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Welded Continuous Frames and Their Components

PLASTIC DESIGN OF BRACED MULTI-STORY FRAMES

by

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ABSTRACT

A procedure for the plastic design of multi-story frames is presented. Recourse is made only to procedures which are permitted by present (1960) specifications. Beams in the frame are designed for their full plastic strength, while columns are designed by conventional methods.

In the design procedure it is assumed that sufficient bracing will be provided to prevent the formation of a sway mechanism and instability of the frame. This will cause the dead and live floor loads to control the design.

In order to study possible economy a ten story frame is designed by the plastic method. The weight of steel required by plastic design is compared with the weights required by a simple beam elastic design and a continuous elastic design. It is shown that some savings can be effected by the plastic design.

INTRODUCTION

This paper is concerned with the design of planar multi-story multi-bay rectangular frames in tier buildings. Each frame is assumed to be subjected to the vertical and horizontal loading distributed on tributary areas of adjacent floors and walls. The vertical loading is transmitted through a slab to floor beams, which in turn transmit it to the frame. The horizontal forces are transmitted to the frame by walls.

A design procedure must attempt to fullfil various requirements. It must have a rational theoretical basis; it must result in an economical structure; it must produce a safe design; and its application must not be excessively complicated. This paper attempts to divide the general problem of design into several basic structural mechanics problems, and to organize the solutions of these problems in a useful manner. The philosophy adopted is one already used by Horne[(2)].

It is assumed that the columns will be designed so that they will not fail in any manner prior to the formation of plastic mechanisms in the beams. It is assumed that some structural element, such as a braced bent, is provided to resist wind or other lateral forces. The frame, including bracing, must be adequate to prevent the formation of sway mechanisms, to insure the stability of individual members, to limit drift and to prevent overall frame instability.

The text of this paper treats the design of beams, several problems related to and including the design of columns, and the effect of wind. The proposed methods are illustrated by an example. Only procedures permitted by present (1960) Specifications are included.

1. DESIGN OF BEAMS

Beams must be designed to carry safely the weight of a roof or floor system and additional loads transmitted through these. If sidesway of the frame is restricted, only local mechanisms can form. Each beam can, therefore, be designed independently to carry its working load multiplied by a load factor. A load factor of 1.85 for dead and live loads will be used in this report. The effects of shear and axial loads on the strength of beams will be neglected. A check can be performed after the entire frame has been designed to determine whether revision is necessary to resist these effects.

Fig. 1 shows an example of a typical beam subjected to a uniformly distributed load. The necessary design calculations would be as follows:

w_w = working load per ft.

w_u = ultimate load per ft.

M_p = plastic moment of the beam

L = length of the beam

$$w_u = 1.85\, w_w$$

$$M_p = \frac{w_u L^2}{16}$$

A plastic moment table for rolled sections commonly used as beams is included in Ref. 10. From this table a beam adequate to supply the plastic moment M_p may be selected.

Throughout this report the examples will assume a uniformly distributed load on the beams. If concentrated loads or combinations of these and uniform loads are the case, the plastic moments can easily be determined by simple plastic analysis.

It can therefore be seen that once the lengths of the beams and their working loads are specified their design is a simple routine. As will be seen later, it is convenient to determine their dead loads and live loads, and tabulate these quantities separately.

2. DISTRIBUTION OF MOMENTS TO COLUMN SEGMENTS WITH FULL LOADING THROUGHOUT THE FRAME

This article is concerned with the distribution of moments to column segments which are subjected to the maximum possible axial loads. This occurs when the entire frame is subjected to its full live loading.

For use in subsequent discussions the following definitions will be made:

1. A column will be defined as a vertical member which spans the entire height of a frame.
2. A column segment will be defined as a vertical member which spans only a single story height.
3. A subassemblage will be defined as a connected group of members which form a part of a frame.

It will be assumed that the axial load on a column segment is equal to the sum of the end reactions of the beams framing into its top plus the thrust in the column segment above (See Fig. 2).

When a frame is subjected to its full ultimate loading (live load plus dead load times a load factor of 1.85) all of the beams will form mechanisms. This in effect isolates the several columns in the frame. (Fig. 3). Each of these separate columns is then similar to a beam continuous over a number of supports and subjected to thrust. In general, external moments will be acting at the supports. (See Fig. 4). These moments are equal to zero whenever the beams adjacent to the column have equal plastic moments. If this is true for all pairs of beams framing into a column, it will be axially loaded only. In such a case the design of each segment is usually achieved by considering it as a pin-ended, axially-loaded compression member.

In some cases when the beams meeting at a joint have unequal plastic moments there will be unbalanced moments at the supports of the column. In such a case, before a segment can be designed, its end moments must be determined.

An exact distribution of the external moments applied to the column at its supports could be made (neglecting the effect of axial loads on stiffness). This, however, is unwarranted for design, as an approximate distribution can be made with reasonable accuracy. The distribution of moments can be approximated by considering only the restraint contributed by the column segments above and below the one to be designed when these restraining segments are assumed to have appropriate extreme support conditions. In general it will be possible to consider the effect of those segments not included in the three-member subassemblage as that afforded by elastic springs. These elastic springs will be located at the extreme ends of the restraining segments of the subassemblage. (See Fig. 5) If the two spring constants could be evaluated, then the distribution of moments would be exactly the same as that which would result from a distribution on the entire frame. The magnitudes of the spring constants depend on the loading as well as on the geometry and sectional properties of the omitted parts of the column. The work involved in such a determination is not necessary as the proper assumption of limiting cases of end restraints and loadings will give results well within engineering accuracy.

Two factors are neglected when the end conditions of the subassemblage are idealized. The first factor is that the effect of moments acting at supports not included in the assemblage is neglected. The second factor is that the restraint of the column segments not included in the assemblage is either neglected or exaggerated. The fact that the designer can always choose those end conditions of loading and restraint which result in the closest conservative approximation of a design situation, justifies the use of the moment distribution procedure subsequently developed. Also, it must be realized that the error resulting from moment distribution will not have as large an effect on the design of a column segment as its magnitude implies.

2.1 EXPRESSIONS FOR DISTRIBUTION OF UNBALANCED MOMENTS

When the beams framing into a continuous column have formed plastic hinges but have different M_p values, an unbalanced moment must be resisted by the column at that joint.

The unbalanced moment is not distributed to the beams, as the beam moments remain constant at their plastic hinge values. A calculation of that portion of the unbalanced moment resisted by each column segment which frames into a joint can be made by considering a subassemblage of members immediately adjacent to the joint. End conditions for the members in the subassemblage are assumed. This results in approximations which take into account the effects of the ommitted parts of the structure with reasonable accuracy. In the absence of such accuracy the analysis of the subassemblage yields results which safely overestimate the severity of the moments on the column segment to be designed.

Six different subassemblages are analyzed in this article. These are shown in Fig. 6. Figures 6a through 6c are concerned with symmetrical single curvature bending of the center segment of the subassemblage. The end conditions of the restraining segments are:

a. Far ends pinned, but subjected to moments such that the restraining segments are bent in symmetrical single curvature.

b. Far ends pinned, but subjected to moments such that the restraining segments are bent in antisymmetrical double curvature.

c. Far ends of restraining segments fixed.

Figs. 6d through 6f are concerned with assemblages whose center segment is bent in antisymmetrical double curvature. The end conditions of the restraining segments are the same as those in 6a, 6b, and 6c respectively. Most cases encountered in frames will approximate one of these subassemblages.

As shown in Fig. 6, the unbalanced external moments which are applied at the two interior joints B and C of a subassemblage are designated by ΔM_p. The moments induced at the extreme ends A and D of the restraining segments produce either symmetric or antisymmetric bending and are designated as M_r. The internal moments which are distributed to the column segment at B and C are designated as M. These are the moments for which the column segment BC must be designed. The relative stiffnesses (ratios of the moments of inertia to the lengths) of the interior columns of subassemblages will be designated as K_c; while the ratio which corresponds to the exterior restraining segments will be designated as K_r. The expressions for M in terms of K_c, K_r and ΔM_p can be stated as follows:

<u>Case a</u> of Fig. 6.

$$\frac{M}{\Delta M_p} = \frac{K_c}{K_r+K_c}$$

<u>Case b</u>

$$\frac{M}{\Delta M_p} = \frac{K_c}{3K_r+K_c}$$

<u>Case c</u>

$$\frac{M}{\Delta M_p} = \frac{K_c}{2K_r+K_c}$$

<u>Case d</u>

$$\frac{M}{\Delta M_p} = \frac{3K_c}{K_r+3K_c} \qquad (1)$$

<u>Case e</u>

$$\frac{M}{\Delta M_p} = \frac{3K_c}{K_r+K_c}$$

<u>Case f</u>

$$\frac{M}{\Delta M_p} = \frac{3K_c}{2K_r+3K_c}$$

These equations are the results of a moment distribution for each subassemblage, taking full advantage of symmetry or antisymmetry. In some instances the actual design situation may fall between two of the subassemblages which have been solved. The designer will then wish to know which subassemblage will result in a conservative design. Figure 7, in which a curve of $M/\Delta M_p$ versus K_c/K_r has been plotted for each of the six subassemblages, illustrates which of any two of the six will yield larger moments. It can be seen that the main member will be subjected to the greatest internal moment when the restraining members are bent in single curvature. Therefore, subassemblages (a) and (d) of Fig. 6 will result in the most severe condition for symmetric and antisymmetric bending respectively. Equations 1a and 1d present the moment distribution factors for these two cases.

Loadings in actual frames which will approximate the subassemblages are illustrated in Figs. 8 and 9. Fig. 8 shows that if only two different magnitudes of loading w and w* exist on either side of a column, and if their positions are such that certain conditions are fulfilled, the resulting situation is approximated by the subassemblage of Fig. 6a. The conditions referred to are:

1) Two beams adjacent to a column at any story level will not have the same loading, i.e., one will be subjected to w and the other to w*.
2) If any given beam is subjected to a loading, say w, then neither the beam above nor the beam below it will be subjected to that same loading, but they will be subjected to the other one w*.

Loading which has alternate magnitudes from bay to bay and floor to floor will be referred to as "checkered" loading. Loading conditions which approximate the remaining five subassemblages (b to f) of Fig. 6 are illustrated in Fig. 9.

2.2 ERRORS DUE TO APPROXIMATIONS

When a structure is reduced to subassemblages in order to simplify the calculation of the distribution of moments, some errors are introduced. To estimate the seriousness of these errors, comparisons will be made between the distributions obtained for subassemblages and for structures in which the effects of additional spans are considered. For this purpose consider a prismatic member, continuous over several supports and subjected to external moments applied at the supports. Three loadings of the member will be studied:

1. Two equal and opposite moments applied at the supports of a single span of the member. (Fig. 10b)
2. Two equal moments applied in the same direction at the supports of a single span of the member. (Fig. 10a)
3. A single moment applied at one of the supports. (Fig. 11)

In Fig. 10a, bar graphs show the ratio of resisted moment to applied moment in a segment bent in double curvature and restrained by one, two, and three simply-supported continuous spans on each side. The moment at each end of an interior segment, restrained by more than one continuous span on each side, approximates that which results when the restraint is afforded by a single pair of fixed ended members. The moment calculated within the segment of a subassemblage with fixed ends will approximate within about three percent the comparable moment in a five to seven span continuous member. The bar-graph for one pin-ended span indicates that the ratio of internal to applied moment is about seven percent higher than that corresponding to a fixed-ended assemblage. The value of the moment ratio for the ends of the central spans, as the number of restraining spans increase, appears to converge asymptotically on the result of the fixed-ended subassemblage. Results of a similar nature can be expected for unequal spans and member sizes.

Fig. 10b gives comparable results for a continuous member in which equal moments are applied to cause single curvature bending in the interior segment. Again it is found that the magnitude of the interior segment moments converge on that obtained from a fixed-ended subassemblage. The result differs from the moment in a pin-ended subassemblage by only six percent of the applied moment. The result of Fig. 10b will approximate the condition depicted in Fig. 9b while that in Fig. 10a will approximate the structural condition in Fig. 9e.

Fig. 11 gives a further demonstration of the small effect of things happening a number of spans away from a loaded joint. The internal moments induced by an external moment applied at just one joint of a multi-span member are shown by bar graphs. These graphs indicate that an internal moment induced at any joint more than two spans away from the applied moment is less than one percent of the applied moment.

The graphs in Fig. 10 and 11, which result from simple calculations, lend support to the belief that subassemblages, with one span on each side of the member being designed, will adequately approximate the situation in a more extended continuous structure.

3. DISTRIBUTION OF MOMENTS TO COLUMNS WITH PARTIAL LOADING THROUGHOUT THE STRUCTURE

It is often the case that the most critical loading condition for a column is that for which only certain parts of the structure are subjected to live loads. This is due to the fact that, although the axial load on the column is somewhat reduced, there may result a relatively large increase in moment. In this article, the effect of "checkered" loading, which was defined in Art. 2.1, will be studied. This "checkered" loading will be assumed in the vicinity of the column segment to be designed (Column AB in Fig. 12). In this discussion all loadings will be considered as the ultimate load obtained by increasing the working loads proportionally by the load factor. On beams subjected to the full dead plus live load, thus causing mechanisms, the load will be designated as w_{D+L} and on those which are only subjected to the ultimate load corresponding to dead load without causing mechanisms, the load will be designated as w_D. Further, it will once again be assumed that only those members which frame directly into the column to be designed contribute to its restraint. The far ends of the framing members will be connected to rotational restraints which will represent the excluded portion of the structure. Since mechanisms will have formed in all beams loaded with w_{D+L}, and the loading is "checkered", a subassemblage of the type shown in Fig. 13 will result.

The subassemblage shown in Fig. 13 represents the most general case that will be considered. The problem to be solved is the distribution of moments to a column segment AB which is subjected to unbalanced moments and which is restrained by members AC, AF, BG and BD. The unbalanced moments will be assumed equal to the difference between the plastic hinge moment on one side of the column and the fixed end moment of the unyielded beam on the other side.

$$M_{UA} = (M_p)_{EA} - \left(\frac{w_D L^2}{12} \right)_{AF}$$

$$M_{UB} = (M_p)_{BH} - \left(\frac{w_D L^2}{12} \right)_{BG} \qquad (2)$$

where M_{UA} = Unbalanced moment at A

M_{UB} = " " B

and the subscripts outside of the parentheses refer to the beam to which the expression belongs.

3.1 DISTRIBUTION OF MOMENTS

Let several members meet at a joint and have their far ends fixed. If an external moment is applied at that joint it will be distributed to the various members in proportion to their relative stiffnesses. For prismatic members the relative stiffness is equal to the ratio of their moments of inertia and their lengths (neglecting axial loads). If M_U designates the unbalanced moment at a joint, the moment M_1 distributed to any member is equal to:

$$M_1 = M_U \left[\frac{\frac{I_1}{L_1}}{\frac{I_1}{L_1} + \frac{I_2}{L_2} + \frac{I_3}{L_3} + \ldots \frac{I_n}{L_n}} \right] \tag{3}$$

If the far end of any member is pinned (instead of fixed), or if there is a definite ratio between the moments at the far and near ends, then it is possible to modify the relative stiffnesses by a factor α. The factor α has the following values for prismatic members:

$\alpha = 3/4$ for far ends pinned

$\alpha = 1/2$ for symmetric bending of the member

$\alpha = 3/2$ for anti-symmetric bending of the member

Equation 3 can now be written:

$$M_1 = M_U \left[\frac{\alpha_1 \frac{I_1}{L_1}}{\alpha_1 \frac{I_1}{L_1} + \alpha_2 \frac{I_2}{L_2} + \alpha_3 \frac{I_3}{L_3} \ldots \alpha_m \frac{I_m}{L_m}} \right] \tag{4}$$

The expression in parentheses will be referred to as the distribution factor D for a joint with respect to a given member. For example, D_{AB} will mean the distribution factor for joint A of member AB. At this point it is important to remember that a member which forms a plastic hinge at its end is assumed not to afford any restraint to the rotation of the joint at that end. Thus, in Fig. 13 members BH and AE are not included in the distribution factors of joints B and A respectively.

It is shown in Appendix A that when the unbalanced moments on the system are M_{UA} and M_{UB} and the distribution factors are D_{AB} and D_{BA} the moments M_{AB} and M_{BA} sustained by a member AB, are:

$$M_{AB} = M_{UA} \frac{D_{AB}(1-\frac{1}{4} D_{AB})}{(1-\frac{1}{4} D_{AB} D_{BA})} - M_{UB} \frac{D_{BA} (1 - D_{AB})}{2(1-\frac{1}{4} D_{AB} D_{BA})}$$

$$M_{BA} = M_{UB} \frac{D_{BA}(1-\frac{1}{4} D_{AB})}{(1-\frac{1}{4} D_{AB} D_{BA})} - M_{UA} \frac{D_{AB}(1 - D_{BA})}{2(1-\frac{1}{4} D_{AB} D_{BA})}$$

..... (5)

The moments M_{UB} and M_{UA} are assumed to be acting in opposite directions, corresponding to "checkered" loading in the vicinity of the central column AB in Fig. 13.

For use in later articles, the problem in simplified to the case where the geometry and loading are symmetrical. This means that:

1) $M_{UA} = M_{UB} = M_U$
2) The upper and lower restraining column segments have equal lengths, cross sections, and end conditions.
3) The restraining beams will have equal spans, cross sections and end conditions, and their loadings w_D are the same.

Conditions 2 and 3 require that $D_{AB} = D_{BA}$.

Equations (5) thereby reduce to:

$$M_{AB} = M_{BA} = M_U \frac{\frac{1}{2} D_{AB}}{1 - \frac{1}{2} D_{AB}} = D''_{AB} M_U$$

Here D''_{AB} is a distribution factor which takes into account the symmetric single curvature bending of AB. It is understood that D''_{AB} applies to the central column segment AB of the assemblage when conditions (1) to (3) above are satisfied.

Extreme conditions of end restraint of the framing members are investigated next. These are shown in Fig. 14. Case (a) represents the situation in which the far ends of Fixed the restraining members are pinned; (c) is the case where there are far end moments which cause symmetrical bending of the framing members, and (d) is identical to (c) with the exception that the far ends of the restraining columns are fixed. A structural analysis of the four cases will reveal the following:

a) Far Ends Fixed (Fig. 14a)

$$\frac{M}{M_U} = \frac{\frac{1}{2} K_c}{K_{rc} + K_{rb} + \frac{1}{2} K_c} \qquad (6a)$$

$$K = \frac{I}{L}$$

$$M_U = M_p - \frac{w_D L^2}{12}$$

b) Far Ends Pinned (Fig. 14b)

$$\frac{M}{M_U} = \frac{\frac{1}{2}K_c}{\frac{3}{4}K_{rc} + \frac{3}{4}K_{rb} + \frac{1}{2}K_c} \qquad (6b)$$

$$M_U = M_p - \frac{w_D L^2}{8}$$

c) Framing Members Symmetrically Bent (Fig. 14c)

$$\frac{M}{M_U} = \frac{K_c}{K_{rc}+K_{rb}+K_c} \qquad (6c)$$

$$M_U = M_p - \frac{w_D L^2}{12}$$

d) Framing Columns With Ends Fixed and Beams Symmetrically Bent (Fig. 14d)

$$\frac{M}{M_U} = \frac{\frac{1}{2}K_c}{K_{rc}+\frac{1}{2}K_{rb}+\frac{1}{2}K_c} \qquad (6d)$$

$$M_U = M_p - \frac{w_D L^2}{12}$$

In these expressions K_c denotes the relative stiffness of the column segment, and K_{rb} and K_{rc} are the relative stiffnesses of the restraining beams and columns respectively. M_p is the plastic hinge moment for the beams and M is the moment which must be resisted at each end of the column segment under consideration.

This article has presented the solutions for several subassemblages which will be of use in subsequent articles.

4. FULL LOADING VS PARTIAL LOADING OF THE STRUCTURE

The means of distributing moments to the columns for the cases when the structure is fully and partially loaded are available. A criterion which tells the designer the loading condition that should be designed for must be developed. It is subsequently shown that the ratio of dead load to full load determines whether or not it is necessary to consider partial loading of the structure.

Let it be assumed that the steel skeleton of a structure is gradually loaded in a uniform manner throughout. For illustration, a portion of the structure, where the girders on either side of the column are of equal length, is chosen. The subassemblage which best approximates the conditions of loading and geometry stipulated, is that in which the far ends of the framing members are fixed. (See Fig. 15)

It is assumed that all loadings include their load factors (1.85). Let w_u designate the ultimate load of a beam, (i.e., that load under which a mechanism will form), and w_D the dead load on a beam times its load factor. Then, due to symmetry in geometry and loading, the moments at the far ends of the beams, for any level of loading w are

$$M = \frac{wL^2}{12}$$

The moments at the centers of the beams for the same loading are equal to $\frac{wL^2}{24}$. No moments are transmitted to the columns due to symmetry. The first hinges to form will be those at the ends of the beams. This will occur if those moments are ever equal to M_p. If the w_D is such that

$$\frac{w_D L^2}{12} \geq \frac{w_u L^2}{16}$$

the hinges form at the ends of the beams (with only the overload corresponding to the dead load times the load factor, and prior to the application of load representing live load). Any further loading on either one of the two adjacent beams does not transmit further moment past the hinge to the column. (Except the difference in shear times 1/2 the depth of the column.) This is demonstrated graphically in Fig. 15. Therefore it is possible to conclude that as long as

$$\frac{w_D}{w_u} \geq \frac{3}{4} \qquad (7a)$$

partial loading of the structure need not be considered for the case of symmetrical loading and geometry.

The criterion for the case where the beams on either side of the column are neither equal in length nor in loading, is derived in appendix B. The notation used is represented in Fig. 16. This criterion is in the form of the two inequalities:

$$\frac{w_{DR}}{w_{UR}} \geq \frac{\frac{3}{4}}{1 + \left(\frac{w_{DL}}{w_{DR}} \frac{L^2{}_L}{L^2{}_R} - 1\right) D_R} \qquad (b)$$

(7)

$$\frac{w_{DL}}{w_{UL}} \geq \frac{\frac{3}{4}}{1 + \left(\frac{w_{DR}}{w_{DL}} \frac{L^2{}_R}{L^2{}_L} - 1\right) D_L} \qquad (c)$$

$$w_{DL} \; L^2{}_L \leq w_{DR} \; L^2{}_R$$

w_{DR} signifies the uniform dead load on the beams framing in from the right while w_{DL} means the uniform dead load on the beams framing in from the left. L_R and L_L signify the lengths of the right and left beams respectively. D_R and D_L are the distribution factors for the right and left side beams respectively.

Equation (7b) is plotted in Fig. 18. Since $w_{DL}L^2{}_L$ is less than or equal to $w_{DR}L^2{}_R$ equation (c) will necessarily be violated if equation (b) is violated. Therefore it is only necessary to be concerned with equation 7b. This equation is plotted in Fig. 18 for four different values of D_R i.e., 0.3, 0.4, 0.5 and 0.6. These four values of D_R span the range of probable actual values.

5. COLUMNS: GENERAL CONCEPTS

In general, column segments in frames are subjected to external moments and axial forces, but are also restrained against end rotation by adjoining members. (Fig. 17) Restrained column segments subjected to external moments at their joints are usually bent either in single or double curvature. The extreme case of single curvature bending occurs when the moments at the ends of a column segment are equal and in opposite directions. The extreme case of double curvature bending occurs when the end moments are equal but in the same direction. The end moments referred to, are the differences between the external applied moments and the restraining moments introduced by the adjacent members. The case, where one end of the column is pinned and free of moment, is the dividing one between those of single and double curvature (See Fig. 19). It can be shown (Ref. 3) that for a given cross section, length, magnitude of larger end moment and axial load, the extreme case of single curvature is the most critical for column instability.

The present 1960 AISC specifications allow two different criteria for the design of columns. One of these is the well-known rule for combined axial and bending stresses. When expressed in terms of axial load and bending moment this rule requires that the following inequality is not violated:

$$\frac{P}{P_{all}} + \frac{M}{M_{all}} \leq 1 \qquad (8)$$

P = Axial load on the column.

M = Larger of two end moments acting on the column.

P_{all} = Allowable axial load on the column if it is not subjected to the end moments.

M_{all} = Allowable moment on the column if it is not subjected to axial load.

The other design criterion is presented in the column formuli of the "Rules for Plastic Design and Fabrication" of the AISC. (Ref.10) This paper will adopt the first of the two criteria. However, the procedure developed subsequently can be adapted to either design criterion.

Several points implied in the use of the interaction equation (Eq. 8) will be reviewed:

1) Every column loading is reduced to one with equal and opposite end moments. The magnitude of the assumed moment is equal to the larger of the two which are acting at the ends of the column. (Fig. 20a)

Since the worst possible bending condition for a column is that of extreme single curvature, this procedure imposes a more severe loading to the column and is thus on the safe side.

2) The interaction formula assumes a linear relationship between an allowable end moment and an allowable axial load. (Fig. 20b)

The assumption of linearity does not take into account the secondary bending effects. The consideration of these effects results in a curve which intersects the M and P axes at the same points as the straight line. (See Fig. 20b, also Ref. 3). This assumption of linearity can be somewhat unconservative.

3) An elastic moment distribution to the column of the unbalanced moment neglects the reduction of stiffness due to the effects of axial load and plastification.

4) Any design of a column, prevented from sidesway, for its pin-ended strength will usually result in the selection of a member size which is larger than that determined on the basis of restrained ends.[6] [7]

From a plastic analysis of the structure and application of the principles of distribution of moments given in Arts. 2 and 3, values of the thrust and bending moment at ultimate load within any column segment may be determined.

To design the column segment on an allowable stress basis, it is necessary to use values of thrust and moment resulting from working loads. Since determination of working load thrusts and moments would require a complete elastic analysis, an approximation obtained by dividing ultimate load values by the load factor will be considered instead. Using these approximate thrusts and moments, column sections may be selected to satisfy the conventional interaction equation with conventional working stresses.

An alternate procedure for designing column segments can take advantage of the AISC plastic design recommendations for columns. Loading for the columns would be that resulting from the moments distributed to each column segment assuming linear moment-rotation relationships for the member at maximum load. Members would be selected on the basis of the recommendations of the "Rules for Plastic Design and Fabrication" of the AISC for the loadings thus determined.

