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Welded Continuous Frames and Their Components

INTERIM REPORT NO. 30

THE PLASTIC BEHAVIOR OF STRUCTURAL MEMBERS AND FRAMES

by

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THE PLASTIC BEHAVIOR OF STRUCTURAL MEMBERS AND FRAMES

(A Summary Report of Demonstration Tests conducted during the Summer Course, "Plastic Design in Structural Steel", sponsored by Lehigh University and the American Institute of Steel Construction)

by

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CONTENTS

1. INTRODUCTION

In September 1955, a Summer Course in PLASTIC DESIGN IN STRUCTURAL STEEL was given at Lehigh University, sponsored by the American Institute of Steel Construction in cooperation with the American Society for Engineering Education and the Structural and Engineering Mechanics divisions of the American Society of Civil Engineers. This course was offered to educators and structural engineers in order to present a new design concept based upon the "ultimate" strength of structural steel as revealed by analysis of its plastic rather than elastic behavior under load.

A comprehensive series of lectures covering this concept was presented to the group(1). To supplement the lectures, a series of tests was conducted to illustrate the principles of plastic behavior of structures. The general plan of the tests is given in Table 1 together with an indication of the principle demonstrated by each.

First, a tensile coupon test was performed to illustrate the manner in which the mechanical properties of steel are determined and to show the stress-strain characteristics of structural steel.

Then, the two basic principles of plastic analysis, plastification of cross section and redistribution of moment, were shown in a pair of beam tests. A test of a simply-supported control beam (T-1) demonstrated the additional reserve load capacity due to plastification of cross section and the ability of a beam to form a plastic hinge. A test of an indeterminate beam, continuous over two spans (T-2), showed the effect of redistribution of moment in adding to the carrying capacity of a member.

Further tests were made to point out certain modifications which must be considered in plastic design.

Plastic lateral buckling was illustrated by a test on a simple beam (T-3) loaded with two concentrated loads such that a section of constant maximum moment occurred over a length purposely made long enough for buckling to be critical. An additional test made on an identical beam (T-5) (with the exception that only a single center concentrated load was applied) showed the effect of a moment gradient in reducing the tendency toward plastic lateral buckling.

Three beam-column tests had as one of their objectives the demonstration of the reduction of plastic moment due to axial load. The test specimens here were made short enough so that buckling would not be a factor.

The first of these tests was a direct axial load test (T-6) on a short column made prior to the summer course to obtain the compressive properties of the section and show its load-carrying capacity in the absence of bending moment. The second test, also made prior to the summer course, was a column (T-7) loaded with small eccentricity to show the effect of high axial load on plastic bending, The demonstration specimen (T-8) was loaded with a direct load at very large eccentricity to produce a case with larger bending moment and smaller axial load.

These three tests, plus the test of the control beam taken from the same material, made it possible to show how the presence of direct stress affects the ability of a structural member to develop the plastic moment.

The test of a welded corner connection was performed to demonstrate the ability of this important structural component to behave as assumed in the theory. A square connection (T-4) of the type which might be found in a flat-roofed building frame, was fabricated from 30-inch wide-flange shapes. In loading the connection in such a manner as to cause closing of the corner, the ability of a large size rolled section to develop a plastic hinge under combined bending and axial load was shown.

The final demonstration was a full-scale test of a 40-foot span gabled portal frame (T-9) subjected to vertical roof loads and horizontal loads simulating the action of wind. In this test there was opportunity to observe most of the major points emphasized in the lectures, including formation of plastic hinges, redistribution of moment, effect of axial forces, magnitude of lateral support forces, effect of lateral buckling, appearance of the structure at working, yield and ultimate loads, and accuracy of calculation of important results.

A phase of the demonstrations which did not include large-scale structural testing was the use of models in plastic analysis. By applying small weights to a model having spring-loaded hinges at all critical joints, it was possible to determine the failure mechanism of a structure.

2. DESCRIPTION OF TESTS

The general plan of the demonstration tests is given in Table 1 which lists each test, shows a schematic diagram of the test and indicates the principle illustrated by each. In the following sections, a detailed description of each test is given. The results of the ~~tests are discussed in S~~ection 3.

2.1 Tensile Coupons

Three different wide-flange rolled shapes were used in the test specimens. These were the 12WF36, 14WF38, and 30WF108 sections. The cross-sectional dimensions of each were measured and recorded for comparison with handbook values and for use in calculating predicted loads. A summary of these values is shown in Table 2.

