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WELDED PLATE GIRDERS

PLATE GIRDERS UNDER COMBINED BENDING AND SHEAR

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Fritz Engineering Laboratory Report No. 251-21

PLATE GIRDERS UNDER COMBINED BENDING AND SHEAR

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by

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FOREWORD

This paper is the third report on the strength of plate girders. While the first dealt with the bending strength only and the second exclusively with the shear strength, this last paper covers the interaction between bending and shear. Reference should be made to the Foreword of the first for the scope of the entire investigation.

SYNOPSIS

A study of possible interaction between bending moments and shear forces on the carrying capacity of plate girders is presented. Based on theoretical considerations and experimental results approximations suitable for design use are suggested.

1. INTRODUCTION

Most plate girders are subjected to a combination of bending and shear. It is possible that a girder section can be subjected to bending moments alone, but not to shear alone. To postulate that no bending moments should occur over a girder panel would exclude shear forces likewise, since shear force is the rate of change of bending moment. Nevertheless, it is safe to disregard moments in the treatment of shear as long as they do not exceed a certain magnitude to be determined here.

As the expression "plate girder" implies, flanges which bound the web are always present and thus prevent a collapse should the web be unable to carry its part of the moment. Interaction will be concerned with such a rearrangement of stress for two different reasons. In very slender webs the stress rearrangement is predominantly due to web deflections; slight deflections of the web from a plane result in a transfer of the bending moment resistance from the web to the flange, as described in Sec. 1.2 of Ref. 3. This is achieved without a loss in shear carrying capacity which is essentially contributed by a tension field. In girders with stockier webs, however, the bending moment which cannot be carried by the web, because of high concurrent shear, is transferred to the flange through yielding.

For these reasons, compatibility conditions can be ignored to a great extent when determining the carrying capacity of plate girders. The procedure will thus be similar to plastic analysis, where a lower bound of the carrying capacity is obtained by considering a possible state of stress which is in equilibrium with the applied moment and shear yet nowhere violates the yield condition.

2. THE INTERACTION FORMULA

Subsequently the following notations for certain reference values are used. M_f, the flange moment, is defined as the moment carried by the flanges alone when the stresses over the entire flange are equal to the yield stress σ_y. The yield moment, M_y, is the moment initiating yielding at the centroid of the compression flange. The resisting moment of a fully yielded cross section is denoted as the plastic moment, M_p[1]. Approximating the distance between the flange centroids as equal to the web depth b and designating the area of a single flange as A_f and the web area as A_w, the three reference moments of a symmetrically proportioned girder cross section can be expressed as

$$\left.\begin{aligned} M_f &= \sigma_y b A_f \\ M_y &= \sigma_y b(A_f + \tfrac{1}{6}A_w) \\ M_p &= \sigma_y b(A_f + \tfrac{1}{4}A_w) \end{aligned}\right\} \quad (1)$$

The shear force V and the bending moment M give information as to their relative importance only if they are compared with girder properties. For this reason, and also to non-dimensionalize V and M, the shear force will be expressed in terms of the ultimate shear force V_u[2] and the moment in terms of the yield moment M_y. The shear force and the bending moment are not independent of each other.

When the loading condition is fixed, the shear force and bending moment at any cross section of a particular girder depend on the common parameter P which denotes the load intensity. Therefore, the ratio M/V is independent of the load and characterizes the loading condition. If a Cartesian coordinate system has abscissa and ordinate of M/M_y and V/V_u, respectively, there is then an associated polar system whose length of radius vector is directly proportional to the load intensity P. The interaction curve C in such a coordinate system, Fig. 1, is defined as the boundary between points on the safe side and those which lead to failure. Because the vector length may be interpreted as the load intensity, the ultimate load P_u for a cross section subjected to bending and shear is by definition the intersection of its particular ray with the curve C. With this preparation the derivation of the interaction curve follows.

As pointed out in the introduction, the web can transfer its allotted moment to the flanges and retain its shear strength, provided that the moment capacity of the flange is not exceeded. This means that, in the coordinate system explained previously, the failure curve is represented by a straight line

$$\frac{V}{V_u} = 1 \qquad (2)$$

as shown in Fig. 2.

Since a web which carries the ultimate shear force is utilized up to yielding, the flanges are the sole carriers of the bending moment. If it is assumed for the time being that these flanges are proportioned and laterally stiffened such that the yield stress can be reached, then the limiting moment which they can take is the flange moment M_f. If there were no shear present, the maximum moment that could be expected under the most favorable circumstances, disregarding strain-hardening, is the plastic moment M_p. The only portion on the moment scale where bending moments affect the shear carrying capacity is therefore that between M_f and M_p. Thus, an interaction curve must pass through the points $Q_1(M_f/M_y, 1)$ and $Q_2(M_p/M_y, 0)$. Since very small quantities of shear hardly affect the moment carrying capacity, the interaction curve should also start off at right angles to the abscissa at point Q_2.

The simplest set of interaction curves fulfilling these conditions is that given in Eq. 3a, with the exponent n greater than unity.

$$\left(\frac{V}{V_u}\right)^n + \frac{M-M_f}{M_p-M_f} = 1 \qquad (3a)$$

$$\left(\frac{V}{V_u}\right)^n + \left(\frac{M-M_f}{M_p-M_f}\right)^m = 1 \qquad (3b)$$

Should the curve also be tangent to the line $V/V_u = 1$ at point Q_1, an interaction formula of the type of Eq. 3b would be required with m and n greater than unity.

For an exponent $n = m = 2$, possible states of stress leading to Eqs. 3a and 3b respectively are shown in Fig. 2. In approach (3a) it is assumed that the portion of the web which participates with the flanges in resisting moment is unable to carry shear. In approach (3b) normal stresses σ and shearing stresses τ act over the entire web depth but are interrelated with Mises' yield conditions: $\sigma^2 + 3\tau^2 = \sigma_y^2$.

In view of tension field action the more conservative approach (3a), or (3b) with $m = 1$, is preferred. The choice of an exponent $n = 2$ for girders with very slender webs may be somewhat hypothetical. But in evaluating the strength of girders subjected to pure bending[3], it was shown that little more than the flange moment is preserved in slender web girders. Therefore, most of the interaction curve 3a is cut off by the requirement

$$\frac{M}{M_u} = 1 \qquad (4)$$

where M_u is the ultimate bending moment evaluated from Eq. 12a of Ref. 3. In Fig. 2, Eqs. 2, 3 and 4 are plotted for the case of $M_f = 0.80\, M_y$, $M_u = 0.95\, M_y$ and $M_p = 1.10\, M_y$.

