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Welded Continuous Frames and Their Components

LOAD-DEFORMATION RELATIONSHIPS FOR SIMPLE FRAMES

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LOAD-DEFLECTION RELATIONSHIPS

FOR SIMPLE FRAMES

by

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Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pennsylvania

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SYNOPSIS

A method is presented for the determination of the load-deflection relationship for simple frames. The principles of equilibrium and compatibility, as expressed by the Moment-Curvature-Thrust relationship, are used directly in the analysis. The procedure is used to determine the relationship between horizontal load and deflection for a frame subjected to constant vertical loads. Inelastic frame buckling and the analysis of frames subjected to proportional loads are also discussed.

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INTRODUCTION

An increased understanding of the inelastic beam-column problem coupled with the development of rapid methods for its solution[1,2] has made possible the prediction of the ultimate strength of structures loaded into the plastic range. The present investigation is concerned with the behavior of a pinned base, single story, single bay, frame, as shown in Fig. 1. The frame is subjected to a vertical uniformly distributed load, w, along the beam, concentrated vertical loads, F, at each column top and a concentrated lateral load, V, at the beam-to-column joint. It is reasonable to first investigate the behavior of such relatively simple structures with the hope that the knowledge gained will provide some insight into the behavior of multi-story frames.

In attempting to assess the load-deformation behavior several concepts are relevant.[3] Consider the graph of the uniform load, w, versus the sway angle, ρ , shown in Fig. 2. In this figure the concentrated loads are assumed to be proportional to the uniform load, w. At low values of w, before the stress reaches the yield point, the load-deformation relationship may be obtained by conventional elastic analysis procedures[4] (first order elastic curve). If the reduction in stiffness due to axial load is considered along with the secondary moments due to the sway of the structure[5], the second-order elastic curve is obtained. This curve is asymptotic to the value of w which represents the buckling load of the frame. To obtain this buckling load, w_b, the horizontal force, V(w) is removed and the vertical loads are increased until the frame buckles in a sidesway mode.[5][6]

If the frame is considered from the point of view of ultimate strength analysis, the simple plastic theory (panel mechanism) would predict a load of w_{sp}. Under the assumptions implicit in this type of analysis the load is independent of the sway angle.[7] If the reduction in plastic moment capacity due to axial load is considered, the ultimate load drops to a value of w_p. However, if now the work done by the vertical load as the frame sways in a panel mechanism is also considered, a relationship between the sway angle and the load may be obtained.[3] This is denoted in the figure as the second-order plastic mechanism.

The true deformation path of the structure must initially follow second-order elastic theory and finally the second-order plastic mechanism. The problem to be solved then is the determination of the transition curve between the two.

This curve is denoted in Fig. 2 as the true load-deformation behavior. It should be noted that at some point the deformations involved would lead to local buckling of the members[8] (see dashed curve).

The method to be described here is not intended for routine design, but it is believed that some application may be found in the following areas:

(i) The determination of the complete load-deflection diagram is of importance as the area under this curve is used in some cases as a measure of the energy absorbed by a structure during deformation[9]. This is of particular significance in earthquake analysis.

(ii)   The frame buckling problem in the inelastic range is at present analyzed by first determining the stiffness of the frame as it deforms in a symmetrical mode, then analyzing an equivalent elastic structure for sidesway buckling[10].   An alternative method would be to subject the structure to an initial set of vertical loads, then to calculate the load-deflection relationship for the structure as a horizontal load is applied.   If this horizontal load is in the same direction as the sway, the structure is stable.   If not, the frame has buckled at a vertical load less than that originally assumed in the analysis.

(iii)   The exact load-deformation curve will yield the true ultimate load for the frame.   Presently used assumptions and design methods, particularly those involved in simple plastic theory, may then be checked.

(iv)   The procedures used in the analysis of the simple frames considered herein may point toward a possible procedure for the analysis of more complex structures.

EQUILIBRIUM AND COMPATIBILITY

The principles of equilibrium and compatibility are basic to the analysis of structures. In most cases the stress-strain diagram obtained from the tensile coupon test is used as the connecting link between the two. However, in the inelastic range the use of the relationship between the moment and the curvature for a particular axial thrust (the M-$\emptyset$-P relationship) has proven convenient.[11]

The principles of equilibrium and compatibility have been used directly to analyze the behavior of beam-columns restrained by framing members attached at the ends.[12] By combining the moment rotation response of the beam with that of the column, the response for the entire assemblage can be estimated. Experimental results[12] show excellent agreement with this theory.

The situation in a simple braced frame is slightly different.[13] Consider the frame shown in Fig. 3. Equilibrium requires that at the joints the beam moment, M_{BC}, must be equal and opposite to the column moment, M_{BA}. Compatibility requires that the rotation, θ_B, of the beam and column at each joint be the same in magnitude and direction. These conditions are illustrated in Fig. 3. Here, P_A represents the axial force in the column and V_A the column shear corresponding to M_{BA}. $M_{\mathbb{Q}}$ represents the beam moment acting at mid span. The solution obtained assumes that the frame is braced so that no sway occurs and therefore the deflected shape is symmetrical about the mid-span of the beam.

When sway is involved the situation becomes more complex.
Except in special cases, advantage cannot be taken of symmetry. One
special case that has been solved is that of the pinned-base frame
without beam loading.[14][15] This situation is shown in Fig. 4 (a).
In this figure V_A, P_A, V_E and P_E represent the axial loads and shears
present in the left and right hand columns.

If the horizontal load is assumed to be significantly less
than the vertical loads the axial force in the column can be assumed
equal to the vertical load, F. This assumption also neglects the
component of the axial load due to the sway of the frame. Under these
conditions the problem reduces to that of the restrained beam-column
shown in Fig. 4 (b).

This problem has been solved by the methods of Reference 14
with the additional assumption that the beam was completely elastic
throughout the loading range. Reference 15 also assumed an elastic
beam but did consider the variation in axial load in the columns.

The equilibrium and compatibility conditions in the restrained
beam-column subject to sway are illustrated with reference to Figs. 4 (b)
and 4 (c). Equilibrium at the beam-to-column connection requires that
the beam moment, M_{BD}, be equal and opposite to the column moment, M_{BA}.
For compatibility to be satisfied the rotation of the column top from
its chord, $\theta_B{}^*$, is given by:

$$\theta_B{}^* = \rho - \theta_B$$

where Θ_B is the beam rotation at the joint.

The column shear, $V/2$, can be computed from equilibrium of the column as a whole and the horizontal load, V, is then the sum of the two column shears. In Reference 15 the more general problem shown in Fig. 1 has also been treated. However, because of the assumption of a completely elastic beam, only very small distributed loads can be treated.

The method to be presented in the present report uses the principles of equilibrium and compatibility essentially as shown in Fig. 4 (c). However, these are modified so that the more general problem shown in Fig. 1 may be treated. The computational process used is similar to that described elsewhere[15] but the method has been extended to allow treatment of the more practical problem where both the beam and columns are in the inelastic range.

ASSUMPTIONS

In the method used in this report the following assumptions are necessary:

1. The deflections involved are small so that the relations of small deflection theory are valid.

2. The frame is braced to prevent out-of plane behavior.

3. The material has a stress-strain diagram similar to that of ASTM-A7 structural steel[7].

4. The generally accepted form of the M-∅-P relationship, derived from the stress-strain curve is valid[11][16]. This neglects strain-hardening, but includes the influence of residual stresses.

