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NEW ASPECTS CONCERNING INELASTIC INSTABILITY OF STEEL STRUCTURES

by

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ABSTRACT

Plastic design procedures necessitated a reconsideration of the problem of inelastic instability of steel structures. Theories developed for members of continuously strain-hardening materials can not be applied indiscriminately to structural steel sections. For this material exhibits an extended yield level at a constant stress before the onset of strain-hardening. Besides, residual stresses introduced by rolling and fabrication procedures have a marked influence on the buckling strength.

Recent developments in the field of column and plate buckling will be discussed with respect to the above mentioned effects. The column buckling problem has been solved for the entire elastic and inelastic range. For plates, a solution for the beginning of strain-hardening has been derived using the theory of orthotropic plates with appropriate moduli developed from theoretical and experimental considerations.

After mentioning the shortcomings of the linear buckling theory in some cases of plate and shell buckling, it is indicated that this theory is unable to predict the static strength of plate girders.

I. INTRODUCTION

Intensive theoretical and experimental studies have been pursued in recent years to replace the time honored "Allowable Stress Design" by methods based on the carrying capacity of steel structures. It should be mentioned that the carrying capacity is not the only criterion by which the usefulness of a structure is measured. However, it is certainly the most important one leading to a definite margin of safety against overloads.

The types of failure associated with the ultimate load of a steel structure can be arranged in essentially four categories:

1. Instability,
2. Brittle Fracture,
3. Fatigue,
4. Ductile Fracture.

Of these four modes, instability is probably the most common cause of failure. Collapse may occur due to overall instability or may be triggered by buckling of some local element. A proper recognition of instability failures is especially important regarding the applicability of Plastic Design Methods. For these methods postulate that the strength of a structure is exhausted if a sufficient number of plastic hinges have developed to form a mechanism. This implies two conditions to be met, namely, no instability failure prior to the formation of the mechanism and no increase in strength due to strain-hardening of the material. In regions of plastic hinges large strains occur. They lead to a considerable drop in the bending and torsional rigidities of the affected cross sections. However, by selecting appropriate geometrical dimensions instability prior to the development of a mechanism can be avoided. On the other hand, for structures of practical dimensions, it almost becomes impossible to delay instability beyond this point. Hence Plastic Design Methods can neglect strain-hardening effects because they can not be realized due to instability.

In the following, results of recent investigations on the inelastic instability of steel members will be reported. A survey of the literature on buckling would indicate that yielding constitutes the upper limit which structural steel members can reach. However, it will be shown that for definite geometric proportions this limit can be exceeded. Furthermore, practically no information can be found concerning the influence of residual stresses on buckling. Definite theoretical and experimental answers to these important problems have been developed.

II. INFLUENCE OF RESIDUAL STRESSES ON COLUMN BUCKLING

The interpretation of results from buckling tests on steel columns with slenderness ratios smaller than 120 has always presented some problems due to the considerable scatter of these results. More or less convincing justifications have been proposed to explain this scatter such as accidental end eccentricities, initial crookedness of the specimens, variations in the stress-strain properties of the material, etc.

If, however, consideration is given to residual stresses present in rolled and welded steel members, it can be shown that the scatter is caused by the difference in magnitude and distribution of the residual stresses. This situation can readily be explained using a simple column model made from structural steel. Figure 1 shows a typical stress-strain curve of a steel coupon (A7 Steel) in tension or compression. Two "familiar" points seem to be missing, one being the proportional limit σ_p, the other one the upper yield point. The latter is entirely dependent on the straining speed under which the test is performed and disappears under static loading conditions. The former is practically indistinguishable from the yield stress if the coupon does not contain residual stresses nor is loaded accidentally with an eccentricity(1). The yield stress σ_y, the strain ε_{st} at the onset of strain-hardening and the corresponding modulus E_{st} may vary somewhat from the average values shown in the figure.

The column model, Fig. 2, consists of three parts interconnected in such a way that they act integrally. The material of each part follows the stress-strain curve of Fig. 1. By an appropriate procedure a state of stress was built into the model as indicated in the figure, stressing parts (1) and (3) to half the yield stress in compression and part (2) to half the yield stress in tension. The entire model including the three parts and the two end blocks is in equilibrium without external forces such that the imposed stresses constitute a residual stress system.

A compression test of the model specimen will furnish the following average stress-strain curve. Initially the sum of the residual and loading stresses will remain within the elastic range. However, for

$$P_p = \frac{1}{2}\,\sigma_y A \tag{1}$$

where A = bh is the cross sectional area, the sum of the residual and loading stresses in parts (1) and (3) equals the yield stress σ_y in compression as illustrated in Fig. 3. If the load is further increased these parts will yield under constant stress such that the elastic part (2) must absorb this entire increase of load. Full yielding of the specimen is reached when the yield stress σ_y in part (2) is also developed or

$$P_y = \sigma_y A \tag{2}$$

An average stress-strain diagram will reflect this situation as follows (Fig. 4). The proportional limit corresponding to P_p of Eq. (1) will be reached for

$$\sigma_p = \frac{P_p}{A} = \frac{1}{2}\,\sigma_y \tag{3}$$

Thereafter the strain will increase at twice the elastic rate. Full yielding develops for a strain $\varepsilon = 1.5\varepsilon_y$, the latter being the yield strain of the material. Figure 4 also shows the Tangent Modulus E_t corresponding to such a stress-strain curve. At the proportional limit, $\sigma_p = \frac{1}{2}\sigma_y$, a sudden change takes place.

Considering now buckling of the model column, the critical load P_{cr} within the elastic range is given by the Euler formula:

$$P_{cr} = \frac{\pi^2 EI}{l^2} \tag{4a}$$

or in terms of stress:

$$\sigma_{cr} = P_{cr}/A = \frac{\pi^2 E}{(l/r)^2} \tag{4b}$$

If no residual stresses would be present, this equation would be valid up to the yield stress. However, for the assumed conditions, parts (1) and (3) of the model will start to yield when σ_{cr} reaches the proportional limit $\sigma_p = 1/2\ \sigma_y$. This in turn will lead to a sudden drop in the bending stiffness. Following the "Tangent Modulus Concept", which assumes that no strain-reversal takes place, only part (2) provides bending resistance as indicated in Fig. 5 once the proportional limit σ_p is exceeded. The reduced resistances with respect to the x-x and y-y axes are:*

$$(EI_x)_t = E\frac{(b/2)h^3}{12} = \frac{1}{2}EI_x = \tau EI_x \tag{5}$$

$$(EI_y)_t = E\frac{(b/2)^3h}{12} = \frac{1}{8}EI_y = \tau^3 EI_y \tag{6}$$

In the above equations I_x and I_y are the moments of inertia of the cross section, (b·h), with respect to the x-x and y-y axes. The parameter

$$\tau = E_t/E = 1/2 \tag{7}$$

is the ratio between the tangent modulus E_t and the modulus of elasticity E, the former being obtained from the average stress-strain diagram of the entire specimen containing the residual stresses as given in Fig. 4. Experimentally this curve can be obtained by compressing a stub column sufficiently long such that it contains the full residual stresses within the length of observation, but short enough, such that buckling will not occur. It should be emphasized that E_t is not the tangent modulus obtained from measurements on a coupon of the material. The latter would exhibit perfect elasticity according to Fig. 1 up to the yield stress.

From the above considerations it follows that the critical load P_{cr} beyond the proportional limit can be obtained

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* Subscript t in $(EI_x)_t$ is used if Tangent Modulus Concept is applied; subscript r, introduced subsequently is used if Reduced Modulus Concept is applied.

by replacing EI in Eq. (4a) with the reduced stiffnesses of Eq. (5) or (6). Hence for buckling about the x-x axis

$$P_{cr} = \frac{\pi^2 \tau E I_x}{l^2} \quad , \tag{8a}$$

or

$$\sigma_{cr} = \tau \frac{\pi^2 E}{(l/r_x)^2} \qquad \text{(x-x Axis).} \tag{8b}$$

Similarly for buckling about the y-y axis

$$P_{cr} = \frac{\pi^2 \tau^3 E I_y}{l^2} \quad , \tag{9a}$$

or

$$\sigma_{cr} = \tau^3 \frac{\pi^2 E}{(l/r_y)^2} \qquad \text{(y-y Axis).} \tag{9b}$$

These results are plotted in Fig. 6 in such a form that they are independent of the yield stress level σ_y. This was accomplished by dividing the critical stress σ_{cr} by the yield stress σ_y and plotting as abscissa the slenderness parameter α. The latter is defined as

$$\alpha = \frac{l/r}{\pi \sqrt{E/\sigma_y}} \tag{10}$$

The denominator of Eq. (10) is an ideal slenderness ratio corresponding to the condition of no residual stresses and $\sigma_{cr} = \sigma_y$, or:

$$\sigma_{cr} = \sigma_y = \frac{\pi^2 E}{(l/r)^2}$$

Hence: $\quad l/r = \pi \sqrt{E/\sigma_y}$

For discussing the behavior of the present model such a nondimensional representation is quite unnecessary. But it is absolutely required for a proper comparison of test results obtained on structural steel columns since the actual yield stress of different columns can vary considerably.

Inspection of Fig. 6 shows that for $\sigma_{cr}/\sigma_y < 0.5$ and $\alpha > \sqrt{2}$, a unique elastic solution exists. At $\sigma_{cr}/\sigma_y = 0.5$ a sudden reduction in the slenderness parameter α takes place. Furthermore, this reduction depends on the axis about which buckling takes place. From Fig. 5 it can easily be seen that yielding of parts (1) and (3) will lead to a much greater drop in stiffness for buckling about the y-y axis. Upon reaching the yield stress, or $\sigma_{cr}/\sigma_y = 1$, a further abrupt reduction occurs leading again to a single solution. This possibility of a column to carry a stress beyond the yield stress, as indicated in Fig. 6, will be discussed in the following section.

The dashed curves shown in the same Fig. 6 were derived using the "Reduced Modulus Theory". Following the assumption that no change in load takes place, the location of the bending axis a-a indicated in Fig. 5 is fixed. The bending stiffness for buckling about x-x or y-y is provided by the cross hatched areas leading to:

$$(EI_x)_r = 0.687EI_x \tag{11}$$

and

$$(EI_y)_r = 0.422EI_y \tag{12}$$

respectively. Accordingly, the critical stresses are

$$\sigma_{cr} = 0.687 \frac{\pi^2 E}{(l/r_x)^2} \quad \text{(x-x Axis).} \tag{13}$$

$$\sigma_{cr} = 0.422 \frac{\pi^2 E}{(l/r_y)^2} \quad \text{(y-y Axis).} \tag{14}$$

which are plotted in Fig. 6 as dashed curves. Whereas the tangent modulus curves indicate the situation at which bending will commence, the reduced modulus curves present upper limits for the maximum load. Methods for predicting this maximum load lying between these two curves have been developed[2]. It should be emphasized, at this point, that residual stresses can create a situation where the maximum column load may be appreciably greater than the tangent modulus load.

Having discussed in principle the influence of residual stresses on the buckling load on a very simplified model, a generalization can be readily made. If a continuously varying residual stress distribution is introduced instead of the discontinuous one as in Fig. 2, steady transition curves between the elastic buckling curve and the yield stress can be obtained. The general conclusions to be drawn are:

1. Residual stresses lower the buckling load of steel columns in the inelastic range.

2. The reduction depends not only on the magnitude but also on the distribution of these stresses.

3. The reduction can be related to the average stress-strain curve obtained from a compression test on a short column containing the residual stresses.

4. Residual stresses can lead to a considerable difference between the tangent and reduced modulus load such that the maximum load may be appreciably higher than the tangent modulus load.

An extensive study on the influence of residual stresses on the buckling strength of steel columns has been under way for the last seven years. A typical residual stress pattern measured on an 8WF31 section in the "as rolled" condition is shown in Fig. 7[3]. Of interest are the compressive stresses of about 15 ksi at the flange tips. If compressive loading is applied, the combined residual and loading stresses will initiate yielding at the flange tips. As illustrated in Fig. 8 this will result in a greater reduction of the bending rigidity with respect to the y-y (weak) axis than the x-x (strong) axis. Figure 9 compares tests of as-rolled 8WF31 columns buckling about their strong and weak axes with predictions based on residual stress measurements[3],[4]. In addition the results of two tests on annealed columns are shown. Since annealing eliminates nearly all residual stresses, columns should show greater strength in the inelastic range. This is definitely borne out by the tests. Of historical interest is the fact that consideration of residual stresses resolves the difference between Tetmaier proposing a straight line and Engesser a parabola as transition curves between elastic buckling and yielding. They approximated weak and strong axis buckling of WF-columns respectively.