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For its application to design, it is of advantage to express the interaction equations in terms of stress. Rewriting Eq. 3a with an exponent n = 2 and solving it for the bending moment M leads to Eq. 5a.

$$M = M_f + (M_p - M_f)\left[1 - \left(\frac{V}{V_u}\right)^2\right] \tag{5a}$$

$$\frac{M}{S}\frac{S}{M_y} = \frac{M_f}{M_y} + \frac{M_p - M_f}{M_y}\left[1 - \left(\frac{V}{A_w}\frac{A_w}{V_u}\right)^2\right] \tag{5b}$$

$$\sigma = \sigma_y \frac{1 + \frac{1}{4}\frac{A_w}{A_f}\left[1 - \left(\frac{\tau}{\tau_u}\right)^2\right]}{1 + \frac{1}{6}\frac{A_w}{A_f}} \tag{5c}$$

$$\sigma = \frac{\sigma_y}{N}\,\frac{1 + \frac{1}{4}\frac{A_w}{A_f}\left[1 - \left(\frac{\tau}{\tau_{all}}\right)^2\right]}{1 + \frac{1}{6}\frac{A_w}{A_f}} \tag{5d}$$

Equation 5b is obtained by dividing either side of Eq. 5a by the yield moment M_y and expanding certain fractions, where S denotes the section modulus and A_w the web area. The ratio M/S is the flange stress σ due to bending, M_y/S is the yield stress σ_y, V/A_w is the average shearing stress in the web, and V_u/A_w is the ultimate shear stress. If these values are substituted and the ratios M_f/M_y and $(M_p-M_f)/M_y$ are expressed according to Eqs. 1, the result in Eq. 5c is obtained. Using this expression with various ratios of A_w/A_f and a yield stress σ_y = 33 ksi (A7 steel) the failure envelopes are sketched in Fig. 3.

If a constant factor of safety ($N = 1.65$, AISC) were applied, the coexistent allowable bending and shear stresses are indicated in this figure by the thin lines determined from Eq. 5d. As seen, with the choice of an ultimate bending stress of 20 ksi an interaction check is required only if the shear stress exceeds about 60% of the allowable value. Also, an interaction limit in flange stress is not required below 15 ksi when $A_w/A_f < 2$. Since this ratio of web to flange area is about the upper limit of the generally used girder proportions, the possible interaction rules for girders made of A7 steel are:

$$\text{AISC}: \sigma < 27 - 12 \frac{\tau}{\tau_{all}} \tag{6a}$$

$$\text{AASHO}: \sigma < 24.5 - 11 \frac{\tau}{\tau_{all}} \tag{6b}$$

These two conditions are traced in Fig. 3. The allowable stress range resulting from a factor of safety $N = 1.83$ (AASHO) is plotted in broken lines.

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Thus far, interaction has been treated solely as a stress problem. Conditions caused by local and overall instability of the compression flange must at least be mentioned. This will be done in the remaining part of this paper.

3. CORRELATION WITH TEST RESULTS

In the course of this investigation some girders were tested with the purpose of obtaining information on the interaction between bending and shear. The girder properties, loading arrangement, and the test observations of these girders are presented in Ref. 4. With the help of the flange moment M_f and the plastic moment M_p listed in Table 1.7 of this reference, as well as the ratios V_u^{exp}/V_u^{th} derived in Table 1 of Ref. 2, the interaction diagrams of Fig. 4 are constructed. The reference moment values M_y which were used differ from those given in Table 1.7 of Ref. 4 in that they are taken as the moment value which initiates yielding at the compression flange centroid, and not at the extreme fiber of the compression flange.

Girders G8, G9 and E1 are shear girders. Girders E2 and E4 furnished the most significant interaction data. Girder E5 is predominantly a bending girder. Since on each ray the distance of a point from the origin is directly proportional to the applied load, the relation between the conventionally computed web buckling load, the predicted ultimate load, and the experimentally obtained ultimate load can be easily visualized. The intersection of a ray with the failure envelope gives the predicted ultimate load, the circled points mark the observed ultimate loads, and the conventional buckling theory predicts instability at a load (Table 1.9, Ref. 4) indicated by a short transverse bar.

It must be pointed out that the choice of the cross section for which the moment values were computed is of significance since the bending moment varies throughout the length of the test girders and only the shear force stays constant (Fig. 1.3 of Ref. 4). This section was chosen to be in the failed panel at a longitudinal distance one-half the web depth away from the high-moment end, or at the middle of the longitudinal panel dimensions when its length was less than its depth. The crosses shown in graph E2 of Fig. 4 would represent the test results if the sections with the maximum bending moment were used.

To justify the choice of a section other than at the moment peak, and also to illustrate the uncertainties unavoidably encountered when predicting ultimate loads in general and interaction in particular, the following paragraphs are presented. Of the various possibilities where small errors could occur only those which are associated with the determination of the girder's yield strength and the choice and application of the yield condition will be examined.

The yield stress, on which all the test results discussed in this plate girder investigation are dependent to a great extent, is a material property whose determination depends on the shape of the coupon and the testing speed. To evaluate the accuracy with which the yield strength of a member can be predicted on the basis of coupon results, reference is made to tests conducted at Fritz Engineering Laboratory(5). It was found that for eighteen different wide flange shapes the static yield stress of stub columns deviate between -8.2% and +4.7% from the yield strength predicted by the coupons. This indicates the uncertainty implicit in large scale experiments which should not be overlooked, even though coupon measurements are made on the very steel plates of which the test girder is built and even though both coupon and girder yield stress are obtained under static condition of zero strain rate.

The shear yield stress can only be computed from the tensile yield stress. Different values will be obtained depending on the assumed yield condition. The "yield condition of constant maximum shear stress", or "Tresca's yield condition", gives $\tau_y = 0.50\ \sigma_y$. In this investigation Mises' yield condition was used for which $\tau_y = 0.58\ \sigma_y$, this being 15% higher than predicted according to Tresca. The very fact that it is not known which of these two conditions is the more appropriate points up the much bigger uncertainties connected with yield level than with the steel properties E and ν used in the theory of elasticity. Thus, the scatter of results seen in the first diagram in Fig. 4 is within the range that has to be expected when shear test results on built-up girders are compared with predictions based on coupon tests.