5. Local buckling of the plate elements of the cross-section does not occur. It should be noted in this connection, that if the influence of strain-hardening had been included in the analysis[17], it would be a relatively simple matter to determine the onset of local buckling using methods suggested in Reference 18.

Within the above assumptions the method to be presented is general. It may be modified to deal with unsymmetrical frames and frames having non-prismatic members. The method is approximate in that it uses a numerical process to determine the deflected shape of the frame; however, it does consider the gradual plastification of the cross-section, the secondary moments produced by the sway of the frame and the variation of axial load in the columns.

DEVELOPMENT OF THE METHOD

The basis of the analysis will be the determination of a set of loads which satisfy equilibrium and maintain compatibility in the deflected shape for a given sway angle. This procedure was chosen rather than the reverse because for a given sway there can be only one set of loads that satisfy both equilibrium and compatibility; while for any given set of loads there can be two sway values which satisfy these same conditions. This can be seen from Fig. 2.

The procedure will be illustrated with reference to Fig. 2. To initiate the process a second order elastic analysis and a second order plastic analysis are performed for the frame and loading considered. This is done to provide an envelope within which the more exact analysis must lie and to provide the basis for the estimates which are involved.

Next the sway angle for which the analysis is to be performed is selected and an estimate made of the corresponding loads. For the proportional loading case shown in Fig. 2, this would mean that the value of w must be estimated that corresponds to a particular value of ρ on the true load-deformation curve.

The axial loads in the columns may then be determined and the M-$\emptyset$-P data tabulated for these loads.[16] In actual practice upper and lower limits are estimated for the axial load in each column and the M-$\emptyset$-P data tabulated for these. The M-$\emptyset$-P data for any intermediate

axial load is then determined by linear interpolation. It should be
noted that the M-$\emptyset$-P data is calculated for the actual cross section
used. The M-$\emptyset$ data for the beam is also computed assuming no axial
load,as the axial load in the beam was found to be negligible for the
cases considered.

Values of the moments at the two beam-to-column joints are
then assumed using the elastic analysis as a guide. These moments are
then applied to the beam and the end rotations are determined by a
numerical process similar to that developed by Newmark[19]. The M-$\emptyset$
relationship for the beam is used in this procedure to determine
curvature values in the inelastic range.

These same moments are then applied to the corresponding column
tops along with the axial loads and corresponding shears. The end
rotations from the beam integration program are used as initial guesses
in programs which integrate numerically to find the column end rotations
compatible with the applied forces. This process will be described in
detail in the following section.

In general the joint rotations as found by the column programs
are not those given by the beam program. In order to enforce compatibility
at each joint the assumed end-moments are adjusted and the beam and
column programs repeated until the joint rotations given by the beam
program match those given by the column programs. In this process of
adjustment the rotations as given by the beam and column programs are

plotted against the values of the end moments. The end moment
corresponding to the smallest rotation difference is held constant
while the other is adjusted until reasonable agreement is obtained
for the rotations at that joint. The other end moment is then
adjusted in the same manner. This second adjustment necessarily
disturbs compatibility at the first joint considered. The two
joints are again balanced using the graphs of rotation versus end
moment as a guide. In the example discussed later convergence was
achieved fairly easily. Part of the process for this example is
shown in Fig. 9. From a knowledge of the end moments, the sway,
and the axial loads; the horizontal force on the frame may be
computed from equilibrium. This value must correspond to that
estimated at the start of the program, otherwise the original assumed
load level must be changed and the complete process repeated.
Usually the need for this step can be recognized at some intermediate
point in the analysis and the load level adjusted accordingly.

The satisfaction of so many estimated conditions seems at
first to be an impossible task and, in fact, it would be without the
aid of the digital computer. However, several satisfactory guides
are available. The second order elastic analysis provides an ideal
starting place for the more exact analysis. Then, as the gradual shape
of the load-deflection curve takes form, each preceeding point serves as
a guide for the one to follow. All calculations were performed on the
Lehigh University GE 225 computer.

COLUMN INTEGRATION PROCEDURE

The column integration procedure is shown in Fig. 5. The column height, h, the sway angle, ρ , the axial load, P, the end moment, M, and the assumed slope, θ_o are given. The slope θ_o, measured with respect to the column chord, is calculated from the compatible beam rotation as described in the preceeding section ($\theta_o = \rho - \theta_{beam}$). The shear, V, is calculated from the equilibrium of the deflected column.

The angles and deflections are assumed to be small so that distances along the chord length are equal to their vertical projections. The slopes and deflections are measured using the chord as a base. The column is divided into increments, Δ, which are arbitrarily chosen to be equal to twice the major radius of gyration of the cross-section, r_x.

The procedure starts at the top of the column and works to find a point of zero deflection. The moment is first calculated at the center of an increment.[2]

$$M_{x+\frac{\Delta}{2}} = M + P\left[U_x + \theta_x \frac{\Delta}{2} - \rho\left(x+\frac{\Delta}{2}\right)\right] - V\left(x+\frac{\Delta}{2}\right)$$

The terms in this equation are defined in Fig. 5. The curvature, $\phi_{x+\frac{\Delta}{2}}$, is next computed. The M-ϕ data is fed into the program in tabulated form for the expected upper and lower limits of P. For a given moment, $M_{x+\frac{\Delta}{2}}$, the corresponding curvatures are determined for these limiting axial loads. Then a linear interpolation is used to estimate the curvature corresponding

to the actual value of P. In the performance of the calculations given

in this report the maximum and minimum values of P used in this step were

within 15% of one another so that the linear interpolation was not

assumed to introduce any significant error.

Once the curvature, $\phi_{x+\frac{\Delta}{2}}$, has been computed it is assumed

that this is constant over the increment length, Δ . Moment area

principles are then used to calculate $\theta_{x+\Delta}$ and $\upsilon_{x+\Delta}$ as:

$$\theta_{x+\Delta} = \theta_x - \phi_{x+\frac{\Delta}{2}} \Delta$$

$$\upsilon_{x+\Delta} = \upsilon_x + \phi_{x+\frac{\Delta}{2}} \frac{\Delta^2}{2}$$

When the calculated deflection, υ , becomes negative the

integration stops and the length of the column is calculated. If this

is not equal to the actual column length, h, the value of θ_o is revised

and the process repeated.

If the curvature at any point along the column length exceeds

that associated with the plastic moment of the column, M_{pc}, the program

stops and a hinge is assumed to have formed.

It is possible to choose a value of θ_o that leads to an

inflection point in the column. In this case a smaller value of θ_o is

chosen and the process repeated.

In the column procedures, as well as in the beam procedure, the computer output can be directed to give values of the moments, curvatures, slopes and deflections for all node points along the length of the frame.

LOAD-DEFLECTION RELATIONSHIP FOR FRAME WITH CONSTANT VERTICAL LOAD

As an example of the method the load-deflection curve for the frame shown in Fig. 6 will be computed. The frame is loaded by vertical column loads which remain constant at 0.3 P_y. (P_y signifies the area of the column cross section multiplied by the yield stress level, $A\sigma_y$). This loading condition would be similar to that of the lower story of a multi-story frame acted upon by the lateral forces due to an earthquake.[9]

To simplify the procedure for this example and to provide a check on the work of Reference 14, it is assumed that the beam remains elastic and that the axial load in each column remains constant at 0.3 P_y. Thus the influence of unloading need not be considered. Further, the beam rotations can be related to the end moments through the elastic slope-deflection equations. The analysis is then reduced to that of the restrained beam-column shown in Fig. 4(b).

