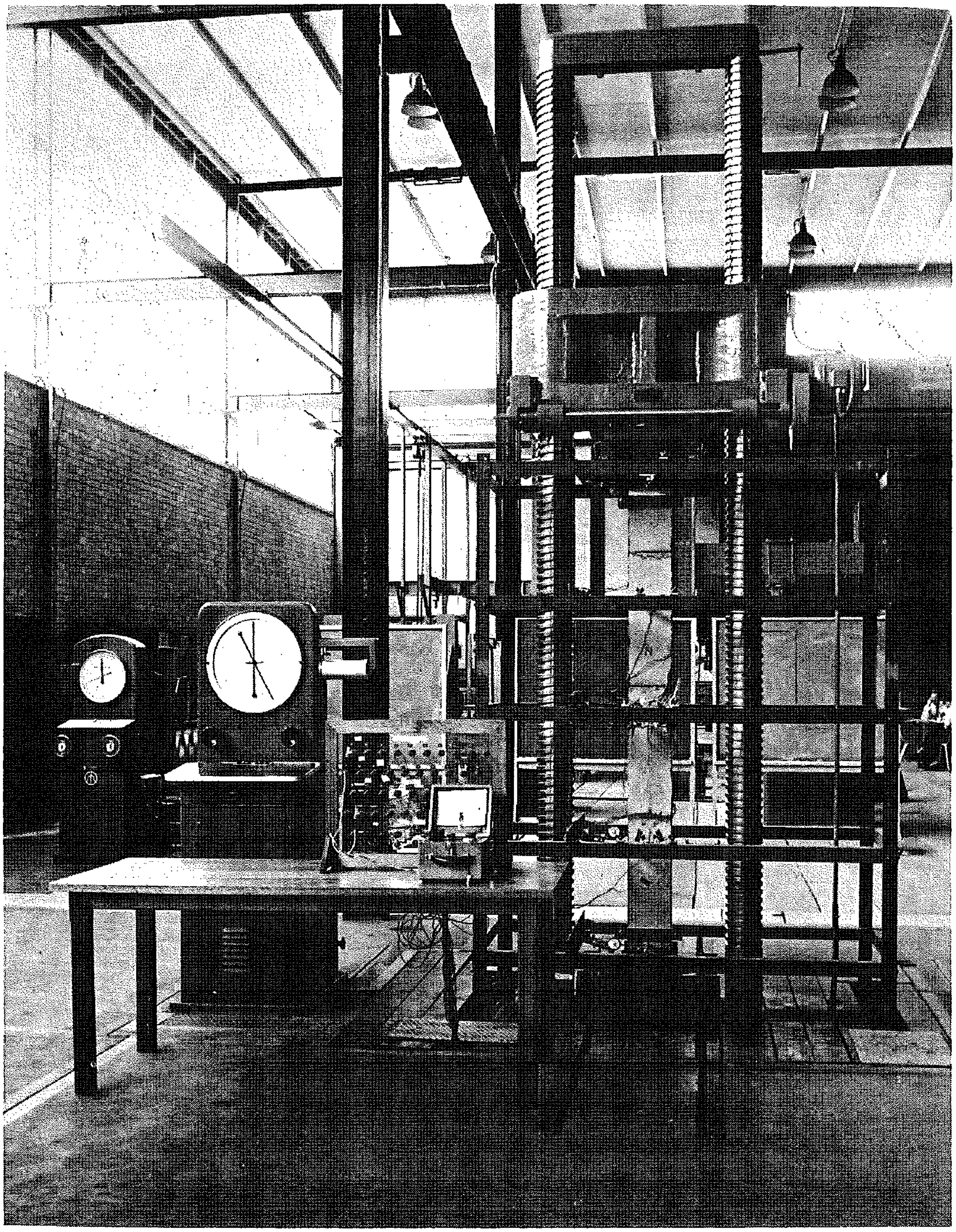
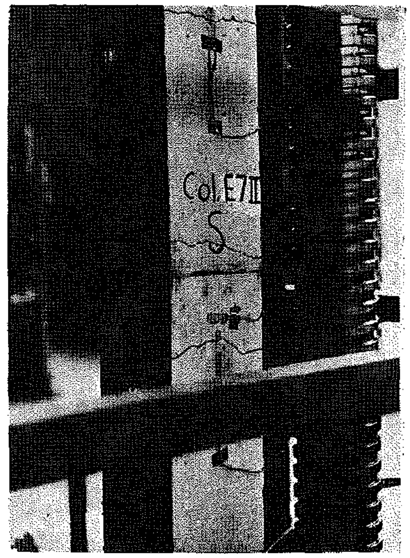


FIG. 11. CONCRETE ENCASED COLUMN SET UP FOR TESTING.



(b)
COLUMN (3)

(a)
COLUMN (1)

FIG. 12. COLUMNS AFTER COLLAPSI

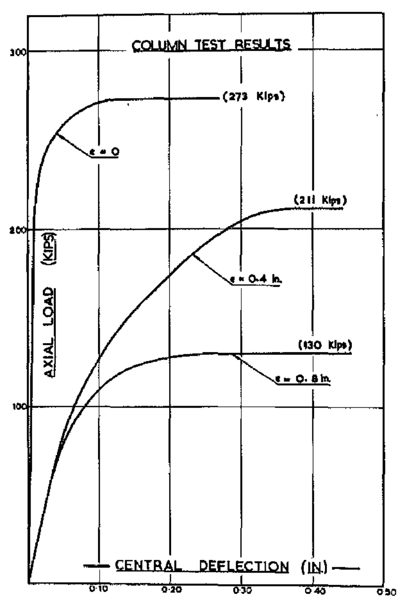


FIG. 13.

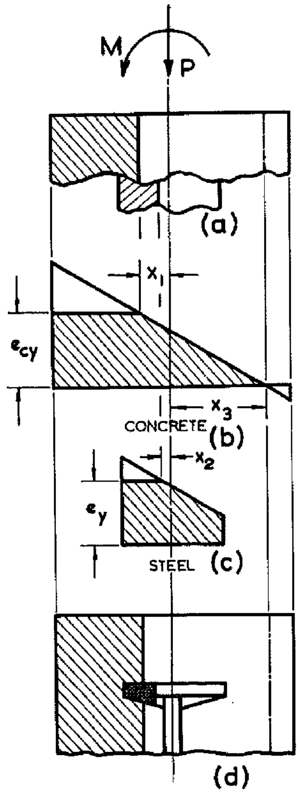

M
P
(a)
x_1
e_{cy}
x_3
CONCRETE
(b)
x_2
e_y
STEEL
(c)
(d)

FIG. 14.

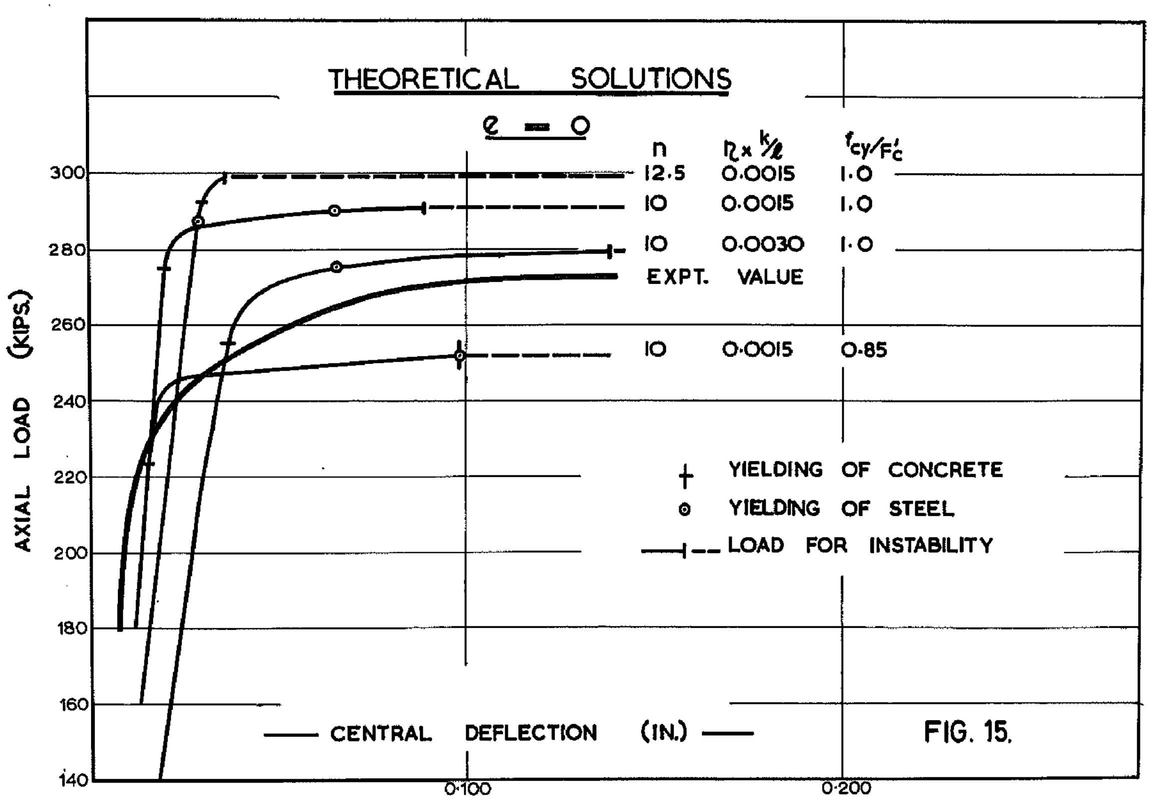

THEORETICAL SOLUTIONS
$e = 0$
n
$\eta \times k/l$
f_{cy}/F'_c
12.5 0.0015 1.0
10 0.0015 1.0
10 0.0030 1.0
EXPT. VALUE
10 0.0015 0.85
YIELDING OF CONCRETE
YIELDING OF STEEL
LOAD FOR INSTABILITY
AXIAL LOAD (KIPS)
300
280
260
240
220
200
180
160
140
CENTRAL DEFLECTION (IN.)
0·100
0·200

FIG. 15.

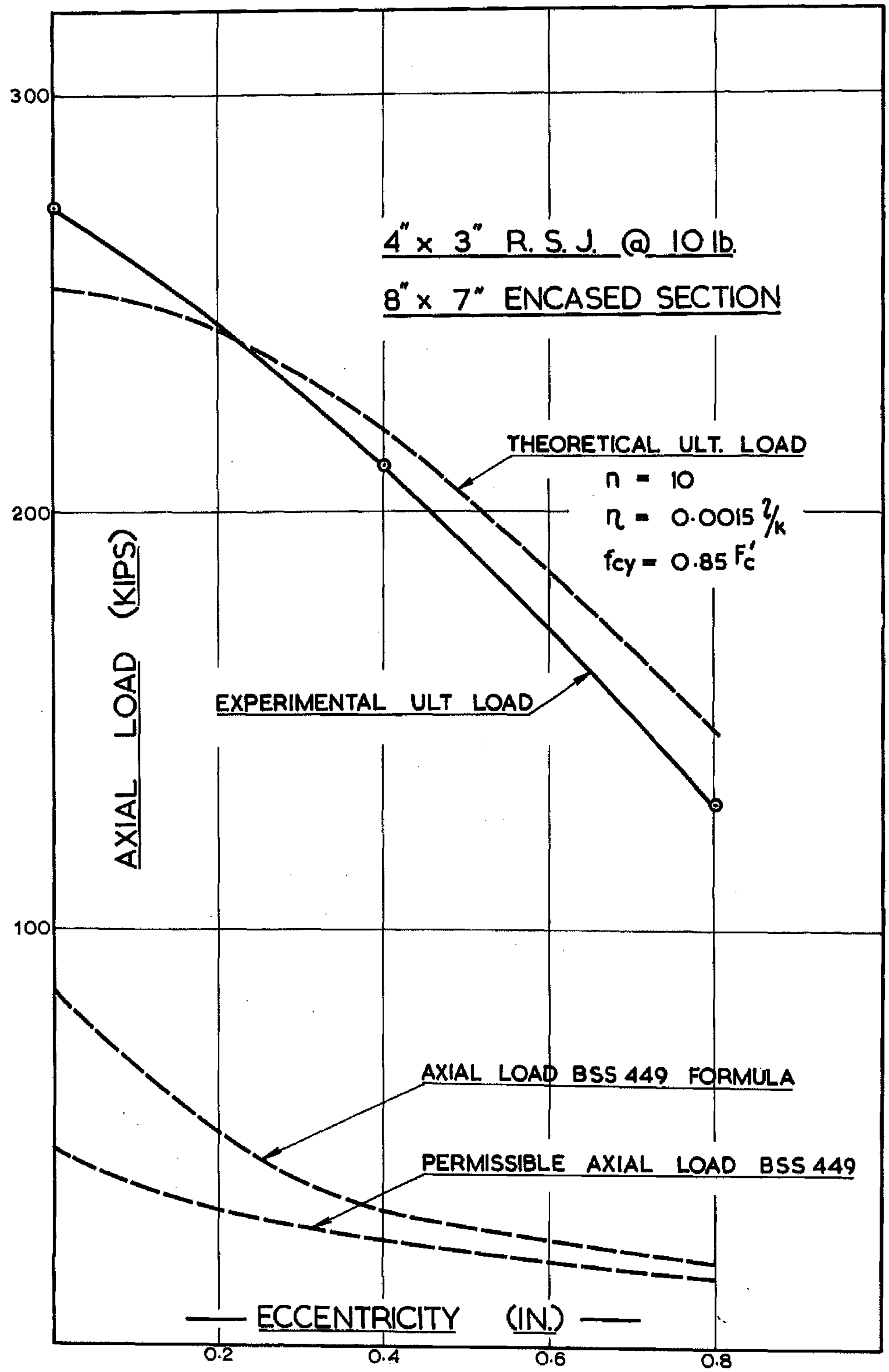

4" x 3" R. S. J. @ 10 lb.
8" x 7" ENCASED SECTION
300
200
100
AXIAL LOAD (KIPS)
THEORETICAL ULT. LOAD
n = 10
η = 0·0015 l/k
fcy = 0·85 F'c
EXPERIMENTAL ULT LOAD
AXIAL LOAD BSS 449 FORMULA
PERMISSIBLE AXIAL LOAD BSS 449
ECCENTRICITY (IN.)
0·2
0·4
0·6
0·8

FIG. 16.

DESIGN PROBLEMS IN THE STEEL-MAKING INDUSTRY

by
Bruce G. Johnston
University of Michigan

During the past twenty-five years the structural engineer has contributed through research and by the preparation of standards to at least four structural design problems in the steel-making industry. These are: (1) overhead traveling cranes; (2) crane girder hooks; (3) hot metal ladles; and (4) steel mill building structures. It is particularly appropriate that these problems be discussed at Lehigh University and in Bethlehem, Pennsylvania, because the research projects underlying the standards and specifications developed in connection with the first three of these topics, under the cognizance of the Association of Iron and Steel Engineers, were carried out at the Fritz Engineering Laboratory. Furthermore, most of the slides to be shown tonight have been selected from a large number that were supplied through the courtesy of the Bethlehem Steel Corporation. Some were also furnished by the Koppers Company.

These research projects on steel mill cranes, hooks, and ladles were noteworthy in particular because of the rapidity with which test results were translated into practical standards. Today we accept as commonplace the rapid introduction of new materials, new forms, and the prompt application of research in design. Structures are in the laboratory today and on their way to the moon or Mars tomorrow. Indeed, one of the outstanding features of the Lehigh work on plastic design has been the rapidity with which research results were written into AISC and CISC specifications and transmuted into the construction of actual structures. Such rapid application of research was almost unheard of forty years ago. At that time we who were then students deplored the fact that nearly all steel building frames were designed as if the beams were simply supported, except when wind was the consideration, but there was nothing much that could be done about it. Buildings built "the way grandfather did it" had proved their worth and nearly all followed the same conventional pattern.

Let us turn now to the first topic under discussion, that of the overhead traveling crane (Fig. 1) which lifts and moves the hot metal ladle or serves in other ways as an integral part of the steel-making process. A F.L. research project involving dynamic tests of actual cranes and static tests of welded and riveted box girders was completed and the research report published in November of 1941. In 1942 the Association of Iron and Steel Engineers issued its Standard No. 6 on this subject. This specification introduced in this country the use of longitudinal stiffeners to improve the buckling characteristics of webs of plate girders. In addition, the advantages of box girder construction in resisting torsion were demonstrated and applied. The resultant saving in steel came at a most appropriate time, just prior to the expansion of steel-making facilities during the last world war and just at a time when steel was in very short supply.

Over and above the direct application of crane girder research into design there came as a by-product the discovery that the buckling stress of plates in compression was lowered by the presence of initial residual stress due to welding. This discovery prompted the later Column Research Council research at Lehigh University on the effect of residual stresses in rolled-steel columns that has since been widely recognized as an important factor in the development of current column design formulas.

The second topic concerns the crane ladle hooks (Fig. 2). The design of these hooks involves a rather simple stress analysis of the curved beam but with complications resulting from distortions caused by temperature differentials. However, the importance of a completely safe design can be appreciated when one thinks of the catastrophic results of the failure of either one of the two hooks used to lift a ladle carrying anywhere from two to five hundred tons of molten metal. When one adds in the weight of the ladle itself, each hook is required to carry safely loads which may be as high as 700 kips.

The research carried on at Lehigh consisted mostly of photoelastic tests of hook models of various configurations. The tests were made in 1948 and the AISE Specifications appeared in 1949.

Concerning Item 3, the hot metal ladle itself, (Fig. 2) again we have an outstanding example of rapid application of research into practice. The research report on the tests of models of hot metal ladles as carried out at Fritz Engineering Laboratory was published in 1949 and the standards were issued by the Association of Iron and Steel Engineers in 1951. This specification has been used as a model in Great Britain, Europe, and elsewhere. As mentioned before, the capacity of single ladles is currently approaching five hundred tons or a million pounds of molten metal. Stress analysis of the ladle might be considered under the category of stiffened shell theory but the boundary conditions and details of construction are so complex as to make any exact mathematical analysis of questionable use. The problem is complicated by severe thermal stress and metallurgical effects, shock, and the erosive process of accidental contact with molten metal. The major problems considered in the tests at Lehigh University, as sponsered by the Association of Iron and Steel Engineers, were the design of the ladle-stiffening rings, which are usually two in number, and the design of the sidewall and bottom plate thicknesses. The location of the trunnion axis is an important problem now under investigation at the University of Michigan. If the axis is located too high in reference to the center of gravity of the combined ladle and molten metal contents, it may cause spillage due to the rotational torque induced by lateral motion. However, too low a position of the trunnion axis could lead to instability.

Fourth, and finally, to bring this discussion down to a matter of more current interest, is the development of a new specification on the overall mill building structure itself. A mill building may be defined as an industrial building in which crane operation is an integral part of the manufacturing process. The new specification currently in preparation will build on the Specification of the American Institute of Steel Construction for Building Design but will supplement it in parts and in some areas will substitute alternate methods of design. In comparison with ordinary building design one must consider floor loads up to and greater than 1000 pounds per square foot and wind forces which involve more unusual structural configurations than in ordinary building construction with the added possibility that the entire side or end of a building may be open. Crane runway loads involve impact, side thrust, longitudinal thrust, and the variable pattern of repeated load that introduces the fatique design problem more explicitly than in most building construction. Furthermore, the impact, side thrust, and longitudinal factors need to be modified in conjunction with different types of mill cranes. Because of the many and varied impact effects, the bracing of the building in the plane of the lower chords of the roof trusses is especially important. It is desired to make the entire structure perform as a three-dimensional space frame to provide distribution of local loads in such a way as to achieve maximum rigidity and stiffness with a minimum total amount of material. The use of stepped beam-columns, as commonly found in mill buildings, with great variation in moment of inertia, introduces additional special design and analysis problems.

A few selected slides will now be shown to illustrate some features of the four problems that have been discussed. However, many of you will be visiting the plants at Bethlehem Steel Corporation during this conference and will see ladles, cranes, and hooks in actual operation in a more vivid way than any picture can depict.

Finally, in closing, I would like to acknowledge the part played by former Fritz Engineering Laboratory researchers on these projects. The research work for the overhead traveling crane studies as well as the specification itself were largely prepared by the late I. E. Madsen who, after being a research engineer at Lehigh University, became editor of the IRON AND STEEL ENGINEER Magazine and research engineer for the Association of Iron and Steel Engineers. The research work on hot metal ladles was largely carried out by Dr. Endre Knudsen and some specific studies for the hot metal ladle specification itself were made by Dr. Bruno Thurlimann, both of whom were former research engineers at Lehigh University. The photoelastic tests on crane ladle hooks were conducted by Dr. F. K. Chang, now an engineer for the firm of Ammann and Whitney.

