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# Load deflection curve of braced and unbraced three-story frame, September 1963

G. C. Driscoll Jr.

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BY GEORGE C. DRISCOLL, JR.

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# LOAD-DEFLECTION CURVE OF BRACED & UNBRACED THREE-STORY FRAME

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George C. Driscoll, Jr.

This report is prepared to present results of a load-deflection computation up to ultimate load for a three-story two-bay frame subjected to proportional gravity and wind loads.

## Problem:

The frame shown in Fig. 1 resulted from a trial design for a typical uniform floor load and wind load. The problem was to make an approximate calculation of deflection for the frame with or without the 6 inch angle braces.

In an elastic slope-deflection solution without bracing, there would be 30 moments to be determined at the ends of the 15 beams and columns; there would be 30 slopes to be determined at the ends of the members and 3 story sways to be determined, for a total for 63 unknowns. To solve for those 63 unknowns there are available 30 slope-deflection equations for the bars, 21 equations for compatibility of rotations of joints, 9 equations of equilibrium of moments at joints, and 3 equations for shear in stories.

With the 6 inch angles added as braces, the number and type of unknowns would not change. The magnitude of force in each diagonal brace could be shown to be a constant times the sway of the story. The only change in the equations for solution of the problem would be a term directly proportional to story sway inserted in each of the 3 equations for story shear to account for the resistance of the brace.

## Assumptions:

For purposes of solution simple plastic theory was assumed. Elastic behavior was assumed at every point on every member except at plastic hinges which were assumed to be concentrated at a point. The effect of axial load on the stiffness of members was neglected. It was assumed that once a plastic hinge had formed the moment at the point neither increased nor decreased.

## Solution:

For each of the two cases the 63 simultaneous equations were formulated. Then they were reduced to 48 simultaneous equations by applying some of the simpler boundary conditions regarding rotations of members meeting at joints. A computer program for the direct solution of up to 48 simultaneous equations with a punched card output was used. For each case 28 end moments and 20 deformations were obtained in terms of a load parameter.

A step-by-step procedure was used to determine the order of formation of plastic hinges and the increments of load, moment, and deformation between formation of hinges.

A subroutine was used to determine the increase in load between formation of each plastic hinge. A trial process was used in which it was determined how much increase in load was required to form each possible plastic hinge. The hinge requiring the least increment of load would form first.

A second subroutine was used to determine the increase in each moment and deformation based on the previously obtained load increment.

After each plastic hinge was determined, the boundary conditions were changed in the original 63 and 48 simultaneous equations so that the increment of behavior of the altered structure could be determined. The cycle of determination of load increment and determination of moments and deformations caused by the load increment was repeated as often as necessary.

When plastic hinges formed under the distributed load on the beams, it was necessary to account for the movement of the location of the hinge along the member as the load increased.

Results:

In Fig. 2 the load on each floor girder in kips is plotted versus the horizontal deflection of the roof in inches for both the braced and the unbraced frames. The deflection of the unbraced frame in the elastic range at 120 kips is more than three times as much as the deflection of the braced frame at the same load. The horizontal deflection of the unbraced frame at 120 kips is nearly twice as much as the deflection of the braced frame at ultimate load. At ultimate load the horizontal deflection of the unbraced frame is nearly 32 times the ultimate deflection of the braced frame. At an approximate working load of about 116 kips (load factor 1.85) the deflections would be approximately 0.0012 and 0.0003 times the height of the unbraced and braced frames respectively. Both of these values are probably quite tolerable from a design standpoint.

Sketches of the braced and unbraced frame showing the location of plastic hinges are given in Fig. 2. Numbers next to each hinge location show the sequence in which the hinges formed. Similar numbers on the load-deflection curves show the state of the structure at the formation of each hinge.