Without going into further details a summary graph of some 18 tests on I-shaped columns is shown in Fig. 10[3],[4],[5]. The symbols refer to the following conditions: (1) "as-rolled" WF section, buckling about weak axis; (2) "as-rolled" WF section, buckling about strong axis; (3) annealed WF section, buckling about weak axis; (4) riveted I-section, buckling about weak axis; (5) welded I-section, buckling about weak axis. The great scatter of all these results would be almost frightening if it could not be explained as a necessary consequence of the difference in residual stresses within these members. Discussion of these results and comparison with theoretical predictions may be found in the above mentioned references. However, the listing of the reduction in the carrying capacity of the different members for the slenderness parameter $\alpha = 0.95$--i.e., slenderness ratio $l/r = 90$ for $\sigma_y = 33$ ksi--may be of interest.

Annealed WF shapes	$\sigma_{cr}/\sigma_y = 0.9$
Riveted I-sections	" $= 0.85$
As-rolled WF shapes	" $= 0.75$
Welded I-sections	" $= 0.60$

This reduction is proportional to the increase in the compressive residual stress between the different columns.

III. BUCKLING OF COMPRESSION ELEMENTS BEYOND YIELDING

Having discussed the influence of residual stresses on the transition from elastic buckling to the yield stress, the question arises as to whether compression elements can reach a stress above yielding without buckling. Such problems are of considerable interest with respect to plastic design methods for reasons mentioned in the introduction. The classical theories imply that yielding cannot be exceeded. However, experience shows that small and sturdy compression coupons can be strained beyond yielding without buckling. In the following a short summary of results obtained on columns and plates is given.

(a) Column Buckling in the Strain-Hardening Range

Inspection of a typical stress-strain curve, Fig. 1, suggests that steel specimens deform homogeneously under the yield stress. However, observation of the actual behavior shows that yielding occurs in extremely thin layers forming successively along the length of the specimen. These slip bands are oriented along the planes of maximum shear stress. The local strain across a band increases instantaneously from ε_y to ε_{st}. Yielding commences at a weak spot (inclusion, stress concentration, etc.) and spreads from this point over the entire length of the specimen. It can therefore be concluded that during yielding part of the material is still elastic whereas other regions have reached the strain-hardening range. Only after complete yielding has taken place throughout the entire length of the specimen do the material properties become homogeneous again.

Considering now a compressed column with no residual stresses and a sufficiently small slenderness ratio, yielding may initiate at the center and spread uniformly toward the two ends as illustrated in Fig. 11(a). In the two end sections the bending stiffness is equal to the elastic stiffness EI, whereas in the middle section, $2\xi l$, yielding reduces the stiffness to $E_{st}I$, E_{st} being the tangent modulus at the onset of strain-hardening. Hence, the problem reduces to the solution of buckling of a column with a reduced bending stiffness over the middle portion. If buckling occurs during yielding the buckling stress is equal to the yield stress. The interesting relationship to observe is that between the average axial strain ε_{cr} at which buckling occurs and the slenderness ratio l/r. In Fig. 12, which is taken from Ref. (6), this average strain ε_{cr} divided by the yield strain ε_y is plotted for two cases, namely, (a) yielding spreading symmetrically from the middle and (b) yielding commencing from both ends (cases (a) and (b) of Fig. 11 respectively). Also plotted are test results obtained on small columns with round and square cross sections varying between 1/2 and 1 inch diameter. Most of the results fall within the area contained between curves (a) and (b), hence substantiating the reasoning presented above. The scatter should be expected as initiation of yielding occurs at random points, spreading from some imperfection anywhere along the specimen. Nevertheless, it is evident that for the range between yielding and strain-hardening, a decrease in the slenderness ratio will lead to an increase of the critical strain ε_{cr}.

For sufficiently small slenderness ratios the column will reach strain-hardening prior to buckling. This ratio can be determined by substituting σ_y and E_{st} for σ_{cr} and E, respectively, in the elastic buckling equation Eq. (4b).

$$\sigma_{cr} = \sigma_y = \frac{\pi^2 E_{st}}{(l/r)^2} \tag{15}$$

With E_{st} = 900 ksi and σ_y = 36 ksi

$$l/r = 15.7 \tag{16}$$

Figure 13 shows results of some tests on specimens with rectangular cross sections 1/2 x 3/4 inches[6]. The maximum stress starts to exceed the yield stress for l/r smaller than 20. The test points are located between the two theoretical curves based on the (a) tangent (no strain reversal) and (b) reduced (full strain reversal) modulus concepts respectively.

As a consequence it follows that compression elements can exceed the yield stress without buckling. The knowledge of the geometric proportions for which this is possible is of great importance for proper structural detailing, especially for designs based on plastic methods.

(b) Plate Buckling in the Strain-Hardening Range:

The previous considerations have been extended to the case of plate buckling(6),(7). Of primary interest is the determination of such geometric proportions for plate elements that large plastic deformations at the yield stress level may occur without buckling. Several theories of plastic buckling have been presented for continuously strain-hardening materials such as aluminum alloys. To solve the problem for structural steel which exhibits a pronounced yield level, a new approach was found to be necessary. Whereas in the elastic range steel behaves as an isotropic material, yielding produces orthotropic properties. Instead of starting from a general theory of plasticity, expressions for the plate buckling stresses based on orthotropic behavior were derived. The advantage of this approach is to clearly indicate the influence of the various stiffness factors that are involved.

As an illustration the solution of a simple but typical problem is described. The flanges of an equal leg angle, Fig. 14 (1), can be considered as plates simply supported along the heel and free along the tips. Assuming the width to thickness ratio b/t of the flange is sufficiently small such that it is possible to compress the angle axially up to the point of strain-hardening, then $\sigma_{cr} = \sigma_y$ and $\varepsilon = \varepsilon_{st}$, and the governing differential equation for buckling is[7]:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = - \frac{12\sigma_{cr}}{t^2} \frac{\partial^2 w}{\partial x^2} \tag{17}$$

where the orthotropic moduli are:

$$D_x = E_x (1 - \nu_x \nu_y)$$

$$D_y = E_y (1 - \nu_x \nu_y)$$

$$D_{xy} = \nu_y D_x$$

$$D_{yx} = \nu_x D_y$$

$$2H = D_{xy} + D_{yx} + 4G_t$$

For the case under investigation the solution of this equation is:

$$\sigma_{cr} = (\frac{t}{b})^2 \left[\frac{\pi^2 D_x}{12} \cdot (\frac{b}{l})^2 + G_t \right] \tag{18}$$

If the specimen is long compared to the flange width ($l \gg b$), the first term in the parenthesis becomes negligible such that

$$\sigma_{cr} = G_t \ (\frac{t}{b})^2 \tag{19}$$

The only modulus entering Eq. (19) is G_t, the Tangent Modulus in Shear at the point of strain-hardening of an axially compressed specimen. Its value has been determined experimentally by axially compressing a thin-walled tube up to the strain $\varepsilon = \varepsilon_{st}$ and then applying a twist[6]. The results are plotted in Fig. 15 and compared with predictions based on an incremental stress-strain relationship. The two tests showed that the initial shearing modulus is practically equal to the elastic shearing modulus. However, a small angle of twist γ leads to a considerable drop of G_t. If G_t enters into a buckling problem the equilibrium at the point of buckling is therefore not just indifferent as under elastic conditions, but due to the rapid drop of G_t with increasing twist γ, the configuration is highly unstable. In the presence of even slight imperfections the initial value of G_t can never be reached. It is therefore obvious that proper account must be given to this situation in selecting the different moduli governing buckling at the point of strain-hardening. In Refs. (6) and (7) theoretical and experimental evidence is presented for the selection of the following values for structural steel (ASTM-A7):

$$E_x = 900 \text{ ksi}$$

$$G_t = 2,400 \text{ ksi}$$

$$D_x = 300 \text{ ksi}$$

$$D_y = 32,800 \text{ ksi}$$

$$D_{xy} = D_{yx} = 8,100 \text{ ksi}$$

Introducing the above value of G_t into Eq. (19) and setting σ_{cr} equal to the yield stress $\sigma_y = 36$ ksi, the value of b/t is derived for which buckling will occur at the onset of strain-hardening:

$$b/t = \sqrt{G_t/\sigma_y} = 8.2 \qquad (20)$$

Similar solutions for other cases have been derived and checked by appropriate tests on angles and WF-shapes in compression and bending[6]. Figure 16 gives the results for five angles tested in compression. It should be noted that an angle can be strained in compression up to the point of strain-hardening, $\varepsilon_{st} = 14 \cdot 10^{-3}$, without torsional buckling provided b/t is sufficiently small.

The results of all column and plate buckling tests are summarized in a single plot, Fig. 17. This is possible by using a non-dimensional representation. As ordinate the ratio of buckling to yield stress is plotted, σ_{cr}/σ_y. The abscissa is equal to the ratio of the actual slenderness ratio to the ideal slenderness ratio corresponding to the yield stress, or:

For columns: $$\alpha = \frac{l}{r}\frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}} \qquad (21a)$$

For plates: $$\alpha = \frac{b}{t}\frac{1}{\pi}\sqrt{\frac{\sigma_y 12(1-\nu^2)}{kE}} \qquad (21b)$$

where k = plate buckling coefficient.

Such a representation has several advantages. First, in the elastic range a single curve for all buckling cases is obtained. Assuming a maximum residual stress equal to half the yield stress, the elastic range holds for $\alpha > \sqrt{2}$ as indicated in Fig. 17. Secondly, the results are independent of the yield stress level which makes a comparison of test results obtained on specimens with different yield stresses possible. Finally, a clearer picture of the slenderness requirements so that the buckling stress of a structural element will exceed the yield stress results. Figure 17 shows that a column goes into strain-hardening for $\alpha = 0.17$, corresponding to $\sqrt{E_{st}/E} = \sqrt{900/30,000} = 0.173$. For a long plate with one simply supported and one free edge, $\alpha = 0.46$ or $\sqrt{G_t/G} = \sqrt{2400/11,500} = 0.456$. The value $\alpha = 0.58$ for a long plate with two simply supported edges, does not correspond to a single modular ratio for the reason that it depends on all three stiffness moduli entering the differential equation (17).

The transition curves between elastic buckling and full yielding depend on the magnitude and distribution of the residual stresses as has been discussed in section II for the case of WF-columns. For the column curve (a) sufficient evidence exists that residual stresses will lead to such a transition curve in the average[3],[4]. Curves (b) and (c) for plates present reasonable interpolations in the absence of test results in this particular region.

Plotted in the same Fig. 17, are test results relevant to the onset of strain-hardening[6]. They are in substantial agreement with the theoretical predictions. It should be mentioned that the rules concerning minimum thickness of compression elements in plastically designed structures[12] are based on these results.

IV. EXTENSION TO OTHER STABILITY PROBLEMS

Considerable work, both analytical and experimental, has been completed on the influence of residual stresses on the carrying capacity of eccentrically loaded WF-columns[(8),(9)]. Similarly, their influence on the lateral-torsional buckling of WF beam-columns loaded eccentrically in the strong plane has been recently studied theoretically[(10)].

An important problem in plastically designed structures is the lateral support of beams, especially in the region of possible plastic hinges. In order to maintain the full plastic bending resistance of the beam through the required hinge rotation, it is necessary to prevent the beam from buckling laterally. Solutions for WF-beams subjected to different end moments and being strain-hardened over part of their length have been worked out. The problem was formulated by means of finite differences and the resulting determinants were solved using a digital computer[(11)]. The results, in greatly simplified form, have found their application in the design rule for lateral bracing of plastically designed structures[(12)].

The elastic buckling strength of longitudinally and transversely stiffened steel panels, used for example in ship construction, has been investigated extensively. However, information has been lacking concerning their inelastic strength and especially the geometric proportions required such that buckling will not occur prior to the onset of strain-hardening. An analytical study into these problems has been recently completed[13]. By delaying instability until the beginning of strain-hardening, a full utilization of the material up to the yield stress is realized. Furthermore, sufficiently large plastic deformations can occur such that the structure is able to redistribute the internal forces under extreme loading conditions. The inherent difficulties concerning inelastic buckling as affected by residual stresses are avoided, for the latter are practically wiped out at the point of strain-hardening. Finally it can be expected that the welding distortions of panels can be better controlled if they are able to undergo plastic straining without buckling.

V. SHORTCOMINGS OF THE LINEAR BUCKLING THEORY

All previous problems were treated on the basis of the linear buckling theory. This theory determines essentially the load at which bifurcation of the equilibrium takes place, e.g., the load of a centrally loaded column under which it starts to deflect laterally. Mathematically, the treatment leads to an "Eigenvalue" problem. However, the linear buckling theory does not give any indication about the behavior beyond this point.