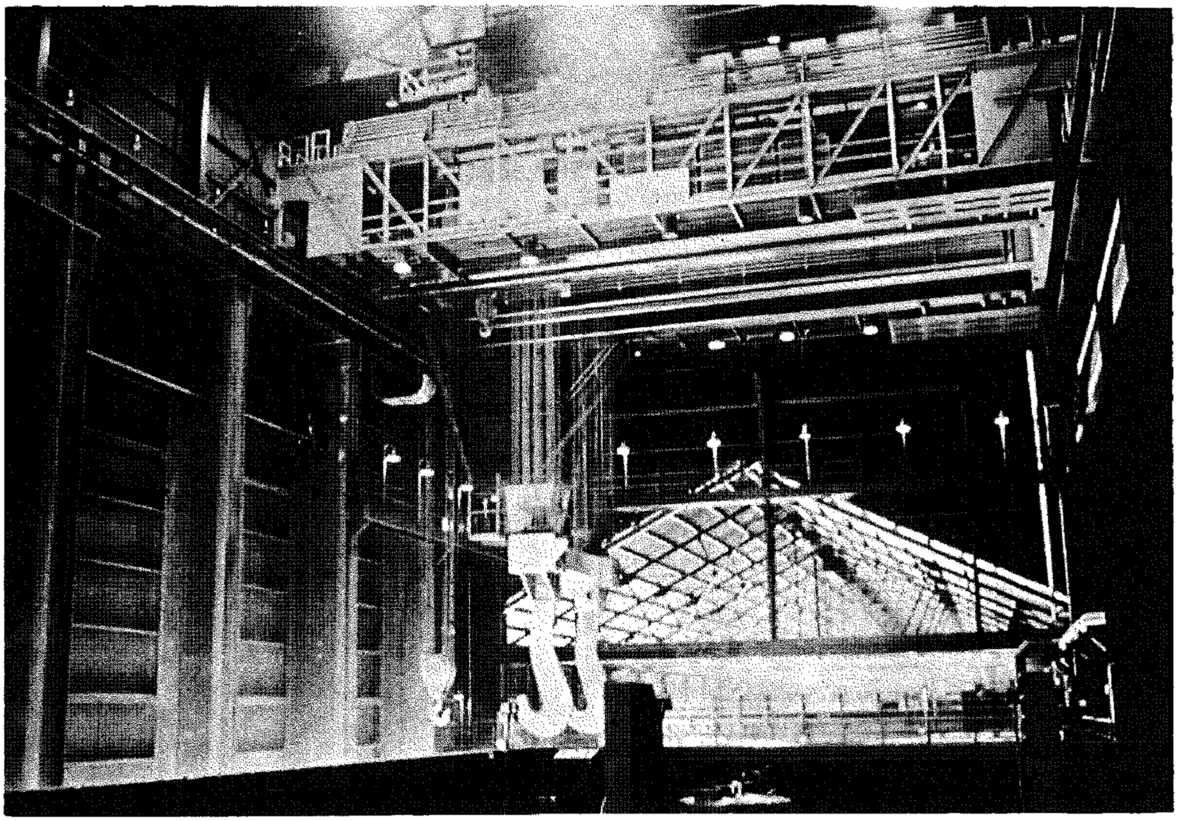


Fig. 1
Courtesy Bethlehem Steel Corp.

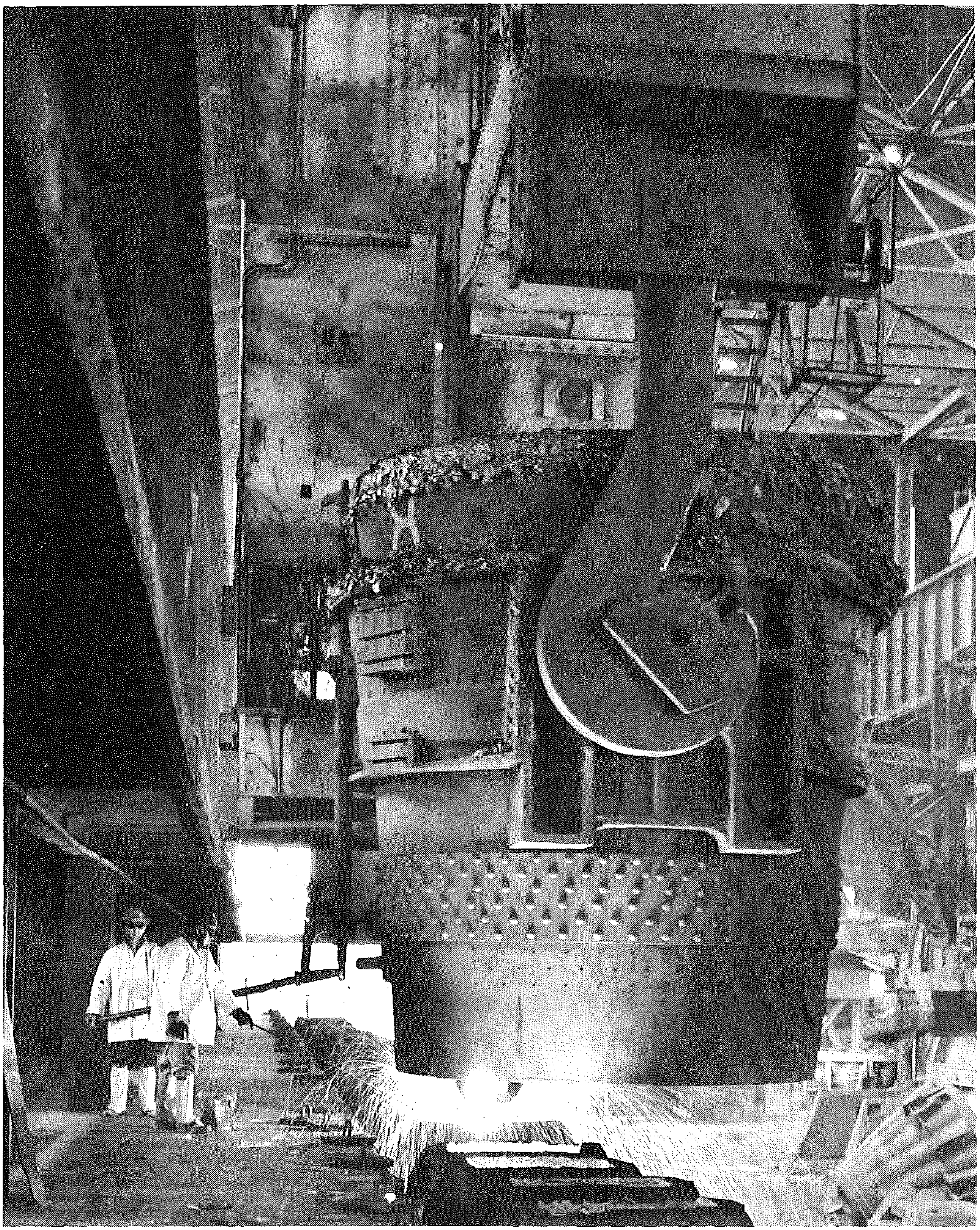


Fig. 2
Courtesy Bethlehem Steel Corp.

EARTHQUAKES AND THEIR EFFECTS ON BUILDINGS

by

Glen V. Berg
Professor of Civil Engineering
The University of Michigan

This evening I would like to talk about earthquakes, what causes them, where they occur, what happens to buildings in earthquakes, and to take a look at some examples of structural damage in two recent earthquakes.

An earthquake can be a terrifying experience. Picture, if you will, the Saada Hotel shown in Figure 1, located in the beautiful port city of Agadir, Morocco. That city was struck by an earthquake the evening of February 29, 1960, and in about 15 seconds the structure you see in Figure 1 was reduced to the pile of rubble shown in Figure 2. A dance was held in the hotel that evening; 400 people died in that one building. A total of 12,000 perished in the city of Agadir, and this was a minor earthquake of a magnitude that occurs somehwere on earth every four or five days. Often they're located under the ocean or in some remote region and nobody pays any attention. Sometimes they are not.

Earthquakes occur all over the globe, but the major ones seem to be concentrated in two specific belts. Figure 3 shows the locations of very large earthquakes for a 50-year period, from 1904 to 1954, and you'll notice that most of them are in a region surrounding the Pacific Ocean, the so-called circum-Pacific belt. There is also a belt that extends across Asia to the Mediterranian, known as the Alpide belt.

Now, if I were to ask you where earthquakes occur in the United States, probably most of you would think first of California, and it is true that more of them occur there than anywhere else in this country. But a look at Figure 4 is a little surprising. Some of the strongest earthquakes ever to have occurred in the United States occurred in New Madrid, Missouri, in 1811 and 12, and in Charleston, South Carolina, in 1886. Not many of us have heard much about these earthquakes, because the regions were not densely populated and the damage was therefore not catastrophic. The shocks that occurred in New Madrid, Missouri, knocked down chimneys in Cincinnati 375 miles away.

What causes earthquakes? There have been many theories running all the way from the supernatural down to something a little more reasonable. The young fellow you see in Figure 5 is Ruaumoko. As far as I know, he is the only god of earthquakes--the earthquake god of the Maoris in New Zealand.

There are several theories about the underlying mechanism of the earthquake. They all agree in many respects and disagree in a few details. All of them are based upon movements of faults, but what causes the movements is sometimes disputed. Figure 6 is a picture of the San Andreas fault in California, probably one of the best known geographic anomalies on the face of the earth. This particular picture was taken near Indio, California, and the fault is quite clearly visible down the center of the picture. It appears that the terrain in the lower left corner of the picture matches that in the center of the picture on the opposite side of the fault. This is not true. The fault is moving the other direction, and the part that matches the terrain in the lower left has moved down out of the picture on the right. The fault moves at an average rate of about two inches a year, and if you extrapolate linearly you might infer that in 15 million years or so, San Francisco will be south of Los Angeles.

Observations in the vicinity of the San Andreas fault about the time of the 1906 San Francisco earthquake gave rise to the Reid elastic rebound theory, which is the most widely accepted theory today on the cause of earthquakes. The mechanism is illustrated in Figure 7. In part A the dark vertical line represents a fault plane down in the crust of the earth, perhaps a few miles to 30 or 40 miles below the surface. The fault plane may be vertical or inclined. Suppose that at a time when the earth's crust is relatively free of stress we scribe some imaginary grid lines perpendicular to this fault plane. With the passage of time there are movements in the crust of the earth. Vaious theories exist to explain the causes of these movements, but the fact that the movements occur is not disputed. As these movements occur, the material in the vicinity of the fault becomes strained, and the grid lines that were originally straight now become distorted, as shown in part B. As this movement continues the state is reached that the stress in the rock at the fault plane becomes sufficient to overcome the strength of the rock at some location and slippage along the fault begins as indicated in part C. This increases the stress in adjacent regions, and the slippage propogates rapidly along the fault. The result is that the strain energy that had accumulated in the rock over the past decades or perhaps centuries is suddenly released and is propogated as a series of shock waves. Finally, after the earthquake is over, the rock again is in a relatively stress-free condition, except now there would be offsets in the grid lines as shown in part D. Observations at the surface in the vicinity of the San Andreas fault lend strong support to this theory. There are a few unresolved questions when it comes to extending this to earthquakes that occur deep in the earth without displaying any recognizable distortion at the surface.

The series of shock waves produced by the energy release is felt at some distant point on the earth's surface as a chaotic and sometimes violent motion in all directions. Figure 8 is an accelerogram for the Puget Sound earthquake of April, 1965, recorded in Seattle by the U. S. Coast and Geodetic Survey. Three components of ground acceleration are recorded, the vertical component and two perpendicular horizontal components, all recorded on a common time base with the time track shown at the top and bottom of the chart.

What does this do to the buildings? The ground moves underneath the buildings both horizontally and vertically. For the most part we are inclined to believe that the vertical ground motion is not too critical, for we are already designing buildings for the vertical acceleration of gravity. The lateral motion is quite a different matter. The ground shakes underneath a building, and the inertia of the building makes it tend to remain in position. If the building has sufficient strength and resilience it will move along with the ground and vibrate; if it does not there will be damage, and possibly collapse.

Now let us look at some of the effects of the earthquakes that occurred in Skopje in 1963, and in Alaska in 1964. These were two quite different earthquakes.

The Skopje earthquake was a small one. Earthquakes of its magnitude occur on the average every five or six days somewhere. This one occurred almost directly beneath the city, and the effects were catastrophic.

Skopje is the capitol of Macedonia, the southernmost of Yugoslavia's six republics. Yugoslavs sometimes speak of it as the Pearl of Macedonia. Figure 9 is an aerial view of the city before the earthquake. The city is divided by the Vardar River into two quite different sectors. On the north bank of the Vardar is the gypsy quarter, with numerous old one-story adobe buildings plus a few modern buildings along the river. On the south bank of the Vardar is the new city, which consists of large modern-type office and apartment buildings with a lot of little one and two-story houses and shops scattered in among them. Notice the great number of small buildings among the large ones in Figure 9.

The earthquake destroyed or rendered unusable some three-quarters of the living accomodations in the city. Perhaps the most photographed building was the Army Club on Tito Square shown in Figure 10. There were many unreinforced masonry buildings, including the one shown in Figure 11, which didn't fare very well.

One of the more modern structures was a shell auditorium at the fairgrounds, shown in Figure 12. It consisted of an elliptic paraboloid shell roof supported on four corner columns. Four horizontal pre-stressed beams served as ties for the edge ribs of the shell. There was a mezzanine floor at half-height. The shell roof was supported on four columns, but the mezzanine floor was supported on twenty-eight. Figure 13 shows more clearly how the structure was built. The ground floor plan shows the twenty-eight columns around the periphery. The mezzanine floor was supported on a grid of beams supported on these twenty-eight columns. The four corner columns extended from the foundation up through the mezzanine to the base of the shell. There was practically nothing above the mezzanine to obstruct view, and there was also practically nothing above the mezzanine floor to resist lateral force. Recognize that the effect of the earthquake is to pull the foundation out from under the structure. All of the inertia of the concrete shell roof had only the shear strength of the four corner columns to resist it. Below the mezzanine floor there was plenty of strength but little inertia force. Only the inertia force of the mezzanine floor and the shear in the four corner columns was transmitted to the lower structure. You can imagine just about what happened. The structure broke off above the mezzanine floor and after the earthquake it looked like Figure 14. I'll leave it to your imagination what might have happened if this earthquake had occurred in the afternoon or evening with a full auditorium instead of occurring at five o'clock in the morning.

At the steel works which is under construction on the outskirts of Skopje, there is a fabricating shop which is a precast, prestressed concrete structure. I don't think Bethlehem Steel builds their shops out of prestressed concrete, but the steel works in Skopje does. The important features of the construction can be seen in Figure 15. There is a line of prestressed columns down the middle supporting a continuous prestressed concrete girder which in turn supports a precast concrete roof. Notice that the center part of the structure is pretty well isolated from the side walls by the two skylights. The inertia force that is generated in the center part of the structure has no way to be transferred to the side walls. The only way it can be transmitted to the foundation is through the columns. As a result the center part of the structure rocked back and forth on the columns. Figure 16 shows the top of a column, and Figure 17 the base of the same column. I'm not sure what was done in the way of repairing this column, but I doubt that the prestress was restored, for that would be rather a difficult chore.

The concept of the foundation being pulled out from under the structure was evident all over town. Figure 18 shows a six-story apartment building which was relatively rigid above the ground floor because of the interior partitions, but the ground floor was mostly open because this area was to serve for shops and other uses that required unobstructed space. There were few partitions to add strength in the ground floor. The building had a concrete frame that was designed for vertical loads only. A note on the plans says that the lateral forces are resisted by the stairwells and elevator shafts. In the earthquake the ground shifted laterally under the building with the result shown in Figure 18. The left edge of the tile veneer in the ground story was originally lined up with the building above it, and it is now offset by about half a foot. The column in the middle is sheared off at the top of the ground story, and the door has become a parallelogram.

There was a housing district on the west side of town, the Karpos housing development shown in Figure 19, in which there were a number of five-story buildings of two different designs and also three fourteen-story apartment buildings. Let's take a look at one of the fourteen-story buildings, and also one of the five-story buildings.

Upon reviewing plans of most of the major buildings in Skopje, I found the fourteen-story buildings of the Karpos development to be the only ones in which earthquake forces were mentioned in the design notes, even though earthquake forces have been specified in the Yugoslav federal building code since 1948. It doesn't do any good to put provisions in the code if the code is not enforced. At any rate, the fourteen-story structure had a reinforced concrete frame which was designed to carry vertical loads only. The center shaft surrounding the lightwell, the elevator core and the stairwells was supposed to act as a unit to resist lateral forces. The performance was not quite satisfactory, and in one corner of the building the corner column in the ground story failed, as shown in Figure 20. This was the worst damage found in the three fourteen-story towers.

Figure 21 shows some of the five-story buildings. This type of building is called the "cut tower" because it has the same floor plan as the fourteen-story tower; however, it is of a different type of construction. The cut tower employs bearing wall construction--reinforced concrete slabs on unreinforced brick walls. The one in the right foreground of Figure 21 fell into a heap, and for the one in the background you can probably visualize what the interior damage must be when it has so much damage visible on the outside.

Early in 1965 I had the opportunity of being in Yugoslavia again, and it is interesting to see the changes that took place in a year and eight months after the earthquake. Figure 22 is an aerial view of the downtown section. In Figure 9 there were many little one and two-story buildings interspersed among the tall buildings. Now the space between the tall buildings is mostly open. A terrific amount of land has been cleared. Most of the major buildings have been repaired, and scaffolding can be seen on some that are still undergoing repair. There were even a few partially collapsed buildings that had not yet been removed. The Karpos development is shown in Figure 23, in which scaffolding is in place on some of the buildings. Figure 24 shows the fourteen-story tower on which the corner column failure occurred. The damage is still evident. The cut towers are being repaired, and Figure 25 shows what one might call building a concrete frame the hard way. The brickwork is channeled out and the reinforced concrete columns are cast and anchored into the floors. The slabs have been in place for several years, and now the columns are being cast. I don't recommend this procedure for economy.

Now let us turn our attention to a different earthquake, the one that occurred in Alaska in March, 1964. This earthquake is different in many respects. In Skopje the earthquake was a small magnitude earthquake that occurred almost in the heart of town. In Alaska the earthquake was very large, the greatest magnitude earthquake ever recorded on the North American continent.

The main shock occurred near Valdez on the north shore of Prince William Sound, about 80 miles east of Anchorage. Most of the damage occurred in Anchorage for two reasons--one was the foundation condition, and the other was that in Anchorage there was more to be damaged. You might have gained the idea from newspaper accounts immediately after the earthquake that the whole city was flattened. Figure 26 is an aerial view that was taken after the earthquake, and we see that while there was a lot of damage, there were also many buildings that escaped damage. There was a great difference between the small residential buildings in Yugoslavia and the small residential buildings in Alaska. In Yugoslavia, most of them were unreinforced masonry with heavy tile roofs. In Alaska, most of them were timber. A wood frame structure with wood sheathing is not very heavy, but it possesses a great amount of reserve strength. This makes a great difference in the way the structure behaves.

You have all seen pictures of downtown Anchorage, and perhaps the Denali Theatre, shown in Figure 27, was one of the more dramatic scenes. The theatre sank about ten feet vertically and left the marquee just above sidewalk level.

Two interesting buildings were the Mount McKinley Building and the 1200L Building. The effects of the earthquake were remarkably similar on both of them. The buildings are of reinforced concrete frames in which the vertical elements are interior columns, the concrete cores surrounding the elevator shaft and stairwell, and vertical piers in the outside wall. The ground shifted mainly in the north-south direction, which was the long direction of the building, although there was strong transverse motion as well. Figure 28 shows the front elevation of the McKinley Building, and a substantial amount of damage can be seen in the concrete piers near the center of the building and in the spandrels between the piers. Figure 29 is a closer view of some of the spandrel beam failure in the east face of the building. One can visualize these piers oscillating back and forth as cantilevers. The spandrels between them were thinner than the piers, so of course they suffered the distress. In the north end of the building one of the concrete piers was fractured and there's actually an air gap between the top and bottom concrete in Figure 30. One can stand inside and look out through the pier. The 1200L Building was located about a mile away, and the damage was remarkably similar, even to the extent of having a fracture in the corresponding vertical pier.

A block away from the McKinley Building was the Cordova Building, the plan of which is shown in Figure 31. This is a steel frame building with lightweight panels covering the exterior walls. The floors are concrete over bar joists. There are some concrete elements that enter into the lateral resistance of the building. There is a concrete wall at the north end, a concrete elevator shaft and stairwell near the middle of the building, and on the southeast corner there is a concrete wall surrounding a stairwell. It was at the southeast corner that the more spectacular damage occurred. Figure 32 is a view of the building showing the southeast corner, the nearest corner in the photograph. Figure 33 is a close-up view of the column in the southeast corner, and we see that both flanges are completely severed and have peeled away from the web, the web is bent into an S shape, and the column has shortened by about an inch and a half.

Another spectacular building was the Alaska Sales and Service Building, shown in Figure 34, which was built of precast prestressed hammerhead columns, supporting precast prestressed roof Tee beams. The column heads were connected end to end in several lines, and the Tee beams spanned between them to form the roof structure. It should be noted that the bond beam around the periphery of the roof had not yet been cast. It has been said that one of the troubles in Anchorage was that there was not enough anchorage. Some of the connections in this structure are sadly deficient, as seen in Figure 35. One could find examples of poor connections in most of the structures that encountered major damage in Anchorage.

Another spectacular failure was a cement bin owned by the Kaiser Permanente Cement Company, shown in Figure 36. This bin was perhaps two-thirds full at the time of the earthquake, The bin had a cylindrical top portion which about 30 feet in diameter and 30 feet high, with a conical bottom, and the ring girder at the base of the cylindrical portion was supported on twelve columns, one of which can be seen in Figure 36. These were all framed into the ring girder at the base of the cylinder, and were supported on reinforced concrete pedestals. Each column bore on a 1-1/4 inch base plate that was set on a one inch leveling plate, and each base plate was tied down by four 2-1/4 inch anchor bolts with double nuts. The base plates stayed right in place. The detail is shown in Figure 37.

Anchorage today is quite a different story than Skopje today. I could embark on a discussion of economic systems at this point, but I won't. The following pictures were taken by

John Gilligan, who visited Anchorage in November, 1964. The Fourth Avenue slide area is the big white expanse in Figure 38. It has all been cleared and graded now. In the background down at the end of the slide area is the McKinley Building. At the time this picture was taken the building very much in the same condition it was in right after the earthquake. So was the 1200L Building. There were some legal questions that were rather complex, for the two apartment buildings were mortgaged and the mortgages were insured by FHA. While legal questions were being settled, not much could be done with the buildings.

The Cordova Building, shown in Figure 39, is back in operation. It was reoccupied perhaps as early as any of the buildings that suffered major damage in Anchorage. Alaska Sales, shown in Figure 40, has been completed. Many of the prestressed, precast concrete units were salvaged. The connections have, I understand, been made substantially stronger than they were originally. And the Permanente Cement Bin, Figure 41, has been replaced by a new one, and this time I hope they remembered to weld the columns to the base plates.