Both procedures result in design moments for columns which are probably more severe than those the columns accept in the actual structure. A graphical explanation of this statement may be made with the aid of Fig. 21. A column segment with equal external moments M_e bending it in single curvature is considered. The combined stiffnesses of the restraining column segments and beams which meet at the joint are designated by the constants k. These are the stiffnesses of all beams and column segments which have not formed a plastic hinge adjacent to the given joint and may be likened to rotation-resisting springs. The assumed linear moment-rotation charactersitics of the column segment and the spring are represented by the two sloping straight lines 0 - 1 and A - 1. The moments induced by a given joint rotation in the column and spring are designated by M_c and M_s respectively. For equilibrium and compatibility it is required that the end of the column and the spring rotate through a common angle θ.

with the sum of the moments M_c and M_s induced being equal to the external moment M_e. Equilibrium is obtained at a rotation θ, as shown in Fig. 21, corresponding to the intersection of the moment-rotation curves for the spring and column.

An actual column demonstrates inelastic behavior as it nears its ultimate load. This results in greater rotation as indicated by the true column curve in Fig. 21. With a greater end rotation θ_2, the "spring" is required to carry more of the distributed moment while the column carries a little less.

An example of the safety of basing the design of an elastically restrained column segment on its pin ended strength will be shown by obtaining the strength of an assemblage. In Fig. 22 the actual moment rotation curve of a pin ended column tested to collapse by increasing end moments under a condition of constant axial load is represented by a'b'c'd'e'. The maximum column moment, on which pin-ended column strength would be based, is given by a point labelled c'. Consider this same column segment as part of an assemblage with restraining members having a total restraining stiffness of k at each end. In the same manner as was done in Fig. 21, the rotation of column and spring due to a given external moment M_e may be determined from the intersection a' of the spring characteristic curve a-a' and the column curve. At this rotation, the moment resisted by the total assemblage is M_e and can be represented as point a" on a moment-rotation curve of the total assemblage. If the restraining springs remain linear and elastic with the same constant k, the rotation versus external moment relationship for the assemblage may be constructed. Intersections b', c', d', and e' on the column curve result in points b", c", d", c"', and e" respectively. The important thing to note from this example is that point c" on the assemblage curve corresponds to the maximum pin-ended column strength c'. However a further increase in the assemblage moment is possible as long as the spring charactersitic can intersect the unloading portion of the column curve. The maximum limit M_e''' is reached when the spring characteristic curve is tangent to the column curve as at point d'. Using the pin-ended column strength for design is thus shown to be on the safe side when the restraint is linear throughout loading. For most cases in braced multistory frames this will also be true when the restrain exhibits non-linear characteristics.

6. CRITICAL LOADING CONDITION FOR COLUMNS WITH PARTIAL LOADING ON THE STRUCTURE

A column segment in a multi-story frame can be bent either in single or double curvature. This depends on the manner in which the members adjacent to it are loaded and on its location in the frame (See Fig. 12). It has already been stated in the previous article that the worst possible bending condition for a column segment is that of single curvature. Also, the least critical bending condition is that of double curvature. The philosophy adopted in this article is the designing of a column segment for the loading which subjects it to the worst condition of bending. The worst axial loading consistant with the worst bending is also used. Whether or not the probability of the occurrence of these "worst" loading conditions justify their use, is beyond the scope of this paper. By "worst loading" is meant that loading which is the most unfavorable of those which can reasonably be expected to occur.

6.1 CENTRAL INNER COLUMN SEGMENTS

A central inner column segment will be referred to as one which has beams framing into it from both sides, and which has a column framing into it from the top as well as from the bottom. Central inner column segments can always be loaded such that they are bent in single curvature. This is achieved by a "checkered" live loading of the four beams framing into it. The structure in Fig. 23 is loaded so that column segments AB and CD are bent in single curvature, and subjected to high axial load. The end conditions which result in probable critical bending are those for which the restraining members are bent in symmetry. This condition exists for the beams if the "checkered" live loading is extended throughout the two stories which bound the column segment. For the restraining columns, this is approximated if the "checkered" live loading is extended one story above and below the two already mentioned. This, however, reduces the axial load on the column (as otherwise the stories above must be fully loaded). If the stories above the two which bound the column segment are fully loaded, the end conditions of the restraining columns are best approximated by full fixity. Therefore, depending on whether or not the extra moment involved is greater than the reduction in axial load which results, assemblages (c) or (d) of Fig. 14 should be used. The difference in moment between assemblages c and d is:

$$\frac{\Delta M}{M_u} = \frac{K_c}{K_{rc}+K_{rb}+K_c} - \frac{K_c}{2K_{rc}+K_{rb}+K_c}$$

$$\frac{\Delta M}{M_u} = \frac{K_c K_{rc}}{(K_c+K_{rc}+K_{rb})\ (2K_{rc}+K_{rb}+K_c)}$$

The difference in axial loads is:

$$\Delta P = \frac{w_L L}{2}$$

where w_L will designate (live load omitted) x 1.85. When the effect of the reduction in axial load is equal to the effect of the increase in moment, either subassemblage results in the same design.

$$\frac{\Delta M}{M_{all}} > \frac{w_L L}{2P_{all}} \qquad \text{Use 14c}$$

$$\frac{\Delta M}{M_{all}} < \frac{w_L L}{2P_{all}} \qquad \text{Use 14d} \tag{9}$$

However, it is conservative to use Fig. 14c, and assume no reduction in P. This is done in this paper.

The design of the column segment now consists of satisfying the following two equations:

$$\frac{M}{M_u} = \frac{K_c}{K_{rc}+K_{rb}+K_c} = \frac{K_c}{\Sigma K} \tag{10}$$

$$\frac{P}{P_{all}} + \frac{M}{M_{all}} = 1$$

This may be done by trial and error, graphs or design tables.

The solution of these two equations is represented graphically by Fig. 25. The lower part of the nomograph performs the moment distribution. Namely, given the ratio $\frac{K_c}{\Sigma K}$ for any given $\frac{M_U}{M_{all}}$, a value of $\frac{M}{M_{all}}$ is defined. The upper portion of the nomograph represents the AISC interaction formula. The design method is as follows:

P and M_U are known:

K_{rc} and K_{rb} are known:

1. - Assume a cross section.

2. - Determine $\frac{K_c}{\Sigma K}$, $\frac{M_U}{M_{all}}$, $\frac{P}{P_{all}}$.

3. - Enter the nomograph with $\frac{M_U}{M_{all}}$.

4. - Draw a horizontal line from $\frac{M_U}{M_{all}}$ to the straight line corresponding to the known value of $\frac{K_c}{\Sigma K}$.

5. - From the point of intersection draw a vertical line to intersect the interaction line.

6. - From the previous intersection draw a horizontal line to intersect the $\frac{P}{P_{all}}$ axis.

7. - If the value of $\frac{P}{P_{all}}$ is equal to the computed one, the section is correct. If not, another section is chosen and the procedure repeated.

Two or three trials are usually sufficient for an experienced designer.

6.2 INNER COLUMN SEGMENTS OF THE FIRST STORY

These column segments have one end fixed. This changes the subassemblage to one where the column itself is not symmetrically bent. A conservative approximation of reality is achieved by subassemblage (a) in Fig. 24. The moment distributed to the column is equal to:

$$\frac{M}{M_U} = \frac{2\,\frac{K_c}{\Sigma K}}{1 + \frac{K_c}{\Sigma K}} \qquad (11)$$

A nomograph for this case is plotted in Fig. 26.

6.3 CENTRAL OUTER COLUMN SEGMENTS

These segments must necessarily be bent in double curvature. It was pointed out in Chapter 5 that this is the least critical condition of bending. Therefore, it is safer to assume that one end of the column segment is pinned. The restraining column segment framing into the bent end of the member to be designed is also pinned at one end. The external moment is equal to the full plastic moment of the adjacent girder. The subassemblage can be seen in Fig. 24-e. The moment distributed to the column segment is equal to:

$$\frac{M}{M_U} = \frac{K_c}{\Sigma K} \qquad (12)$$

Since this is equal to that of the central inner columns, the nomograph of Fig. 25 can be used for design.

6.4 FIRST STORY OUTER COLUMN SEGMENTS

The column segments have one end fixed. The subassemblage used is shown in Fig. 24d. The equation for the distributed moment is equal to:

$$\frac{M}{M_U} = \frac{4 \frac{K_c}{\Sigma K}}{3 + \frac{K_c}{\Sigma K}} \qquad (13)$$

The nomograph of Fig. 27 can be used for design.

6.5 UPPERMOST COLUMN SEGMENTS

These column segments do not have a restraining segment above them. In order to approximate the worst bending condition, the subassemblages shown in Fig. 24c for inner segments, and in Fig. 24f for the outer ones are adopted. The value of $\frac{M}{M_U}$ for the first of the two is the same as those corresponding to Fig. 24b and Fig. 24e. Therefore the nomograph of Fig. 25 can be used for design. The second case receives the entire M_U (See Fig. 24f). Therefore, $\frac{M}{M_U} = 1$. The equations corresponding to all the subassemblages are listed in Fig. 24.

7. THE EFFECT OF WIND

So far, it has been assumed that the frame is not called upon to resist lateral loads. This assumption is justified for frames subjected to vertical loads only. For frames subjected to combined loading (lateral plus vertical) it may be necessary to provide bracing to reduce the effect of lateral loads. The major effects of lateral (wind) loading on frames are:

1. The influence on the type of failure mechanism.
2. The influence on the stability of individual column segments.
3. The influence on overall frame stability.
4. The influence on deflections.

These four, along with a discussion of bracing, are subsequently described.

7.1 THE INFLUENCE OF WIND ON THE TYPE OF FAILURE MECHANISM

For design purposes it is desirable to determine the failure mechanism and obtain a moment diagram for a frame subjected to combined loading. The specific mechanism will depend on geometry, relative member sizes, and the ratio of lateral to gravity load. Two general types of failure are the formation of local beam and sway type mechanisms (See Fig. 28). Local beam mechanisms can be forced to occur if a frame is adequately braced (See Fig. 29). An analysis of the unbraced frame will determine whether a sway type mechanism can occur under a system of gravity and wind loads. If such a mechanism can occur, the analysis will show what specific type it is. This can be done by one of the methods of simple plastic theory. Proper reduction in the plastic moments of the columns must be made to account for the effect of axial loads[5].

Before plastic analysis can be applied in routine fashion, systematic procedures for determination of mechanisms and moment diagrams will have to be devised. If the combined ultimate loads which a structure is capable of resisting are less than 1.40 times the required working loads, bracing is needed. The amount of bracing required is that which will make the load factor of the braced frame at least 1.40 at failure.

If bracing is required, advantage can be taken of the nominal amount ($\frac{l}{r} = 300$) which might be used to guard against frame instability and buckling. The magnitude of wind load, which the nominally braced frame can resist, can be determined by simple plastic theory. Again, however, reductions must be made for the effect of axial loads in column segments. Computations on frames have shown that nominal bracing greatly increases resistance to wind. The increase in wind resistance is such that beam mechanisms are usually forced to govern.

7.2 APPROXIMATE ANALYSIS OF FRAMES SUBJECTED TO COMBINED WIND AND GRAVITY LOADS

Frames subjected to combined loading with a lower load factor (1.40) than is used for vertical loading (1.85) behave in a manner which enables simplifications to be made in the determination of wind resistance.

When beams in frames designed for gravity loads of 1.85 w_w are subjected to gravity loads of 1.40 w_w the full capacity of the beams is not utilized. The unused capacity is available to help resist wind loads applied in combination with the lighter gravity loads. Some limits on the range of distribution of moments can be deduced if it is first assumed that all beams behave as fixed-ended beams. Under uniform vertical loads, the moments at the ends of the beams will be equal to $1.40 \frac{w_w L^2}{12}$ in the elastic range. However,

$$1.40 \frac{w_w L^2}{12} \approx \frac{1.85 \, w_w L^2}{16}$$

Therefore, plastic hinges may form at the ends of the beams upon application of vertical loading equal to 1.40 w_w. Upon application of some horizontal loads, the plastic hinges at the leeward ends of the beams tend to be maintained, while those at the windward ends tend to unload and vanish. The resulting locations of plastic hinges can be seen in Fig. 30a. Upon further application of horizontal loading the tendency to reverse the moment at the windward ends of the beams increases the moment causing tension on the bottom fibers near the center of the beams until a second plastic hinge forms in each one. Once this has occurred the beams have done <u>all</u> that they possibly can to resist wind. Further, their moment diagrams can be uniquely determined. For beams designed to fail under vertical loading equal to 1.85 w_w and subjected to 1.40 w_w, the limiting moment diagram is shown in Fig. 31. The moment at the leeward end is equal to M_p, the moment at 0.425 L from the windward end is equal to M_p, and that at the windward end is equal to 0.1 M_p(4). The directions of the moments are shown in Fig. 31. Equilibrium of moments at each joint in the roof structure will determine the magnitude and direction of the moment at the upper end of each top story column segment. This should not exceed the effective plastic hinge moment of the column segment. Limits on the moments at the lower ends of the top story column segments can be determined by assigning arbitrary values which enable the column segments in the story to resist the total horizontal shear without anywhere exceeding effective plastic hinge values.* Depending on the type of analysis being considered, this method could be used either to assign a set of moments capable of resisting a given story shear or to determine the maximum shear which the columns and beams in a given story are capable of resisting.

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* By "effective plastic hinge value" is meant any reduced value of moment capacity which may be determined by an appropriate column theory.

The remaining stories may be analyzed in the same manner. Namely, joint equilibrium is satisfied at the top of each column segment and at the bottom of the segments sufficient moments are assumed to resist the story shears without anywhere exceeding the effective plastic moment. This method of analysis constitutes a lower bound solution because a system of moments is obtained which can carry the system of loads in equilibrium without anywhere exceeding the effective plastic hinge moment. However, the structure is still subject to such secondary design checks as local, lateral and column buckling; bending instability of columns and frame instability.

As an example of the application of this procedure consider the frame of Fig. 33. This is a two story two bay frame of equal story heights h, and equal spans L. The beams are loaded with uniformly distributed loads w. These beams were designed to fail under a loading of $\frac{1.85}{1.40}$ w. The members of the frame all have plastic moment capacities equal to M_p, except the lower story column segments. These have plastic moment capacities equal to 3 M_p. Lateral loads H_A, and H_D are applied at joints A and D respectively.

The limiting moment diagrams of these beams are as shown in Fig. 33. The unbalanced beam moments are 0.1 M_p at joints A and D, 0.9 M_p at joints B and E, and M_p at joints

C and F. By equilibrium the moments at the tops of column segments AD, BE, and CF are determined. Moments of magnitude M_p are assigned at the bottoms of columns AD, BE and CF in the direction in which they will contribute the greatest shear resistance as shown in Fig. 33. The moments at the tops of column segments DG, EH, and FI are determined by equilibrium of the joints and are shown in Fig. 33. Finally the plastic moment values are assigned to the bottom of column segments DG, EH, and FI in the direction in which they will tend to resist wind. This results in the moments shown at the bottoms of the column segments in Fig. 33. Summation of shears at each story level yields a load of $\frac{4.8\ M_p}{h}$ acting at joint A and one of $\frac{3M_p}{h}$ acting at joint D. If these values result in story shears equal to or greater than H_A and $(H_D + H_A)$ respectively, the structure may be expected to resist the applied loads.

7.3 SWAY BRACING

Bracing of frames in the loading plane can be provided by diagonal truss-type members located in a sufficient number of bays. This bracing has the functions of preventing frame instability or sidesway due to vertical loads and of helping the rigid frame resist sway due to lateral loads. In many cases very slender bracing having ℓ/r ratios up to about 300 may be adequate. In these cases the members may only be capable of carrying tension forces adequately. Therefore cross members should be provided to resist forces in any direction by tension. If bracing slenderness ratios are restricted to about 200, members can take both compression and tension. This can either eliminate the need for cross bracing or allow forces to be shared by tension and compression braces in a panel.

In analyzing a braced frame by the approximate method of Art. 7.2, the shear resistance of panels may be determined by considering the horizontal resistance of bracing members along with the horizontal shear of the columns in any story. It is proposed to discuss the behavior and design of bracing in greater detail in a future report.

7.4 COLUMN STABILITY UNDER THE INFLUENCE OF WIND

Once an applicable distribution of moments is made to the columns, they (columns) must be checked to insure stability. Presently applicable elastic and plastic procedures for the analysis and design of individual column segments are based on the assumption that the columns remain essentially vertical. This is best assured by sway bracing.

Certain assumptions may be used to apply conventional methods to check the adequacy of column segments subjected to the approximate moment distributions obtained by the method of Art. 7.2. It may be assumed that columns remain continuously elastic with no plastic hinges forming except at the bases. This continuity is consistent with other assumptions of plastic design which imply that moments at some points can approach effective hinge values without behaving inelastically. For additional safety in assuring that columns will not participate in the mechanism, an alternative could be to assume a limit somewhat lower than the effective plastic hinge value in assigning arbitrary column moments in the analysis of Art. 7.2. For columns remaining elastic, loads and moments suitable for the AISC interaction formula may be obtained by dividing ultimate load results of Art. 7.2 by the load factor.

The approximate ultimate load results of Art. 7.2 (not divided by load factor) may be used directly for the application of the AISC plastic design column formulas to check the adequacy of column segments in braced frames.*

Results of work on restrained columns[6] show promise of adaptation to the analysis of column stability under the influence of wind, as well as of economies in the design of braced multi-story frames. Another report will discuss this work in detail.

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* The validity of using the results of Art. 7.2 to check the stability of column segments rests on the assumption that these results represent conditions which are not too far removed from the worst which can be expected.

7.5 FRAME INSTABILITY

When columns in a frame are permitted to sway from the vertical position, the stability problem becomes a case of overall frame instability. A method of solving the frame instability problem for single-story single-span frames with symmetric vertical loading is available. It is planned to extend this work to multi-story frames subjected to vertical loads. The problem of the stability of frames subjected to lateral as well as vertical loads has not yet been solved. It is necessary to solve this problem before a completely satisfactory design method can be obtained for unbraced multi-story frames.

No solution has yet been obtained for the stability of braced frames. However, it is fairly certain that a nominal amount of diagonal bracing included in each bent and made up of members having a maximum allowable slenderness will be adequate for most frames subjected to vertical loads. The fact that a truss diagonal is much more efficient in resisting sway than a rigid frame panel also gives promise that minimum bracing will be adequate to prevent sway in many frames which are subjected to horizontal loads. It follows that some frames may be stiff enough not to require bracing either to maintain frame stability or to resist the horizontal loads which are likely to be applied.

the beams might simplify the computations somewhat. Experience to be gained by calculating a few examples may point to some simplifying assumptions which can be used in calculating deflections.

It is possible that estimates of deflections in the working range may be made by obtaining approximate moment deistributions by the method of Art. 7.2. In this case the limiting moments to be assumed would be based on appropriate elastic moment capacities. Calculation of the sway of the columns should be relatively easy due to full continuity.

Sample details of all the calculations for the four comparative designs are presented in Appendix C.

8.1 COMPARISON OF FINAL DESIGNS

The member sizes and weights which result from each of the four designs are tabulated in design sheets 23 and 24 of Appendix C. It can be seen that the plastic design of beams and conventional design of columns results in the most economy. The relatively uneconomical member sizes which result from the plastic design of beams and ultimate design of columns is due to the $\frac{P}{P_y} \leq 0.6$ limitation imposed by the AISC Plastic Design Code[10]. The approximate continuous elastic design is not as economical as the plastic design of beams and elastic design of columns. The weight saving of the latter over the former is 5.4%. The weight saving of the plastic design of beams and elastic design of columns over the simple beam elastic design is 17.1%.

The use of live load reductions in the design of girders would result in a further saving of steel. This reduction refers to the statement on page 344 of the Steel Construction Manual of the AISC[9] which states that: "For live loading of 100 pounds or less per square foot, the design of live load on any member supporting 150 square feet or more may be reduced at the rate of 0.08% per square foot, except for areas of public assembly". The total reduction is limited as specified in the same paragraph of the manual. However, the relative economies would be about the same for all methods. The resulting steel weights of the comparative design are shown in Fig. 36.

9. SUMMARY

A plastic design procedure is presented for braced multi-story multi-bay frames subjected to vertical dead and live loads. In this procedure, floor and roof beams are proportioned to form mechanisms at ultimate load while columns are proportioned to remain continuous. Because the continuous columns will remain highly indeterminate even after mechanisms have formed in beams, an approximate method for distribution of bending moments in columns is developed. To simplify the design of each column segment, isolated subassemblages consisting of the segment to be designed and the restraining beams and columns joining it are considered. Moment distribution factors are derived for several types of assumed boundary conditions for the restraining members. Sets of distribution factors for structures loaded fully and partially are presented. These factors represent bending conditions, which in combination with compressive loads, are apt to be the most severe for an individual member. It is found that the most severe bending condition will occur within a column segment when the restraining members are bent in single curvature. Another derivation shows that the design of columns with all bays of the structure loaded will be the only case to be considered if the dead load exceeds a certain fraction (in the order of 75 percent) of the total dead plus live load.

As partial justification for using the subassemblage concept to design columns, calculations are made to determine the effects on the bending moment at a joint in a continuous member from moments and restraints a number of spans away. It is shown that the errors introduced by considering the subassemblage are in the order of one to six percent of the applied moment.

The design procedure proposed in this paper consists of the following steps:

1. Design all beams plastically as fixed-ended beams subjected to appropriate dead plus live load distributions with a load factor of 1.85.

2. Determine the axial load and bending moment for each column segment. Divide these by a load factor of 1.85 to obtain an approximate working value. Select column sections based on present AISC conventional design specifications.

3. Bracing should be provided to prevent frame instability and resist the effects of wind. The minimum bracing requirement should be such that:

   a) $\frac{\ell}{r} = 300$ if the bracing is subjected to tension only. In such a case counters should be used.

   b) $\frac{\ell}{r} = 200$ if the bracing is expected to act as a compression member at least part of the time.

Step 2 comprises the following sub-steps:

a) In order to determine whether partial loading need be considered equation 7-b or B-4 must be applied. If the loading and geometry in the vicinity of a column are symmetric equation 7-b reduces to equation 7-a. If it is determined that partial loading must be designed for, it will be necessary to check the resulting columns for full loading.

b) The unbalanced moment due to partial loadings is computed by Equation (2).

c) The choice of distribution factor for each column segment for partial loading depends on the location of the column in the frame. Top story exterior columns are considered as part of the subassemblage shown in Fig. 24f, and the distribution factor for these columns is equal to unity. Top story interior columns correspond to Fig. 24c and are designed with the distribution factor of Eq. 12. The central exterior and interior columns correspond to Fig. 24e and Fig. 24b respectively and their distribution factor is given by Eq. 12. The exterior columns of the bottom story correspond to Fig. 24d and have a distribution factor given by Eq. 13. The interior columns of the first story correspond to Fig. 24a and their distribution factor is given by Eq. 11.

d. The resulting axial loads and column end moments are substituted in Equation 8 in order to check the adequacy of trial sections. This is in accordance with the present AISC conventional design specifications.

A design of a ten-story five-bay frame made according to the recommendations of this report is compared with simple and continuous elastic designs and with a plastic design using different assumptions for the design of columns. The frame with beams designed plastically and columns designed elastically shows savings in weight of steel of 5.4 percent over a continuous elastic design and 17.1 percent over a simple beam elastic design (both designed according to AISC specifications). In the final plastic design, the axial forces and moments are distributed to the columns in the same manner as that recommended in this report but the columns are proportioned according to the AISC rules for plastic design. The resulting weight of steel is greater than that for the frame with plastically designed beams and elastically designed columns. The reason the plastic design of columns proves to be uneconomical is that the axial force in columns at ultimate load is limited to 0.6 P_y by the AISC provisions. The study shows the need for additional information about the design of columns.

Another result of the studies leading to this report is that there is a need for additional information about several topics. These topics are mentioned only briefly in this report. Basic studies for some of this information have been carried out and need only be applied whereas new work needs to be done for other topics. Topics which should be covered in future reports are:

(1) Column design curves for axial loads greater than 0.6 P_y.

(2) Design of columns for multi-story frames by restrained column theory.

(3) Study of behavior of bracing in multi-story frames under the action of lateral loads.

(4) Analysis of unbraced and braced multi-story frames subjected to lateral loads.

(5) Frame stability of multi-story frames subjected to vertical loads only and of frames subjected to lateral loads as well.