To obtain the mechanical properties of the steel used, tensile coupons from each section were tested. The specimens were machined from the web and flanges of the rolled sections in the shape ordinarily used for plate specimens.

The specimens were tested with flat wedge grips in a 60,000-lb. capacity hydraulic universal testing machine. Load and elongation over an 8-inch gage length were measured and plotted by means of a low-magnification automatic stress-strain recorder. This instrument could record strains well into strain hardening without resetting, but the strain readings were not precise enough to permit calculation of the modulus of elasticity.

The rate of application of load was about 30 micro inches per inch per second in the plastic range, a rate much lower than the usual standard mill test rate. This reduced rate of loading was used because the results were to be used to predict values for static tests where equilibrium of load and deformation would be obtained at each load increment before readings would be taken.

A unique feature of these tests was the taking of "static yield load" readings in the yield range. After the yield region had been reached, but before strain hardening had commenced, the strain rate was reduced to zero for a period of a few minutes to allow the load to reach an equilibrium point. The technique used to accomplish this was to mount a dial gage between the fixed and the moving cross heads of the testing machine. The loading value was closed until the increase in deformation and the decrease in load both approached zero. "Static" readings were then taken. From this reading the lowest possible yield stress could be calculated, thus insuring that in the actual test structure the yield stress would be equal to this or greater. In Figs. 1 to 4 the yield levels at different strain rates as well as at zero strain rate are indicated.

All other usual data such as ultimate load, per cent elongation, and reduction of area, were taken.

2.2 Control Beam Test (T-1)

Since several of the demonstration structures were to be fabricated from 12WF36 sections, a length of this section was tested as a control beam to obtain the relationship between moment and curvature under pure bending. The reserve of moment above the yield point due to plastic yield of the cross section was the other important phenomenon demonstrated by this test.

The specimen tested was a 13-foot length of 12WF36 with vertical stiffeners welded at support and load points to prevent web crippling (Fig. 5). A 300,000-lb. hydraulic universal testing machine was used to apply load to the specimen supported on rollers at a 12-foot span. A heavy 14-inch wide-flange section was used as a base beam to carry the reactions of the specimen to the testing machine table. Loads were applied at third points of the span using a spreader beam to distribute the testing machine load to two rollers.

Instrumentation used to measure the important deformations consisted of an automatic plotting board*, plotting load versus center deflection; a rotation indicator, measuring rotation in the maximum moment section; and a dial indicator, reading center deflection.

Load was applied continuously at a moderate rate which allowed gage readings to be taken and the data plotted without interrupting the loading. After the plastic range had been reached, it was necessary to halt the application of load twice to allow dial indicators to be reset. At these points a short time was taken to allow the load and strains to reach equilibrium before the resets were made.

Loading was continued until the ultimate load was reached, followed by a gradual drop in load when deformations had become so great as to cause lateral instability.

2.3 Continuous Beam Test (T-2)

A two-span continuous beam, statically indeterminate to the first degree, was tested to show the increase in ultimate load above that at first yield, an increase achieved as a result of redistribution of moment combined with plastification of cross section.

The beam was symmetrically loaded at the third-points of two 10-foot spans (Fig. 10). The center support was a fixed rocker and the two end supports were expansion rockers. Two 100-ton hydraulic bridge jacks were used for loading. Each was supported on a spreader beam which distributed the load to two points by means of rollers. Each jack reacted

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* Certain instruments used on the different tests are described in detail in Appendix.

against a frame consisting of a cross beam and two 3-inch diameter steel rods anchored to the test floor of the laboratory. SR-4 gages attached to each of the four tie rods converted them to dynamometers, allowing the loads to be read to greater accuracy and sensitivity than permitted by the pressure gage of the jacks.

Lateral support was provided for the compression flange of the beam at the load-points by means of 6-in. light beam sections welded to the web stiffeners. See Fig. 14. Lateral reaction was furnished by a beam fastened to the wall of the laboratory. The magnitude of the lateral reaction for each beam was measured by a flexible aluminum dynamometer which also allowed vertical deflection of the lateral support beams without giving restraint to the vertical deflection of the beam specimen. Lateral support of the compression flange of the beam at the support points was provided by friction on the supports.