Fig. 1 Saada Hotel, Agadir, Morocco (AISI)

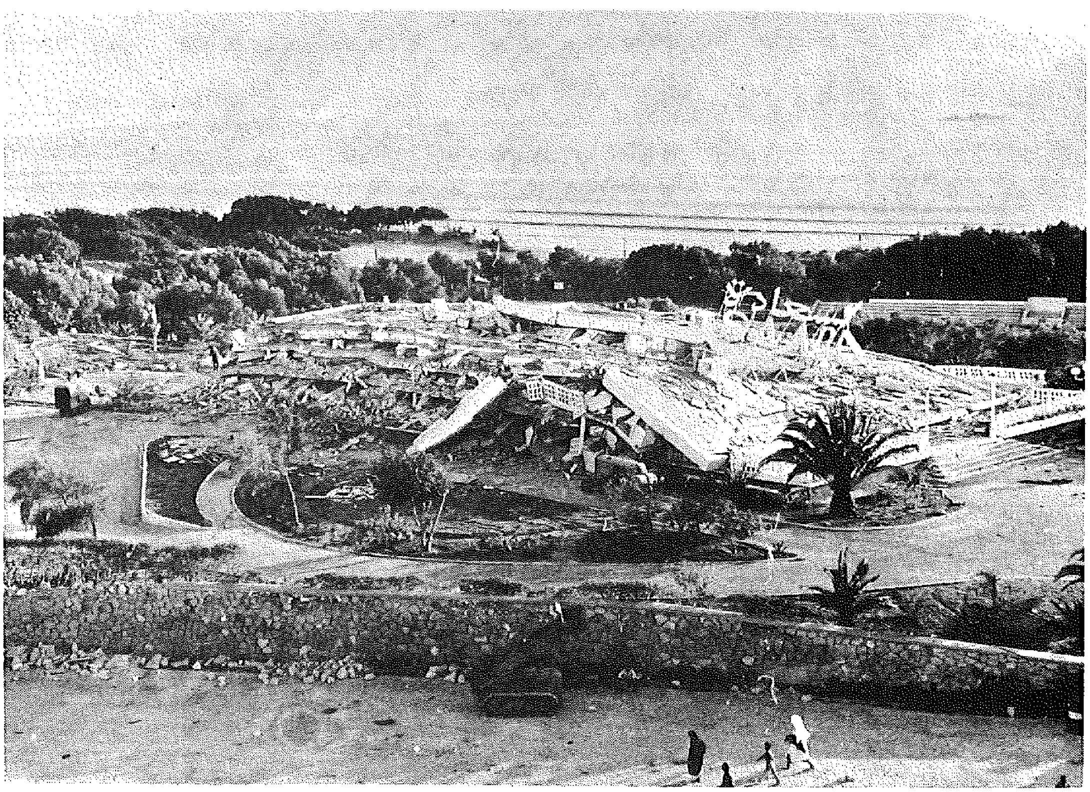


Fig. 2 Saada Hotel After Earthquake (AISI)

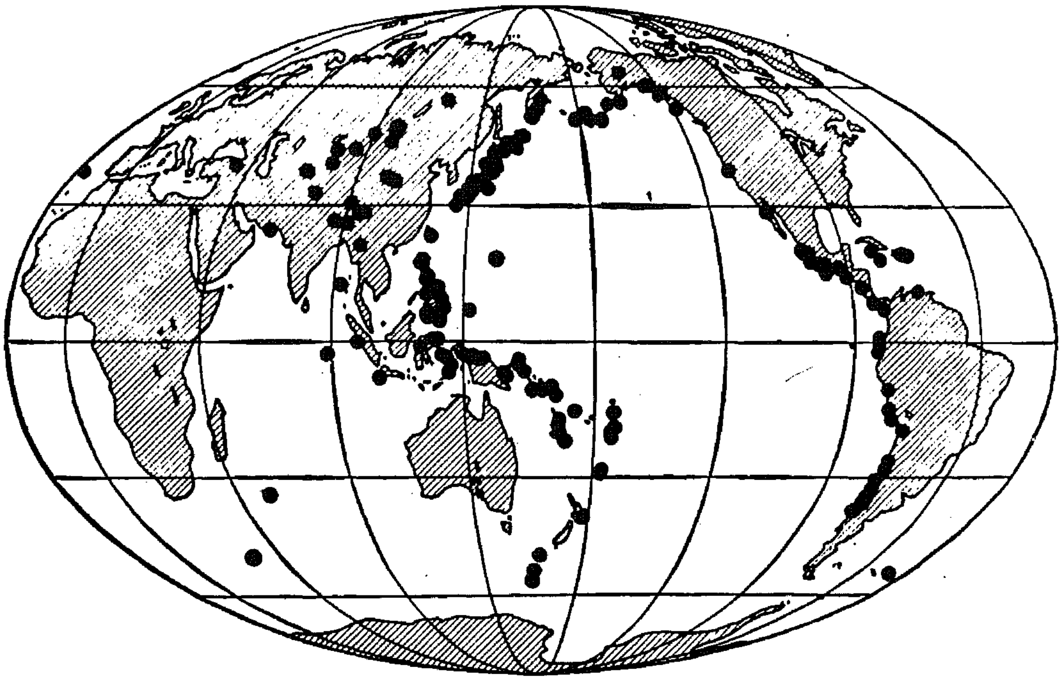


Fig. 3 Major Earthquakes of the World (After Gutenberg and Richter)

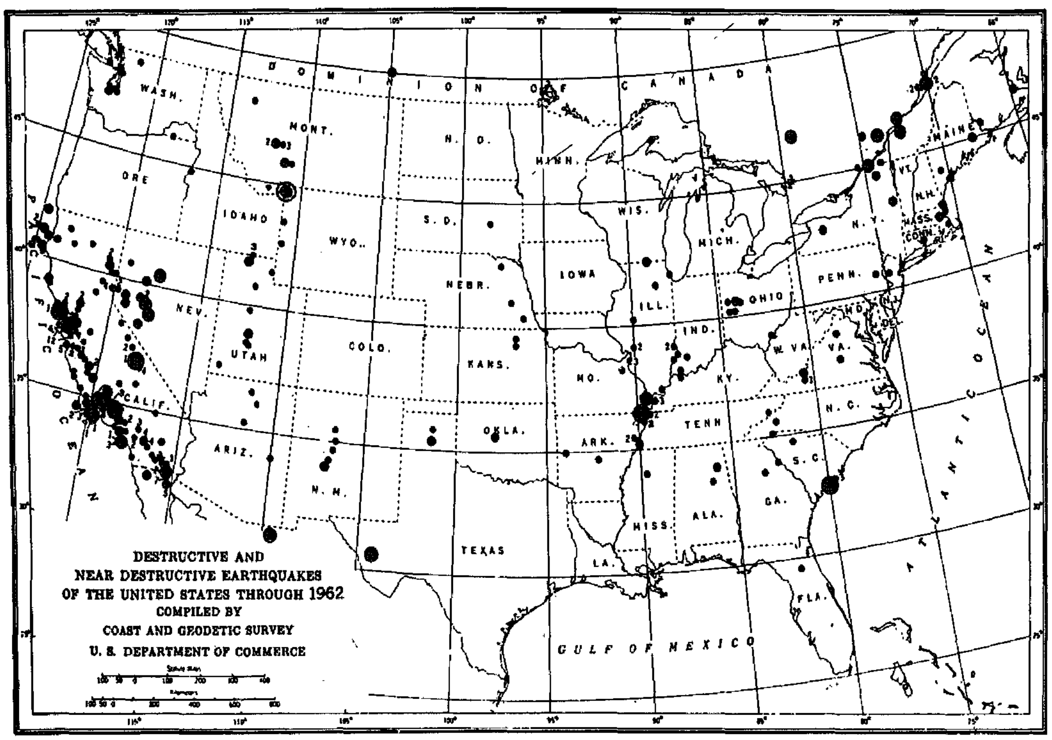


Fig. 4 United States Earthquakes (From U. S. Coast and Geodetic Survey)

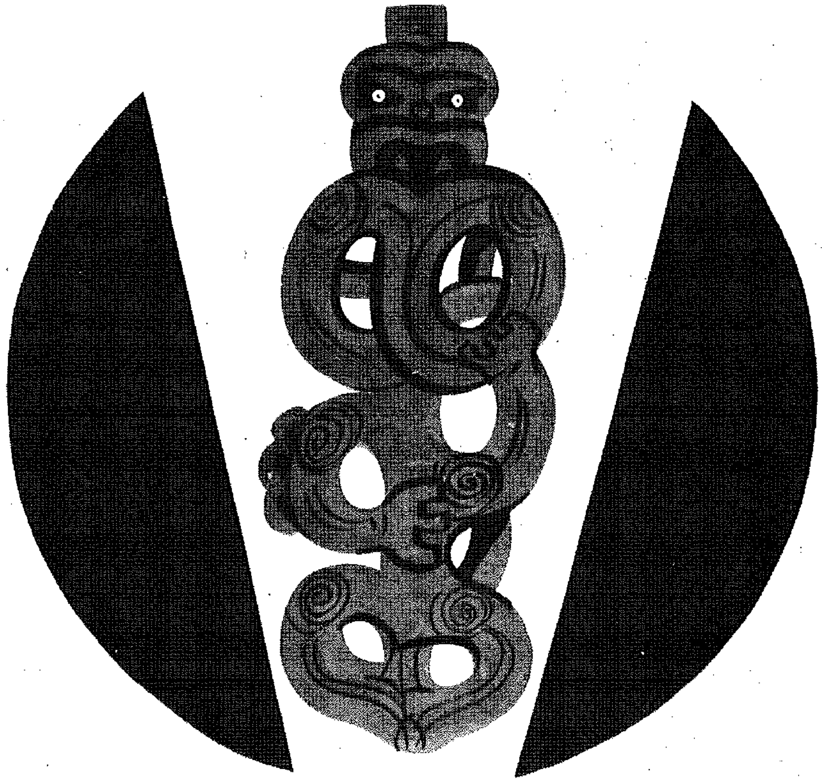


Fig. 5 Ruaumoko, Maori God of Earthquakes

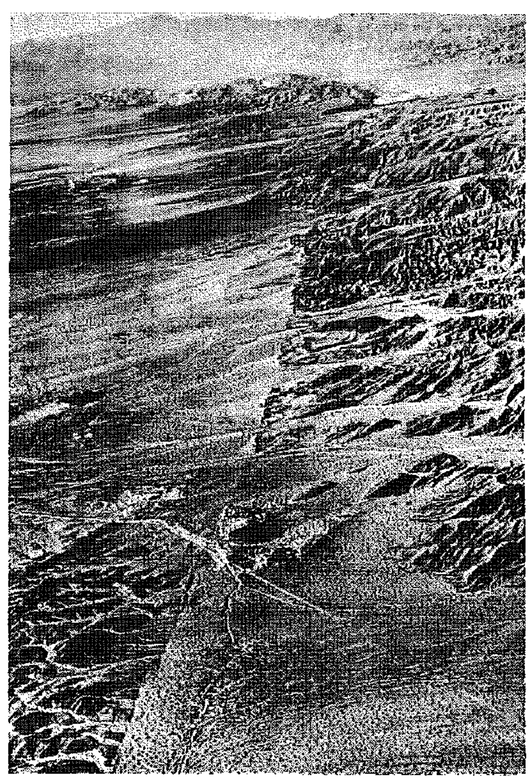


Fig. 6 San Andreas Fault

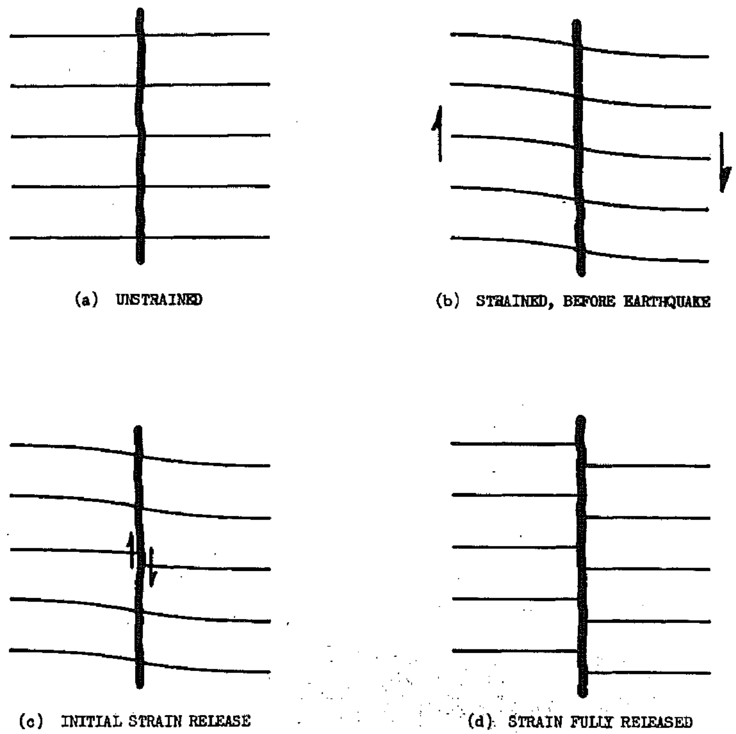


Fig. 7 Reid Elastic Rebound Theory

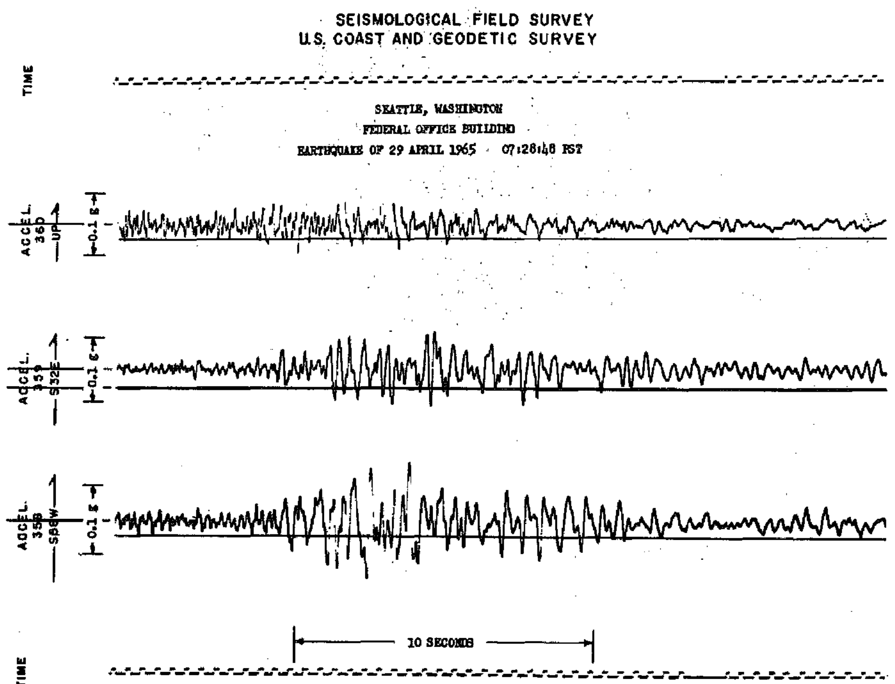


Fig. 8 Accelerogram Recorded in Seattle, April 29, 1965

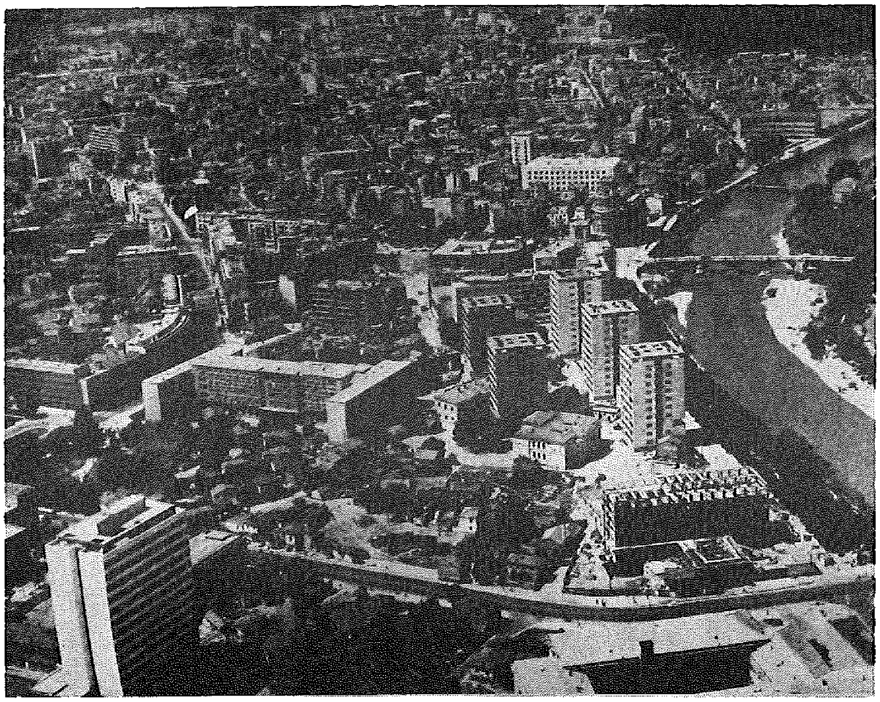


Fig. 9 Aerial View of Skopje, Yugoslavia, Before Earthquake

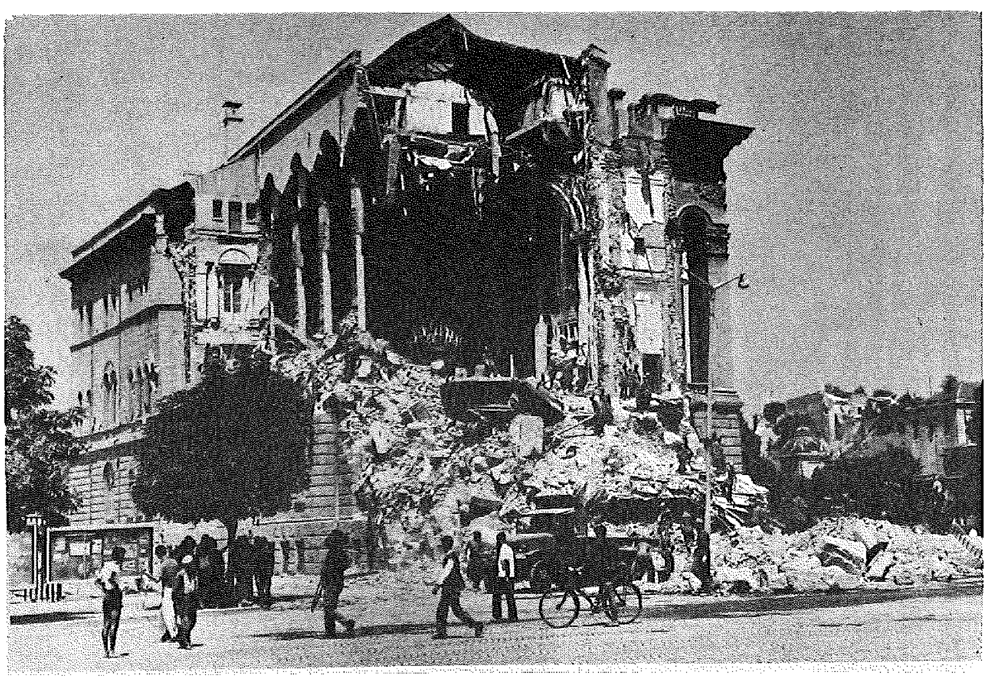


Fig. 10 Army Club (AISI)

Fig. 11 No. 38, Djuro Salaj (AISI)

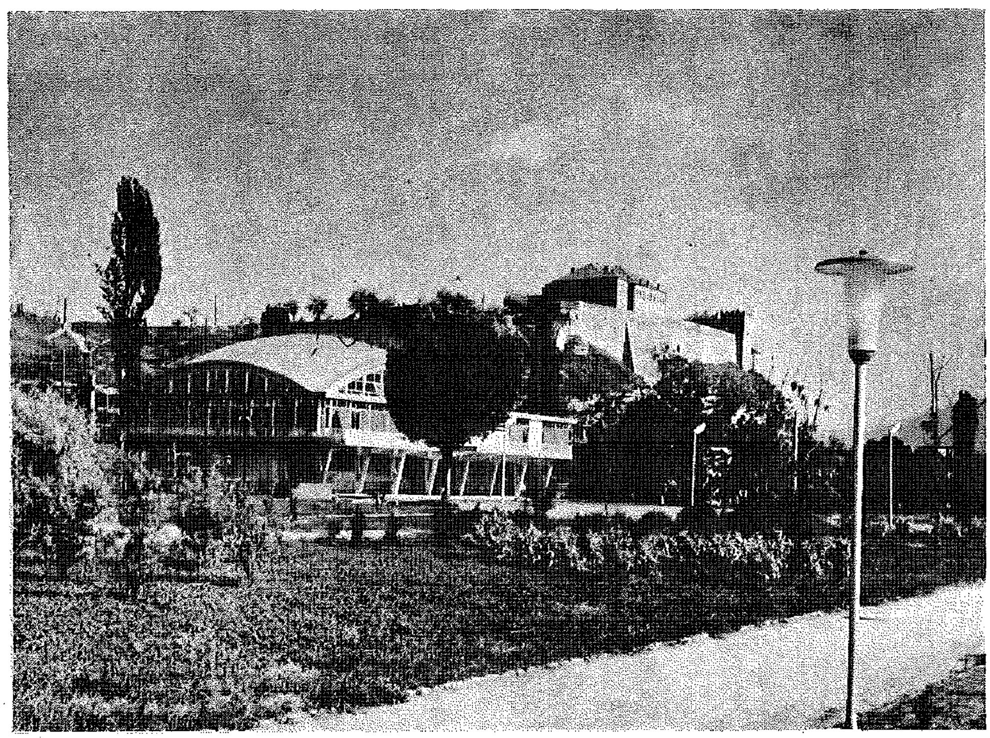


Fig. 12 Fairgrounds Auditorium, Before Earthquake

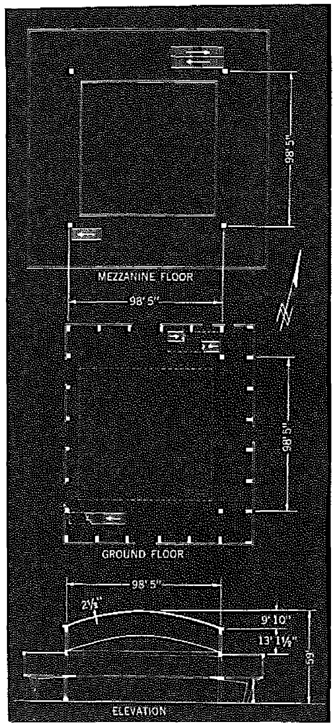


Fig. 13 Plan of Auditorium (AISI)