Mises' yield condition finds its application in most European specifications when considering interaction between bending and shear in plate girders. Here the stress intensity

$$\sigma_g = \sqrt{\sigma_u^2 + \sigma_v^2 - \sigma_u\sigma_v + 3\tau^2} \qquad (7)$$

must not exceed a specified stress level at any point in the web. Thus, the same margin against incipient yielding is obtained as in a test coupon subjected to the normal stress σ_g. This method is, however, unsatisfactory for an interaction check of plate girders. Along the panel borders are residual stresses of unknown magnitude which are always neglected in the application of Eq. 7. Even if their magnitude were known, an estimate of the static carrying capacity could not be made, since the load producing yielding at one point is not in a constant relationship with the load causing such exhaustion of ductility that failure of the structure occurs.

It would be better to accept as a criterion for carrying capacity that the yielding must spread over an entire girder cross section. Then it would be justified to disregard residual stresses due to fabrication since their resultant over an entire cross section vanishes. Still, a failure mechanism is only theoretically obtained by postulating an ideal elastic - plastic stress-strain relation. As soon as a pronounced moment gradient (shear force) is involved test results are likely to exceed the predictions based on maximum moment. This has been observed with beams (Fig. 5.7, Ref. 1) as well as with plate girder as pointed out before, and is due to the strain-hardening effect. Barring premature failure due to primary instability, failure of a statically determinate girder only occurs when yielding has progressed not only over the entire cross section at peak moment but also over a certain length of the girder as well, after which failure is triggered by local inelastic buckling of a compression element. It must be noted that, due to the requirement of transverse stiffeners at places where concentrated loads are introduced, the maximum bending moment occurs always at the end of a panel. At this cross section, however, the web as well as the compression flange is restrained against local instability by the transverse stiffener which allows this yielded zone of limited length to strain-harden. Therefore, local torsional buckling of the flange occurs at cross sections where the moment value is smaller than that at the theoretical reaction line or cover plate end, as is illustrated in Fig. 5.

Consequently, in presenting the test results a "significant cross section" is chosen rather than the loading point at the end of a panel where the moment is highest. This beneficial effect of a moment gradient will be considered again while discussing the influence of torsional buckling on interaction.

4. THE INFLUENCE OF FLANGE INSTABILITY

So far it has been assumed that failure would occur by shear exhaustion of ductility, or at least that the flanges could be strained up to the yield level without a premature instability failure due to lateral, torsional or vertical buckling of the compression flange. This requirement will now be dropped and the question raised as to how the results obtained by analyzing these three failure modes on girders subjected to pure bending(3) must be modified in the case of a combination of bending and shear.

The presence of shear has both a detrimental and a beneficial aspect. The beneficial aspect is due to the fact that shear forces always imply a moment gradient and, therefore, only a short girder portion is affected by the maximum moment. The adverse aspect is that a web which is exhausted by shear cannot simultaneously take its allotted bending moment and the flanges will have to compensate for it, resulting in a higher flange stress than computed by the section modulus concept.

a) Lateral Buckling. From Fig. 7 of Ref. 3 it is seen that the overturning moment (torsion) causing lateral buckling is made of a contribution by the compression flange and another by the web. A rearrangement of stresses between the web and the flange, however, does not change the overall or resulting overturning moment. And since the resisting moment, which is dependent on the lateral stiffness of the compression flange, is only slightly affected by the higher stress level, the adverse influence mentioned above can be neglected in an analysis of lateral buckling. Calling the ultimate bending moment due to flange instability M_u, the failure condition given by Eq. 4, which is independent of shear, applies also to lateral buckling.

It remains to discuss the beneficial aspect of a moment gradient. This can be evaluated in the way proposed by Clark and Hill[6] and advocated in the Guide to Design Criteria for Metal Compression Members[7], namely, by multiplying by a factor C_1 the elastic critical stress which would result if the entire girder section were subjected to pure bending. Thus, the lateral buckling expression as represented in Eq. 7 of Ref. 3 is generalized as follows:

$$\frac{\sigma_{cr}}{\sigma_y} = 1 - \frac{\lambda^2}{4C_1} \qquad 0 < \lambda < \sqrt{2C_1} \tag{8a}$$

$$\frac{\sigma_{cr}}{\sigma_y} = \frac{C_1}{\lambda^2} \qquad \lambda > \sqrt{2C_1} \tag{8b}$$

with $$\lambda = \frac{\ell}{r}\sqrt{\frac{\varepsilon_y}{\pi^2}} = \ell\sqrt{\frac{\varepsilon_y}{\pi^2}\,\frac{A_f + \frac{1}{6}A_w}{I_f}}$$

Pursuant to the Guide's recommendation (Eq. 4.6, Ref. 7), the effective inelastic buckling stress is obtained on the basis of an equivalent column slenderness, and is reduced from Euler's curve in the same way as the basic column curve, Sec. 2.2, Ref. 7. Equation 8a fixes the thusly derived critical stress in the inelastic range. This reduction in the inelastic range is graphically indicated in Fig. 6, where the buckling stress curves are plotted for various values of C_1.

As explained when discussing the case of pure bending[3], the standard slenderness abscissa λ in Fig. 6 can be supplemented by one for ℓ/r and $\ell/2c$, the former being the slenderness ratio obtained by considering the compression flange together with 1/6 of the web as a column, while the latter is simply the ratio of buckling length to flange width, applicable only if the flange is a rectangle. Both of these abscissa are plotted for a yield strain $\varepsilon_y = 33/30000$; the λ scale, however, is appropriate for any yield strain. Instead of the lateral bracing distance ℓ the "k-length" $k\ell$, an effective lateral buckling length, can be introduced to account for restraining influences offered by neighboring sections. Since St. Venant torsion is neglected, the value k is exactly the same as for columns subjected to identical axial stresses and end restraint as the compression flange, and also has the same physical significance, Sec. 2.2b, Ref. 3.

Denoting as $\varkappa$ the ratio of the smaller end moments of a longitudinal girder segment free from interspan loads to the larger end moment, Eq. 4.13 of the Guide[7] gives the following expression for the coefficient C_1:

$$C_1 = 1.75 - 1.05\varkappa + 0.3\varkappa^2 \quad (-0.5 < \varkappa < +1) \qquad (9)$$

This relationship between stress raising coefficient C_1, and the moment gradient is based on solutions obtained by Salvadori[8]. With help of the sketches to the right of Fig. 6 the beneficial influence of a moment gradient can be readily studied.