The dimensions and sections for this example have been chosen so that the results of Reference 14 could be used as a check. The yield stress is 33 ksi and the modulus of elasticity 30,000 ksi. The M-$\emptyset$-P relationships used for this example are based on a residual stress pattern which has a maximum value of $0.3\sigma_y$ compression at the flange tips.[16] The results of the analysis are given as the curve of applied horizontal force, V, versus the sway angle, ρ , shown in Fig. 7. The maximum value obtained for V was 26.2 kips. The analysis performed in Reference 14 produced a value of 26 kips. Both analyses show this maximum to occur at a sway angle of 0.02 radians. It should be noted that for this particular example the

assumptions used are the same as those in Reference 14. In the present

method the deflected shape of the column was obtained by numerical

integration.

In Reference 14 this shape was taken as a portion of a Column

Deflection Curve[2] and an algebraic expression was fitted to the

resulting moment-rotation curve. The load-deflection relationship for

the restrained beam-column was then expressed in closed form and the

maximum value of V obtained. A further check on the validity of the present

method is provided by the fact that the true $V - \rho$ curve is tangent

to the second order elastic curve for low values of ρ and to the second

order plastic curve once a hinge has formed at the column tops. This

coincides with the point at which the computer program cannot find a

curvature at the column top which is compatible with the moment and

sway angle. The analysis could of course be extended by determining

the $M-\phi-P$ relationship in the strain hardening range and using this to

eliminate the discontinuity which occurs due to the assumptions of

elastic -perfectly plastic material.

It should be noted that for the particular example chosen, with

a maximum compressive residual stress in the beam of $0.3\sigma_y$, the beam

would begin to plastify at a sway angle which lies between 0.0175 and

0.0200 as shown in Fig. 7. The axial load in the left column decreased

from 0.30 P_y at zero horizontal load to 0.27 P_y at the sway corresponding

to the maximum horizontal load. The load in the right column increased

correspondingly. The change in axial load is 10%. Thus, unloading of
the left hand column does not seriously influence the results of the
analysis.

INELASTIC FRAME BUCKLING

Two different approaches are available to deal with buckling problems. The first is to seek the two infinitely close equilibrium positions which the structure may assume at the point of neutral equilibrium. Associated with this approach is the eigenvalue type of solution.

The second method assumes some initial imperfection and then solves the resulting second order instability problem. As the assumed initial imperfection approaches zero, the bifurcation load of the first method is approached.

Lu[10] has used the first approach to determine the inelastic buckling load for the structure pictured in Fig. 8 (a). The approach selected was to first assume a value of w in the stable range. Then, by assuming a symmetrical deflection configuration such as that shown in Fig. 8 (b), the moments and resulting stiffnesses throughout the frame could be computed.[13] Using the stiffnesses so obtained and assuming the total vertical loads concentrated on the column tops an elastic analysis was performed. This determined the load at which the frame would buckle into the sidesway configuration shown in Fig. 8 (c). If this load coincided with the assumed load the correct inelastic buckling load had been reached. For the frame shown in Fig. 8 (a), w_b was found to be 2.28 kips/ft. The maximum load for a similar braced frame was

found to be 2.38 kips/ft. by the methods of Reference 13.

In applying the second approach to the inelastic frame buckling problem it is realized that for a frame which is stable under vertical loads it will be possible to subject it to an arbitrary (small) sway and to calculate the resulting horizontal load, V. This horizontal load is in the same direction as the sway. At the vertical load corresponding to the limit of stability, the horizontal load corresponding to an arbitrary sway will be equal to zero.

The frame shown in Fig. 8 (a) was subjected to a vertical uniformly distributed load of w = 1.80 kips/ft. and column top loads, F, of 158.5 kips. The total axial load in each column was 0.190 P_y. An analysis was carried out in the unswayed condition to determine the extent of plastification, the joint rotations and moments. At this stage the top portions of both columns had partially yielded as well as the center and ends of the beam. The moment was 9,300 in. kips at the corners and 11,640 in. kips at the center of the beam.

A sway angle of 0.01 radians was then selected as the arbitrary sway and the frame analyzed to determine the corresponding horizontal load, V. In this swayed position the moment at the left corner dropped to 5340 in. kips and that at the right corner increased to 12,750 in. kips. In this condition the axial load in the left column dropped to 0.187 P_y while that in the right column had a corresponding increase to 0.193 P_y. The right end and center portions of the beam were yielded as well as the top half of the

right column. The left column was completely elastic. The horizontal
load, V, was computed to be 4.59 kips in the direction of the sway.

The load w was then increased to 2.22 kips/ft. and a procedure
similar to that outlined above performed. In the zero sway condition the
moments at the corners were 11,500 in. kips and the axial load in each
column was 0.234 P_y. At a sway of 0.01 radians the moment at the left
corner was 8,950 in. kips while that at the right 13,500 in. kips. Due
to the low value of the horizontal load the change in the axial column
load was negligible. The compatible horizontal load was 0.11 kips,
opposing the sway.

The above analyses were performed using the procedure outlined
earlier in the report. The beam rotations were determined by a numerical
process that considered the gradual yielding when the beam was subjected
to the distributed load and end moments. The numerical procedure for
determining the deflected column shape considered the change in axial
load induced by the horizontal load on the frame as well as that due to
sway.

To enforce compatibility at the beam-to-column connections it was
necessary to change the assumed values of the joint moments until the
rotations as given by the beam process were equal (within 3%) to those
given by the column process. For each value of w approximately eight
trials were necessary in the swayed position.

The first and last steps of the trial and error process for

w = 1.80 kips/ft. are illustrated in Fig. 9. Here assumed values of the

moment at the left beam-to-column joint, M_L, are plotted against the

resulting rotations, θ_L, as determined by the beam and left column

integrations. This particular plot is for w = 1.80 kips per foot and

ρ = 0.01 radians. The initial estimate of M_R, the moment at the right

hand beam-to-column joint was 11,200 in. kips.

The points for M_R = 11,200 in kips were joined by the lines

shown. The value of M_L corresponding to zero error, in this case

5,700 in. kips, was used to begin a second series of trials to obtain a

closer estimate of M_R. This process was repeated eight times to obtain

convergence. The final stage in the process, trial 8, is also shown in

Fig. 9. At this point M_R = 12,750 in kips and θ_R has been determined

as 0.0100 radians. These values have been obtained by assuming M_L =

5,340 in. kips and were taken as final values. From Fig. 9 the average

value of θ_L is 0.0170 radians and the difference between the rotation

given by the beam program and that given by the column program is 2%.

For each set of values of M_L and M_R there is a corresponding

value of V, the horizontal load on the frame. As the trial and error

procedure described above progresses, the value of V could be estimated

closely and the axial loads in the columns adjusted accordingly. Thus,

the axial loads at the final stage were close to their required values and

no further adjustment was made.

The results of the computations are shown in Fig. 10. This figure presents the relationship between w and V, the horizontal load obtained for a sway angle of 0.01 radians. It can be seen that at w = 1.80 kips/ft the frame is stable and that at w = 2.22 kips/ft. it is unstable. Thus, the critical load is bounded by these values. The value obtained by Lu[10] for this same frame was 2.28 kips/ft. A linear interpolation as shown by Fig. 10 would give a critical value of w = 2.21 kips/ft.

It should be noted that as the frame moves from the unswayed to the swayed position the left hand column and adjacent beam portions unload. This unloading has not been considered in the analysis and at present there is no way to evaluate its influence.