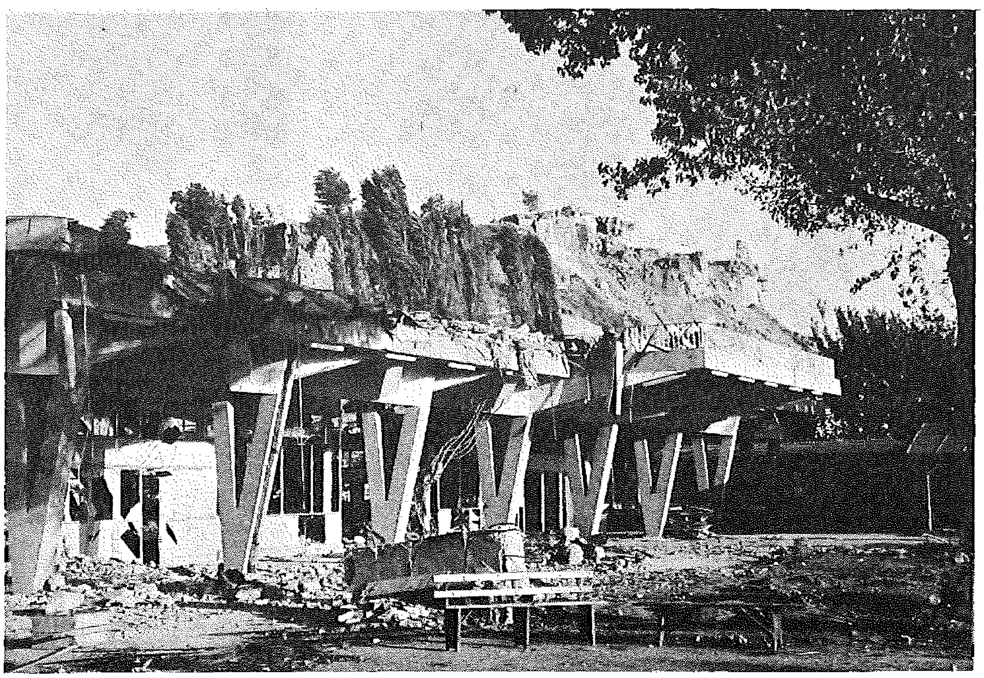


Fig. 14 Fairgrounds Auditorium (AISI)

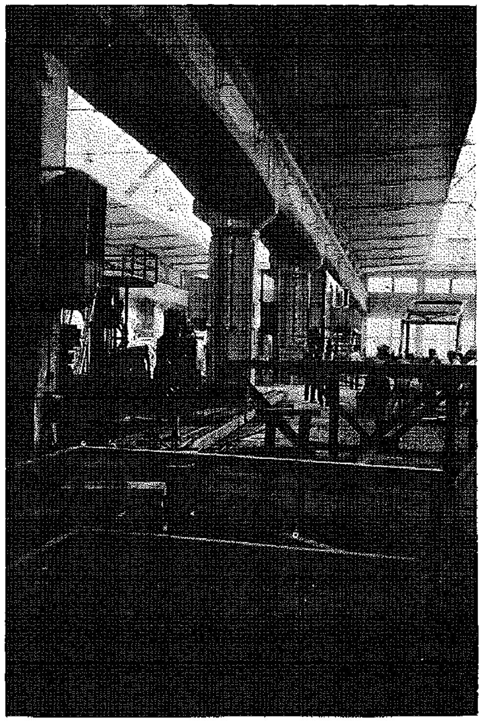


Fig. 15 Fabricating Shop at Steel Works (AISI)

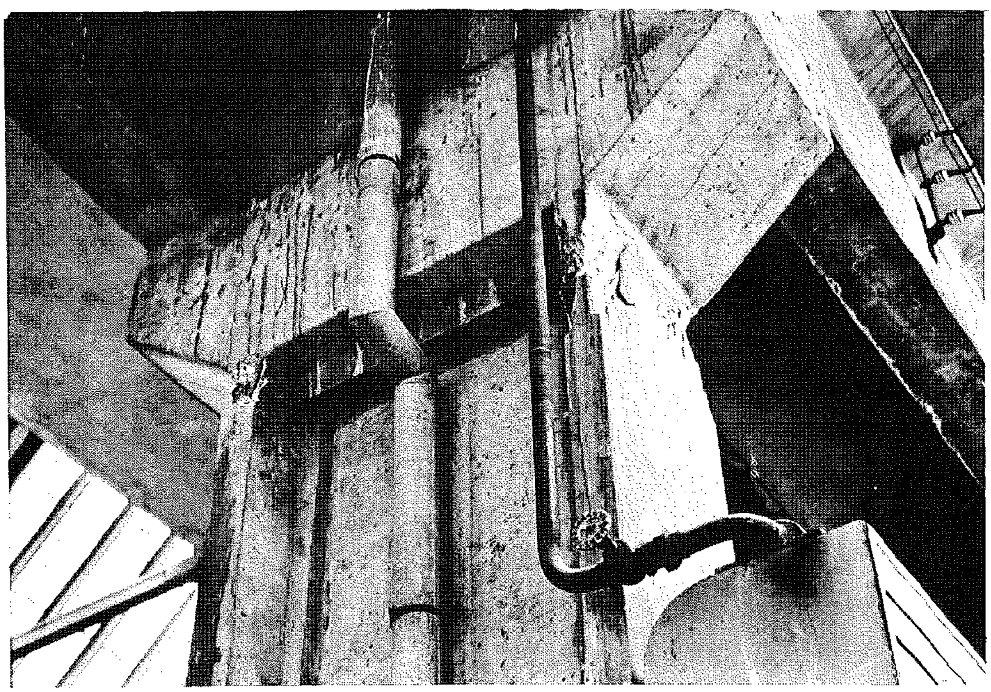


Fig. 16 Column Head (AISI)

Fig. 17 Column Base (AISI)

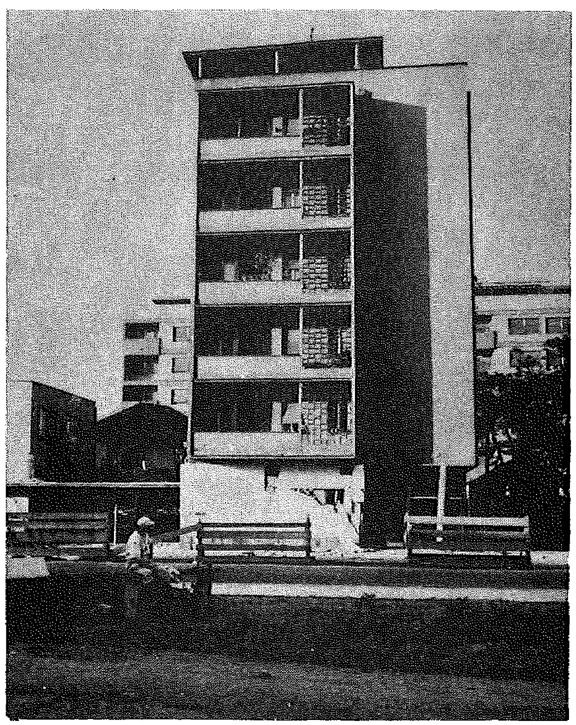


Fig. 18 Apartment Building (AISI)

Fig. 19 Karpos Housing Development, Before Earthquake (AISI)

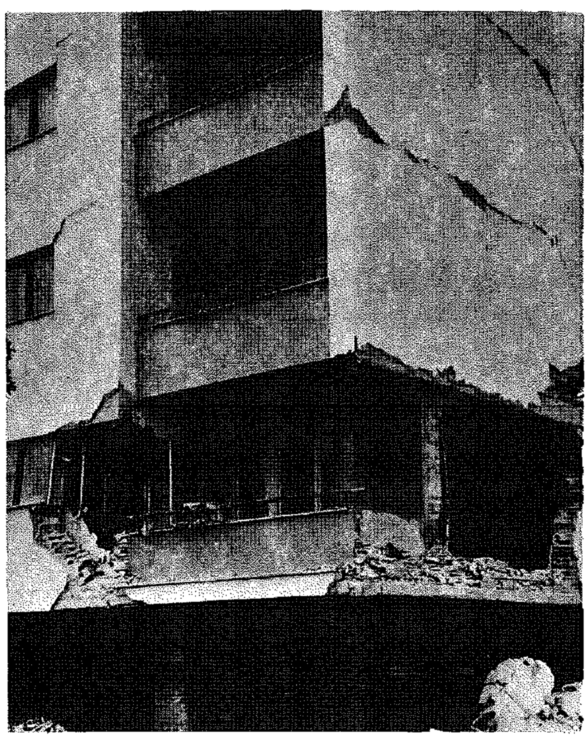


Fig. 20 Karpos Tower (AISI)

Fig. 21 Karpos "Cut Tower" (AISI)

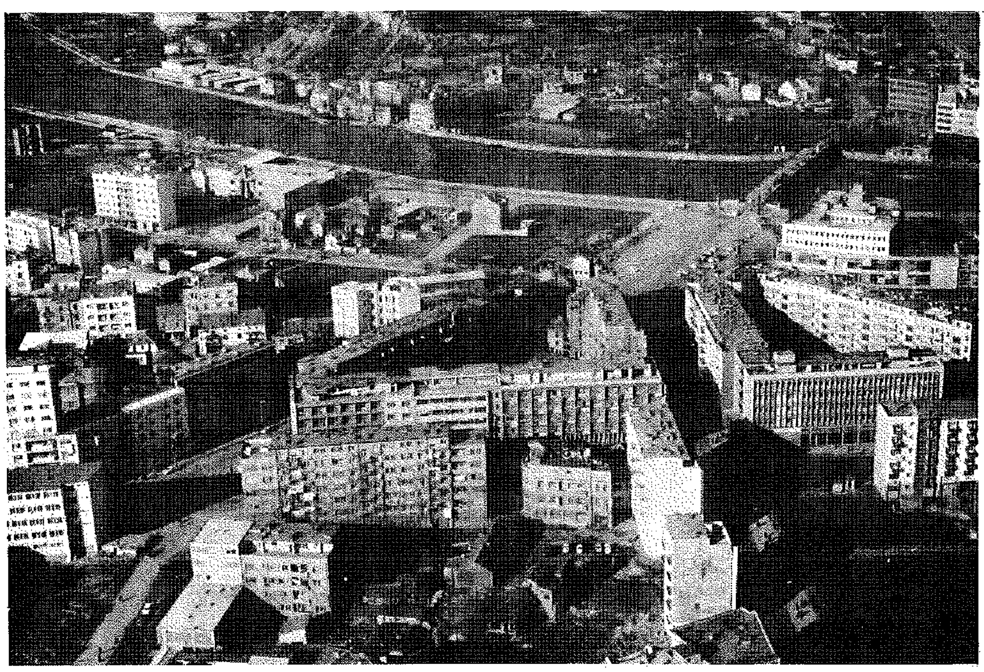


Fig. 22 Aerial View of Skopje, March, 1965

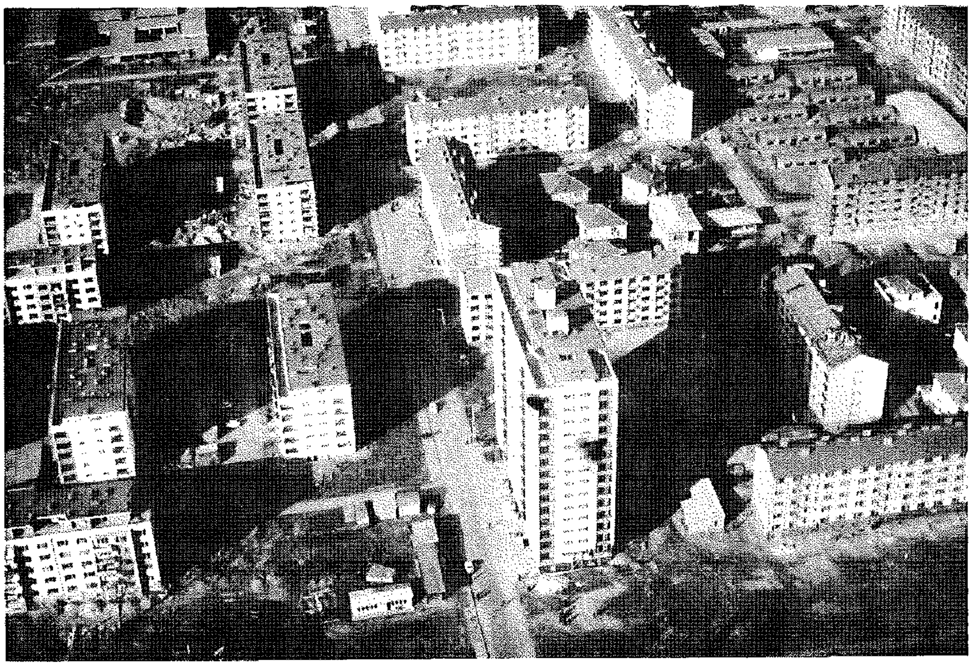


Fig. 23 Karpos Area, March, 1965

Fig. 24 Karpos Tower, March, 1965

Fig. 25 Karpos Cut Tower Repair, March, 1965

Fig. 26 Aerial View of Anchorage, After Earthquake (AISI)

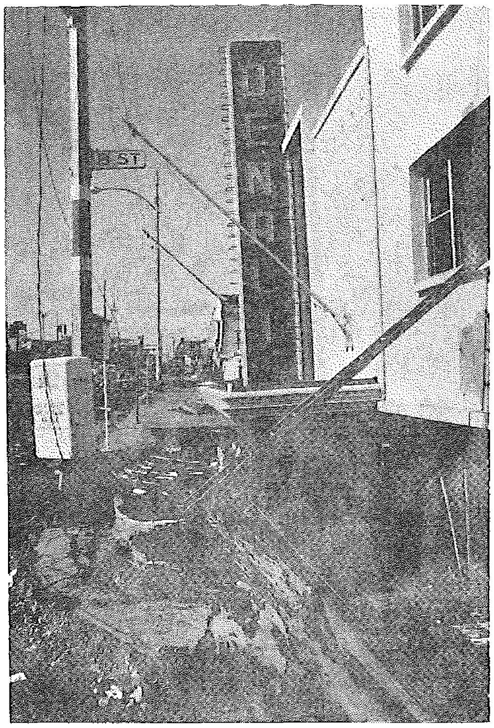


Fig. 27 Denali Theater

Fig. 28 Mount McKinley Building (AISI)

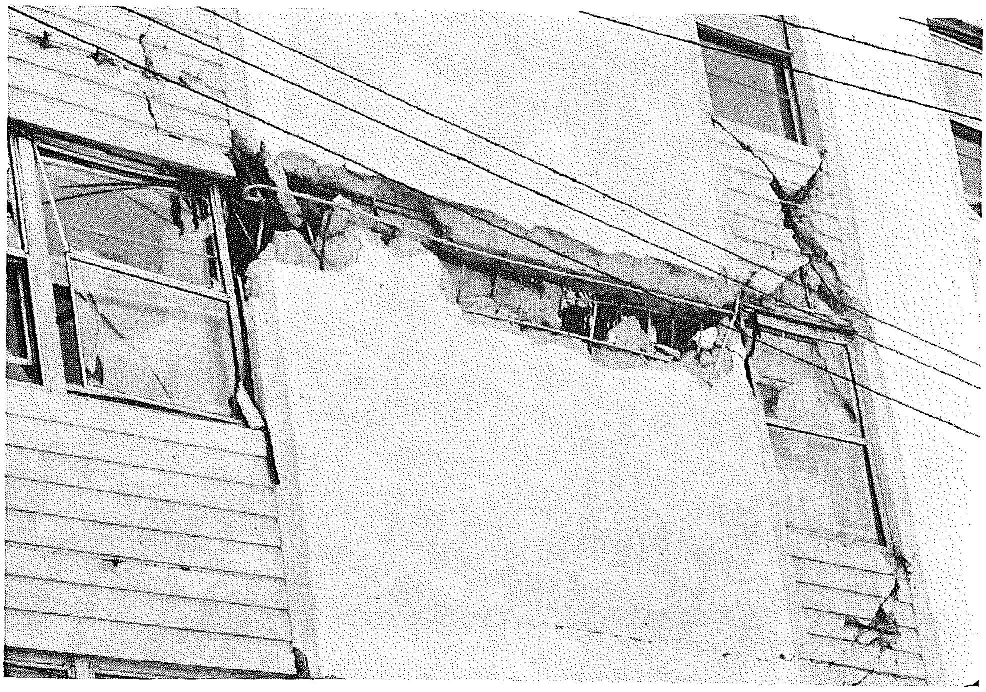


Fig. 29 Spandrel Failure (AISI)

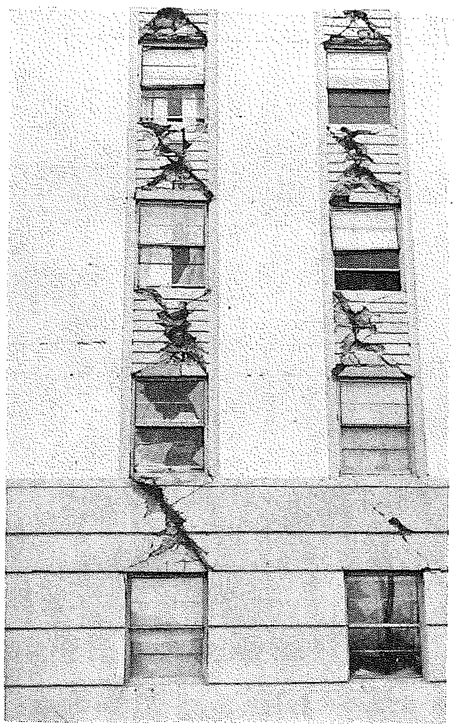


Fig. 30 Pier Failure (AISI)

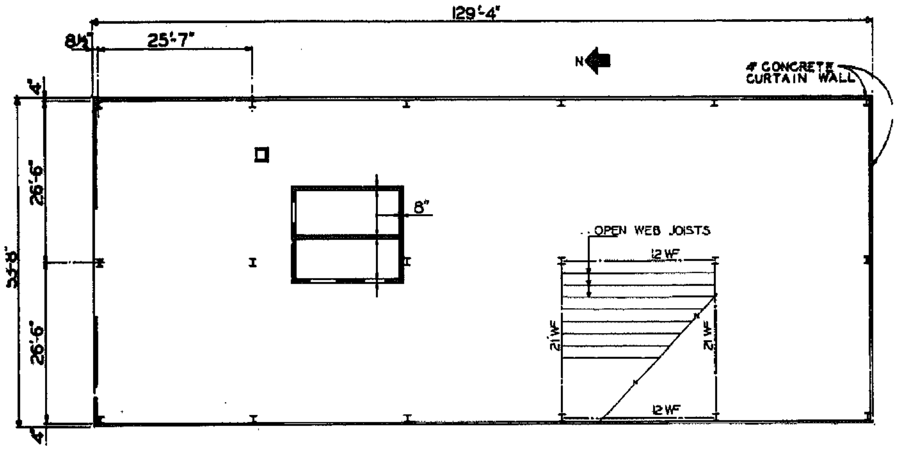


Fig. 31 Plan of Cordova Building

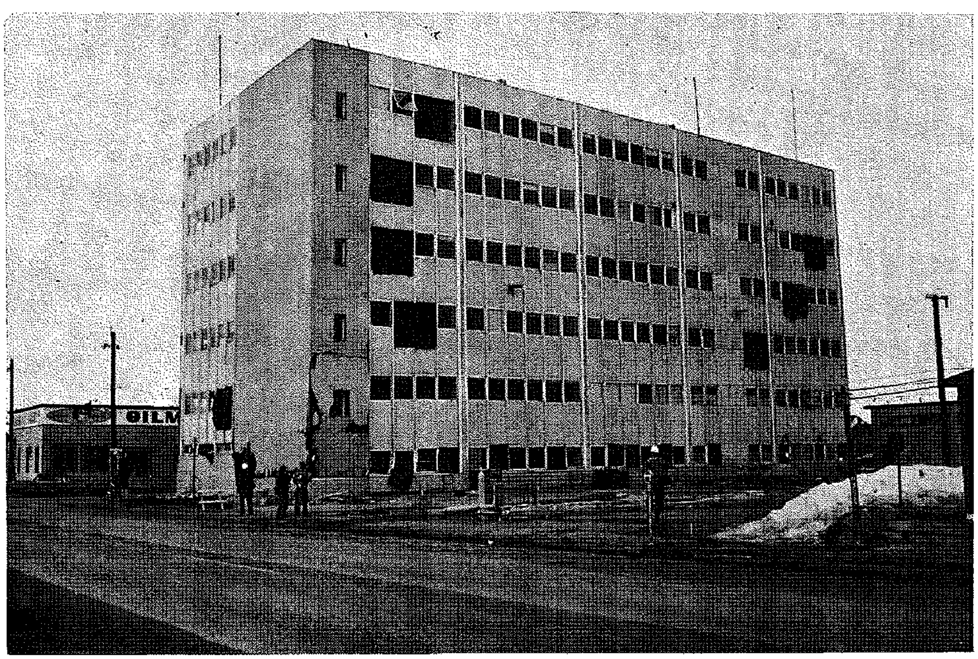


Fig. 32 Cordova Building (AISI)