A few cases of a girder with interspan loads are also inserted in Fig. 6. Here the critical stress is often further modified to include the effect of location of load application (top flange or bottom flange). If a load is suspended from the bottom flange, the buckling stress curves presented are conservative; if loaded on top, a tipping effect could make the result unconservative. Although it makes a significant difference whether the load is acting through the shear center or not, it must be pointed out that, for the case of plate girders, the tipping effect is more an academic than a real problem. In most cases the points of load application are simultaneously points of lateral bracing. Even in the case where the loading beam is laterally unstayed the physical picture still does not correspond to the condition pictured in Fig. 7a, since a cross beam offers torsional restraint to the girder. As seen from Fig. 7b, the cross beam does not even need to be tied to the girder by bolts in order to exert a torsional restraint. Transverse stiffeners are required under concentrated loads, hence theoretical knife edge load application at flange center is rather unlikely to occur. But when it does, as is the case of a crane girder, concurrent lateral forces are usually taken into account, and the analysis changes from an eigenvalue into a boundary value problem, i.e. stress limitations at the flange tips govern the design rather than lateral buckling stress. For these reasons no further provision against tipping seems required in plate girder specifications.

b) Torsional Buckling. According to the analysis of girders subjected to pure bending given in Sec. 2.3 of Ref. 3, torsional buckling is preceeded by lateral buckling if the ratio of flange width to thickness is smaller than twelve plus the ratio of lateral buckling length to flange width. For larger flange width-thickness ratios, a critical stress can be obtained by entering Fig. 6 with the buckling length that fulfills this condition.

The beneficial effect of a moment gradient applies also to torsional buckling of a long, hinged plate under longitudinal edge compression since its wave length extends also over the entire plate length. But the slight improvement in critical stress when the buckling length exceeds two or three times the plate width is less pronounced than in the case of lateral buckling; torsional buckling of the compression flange plate is of more local nature. Furthermore, the increase of flange stress, resulting from an exhausted web, should be accounted for. Rather than create further design provisions, the relation cited above from Ref. 3 might be used. The fact that, at the most stressed cross section of a panel, the compression flange is prevented from torsional buckling (Fig. 5) might be regarded as a compensation for any adverse influences.

This, of course, is only a step in the right direction. However, to compare numerically the mutually canceling effects would amount to an effort beyond the scope of this investigation. Not only would an exact analysis require knowledge of the combined failure mode of lateral and torsional buckling of the compression flange but strain-hardening and residual stresses would also have to be considered. Allowing lateral buckling alone White attempted to include strain-hardening(9), while Galambos considered the influence of residual stresses in the absence of moment gradient.(10) Torsional buckling of the flange plate in the inelastic range was studied by Haaijer and Thürlimann disregarding possible interaction between lateral and torsional buckling.(11)

c) Vertical Buckling. To prevent a premature failure due to vertical buckling of the flange plate, a limit for the web's depth-to-thickness ratio was proposed in Sec. 2.1, Ref. 3. When the girder web is slender, a prerequisite for vertical buckling, the shear is carried principally in tension field manner. The question is whether or not the web's tension field would pull the flanges of a girder with I-shaped cross section into the web. In deriving expressions to predict the shear carrying capacity[(2)], no such intentional use of the flanges as supporting boundary members of a diagonal tension field was made. The interaction tests carried out on plate girders developed no detrimental effect prior to application of ultimate load. As an illustration, Fig. 8 shows a girder after tests which caused failure in different panels.[(4)] It had a 50 x 3/16 inch web plate and 12 x 3/4 inch flanges. Of course, a straining beyond the ultimate load, that is, into the unloading range, would reach a point where the compression flange plate suddenly buckles into the web. But this is not the primary cause of failure. Rather it is a factor limiting girder deformation capacity under bending after the ultimate loading has been reached. While certain girders exhibit a pronounced yield plateau in their load versus centerline deflection diagram, slender-web girders under a combination of bending and shear may lack this favorable property.

It is conceivable that shear stress might be a factor in initiating vertical buckling of compression flange in the plane of a girder web. This was not discernible in any of the tests reported in Ref. 4 but these did not cover the entire interaction range with slender-web girders. However, precautionary measures have been taken in the derivation of Eqs. 6, from Eq. 5d, by the use of a relatively large value for the ratio A_w/A_f as representative of all girders, whereas the derivation given in Sec. 2.1 of Ref. 3 has indicated that girders prone to vertical buckling of compression flange have a low A_w/A_f ratio.

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Interaction between bending and shear occurs only when both types of stresses simultaneously reach high values. Generally, this will only occur at interior supports of continuous girders. Due to the stress concentrated at these points, strain-hardening is possible, (Fig. 5). This favorable effect is not reflected in the conventional design rules which prescribe the same maximum bending stress limits, regardless whether it occurs only at one cross section or over a certain length of the girder. As far as the compression flange is concerned, this effect was used to compensate for an unconservative provision for torsional buckling. In the tension flange, however, local torsional buckling cannot occur and the flanges can strain-harden. Also, the tension flange does not require provisions against vertical buckling, such as the relatively restrictive Eqs. 6 which, although derived for $A_w/A_f = 2.0$ would be applied to girders with a lower ratio of A_w/A_f, as pointed out above.

For these reasons it is suggested that the interaction Eqs. 6 be waived for the tension flange stress at interior reaction points of fully continuous girders. This may result in the choice of unsymmetrical cross sections having a smaller tension flange area than compression flange area, as advocated previously(12). Due to this measure, however, yielding in the tension flange would be initiated prior to instability of the compression flange, resulting in a beneficial redistribution of moments. Signs of imminent failure through tension yielding would be much better than a sudden collapse, triggered by compression flange instability.

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NOMENCLATURE

b : Depth of girder web

c : Half of flange width

ℓ : Lateral buckling length

r : Radius of gyration (in lateral direction, by considering compression flange and one sixth of the web as column cross section)