Vertical deflection was measured at one of the outer load-points, and rotations were measured across the plastic hinges at the center support and one of the outer load-points.

Load was applied by hand-pumping of the hydraulic jacks and stopped momentarily to take readings of the instruments. Loading was continued until the maximum had been reached and started to drop off, after which the test was discontinued since further deformation served no useful purpose.

2.4 Lateral Buckling Beam (T-3)

In order to show the effect of lateral buckling on a beam loaded into the plastic range, a 14WF38 beam was tested much the same as the control beam, with the exception that the unsupported length was made sufficiently long so that plastic lateral buckling should occur prior to the onset of strain-hardening.

The simply-supported span was 15 feet and the loads were centered 6 feet apart (inset, Fig. 19). The unsupported length was short enough so that buckling would not occur in the elastic range, but long enough so it would occur once the plastic moment had been reached.

Instrumentation included a center deflection gage, rotation indicator, and the plotting board - the same as was used in the control beam test. In addition, a twist indicator was added to aid in the detection of lateral buckling.

The test was conducted in the same manner as the control beam test, with load being applied continuously at a rate slow enough to allow readings of all the instruments to be taken at intervals until the test was completed. Although the load dropped off gradually after reaching its maximum value, straining was continued until the center deflection was four times the value at the maximum load.

2.5 Centrally Loaded Beam (T-5)

A beam identical to the one used to show the effects of lateral buckling was loaded at its center-point to demonstrate the way a steep moment gradient reduces the tendency toward lateral buckling.

This 16-foot length of 14WF38 section was also simply-supported on a 15-foot span, but the only load was a single concentrated load applied through a roller at center span (Fig. 23). The altered loading pattern actually increased the laterally unsupported length by 18 inches.

The instrumentation on this test comprised a center deflection gage, rotation indicator, and a twist indicator.

The load was again applied continuously with instrument readings being taken without stopping the test. In this case, the load did not level off after the plastic hinge had been formed, but continued to increase somewhat due to strain-hardening. Loading was continued through to strain-hardening region, the maximum being about 15 per cent greater than the predicted ultimate load. This occurred at about 6 times the yield deflection. Further deformation by the machine caused a drop in load (due to lateral buckling) until it equalled the predicted maximum load at about 10 times the yield deflection. The test was discontinued at this point in fear that a complete loss of stability might endanger spectators or equipment.

2.6 Cross-Section Test (T-6)

A cross section test was made to find the compressive stress-strain curve of a full cross section of the 12WF36 material used for many of the test specimens. This was an axial compression test which gave the integrated effect of different web and flange strengths in one test. The main purpose of the test was to obtain a compressive yield value to use in predictions for eccentrically loaded compression tests.

The specimen was a 3-foot length of rolled section with both ends milled parallel in planes perpendicular to the centroidal axis. The short length was used to keep the L/r ratio below 25, reducing the tendency toward lateral buckling.

An 800,000-lb. screw-type universal testing machine was used for loading. Adjustable bearing blocks were used to furnish flat bearing for the ends of the specimen (Fig. 27). Several SR-4 strain gages and two dial indicators, fastened to lugs welded on 20-inch gage lengths, were provided for the measurement of strains. Prior to the stress-strain test, the specimen was aligned to insure concentric loading.

In the stress-strain test of the cross section, load was applied in selected increments until yielding occurred. Thereafter further straining was controlled by limiting the magnitude of strain increments. Loading and straining were halted long enough to take readings of the several measurements at each of these increments. Loading of the specimen was continued past the ultimate load until buckling of web and flanges caused the load to show a definite downward trend.

2.7 Eccentrically Loaded - Beam-Column Tests (T-7, T-8)

Two compression members were tested with the loads applied at eccentricities such that, in effect, the tests were bending tests with high axial loads. Through the results of these tests, it was intended to show the effect of axial load on the plastic moment.

The specimens used were the same size as the cross section specimen previously tested, being 3-foot lengths of 12WF36. Since a component of tensile force was to be expected in a bending test, it was necessary to weld end plates to the specimens. The distinguishing feature between the two tests was that T-7 was to have an eccentricity of 1.75 inches and T-8 an eccentricity of 5.10 inches. Fig. 31 shows the extent to which axial load should theoretically reduce the hinge moment capacity for each of these specimens.