In future, the influence of unloading will have to be accounted for. This could be done by first neglecting that unloading caused by a decrease in axial load. This is justified because the change in axial load comes about primarily due to an increase in the horizontal load on the frame. This horizontal load is usually significantly smaller than the vertical loads. In addition, for a partially yielded section, the change in curvature due to a change in axial load is small compared to that for a corresponding change in moment.

If only the unloading due to a change in moment is to be considered the computer program could be easily altered so that it would compare the moment at a given station with the moment at the same station for the previous load increment; then calculate the curvature, either

by using the M-$\emptyset$-P relationship for loading stations, or by using

elastic principles for unloading stations.

PROPORTIONAL LOADING

In simple plastic theory[7] failure is assumed to occur
through the development of sufficient "plastic hinges" at points of
maximum moment so that a mechanism is formed. No recognition is
given to the secondary moments produced by the loads on the structure
acting through the displacements of their points of application. In
reality, this so called $P\Delta$ effect, plus the spreading of the zones of
partial plastification, serve to reduce the ultimate load of a frame
below the value given by simple plastic theory.

Some estimate of this reduction may be gained by computing
the point of intersection of the second order elastic and plastic
analyses on a load-deformation diagram. This is given as point E in
Fig. 2. However, an additional reduction will also occur, and the
methods of this report may be utilized to determine the magnitude of
this reduction. It may be found that the reduction in strength from
that value given by simple plastic theory will be significant only for
very slender structures or for structures under high axial loads.

The method used for the analysis of this proportional loading
case will be essentially that outlined in previous sections. In fact
the value of $V = 4.59$ kips obtained in the previous example for $w = 1.80$ kips/ft.
and $\rho = 0.01$ radians is essentially one point on the load-deflection
curve for the same frame under proportional loading conditions with $V = 0.04$ wh.

The limits of the axial load in the columns would change significantly

with each new value of ρ chosen, so that new limiting sets of M-$\emptyset$-P

data would have to be inserted for each run of the program.

EXTENSION TO OTHER CASES

In theory, these same basic principles of equilibrium and compatibility that have been applied to the analysis of the simple pinned base frame could be applied to more complex structures. In application, however, this would involve so many initial assumptions that the iteration processes involved would render the analysis impractical. Thus, extension of the procedure outlined in this report to other more complex cases does not appear to be warranted.

However, some of the ideas that are incorporated in this study may find a place in the analysis of multi-story frames if a suitable procedure can be found for their utilization. The column integration process in particular may find a place in the analysis of more complex structres.

SUMMARY AND RECOMMENDATIONS

The pertinent results of the work described in the previous sections may be summarized briefly as follows:

1. A method has been presented which allows the determination of the load-deformation curve for a simple pinned-base frame. The method uses directly the principles of equilibrium and compatibility as expressed by the M-ϕ-P relationship.

2. The method accounts for the partial yielding along the length of the members, the secondary moments caused by the sway of the frame, and the changes in axial load in the columns.

3. The method was used to analyze the restrained beam-column subjected to sway. The results coincide almost exactly with those obtained in a previous report.[14]

4. The inelastic buckling load for a frame subjected to the loads shown in Fig. 8 (a) was bounded between 1.80 kips/ft. and 2.22 kips/ft. with the expected value close to the latter. The load as previously determined was 2.28 kips/ft.[10]

5. A procedure has been outlined for the analysis of the proportional loading case.

Principles for the analysis of simple frames have been presented; however, few frames have been analyzed. Some of the obvious extensions that might be considered are:

1. The strain-hardening portion of the stress strain curve could be easily included in the analysis.

2. The influence of unloading could be determined.

3. The determination of the sidesway buckling load for
inelastic frames is tedious and it has been suggested that approximate
methods should be developed which would eliminate the need for such a
calculation. The procedure presented herein could be utilized to
determine the critical loads for a number of frames under vertical
loading and also the load-deflection relationships for these same
frames under combined lateral and vertical loads. Using these
results as a basis, an approximate method of analysis for the frame
under an equivalent combined loading would be sought which would
replace the frame buckling analysis.

4. Computer analysis of frames subject to proportional
loading could be initiated to fully evaluate simple plastic theory
as it is now applied.

5. Thought could be given to the modification of the
procedures outlined in this report for use in multi-story frame
analysis.

ACKNOWLEDGEMENTS

NOMENCLATURE

σ_y static yield stress

σ_{RC} maximum compressive residual stress

w   uniformly distributed vertical load on beam

F   concentrated vertical load at column top. F (w)

           indicates that this load is a function of w.

V   concentrated horizontal load at beam-to-column joint

A   area of cross-section

L   length of beam

h   column height

I_B moment of inertia of beam about major axis

I_C moment of inertia of column about major axis

r_X radius of gyration about major axis

P   total axial load

P_y $A \cdot \sigma_y$

M   bending moment

$M_{\mathbb{C}}$ bending moment at mid-span of beam

M_y $\dfrac{2 \cdot \sigma_y \cdot I}{d}$ where, d, represents the depth of the section and,

                            I, its moment of inertia about the major axis.

ϕ curvature

ϕ_y $\dfrac{2\sigma_y}{dE}$ where, E, is the modulus of elasticity

θ rotation

θ_o rotation at top of column, measured with respect to the chord

θ_X rotation measured with respect to chord at a distance x

            from top of column

Δ increment of column length

ν_X deflection at a distance X from top of column, ν_X is measured

            from chord

w_b the value of w at which a frame buckles into a sidesway mode.

w_{SP} the value of w given by simple plastic theory

w_P the value of w given by simple plastic theory and modified to

            account for the reduction in moment capacity due to the presence

            of axial load.