Considerable fundamental research, done primarily in the field of aeronautical engineering, shows that the linear theory underestimates, in general, the carrying capacity of compressed plate elements but overestimates the resistance of shell structures considerably (see Ref. (14) for a summary presentation). The situation is illustrated schematically in Fig. 18. According to the linear buckling theory a perfect specimen - it may be a column, plate or shell - buckles at a critical stress σ_{cr}. For stresses $\sigma < \sigma_{cr}$ no lateral deflection δ is possible. Under a stress $\sigma = \sigma_{cr}$ the specimen deflects along line A-A starting from point $\sigma/\sigma_{cr} = 1$ and $\delta/t = 0$, t being a representative thickness of the specimen. However, large deflection theory shows that only a column will follow line A-A provided it remains elastic. On the other hand a flat plate under appropriate support conditions will exhibit a postbuckling strength according to curve B-B. Contrarily a cylinder will show an immediate drop in its resistance along curve C-C. If any initial imperfection δ_o/t is present, the bifurcation point ($\sigma/\sigma_{cr} = 1$; $\delta/t = 0$) completely loses its significance except as an upper limit for the carrying capacity of a column. It is therefore obvious that only the large deflection theory can adequately describe the behavior and strength of plate and shell elements. However, its application presents two main obstacles. First, this theory leads generally to involved mathematical problems, the solution of which can only be justified in exceptional cases, certainly not for a routine design. A more serious problem arises because the dimensions of ordinary steel structures are such that inelastic behavior takes place. Under these circumstances the application of the large deflection theory to plate and shell problems becomes practically impossible. It is felt that for such cases the development of "upper and lower bound techniques" may prove useful approaching the true carrying capacity from above and below.*

Finally, the static behavior of plate girders is singled out to demonstrate that under certain conditions a rearrangement of the internal forces can lead to a much higher strength than predicted by the linear buckling theory. Under present specifications such as AASHO, AISC, AREA, German DIN 4114, etc., the provisions against buckling of the web govern essentially the design. The new British Standards recognize to some degree the web's own postbuckling strength. Recently a comprehensive investigation has been initiated at Fritz Engineering Laboratory, Lehigh University, with the objective to determine the static load carrying capacity of welded plate girders(15). Systematic tests have already been completed showing conclusively that the linear buckling theory is unable to predict the strength of such members(16),(17). In 15 ultimate load tests on seven full-size girders the observed loads exceeded the conventionally computed critical loads anywhere from 15% to 800%. From these tests, it became evident that the web should not be considered as an isolated element in the design of such members. Due to the

* "Upper and lower bound theorems" used in Plastic Analysis do not apply to stability problems. They are based on the assumption that equilibrium is formulated on the undeformed structure.

presence of the flanges and vertical stiffeners framing the web a gradual rearrangement of the stress pattern predicted by the beam theory to a more favorable one takes place. This transfer of stresses is the important and governing contribution to the postbuckling strength of plate girders. An analytical study taking into account the actual behavior of plate girders is under way with the objective to predict their static strength.

VI. SUMMARY

Recent developments connected with studies on the static carrying capacity of structural steel members have necessitated the introduction of new aspects into the classical buckling theory. In some instances it was found that the theory was completely inadequate to describe the strength and a new approach became necessary. In this paper a survey of this situation has been presented describing specifically the following findings:

1. Residual stresses govern the transition curve between elastic buckling and yielding of steel columns. Their influence has been studied analytically and confirmed by tests.

2. Contrary to accepted opinion, properly proportioned steel compression elements, such as columns and plates, can be compressed up to the point of strain-hardening without premature buckling. The corresponding slenderness ratios have been computed

analytically and checked by appropriate tests.

3. Extending the above findings, the influence of residual stresses on the strength of eccentrically loaded columns and the lateral torsional buckling of WF-beams loaded in the strong plane have been studied. A further application was made by deriving the geometric proportions of longitudinally and transversely stiffened panels, e.g. ship plating, such that buckling will not occur prior to the point of strain-hardening. The possible advantages of such a design criterion were pointed out.

4. After indicating the inadequacy of the linear buckling theory in describing the strength of shells and the postbuckling behavior of plates, the problem of the carrying capacity of plate girders was singled out. A recently started investigation indicates that the strength of such members cannot be related to the critical web buckling stress of the linear buckling theory. Due to the presence of flanges and vertical stiffeners framing the web a rearrangement of the internal forces can take place leading to a more favorable configuration.

VII. ACKNOWLEDGEMENT

Most of the findings described in this paper were obtained over the past eight years in the course of research by members of the staff of Fritz Engineering Laboratory, Lehigh University. Professor W. J. Eney is director of the Laboratory and head of the Civil Engineering Department. The organizations that have sponsored this research are: American Institute of Steel Construction, American Iron and Steel Institute, Association of American Railroads, Bureau of Public Roads, Column Research Council, Department of the Navy, National Science Foundation, Pennsylvania Department of Highways and Welding Research Council.

VIII. REFERENCES

1. Tall, L. and Ketter, R. L.
"On the Yield Properties of Structural Steel Shapes"
Fritz Laboratory Report No. 220A.33, Lehigh University, 1958

2. Duberg, J. E. and Wilder, T. W.
"Inelastic Column Behavior"
NACA, Technical Note No. 1072, 1952

3. Huber, A. W. and Beedle, L. S.
"Residual Stress and the Compressive Strength of Steel"
Welding Journal 33 (12), 1954, p. 589-s

4. Thürlimann, B.
"Der Einfluss von Eigenspannungen auf das Knicken von Stahlstützen"
("Influence of Residual Stress on the Buckling of Steel Columns")
Schweizer Archiv 23 (12), 1957, p. 388

5. Fujita, Y.
"Built-up Column Strength"
Ph.D. Dissertation, Lehigh University, 1956

6. Haaijer, G. and Thürlimann, B.
"On Inelastic Buckling in Steel"
ASCE, Proc. Paper 1581, Vol. 84, EM2, April 1958

7. Haaijer, G.
"Plate Buckling in the Strain-Hardening Range"
ASCE, Proc. Paper 1212, Vol. 83, EM2, April 1957

8. Galambos, T. V. and Ketter, R. L.
"Columns under Combined Bending and Thrust"
ASCE, Proc. Paper 1990, Vol. 85, EM2, April 1959

9. Ketter, R. L. and Huber, A. W.
"The Influence of Residual Stress on the Carrying Capacity of Eccentrically Loaded Columns"
International Association of Bridge and Structural Engineering, Publications, Vol. 18, 1958, p. 37

10. Galambos, T. V.
"Inelastic Lateral-Torsional Buckling of Eccentrically Loaded Wide Flange Columns"
Ph.D. Dissertation, Lehigh University, 1959

11. White, M. W.
"The Lateral Torsional Buckling of Yielded Structural Steel Members"
Ph.D. Dissertation, Lehigh University, 1956

12. AISC
"Plastic Design in Steel"
American Institute of Steel Construction, New York, 1959

13. Kusuda, T.
"Buckling of Stiffened Panels in Elastic and Strain-Hardening Range"
Ph.D. Dissertation, Lehigh University, 1958

14. Gerard, G. and Becker, H.
"Handbook of Structural Stability, Part III: Buckling of Curved Plates and Shells"
NACA, Technical Note 3783, 1957

15. Thürlimann, B.
"Strength of Plate Girders"
National Engineering Conference, 1958 Proc. AISC, 1958

16. Basler, K. and Thürlimann, B.
"Plate Girder Research"
National Engineering Conference, 1959 Proc. AISC, 1959

17. Basler, K. and Thürlimann, B.
"Buckling Tests on Plate Girders"
6th Congress, International Association of Bridge and Structural Engineering 1960, Preliminary Report (in print)

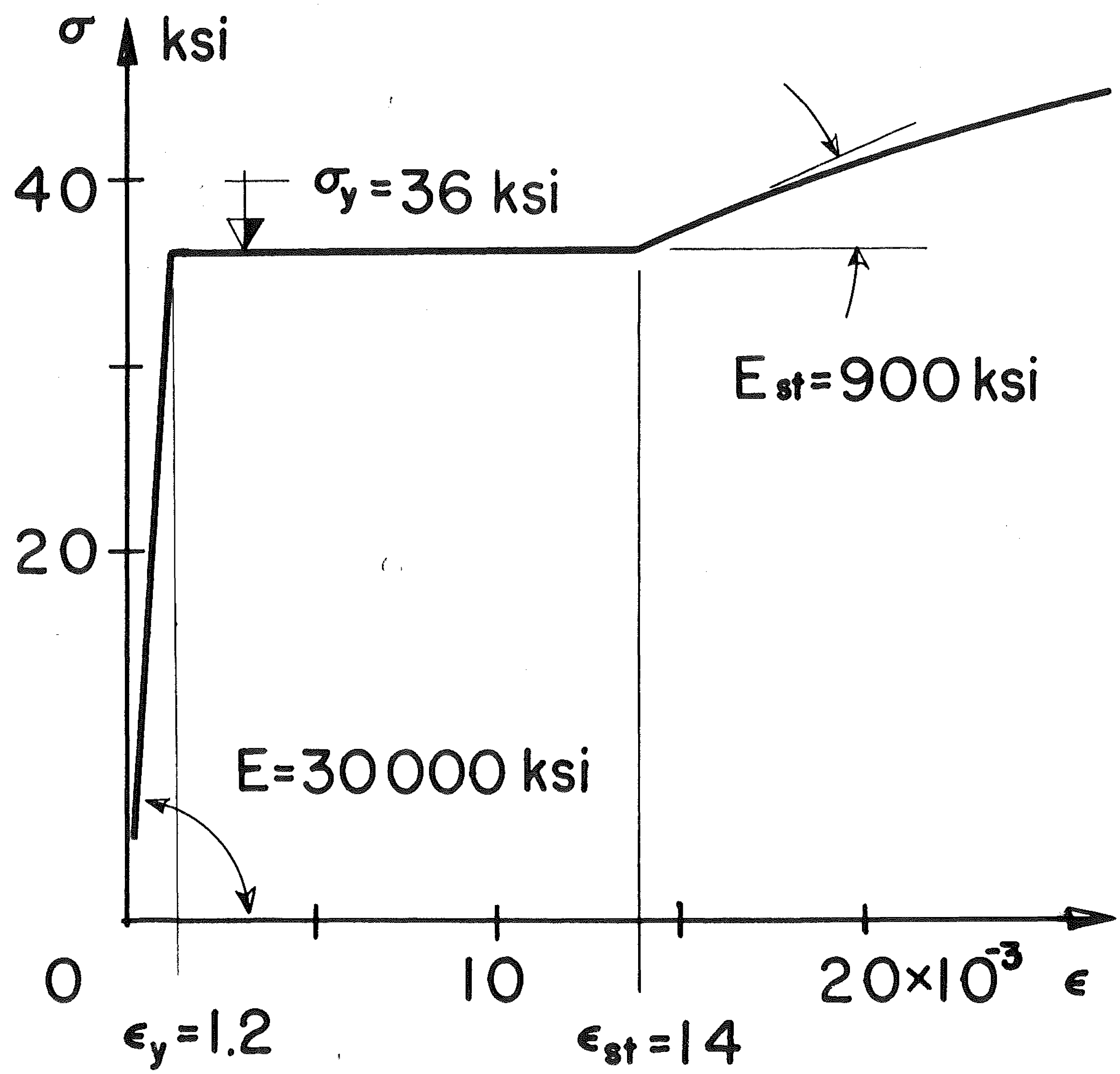


FIG. 1 STRESS-STRAIN CURVE FROM COUPON OF STRUCTURAL STEEL (ASTM-A7)