Special column testing fixtures were used to hold the specimens in position for the proper application of loads (Fig. 28). These fixtures allowed rotation of the ends in a pin-ended condition in the plane of bending, but held the ends fixed against rotation perpendicular to that plane. For each beam-column test, the assembly of specimen and end fixtures was mounted in an 800,000-lb. screw-type universal testing machine.

On each test, the specimen was equipped with a number of SR-4 gages for strain measurements and alignment, a rotation indicator, and dial gages for measuring center deflection. Prior to the start of the ultimate load test, an alignment procedure was carried out to insure that the eccentricity at each end of the specimen was equal.

In the actual test of each specimen, the usual procedure followed in tests to be carried through the plastic range was used. Increments of load were applied and data taken at planned intervals; after yield and occurred, "load" criterion was changed to a "deflection" criterion. Deformation was continued until the maximum load was passed and the specimen was showing definite signs of reduction of load capacity.

2.8 Corner Connection Test (T-4)

In testing a corner connection made from 30WF108 members, the opportunity was presented to observe the behavior of a fair sized fabricated structural component. The manner of testing the connection introduced the effects of axial load with bending moment, bending with moment gradient, and magnitude of lateral support forces into the testing program. The test was similar in type to many which are reported in the earlier Lehigh work.[2]

The specimen tested was a square corner connection of a type which could be used in framing a flat-roofed industrial building. The rolled sections joined were carried a distance of about four times the beam depth outside the knee. Six-inch diameter rollers were welded to the ends of these members to act as bearing surfaces.

Figure 34 shows the position in which the specimen was erected in an 800,000-lb. screw-type universal testing machine. When the specimen was in this position, vertical motion of the testing machine head would apply a force which caused stresses on the knee that could be considered as a resultant of shear, axial force, and bending moment in proportions typical of those found in rigid portal frames. To stabilize the structure against lateral deflection, tie rods hinged to outriggers fastened to the testing machine columns were attached at the inner and outer corners of the knee. (See Fig. 37) Dynamometers for measuring the lateral forces were a part of the tie rods.

In addition to using the plotting board to graph load against deflection across the ends of the legs, instruments were mounted for measuring the rotation of the knee and the forces in the lateral supports.

Testing was done in the fashion just described for the preceding tests. After the connection had been deformed beyond usefulness, it finally buckled laterally and the test was stopped.

2.9 Gabled Portal Frame Test (T-9)

The 40-foot single span gabled portal frame combined a display of all the principles covered in the earlier tests. It was a welded fully-rigid structure fabricated from 12WF36 members with a layout and dimensions as shown in Fig. 40. The column bases were welded to 2-1/2 inch thick end plates and bolted to the test bed floor of the laboratory with two 3-inch studs in each plate.

Lateral support was furnished at points 4, 5, A, 6, 7, 8, B, 9, and 10 shown in Fig. 41, by means of light beams simulating purlins. These purlins were fastened by flexible aluminum bars at their far ends to spreader beams attached to the wall columns of the laboratory.

Hydraulic tension jacks bolted to the test bed floor were used to apply a combination of vertical and horizontal loads through a system of linkages. Four vertical loads and two horizontal loads were programmed to be approximately proportional to the loads which would exist with 60 psf vertical load and 20 psf horizontal load on a building with 18 foot bay lengths.

Various instruments were used to measure the behavior of the structure. A surveyor's transit focused on a 1/100th inch scale suspended from the peak of the gable was used to measure the vertical deflection. The horizontal deflections of the two knees were obtained from 1/100th inch scales measuring the movement of plumb lines suspended from the knees. Rotation indicators at points 1, 4, 6, 8, 10, and 11, (Fig. 41) measured the rotations at these important points. The magnitudes of the applied loads were measured by aluminum tube dynamometers, and the lateral support forces in purlins 4, 5, 7, 9, and 10 were measured by dynamometers on the flex bars. A curve of vertical deflection of the roof peak versus load was plotted by the automatic plotting board in order to keep the audience posted on the progress of the test.

The test was carried on with a squad of pumpers applying and maintaining the loads by hand while gage readers mounted on ladders and scaffolds took data at each load point. (Fig. 49)

An announcer described the test to the group and answered questions from the floor while additional personnel plotted large scale graphs of deflections and rotations as the test progressed. The test was carried to the ultimate load (accompanied by the formation of a mechanism) and continued far beyond this point until a combination of lateral buckling of the frame and buckling of one of the lateral support flex bars indicated that further deformation might endanger personnel. At this point, the load was still at the maximum value and the deflection about twice that at ultimate load.