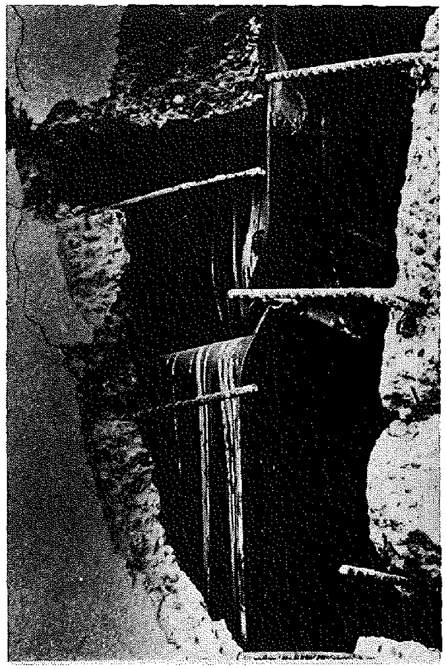


Fig. 33 Corner Column (AISI)

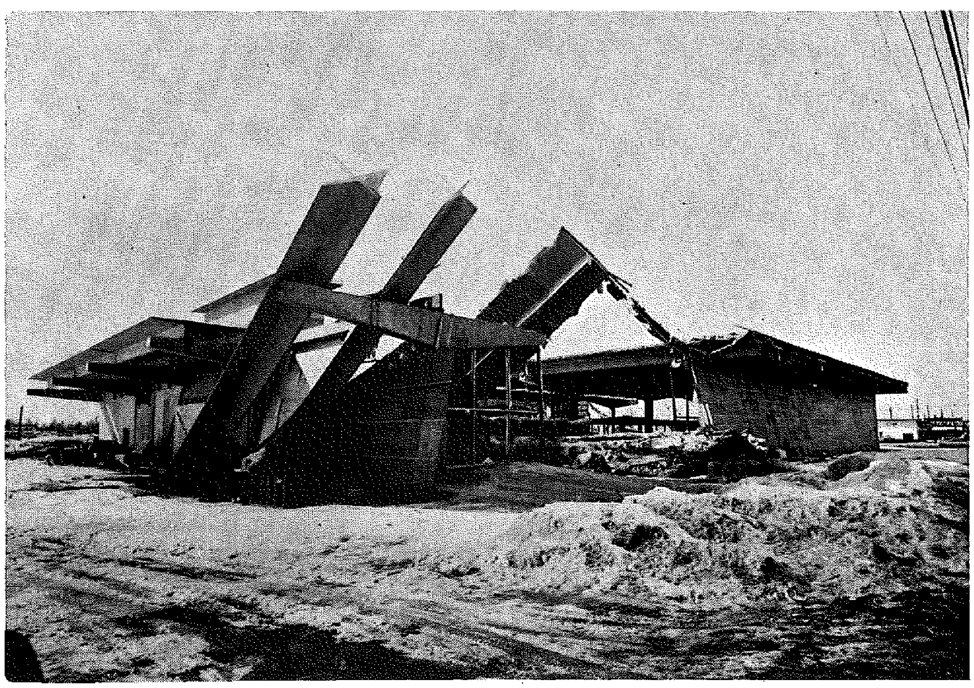


Fig. 34 Alaska Sales and Service Building (AISI)

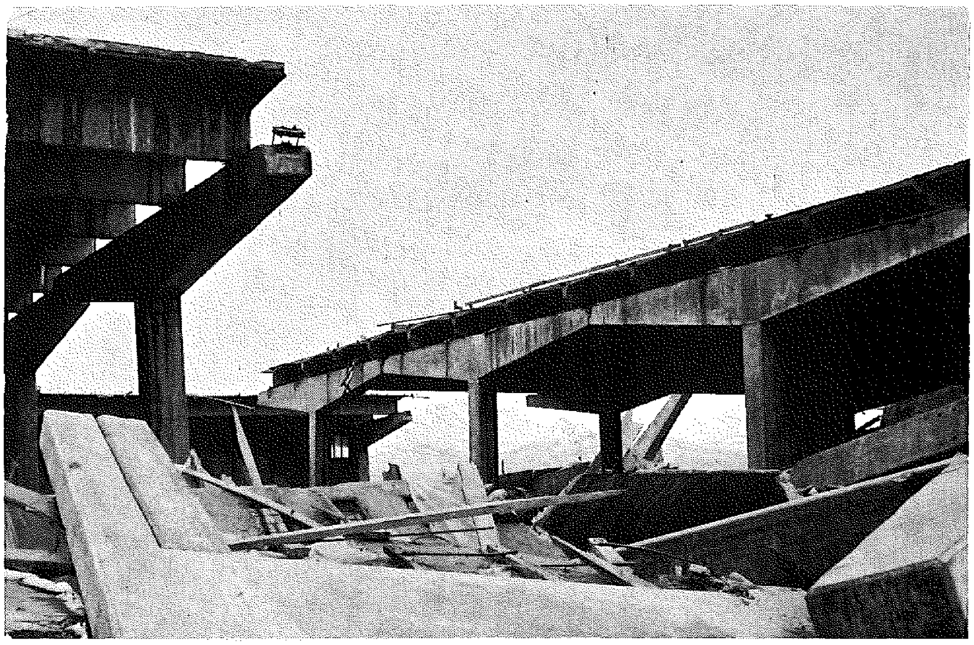


Fig. 35 Alaska Sales and Service Building (AISI)

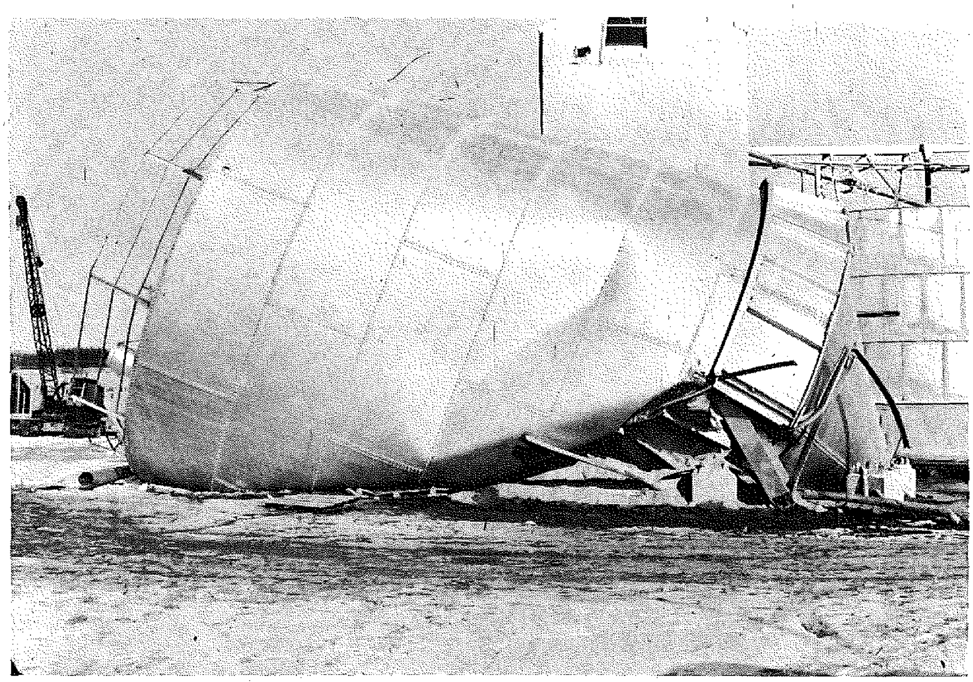


Fig. 36 Kaiser Permanente Cement Bin (AISI)

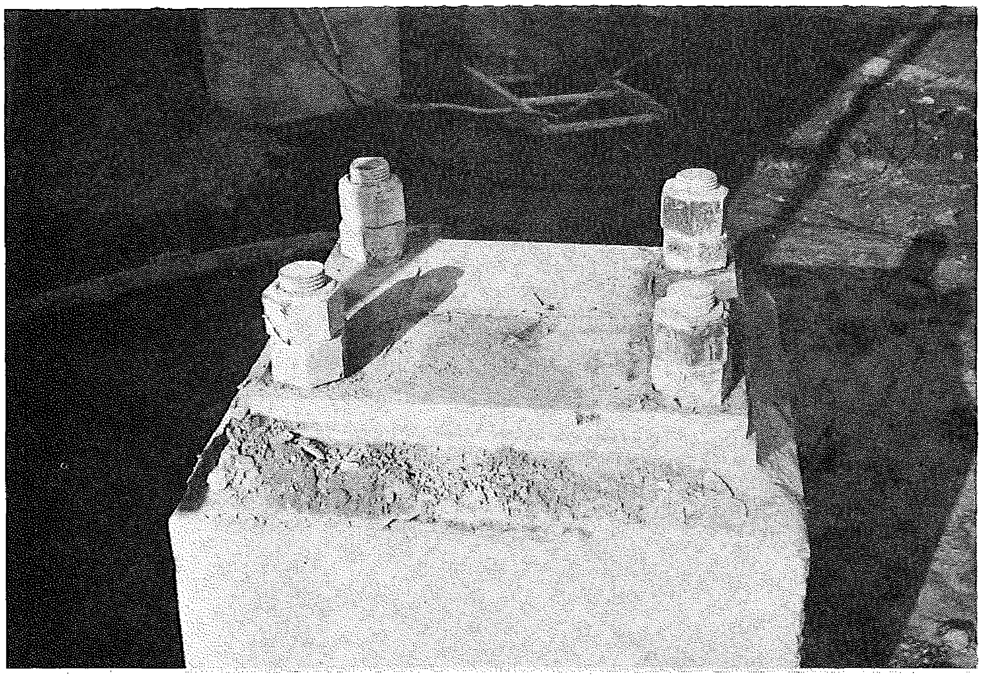


Fig. 37 Column Base Detail (AISI)

Fig. 38 Fourth Avenue Slide Area, November, 1964

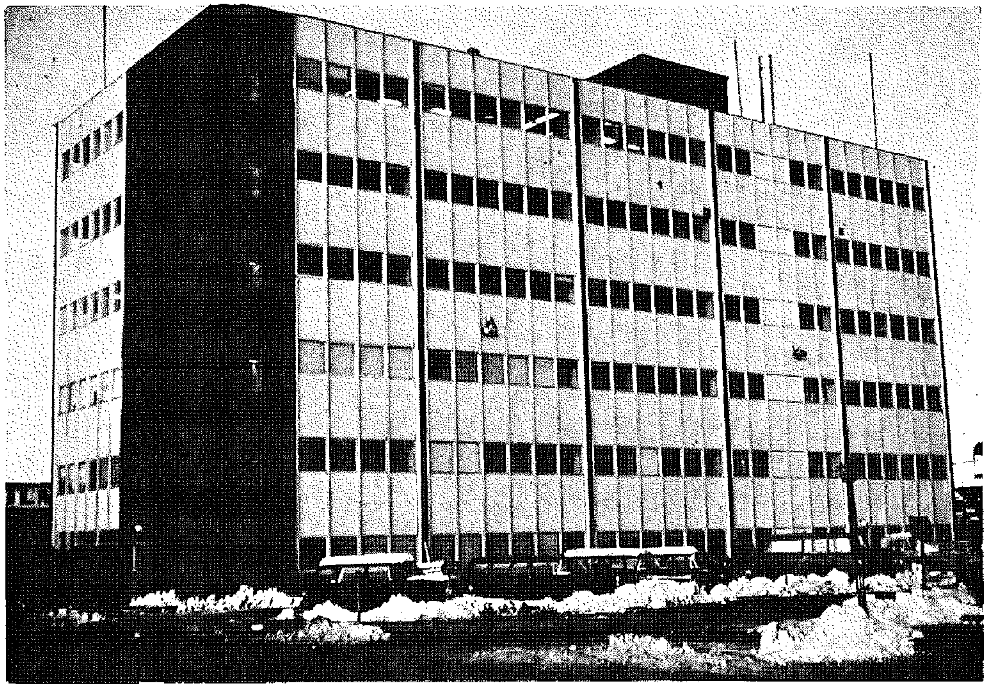


Fig. 39 Cordova Building, November, 1964

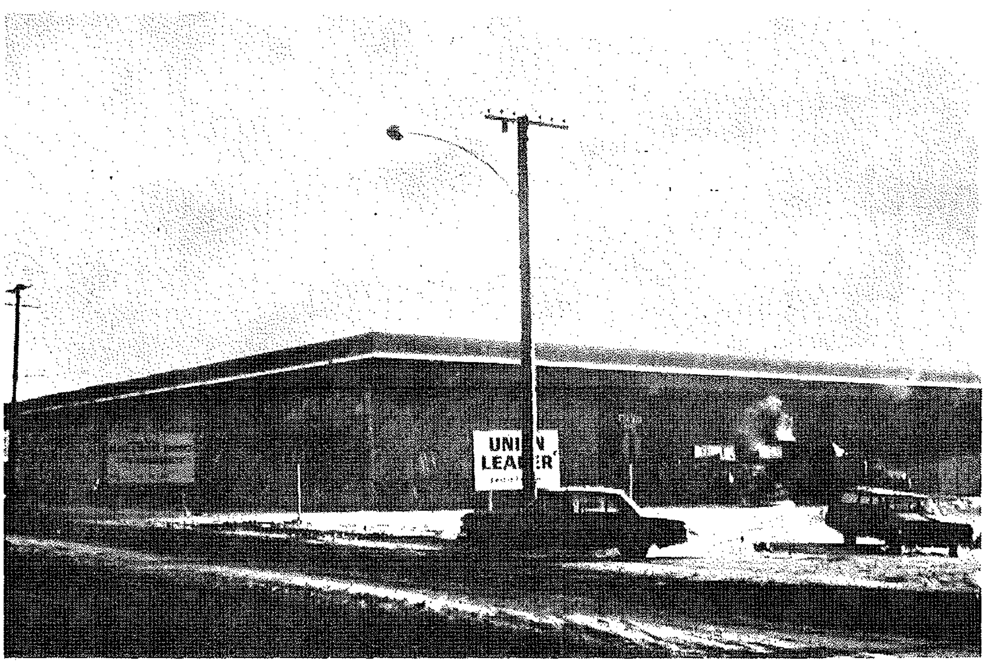


Fig. 40 Alaska Sales and Service Building, November, 1964

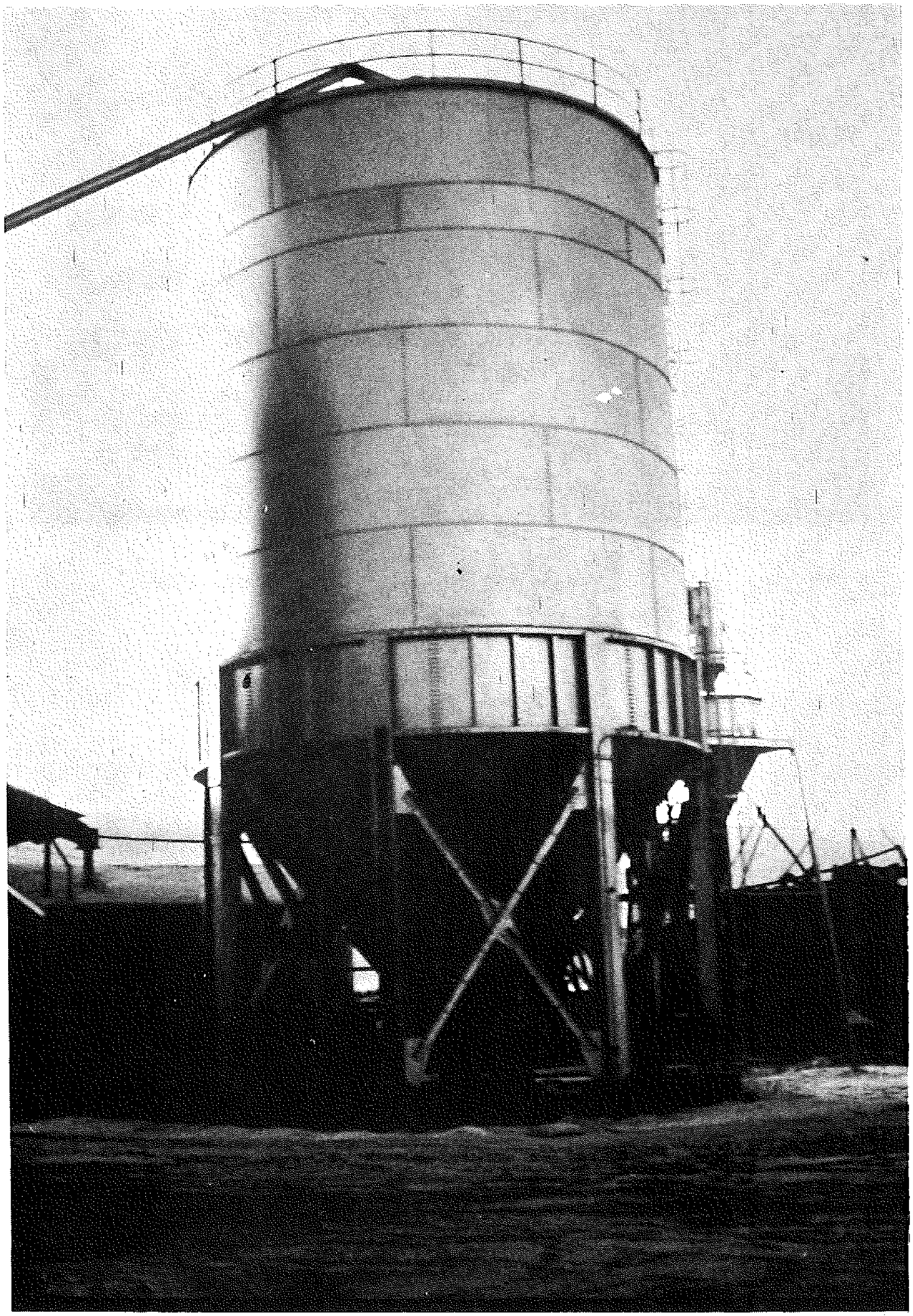


Fig. 41 Kaiser Permanente Cement Bin, November, 1964

BEHAVIOR OF STEEL BEAM-TO-COLUMN CONNECTIONS UNDER REPEATED AND REVERSED LOADING

by E. P. Popov[1]

SYNPOSIS

During seismic disturbances, building frames respond to ground motion in a vibratory manner. This causes repeated and reversed loading to act on a building in a horizontal direction. Large cyclically applied bending moments are thus induced in beam-to-column connections. Experimental evidence on the behavior of connections under repeated and reversed loadings simulating an earthquake is essential for safe design of structures. This paper describes one such experiment. Further work along the lines described is in progress.

[1] Prof. of Civil Engineering, University of California, Berkeley, California

INTRODUCTION

Prior to and for nearly two decades after the 1906 San Francisco earthquake, buildings in seismic regions were designed principally on the basis of horizontal wind loads applied statically in addition to conventional gravity loads. After the 1925 Santa Barbara earthquake, the 1927 Uniform Building Code[2] revised the above approach and employed the concept of lateral earthquake forces proportional to masses.

Strong laws governing the design for lateral forces of public schools (Field Act) and of all buildings for human occupency (Riley Act) were passed in California almost immediately after the March 10, 1933 Long Beach earthquake. This event also gave strong motivation for further studies and in 1943 a new Los Angeles Building Ordinance was adopted. This ordinance not only recognized that lateral force is proportional to the mass of the building, but also introduced the influence of flexibility of the structure into the earthquake design coefficients. In 1946 the Uniform Building Code, and in 1953 the State Division of Architecture, incorporated the same

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2 "Recommended Lateral Force Requirements and Commentary" by Seismology Committee, Structural Engineers Association of California, 1960.

features into the design regulations.

In San Francisco in 1948, a joint committee was formed of the San Francisco section of ASCE and the Structural Engineers Association of Northern California. Their work resulted in the excellent paper on "Lateral Forces of Earthquake and Wind".[3] This study was followed by the appointment in 1957 of a statewide committee of the Structural Engineers Association of California to review and recommend further improvements in aseismic design. Their work produced the "Recommended Lateral Force Requirements and Commentary". The 1961 Uniform Building Code incorporates these recommendations. Moreover, since the latest thinking exhibited in these recommendations considers the earthquake as a dynamic phenomenon, response of structural materials, members, and connections must be more carefully scrutinized with regard to cyclic loadings. The work discussed here is motivated by the above considerations.

DESIGN OF EXPERIMENT

In a comprehensive experimental program concerned with the behavior of steel beam-to-column connections under cyclic load, numerous types of connections must be investigated. In this

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3 By Anderson, Blume, Degenkolb, Hammill, Knapik, Marchand, Powers, Rinne, Sedgwick, and Sjoberg. ASCE Trans., Vol. 117, 1952.

particular program four widely used connections were selected for study. In three of these, the beam connects to the flange of the column; in one, to the web. Both welded and bolted joints will be investigated in this series of experiments.

The type of connection tested first, and reported here, is shown pictorially in Fig. 1. The more complete details of the specimen are shown in Fig. 2. The 8 WF 20 beam is welded directly to the 8 WF 48 column stub. The proportions of the beam are such that similitude with full-size structural members encountered in high-rise buildings is achieved rather well. Since formation of a plastic hinge was wanted in the beam, a substantial column stub was chosen, and its length was made for convenience of installation in the machine. This type of specimen is designated F1.

The machine for testing the specimens consists of a frame and a double-acting hydraulic jack which connects to the end of the cantilever being loaded as shown diagrammatically in Fig. 3. The actual testing arrangement is shown in Fig. 4. This set-up is a strengthened and modified machine used earlier with some smaller specimens.[4] Note the vertical guides at the end of the cantilever and at mid-span which provide lateral

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4 Bertero, V.V., Popov, E.P., "Effect of Large Alternating Strains on Steel Beams", ASCE Proceedings, Vol. 91, ST1, February 1965, Paper No. 4217.

support for the beam. Since the point of load application by the jack corresponds to an inflection point in an actual structure, complete vertical guiding for the beam is provided.