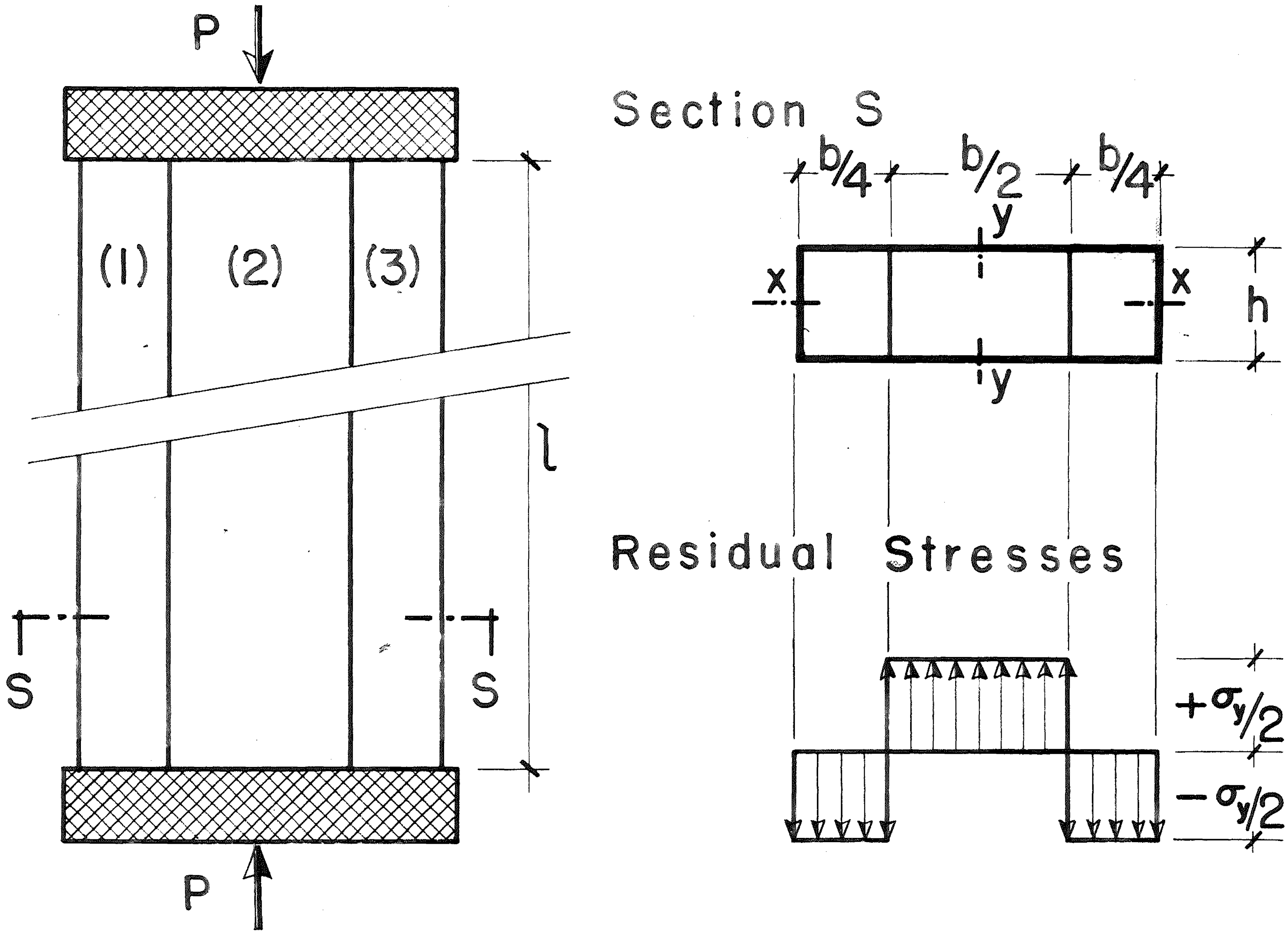


FIG. 2 MODEL COLUMN WITH RESIDUAL STRESSES

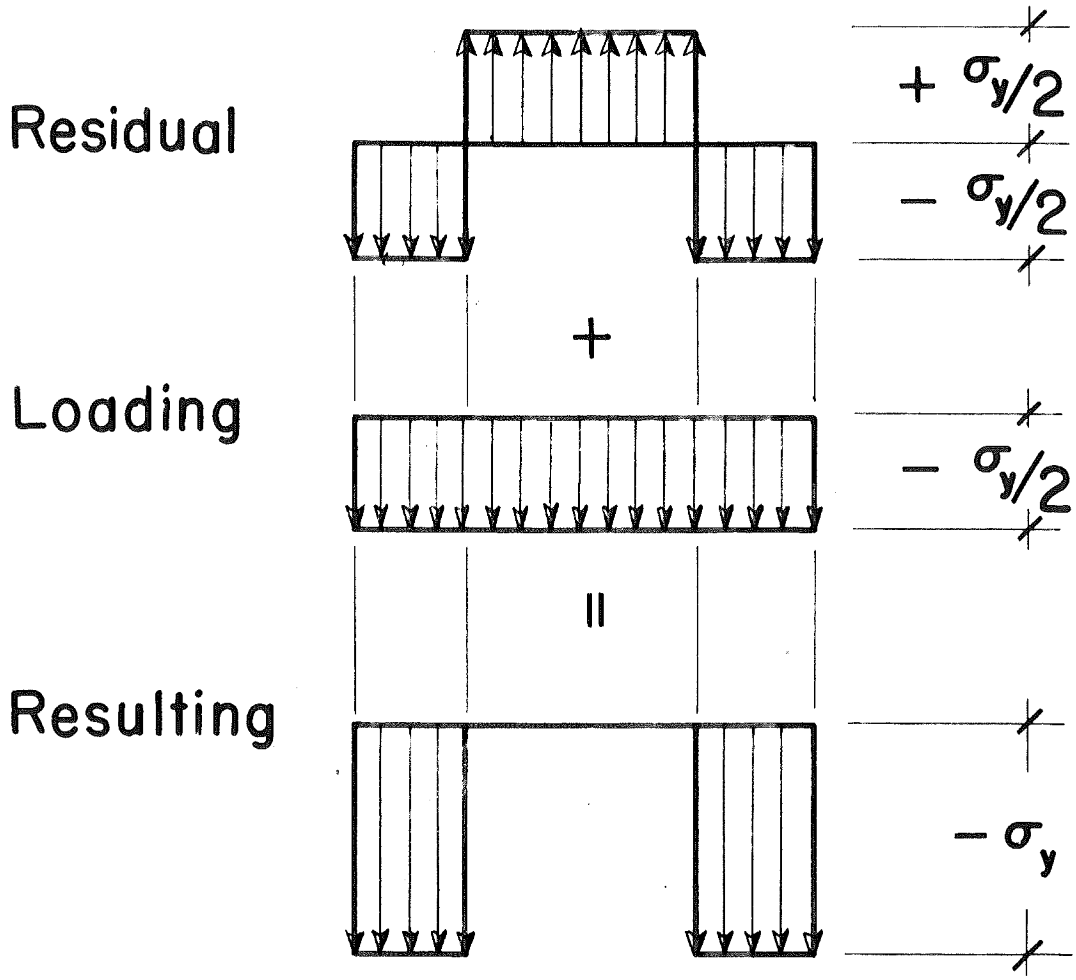


FIG. 3 SUPERPOSITION OF RESIDUAL AND LOADING STRESSES

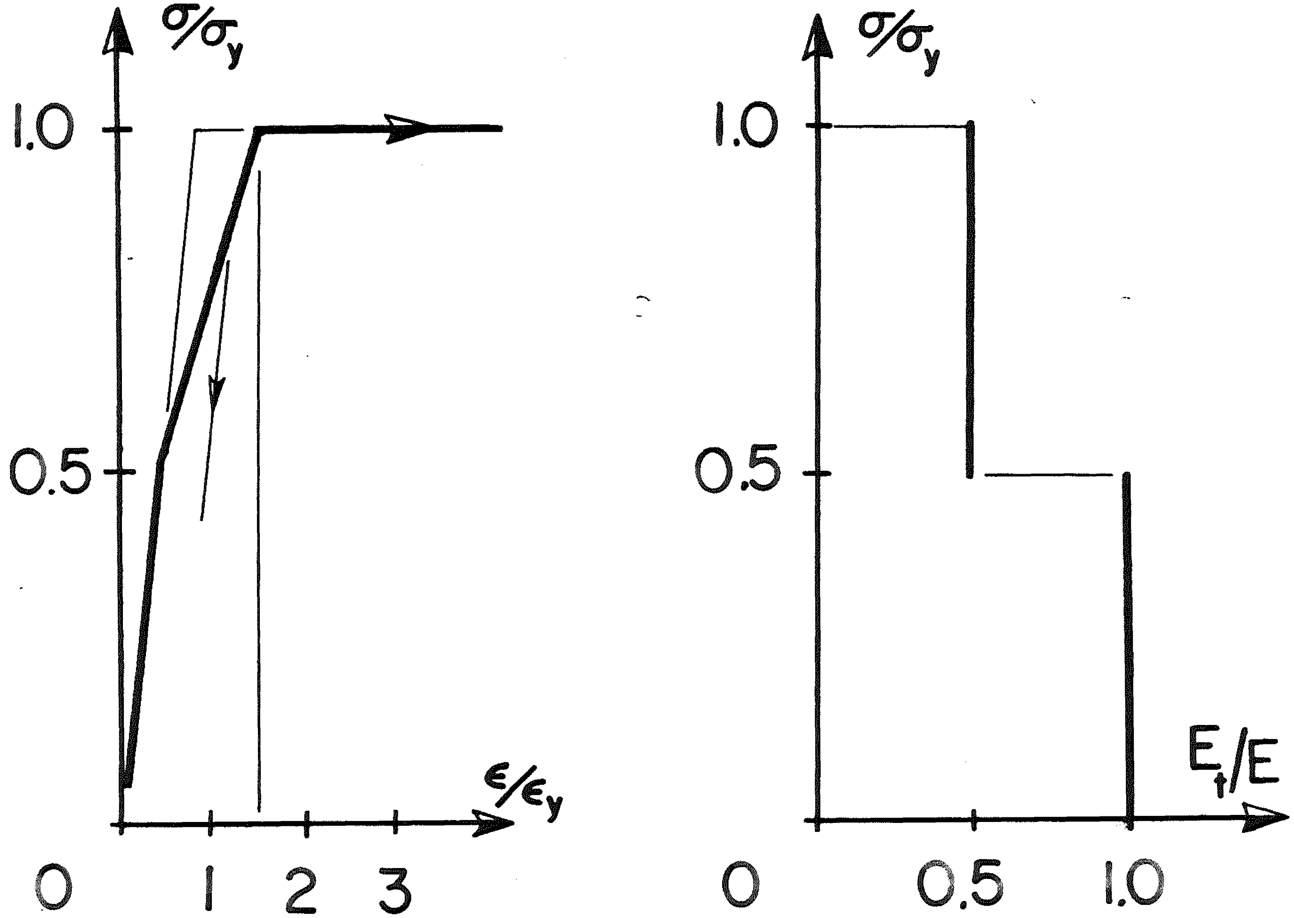


FIG. 4 AVERAGE STRESS-STRAIN CURVE FOR MODEL COLUMN; CORRESPONDING TANGENT MODULUS E_t

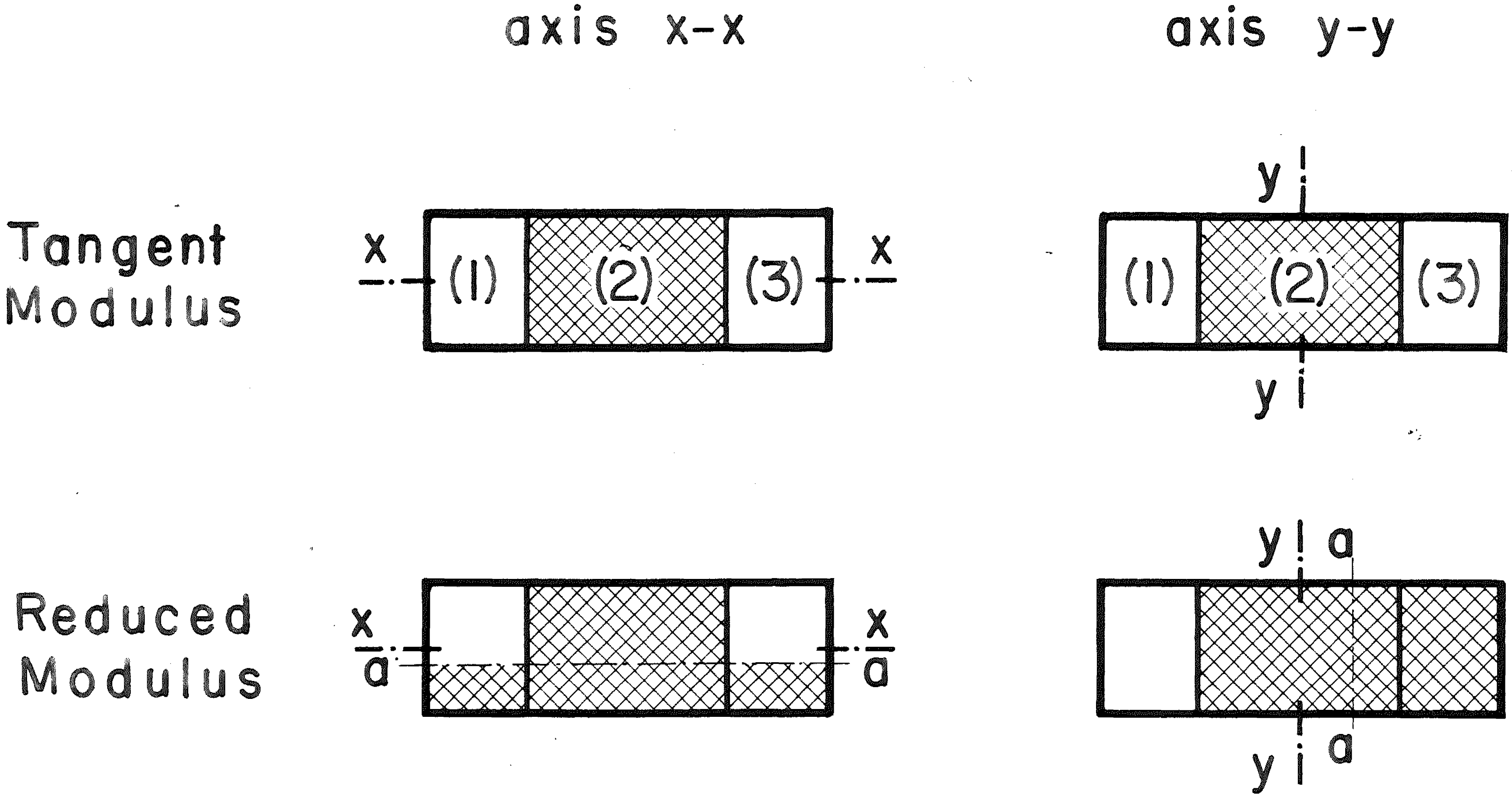


FIG. 5 EFFECTIVE CROSS SECTIONS OF MODEL COLUMN
AFTER INITIATION OF YIELDING