Symbols

K_{rc}	relative stiffness of a restraining column
ℓ	length of a diagonal bracing member
L	length of a member
M	moment acting at the cross section of a member
M_{all}	allowable moment for a member
M_c	moment resisted at end of a column segment
M_D	moment resulting from dead loading only
M_e	external moment applied at a joint
M_p	plastic moment capacity of a member in bending
M_{pc}	reduced plastic moment capacity of a member under compression and bending
M_r	moment resisted by a restraining member
M_s	moment resisted by a rotational spring
M_U	unbalanced moment acting at a joint
P	axial load acting on a column segment
P_{all}	allowable working axial load of a column segment
P_y	axial load required to plastify a cross section
r	radius of gyration of a cross section
t	thickness of flange of a WF section
V	shear force
V_p	shear force acting at a plastified section
V_D	shear force due to dead loading only

Symbols

w	uniformly distributed load
w_D	uniformly distributed dead load
w_{D+L}	uniformly distributed dead plus live load
w_L	uniformly distributed live load
w_U	ultimate uniformly distributed load
w_W	working uniformly distributed load
α	modification factor for relative stiffness
δ	lateral deflection
ΔM	increment of moment
ΔP	increment of axial load
ϵ	error
θ	end rotation of a member
σ_{all}	allowable stress
σ_y	yield stress

APPENDIX A

DISTRIBUTION OF UNBALANCED MOMENTS TO AN ELASTICALLY RESTRAINED COLUMN SEGMENT

This derivation uses notation based on the letters assigned to the subassemblage in Fig. 13 with the following modifications:

1. It is assumed that the far ends of the framing members are fixed.
2. $\left(\frac{w_D L^2}{12}\right)_{AF} = M'_A$
3. $\left(\frac{w_D L^2}{12}\right)_{BG} = M'_B$
4. $(M_p)_{AE} = M_{PA}$
5. $(M_p)_{BH} = M_{PB}$

By the method of slope deflection and adopting the sign convention that on a joint a clockwise moment is positive.

$$
\begin{aligned}
M_{AC} &= 2EK_{AC}\ (2\theta_A) \\
M_{AF} &= 2EK_{AF}\ (2\theta_A) + M'_A \\
M_{AB} &= 2EK_{AB}\ (2\theta_A + \theta_B) \\
M_{BD} &= 2EK_{BD}\ (2\theta_B) \\
M_{BA} &= 2EK_{BA}\ (2\theta_B + \theta_A) \\
M_{BG} &= 2EK_{BG}\ (2\theta_B) - M'_B
\end{aligned}
\qquad \text{(A-1)}
$$

where:

$$K_{AC} = \frac{I_{AC}}{L_{AC}} \quad , \text{etc.}$$

Equilibrium of moments at joints A and B requires:

$$\begin{aligned} M_{AC} + M_{AF} + M_{AB} - M_{PA} &= 0 \\ M_{BD} + M_{BG} + M_{BA} + M_{PB} &= 0 \end{aligned} \qquad \text{(A-2)}$$

Substitution from equations (1) into equations (2)

$$4E\theta_A(K_{AC} + K_{AF} + K_{AB}) + 2E\theta_B K_{AB} = M_{PA} - M_A'$$

$$4E\theta_B(K_{BD} + K_{BG} + K_{BA}) + 2E\theta_A K_{BA} = -M_{PB} + M_B'$$

Letting

$$E\theta_A = A$$

$$E\theta_B = B$$

$$(K_{AC} + K_{AF} + K_{AB}) = \sum K_A$$

$$(K_{BD} + K_{BG} + K_{BA}) = \sum K_B$$

and substituting:

$$M_{PA} - M_A' = M_{UA}$$

$$M_{PB} - M_B' = M_{UB}$$

The resulting equations are:

$$\begin{aligned} 4A\sum K_A + 2BK_{AB} &= M_{UA} \\ 4B\sum K_B + 2AK_{AB} &= -M_{UB} \end{aligned} \qquad \text{(A-3)}$$

Solving equation (3) for A and B.

$$A = \frac{M_{UA} + \frac{1}{2}\frac{K_{BA}}{\sum K_B} M_{UB}}{\sum K_A \left(4 - \frac{K_{AB}}{\sum K_A}\frac{K_{BA}}{\sum K_B}\right)}$$

$$B = \frac{-M_{UB} - \frac{1}{2}\frac{K_{AB}}{\Sigma K_A} M_{UA}}{\Sigma K_B \left(4 - \frac{K_{BA}}{\Sigma K_B}\frac{K_{AB}}{\Sigma K_A}\right)}$$

Since member AB is prismatic $K_{AB} = K_{BA} = K$

Also $\frac{K_{AB}}{\Sigma K_A} = D_{AB}$

and $\frac{K_{BA}}{\Sigma K_B} = D_{BA}$

Therefore:

$$E\theta_A = A = \frac{M_{UA} + \frac{1}{2} D_{BA} M_{UB}}{\Sigma K_A (4 - D_{AB} D_{BA})}$$

$$E\theta_B = B = -\frac{D_{BA}}{2K} \frac{(2M_{UB} + D_A M_{UA})}{(4 - D_{AB} D_{BA})}$$

Substituting into the two pertinent equations (1) the following is obtained:

$$M_{AB} = M_{UA} \frac{D_{AB}(1 - \frac{1}{4} D_{BA})}{(1 - \frac{1}{4} D_{AB} D_{BA})} - M_{UB} \frac{D_{BA}(1 - D_{AB})}{2(1 - \frac{1}{4} D_{AB} D_{BA})}$$

(A-4)

$$M_{BA} = M_{UB} \frac{D_{BA}(1 - \frac{1}{4} D_{AB})}{(1 - \frac{1}{4} D_{AB} D_{BA})} - M_{UA} \frac{D_{AB} (1 - D_{BA})}{2(1 - \frac{1}{4} D_{AB} D_{BA})}$$

It should be noted that this derivation could have begun with different assumptions for end conditions. The effect of different end conditions may be recognized by using modified values of the relative stiffnesses as indicated in Art. 3.1 of the text.

APPENDIX B

THE EFFECT OF DEAD LOAD RATIOS ON PARTIAL LOADING

Consider the subassemblage represented in Fig. 16. It is assumed that the columns above and below the one to be designed are of equal lengths and cross sections, so that the distribution factors to beams AE and AF will be:

$$D_{AE} = \frac{K_{AE}}{K_{AE}+K_{AF}+K_{AC}+\frac{3}{2}K_{AB}}$$

$$D_{AF} = \frac{K_{AF}}{K_{AE}+K_{AF}+K_{AC}+\frac{3}{2}K_{AB}} \qquad \text{(B-1)}$$

For a given level of loading w_{AE} and w_{AF} the moments at the end framing into the columns of beams AE and AF are:

$$M_{AE} = \frac{w_{AE}L^2_{AE}}{12} + \left(\frac{w_{AF}L^2_{AF}}{12} - \frac{w_{AE}L^2_{AE}}{12}\right) D_{AE}$$

$$M_{AF} = \frac{w_{AF}L^2_{AF}}{12} - \left(\frac{w_{AF}L^2_{AF}}{12} - \frac{w_{AE}L^2_{AE}}{12}\right) D_{AF} \qquad \text{(B-2)}$$

Let

$w_{AE} = w_L$

$w_{AF} = w_R$

$D_{AE} = D_L$

$D_{AF} = D_R$

The criterion for determining whether or not partial loading must be considered will depend on whether or not the moments M_L and M_R will be equal to the plastic moments M_{PL} and M_{PR} upon application of the loading level w_{DL} and w_{DR}.

$$\frac{w_{UL}L_L^2}{16} \leq \frac{w_{DL}L^2_L}{12} + \left(\frac{w_{DR}L^2_R}{12} - \frac{w_{DL}L^2_L}{12}\right) D_L$$

$$\frac{w_{UR}L^2_R}{16} \leq \frac{w_{DR}L^2_R}{12} - \left(\frac{w_{DR}L^2_R}{12} - \frac{w_{DL}L^2_L}{12}\right) D_R \qquad \text{(B-3)}$$

Inequalities 3 can be expressed as follows:

$$\frac{w_{DL}}{w_{UL}} \geq \frac{\frac{3}{4}}{1 + \left(\frac{w_{DR}L^2_R}{w_{DL}L^2_L} - 1\right) D_L}$$

$$\frac{w_{DR}}{w_{UR}} \geq \frac{\frac{3}{4}}{1 - \left(1 - \frac{w_{DL}L^2_L}{w_{DR}L^2_R}\right) D_R} \qquad \text{(B-4)}$$

It can be seen that if $\frac{w_{DL}L^2_L}{w_{DL}L^2_R} = 1$ as in Art. 4 then Eq. B-4 becomes $\frac{3}{4}$, as was expected.

APPENDIX C

COMPARATIVE DESIGNS OF A TEN STORY FIVE BAY FRAME

Four different designs of an interior frame of a ten story five bay building are made. The four different designs are compared for weight of steel. The frame has story heights equal to 12 ft. except the first story which is 15 ft. high. The spans of the bays are 30 ft. in length. An elevation and plan of the building frame are given in Figs. 34 and 35. The four different design methods are described in Art. 8. This appendix contains the description of the computations which comprise each of the designs. Sample computations and tabulations are shown in the design sheets at the end of the appendix.

The loadings on the structure are listed on design sheets 1 and 2. These loadings are the same for all floors except the roof. The uniform loads per square foot of floor area are converted to pounds per linear foot of beam by multiplication by the distance between frames (24').

C1 METHOD 1 - PLASTIC DESIGN OF BEAMS - CONVENTIONAL (ELASTIC) DESIGN OF COLUMNS

C1.1 Design of Beams - Sheet 2

Beams are designed plastically for full end restraint.

The design load w_U is the total dead plus live loading times a factor of 1.85. The values of $\frac{I}{L}$ for the beams and of $\frac{w_D L^2}{12}$ are computed for future use. Having determined the required M_p for the beams, the member sizes are determined from the plastic moment tables of the Plastic Design Supplement of the AISC (10).

C1.2 Unbalanced Moments Acting at Column Joints - Sheets 3 and 4

The unbalanced moment acting at each column joint is due to partial floor or roof loading in its vicinity. It is made up of two parts. (a)* One part is the difference between the plastic moment M_p of the beam on the right side of the column and the dead load moment M_D of the beam on the other. M_D is equal to $\frac{w_D L^2}{12}$. (b) The other part of the unbalanced moment is the difference in shears at the opposite faces of a column multiplied by half the depth of the column. The column depths are taken equal to 14 in. V_P is the shear acting at the face of a column due to full live plus dead load times a factor of 1.85. V_D is the shear acting at the face of a column due to dead load times a factor of

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* Comments refer to calculations identified by the same letters in design sheet 3.

1.85 (See Fig. 37). (c) Once the ultimate unbalanced moment is obtained, an effective working value is calculated by dividing by the load factor.

C1.3 Distribution of Moments to Column Segments - Sheet 4

The equations for the distribution factors to be used in the design of each column segment are tabulated. These are taken from Art. 6. They take into account partial loading wherever it is a more severe condition than full loading.

C1.4 Design Axial Loading for Column Segments - Sheet 5

Design axial loadings and unbalanced moments for column segments are tabulated. Design axial loadings for interior columns B, C, D and E are all the same. Design axial loadings for exterior columns A and F are the same.

(a) The floor or roof dead load contributed to each column segment (column 1) is the uniform floor load on a tributary area. For the interior columns this area has a width equal to the 24 ft. bent spacing and a length equal to the 30 ft. bay span. For exterior columns the width is the same but the length is equal to half the bay span 15 ft. The magnitudes of the floor loads are taken from Design Sheet 1. The exterior columns, in addition to the loads from their tributary floor areas, must support a wall weight. The wall weight is 80 lb per square foot. The wall area supported by each column has a height equal to the distance between stories of 12' and a width equal to the bent spacing of 24'.

(b) The floor or roof live load (column 2) is calculated from the uniform live loads given in Design Sheet 1 applied to the same tributary areas.

(c) Due to the improbability of the entire live load acting on the frame at any one time, columns are designed for only certain percentages of the live load contribution from each story. The percentages of live load specified (Column 3) are those allowed by the New York City Building Code.

(d) The reduced live loads from floors are calculated by taking the percentage of live load contributions (Column 2) specified by the New York City Building Code (Column 3). These calculations are tabulated in Column 4.

(e) The total contribution from each floor (Column 5) is obtained by adding the dead and reduced live load contributions (Columns 1 and 4).

(f) The loading to be carried by each column segment (Column 6) is equal to the sum of all the loading contributed by tributary floor areas to it and to the segments above it. This is equivalent to a sub-total of Column 5.

(g) Since each interior column segment is to be designed for "checkered" loading in its immediate vicinity, one of the beams which frame into its top will only be subjected to dead load. Therefore, a reduction to the total load acting on each interior column segment must be made. This reduction is equal to the total live load distributed over one half the tributary area of the partially loaded beam framing into each segment's top (Column 7).

(h) The reduced total loading on each column segment (Column 8) is equal to the total axial load for full loading (Column 6) less the reduction due to partial loading (Column 7).

C1.5 Design of Columns - Sheets 6 to 10

Columns are designed by using the AISC interaction formula. The axial loadings on the column segments are the values taken from the table in Design Sheet 5 plus the weight of the column segment, the segments above and fireproofing.

(a) In the sample calculations, the weight of the column segment plus the column segments above is designated C.W., while the weight of the fireproofing of these columns is designated F.P. The weight of fireproofing for a twelve foot column is taken as 1.1 kips.

(b) The moment which acts at the ends of the column segment is obtained by multiplying the corresponding unbalanced moment at its joint by the appropriate distribution factor. The unbalanced moments at the ends of each column segment are tabulated in Design Sheet 5 (Column 9). The appropriate formula for the distribution factor is obtained from Design Sheet 4. For practical reasons, columns are usually taken to have the same cross section for two story lengths. The lower of the two stories is the one designed, since it has the higher axial load, and it has unbalanced moments which are equal to or larger than those for the column segment above (in this particular frame). Therefore, the restraining column segment has the same moment of inertia as the one to be designed.

(c) Once the end moments and the axial load for a trial section of a column segment are known, the AISC interaction formula is applied. The moment is divided by the section modulus of the segment, from which the applied bending stress f_b is obtained. The axial load is divided by the area; therefore, the applied axial stress f_a is obtained. The allowable bending stress F_b is 20 kips unless $\frac{Ld}{bt} > 600$ (9).

(d) The allowable axial stress can be determined for a given $\frac{L}{r}$ from the AISC Design Manual (9).

(e) The expression $\frac{f_a}{F_a} + \frac{f_b}{F_b}$ is then obtained. If it is equal to or less than one, the section is adequate. Sample designs of column segments are shown in Design Sheets 6 to 10.

C2 PLASTIC DESIGN OF BEAMS AND ULTIMATE STRENGTH DESIGN OF COLUMNS

C2.1 Design of Beams

This is done in exactly the same manner as was done for the previous plastic design. Sample calculations are shown in Design Sheet 2 and members are summarized under Method No. 2 on Design Sheet 23.

C2.2 Unbalanced Moments Acting at a Joint

These are determined as described in C1.2. Sample calculations are given in Design Sheets 3 and 4 and the unbalanced moments are summarized in Design Sheet 11.

C2.3 Distribution of Moments to Column Segments

This is done as described in C1.3.

C2.4 Design Axial Loading for Column Segments - Sheet 11

The design axial loadings for column segments which are designed by the formuli of the Plastic Design Supplement of the AISC (10) are tabulated in Design Sheet 11. These values are equal to the working loads of Design Sheet 5 multiplied by the factor 1.85.

C2.5 Design of Columns by the Formuli of the Plastic Design Supplement of AISC - Sheets 12-14

(a) The percentage of yield compressive load acting on a column segment can be calculated once the cross section has been chosen.

(b) The unbalanced moments acting at the joints of the segment are then multiplied by the appropriate distribution factor (Sheet 4), the result being the moments acting at the ends of the column segment. The Plastic Design Manual of the AISC recommends the formula

$$\frac{M}{M_p} = B - G \left(\frac{P}{P_y}\right) \qquad \text{(C-1)}$$

for column segments pinned at one end or bent in double curvature with a hinge to be formed at one end only. For column segments bent in single curvature the recommended formula is

$$\frac{M}{M_p} = 1.0 - K\left(\frac{P}{P_y}\right) - J\left(\frac{P}{P_y}\right)^2 \qquad \text{(C-2)}$$

Interior column segments except for the first story are designed by formula C-2. They are bent in single curvature by the "checkered" loading assumed in their vicinity. All other columns are designed by Eq. C-1, as they either are bent in double curvature or have one end fixed (which is equivalent to double curvature bending).

(c) The values for B, G, K & J can be obtained for a given $\frac{L}{r}$ in the appropriate tables of the Plastic Design Supplement of the AISC (10).

(d) The adequacy of a given trial cross section is determined by the pertinent formula (C-1 or C-2). These formuli are only valid for $P \leq 0.6\ P_y$. Most of the columns designed in this frame violate this limitation. Sample designs are shown in Sheets 12 to 14.

C3 APPROXIMATE CONTINUOUS ELASTIC DESIGN

C3.1 Design of Beams - Sheet 15

The design of the beams is achieved by assuming that inflection points are located at 0.1 L from the ends of the beams.

(a) The beam design moment is $\frac{w_W \times (0.8L)^2}{8}$, where w_W is the working dead plus live loading on the beam.

(b) The section modulus is then obtained by dividing the moment by σ_{all}. The beam section is obtained from the table of economy sections of the AISC Design Manual (9).

C3.2 Unbalanced Moments Acting at Joints - Sheet 15

The unbalanced moment at a joint is determined by finding the difference between the end moments of two beams framing into the joint. This difference occurs when one of the beams is loaded with full dead plus live uniform working load and the other with only dead working uniform load. The moments at the ends of the beams are equal to the moments at the end of cantilever beams of length 0.1L. The cantilever beams are loaded by uniformly distributed loads w_W and shears V, transmitted by pin ended beams of length 0.8L, acting at their end (Sheet 15). This is illustrated in Fig. 38.

(a') The end moments due to a working dead load and to a total dead plus live working load are computed.

(a") Their difference is the unbalanced moment.

C3.3 Distribution of Moments to Column Segments - Sheets 17-20

The assumption of a point of inflection at 0.1L of the ends of the beams has fixed the moments at the column faces. The distribution is, therefore, between the column above and the one to be designed. The formuli of Sheet 4 can be used with $\sum K$ being only the sum of the relative stiffnesses of two column segments.

(a) For all column segments above the first story level the distribution factor will be equal to $\frac{1}{2}$.

C3.4 Design Axial Loads for Column Segments - Sheet 16

The axial loads for the design of column segments are the same as those on Sheet 5. They are retabulated along with the unbalanced moments on Sheet 16.

C3.5 Design of Columns - Sheets 17-20

Columns are designed by the AISC interaction formula. The procedure is the same as that described in C1.5.

C4 SIMPLE BEAM ELASTIC DESIGN

C4.1 Design of Beams - Sheet 21

The beams are designed for full dead plus live working uniform loads as simply supported.

(a) The moment is equal to $\frac{w_w L^2}{8}$.

(b) The section modulus is then determined and the cross section chosen from the economy sizes in the AISC Design Manual.

C4.2 Unbalanced Moments Acting at Column Joints - Sheets 21&22

(a) The unbalanced moments are assumed to be zero for the interior column. For the exterior columns the unbalanced moments are equal to the shear at the end of the beam framing into a joint times half of the depth of the column.

C4.3 Distribution of Moments to Column Segments - Sheets 21 & 22

As in the case of the approximate continuous elastic design, the distribution of moments is only between the two column segments framing into a joint.

(b) The distribution factor will be equal to $\frac{1}{2}$ for all exterior column segments except those of the first story.

C4.4 Design Axial Loading for Column Segments - Sheets 21 & 22

The design axial loads for column segments are not reduced for partial loading. Their values are taken from Column 6 of Design Sheet 5.

C4.5 Design of Column Segments - Sheets 21 & 22

Column segments are designed by the AISC interaction formula as described in C1.5 and C3.5. Design examples are illustrated in Sheets 21 and 22.

C5 DESIGN OF BRACING - Sheet 25

Computations have shown that the frame requires loads much higher than the design loads in order to fail by a sway mechanism. This justifies the assumption that wind will not affect the stability of the columns. In order to guard against frame instability one bay is braced. The bracing is proportioned so that $\frac{\ell}{r} = 300$, and counters are recommended to insure tensile stress only. The AISC Code (9) allows bracing with slenderness ratios of no more than 300 for tension members. The design is done on Sheet 25.

DESIGN SHEET 1

LOADING

Typical Floor

$2\frac{1}{2}$" Concrete	32 psf
3" Cellular Steel Floor	7
$1\frac{3}{4}$" Tile Floor	22
Blown on Fireproofing	2
Hung Ceiling	10
Partitions	20
Mech. & Elect.	4
	97 psf
Floor Beam at 6'	5
Dead Load	102 psf
Live Load	80
Total Load	182 psf

Roof

$1\frac{1}{2}$" Steel Decking	7
$2\frac{1}{2}$" Concrete	32
Blown on Fireproofing	2
1" Insulation	3
Built up Roofing	8
Quarry Tile & Bed $2\frac{1}{2}$"	24
Hung Ceiling	10
Mech. & Elect.	4
	90 psf
Floor Beam at 6'	5
Dead Load	95 psf
Live Load	60
Total Load	155 psf
Wall	80 psf
Wind	30 psf

DESIGN SHEET 2

Loading Cont.

Floor

Total Ld. = 4.4 k/'
Dead Ld. = 2.5 k/'

Roof

Total Ld. = 3.7 k/'
Dead Ld. = 2.3 k/'

PLASTIC DESIGN OF BEAMS

Floor

T.L. = 4.4 k/'

$$\text{Girder} = \frac{0.1}{4.5}$$

$$w_u \quad \frac{}{} \times 1.85 = 8.33 \text{ k/'}$$

$$M_p = \frac{8.33 \times 30^2}{16} = 469 \text{ 'k}$$

Use 21 WF 73

$$\frac{I}{L} = \frac{1600.3}{360} = 4.45$$

$$\frac{w_D L^2}{12} = \frac{2.6 \times 1.85 \times 30^2}{12} = 360 \text{ 'k}$$

Roof

T.L. = 3.7 k/'

$$\text{Girder} = \frac{0.1}{3.8} \times 1.85 = 7.03 \text{ k/'}$$

$$M_p = \frac{7.03 \times 30^2}{16} = 395 \text{ 'k}$$

Use 21 WF 62

$$\frac{I}{L} = \frac{1326.8}{360} = 3.68$$

$$\frac{w_D L^2}{12} = \frac{2.4 \times 1.85 \times 30^2}{12} = 333 \text{ 'k}$$

DESIGN SHEET 3

UNBALANCED MOMENTS ACTING AT COLUMN JOINTS

Joint A_1, F_1

$M_p - M_D = 395$ 'k

$V_p - V_D = 3.8 \times 1.85 \times 15 = 105.5$ k (a)

$V_p e = 105.5 \times \frac{7}{12} = 61.5$ 'k (b)

$M_U = 61.5 + 395 = 456.5$ 'k

Joints B_1, C_1, D_1, E_1

$M_p - M_D = 395 - 333 = 62$ 'k

$V_p - V_D = (3.8-2.4)15 \times 1.85 = 38.9$ k

$(V_p - V_D)\, e = 38.9 \times \frac{7}{12} = 22.7$ 'k

$(M_u)_{ult.} = 62 + 22.7 = 84.7$ 'k

$(M_u)_{work.} = \frac{84.7}{1.85} = 45.8$ 'k (c)

Joints A_2 to A_{11}

$M_p - M_D = M_p = 469$ 'k

$V_p - V_D = V_p = 4.5 \times 1.85 \times 15 = 125$ k

$V_p e = 125 \times \frac{7}{12} = 73$ 'k

$(M_u)_{ult.} = 73 + 469 = 542$ 'k

$(M_u)_{work.} = \frac{542}{1.85} = 293$ 'k

Joints B_2 to B_{11}, C_2 to C_{11}, D_2 to D_{11}, E_2 to E_{11}

$M_p - M_D = 469 - 360 = 109$ 'k

$V_p - V_D = (4.5-2.6)15 \times 1.85 = 52.7$ k

$(V_p - V_D)\, e = 5.27 \times \frac{7}{12} = 30.8$ 'k

DESIGN SHEET 4

$(M_u)_{ult.} = 30.8 + 109 = 139.8$ 'k

$(M_u)_{work} = \frac{139.8}{1.65} = 75.5$ 'k

DISTRIBUTION OF MOMENTS TO COLUMN SEGMENTS

A_1A_2, F_1F_2 $\qquad \frac{M}{M_U} = 1$

B_1B_2, C_1C_2, D_1D_2, E_1E_2 $\qquad \frac{M}{M_U} = \frac{K}{\Sigma K}$

A_2A_3, to A_9A_{10} $\qquad \frac{M}{M_U} = \frac{K}{\Sigma K}$

B_2B_3 to B_9B_{10}, C_2C_3 to C_9C_{10}, D_2D_3 to D_9D_{10}

E_2E_3 to E_9E_{10} $\qquad \frac{M}{M_U} = \frac{K}{\Sigma K}$

$A_{10}A_{11}$, $F_{10}F_{11}$ $\qquad \frac{M}{M_U} = \frac{4\frac{K}{\Sigma K}}{3+\frac{K}{\Sigma K}}$

B_{10},B_{11}, $C_{10}C_{11}$, $D_{10}D_{11}$, $E_{10}E_{11}$

$$\frac{M}{M_U} = \frac{2\frac{K}{\Sigma K}}{1+\frac{K}{\Sigma K}}$$

DESIGN SHEET 5

Column Segment	1 (a) D.L. Contr.	2 (b) L.L. Contr.	3 (c) %L.L. Reduction	4 (d) Red. L.L. Contr.	5 (e) Total Contr.	6 (f) Sub Total	7 (g) Ref. for Partial Ld.	8 (h) Total k	9 M_{unb} 'k
B1B2	72 k	42	100	42	114	114	21.0	93	45.8
B2B3	78	57	85	48.5	126.5	240.5	24.3	216.2	75.5
B3B4	78	57	80	45.7	123.7	364.2	22.9	341.3	75.5
B4B5	78	57	75	42.8	120.8	485.0	21.4	463.6	75.5
B5B6	78	57	70	39.9	117.9	602.9	19.9	583.0	75.5
B6B7	78	57	65	37.1	115.1	718.0	18.6	699.4	75.5
B7B8	78	57	60	34.2	112.2	830.2	17.1	813.1	75.5
B8B9	78	57	60	34.2	112.2	942.4	17.1	925.3	75.5
B9B10	78	57	60	34.2	112.2	1054.6	17.1	1037.5	75.5
B10B11	78	57	60	34.2	112.2	1166.8	17.1	1149.7	75.5
A1A2	36	21	100	21	57.0	57.0	-	57.0	214.0
A2A3	62	28.5	85	24.2	86.2	143.2	-	143.2	293.0
A3A4	62	28.5	80	22.8	84.8	228.0	-	228.0	293.0
A4A5	62	28.5	75	21.4	83.4	311.4	-	311.4	293.0
A5A6	62	28.5	70	20.0	82.0	393.4	-	393.4	293.0
A6A7	62	28.5	65	18.5	80.5	473.9	-	473.9	293.0
A7A8	62	28.5	60	17.1	79.1	553.0	-	553.0	293.0
A8A9	62	28.5	60	17.1	79.1	632.1	-	632.1	293.0
A9A10	62	28.5	60	17.1	79.1	711.2	-	711.2	293.0
A10A11	62	28.5	60	17.1	79.1	790.3	-	790.3	293.0

D.L. of Columns not included

AXIAL LOADING & UNBALANCED MOMENTS

FOR COLUMN SEGMENTS

DESIGH SHEET 6

DESIGN OF COLUMNS BY AISC INTERACTION FORMULA

14 x 14 Column Fireproofing = 90 lb. per ft.