ρ the angle between the column chord and the vertical after sway
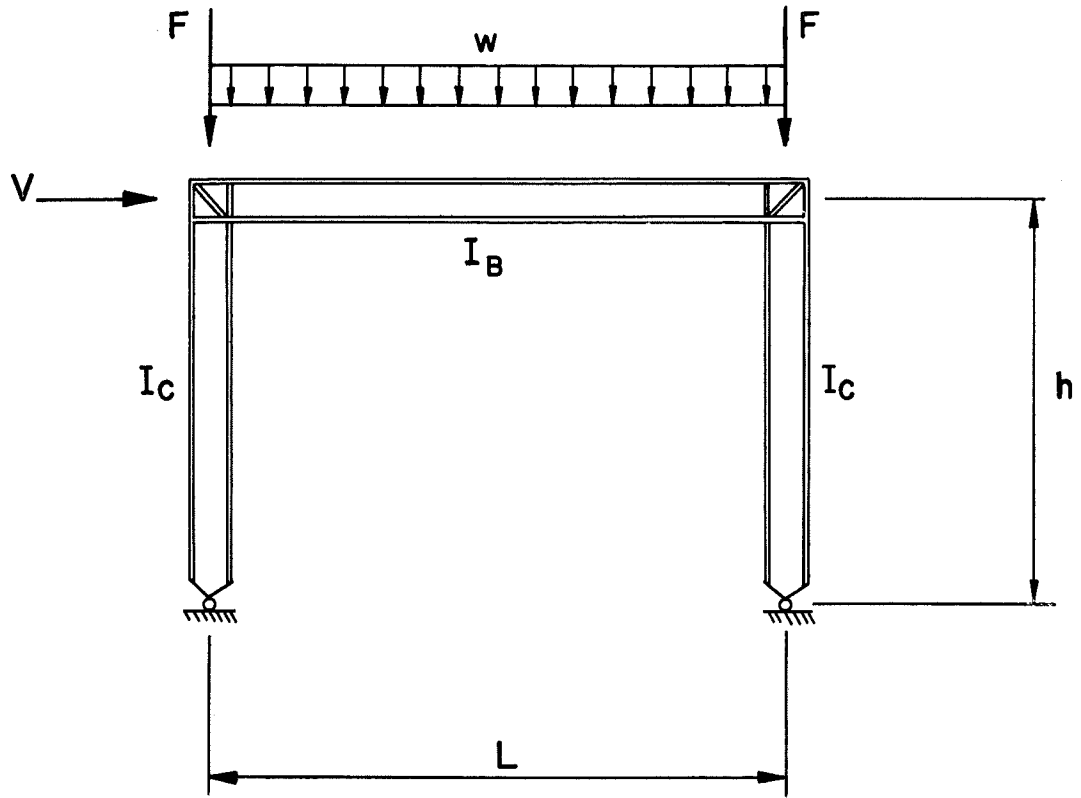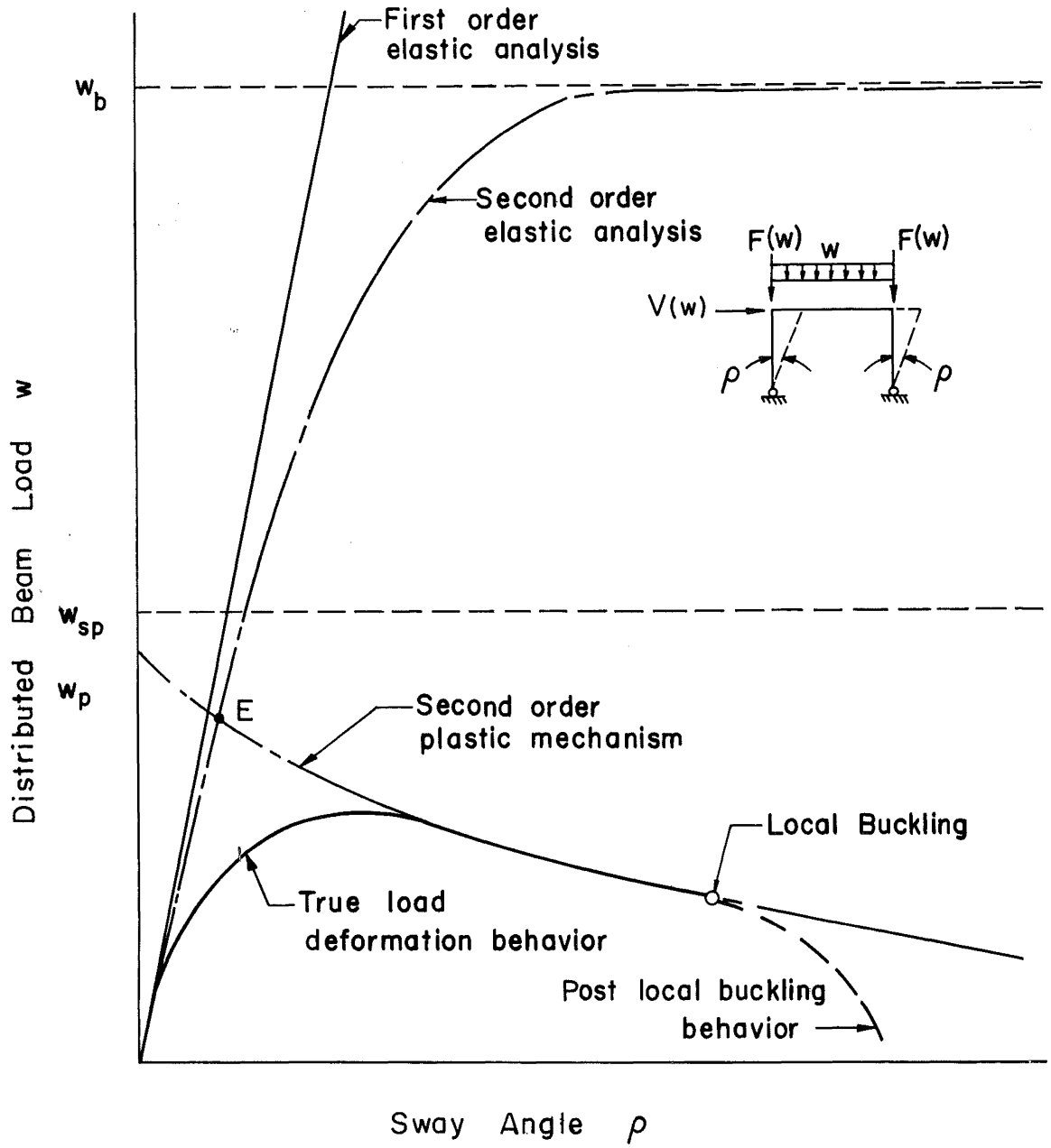
            has occurred

Fig. 1   Frame and Loading
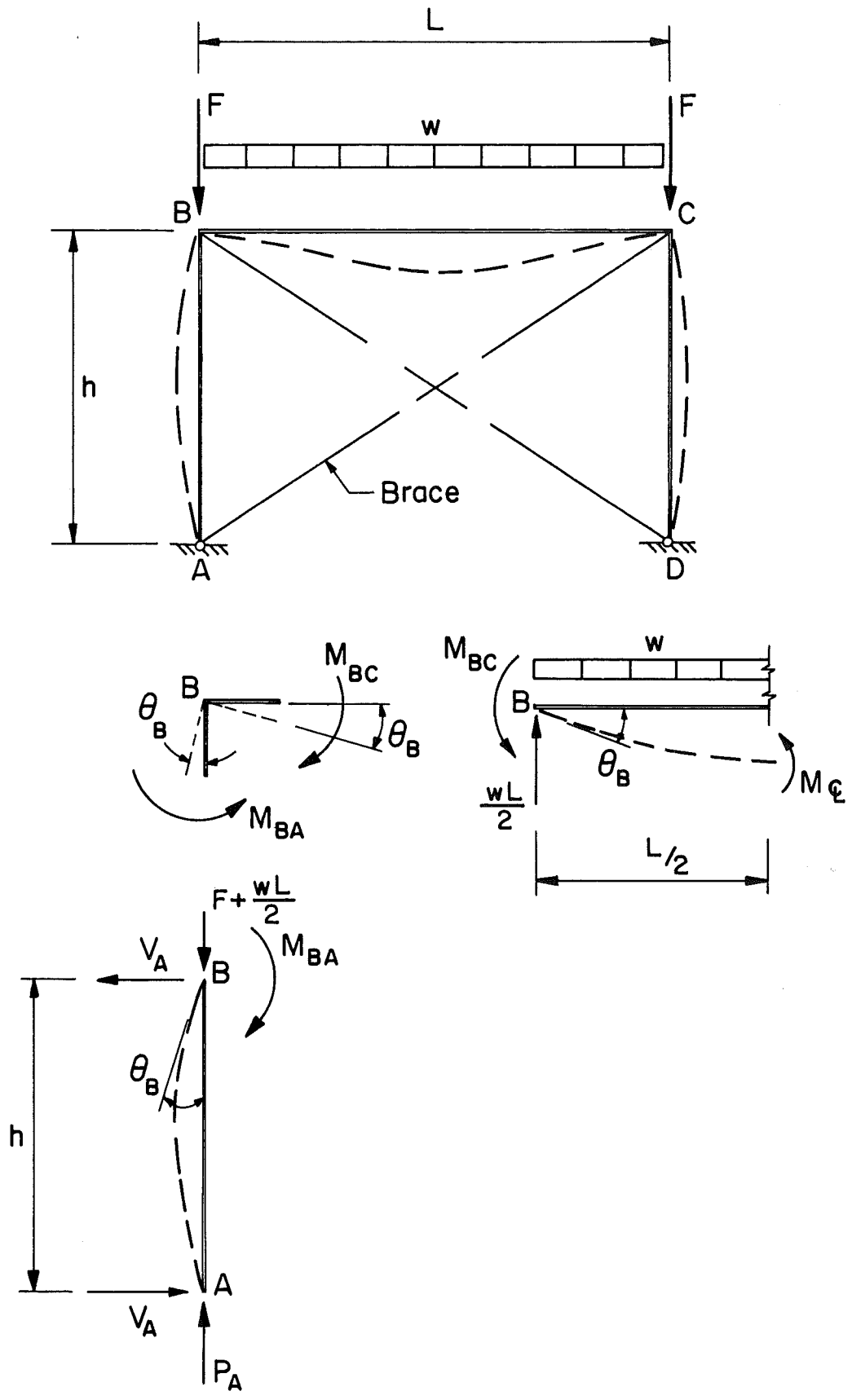
Fig. 2   Load-Sway Relationship