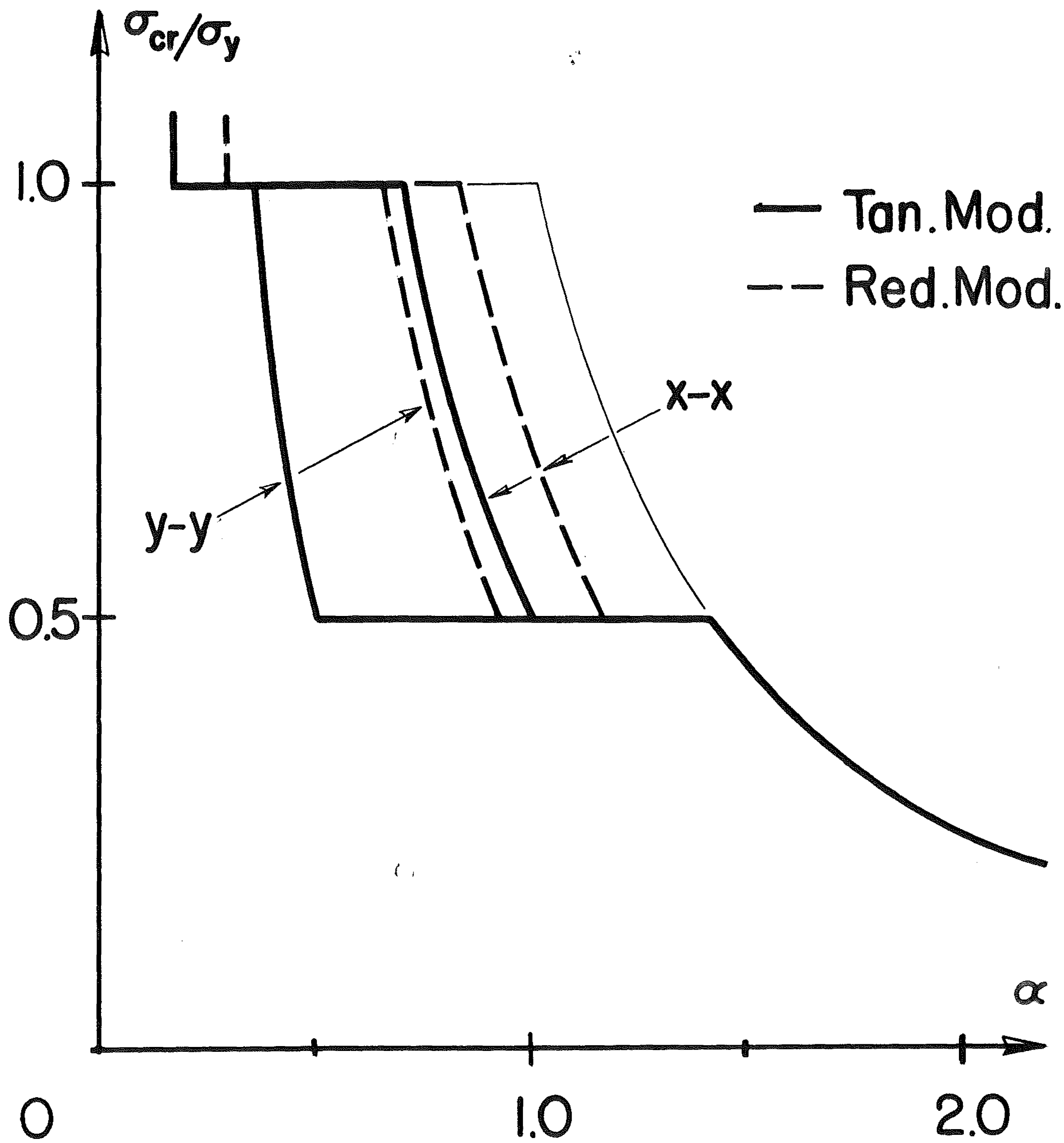


FIG. 6 COLUMN CURVES FOR MODEL COLUMN

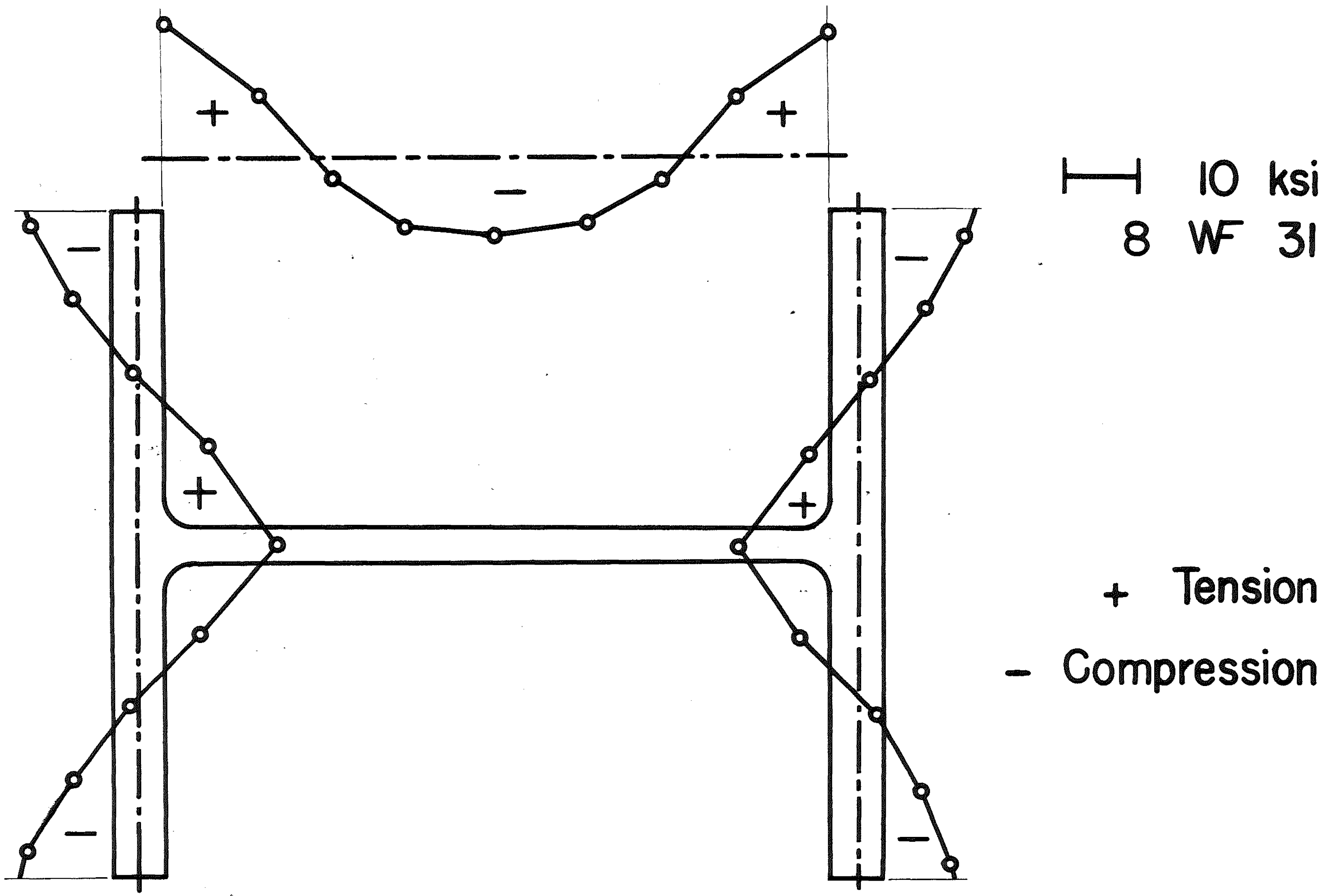


FIG. 7 MEASURED AXIAL RESIDUAL STRESSES IN "AS-ROLLED" 8WF31 SECTION

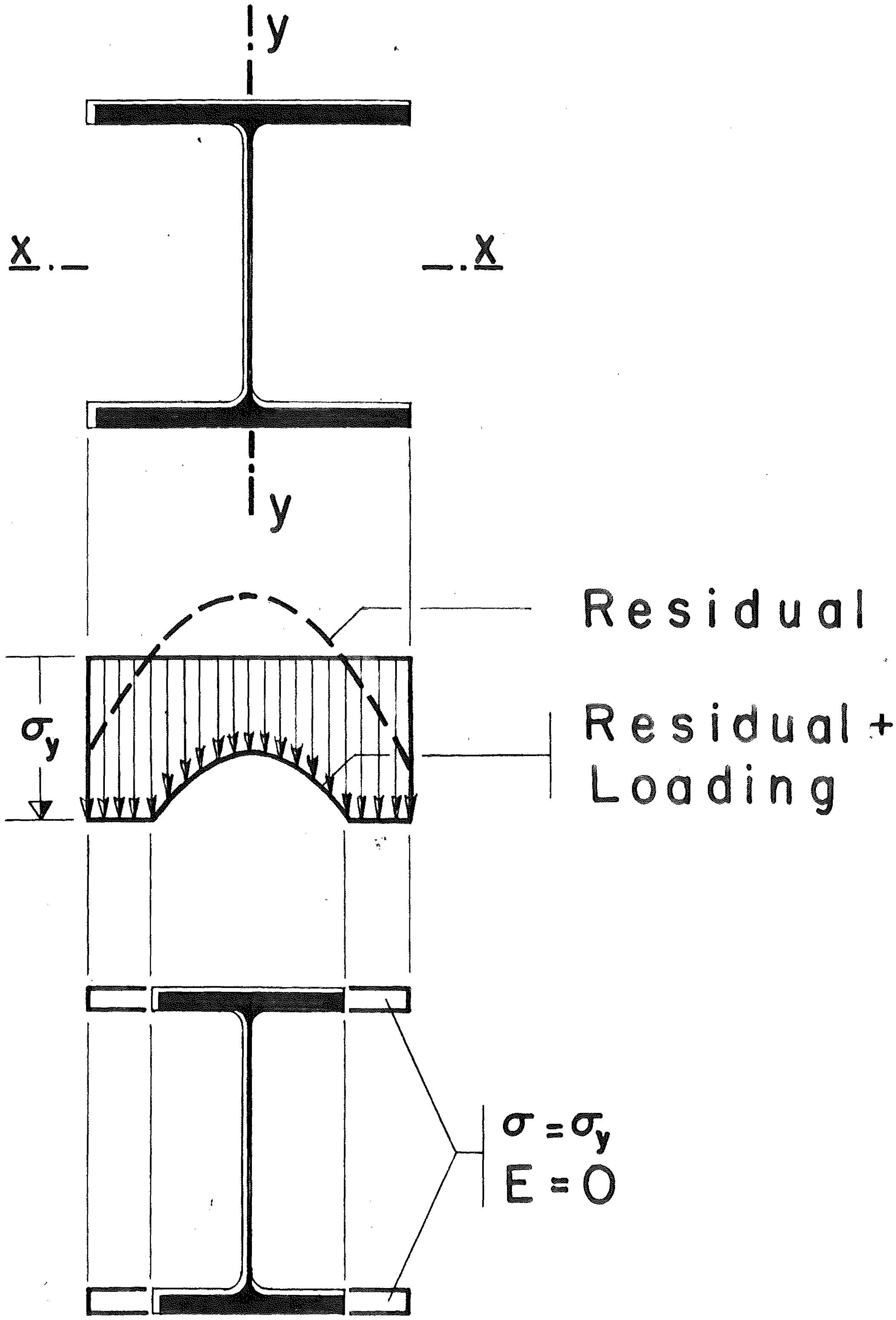


FIG. 8 EFFECTIVE CROSS SECTION OF WF SECTION AFTER INITIATION OF YIELDING

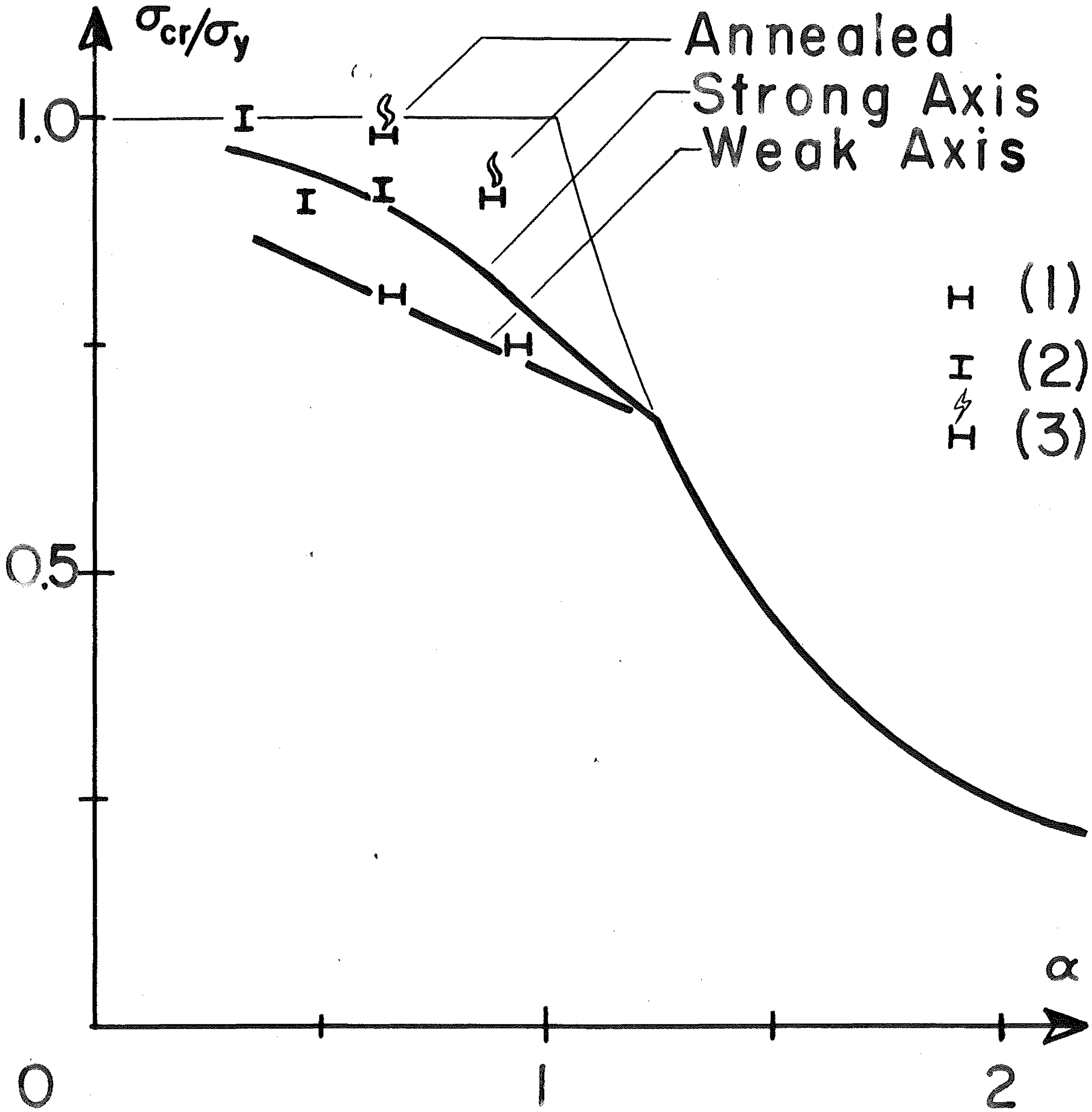


FIG. 9 BUCKLING TESTS OF 8WF31 COLUMNS COMPARED WITH THEORETICAL PREDICTIONS