At mid-span of the cantilever only the top flange is guided, see Fig. 5. This simulates the lateral support of the top flange by a floor system. To be on the conservative side, no guiding is provided for the bottom flange. The guide sliding between the verticals is connected to the beam only by means of a longitudinal pin. Two small lugs welded to the beam hold the pin in position.

The mechanical arrangement described above, together with dial gaging for measuring beam deflections and curvatures, performed very well during experiments. Numerous electric strain gages also functioned satisfactorily.

It was difficult to reach a decision on the most meaningful loading cycles for the first experiment. Upon reflection, it was concluded that the most valuable information for this purpose is of very recent origin. The Alaska Earthquake of March 27, 1964 showed the severity of load reversals which may be experienced, in extreme cases, by steel members. Figs. 6 and 7 show photographs[5] of damaged columns in the Cordova Building

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[5] Berg, G.V., and Stratta, J.L., "Anchorage and the Alaska Earthquake of March 27, 1964", AISI 1964 (see Figs. 91 and 94 of this paper).

at Anchorage, Alaska. These may be compared with the photographs of WF members failed by repeated and reversed loadings shown in Figs. 8 and 9, taken from Reference 4. The similarity in shape of the member distortions is striking. Based upon this evidence, it was decided to perform the first exploratory test for cyclic loading by scheduling it as shown in Fig. 10.

The scheme adopted for cyclic testing of specimens, designated as C1 series*, consists of first subjecting a specimen to three elastic cycles followed by a sequence of cycles at progressively increasing control strains. For the elastic case the <u>stress</u> of 24 ksi is reached at the control section; for inelastic cases the control is based on <u>strain</u>. A loading cycle in the plastic range is defined as the process of loading the beam downward until the desired positive control strain is reached, unloading, then reverse loading the beam upwards until the desired negative control strain is reached and finally unloading. For each plastic cycle two excursions into the plastic region take place.

To provide a basis of comparison for the cyclic experiment, an F1 specimen was also tested statically to failure under a single load application. This experiment is identified by adding

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* The superseded designation D1 is seen on the photographs.

a letter S after the specimen type, i.e., F1-S.

EXPERIMENTAL RESULTS

The specimen F1-S was tested by applying a downward load. The control section for measuring strains was located 1-1/2 in. from the face of the column. Strain hardening began at about 1-1/2% strain, and the maximum strain reached was about 5-1/2%. Buckling of the compression flange was observed near 1% control strain. The failed specimen is shown in Fig. 11 where the characteristic buckle of the lower flange may be seen. The calculated and the observed yield moments agreed within 1%; the observed fully plastic moment of 732 kip inches exceeded the theoretical by 6.5%; the maximum moment of 943 kip-inches was 137% of the theoretical plastic moment. The specimen was fabricated of ASTM A36 rolled steel section with a flange yield stress of 39.2 ksi.

The specimen F1-C1, fabricated from the same material stock as F1-S, was tested following the schedule given in Fig. 10. The control section was located 5-1/2 in. from the face of the column. Strain output from the control gage versus the output from a transducer on a load cell were automatically recorded on an X-Y plotter. Other gage outputs and deflection data were obtained mostly by observers. A summary of the results is given in Table 1. This F1-C1 specimen failed after 28 <u>complete</u> plastic cycles.

Table 1

Summary of Results

F1 Specimen — C1 Strain Cycles

28 Cycles to Failure

No. of Cycles / Strain*	5	5	5	13
Nominal	1/2%	1%	1-1/2%	2%
Actual	+0.4% -0.1%	+1.2% -0.3%	+1.6% -0.8%	±1.8%
Average Max. Load	12k	14k	15k	15k

*At 5-1/2 in. from face of column

It can be noted from the summary that the achieved strains were lower than planned. This is due to the fact that control problems are intricate. Coordination between dial readings giving beam curvature and electric strain gages was not ideal. This problem is further complicated by the gradual stretch of the control fibres. However, since the goals set by the nominal strain requirements are rather arbitrary, this lack of strain correlation is not considered serious.

The physical performance of the beam under cyclic loading is best understood by referring to Figs. 12 through 16. The compression buckle shown in Fig. 12 actually occurred during the 14th plastic cycle at 1-1/2% control strain. In fact, during the cycling at ± 1/2% and ± 1% there was no apparent buckling of the flanges and were no visible signs of damage. Flange buckling began during the first cycle (the eleventh plastic cycle) at ± 1-1/2% nominal control strain.

In Fig. 13 buckling of the beam during the second cycle of ± 2% nominal strain is shown. In Fig. 14 the appearance of the buckle during the third cycle at ± 2% strain may be seen. In Fig. 15 the beam is shown later in the same cycle, but during an upward application of load. Note how the lower buckle has straightened out. The close up view of the beam at the end of the experiment is shown in Fig. 16. Cracking of the bottom flange at the top of the buckle precipitated the final failure.

As noted earlier, strain at the control gage versus the magnitude of the applied load were automatically recorded. This was done at all strain levels. The record for 1% control strain is re-drawn in Fig. 17. Note the remarkable reproducibility of these hysteresis loops, i.e., their shape remains essentially the same during the consecutive loading cycles. This fact demonstrates that the energy absorption capacity of the material is highly dependable. This property is desirable for structural damping of motions. Moreover, since the shapes of these hysteresis loops are reproducible at successive cycles, the basic structural stiffness of the system does not deteriorate.

Analogous hysteresis loops were obtained at other levels of control strain. These are shown in Fig. 18 in a slightly idealized form. The only essential difference on this plot from the original data consists of placing the origin in the middle of each one of the loops. This is permissible since the location of the origin around which the reversal takes place seems to be arbitrary, particularly for steel. The hysteresis loop for 2% nominal strain, which is of similar shape and encloses a still larger area, is not shown on the diagram.

CONCLUSIONS

In this paper is given a preliminary report on the first in a series of experiments on cyclic loading of structural steel connections. Of necessity, therefore, the conclusions must be considered tentative. Nevertheless, some of the results are so striking that some conclusions are possible:

1. The welded connection tested showed a remarkable ability to withstand a severe cyclic loading comparable in intensity with the extreme cases which may be encountered during an earthquake.

2. The onset of flange buckling does not precipitate an immediate loss of load carrying capacity. The buckles tend to appear and then disappear under the application of cyclic loading.

3. The hysteresis loops are remarkably stable and are reproducible under the cyclic loading. This fact suggests that overall structural stiffness does not deteriorate rapidly and that welded steel connections intrinsically possess good damping capabilities.

ACKNOWLEDGEMENTS

The author wishes to acknowledge with gratitude the financial support of AISI which is making this investigation possible. The AISI Advisory Committee and the Committee on Seismology of the Structural Engineers Association of California offered much good advice. Messrs. R. Binder, H. Degenkolb, A. Johnson, H. S. Kellam, R. Napper, I. Viest, and C. Zwissler from these groups were particularly helpful. Graduate students H. A. Franklin and R. B. Pinkney gave valuable assistance in performing these experiments.

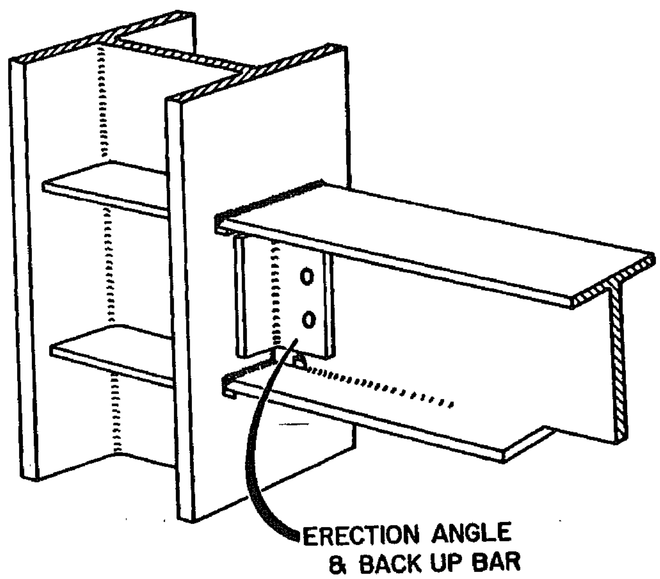


FIG. 1 GENERAL VIEW OF CONNECTION F1

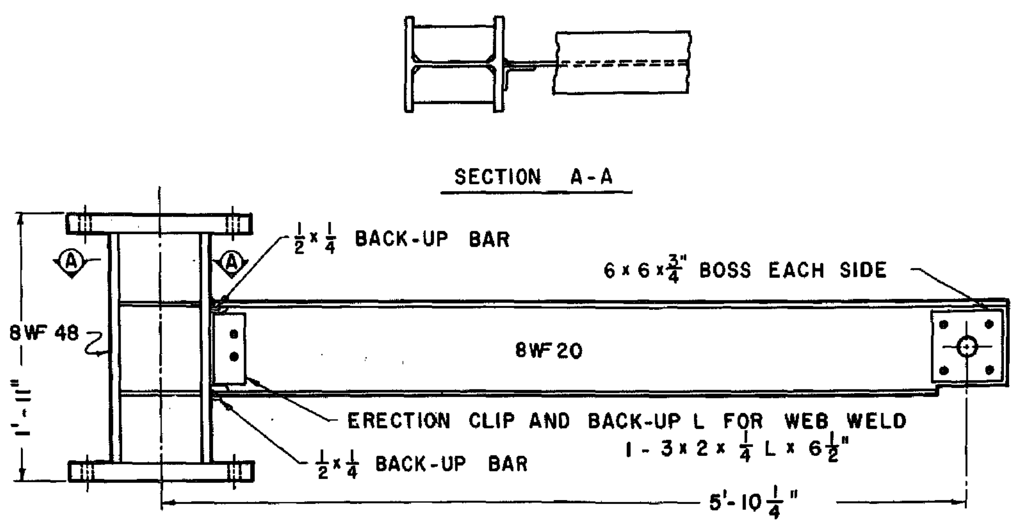


FIG. 2 TYPE F1 WELDED BEAM TO COLUMN CONNECTION

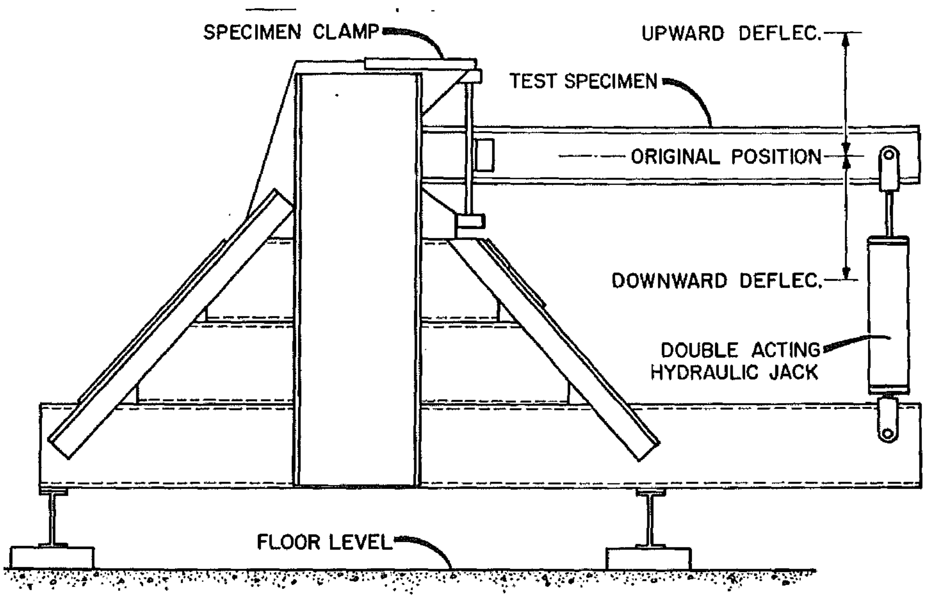


FIG. 3 TEST FIXTURE WITH SPECIMEN

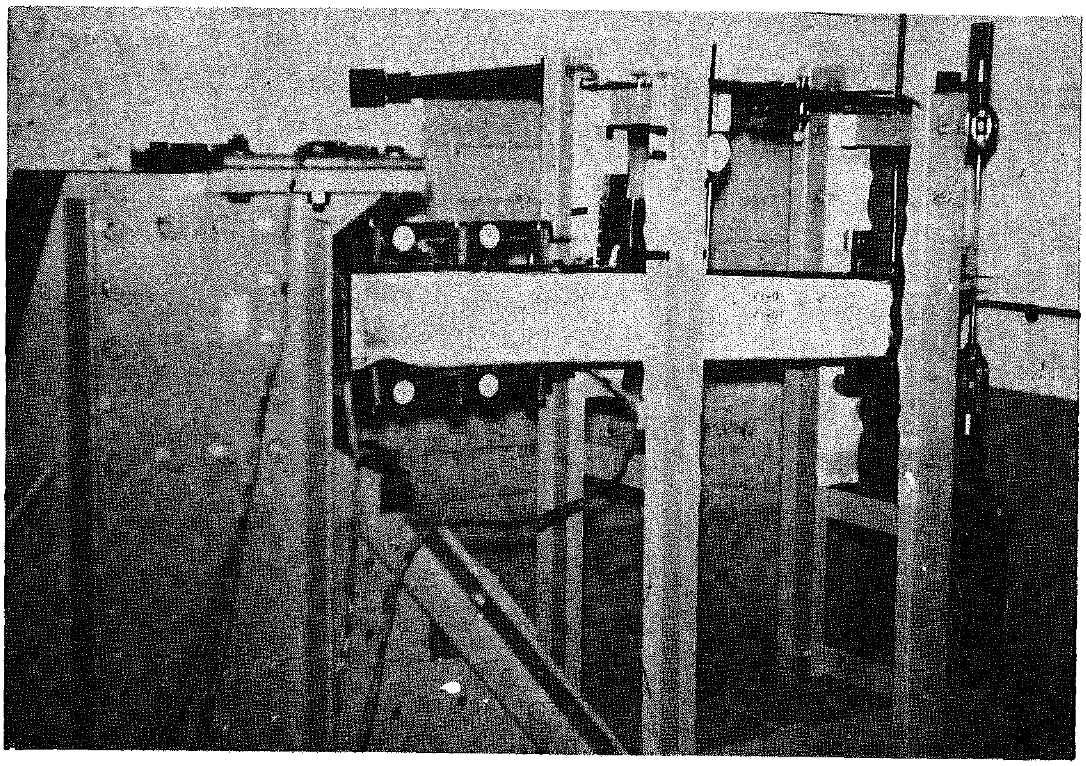


FIG. 4 GENERAL VIEW OF THE TESTING MACHINE

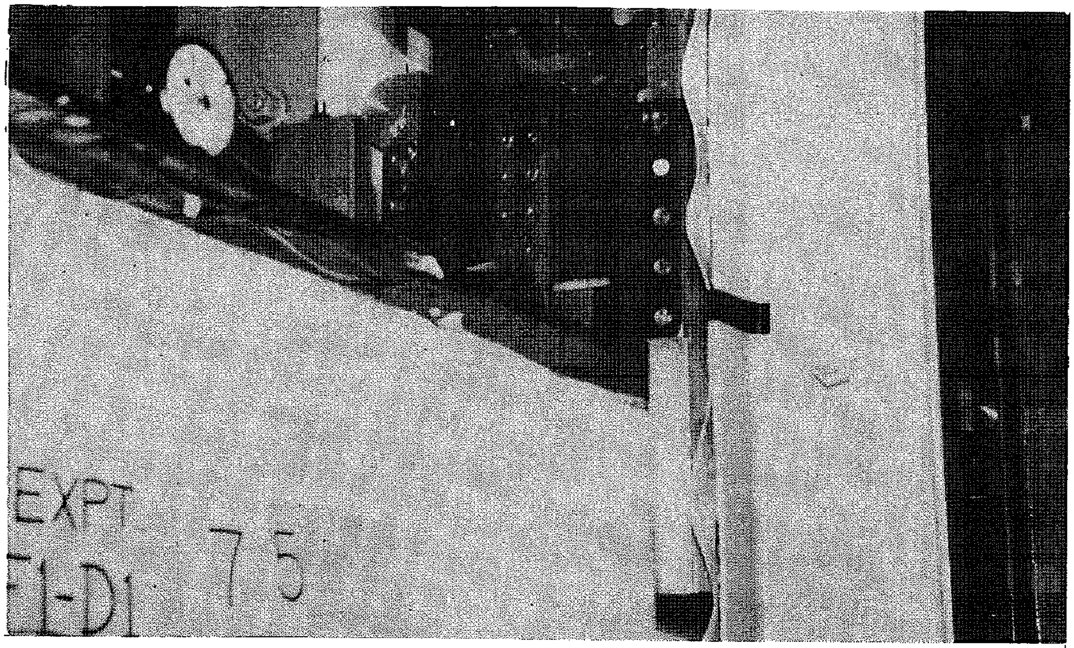


FIG. 5 DETAIL OF THE CENTER GUIDE

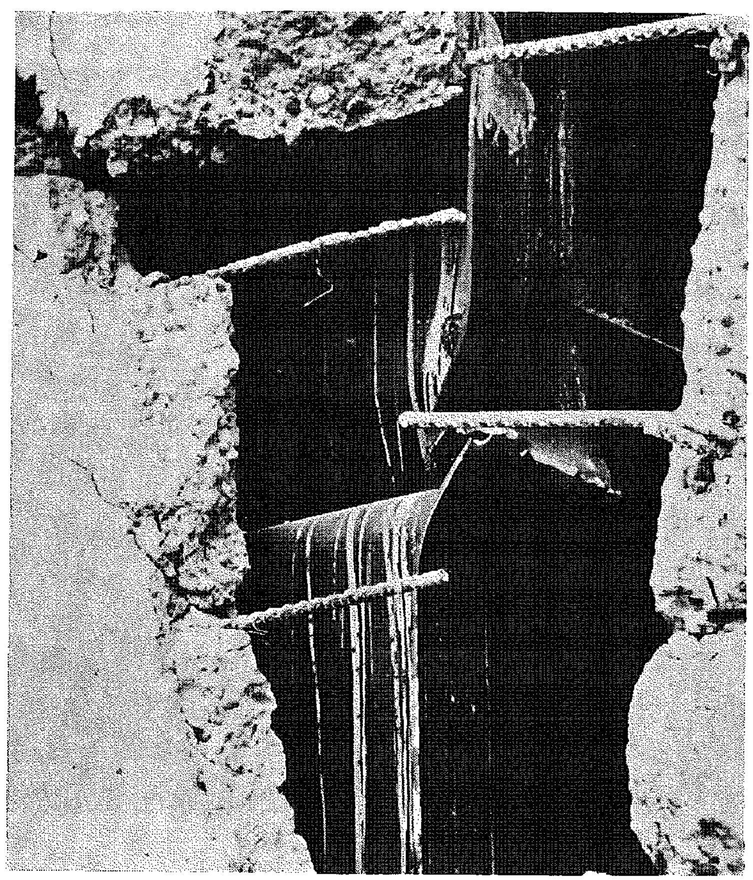


FIG. 6 CORNER COLUMN CORDOVA BUILDING

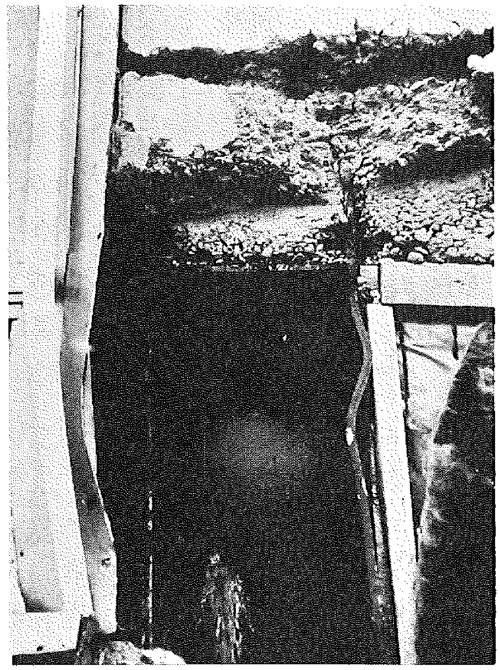
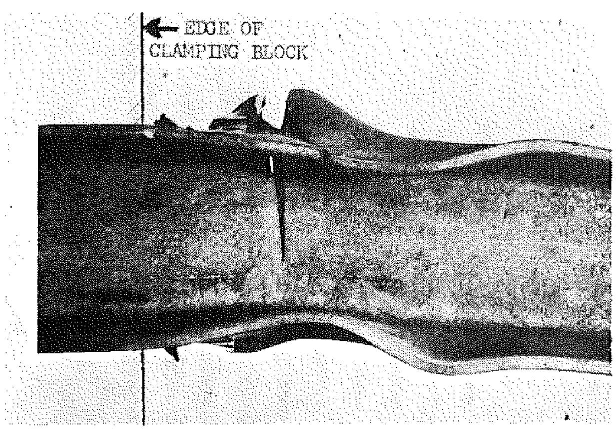


FIG. 7 CENTER COLUMN CORDOVA BUILDING



SPECIMEN NO.10 AFTER TEST

CONTROLLING STRAIN: ± 0.025

BUCKLING OF FLANGES WAS OBSERVED DURING 2ND HALF OF 1ST CYCLE.

FIRST CRACK BECAME VISIBLE DURING 9TH CYCLE.

FAILED AFTER 15 CYCLES.