B_1B_2, B_2B_3, C_1C_2, C_2C_3, D_1D_2, D_2D_3, E_1E_2, E_2E_3

$$\begin{aligned} P &= 216.2 \text{ k} \\ F.P. &= 2.2 \\ C.W. &= 1.4 \end{aligned} \qquad \text{(a)}$$

$$P = 219.8 \text{ k}$$

$$M_U = 75.5 \text{ 'k}$$

Try 14 WF 61

$$\begin{aligned} A &= 17.94 \text{ in}^2 \\ I &= 641.5 \text{ in}^4 \\ S &= 92.2 \text{ in}^3 \\ r_y &= 2.45 \text{ in} \end{aligned}$$

$$\frac{Ld}{bt} = \frac{144 \times 13.91}{10 \times 0.64} = 313$$

$$F_a = 20 \text{ ksi}$$

$$\left(\frac{I}{L}\right)_{col} = \frac{641.5}{144} = 4.45$$

$$\left(\frac{I}{L}\right)_{beam} = 4.45$$

$$\frac{K}{\Sigma K} = \frac{1}{3}$$

$$M = \frac{75.5}{3} = 25.2 \text{ 'k} \qquad \text{(b)}$$

$$\frac{f_b}{F_b} = \frac{M}{F_a S} = \frac{25.2 \times 12}{92.2 \times 20} = 0.164 \qquad \text{(c)}$$

$$\frac{P}{A} = \frac{219.8}{17.9} = 12.25 \text{ ksi}$$

$$\frac{L}{r} = \frac{144}{2.45} = 58.3$$

$$F_a = 15.31 \text{ ksi} \qquad \text{(d)}$$

$$\frac{f_a}{F_a} = \frac{\frac{P}{A}}{F_a} = \frac{12.25}{15.31} = 0.80$$

$$\frac{F_a}{f_a} + \frac{f_b}{F_b} = 0.96 < 1 \qquad \text{(e)}$$

Use 14 WF 61

DESIGN SHEET 7

Columns B_9B_{10}, $B_{10}B_{11}$, C_9C_{10}, $C_{10}C_{11}$
D_9D_{10}, $D_{10}D_{11}$, E_9E_{10}, $E_{10}E_{11}$

P = 1149.7 k

F.P.= 11.0
C.W.= 1.4
2.7
3.9
5.0
6.6

P = 1180.3 k

$M_U = 75.5$ 'k

$$\frac{M}{M_U} = \frac{2\frac{K}{\Sigma K}}{1 + \frac{K}{\Sigma K}}$$

Try 14 WF 287

A = 84.37 in² I = 3912.1 in⁴ S = 465.5 in³ r = 4.17 in.

$$\frac{I}{L} = \frac{3912.1}{180} = 16.15$$

$$\Sigma K = 32.3 + 4.45 = 36.75 \text{ in}^3$$

$$\frac{K}{\Sigma K} = \frac{16.15}{36.75} = 0.44$$

$$M = 75.5\,\frac{2 \times 0.44}{1.44} = \frac{75.5 \times 0.88}{1.44} = 46.2 \text{ 'k}$$

$$\frac{f_a}{F_a} = \frac{46.2 \times 0.6}{465.5} = 0.06$$

$$\frac{P}{A} = \frac{1180.3}{84.37} = 14.0 \text{ ksi}$$

$$\frac{L}{r} = \frac{180}{4.17} = 43.2 \quad F_b = 16.06 \text{ ksi}$$

$$\frac{f_b}{F_b} = \frac{14.0}{16.06} = 0.872$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 0.872 + 0.06 = 0.93$$

Use 14 WF 287

DESIGN SHEET 8

Columns A_1A_2, A_2A_3

$$P = 143.2 \text{ k} + 2.2 + 2.0 = 147.4 \text{ k}$$

M_U = 293 'k

$M = \frac{293}{2} = 146.5$ 'k

Try 14 WF 84

A = 24.71 in^2 I = 928.4 in^4 S = 130.9 in^3 r = 3.02 in.

$$\frac{f_a}{F_a} = \frac{146.5 \times 0.6}{130.9} = 0.672$$

$$\frac{P}{A} = \frac{147.4}{24.71} = 5.97 \text{ ksi} \qquad \frac{L}{r} = \frac{144}{3.02} = 47.7 \qquad F_b = 15.88 \text{ ksi}$$

$$\frac{f_b}{F_b} = \frac{5.97}{15.88} = 0.375$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 0.38 + 0.67 = 1.05$$

Use 14 WF 87

DESIGN SHEET 9

Columns A_9A_{10}, $A_{10}A_{11}$

P = 790.3
11.0
2.0
2.9
4.0
4.8
6.0
821.0 k

$M_U = 293.0$ 'k

$$\frac{M}{M_U} = \frac{4\frac{K}{\Sigma K}}{3 + \frac{K}{\Sigma K}}$$

$$\frac{K}{\Sigma K} = \frac{1}{2} \times \frac{4}{3} = 0.4$$

$$M = \frac{1.6}{3.4} \times 293.0 = 138.0 \text{ 'k}$$

Try 14 WF 246

$A = 72.33 \text{ in}^2$ $S = 397.4 \text{ in}^3$ $r = 4.12$ in.

$$\frac{f_a}{F_a} = \frac{138.0 \times 0.6}{397.4} = 0.208$$

$$\frac{P}{A} = \frac{821.0}{72.33} = 11.35 \text{ ksi} \qquad \frac{L}{r} = \frac{180}{4.12} = 43.7 \qquad F_b = 16.06 \text{ ksi}$$

$$\frac{f_b}{F_b} = \frac{11.35}{16.06} = 0.71$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 0.91$$

Try 14 WF 237

$A = 69.69 \text{ in}^2$ $S = 382.2 \text{ in}^3$ $r = 4.11$ in.

$$\frac{f_a}{F_a} = \frac{138.0 \times 0.6}{382.2} = 0.212$$

$$\frac{P}{A} = \frac{821.0}{69.69} = 11.65 \text{ ksi} \qquad F_b = 16.06 \text{ ksi}$$

$$\frac{f_b}{F_b} = \frac{11.65}{16.06} = 0.726$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 0.937 = 0.94$$

DESIGN SHEET 10

Try 14 WF 228

$A = 67.06 \text{ in}^2$ $S = 367.8 \text{ in}^3$ $r = 4.10 \text{ in.}$

$\frac{f_a}{F_a} = \frac{138 \times 0.6}{367.8} = 0.227$

$\frac{P}{A} = \frac{821.0}{67.06} = 12.2 \text{ ksi}$ $\frac{L}{r} = \frac{180}{4.10} = 44.0$ $F_b = 16.06 \text{ ksi}$

$\frac{f_b}{F_b} = \frac{12.2}{16.06} = 0.760;$ $\frac{f_a}{F_a} + \frac{f_b}{F_b} = 0.760 + 0.227 = 0.987$

Use 14 WF 228

DESIGN SHEET 11

DESIGN OF COLUMNS BY THE AISC PLASTIC DESIGN SUPPLEMENT

Ultimate Loads		
Columns	Axial Load k	M_U 'k
B_1B_2	172.05	84.7
B_2B_3	400.00	139.7
B_3B_4	631.40	139.7
B_4B_5	857.70	139.7
B_5B_6	1078.60	139.7
B_6B_7	1293.90	139.7
B_7B_8	1504.20	139.7
B_8B_9	1711.80	139.7
B_9B_{10}	1919.40	139.7
$B_{10}B_{11}$	2126.90	139.7
A_1A_2	105.45	456.0
A_2A_3	264.90	542.1
A_3A_4	421.80	542.1
A_4A_5	576.10	542.1
A_5A_6	727.80	542.1
A_6A_7	876.70	542.1
A_7A_8	1023.10	542.1
A_8A_9	1169.40	542.1
A_9A_{10}	1315.70	542.1
$A_{10}A_{11}$	1462.10	542.1

DESIGN SHEET 12

Columns B_1B_2, B_2B_3, C_1C_2, C_2C_3
D_1D_2, D_2D_3, E_1E_2, E_2E_3

P = 400 k
F.P. 2.2
C.W. 1.4
3.6 x 1.85 = 6.66 k

M_U = 139.7 'k

$\frac{M}{M_U} = \frac{K}{\Sigma K}$

P = 406.7 k

$\frac{P}{P_y} \leq 0.6$; $P_y = \frac{406.7}{0.6} = 675$ k

Use 14 WF 74

P_y = 718 k

$\frac{P}{P_y} = \frac{406.7}{718} = 0.565$ (a)

$\left(\frac{P}{P_y}\right)^2 = 0.3200$

r_x = 6.05 in. $\frac{L}{r} = \frac{144}{6.05} = 23.8$ M_p = 345.4 'k

K = 0.746, J = 0.418

$\frac{I}{L} = \frac{796.8}{144} = 5.5$ in³ $\frac{K}{\Sigma K} = \frac{5.5}{15.45} = 0.357$ (b)

M = 0.357 x 139.7 = 49.7 'k

$\frac{M}{M_p} = \frac{49.7}{345.4} = 0.144$

$\frac{M}{M_p} \leq 1 - K\left(\frac{P}{P_y}\right) - J\left(\frac{P}{P_y}\right)^2$

0.144 ≤ 1 - 0.42 - 0.13 = 0.45 (c) (d) ok

Use 14 WF 74

DESIGN SHEET 13

Columns B_3B_4, B_4B_5, C_3C_4, C_4C_5

D_3D_4, D_4D_5, E_3E_4, E_4E_5

P = 857.7 k

F.P. 4.4

C.W. 1.4

2.7

8.5 x 1.85 = 15.7

M_U = 139.7 'k

P = 873.4 k

$$\frac{P}{P_y} \leq 0.6$$

$$P_y = \frac{873.4}{0.6} = 1450 \text{ k}$$

Use 14 WF 150

Columns B_5B_6, B_6B_7, C_5C_6, C_6C_7

D_5D_6, D_6D_7, E_5E_6, E_6E_7

P = 1293.9 k

F.P. 6.6

C.W. 1.4

2.7

3.9

14.6 x 1.85 = 27

M_U = 139.7 'k

P = 1320.9 k

$$\frac{P}{P_y} \leq 0.6$$

$$P_y = \frac{1320.9}{0.6} = 2200 \text{ k}$$

Use 14 WF 228

DESIGN SHEET 14

Columns A_1A_2, A_2A_3

$P = \underline{264.90}$ k $\qquad M_U = 542.1$ 'k

F.P. 2.2 $\qquad M_o = 271.0$ 'k (b)

C.W. 2.0

$4.2 \times 1.85 = 7.76$ k

$P = 272.7$ k

Try 14 WF 78

$M_p = 368.5$ 'k $\quad P_y = 757$ k $\quad I = 851.2\ in^4$ $\quad r_x = 6.09$ in.

$$\frac{M}{M_p} = \frac{271.0}{368.5} = 0.735$$

$$\frac{P}{P_y} = \frac{272.7}{757} = 0.36 \quad (a) \qquad \frac{L}{r_x} = \frac{144}{6.09} = 23.6$$

$$\frac{M}{M_p} = B - G\frac{P}{P_y} \qquad B = 1.144 \qquad G = 1.191 \quad (c)$$

$1.144 - 1.191 \times 0.36$

$1.144 - .429 \qquad = 0.715 < 0.735$ (d)

Use 14 WF 84

Columns A_5A_6, A_6A_7

$P = \underline{876.7}$ k $\qquad M_U = 542.1$ 'k

F.P. 6.6 $\qquad M = 271.1$ 'k (b)

C.W. 2.0

3.0

3.6

$15.2 \times 1.85 = 28.1$ k

$P = 904.8$ k

$$\frac{P}{P_y} \leq 0.6 \quad ; \qquad P_y = \frac{904.8}{0.6} = 1505 \text{ k}$$

14 WF 158

DESIGN SHEET 15

ELASTIC DESIGN OF BEAMS

Floor (Typical)

Assume inflection points at 0.1 x L from the ends of the beams.

$L = 30 - 2 \times 0.1 \times 30 = 24'$

Total Load = 4.4 k/'
Girder = 0.1
4.5 k/'

$M = \frac{w_w L^2}{8} = \frac{4.5 \times 24^2}{8} = 324\ 'k$ (a)

$V = 4.5 \times 12 = 54\ k$

$M_{col.} = 54 \times 3 + 3 \times 4.5 \times 1.5 = 162 + 20.2 = \underline{182.2}\ 'k$ (a')

$(M_{col.})\ D.L. = \frac{2.6}{4.5} \times 182.2 = 105.3\ 'k$ (See Fig. 38)

$M_U = 182.2 - 105.3 = \underline{76.9}\ 'k$ (a")

$S = \frac{M}{\sigma_{all}} = \frac{324 \times 12}{12} = 194.5\ in^3$ (b)

24 WF 84

Roof

Total Load = 3.8 k/'

$M = \frac{w_w L^2}{8} = \frac{3.8 \times 24^2}{8} = 274\ 'k$ (a)

$V = 3.8 \times 12 = 45.6\ k$

$M_{col.} = 45.6 \times 3 + 3 \times 3.8 \times 1.5 = \underline{154.1}\ 'k$

$(M_{col.})\ D.L. = \frac{2.4}{3.8} \times 154.1 = \underline{97.4}\ 'k$ (a')

$M_U = 154.1 - 97.4 = \underline{56.7}\ 'k$ (a")

$S = \frac{M}{\sigma_{all}} = \frac{274 \times 12}{20} = 164.4$ (b)

24 WF 76

DESIGN SHEET 16

DESIGN OF COLUMNS CORRESPONDING TO APPROXIMATE CONTINUOUS ELASTIC DESIGN OF BEAMS

Working Loads		
Columns	Axial Load k	M_U 'k
B_1B_2	93	56.7
B_2B_3	216.2	76.9
B_3B_4	341.3	76.9
B_4B_5	463.6	76.9
B_5B_6	583.0	76.9
B_6B_7	699.4	76.9
B_7B_8	813.1	76.9
B_8B_9	925.3	76.9
B_9B_{10}	1037.5	76.9
$B_{10}B_{11}$	1149.7	76.9
A_1A_2	57.0	154.1
A_2A_3	143.2	182.2
A_3A_4	228.0	182.2
A_4A_5	311.4	182.2
A_5A_6	393.4	182.2
A_6A_7	473.9	182.2
A_7A_8	553.0	182.2
A_8A_9	632.1	182.2
A_9A_{10}	711.2	182.2
$A_{10}A_{11}$	790.3	182.2

DESIGN SHEET 17

Columns B_1B_2, B_2B_3, C_1C_2, C_2C_3, D_1D_2, E_1E_2, E_2E_3

$P = 216.2$ k
$\quad 2.2$
$\quad 1.4$
$\quad 219.8$ k

$M_U = 76.9$ 'k
$M = 38.5$ 'k (a)

Try 14 WF 61

$A = 17.94$ in^2 $\quad S = 92.2$ in^3 $\quad r_y = 2.45$ in.

$$\frac{f_a}{F_a} = \frac{38.5 \times 0.6}{92.2} = 0.25$$

$$\frac{P}{A} = \frac{219.8}{17.94} = 12.25 \text{ ksi}$$

$$\frac{L}{r_y} = \frac{144}{2.45} = 58.3$$

$$F_b = 15.31 \text{ ksi}$$

$$\frac{f_b}{F_b} = \frac{12.25}{15.31} = 0.80$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1.05$$

Try 14 WF 68

$A = 20.00$ in^2 $\quad S = 103.0$ in^3 $\quad r_y = 2.46$ in.

$$\frac{f_a}{F_a} = \frac{38.5 \times 0.6}{103} = 0.224$$

$$\frac{f_b}{F_b} = \frac{219.8}{20 \times 15.31} = 0.717$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 0.941$$

Use 14 WF 68

DESIGN SHEET 18

Columns B_9B_{10}, $B_{10}B_{11}$, C_9C_{10}, $C_{10}C_{11}$, D_9D_{10}, $D_{10}D_{11}$

E_9E_{10}, $E_{10}E_{11}$

$P = 1180.3$ k

$M_U = 76.9$ 'k

$$\frac{K}{\Sigma K} = \frac{\frac{I}{15}}{\frac{I}{12} + \frac{I}{15}}$$

$$= \frac{1}{\frac{15}{12} + 1} = \frac{1}{2.25} = 0.45$$

$$\frac{M}{M_U} = \frac{0.9}{1.45} = 0.62$$

$M = 0.62 \times 76.9 = 47.7$ 'k (a)

Try 14 WF 287

from sheet 7

$$\frac{f_b}{F_b} = 0.872$$

$$\frac{f_a}{F_a} = \frac{47.7 \times 0.6}{465.5} = 0.062$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 0.934$$

Use 14 WF 287

DESIGN SHEET 19

Columns A_1A_2, A_2A_3

$P = 147.4$ k $M = 91.1$ 'k (a)

Try 14 WF 68

$A = 20.00\ in^2$ $S = 103.0\ in^3$ $r_y = 2.46$ in.

$$\frac{f_a}{F_a} = \frac{91.1 \times 0.6}{103} = 0.53$$

$$\frac{P}{A} = \frac{147.4}{20} = 7.37 \text{ ksi}$$

$$\frac{L}{r} = \frac{144}{2.46} = 58.5$$

$F_b = 15.31$ ksi

$$\frac{f_b}{F_b} = \frac{7.37}{15.31} = 0.48$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1.01$$

Use 14 WF 68

DESIGN SHEET 20

Columns A_9A_{10}, $A_{10}A_{11}$

P = 821.0 k

M_U = 182.2 'k

$\frac{K}{\sum K} = \frac{1}{2.25} = 0.445$

$\frac{M}{M_{unb}} = \frac{1.8}{3.45} = 0.52$

M = 0.52 x 182.2 = 95.4 'k (a)

Try 14 WF 211

A = 62.07 in^2 S = 339.2 in^3

$\frac{L}{r} = \frac{180}{4.07} = 44.3$

F_b = 16.06 ksi

$$\frac{f_b}{F_b} = \frac{821.0}{62.07 \times 16.06} = 0.823$$

$$\frac{f_a}{F_a} = \frac{95.4 \times 0.6}{339.2} = 0.169$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 0.823 + 0.169 = 0.992$$

Use 14 WF 211

DESIGN SHEET 21

PIN-ENDED ELASTIC DESIGN OF BEAMS

Floor Beams

$$M = \frac{w_w L^2}{8} = \frac{4.5 \times 30^2}{8} = 507 \text{ 'k} \qquad \text{(a)}$$

$$S = \frac{M}{\sigma_{all}} = \frac{507 \times 12}{20} = 304 \text{ in.}^3 \qquad \text{(b)}$$

Use 30 WF 116

Roof Beams

$$M = \frac{3.9 \times 30^2}{8} = 439 \text{ 'k} \qquad \text{(a)}$$

$$S = \frac{439 \times 12}{20} = 265 \text{ in.}^3 \qquad \text{(b)}$$

Use 27 WF 102

COLUMNS CORRESPONDING TO PIN-ENDED BEAMS

Columns B_1B_2, B_2B_3, C_1C_2, C_2C_3, etc.

$P = 240.5$ k $\qquad M = 0$ (a)

Use 14 WF 61

Columns A_1A_2, A_2A_3

$V = 67.5$ k

$$M = \frac{67.5 \times 7}{12} = 39.4 \text{ 'k} \qquad \text{(a)}$$

$\frac{1}{2} M = 19.7$ 'k (b)

$P = 147.4$ k

DESIGN SHEET 22

Try 14 WF 43

$$\frac{f_b}{F_b} = \frac{19.7 \times 0.6}{62.7} = 0.189 \qquad \frac{L}{r} = 76, \; F_a = 14.2 \text{ ksi}$$

$$\frac{P}{A} = \frac{147.4}{12.65} = 11.7 \text{ ksi}$$

$$\frac{f_a}{F_a} = \frac{11.7}{14.2} = 0.823$$

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1.01$$

Use 14 WF 43

METHOD No. 1

PLASTIC DESIGN OF GIRDERS-COLUMNS AISC INTERACTION

Member	Section	Length ft.	No.	Total Weight lb.
Floor Girders	21WF73	30'	45	98,550
Roof Girders	21WF62	30'	5	9,300
Inner Col.	14WF61	12'	8	5,856
	14WF111	12'	8	10,656
	14WF167	12'	8	16,032
	14WF219	12'	8	21,024
	14WF287	13.5	8	30,996
Outer Col.	14WF87	12'	4	4,176
	14WF127	12'	4	6,096
	14WF158	12'	4	7,584
	14WF193	12'	4	9,264
	14WF228	13.5	4	12,312
			Girders	107,850
			Columns	123,996
			Total	231,846

METHOD No. 2

PLASTIC DESIGN OF GIRDERS-COLUMNS ULTIMATE STRENGTH (AISC)

Member	Section	Length ft.	No.	Total Weight lb.
Floor Girders	21WF73	30'	45	98,550
Roof Girders	21WF62	30'	5	9,300
Inner Col.	14WF74	12'	8	7,104
	14WF150	12'	8	14,400
	14WF228	12'	8	21,888
	14WF314	12'	8	30,144
	14WF398	13.5	8	42,984
Outer Col.	14WF84	12'	4	4,032
	14WF119	12'	4	5,712
	14WF158	12'	4	7,584
	14WF211	12'	4	10,128
	14WF264	13.5	4	14,256
			Girders	107,850
			Columns	158,232
			Total	266,082

METHOD No. 3

CONTINUOUS ELASTIC DESIGN OF GIRDERS

COLUMNS AISC INTERACTION

Member	Section	Length ft.	No.	Total Weight lb.
Floor Girders	24WF84	30'	45	113,400
Roof Girders	24WF76	30'	5	11,400
Inner Col.	14WF68	12'	8	6,528
	14WF119	12'	8	11,424
	14WF167	12'	8	16,032
	14WF211	12'	8	20,256
	14WF287	13.5	8	30,996
Outer Col.	14WF68	12'	4	3,264
	14WF103	12'	4	4,944
	14WF142	12'	4	6,816
	14WF176	12'	4	8,448
	14WF211	13.5	4	11,394
			Girders	124,800
			Columns	120,102
			Total	244,902

METHOD No. 4

GIRDERS DESIGNED AS PIN ENDED ELASTIC BEAMS

COLUMNS AXIALLY LOADED (Except Outer Col.)

Member	Section	Length ft.	No.	Total Weight lb.
Floor Girders	30WF116	30'	45	156,600
Roof Girders	27WF102	30'	5	15,300
Inner Col.	14WF61	12'	8	5,856
	14WF111	12'	8	10,656
	14WF158	12'	8	15,168
	14WF202	12'	8	19,392
	14WF264	13.5	8	28,512
Outer Col.	14WF43	12'	4	2,064
	14WF78	12'	4	3,744
	14WF111	12'	4	5,328
	14WF142	12'	4	6,816
	14WF176	13.5	4	9,504
			Girders	171,900
			Columns	107,040
			Total	278,940

DESIGN SHEET 25

DESIGN OF BRACING

$$\frac{\ell}{r} = 300$$

$\ell = 32.31'$ for upper nine stories

$$r = \frac{32.31 \times 12}{300} = 1.29 \text{ in.}$$

2∠s $2\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$

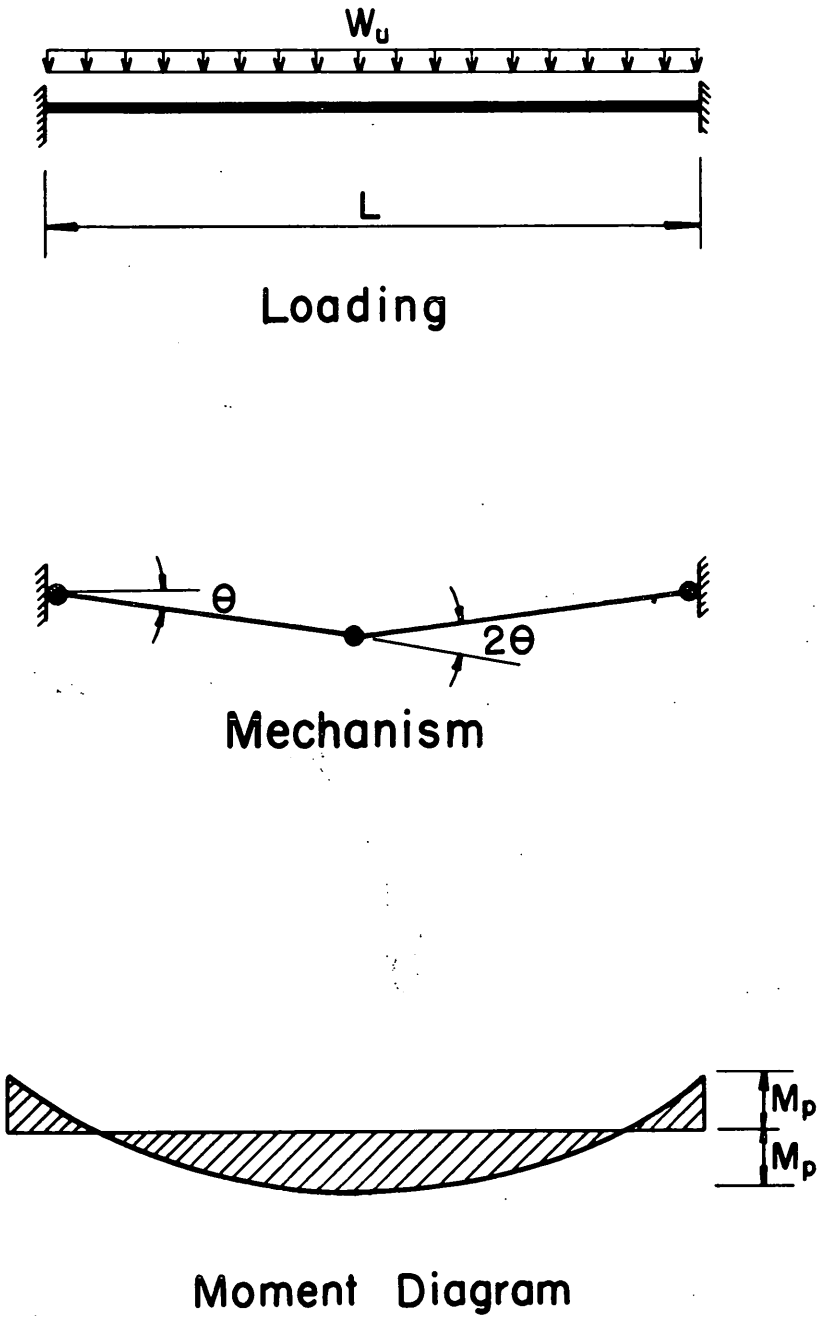
$r_y = 1.40 > 1.29$ in.