Legend: See Fig. 10

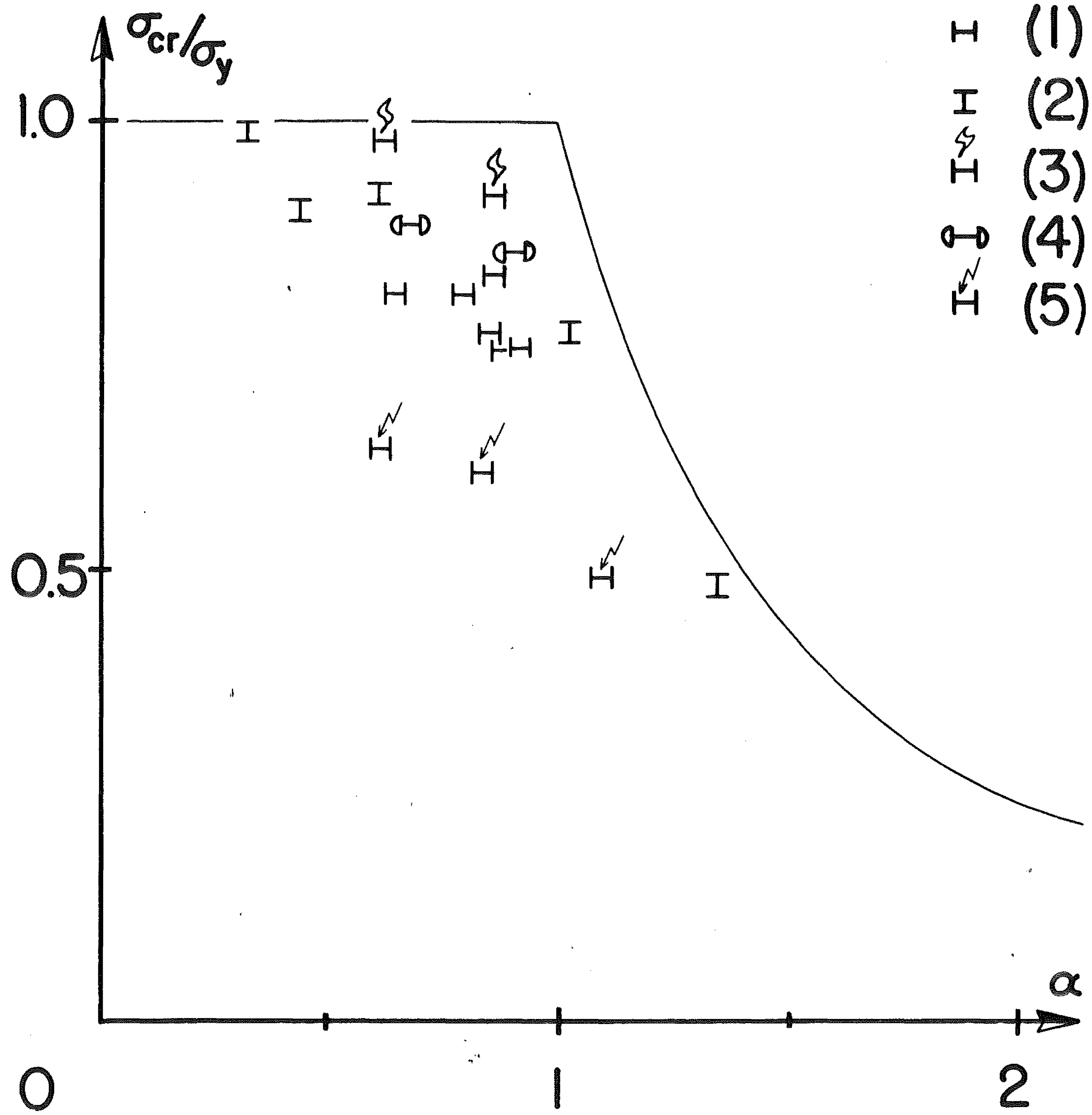


FIG. 10 SUMMARY OF BUCKLING TESTS ON I-SHAPED COLUMNS SHOWING SCATTER CAUSED BY RESIDUAL STRESSES

Legend (1) WF Shape, as-rolled, weak axis buckling
Legend (2) WF Shape, as-rolled, strong axis buckling
Legend (3) WF Shape, annealed, weak axis buckling
Legend (4) Riveted I-Section, weak axis buckling
Legend (5) Welded I-Section, weak axis buckling