Fig. 3   Braced, Single Story

Frame Relationships

Fig. 4 (a)   Unbraced Frame

Fig. 4 (b)   Restrained Beam-Column

$$\theta_B^* = \rho - \theta_B$$

Fig. 4 (c)   Equilibrium and Compatibility
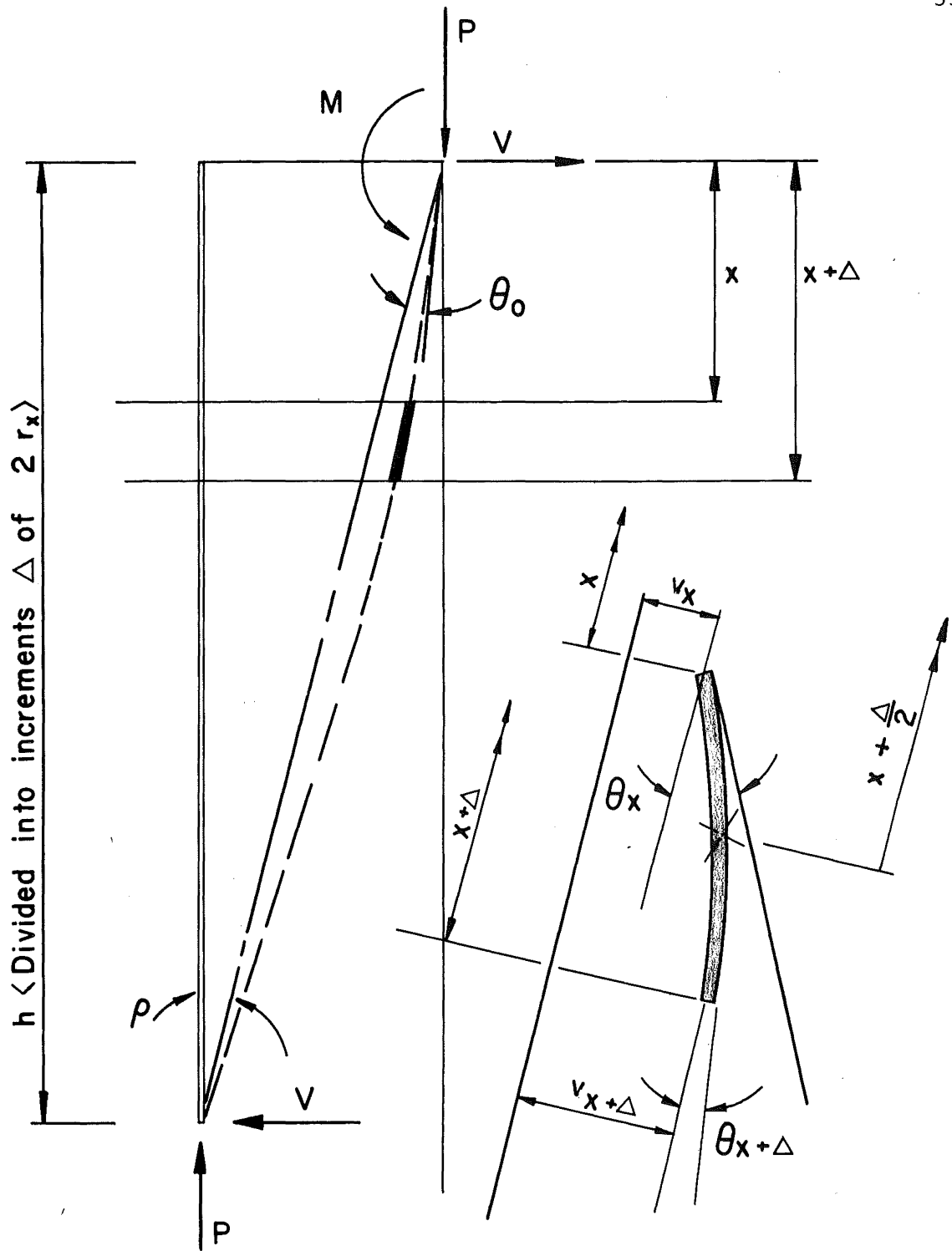
Fig. 5   Column Integration Process
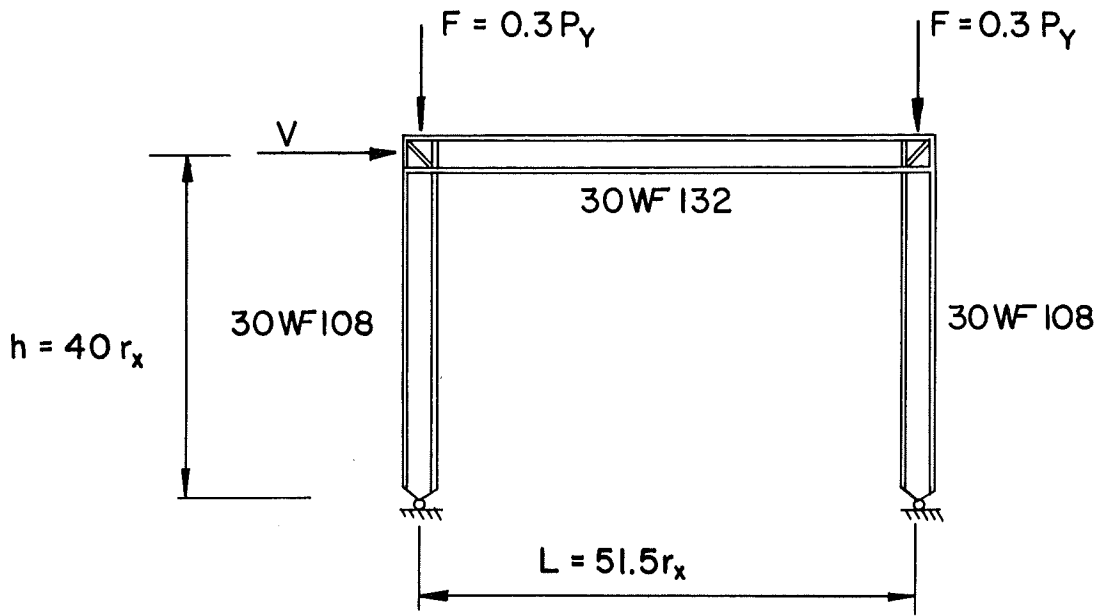
Fig. 6   Frame Subjected to Concentrated Loads