3. TEST RESULTS AND DISCUSSION

Test results and photographs of the tests are presented in Fig. 1 through 53, and test results are summarized in Fig. 54. The results are grouped by tests in the order in which the tests were performed.

3.1 Tensile Coupons

Stress-strain curves of all tensile specimens tested are shown in Fig. 1 to 3. A more detailed curve of coupon P-2 is shown in Fig. 4, which points out the static yield readings and the yield stress levels at different strain rates. The results of the tests are summarized in Table III.

These coupon stress values, together with the measured cross section dimensions shown in Table II, were used to calculate the theoretical curves shown on each plot of test results.

3.2 Control Beam Test (T-1)

A load-deflection curve for the control beam is shown in Fig. 6, and an M-Ø curve is shown in Fig. 7. These two curves demonstrate that the simple plastic theory is a reliable basis for predicting the behavior of a beam in bending.

3.3 Continuous Beam Test (T-2)

Results of the continuous beam test are shown in Fig. 11, 12, 13, and 14. The load-deflection curve of the beam is shown in Fig. 11. The rotations at the two plastic hinges are plotted in Fig. 12 and 13. In Fig. 14 are shown the forces in the lateral supports at each point of load application.

It may be noted that the agreement between the theoretical curves and the experimental results is much poorer in this test than for the control beam test. This may be attributed not to a shortcoming of the theory, but to lack of application of known theory.

The difference lies in the neglect of the effect of shear on deflection. In the control beam test, the nominal web shear stress at ultimate load was about half the shear yield stress. Therefore the shear deflections would be small, and neglecting them as is customary, causes no serious discrepancy. However, in the case of the continuous beam test, it was necessary to change the spans originally planned to shorter spans because of limitations of the testing equipment. This created a situation in which the shear stresses, even in the elastic range, were sufficient to cause deflections in the same order of magnitude as the flexure stresses. In addition, a maximum principal shearing stress equal to the shear yield stress could be predicted near the center support at a load of about 143 kips, or less than the yield load in flexure. Naturally the reduced shearing rigidity at loads above this point could be expected to result in larger deflections than those calculated by considering bending alone.

It is quite significant, however, that despite the premature shear yielding, the continuous beam eventually managed to carry its predicted ultimate load. (Fig. 11)

Another interesting result obtained in this test is the magnitude of the lateral support forces. In Fig. 14, bar graphs show the lateral support force at each of the load points for two different load conditions typical of the plastic range. Load condition "A" indicates the lateral support forces at the formation of the final mechanism and load condition "B" shows the lateral support forces after the mechanism has deflected six times as much as at "A", while continuing to maintain and even increase the load on the beam. Markings on Fig. 11 show the relative locations of these lateral support readings in the load-deflection history of the beam.

Of importance is the fact that at both readings, the loads in the lateral supports were no greater than a maximum of 1750 lb., indicating that adequate support could be provided by very light members, probably limited in size by slenderness more than by load.

3.4 Lateral Buckling Beam Test (T-3)

Load versus center deflection of the lateral buckling beam is shown in Fig. 19. Moment versus ϕ and lateral twist versus ϕ are shown in Fig. 20.

Comparison of the general shape of these curves with the corresponding curves for the laterally stable control beam shown in Fig. 6 and 7 immediately demonstrates the effect of a long laterally-unsupported span.

In the lateral buckling test, T-3, load began to drop off immediately after the mechanism had formed. This coincided with the rapid increase in lateral twist. In the control beam test, T-1, load remained reasonably constant and even increased slightly after the mechanism had formed. To compare the magnitudes of the relative effects, it will be noted that the load did not drop off in test T-1 until about 10 times the yield Ø, whereas it dropped off at only 2 times yield Ø in test T-3. Though twist was not measured in test T-1, past experience indicates that measurements would have shown that it would not begin to twist rapidly until it neared the larger Ø value which was reached before the drop in load.

3.5 Centrally Loaded Beam Test (T-5)

Fig. 24 presents the load versus center deflection curve of the centrally loaded beam, T-5. Though this beam was identical to specimen T-3 which behaved so poorly after the mechanism had formed, it passed its predicted maximum load and continued to increase in load at a uniform rate. This increase in load was achieved even though specimen T-5 had an even greater unsupported span than T-3.