FIG. 8 4 in. BY 4 in. 13 lb. SPECIMEN AFTER TEST

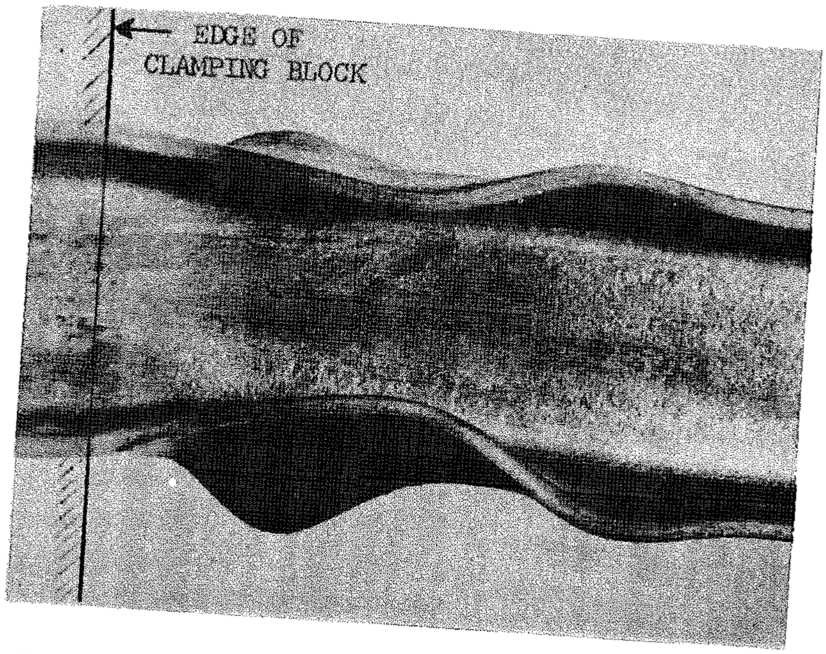


FIG. 9 CHARACTERISTIC BUCKLING OF FLANGES IN A 4 in. BY 4 in. 13 lb. SPECIMEN

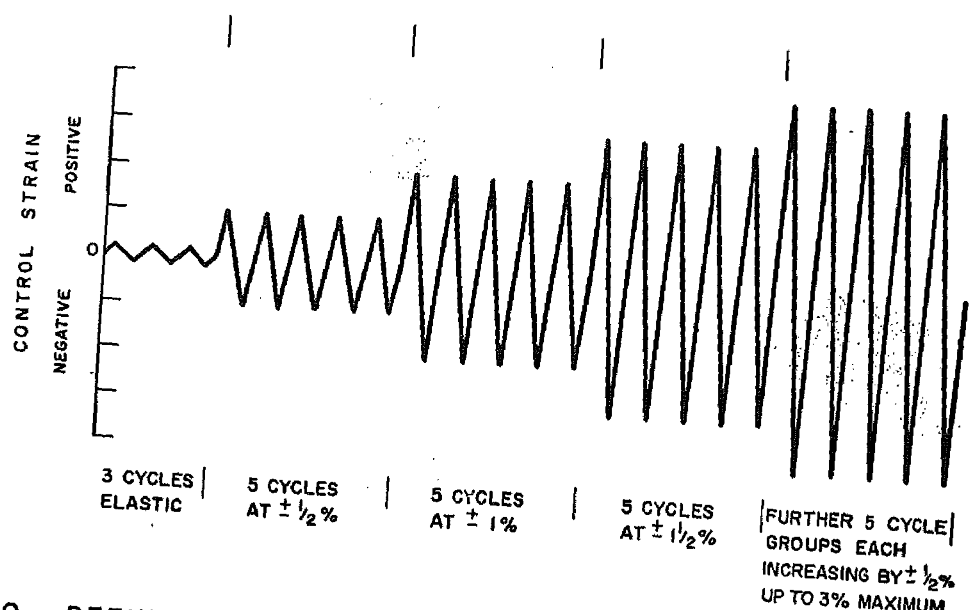


FIG. 10 DEFINITION OF THE C1 SERIES OF NOMINAL STRAIN CYCLES

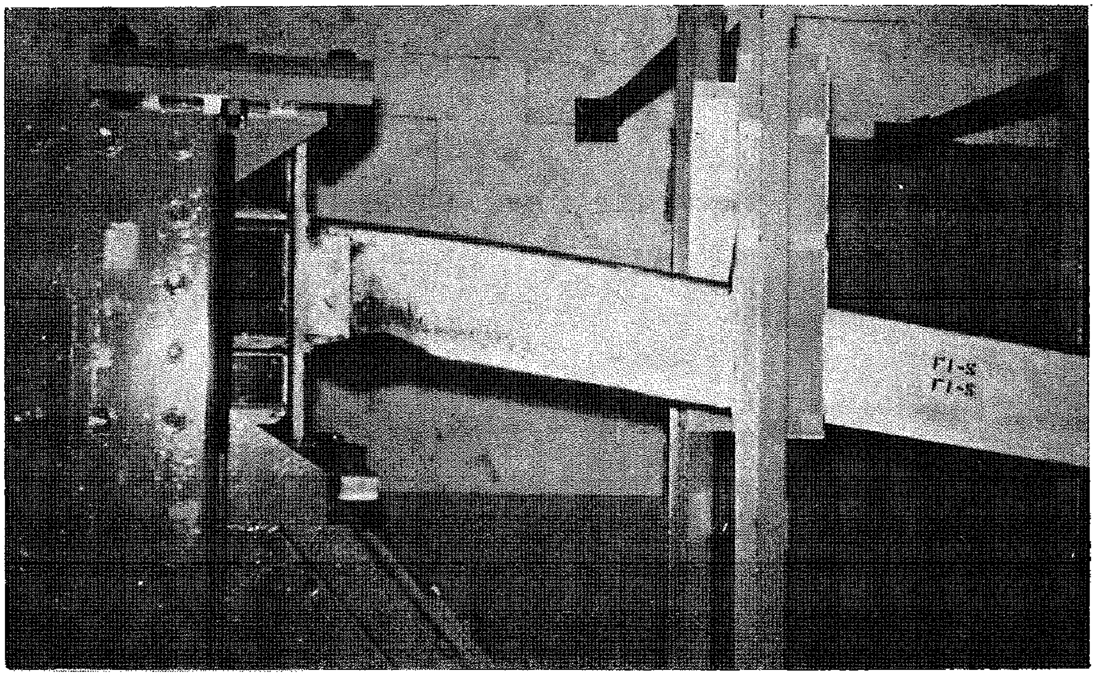


FIG. 11 SPECIMEN F1-S AFTER TEST

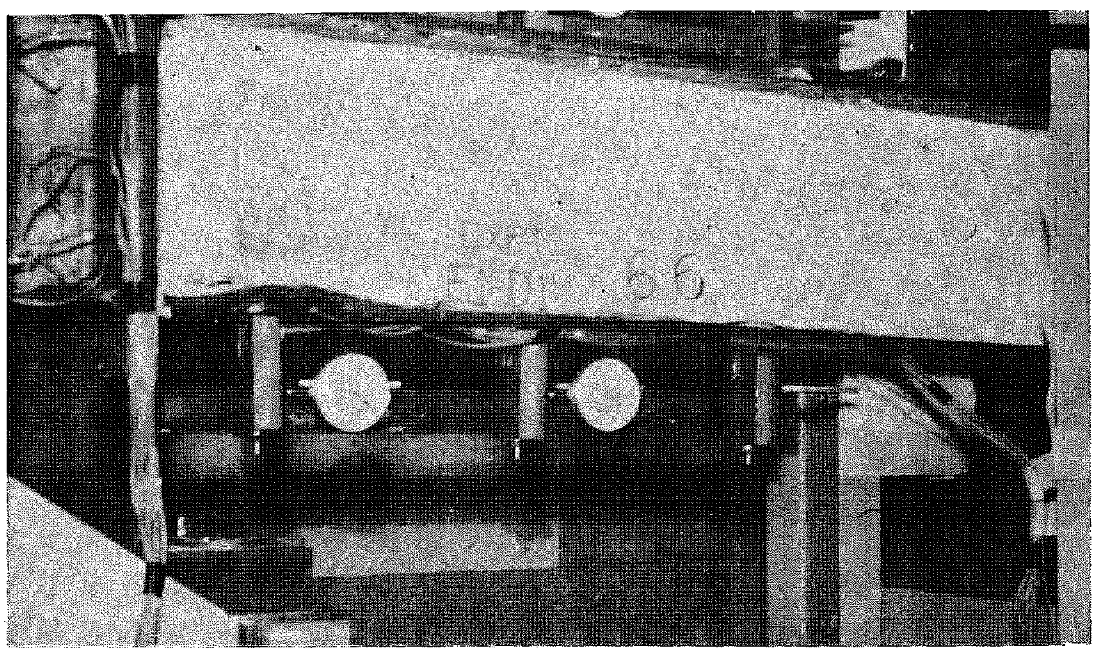


FIG. 12 SPECIMEN F1 DURING THE 14th PLASTIC CYCLE.
1 1/2 % NOMINAL CONTROL STRAIN

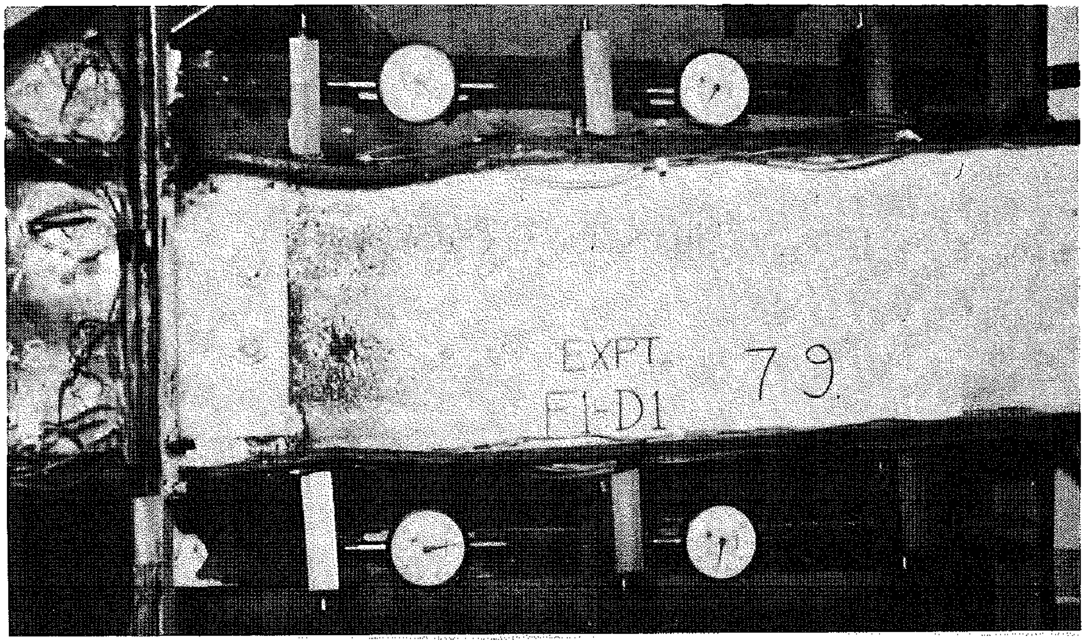


FIG. 13 SPECIMEN F1 DURING THE 17th PLASTIC CYCLE.
2 % NOMINAL CONTROL STRAIN

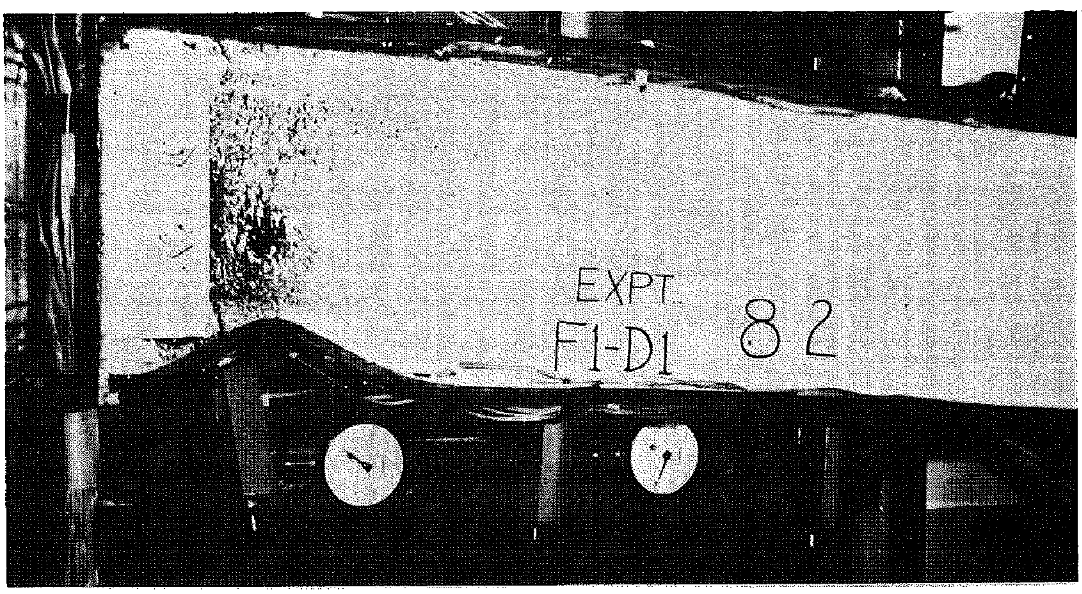


FIG. 14 SPECIMEN F1 DURING THE 18th PLASTIC CYCLE.
2% NOMINAL CONTROL STRAIN. LOAD
APPLIED DOWNWARD

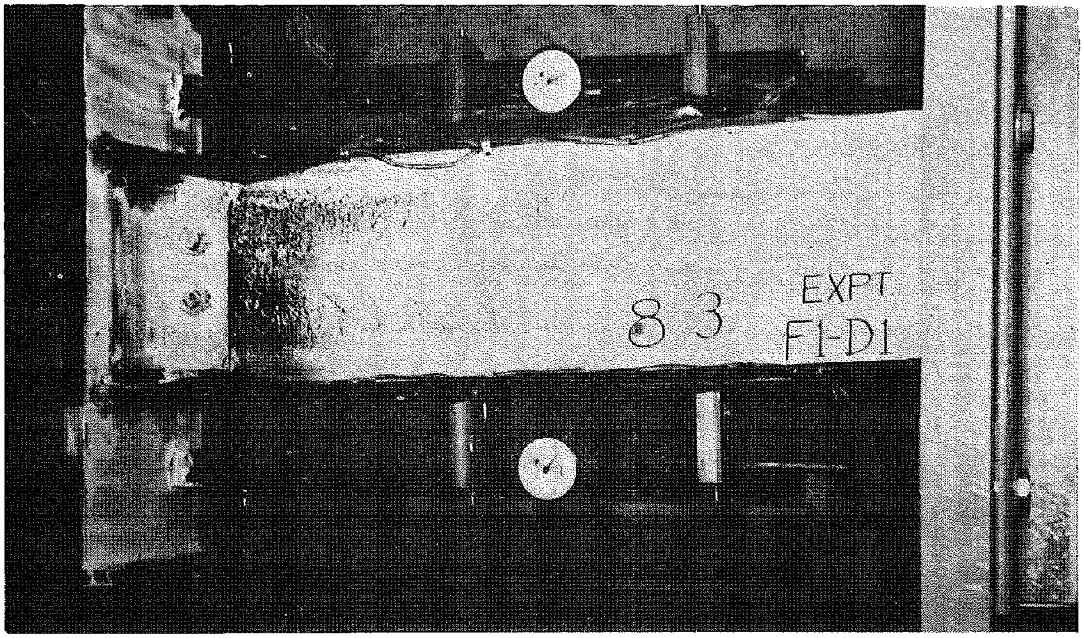


FIG. 15 SPECIMEN F1 DURING THE 18th PLASTIC CYCLE. LOAD APPLIED UPWARDS

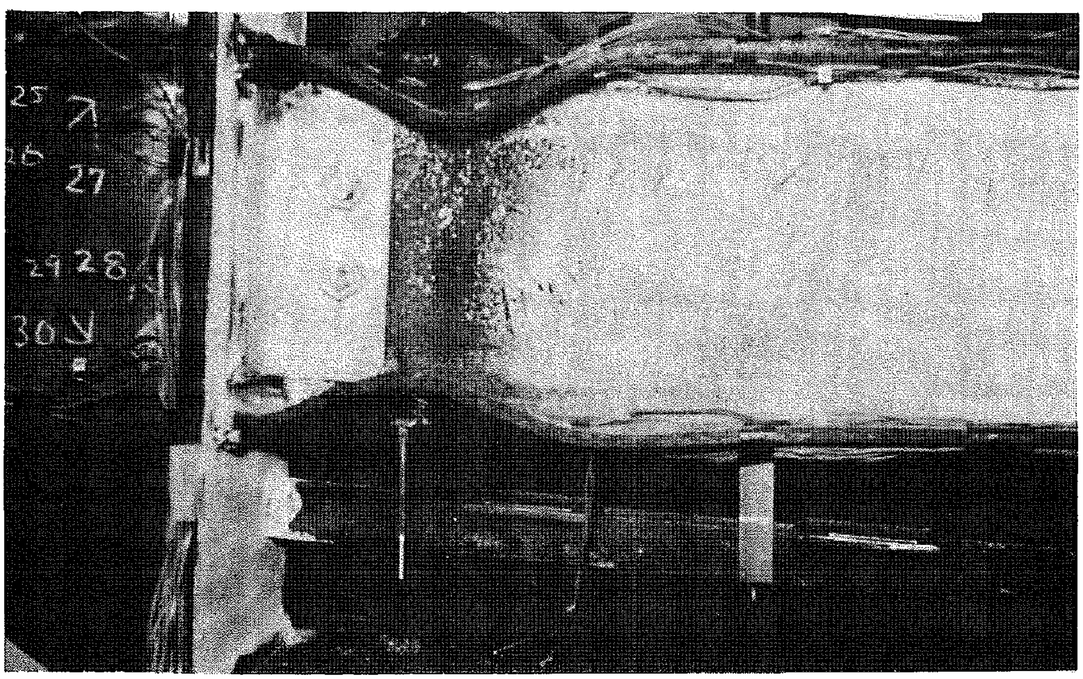


FIG. 16 SPECIMEN F1 AFTER CYCLIC TEST

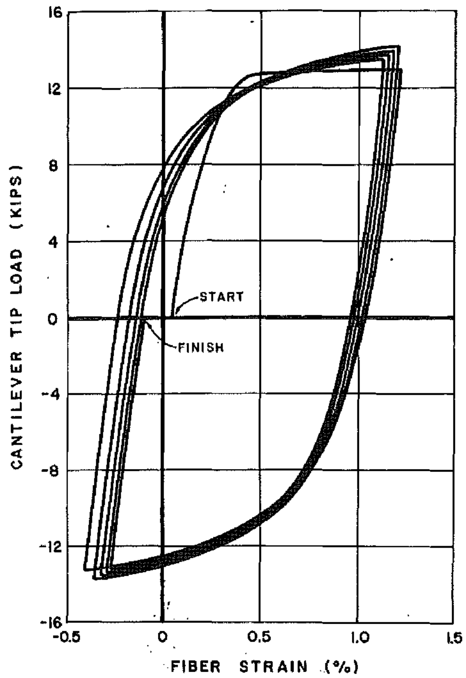


FIG. 17 STABILITY OF HYSTERESIS LOOPS AT ±1% NOMINAL CONTROL STRAIN

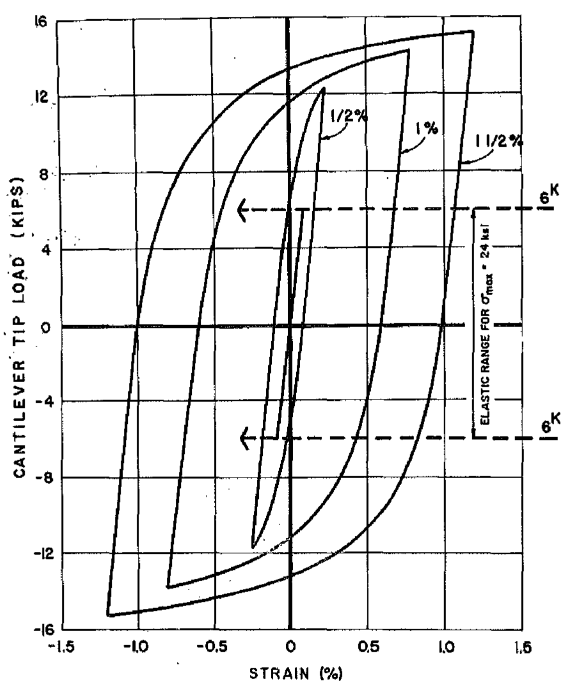


FIG. 18 LOAD - CONTROL FIBER STRAIN HYSTERESIS BEHAVIOR

MULTISTORY FRAME DESIGN IN EARTHQUAKE ZONES

by

Henry J. Degenkolb,[1] F. ASCE

The building code forces specified for providing earthquake resistance are derived principally from the observation of the performance of buildings that have been subjected to earthquakes. This crude approach is still necessary at the present time because the theoretical design factors obtained from the presently available ground motion measurements are much too high for the usual elastic methods of analysis. And, while great progress has been made on analyses based on elasto-plastic performance, these studies have not as yet progressed to the point where they can be used in the design office for the average building. So it is necessary to study the buildings that have performed well in the past in order to determine why they were satisfactory and from these studies to determine the requirements for future designs.

The most valid experience with the performance of tall structures in major earthquakes has been in the San Francisco 1906 Earthquake. This earthquake has been estimated to be of

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[1]Structural Engineer and Pres., H. J. Degenkolb & Assoc., Consulting Engineers, San Francisco, California.

magnitude 8-1/4 on the Richter Scale of magnitude. Downtown San Francisco is about 9 miles from the San Andreas fault when the rupture occurred. Tables 1 and 2 are taken from the ASCE Committee Report on the effects of that earthquake on Buildings.[2] According to this committee report, the buildings in Table 1, those with complete steel frames, performed well. It will be noted that this group of buildings includes one 19-story, one 16-story and several 14-story buildings as well as lower structures.

The buildings in Table 2, which had masonry bearing walls with interior steel frames, were reported by the committee to have suffered more damage by earthquake and fire than those in Table 1.

It is interesting to compare the magnitude of well known earthquakes and the distance to high rise construction.