$\ell = 33.3'$ for first story

$$r = \frac{33.3 \times 12}{300} = 1.33 \text{ in.}$$

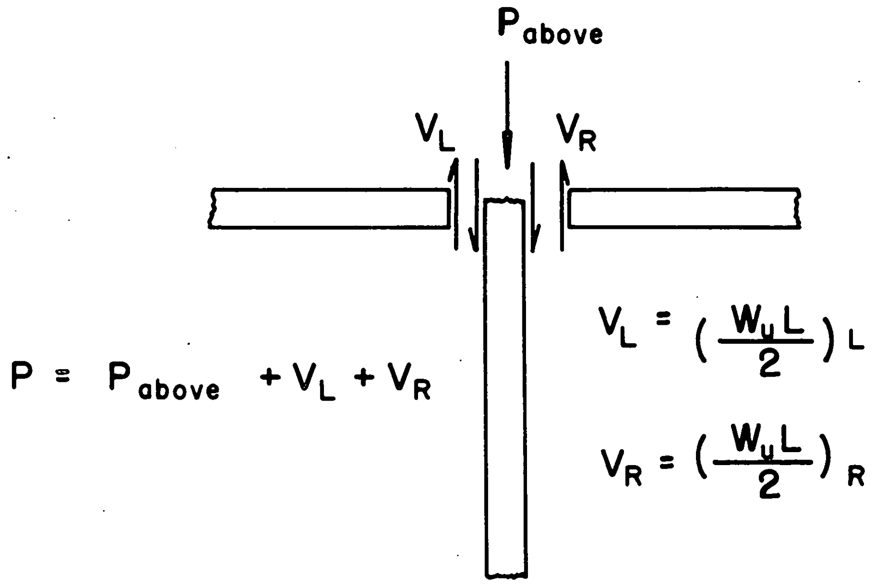
2∠s $2\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$

FIGURES



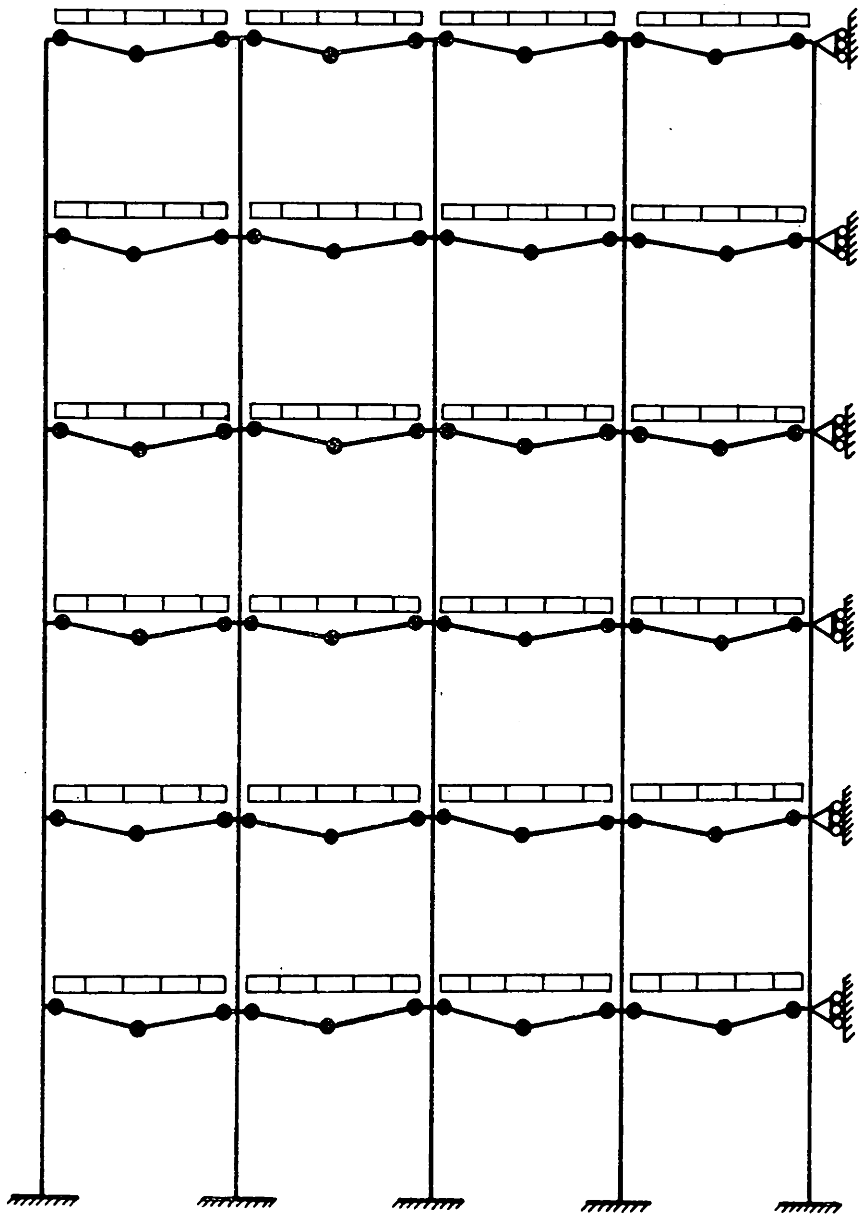
TYPICAL BEAM FAILURE

FIG. 1



AXIAL LOADING ON COLUMNS

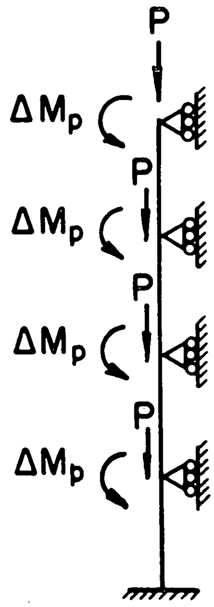
FIG. 2



W_u on all girders

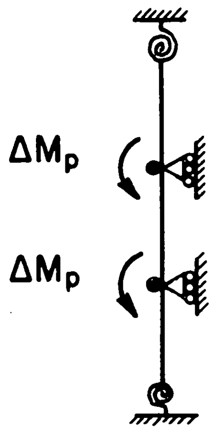
FAILURE MECHANISM OF FRAME UNDER FULL LIVE PLUS DEAD LOAD

FIG. 3



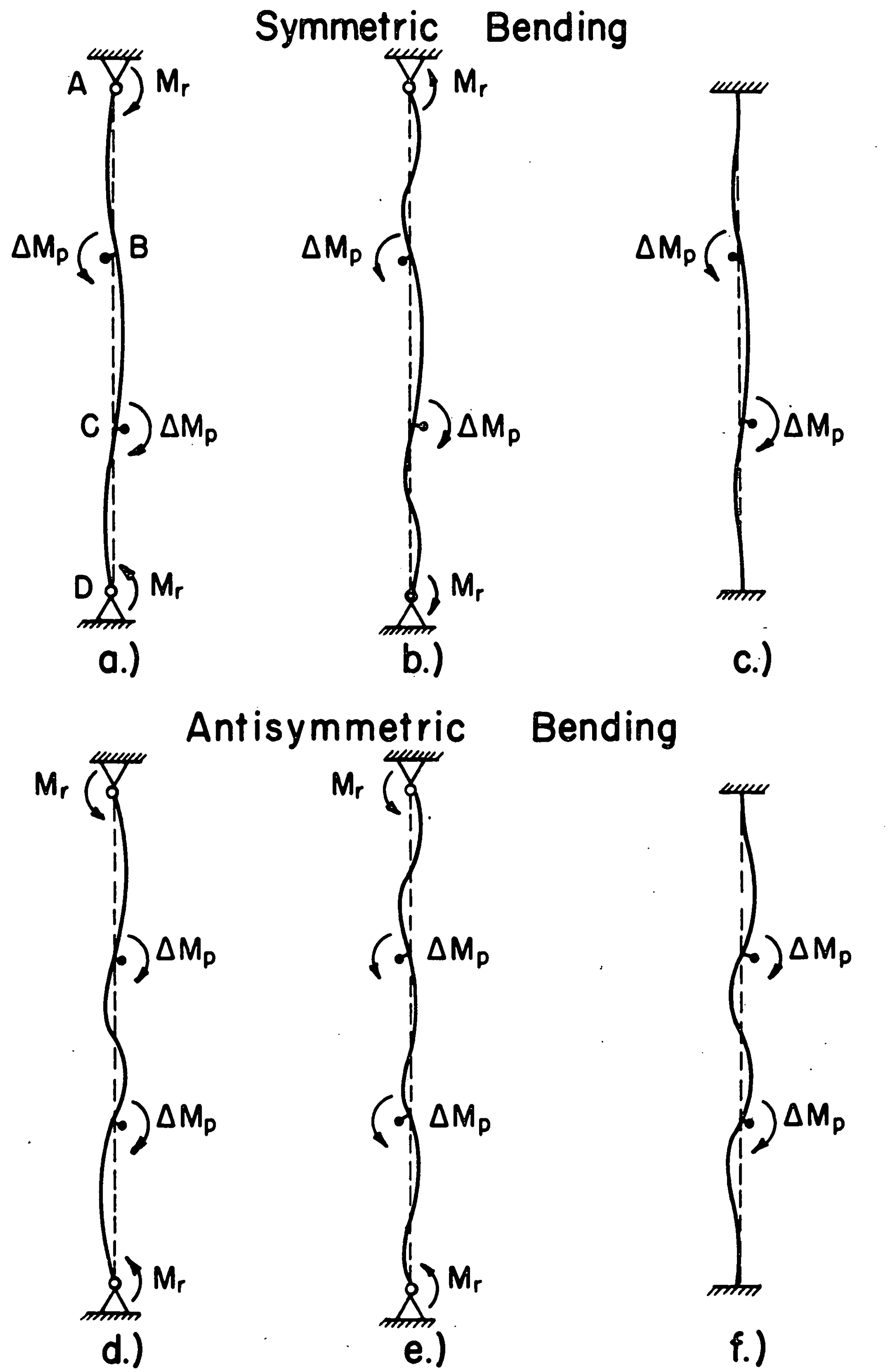
CONTINUOUS COLUMN

FIG. 4



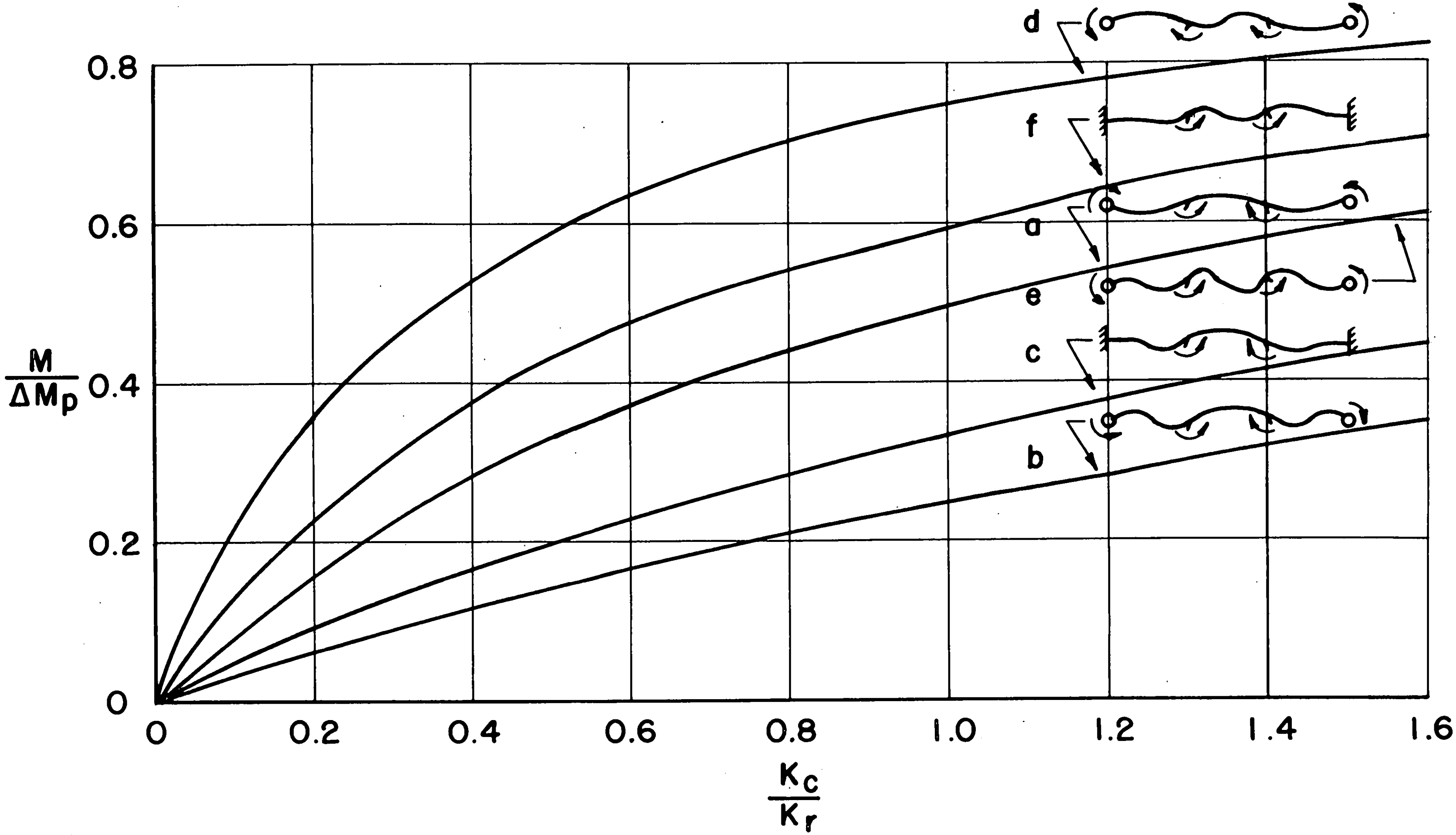
SUBASSEMBLAGE
FOR PURPOSES OF
MOMENT DISTRIBUTION

FIG. 5



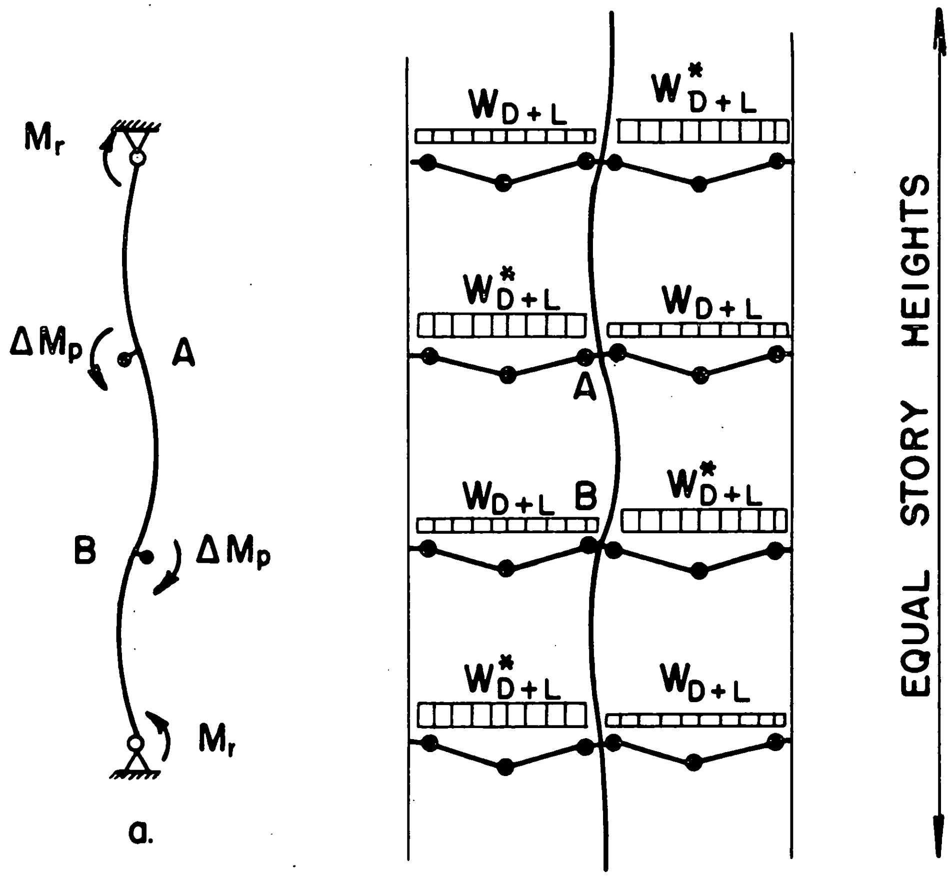
SUBASSEMBLAGES FOR FULL LOADING

FIG. 6



CURVES FOR MOMENT DISTRIBUTION TO SUBASSEMBLAGES

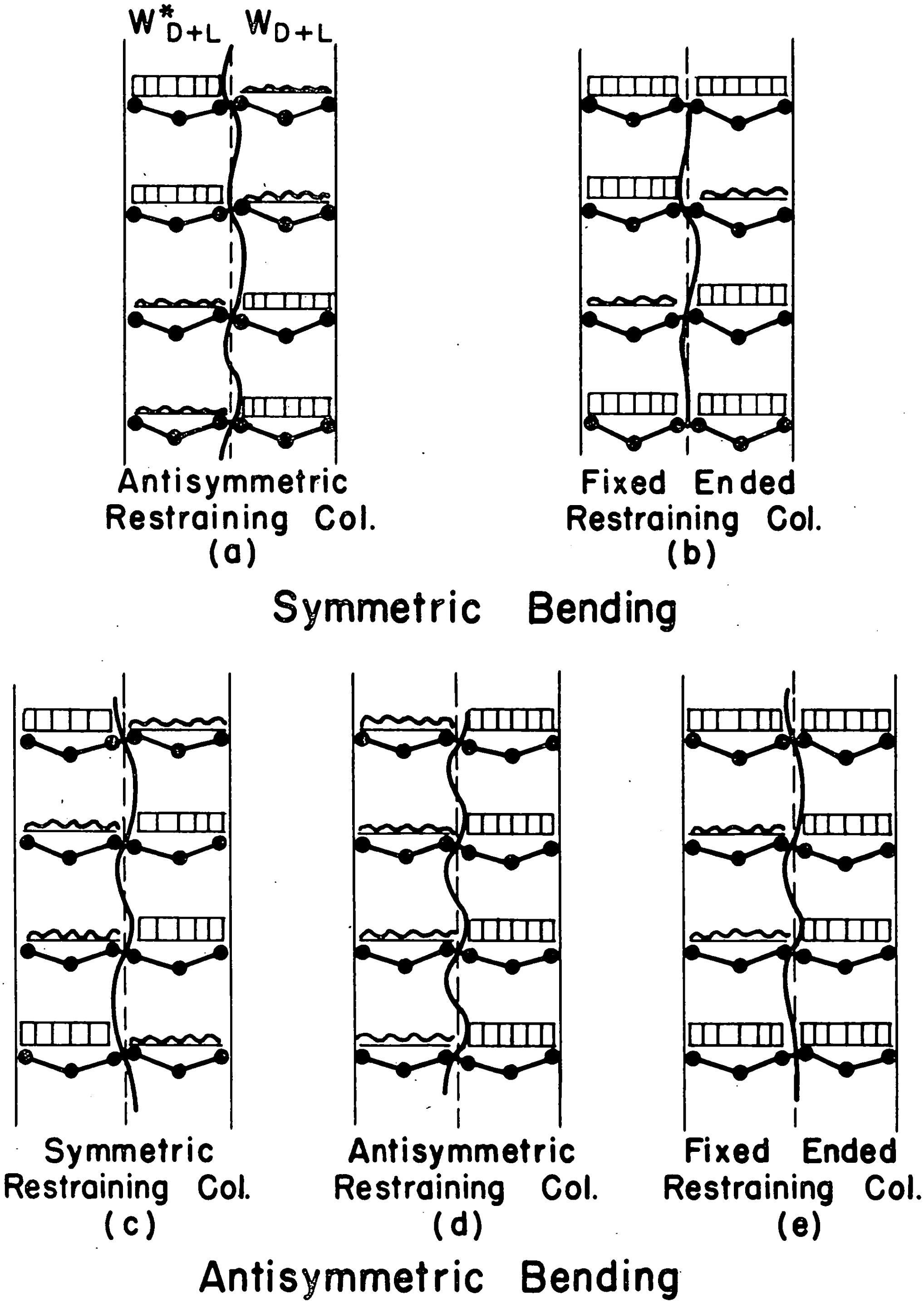
FIG. 7



$W^*_{D+L} > W_{D+L}$

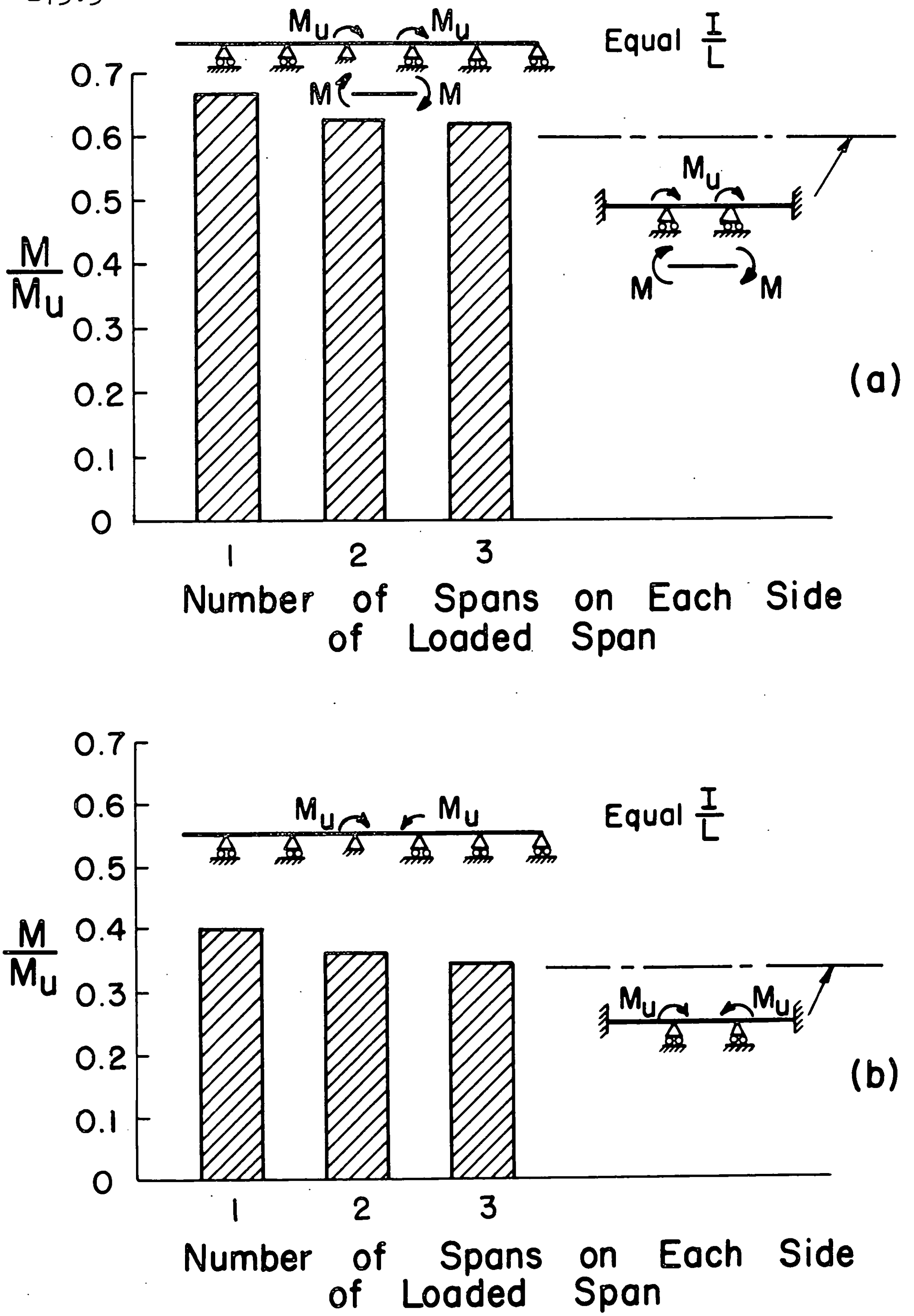
SYMMETRIC BENDING WITH SYMMETRIC RESTRAINING COLUMNS CAUSED BY CHECKERED LOADING