The factor which produced this added resistance to lateral buckling was the steep moment gradient produced by the altered loading condition. The steep moment gradient allowed strain hardening to occur with much more effect than is possible in a long section of constant moment. The resulting increase in moment is a quantity which can actually be calculated by extending the simple plastic theory to include strain hardening.

3.6 Cross Section Test (T-6)

In Fig. 29 is shown the stress-strain curve of specimen T-6, the cross-section specimen. The yield stress level from this curve served as the level to be used in calculating axial loads for the beam-column tests which followed.

The result of the cross section compression test was slightly lower than that predicted (about 5%), using the weighted average of two web and two flange tensile coupons.

The maximum load in this test is used to signify a P/P_y value of 1 in the interaction curve shown in Fig. 31.

3.7 Beam-Column Tests (T-7, T-8)

Figure 30 compares the moment versus $\emptyset$ curves for the three bending members of 12WF36 section which were tested with different amounts of axial load.

Here may be seen the effect of axial load in reducing plastic bending moment. For the case of test T-1, the axial load was zero. For the other cases where the member was loaded as an eccentrically loaded column, the larger the eccentricity, the smaller the axial load accompanying formation of a plastic hinge.

In Fig. 31 is shown the theoretical interaction curve of moment versus axial load calculated from equations 9.10 and 9.11 of Reference 1. The experimental results at ultimate load and yield load are plotted for comparison. Also shown here are bar graphs of tests T-7 and T-8, indicating working loads compared with theoretical yield and ultimate loads, and also with the experimental ultimate loads. Although experimental yield occurred below predicted yield, the ultimate moments exceeded the predicted moments.

3.8 Corner Connection Test (T-4)

The load-deflection curve of the corner connection is shown in Fig. 35, and the moment versus knee rotation curve is shown in Fig. 36. It will be noted that after the mechanism had formed as indicated by the letter "A", the load increased at a uniform rate until a maximum indicated by "B" was reached. This increase in load may be attributed to the strain hardening effect which was hastened due to the steep moment gradient in the same way as in the test of a centrally loaded beam (T-5).

In Fig. 37 are plotted the forces in the lateral support tie rods at the time the mechanism was being formed and at the ultimate load. The required lateral support force again proved to be very small in proportion to the load on the specimen.

3.9 Gabled Portal Frame (T-9)

Experimental results of the gabled portal frame test are shown in Fig. 42 through 45. The horizontal deflections of the knees and the vertical deflection of the peak of the frame are compared with the approximate theoretical curves in Fig. 43. The observed ultimate load was 1% greater than predicted.

Figure 44 shows the rotation of the lee knee which was the location of the first plastic hinge and therefore the most strained part of the frame. In this curve, as in the others, the region of greatest deviation from the theoretical prediction is in the early stages of yielding, in other words, in the region where residual stresses are known to exhibit their greatest effect.

4. SUMMARY AND CONCLUSIONS

The following statements summarize the principles of plastic behavior of structural steel members and frames as demonstrated by the tests conducted:

(1) Test of a "coupon" of structural steel verifies that its stress-strain relationship up to strain-hardening may be characterized by two straight lines, one at a slope E in the elastic region, while beyond that (in the plastic region) deformation occurs at constant stress (σ_y) until the strain is about 15 times the elastic limit strain. (Fig. 4)

(2) On the basis of the "static rate" yield level determined from coupon tests, the moment-rotation relationship of a beam and the resulting load-deflection curve may be determined with satisfactory accuracy. (Figs. 6, 7)

(3) Properly-proportioned members and components will develop the "plastic hinges" that are essential to a realization of ultimate load. Adequate "rotation capacity" was obtained in such members. (Figs. 7, 24, 36)

(4) As would be expected, a beam loaded so as to produce plastification over a considerable distance between points of lateral support may buckle laterally causing a reduction of load-carrying capacity and rotation capacity. (Fig. 19, 20). A steep moment gradient on a long span beam may reduce the tendency towards plastic lateral buckling sufficiently to allow the beam to display adequate rotation capacity at maximum moment, even when there is considerable distance between points of lateral support. (Fig. 24).

(5) Redistribution of moment due to the successive formation of plastic hinges actually occurs in indeterminate structures (Fig. 11 and 43) and thereby permits the realization of the computed load (Fig. 54). (Plastification of the cross-section and redistribution of moment are the two factors that account for the increase in load capacity above "first yield".)