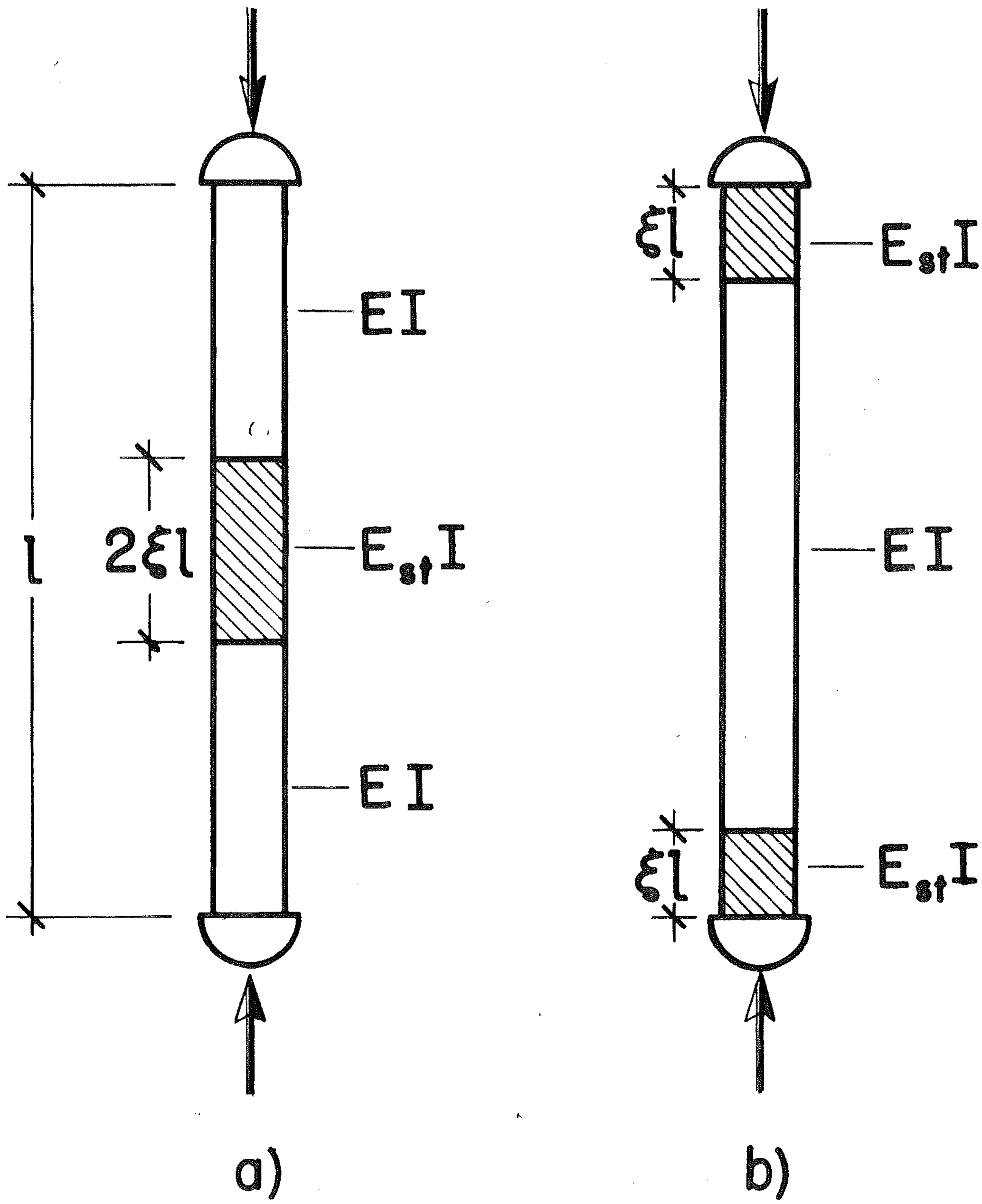


FIG. 11 PARTIALLY YIELDED COLUMN, YIELDING SPREADING FROM (a) THE MIDDLE AND (b) BOTH ENDS

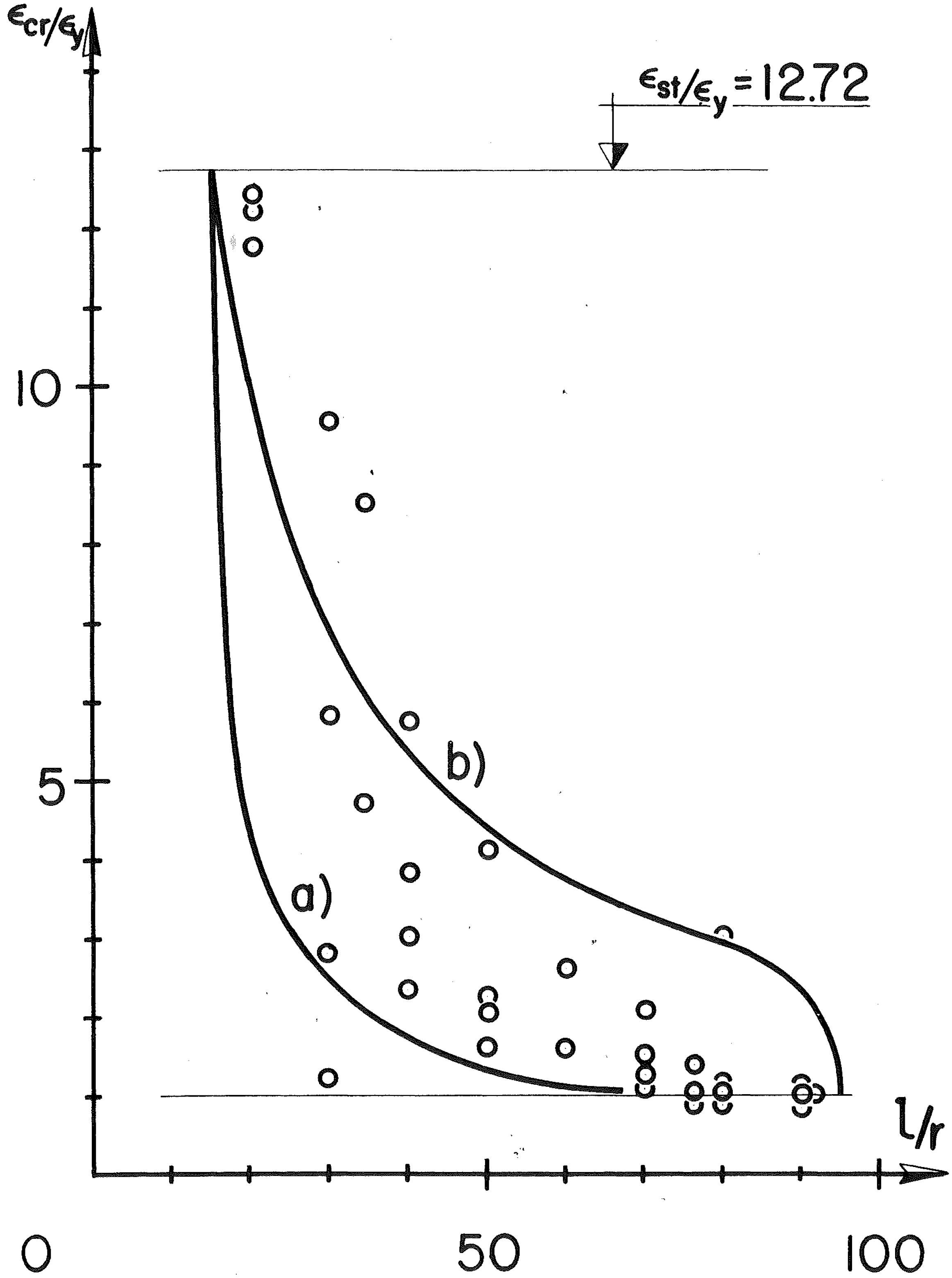


FIG. 12 AVERAGE AXIAL STRAIN OF COLUMNS AT FAILURE COMPARED WITH PREDICTIONS; (a) YIELDING FROM MIDDLE, (b) YIELDING FROM ENDS

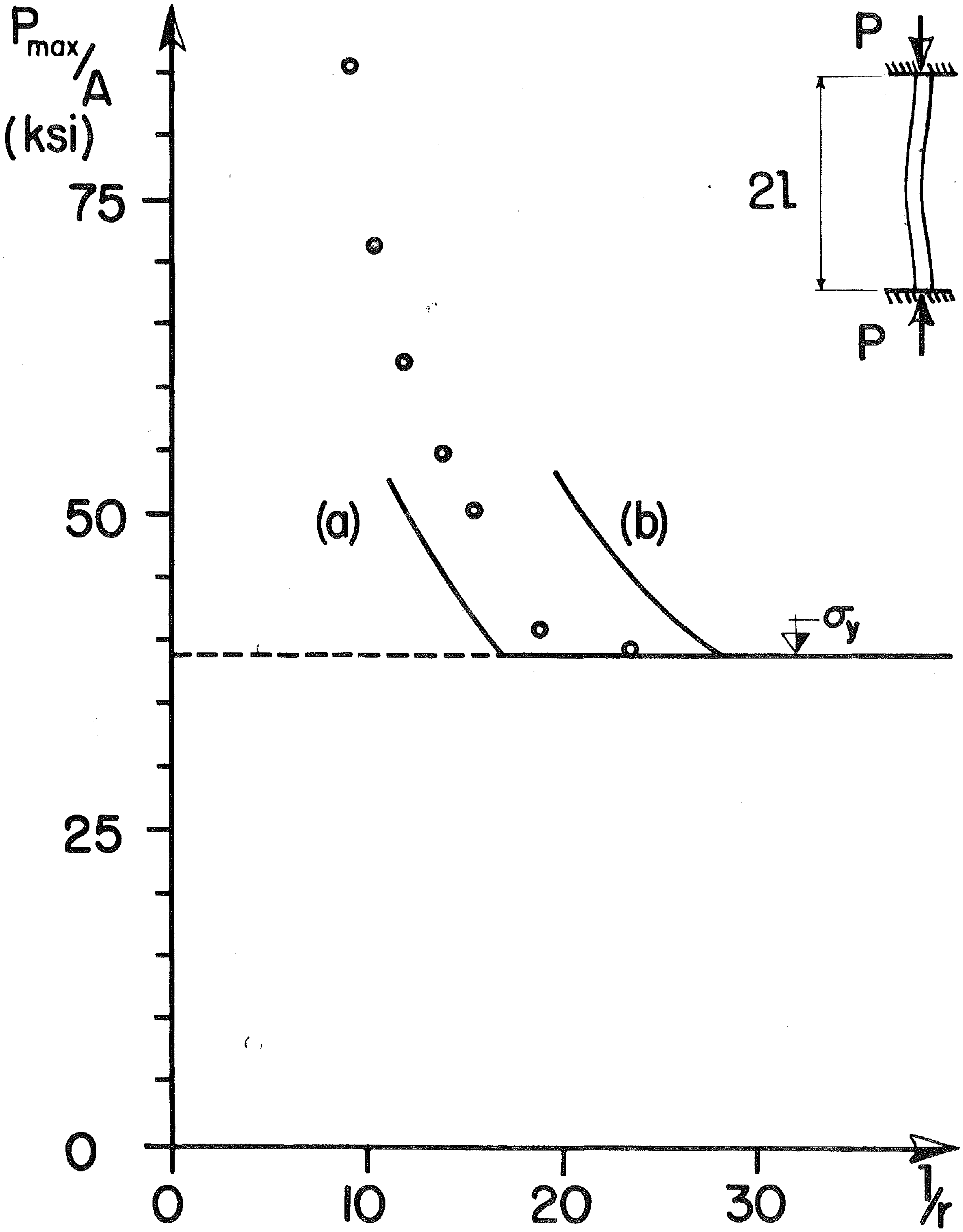
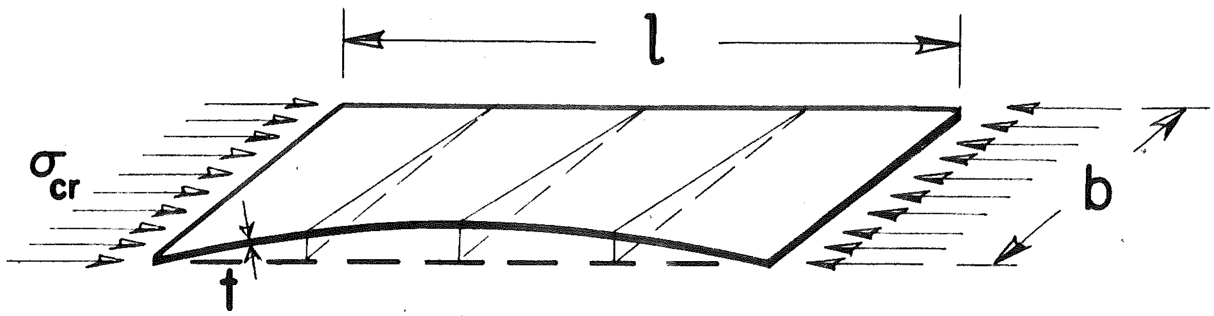