FIG. 8



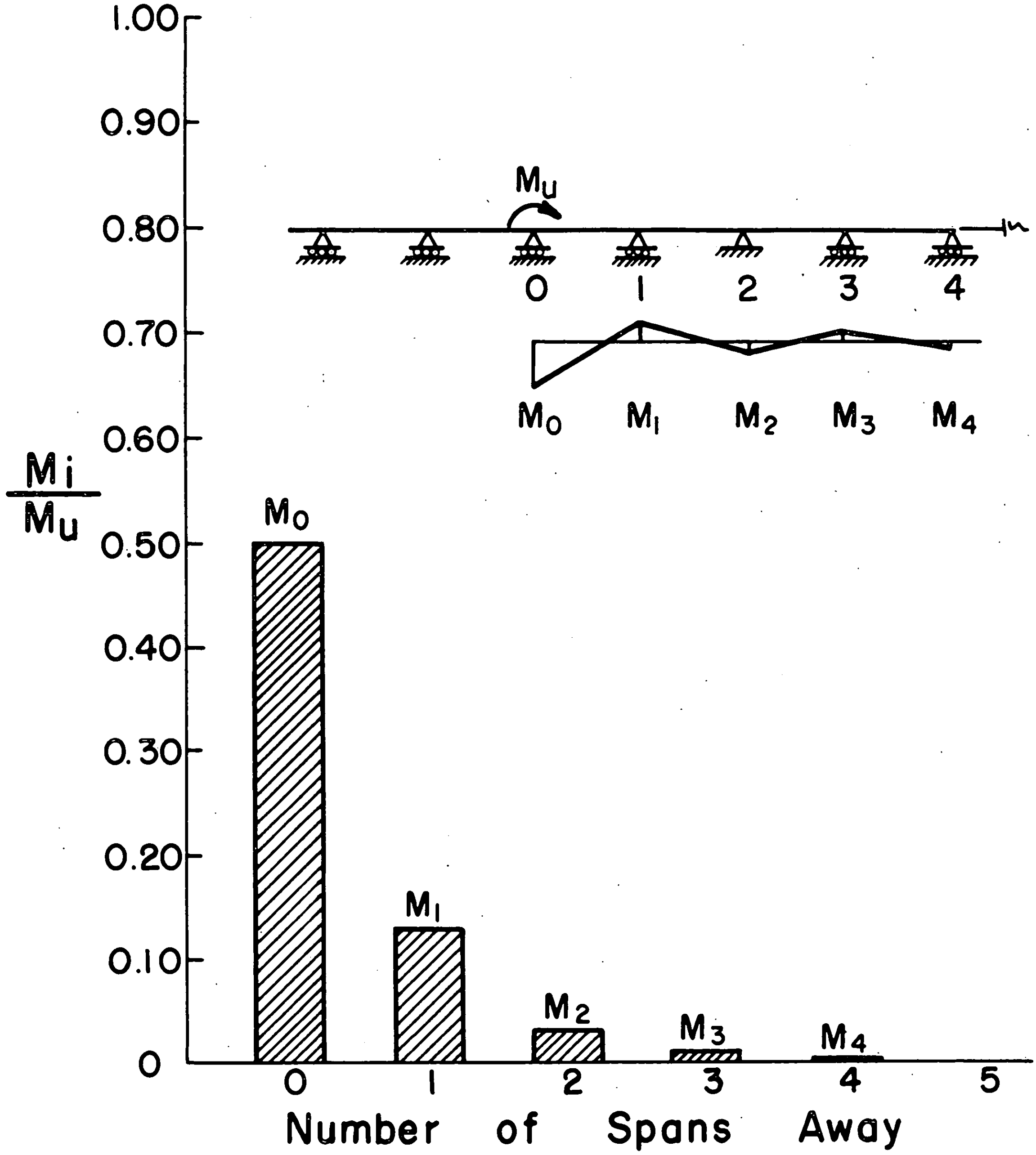
BENDING AND RESTRAINT CORRESPONDING TO VARIOUS CONDITIONS OF LOADING

FIG. 9



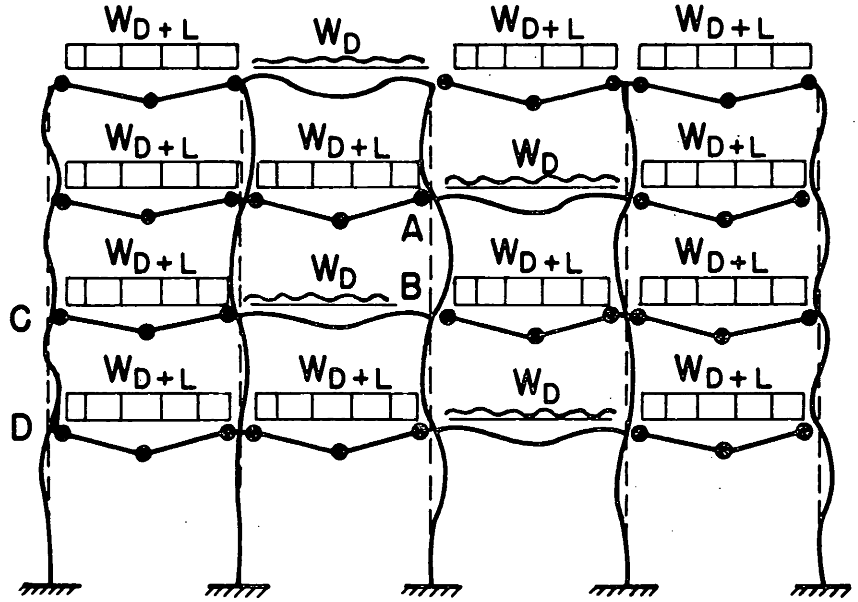
EFFECT OF NUMBER OF SPANS INCLUDED IN THE DISTRIBUTION OF MOMENTS

FIG. 10



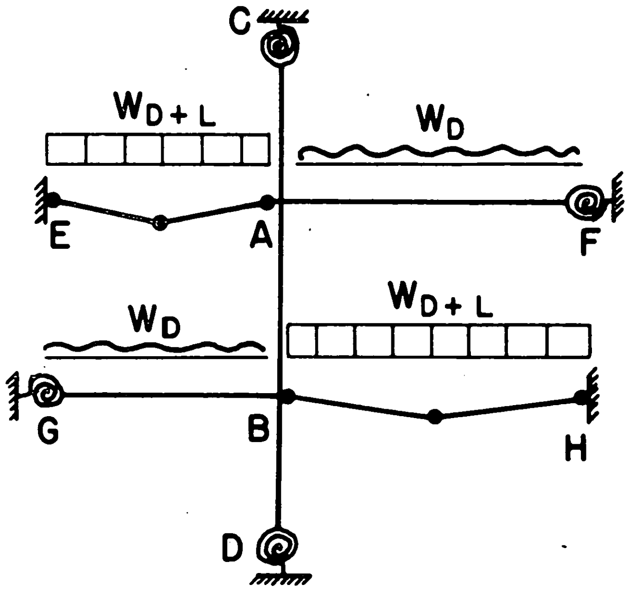
EFFECT OF EXTERNAL MOMENT SEVERAL JOINTS AWAY

FIG. 11



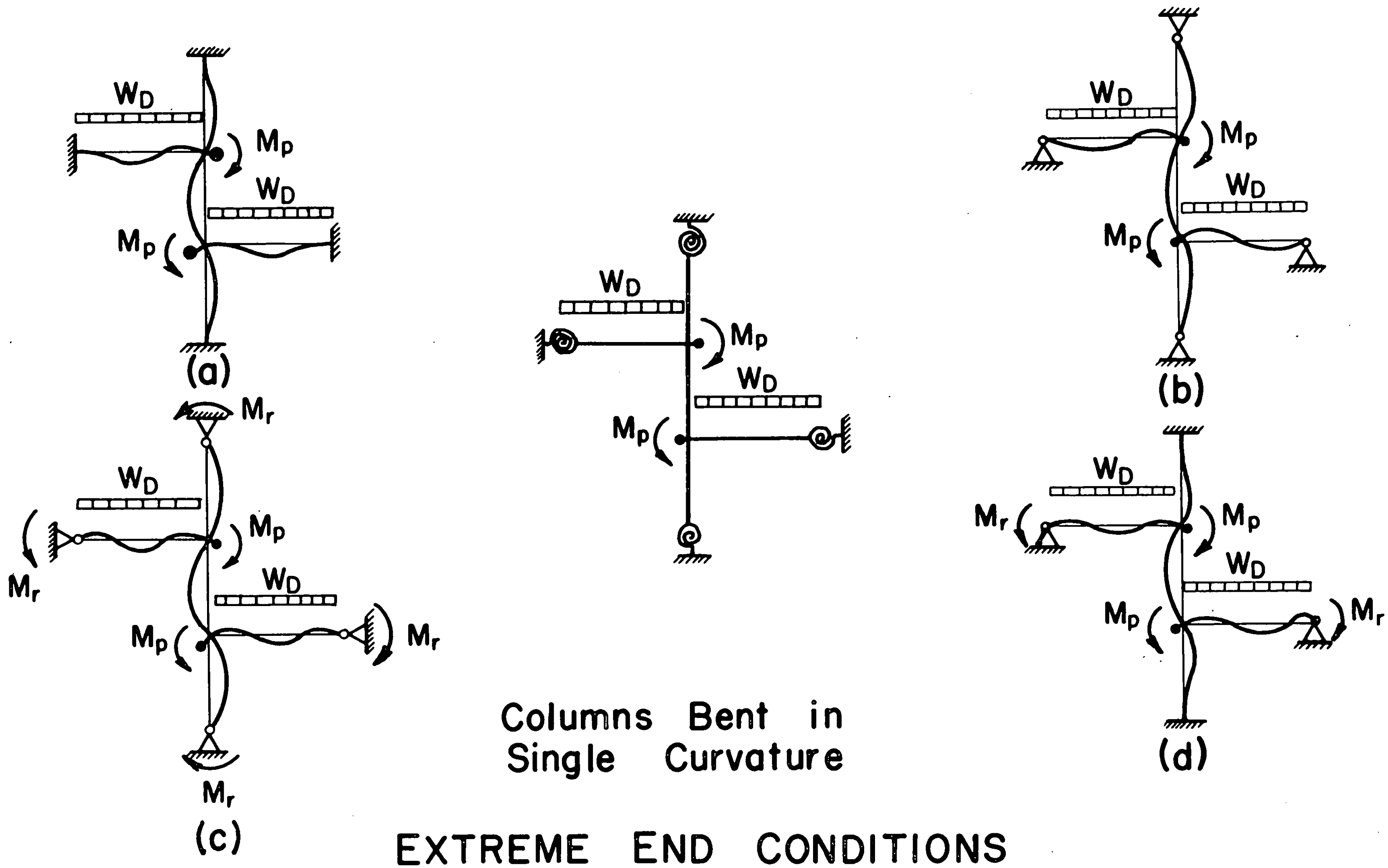
PARTIAL "CHECKERED" LOADING IN NEIGHBORHOOD OF COLUMN SEGMENT

FIG. 12



GENERAL SUBASSEMBLAGE

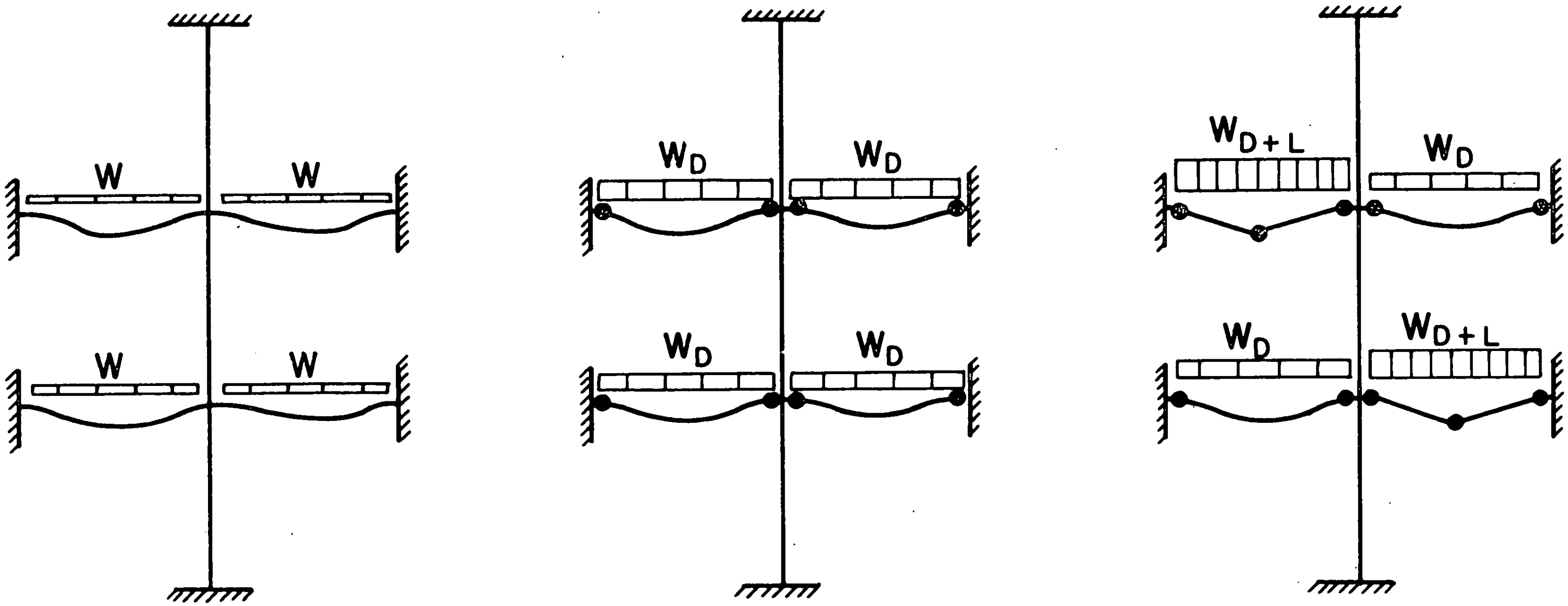
FIG. 13



EXTREME END CONDITIONS
OF RESTRAINING MEMBERS

FIG. 14

$K = \frac{I}{L}$ = Constant for all members



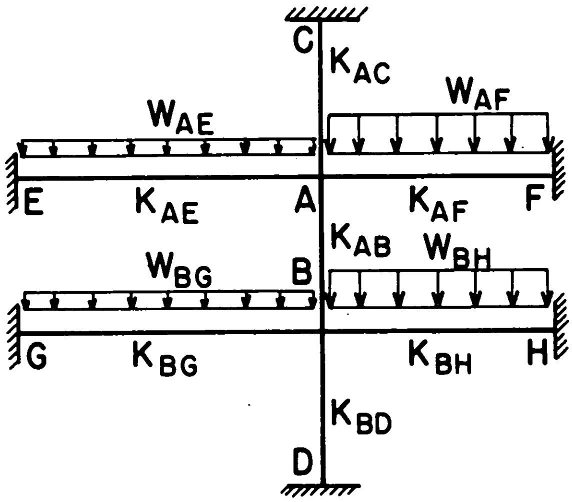
$$\frac{W_{D+L} L^2}{16} > \frac{W L^2}{12}$$

$$\frac{W_{D+L} L^2}{16} = \frac{W_D L^2}{12}$$

Further Loading does not transmit moment to column

HISTORY OF LOADING OF A SUBASSEMBLAGE

FIG. 15



$$W_{AF} = W_{BH} = W_R$$

$$W_{AE} = W_{BG} = W_L$$

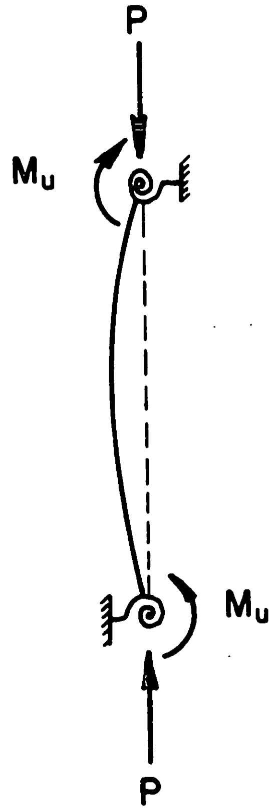
$$K_{AE} = K_{BG}$$

$$K_{AF} = K_{BH}$$

$$K_{AC} = K_{BD}$$

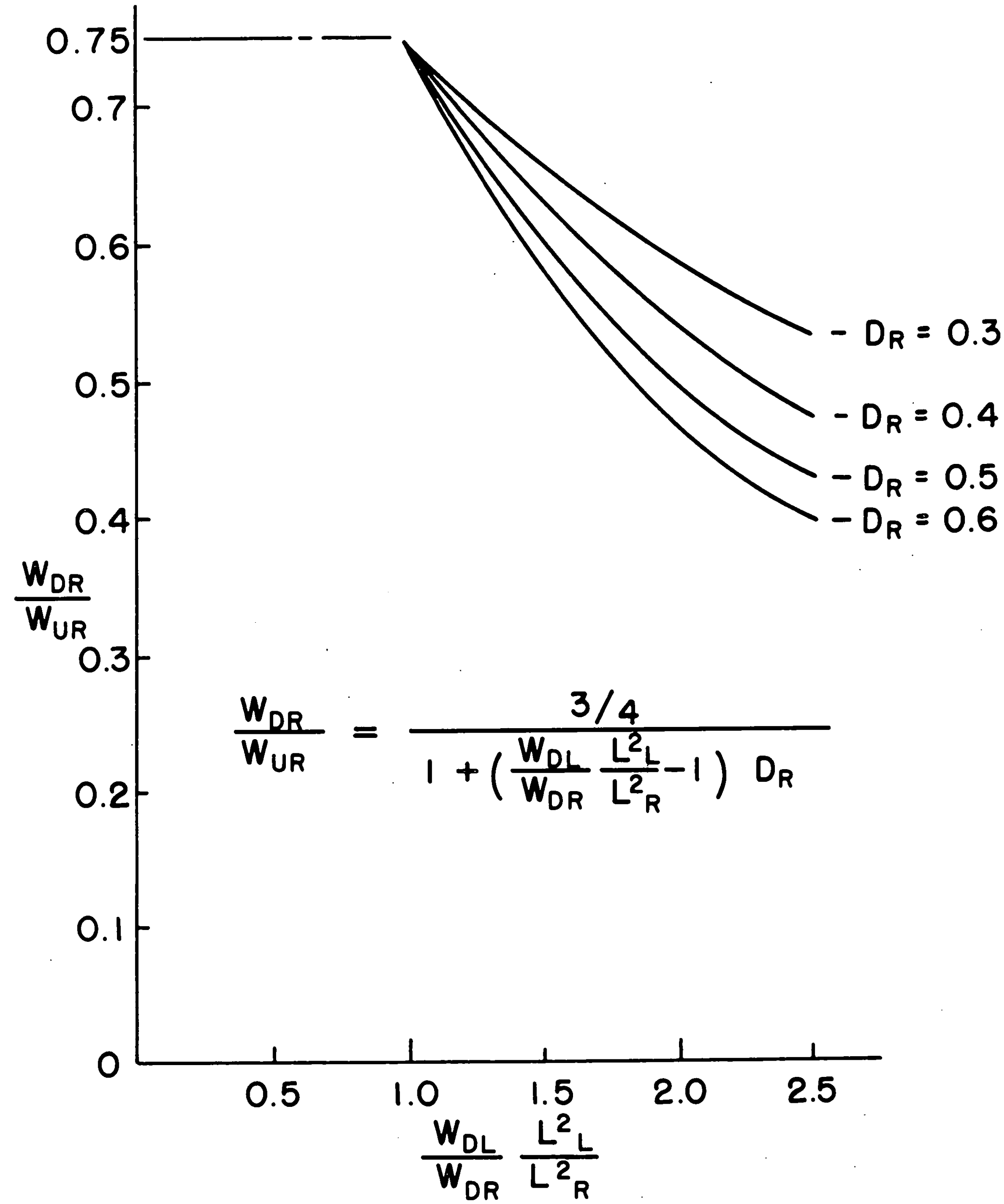
STRUCTURAL SUBASSEMBLAGE

FIG. 16



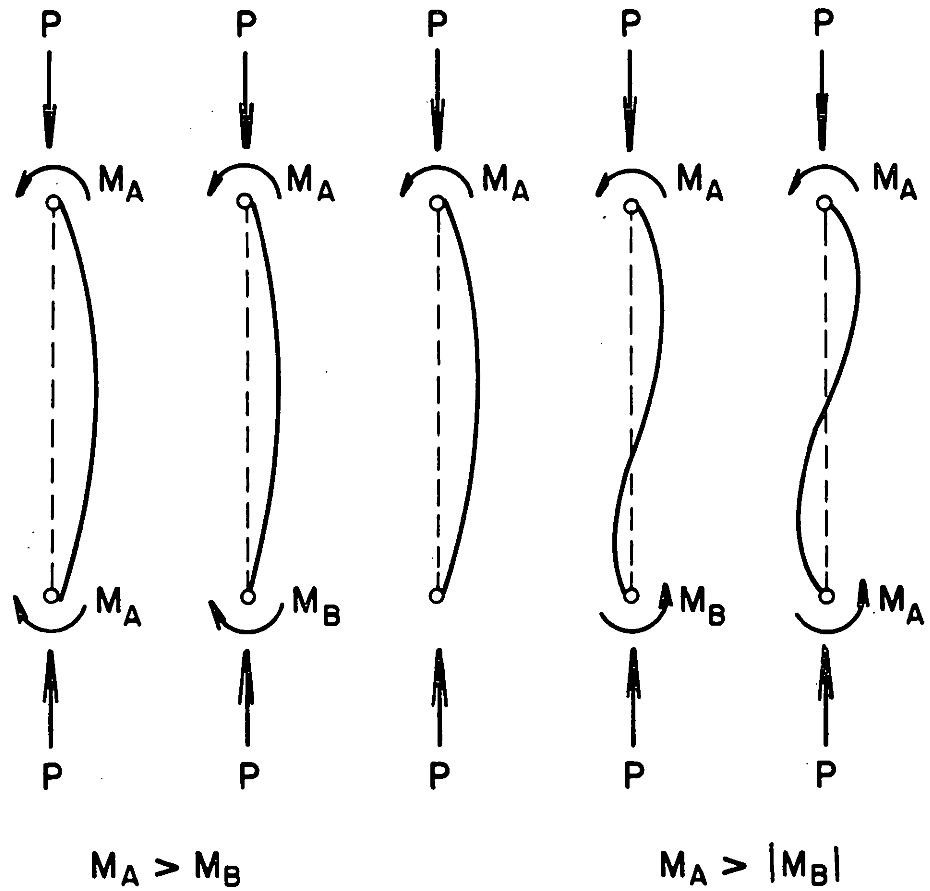
ELASTICALLY RESTRAINED BEAM-COLUMN

FIG. 17



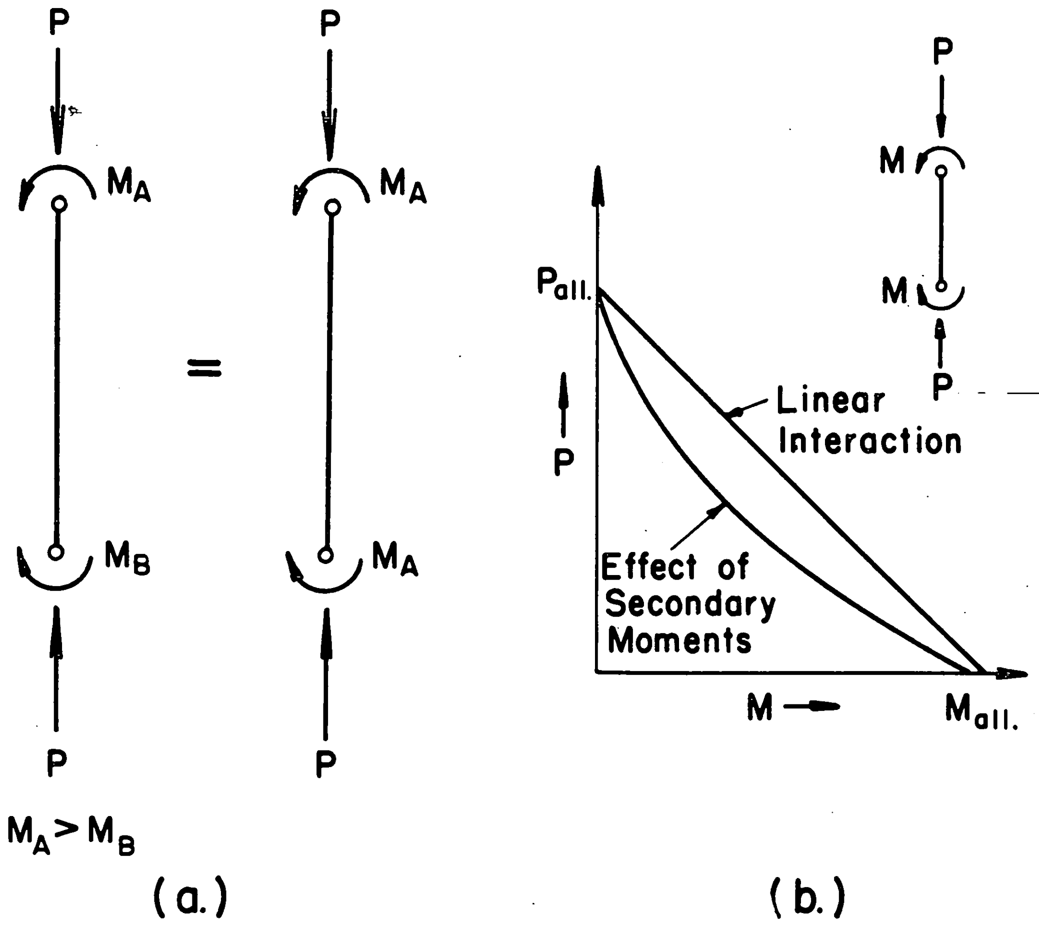
CRITERION FOR HINGE FORMATION
UNDER DEAD LOAD x 1.85

FIG. 18



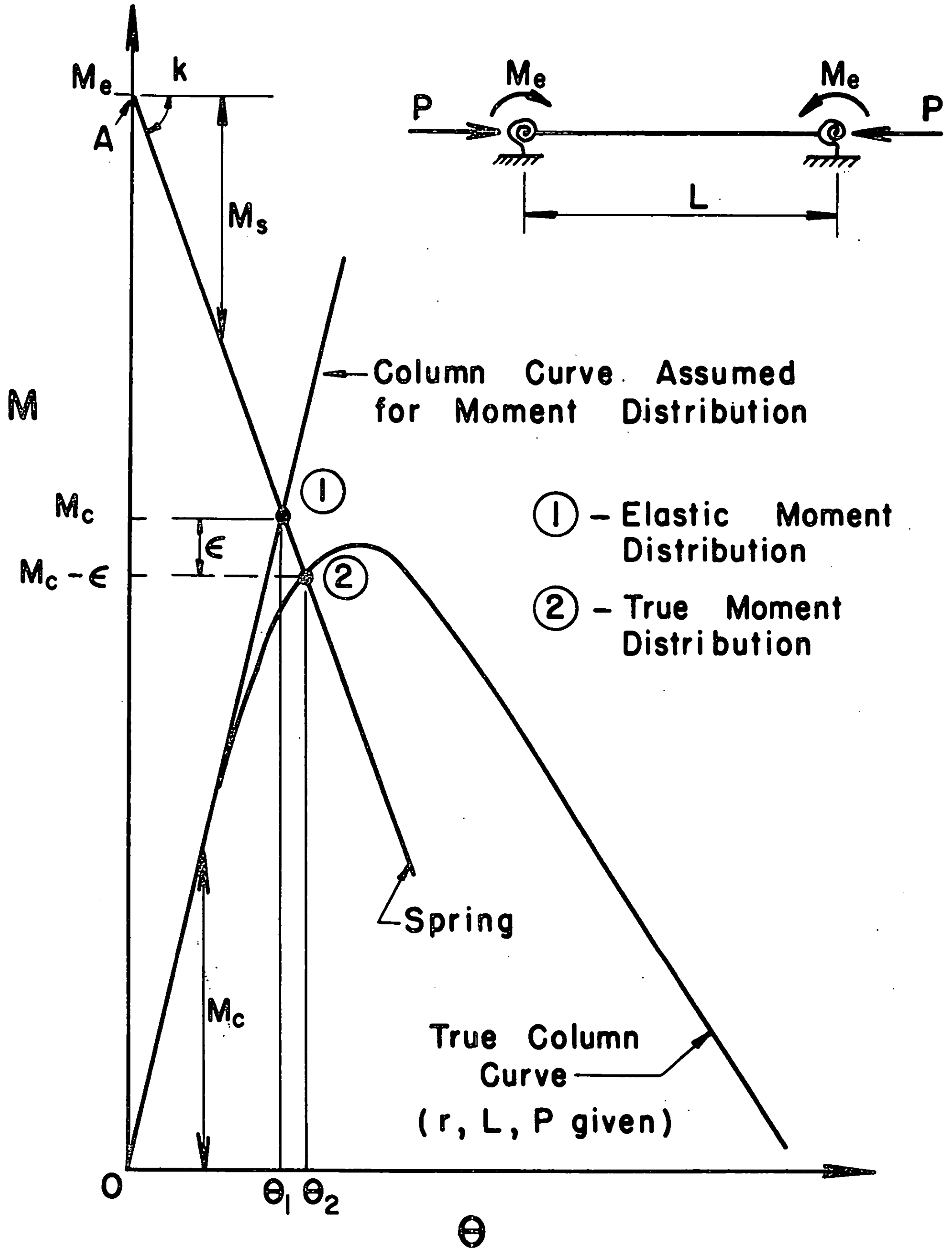
POSSIBLE BEAM-COLUMN BENDING CONFIGURATION