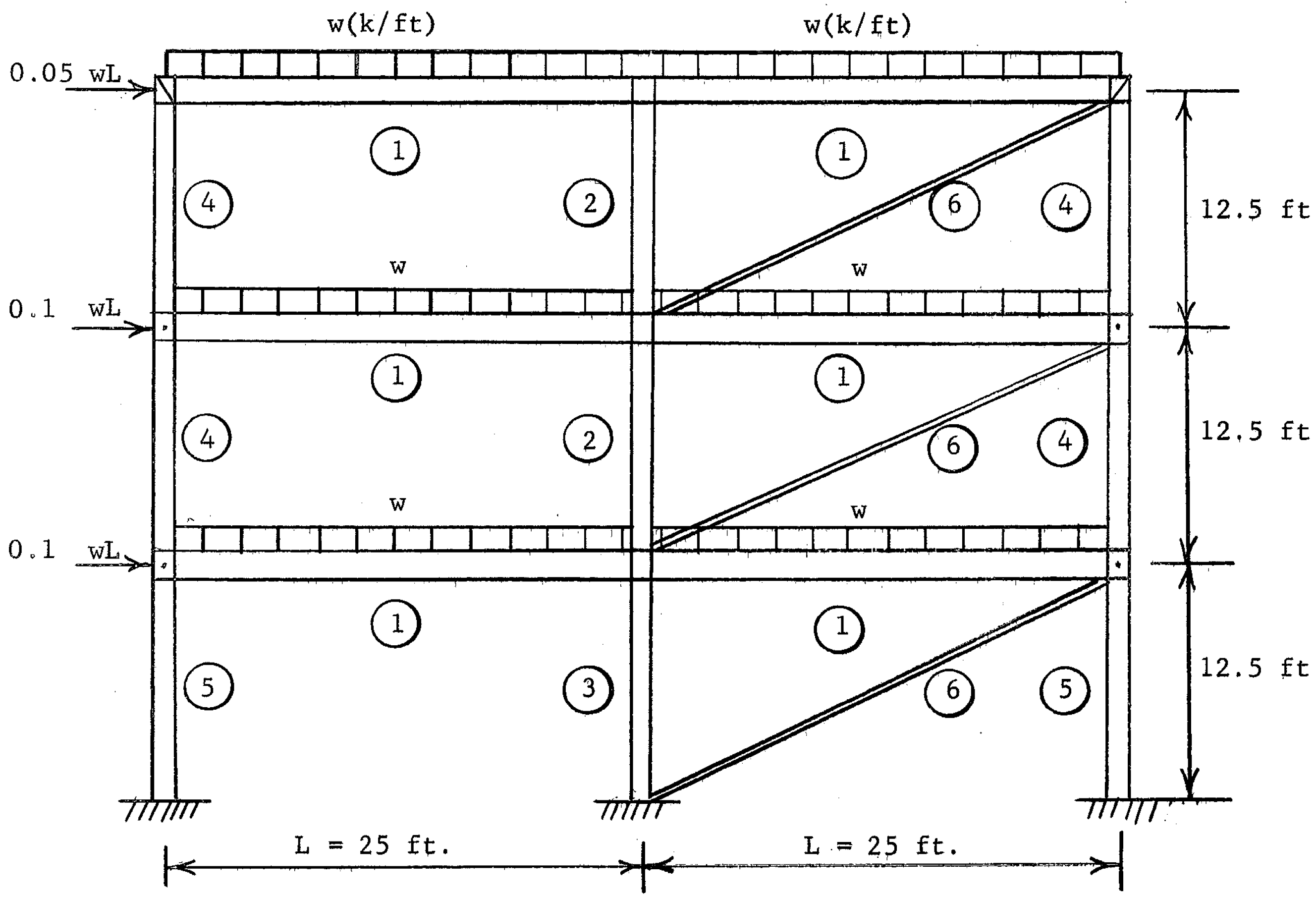
At the formation of the 12th plastic hinge for the unbraced frame 96 percent of the load but only 40 percent of the deflection have been attained. It takes an awful lot of extra deformation to obtain that final 4 percent of load. However, if rotation capacity and frame stability were only adequate to attain the 12th plastic hinge, the actual load factor over working load would still be more than 1.75.

It is interesting to compare the order of plastic hinge formation for the two frames. In the unbraced frame, the trend follows that known to occur in two-bay single-story frames. First, hinges form at the lee ends of the windward girders and then at the lee ends of the lee girders. Succeeding hinges form beneath the uniformly distributed load in the girders. Finally, hinges form at the bases of the columns progressing from the leeward toward the windward side. The only surprise is the hinge forming in the lee column just above the lowest girder.

In the braced frame, six hinges form in the girders adjacent to the center column before any hinges form near the exterior columns. After six additional hinges form at the outer ends of the girders, the final six hinges form simultaneously at the center of the girders. The result is six beam mechanisms.

In Fig. 3, the final moment diagrams for the two frames are given. The moment distributions in the girders are almost identical showing that the unbraced frame was on the verge of forming beam mechanisms as it reached ultimate load. Fig. 3a also shows the forces in the diagonal braces at ultimate load. The 43 kip force in the lower brace represents a unit stress of only 12 ksi although it represents a horizontal shear resistance of 39 kips on the story. This means that the lower story columns of the braced frame resist a shear of only 15 kips as compared with 54 kips resisted by the same columns in the unbraced frame. This explains the large difference in the moment diagrams of the columns in the two frames.