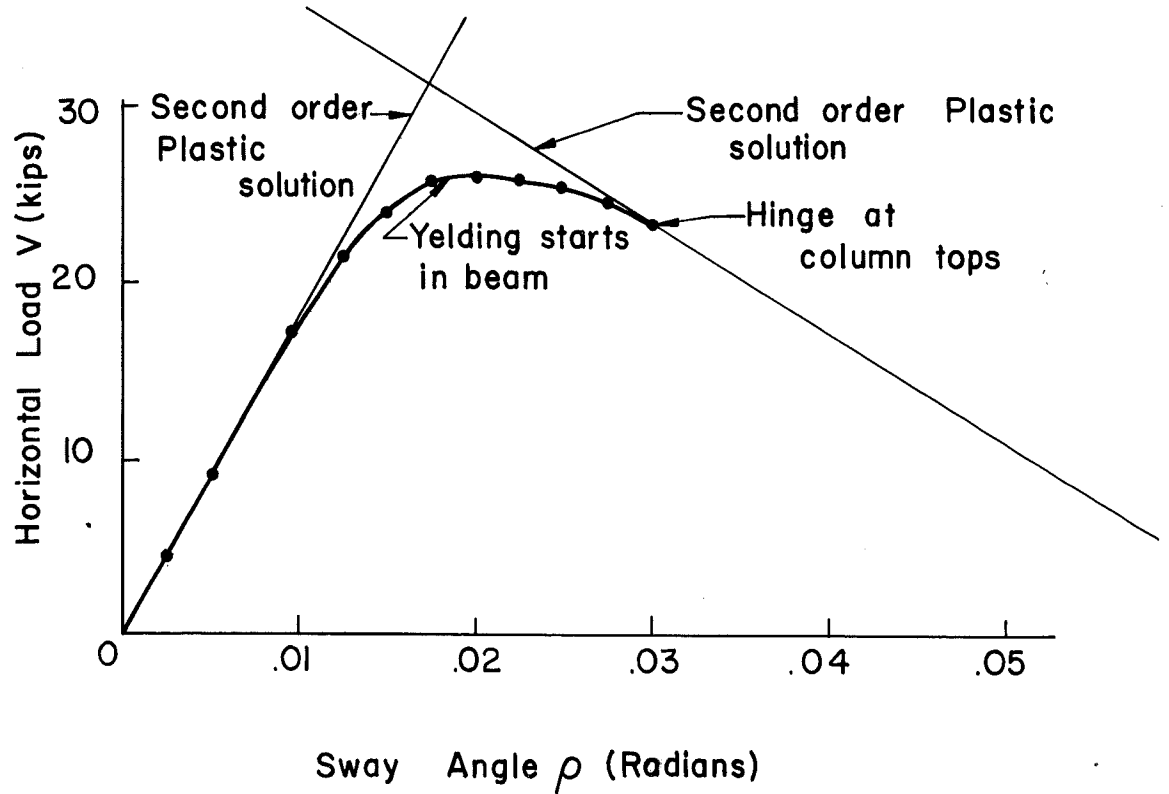Sway   Angle ρ (Radians)

Fig. 7   Load Deflection Relationship for Frame

Subjected to Concentrated Loads
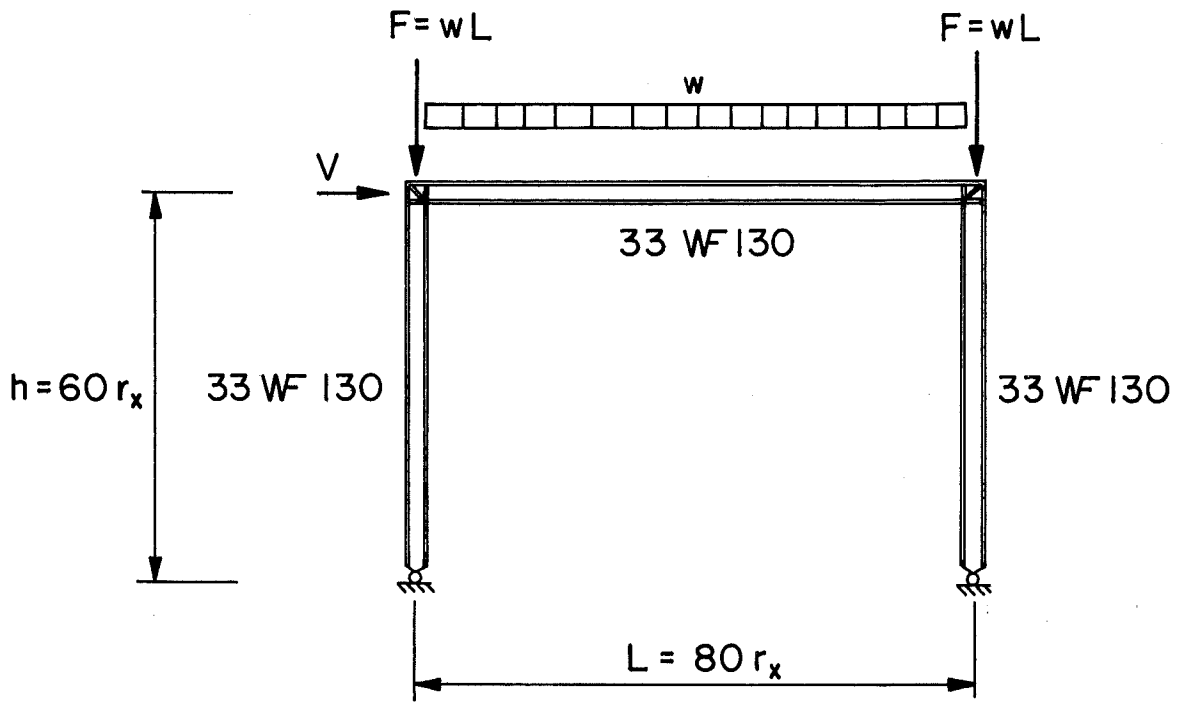
Fig. 8 (a)　　　Frame Buckling Example

Fig. 8 (c)

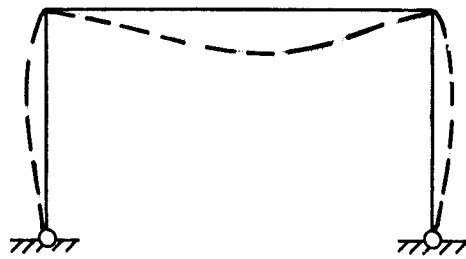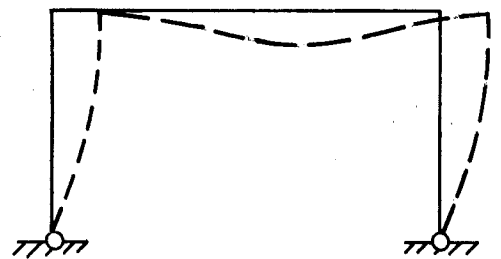Sidesway Deflection Configuration

Fig. 8 (b)