A_f: Flange area

A_w: Web area

C_1: Stress raising coefficient

I_f: Moment of inertia corresponding to r

M : Bending moment

M_f: Flange moment

M_y: Yield moment

M_p: Plastic moment

M_u: Ultimate moment

N : Factor of safety

S : Section Modulus

V : Shear force

V_u: Ultimate shear force

ε : Strain

$\varkappa$: Ratio of smaller to higher end moment

λ : Normalized slenderness ratio

σ : Normal stress

σ_y: Yield stress

σ_{cr}: Critical stress, buckling stress of compr. flange

τ : Shear stress

τ_{all}: Allowable shear stress

τ_y: Shear yield stress

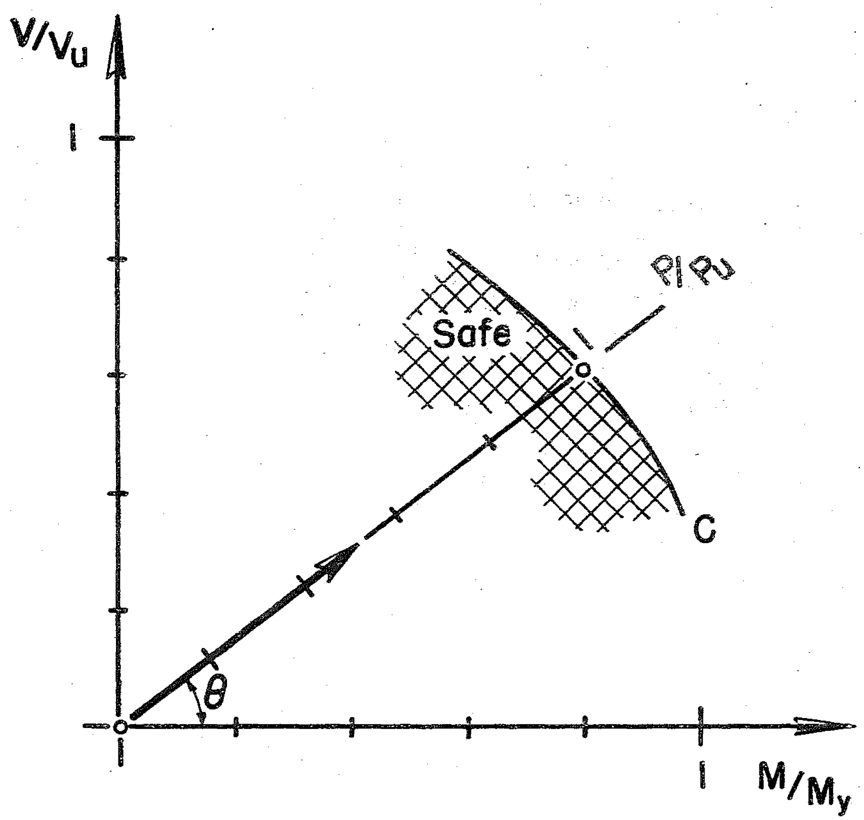