(6) High shear stresses in a continuous beam may cause large deflections not predicted by the simple bending theory, but these additional deflections do not seriously affect ultimate load-carrying capacity. (Fig. 11).

(7) In a beam with a steep moment-gradient (beam with a concentrated load or in a corner connection), the strain-hardening which occurs immediately following plastic yield of the cross-section causes a rising rather than a horizontal load-deflection curve. (Figs. 24, 36). This is an effect that is ignored by the simple plastic theory.

(8) Subjecting a bending member to axial load reduces its ability to withstand bending moment. (Fig. 31). Plastic theory affords an accurate means of predicting the effect; also, as predicted, the necessary hinges are developed (Fig. 30).

(9) Welded connections for continuous frames can be fabricated to provide for the formation of plastic hinges and to develop the strength and rotation properties comparable to ordinary rolled sections. (Fig. 35)

(10) Although lateral support of a structure is necessary for it to develop its ultimate load, the magnitude of the necessary forces is quite small. The maximum value measured was less than 0.5% of the force required to compress the main member to yield point stress. (Fig. 14, 37, 45)

(11) Finally, the simple plastic theory is a reasonable basis for computing the ultimate load of structural members and frames. (Fig. 54) Of all the tests, only one member was weaker than the computed value -- and that by less than 2%.

5. ACKNOWLEDGEMENTS

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D. L. McCullough calculated and plotted the results obtained from the tests.

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TABLE 1
GENERAL PLAN OF DEMONSTRATIONS

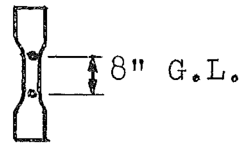
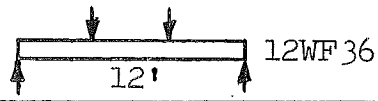
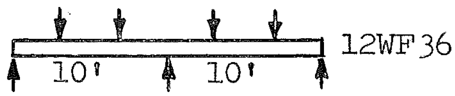
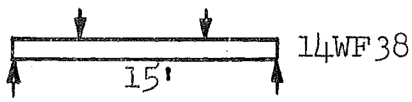
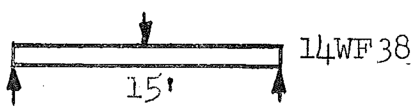
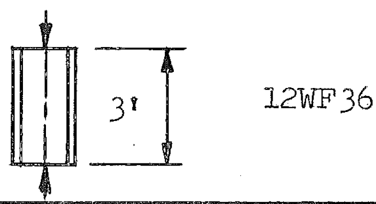
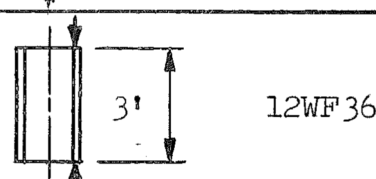
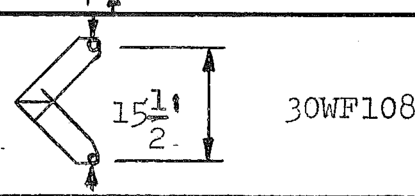
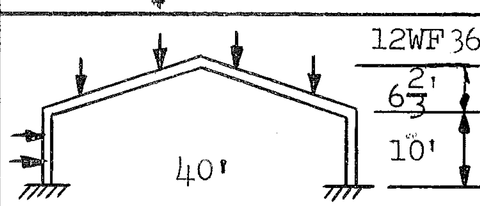
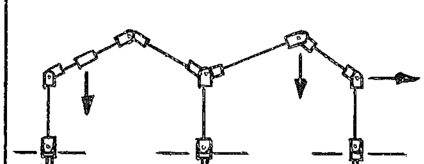
TEST	SKETCH	PRINCIPLE DEMONSTRATED
P-2 Tensile Coupon		Manner of Determining Mechanical Properties
T-1 Control Beam		Plastification Of Cross Section
T-2 Continuous Beam		Redistribution Of Moment
T-3 Lateral Buckling Beam		Lateral Buckling Of Long Laterally Unsupported Beam
T-5 Centrally Loaded Beam		Effect of Moment Gradient In Reducing Tendency Toward Plastic Lateral Buckling
T-6 Cross Section		Compressive Properties Of Steel
T-7 T-8 Eccentrically Loaded Beam-Column		Reduction Of Plastic Moment Due To Axial Load
T-4 Corner Connection		Behavior Of Corner Connection
T-9 Gabled Portal Frame		Behavior Of Complete Portal Frame
Model Of Two-Span Gabled Frame		Use Of Models In Plastic Analysis