FIG. 19



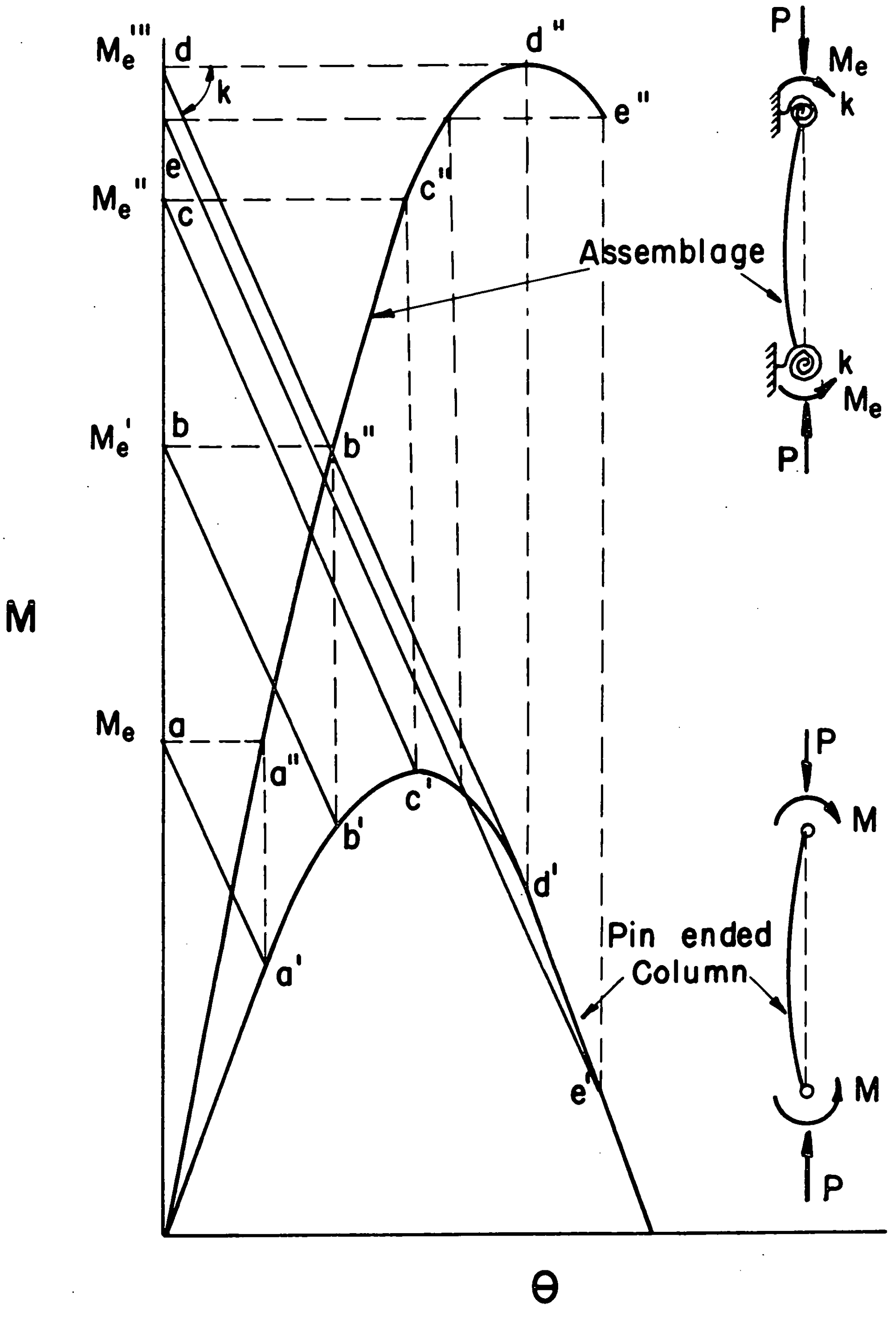
ASSUMPTIONS IMPLIED BY AISC INTERACTION FORMULA

FIG. 20



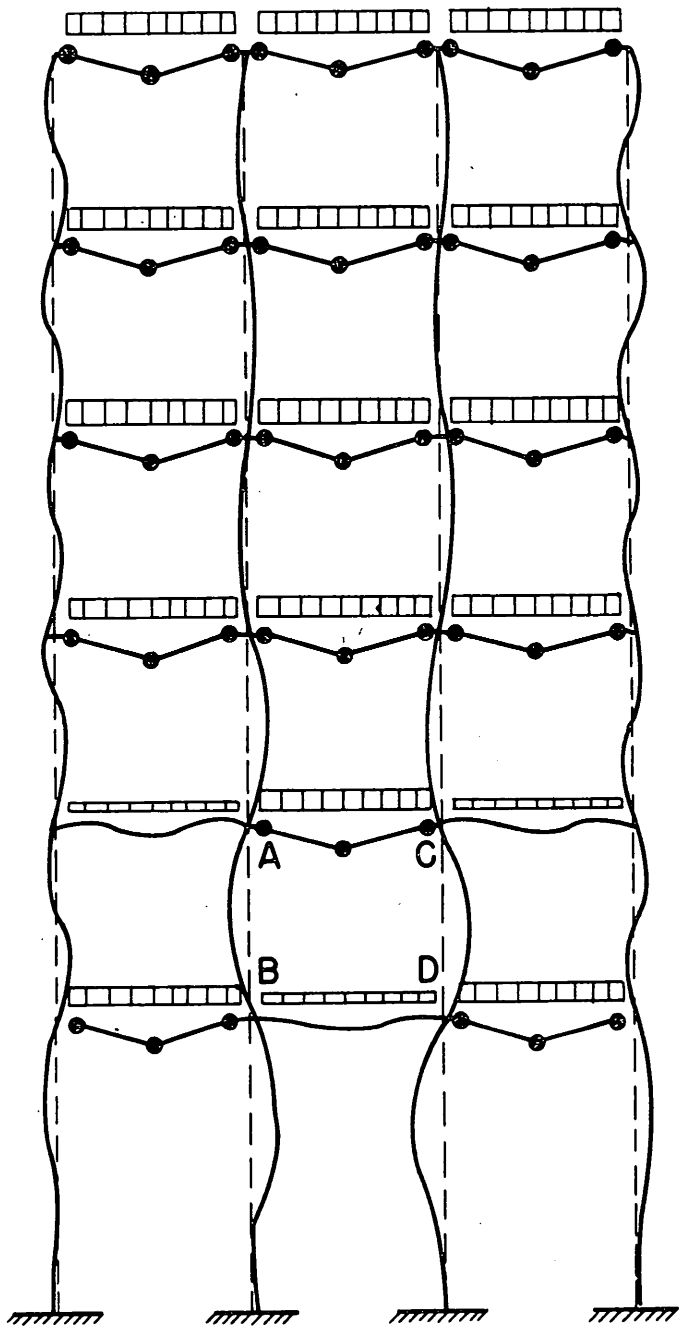
SCHEMATIC REPRESENTATION OF ERROR IN ELASTIC MOMENT DISTRIBUTION

FIG. 21



SCHEMATIC REPRESENTATION OF STRENGTH OF A SUBASSEMBLAGE

FIG. 22



CRITICAL LOADING FOR
INTERIOR COLUMNS (AB,CD)

FIG. 23

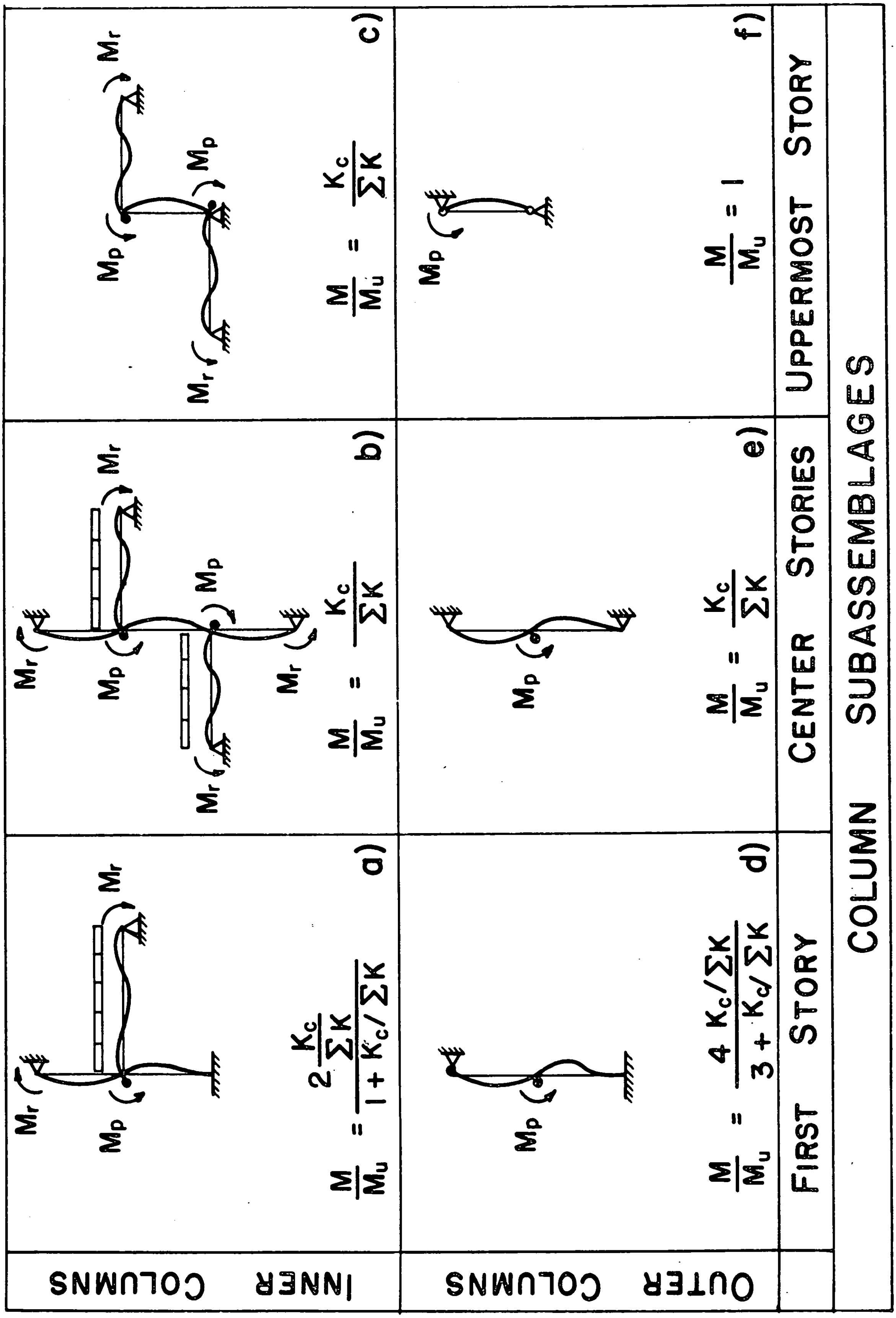
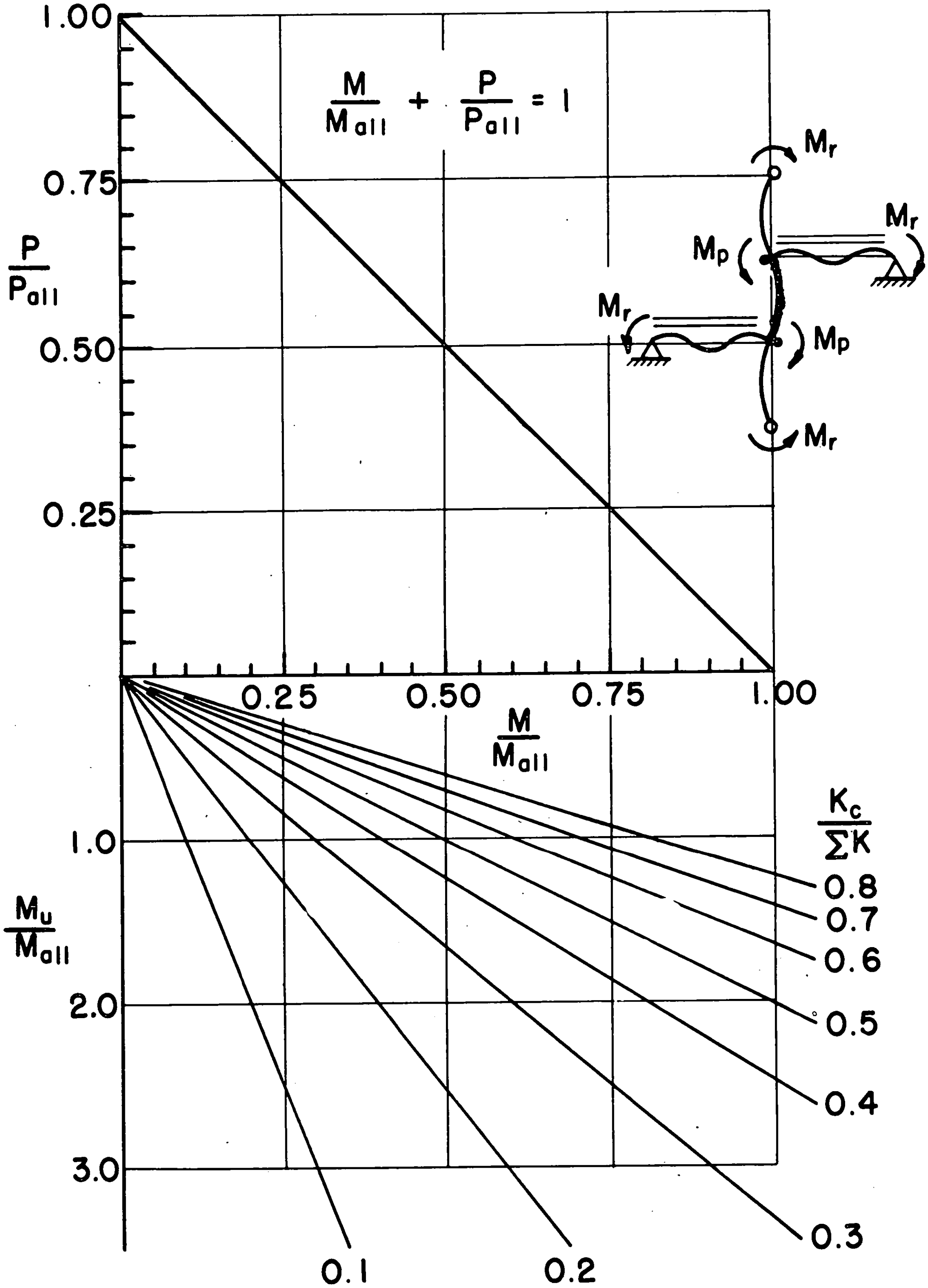
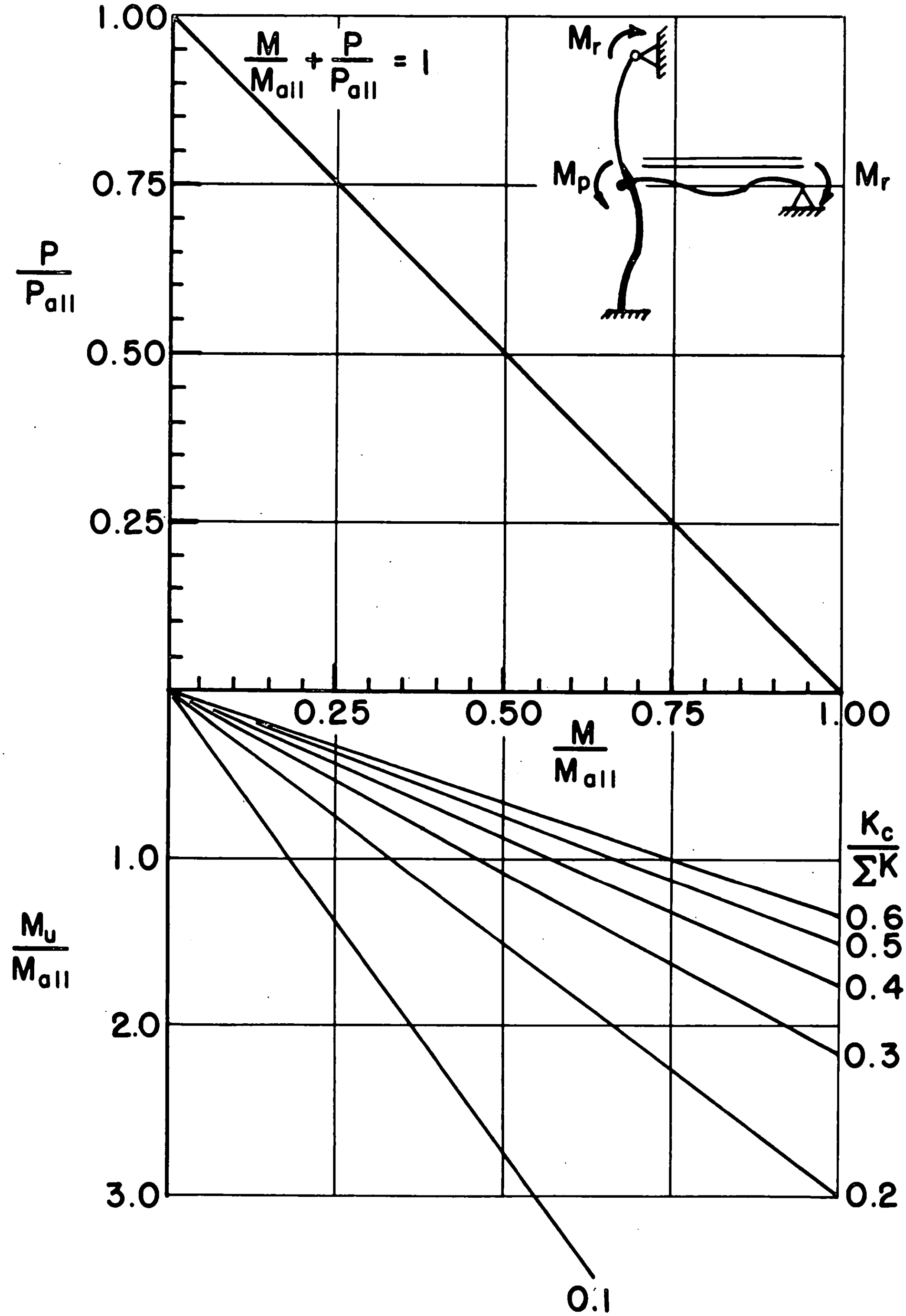

INNER COLUMNS
OUTER COLUMNS
Mr
Mp
Mr
M/Mu = 2 (Kc/ΣK) / (1 + Kc/ΣK)
a)
Mr
Mp
Mr
Mr
Mp
Mr
M/Mu = Kc/ΣK
b)
Mr
Mp
Mp
Mr
M/Mu = Kc/ΣK
c)
Mp
M/Mu = 4 Kc/ΣK / (3 + Kc/ΣK)
d)
Mp
M/Mu = Kc/ΣK
e)
Mp
M/Mu = 1
f)
FIRST STORY
CENTER STORIES
UPPERMOST STORY
COLUMN SUBASSEMBLAGES

FIG. 24



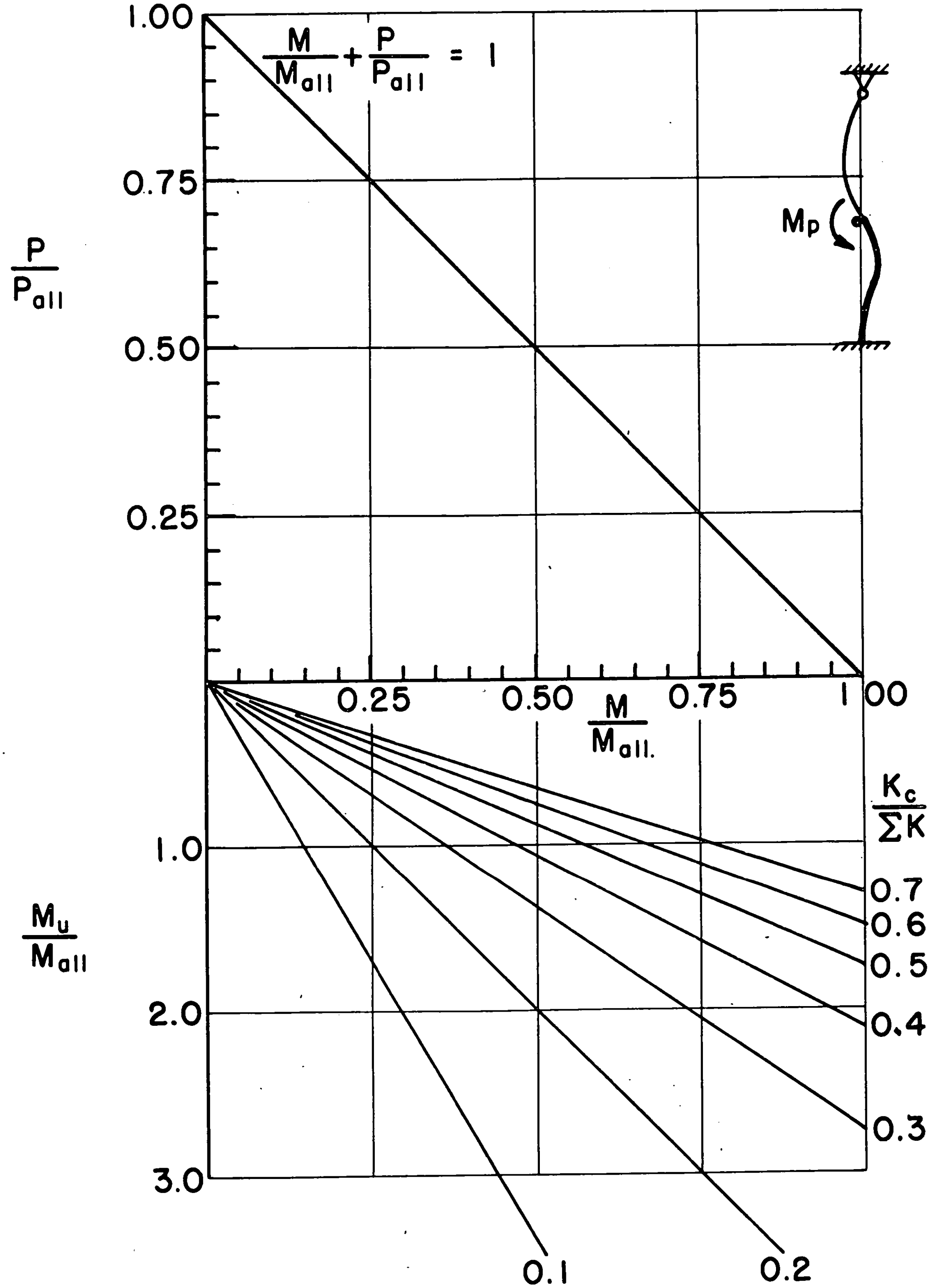
NOMOGRAPH FOR DESIGN OF CENTRAL COLUMNS

FIG. 25



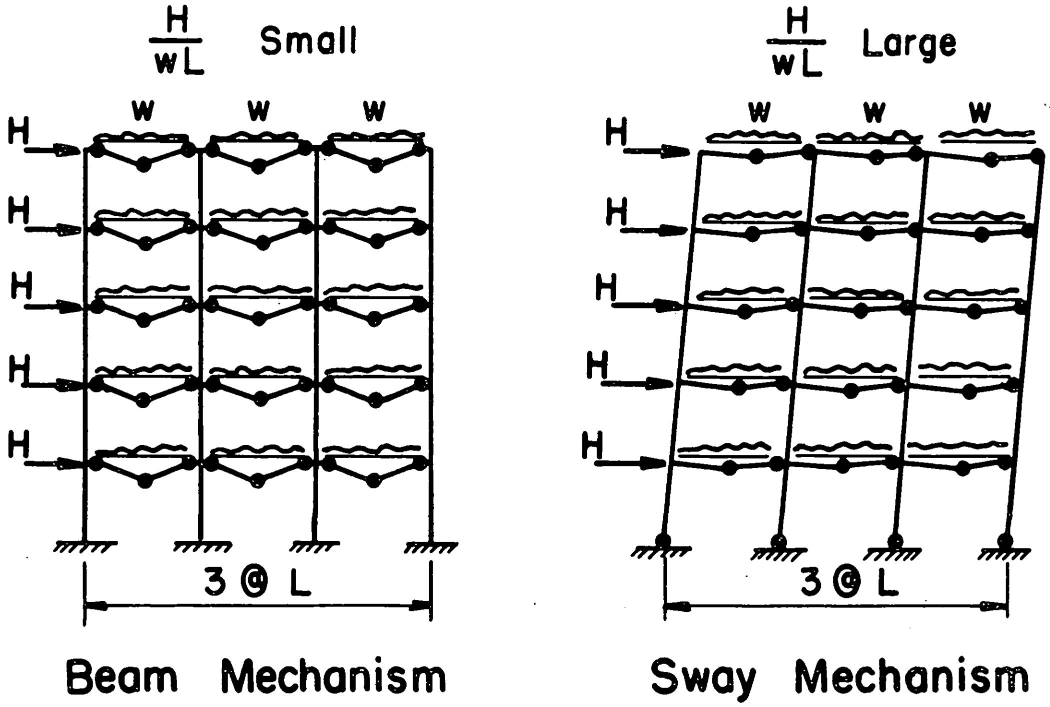
NOMOGRAPH FOR DESIGN OF FIRST STORY INNER COLUMNS