San Francisco 1906	8-1/4 magnitude	Downtown 9 miles from fault
Mexico City, July 1957	7.5 magnitude	Epicenter 170 miles from Mexico City, 60 miles from Acapulco
Alaska 1964	8.4 magnitude	Anchorage 80 miles from fault
El Centro 1940	7.1 magnitude	No tall buildings near

[2] Galloway, Couchot, Snyder, Derleth, Wing - "The Effects of the S.F. Earthquake of April 18, 1906 - Report of Committee on Fire and E.Q. Damage to Buildings." Trans. ASCE Vol. 59 P. 223 (1907).

From this comparison it can be seen that the 1906 San Francisco earthquake has provided the most severe test of high rise construction to date.

In Anchorage, Alaska, there were three 14-story buildings, all seriously damaged in the 1964 Earthquake and one 14-story building in Whittier, also damaged.

Because of the essentially satisfactory performance of the tall structures in San Francisco in a nearby major earthquake, it becomes important to study the characteristics of those buildings to properly determine code requirements.

All of the taller buildings in downtown San Francisco in 1906 used structural steel frames; most of them used semi-rigid connections.

In studying the performance of these steel structures, three basic elements are involved: 1) beam-column connections, 2) bending of structural shapes and 3) column compression and bending. First to be considered are the connections since these were the weakest elements of these older structures. The connections used were generally of the type that would now be classified as Series "A" web connections with top and bottom clip angles, generally 3/8" thick. Because of the past use and excellent performance of these connections, it is somewhat interesting and informative to see how they might perform under test.

A series of connection tests was performed and described by J. C. Rathbun in the Transactions of ASCE in 1936.[3] Table 3 gives the results for web connections without clip angles while Table 4 gives the test results for clip angle moment connections. Unfortunately in those days tests were run for studying the elastic properties of the joints and deformation readings were not taken at high strains. From the rotation data given, and the ultimate resisting moment, it can be assumed that very large rotations were achieved before the ultimate load, an assumption that is substantiated by the pictures included in the report. In order to indicate scale, a rotation of .007 would correspond to a story drift of 1" in a 12'-6" high story -- the drift caused by rotations of the joint itself, not including the bending in columns or girders. The ability of this type of connection to hold together and deform without collapse may be largely responsible for the reputation of steel frames to perform well in earthquakes. If this connection can be assumed to be "plastic" under vertical load and then to perform "elastically" under lateral loads, this type of connection using thicker angles can usually resist a lateral load of about 1%G in moderate height buildings of 8 to 10 stories.

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[3] J. C. Rathbun - "Elastic Properties of Riveted Connections" - Trans. ASCE 101:524-96 (1936).

As the requirements of the building codes regarding lateral forces became more severe, and buildings in earthquake regions became taller it was necessary to provide stronger moment connections than could be designed with top and bottom clip angles. The most commonly used connection uses the split tee top and bottom of the beam, and has been used in most recent buildings designed for major earthquake forces except where all welded construction has been used. This type of connection has great strength and very great ductility when properly designed. Table 5 shows some tension tests on split tee's using rivets as the tension connection between column and tee. These tests were reported in Earl Cope's discussion of Berg's paper in the 1933 Transactions of ASCE.[4]

It is interesting to note, that in these 14 tests, ranging up to a flange stress ultimate tension of 294 kip requiring 4-1 1/4"ø tension rivets, the separation of the "tee" from the column goes up to over 1 1/4" with an average deformation of 0.80". In relating these data to ductility, drift, or total building deformation, this average "tee" deformation of 0.80" corresponds to a story drift of 5" using 24" deep girders in 12'-6" high stories.

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[4] U. T. Berg - "Wind Bracing Connection Efficiency" - Trans. ASCE 98:709-770 (1933).

Other tests have been made on connection deformation of split tees, principally in the elastic range, but time does not permit going into further detail now. Those interested should refer to either Rathbun[3] or Young & Jackson,[5] or Douty & McGuire.[6]

One of the problems in designing split "tee" connections concerns the "prying" action under the tension bolt as discussed in several of the references. In designing the structure for the International Building in San Francisco, the edge margins of the tension bolts were reduced to the minimum possible, eliminating the "prying" action and causing the flanges of the "tee" to bend as simple cantilevers. Twelve samples of the "tees" were tested[7] in 6 back to back tests using enough packing between tees to simulate the thickness of the columns. This was done in order to provide the proper relationship between grip of bolts and the bending properties of the "tee" flange. It is interesting to note that the minimum elongation was 0.98" on Specimen T83 where a tension bolt failed. This

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[5]C. R. Young & K. B. Jackson - "The Relative Rigidity of Welded and Riveted Connections". Canadian Journal of Research Vol. II (July & August 1934).

[6]R. T. Douty & Wm. McGuire - "High Strength Bolted Moment Connections". Proceedings ASCE Vol. 91 ST2 (April 1965).

[7]P. B. Cooper & S. J. Errera - "Static Tension Tests of Structural Tee Joints". Lehigh University Institute of Research. Fritz Engr. Lab Report No. 200.60.345 (August 1960).

minimum elongation corresponds to a story drift of about 5" for a 30" deep girder in 12'-6" story height, again neglecting column and girder bending. The tests indicate that the minimum ratio of ultimate deformation to yield deformation was greater than 20.

There are several items to consider in the design of steel connections, especially with regard to the internal actions of the joint. Consider the moment resisting joint shown in the upper left portion of figure No. 1. "Tees" are provided to develop the full moment capacity of the beam. The web of the tee must have sufficient net section and a sufficient number of bolts acting in shear to develop the chord stress "T". This stress "T" is then transferred through the flange of the Tee in bending, then through tension bolts to the column. In considering the flange of the Tee and the tension bolts, there are two alternate conditions of design as indicated in the portions "A" of figure 1. The more usual condition, shown on the left has generally been assumed in the past since "tees" traditionally have been made by cutting portions of rolled sections. With the sections available for large moments, the thickness of the "tee" flange is limited and consequently a point of inflection (P.I.) has to be assumed between the tension bolts and the web of the "Tee". In order to assume the points of inflection, prying forces "C" must also be assumed, so that the tension force in the tension bolts is larger than the flange stress "T" by the amount of prying force "C". In actual connections, this force "C" may range from 25% to 50% of "T". An alternate condition of design may be as shown in the right hand detail A of Fig. 1. If the edge distance of the tension bolts is kept short, no prying forces can be developed and hence there can be no point of inflection in the flange of the "Tee". The flange of the "Tee", then must be designed as a simple cantilever from the web, requiring more thickness of flange than the first alternate, but permitting smaller tension bolts. For high rise buildings in areas subjected to major earthquakes, this flange thickness may be greater than available in rolled sections so welded built-up tees may be required.

If a view is taken of the column in plan as shown in Section B, it can be seen that there may be additional prying forces as the column flange tends to bend. In the past, these have often been overlooked. However, with the high forces associated with the earthquake stresses in high rise structures, and the general necessity to provide moment connections in the opposite direction (beams connected to web of column) stiffeners will usually be required to reduce column flange bending to acceptable levels. Therefore prying forces in this plane usually do not control. It might be mentioned at this time that although all welded steel construction may eliminate the need for this type of column-beam connection in many cases, it is still a commonly used connection especially where coverplated columns must be used because of heavy loads.

In figure 2, are shown two loading conditions that inexperienced designers frequently overlook. At the left is shown the shear condition in the column web between the beam flanges. Regardless of whether this is a bolted or welded joint, the stiffener plates have tension at one side and compression on the other (considering only the lateral loading portions of the beam moments) so that the shear on the weld to the column web is measured by twice the beam flange stress. Similarly the shear force in the column web is approximately twice the beam flange stress.

When the column is in bending in the weak direction as shown in plan, only the welds to the flanges of the column are fully effective and again the total welding to the flanges must take twice the beam flange stress.

The second factor that influences the overall performance of a steel framed structure is the bending capacity of the beams and girders. In steel, failure is usually by instability and in order to perform satisfactorily in a major earthquake, the moment-rotation curve must develop a long plateau. Ultimate failure is usually not caused by lateral buckling, but is usually triggered by local buckling. This is one of the main concerns of this conference and time would not permit a recapitulation of the conference at this point.

However, there are several factors in earthquake resistant design that are somewhat different from the tests made for plastic design considerations. Probably the most important in beams relates to the moment gradient for the beam. Whereas most tests apply a uniform moment to determine buckling pattern, the girder in a building subjected to earthquakes has the maximum moment at the column where beam flange support is greatest. The beam moments drop off rapidly to about zero at the center of the beam where flange support is usually least. The types of connection details generally used also offer major support where the moments are greatest.

Suffice it to say that rather large rotations can be obtained through proper design.

Similarly with the third major factor affecting earthquake performance - column behavior.

Much of this Conference is devoted to the study of columns under combined axial load and bending with various degrees of restraint, and it is unnecessary to review the subject at this time. Again, however, research has shown that a frame properly designed for earthquake forces will not fail in the columns especially in the lower stories, but that plastic hinges will usually form in the girders.

In conclusion, there are three considerations that are of utmost importance in the design of tall structures, only one of which has been discussed here tonight.

First, the tallest structures that have successfully withstood major earthquakes have been steel framed with a very ductile type of connection. In these structures, all of the ductility was forced into the connections because of their type. While it is difficult to correlate deformation capacities with ductility, the type of joints used in the past probably have ductility ratios in the 10 to 30 range. Considering all factors, beams, columns and connections, a properly designed high rise steel structure probably has a ductility ratio of 8 to 10 or possibly considerably higher.

The second major consideration which has not been discussed tonight is the necessity to maintain a clear and continuous stress path whereby all details, connections, chords, diaphragms, shear walls are correctly interconnected to provide proper shear transfers and an unbroken path for the stresses.

Most of the failures seen in earthquakes are the result of detail failure - a missing link in the flow of stress from one point to another.

And finally, one of the most important considerations is the provision for adequate field inspection. This is a factor that receives the minimum of discussion at seminars such as this and yet it is one of the most important factors toward achieving a successful design. All design considerations and all calculations are worthless unless they are carried out in the field. Now that the engineering profession is extensively using high strength steels, welding, very high strength concretes, high yield reinforcing bars, sophisticated methods of analysis, it is of the utmost importance that the structure is given thorough inspection in the field. The client is buying a structure - not a set of design calculations.

Table No. 1

Buildings in San Francisco with a complete steel frame supporting all wall and floor loads

Building	Stories	Approx. Area	Columns	Floors
Claus Spreckels	19	5,600	Z	Concrete
New Chronicle	16	5,600	Pl & A	Tile
Merchants Exchange	14	25,000	Pl & A	Concrete
St. Francis Hotel	14	22,000	Z	Concrete
Flood Building	12	40,000	Z	Tile
Hamilton Hotel	12	3,100	Chan.	Concrete
Mutual Savings	12	5,000	Z	Concrete
Alexander Hotel	11	4,700	Chan.	Concrete
Kohl	11	9,000	Chan.	Concrete
Shreve	11	9,000	Z	Concrete

11 others listed from 6 to 10 stories

"Damage to steel frames almost negligible." Cracked partitions, walls, tile, stonework. Shifting stone.

Table No. 2

Buildings in San Francisco with steel frames supporting floors but brick and stone bearing walls supporting themselves and some supporting outer bays of floors.

Building	Stories	Approx. Area	Columns	Floors
Crocker	11	10,000	Phoenix	Tile
Mills	11	21,000	Z	Tile
Mutual Life	9	5,000	Z	Tile
Old Chronicle	9	--	Cast Iron	Tile
Union Trust	9	8,400	Channel	Tile

17 others listed from 2 to 7 stories.

Tables No. 1 and 2 based on data in

Galloway, Couchot, Snyder, Derleth, Wing - "The Effects of the S. F. Earthquake of April 18, 1906 - Report of Committee on Fire and E.Q. Damage to Buildings", Trans. ASCE Vol. 59 P. 223 (1907).

Table No. 3

Moment Tests on Web Shear Connections

Test	Member	Number of Vertical Rows	Number of Rivets: Web R	Number of Rivets: O.S. I.R.	Ult. Moment in.kips	Maximum Recorded Rotation
1	6" I	1	2	2	23.5"k	.030
2	8" I	2	4	4	135	.024
3	8" I	2	4	8	159.5	.030
4	12" I	3	3	6	403	.026
5	12" I	3	6	12	529	.022
6	18" I	5	5	10	1225	.010
7	18" I	5	10	20	1300	.007

All rivets 7/8". All angles 3/8".

There was no failure and no definite yield point.
Angles deformed badly without visible distress in rivets or beams.
There were several repetitions of load.
θ=0.030 corresponds to 12'-6" high story drift of 4-1/2".

Table No. 4

Moment Tests on Top & Bottom Clip Angles

All tests on 12" I beam, 7/8" rivets, 3/8" angles.
All tests had 4 rivets from angle to each flange.

Test	Tension Rivets	Ult. Moment in.kips.	Maximum Recorded θ at Moment
8	4 (2 rows)	580"k	.006 @ 268
9	6 (2 rows)	665	.010 @ 430
10	8 (2 rows)	1055	.014 @ 560
11	4 (2 rows)	Not Given	.007 @ 560
12	8 (2 rows)	Above 1026* did not fail	.010 @ 750

From pictures, large deformations were observed but not given.

Tables No. 3 and 4 based on data from

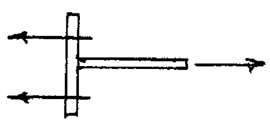
J. C. Rathbun - "Elastic Properties of Riveted Connections." Trans. ASCE 101:524-96 (1936).

Table No. 5

Tests for Mills Tower, San Francisco 1930

Test	"Tee"	Tension Rivets	Ultimate Load	Deformation at Ultimate	Failure
1	24I100	4 3/4"	123 k	0.46"	Rivet Necking
2	"	4 3/4"	120	0.48"	" "
3	"	4 7/8"	176	0.60"	" "
4	"	4 7/8"	181.5	0.60"	" "
5	"	4 1"	206.5	0.82"	" "
6	"	4 1"	212.0	0.65"	" "
7	36B260	4 1-1/8"	209.0	1.46"	" "
8	"	4 1-1/8"	205.0	0.80"	Head snapped off
9	"	4 1-1/4"	253.4	0.90"	" " "
10	"	4 1-1/4"	251.0	0.95"	" " "
11	36B300	4 1-1/8"	205.4	0.74"	Head Sheared
12	"	4 1-1/8"	237.0	1.05"	Rivet Necking
13	"	4 1-1/4"	293.6	0.94"	Head snapped off
14	"	4 1-1/4"	281.6	0.60"	" " "

Average Deformation 0.80"



1. Deformation is separation of flange from column at C.L. of web.
2. 0.80" corresponds to 12'-6" high story drift of 5" using 24" deep girders.
3. Deformations at ultimate <u>many</u> times (10 to 20) that at yield.

Data from

U. T. Berg - "Wind Bracing Connection Efficiency". Trans. ASCE 98:709-770 (1933). Discussion by Cope.

Table No. 6

Tests for the International Building, 1960

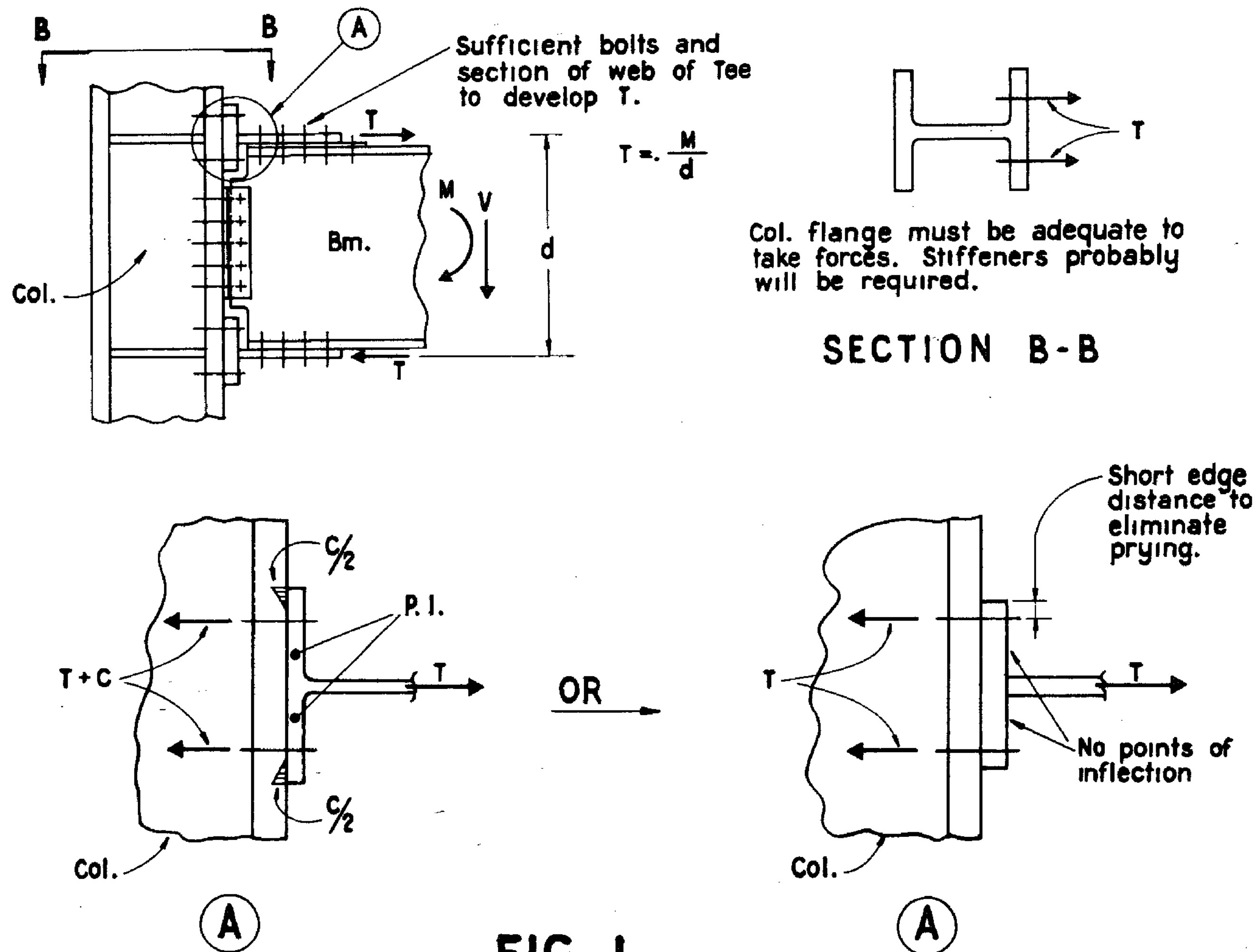
Tension tests on welded, stress relieved "Tees" with extra high strength, chrome molybdenum steel tension bolts.

Test	Stem "t"	Flange PL	Tension Bolts		Design	Loads Yield	inkips Ultimate	Ultimate Deformation
T28	3/8"	14x1-1/2	4	7/8"	79	124	248.5	1.30"
T18	1/2"	14x1-5/8	4	1"	103	158.2	327	1.54"
T77	5/8"	16x1-1/2	8	7/8"	158	270	522	1.46"
T83	7/8"	16x1-5/8	8	1"	206	359.5	706	0.98"
T44	1"	16x2	8	1-1/8"	260	444	825	1.84"
T129	1-1/4"	16x2-1/4	8	1-1/4"	330	561	1017	1.61"

All failures except T83 were thru net section of Stem Pls.
T83 failed thru fracture of bolts.

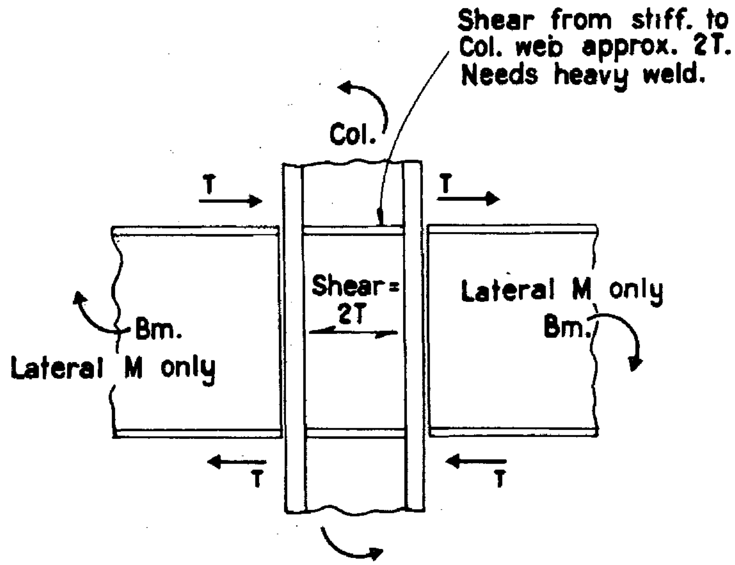
Data from

P. B. Cooper & S. J. Errera - "Static Tension Tests of Structural Tee Joints". Lehigh University Institute of Research - Fritz Engr. Lab Report No. 200.60.345 (August 1960).

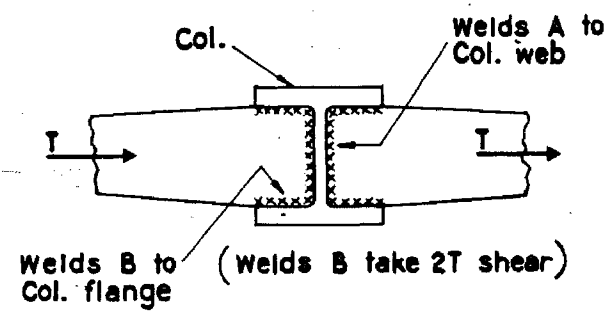


B
B
A
Sufficient bolts and section of web of Tee to develop T.
T
T =. M/d
M
V
Bm.
d
Col.
T
T
Col. flange must be adequate to take forces. Stiffeners probably will be required.
SECTION B-B
C/2
P. I.
T + C
T
C/2
Col.
A
OR
Short edge distance to eliminate prying.
T
T
No points of inflection
Col.
A

FIG. 1



BEAM CONNECTED
TO FLANGES



• PLAN •
PLATE FOR BEAM
CONNECTED TO COL. WEB

FIG. 2