Fig. 1 Interaction Diagram

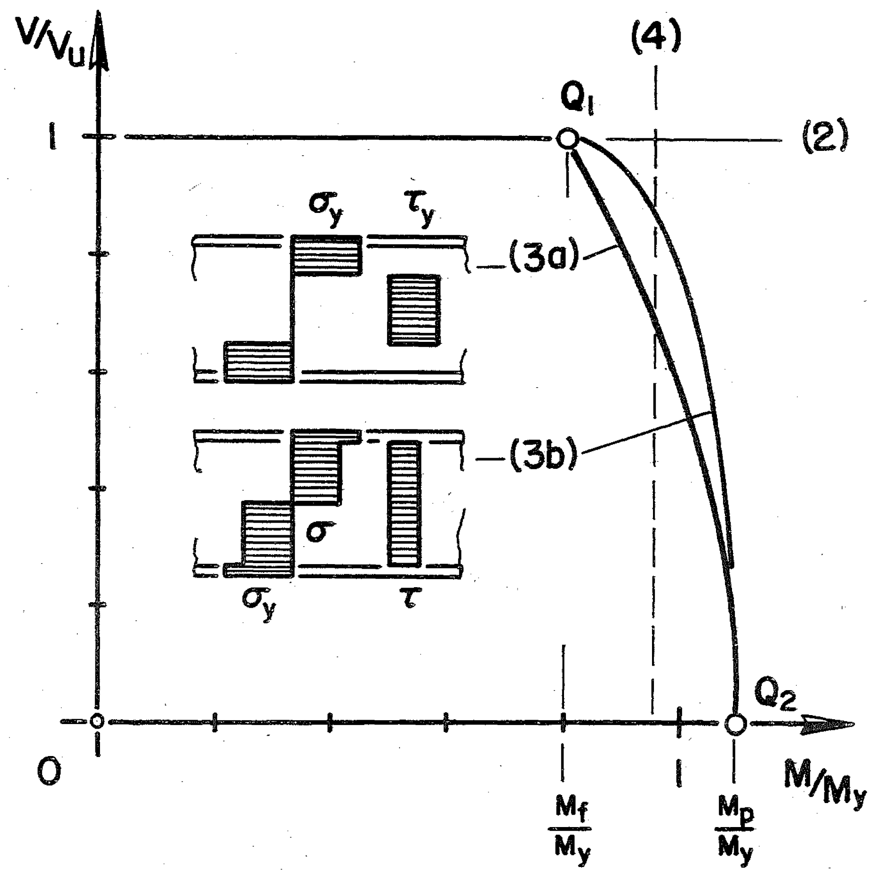

Fig. 2 Possible Interaction Curves

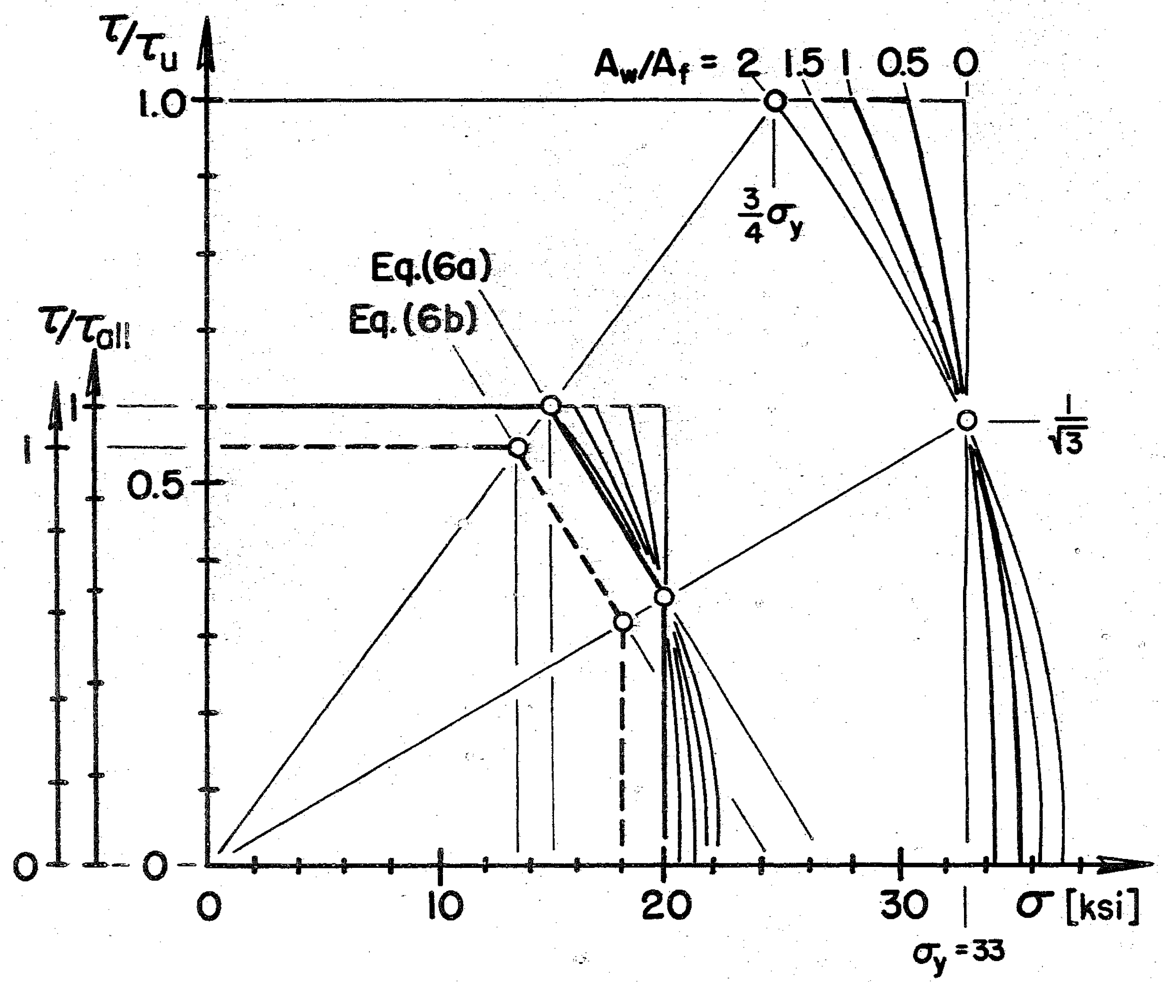

Fig. 3 Failure Envelopes and Allowable Stresses

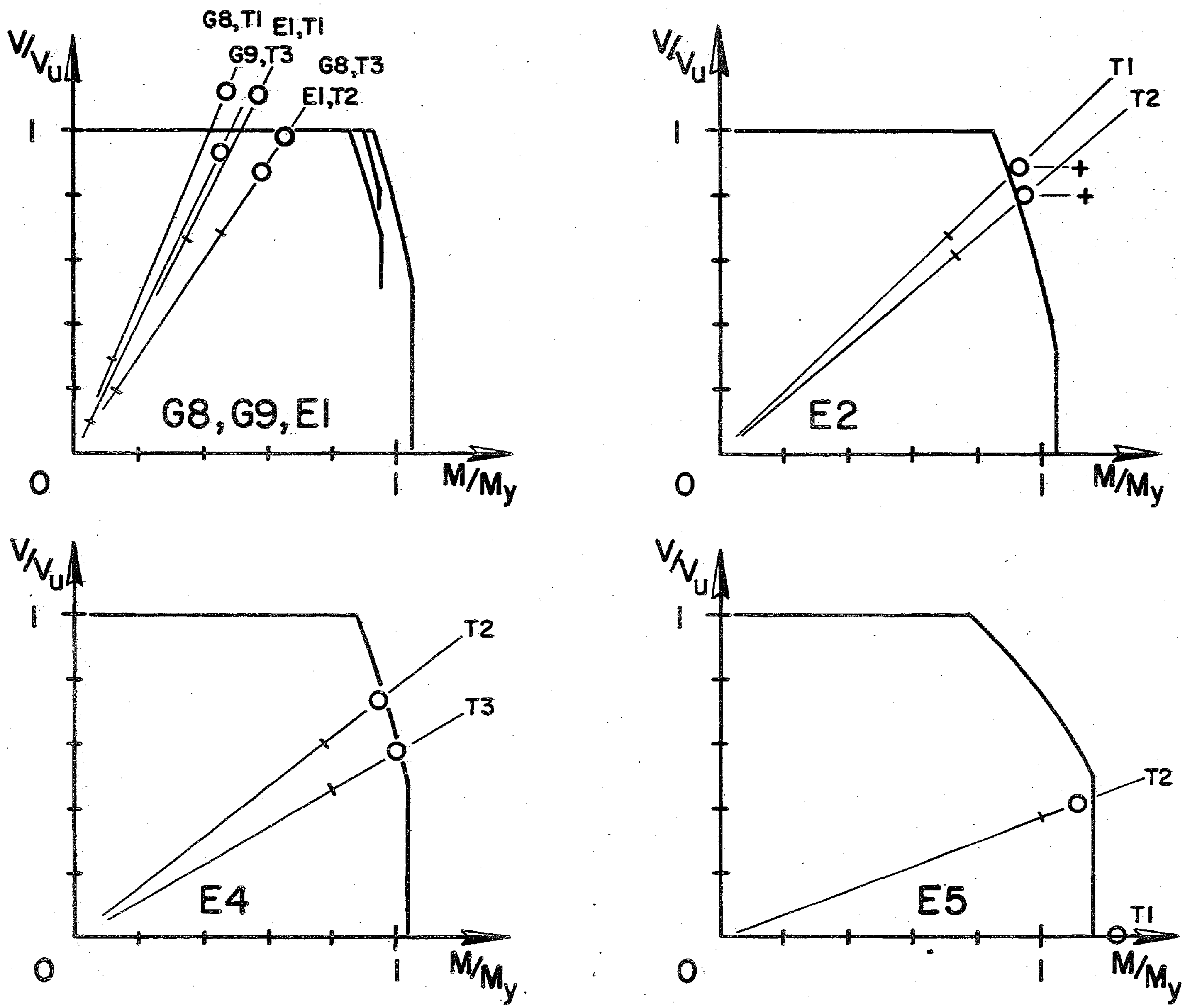

Fig. 4 Test Results

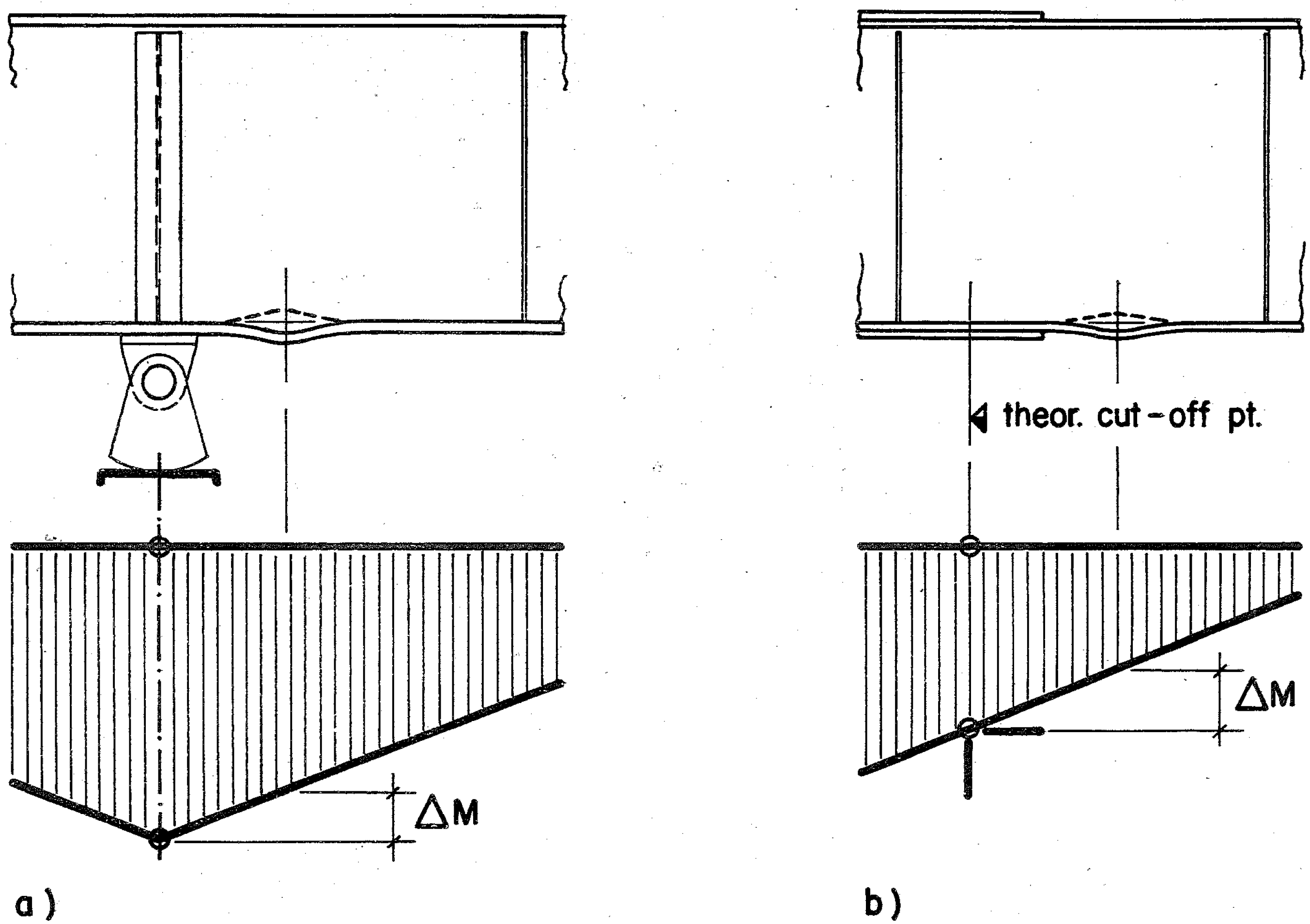