The large deflection of the unbraced frame suggests its greater susceptibility to instability problems.




| Member | | $M_p$ (k-ft) | |
|---|---|---|---|
| (1) | 18 WF 60 | 337 | Circled number |
| (2) | 14 WF 61 | 282 | designates shape |
| (3) | 14 WF 78 | 369 | used. |
| (4) | 14 WF 74 | 345 | |
| (5) | 14 WF 84 | 400 | |
| (6) | Angle 6 x 6 x 5/16 | (Braced Frame Only) | |

Fig. 1. Dimensions and Loading of Braced and Unbraced Frames

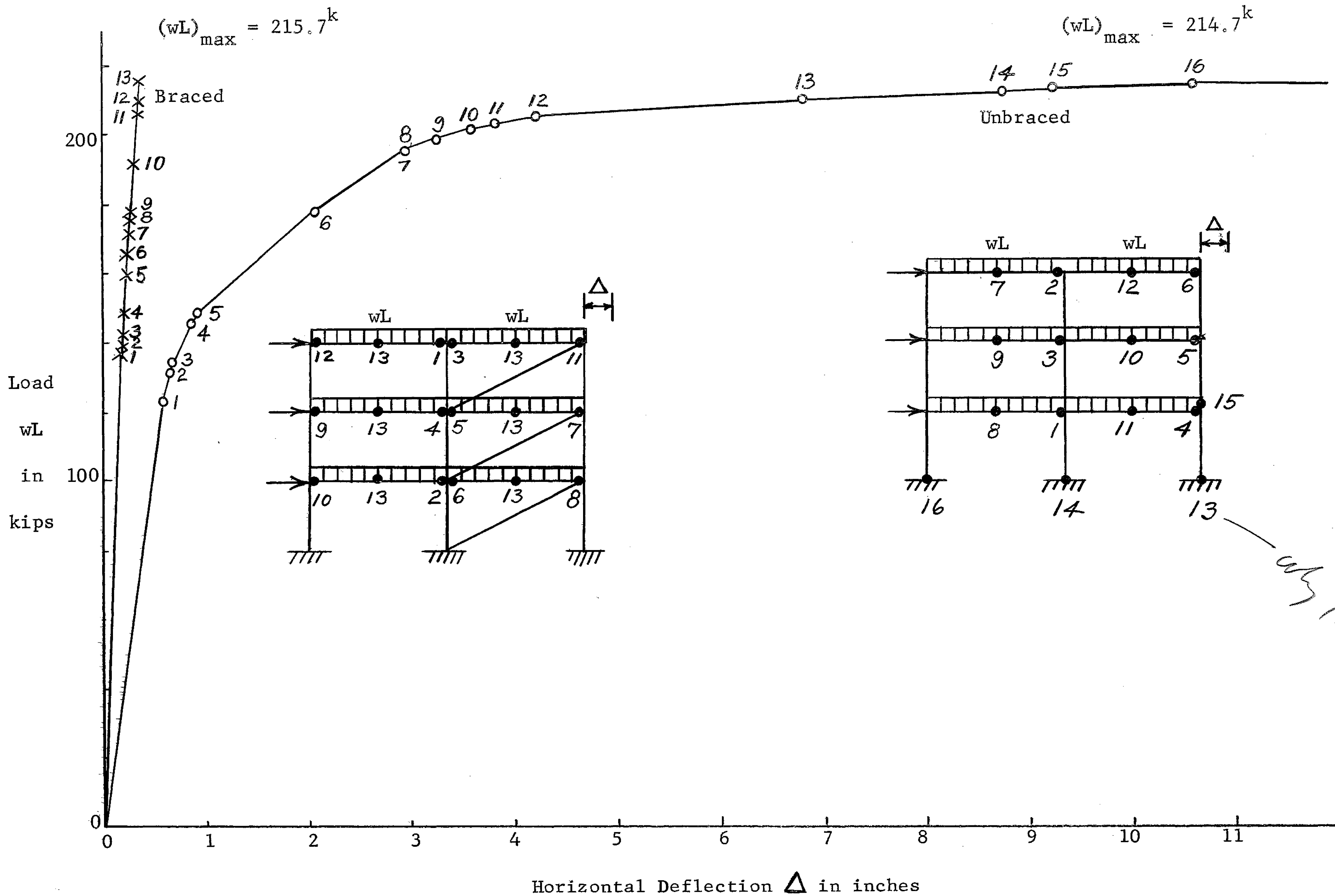



Fig. 2 Load-Deflection Curves for Braced and Unbraced Frames

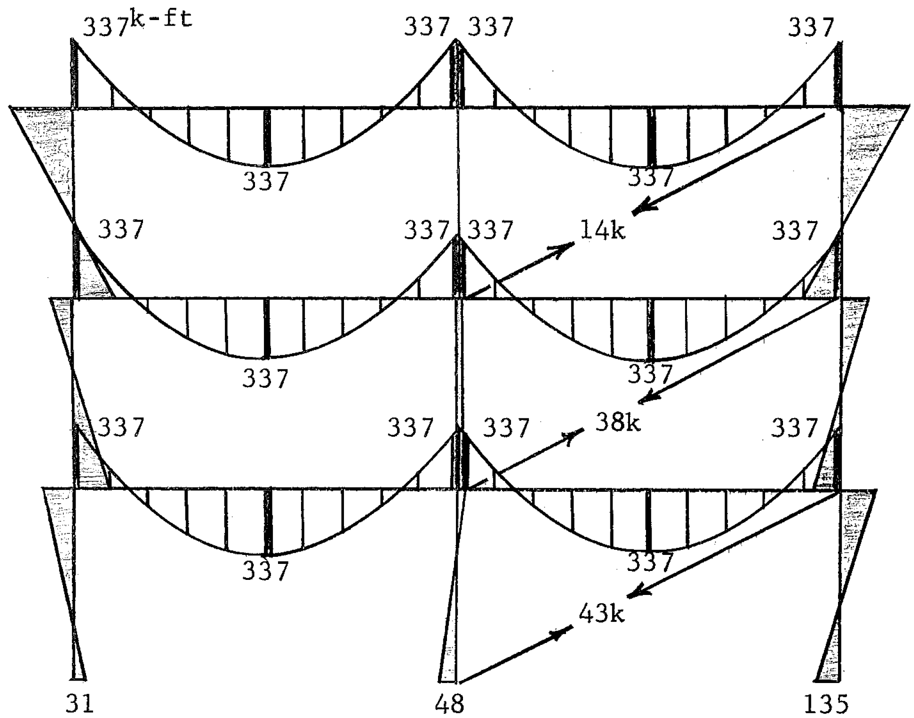



Fig. 3a. Final Moment Diagram for Braced Frame

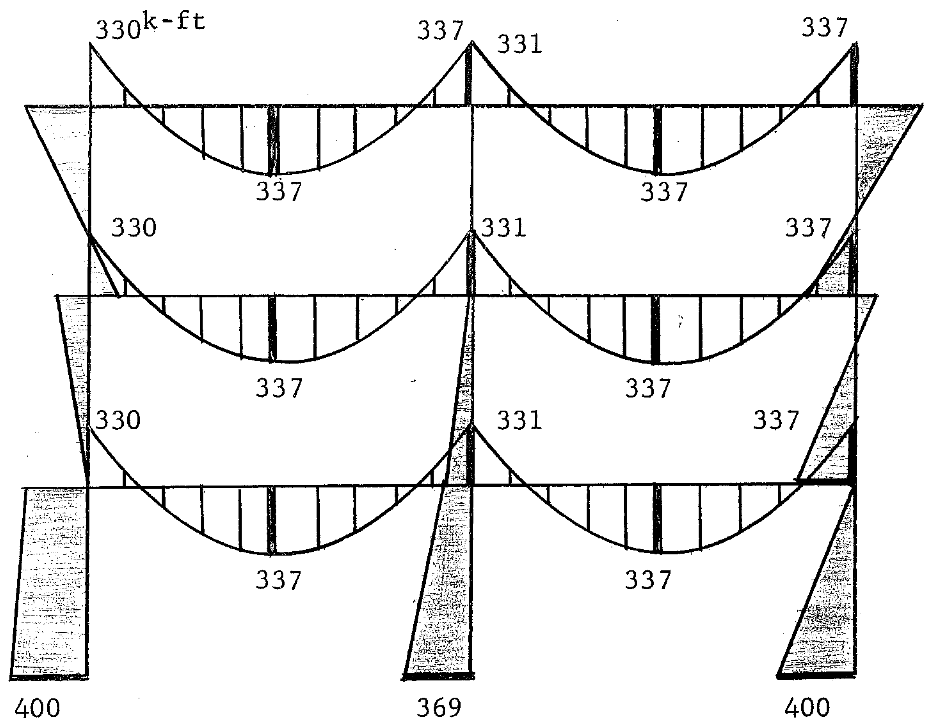



Fig. 3b. Final Moment Diagram for Unbraced Frame