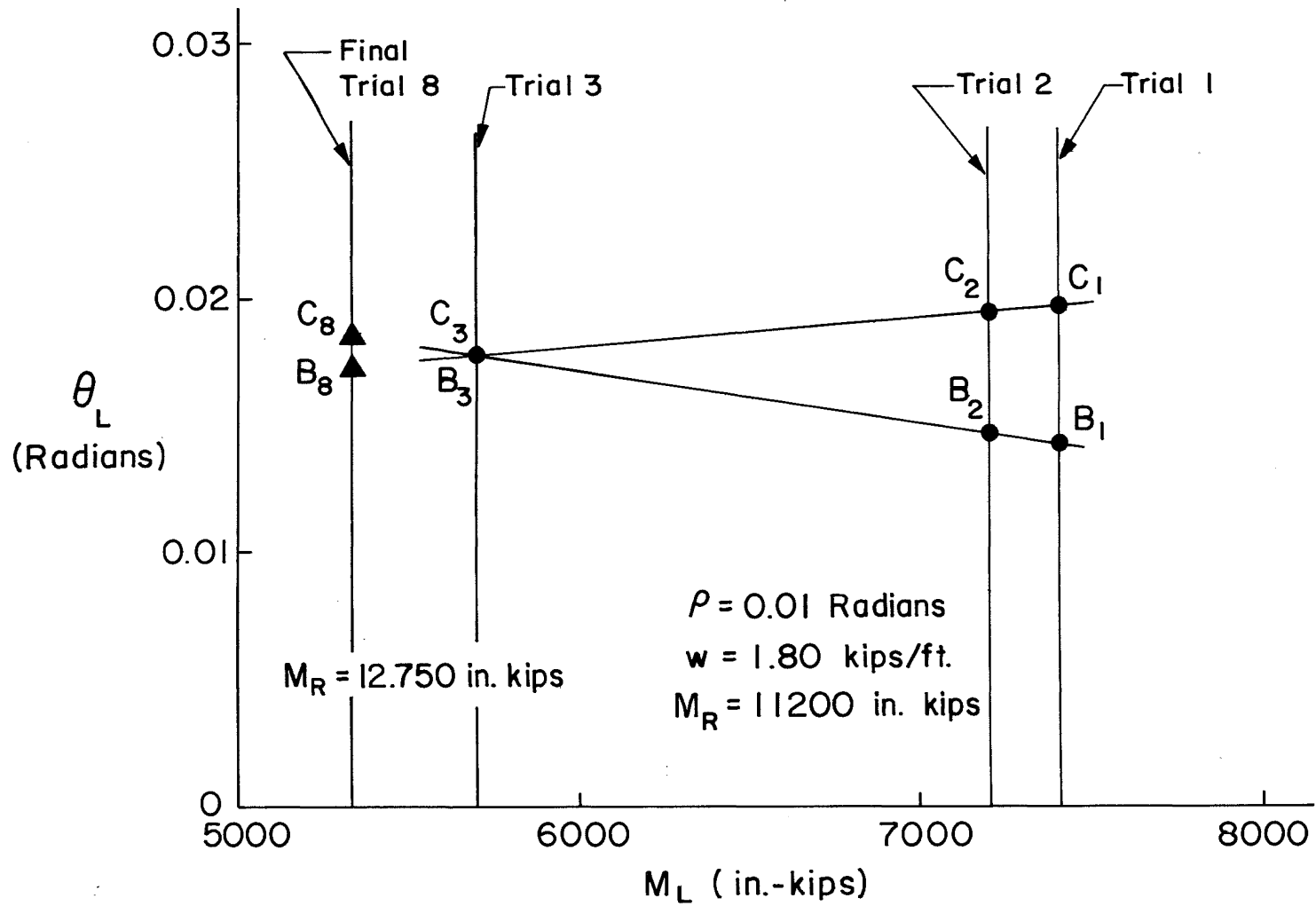Symmetrical Deflection Configuration

Fig. 9   Interpolation Procedure to Enforce Joint Compatibility
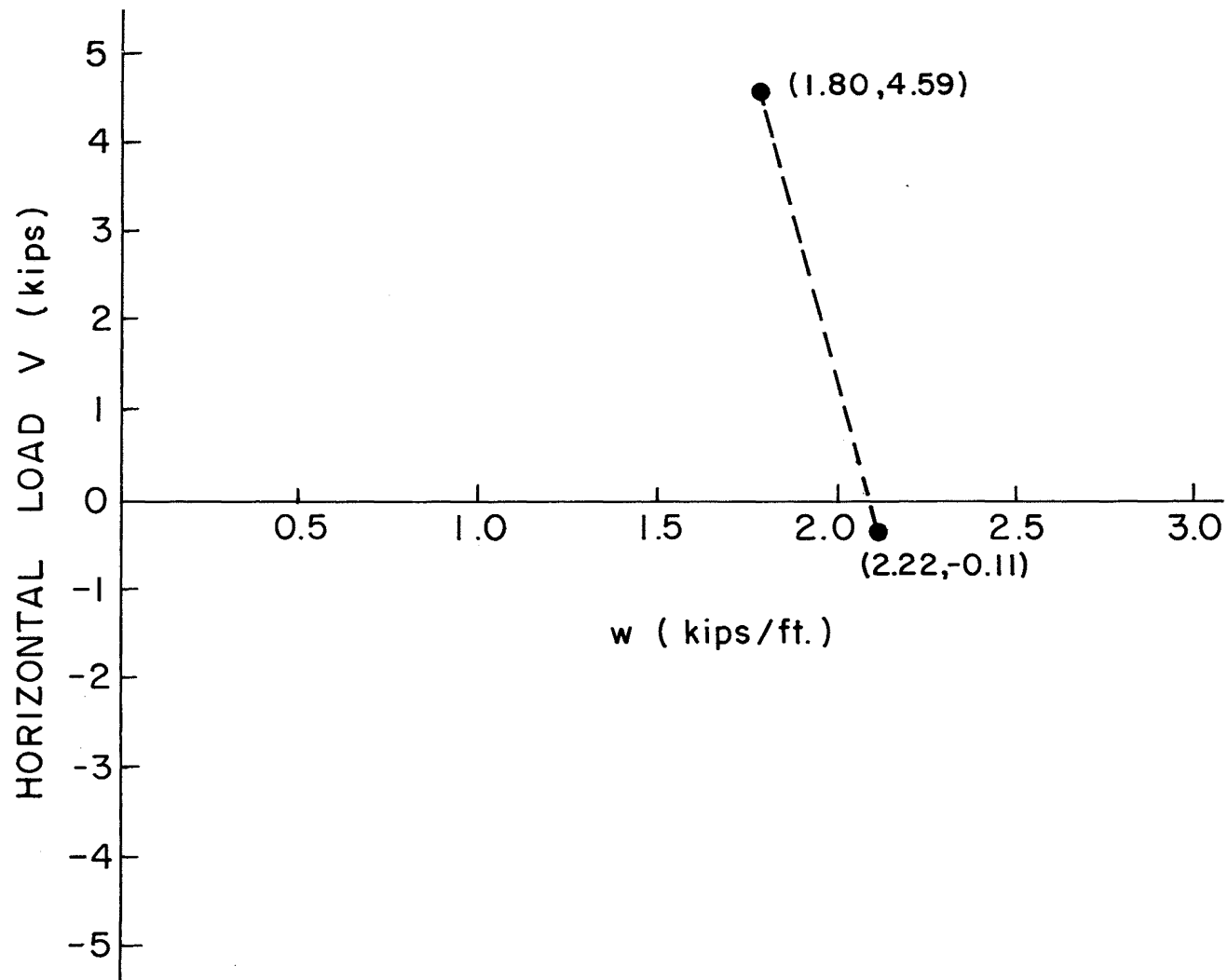
Fig. 10   Relationship Between Distributed and Horizontal Loads

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