FIG. 13 FAILURE LOAD OF SHORT COLUMNS COMPARED TO PREDICTIONS; (a) TANGENT MODULUS, (b) REDUCED MODULUS THEORY



EXAMPLES

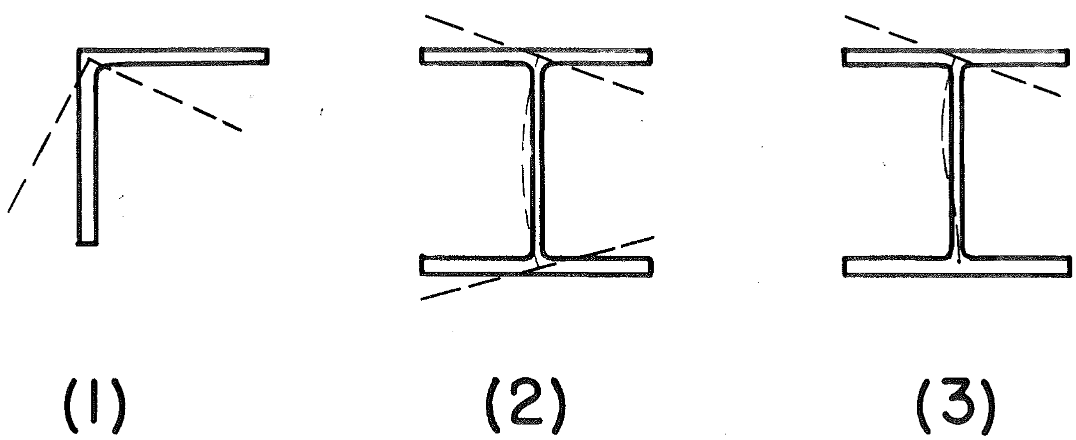


FIG. 14 BUCKLING OF PLATE ELEMENTS; (1) ANGLE IN COMPRESSION, (2) I-SECTION IN COMPRESSION, (3) I-SECTION IN BENDING

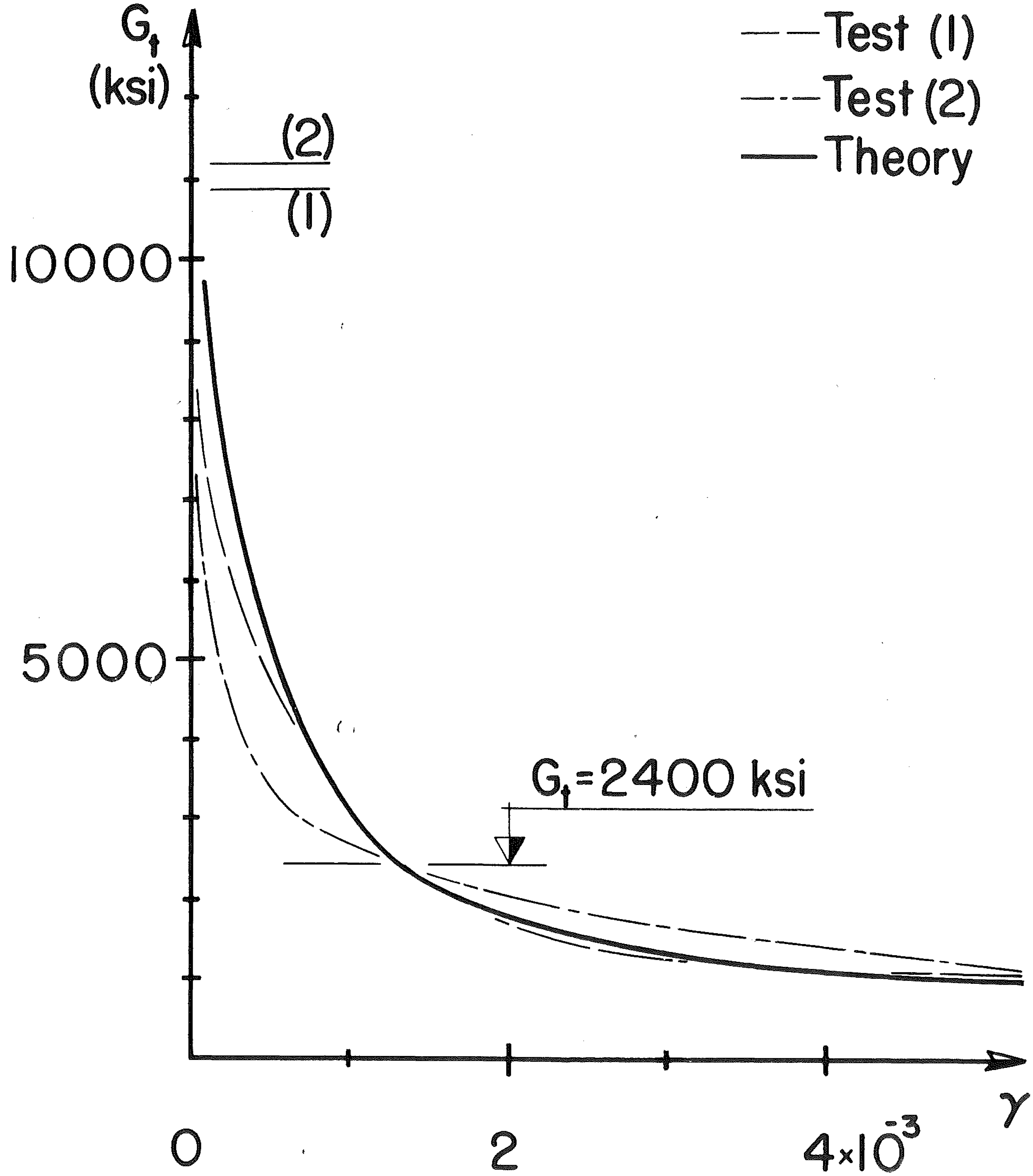


FIG. 15 SHEARING MODULUS G_t AT ONSET OF STRAIN-HARDENING VS SHEARING STRAIN

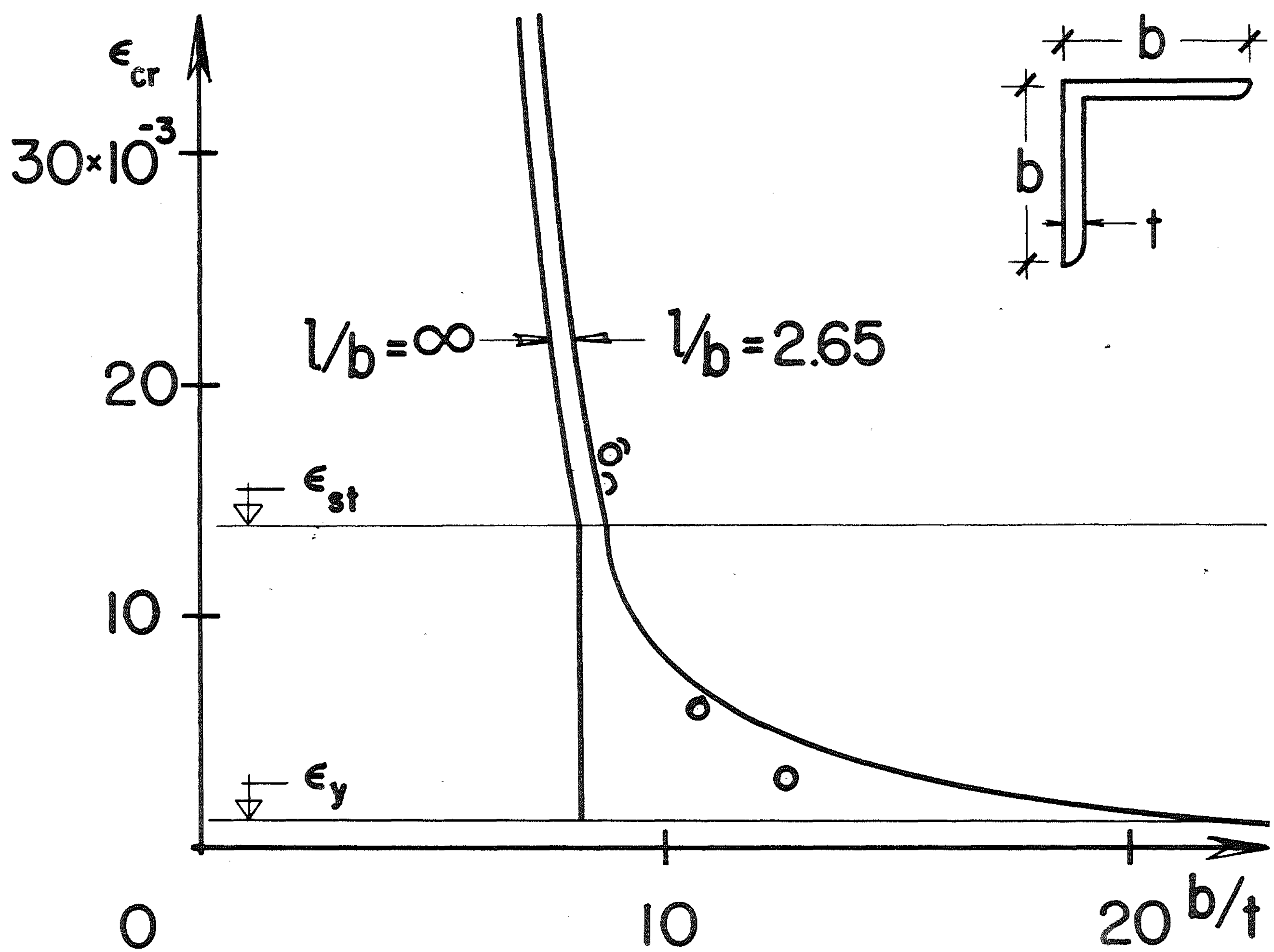


FIG. 16 STRAIN AT BUCKLING OF COMPRESSED ANGLES VS WIDTH/THICKNESS RATIO, TESTS AND THEORY

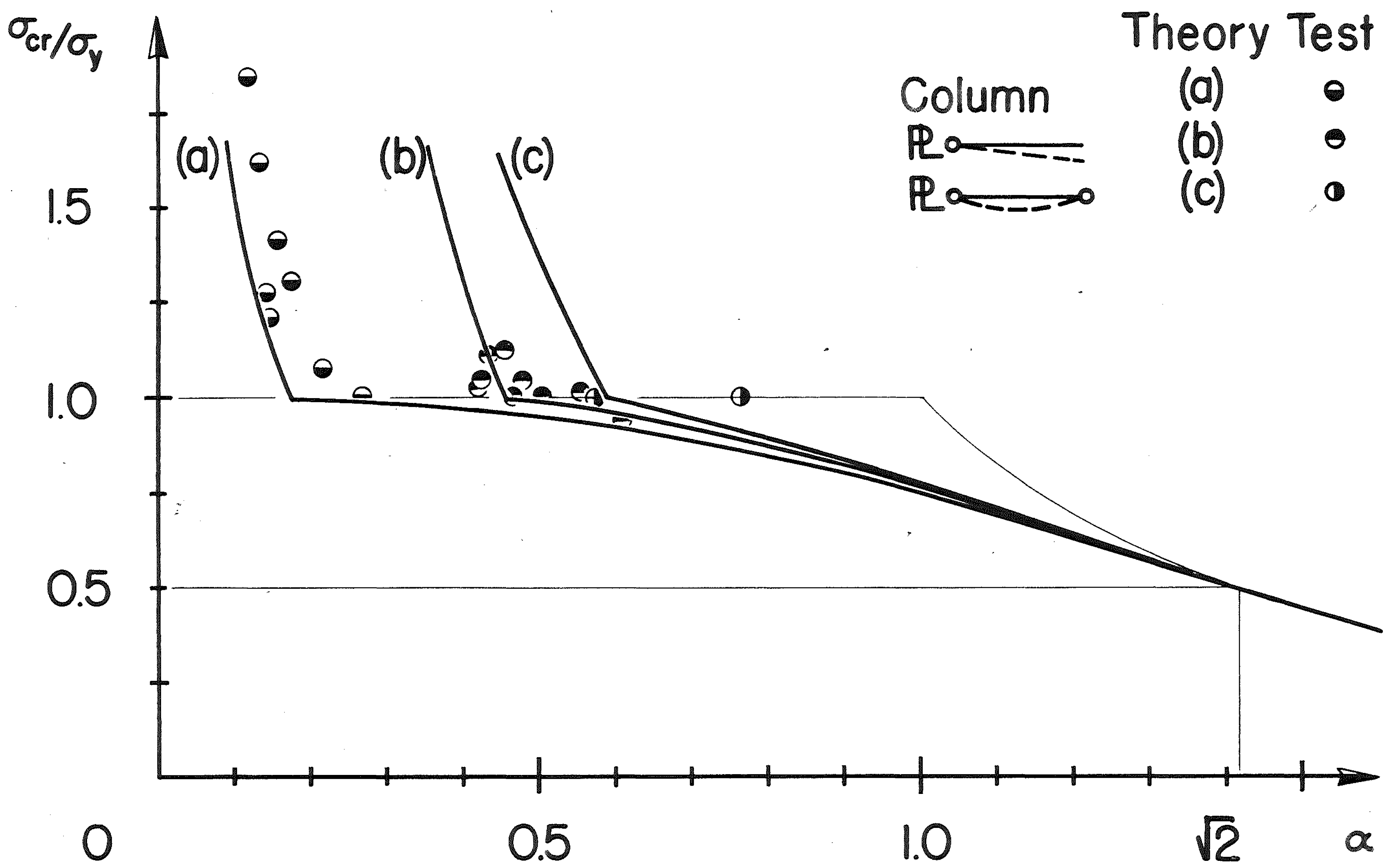


FIG. 17 SUMMARY GRAPH ON INELASTIC BUCKLING OF COLUMNS AND PLATES WITH TEST RESULTS

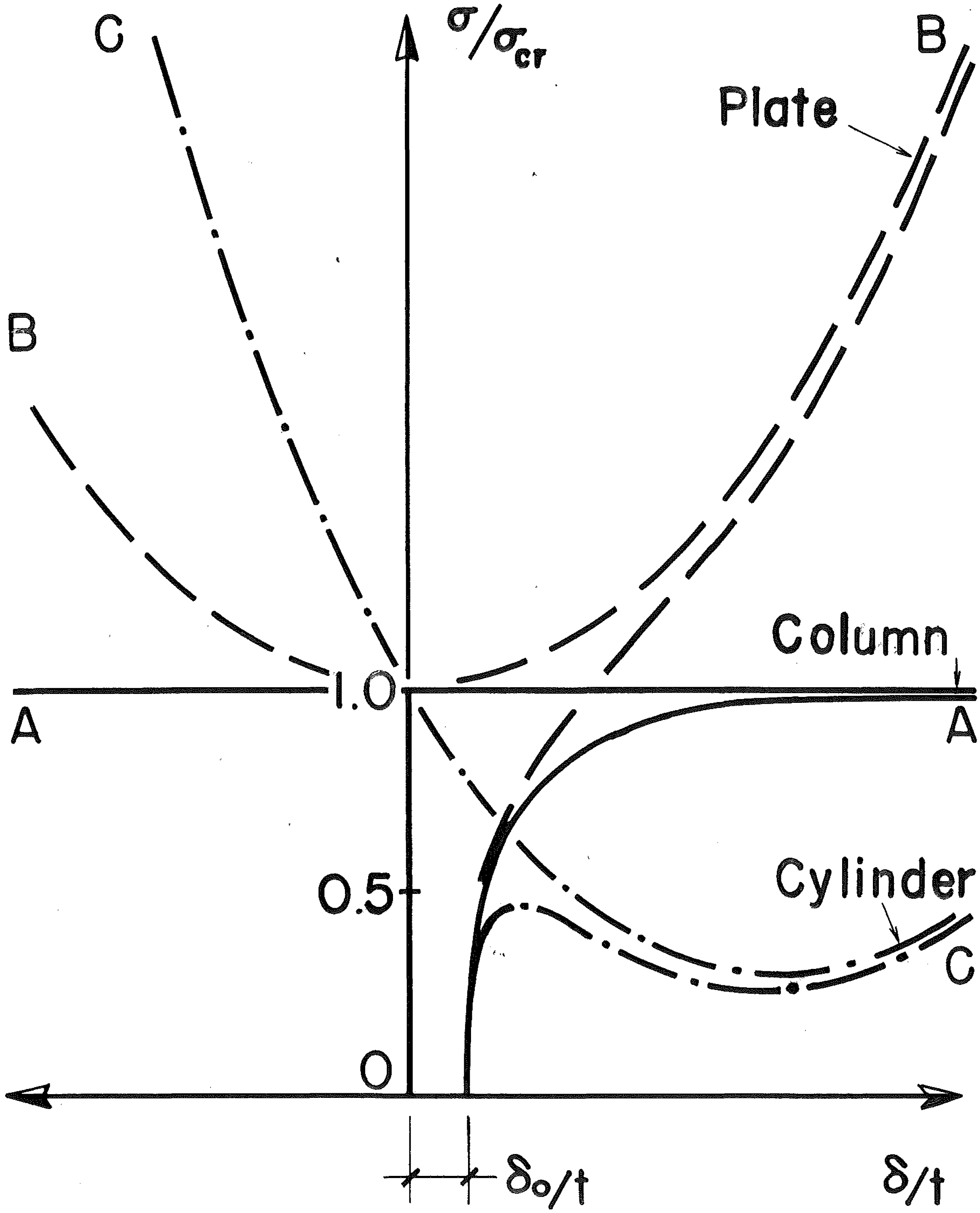


FIG. 18 SCHEMATIC ELASTIC BEHAVIOR OF PERFECT ($\delta_o/t=0$) AND IMPERFECT (δ_o/t) COLUMN, PLATE AND CYLINDER IN COMPRESSION