FIG. 26



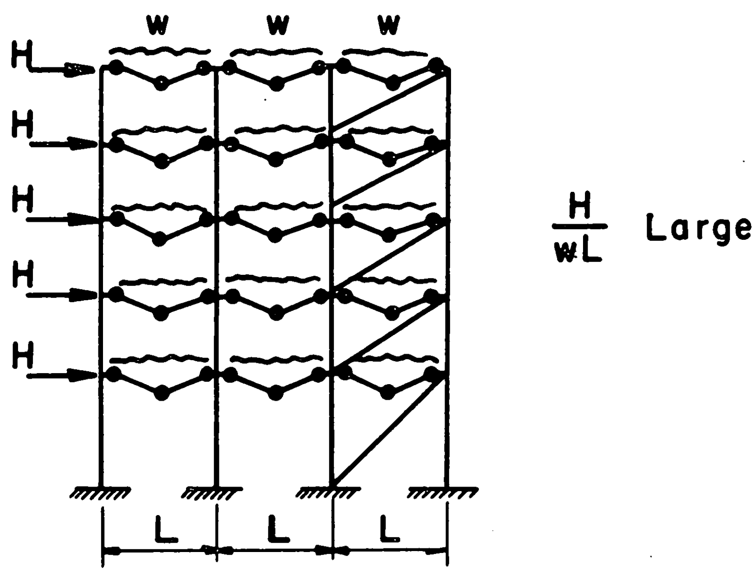
NOMOGRAPH FOR DESIGN OF FIRST STORY OUTER COLUMNS

FIG. 27



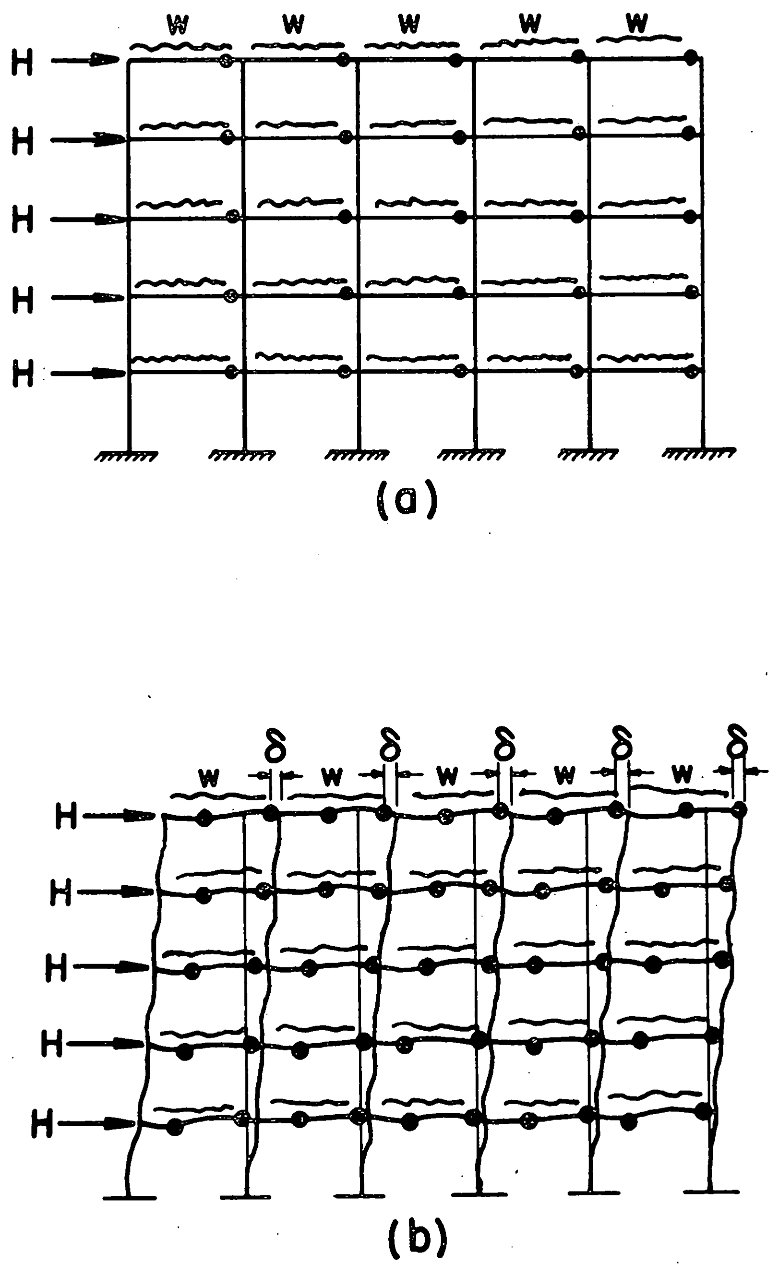
POSSIBLE FAILURE MECHANISMS

FIG. 28



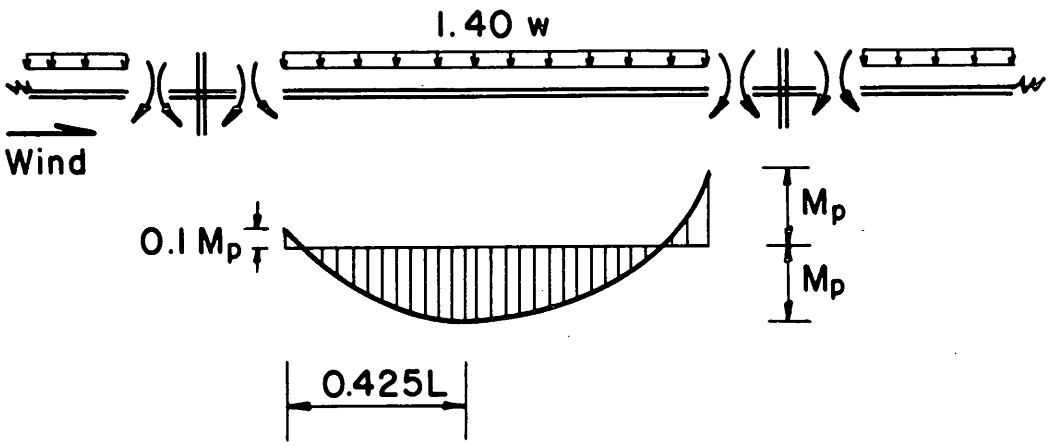
FAILURE MECHANISM OF ADEQUATELY BRACED FRAME

FIG. 29



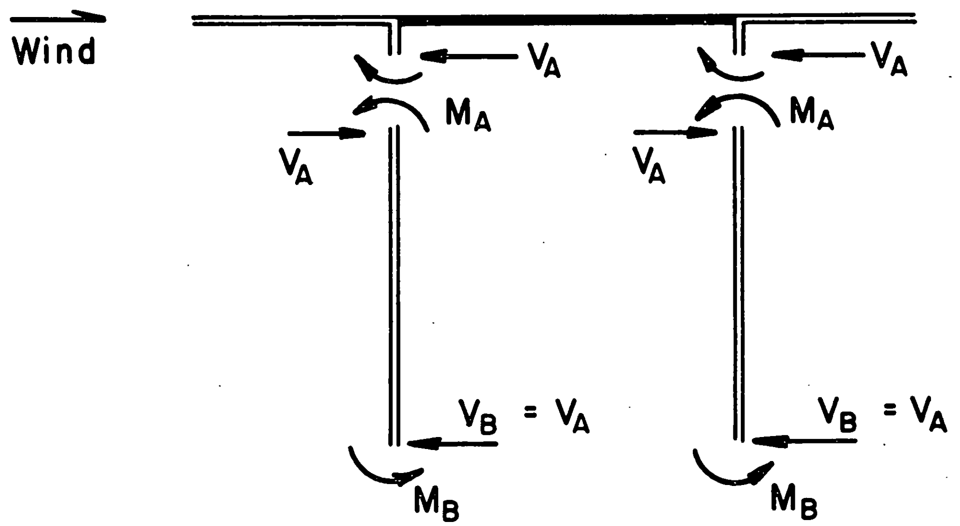
EFFECT OF COMBINED LOADING ON FRAMES

FIG. 30



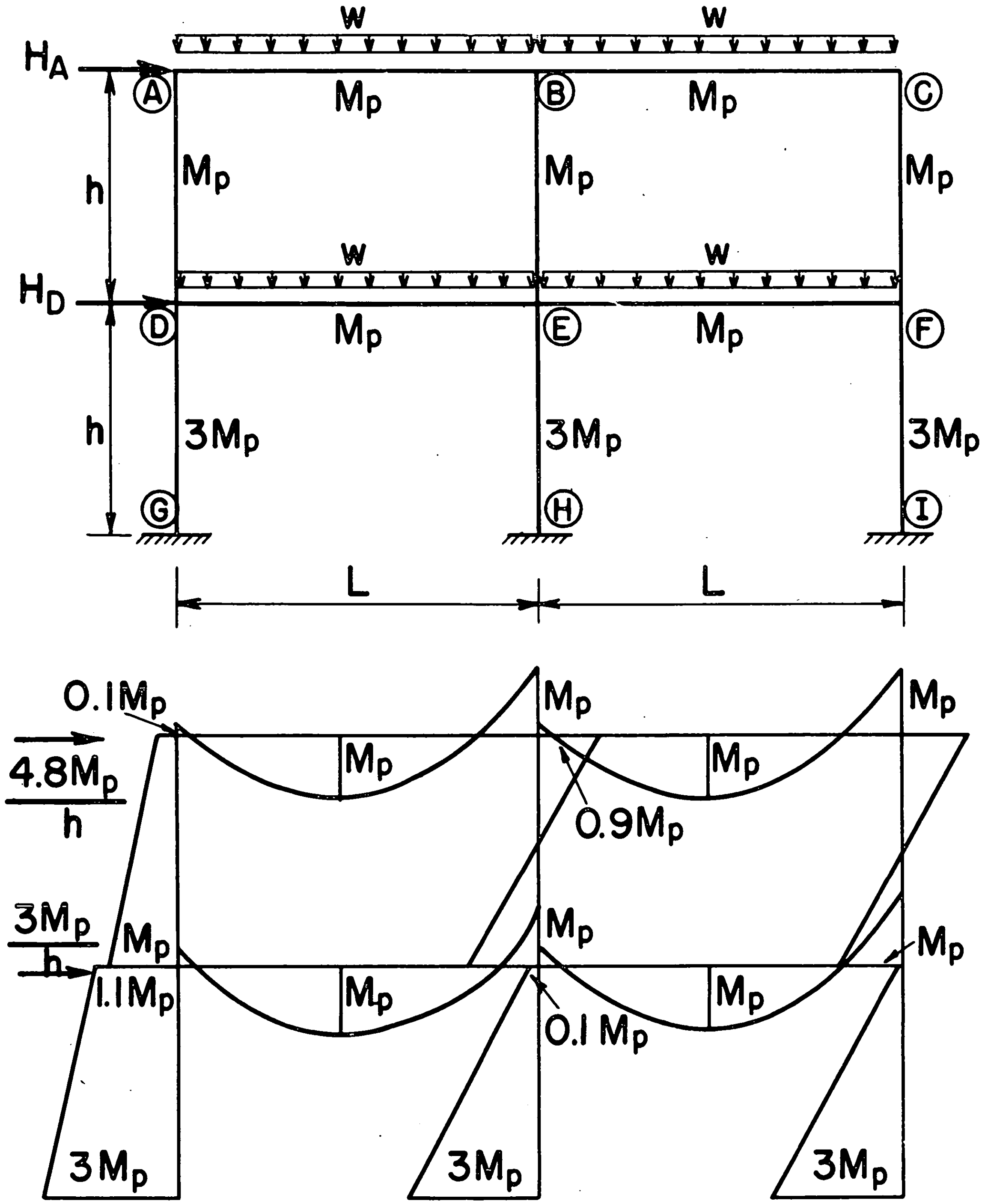
LIMITING MOMENT DIAGRAM OF BEAM IN FRAME UNDER COMBINED LOADING (WIND + GRAVITY)

FIG. 31



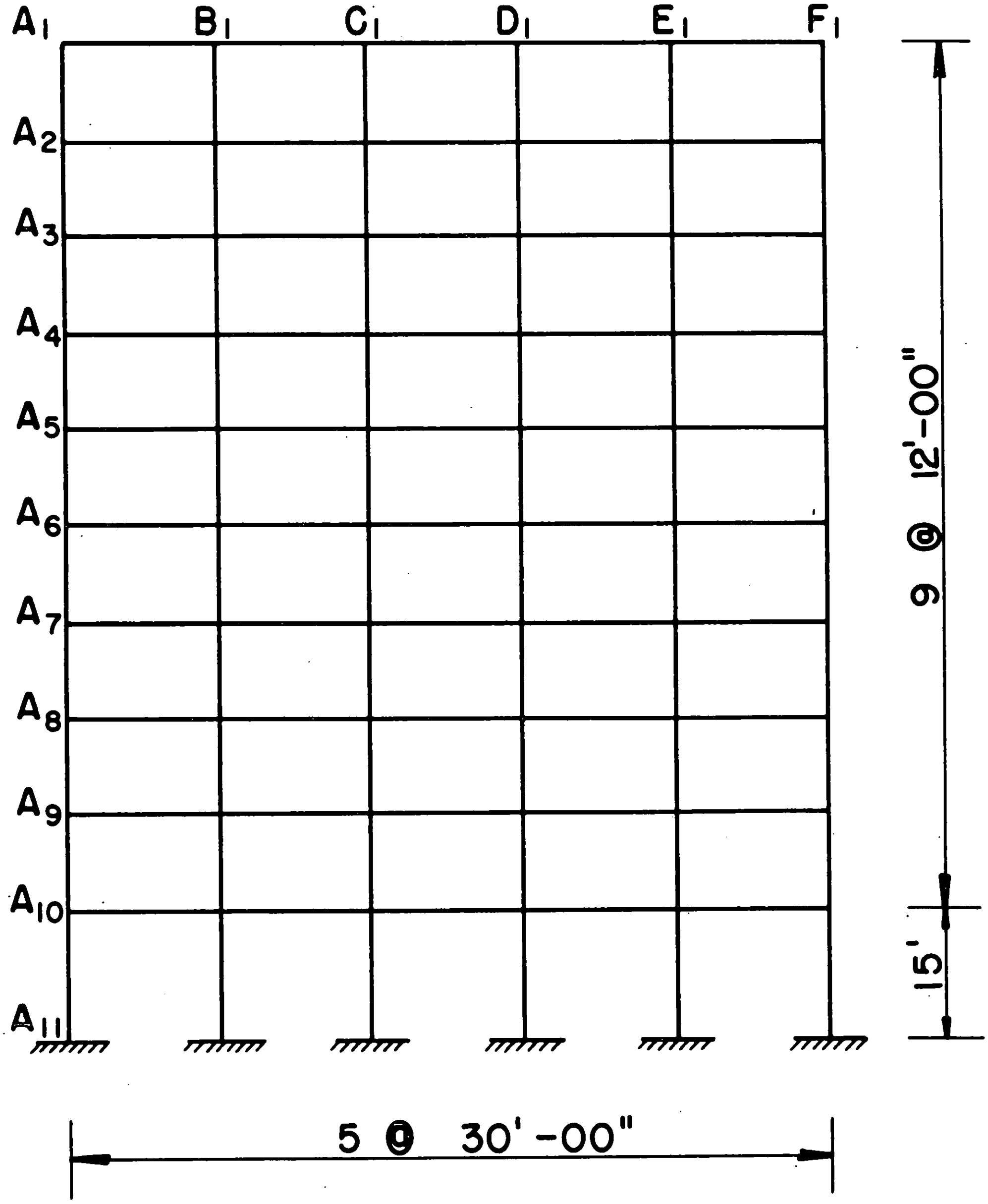
WIND RESISTANT COLUMN MOMENTS

FIG. 32



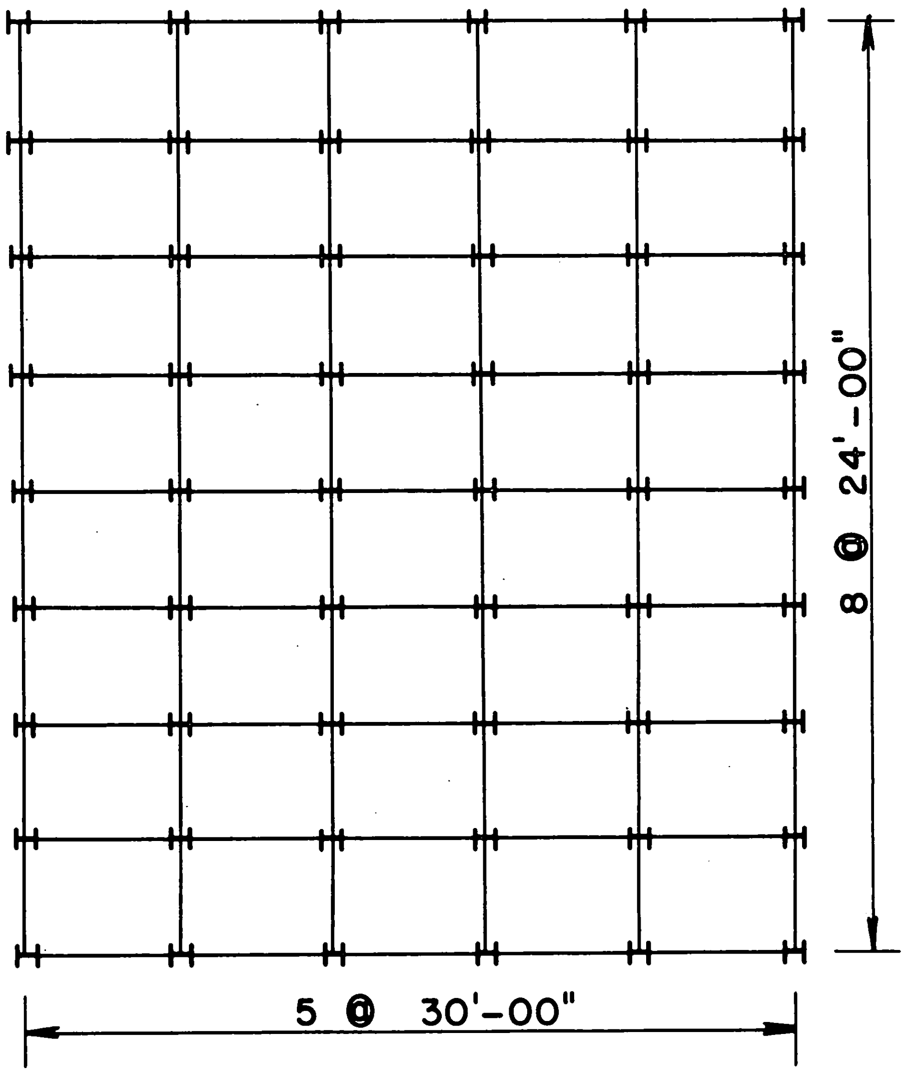
DETERMINATION OF LOWER BOUND FOR FRAME

FIG. 33



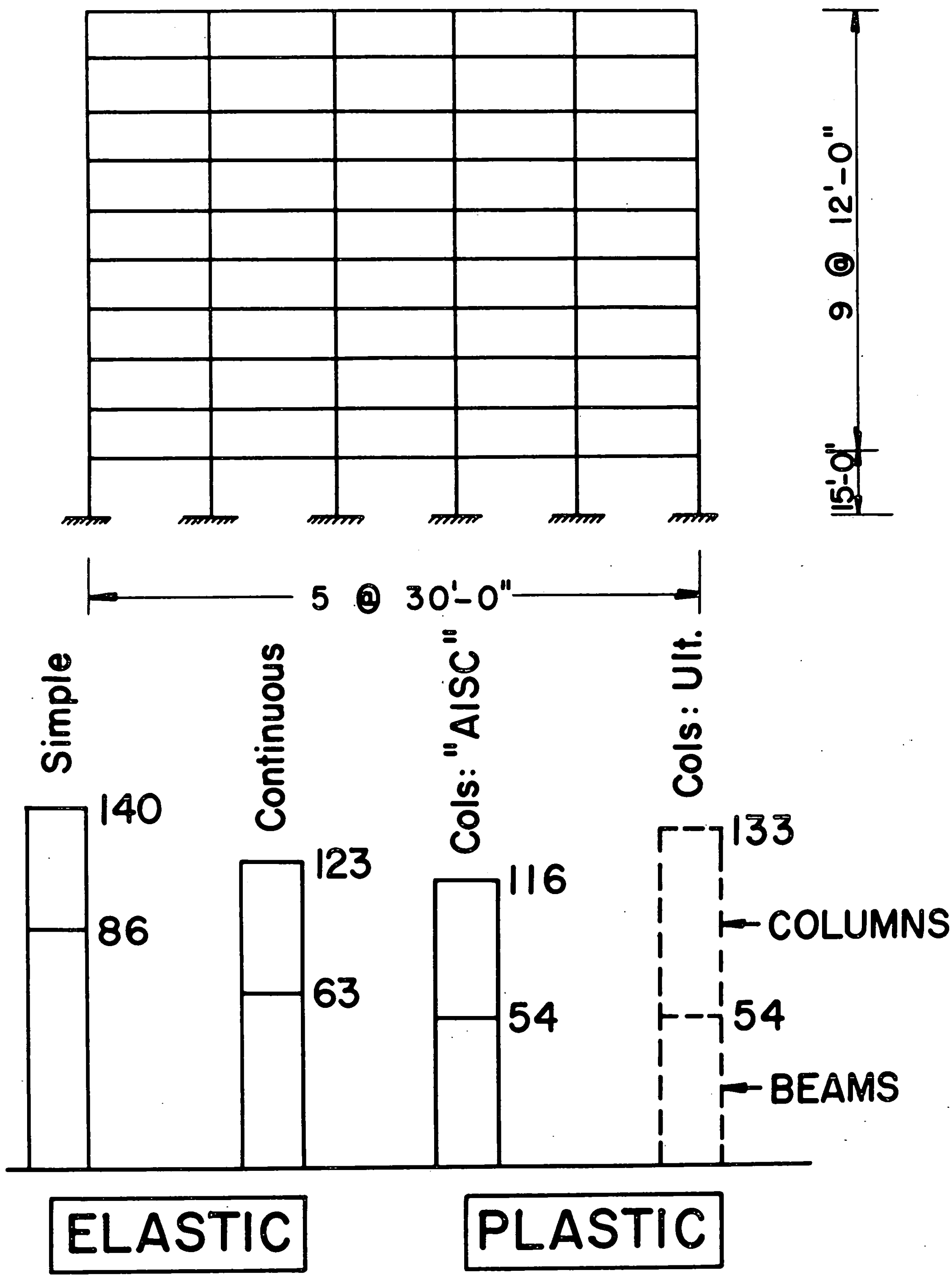
ELEVATION FOR DESIGN EXAMPLE

FIG. 34



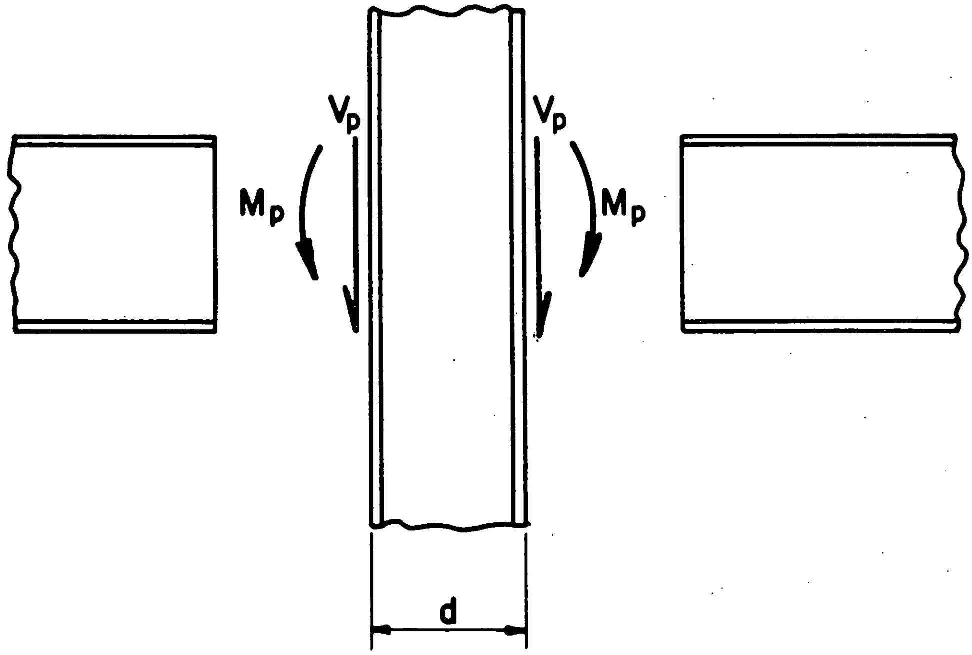
PLAN FOR DESIGN EXAMPLE

FIG. 35



COMPARATIVE DESIGN

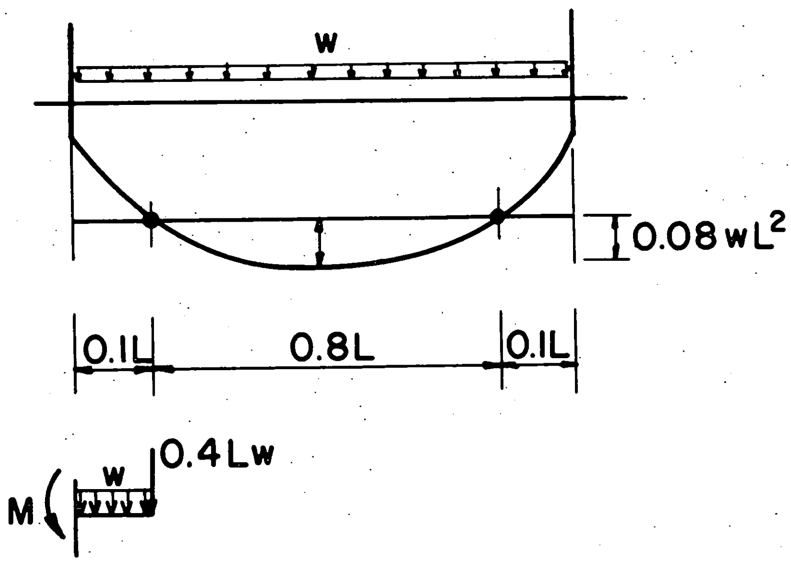
FIG. 36



$$M_u = M_p - M_D + (V_p - V_D)\frac{d}{2}$$

UNBALANCED MOMENT AT A JOINT

FIG. 37



$$M \doteq 0.04\, L^2 w + 0.005 L^2 w$$
$$= 0.045\, L^2 w$$

COLUMN MOMENT DUE TO CONTINUOUS ELASTIC BEAM

FIG. 38

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