Fig. 5 Local Torsional Buckling of Compression Flange

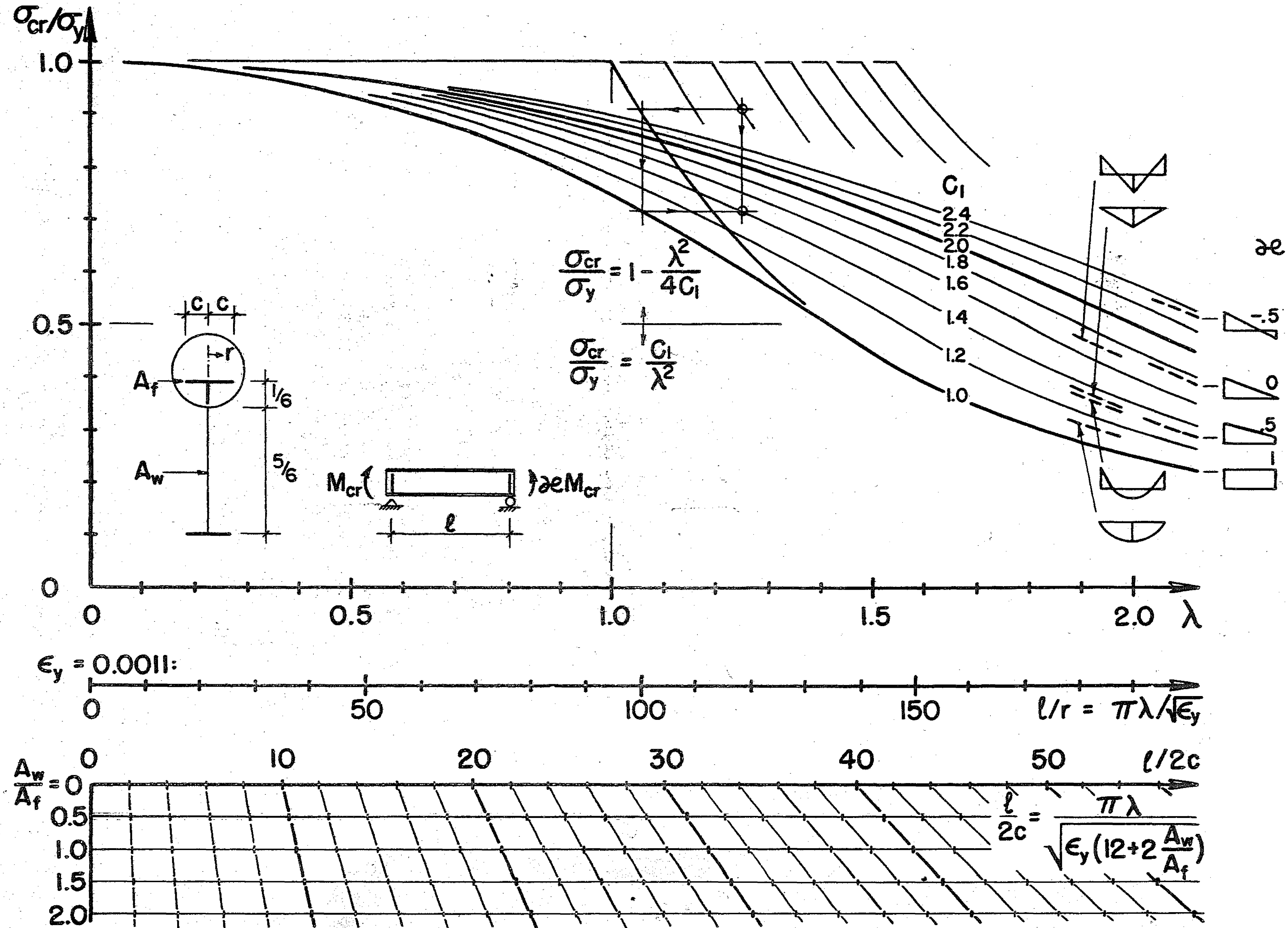

Fig. 6 Lateral Buckling Curves

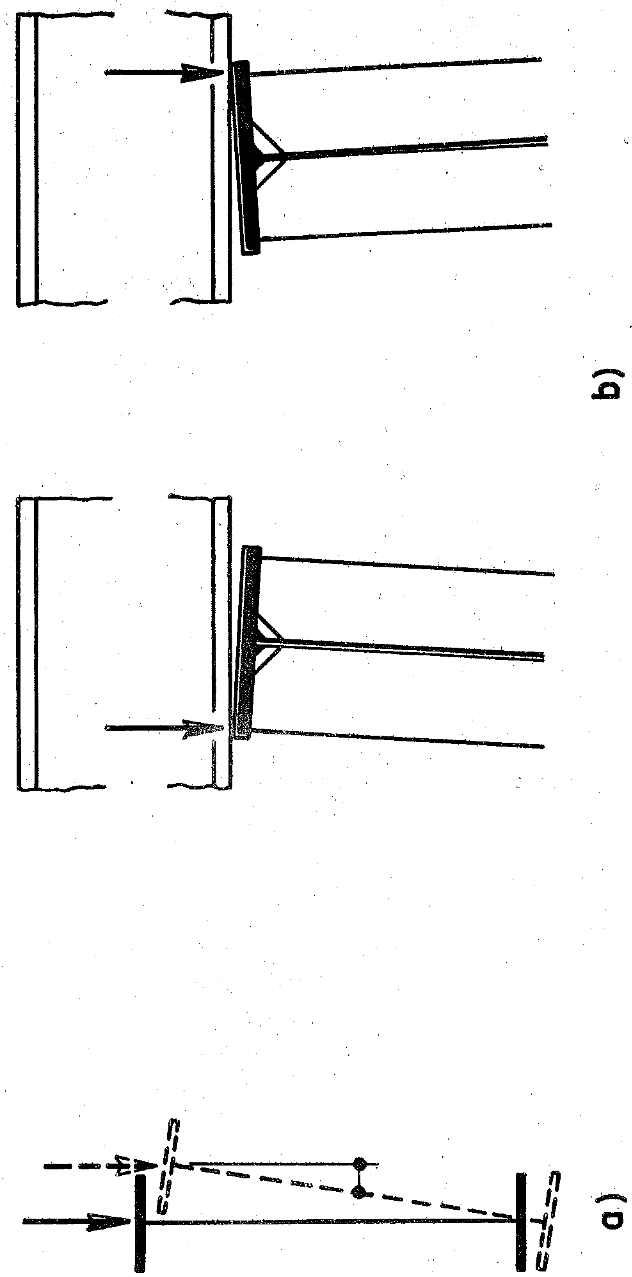

Fig. 7 The Tipping Effect

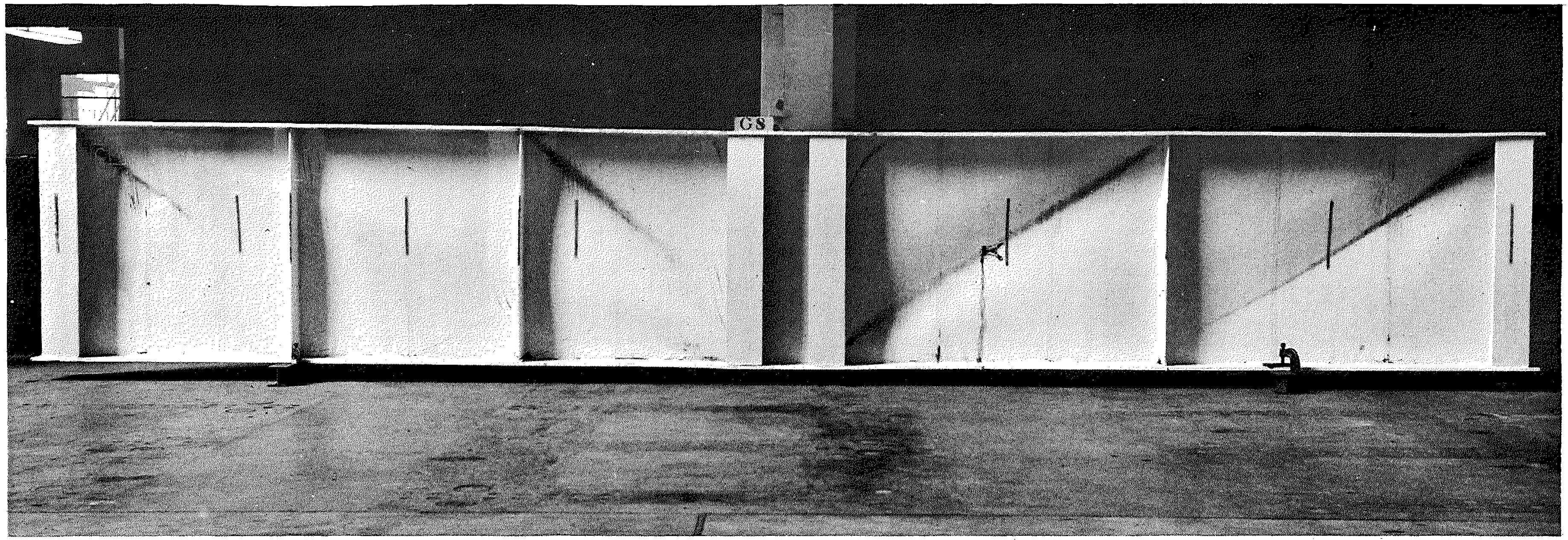

Fig. 8 A Thin-Web Girder after Testing

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