TABLE 2

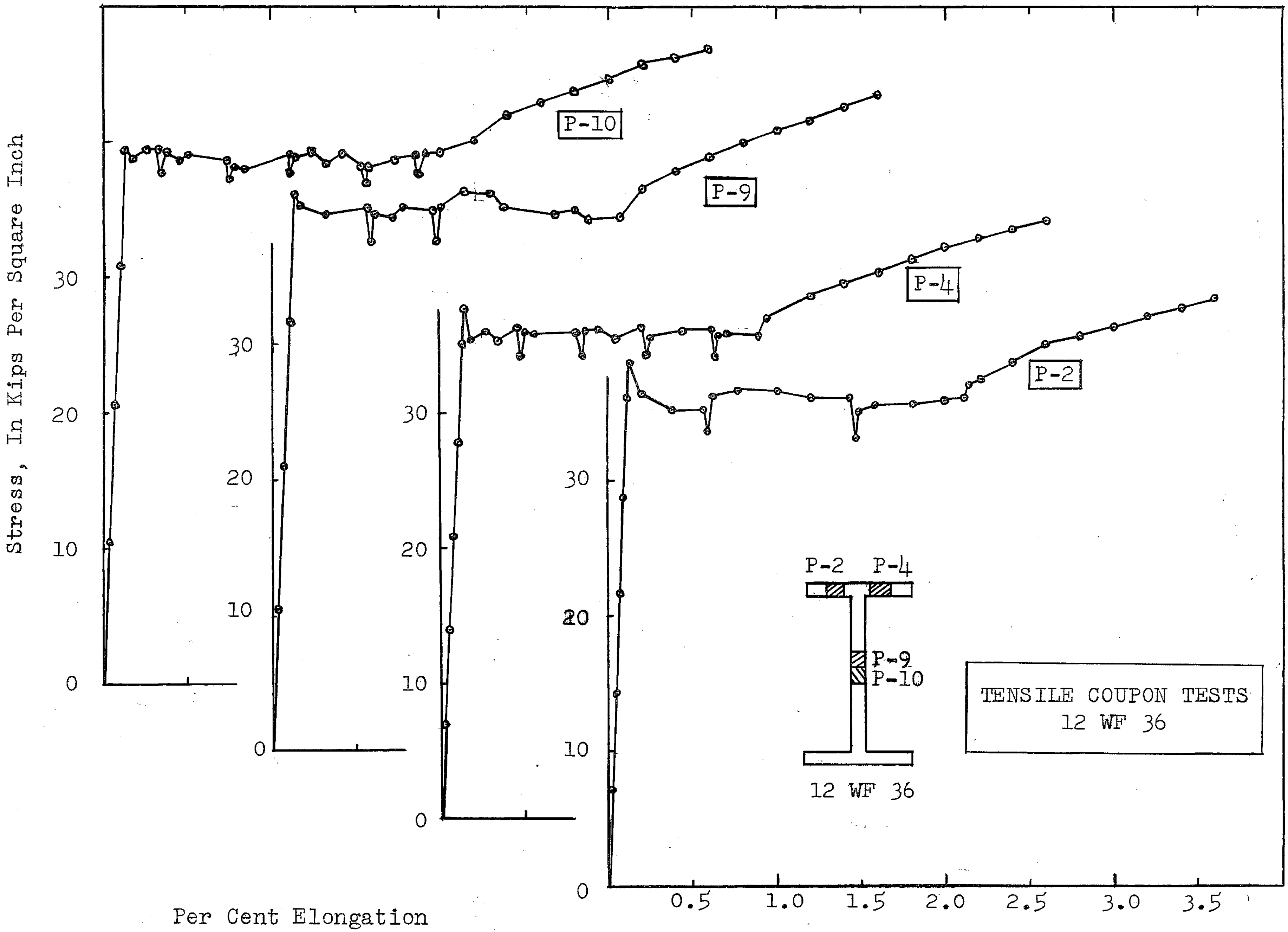
SECTION PROPERTIES OF TEST SPECIMENS

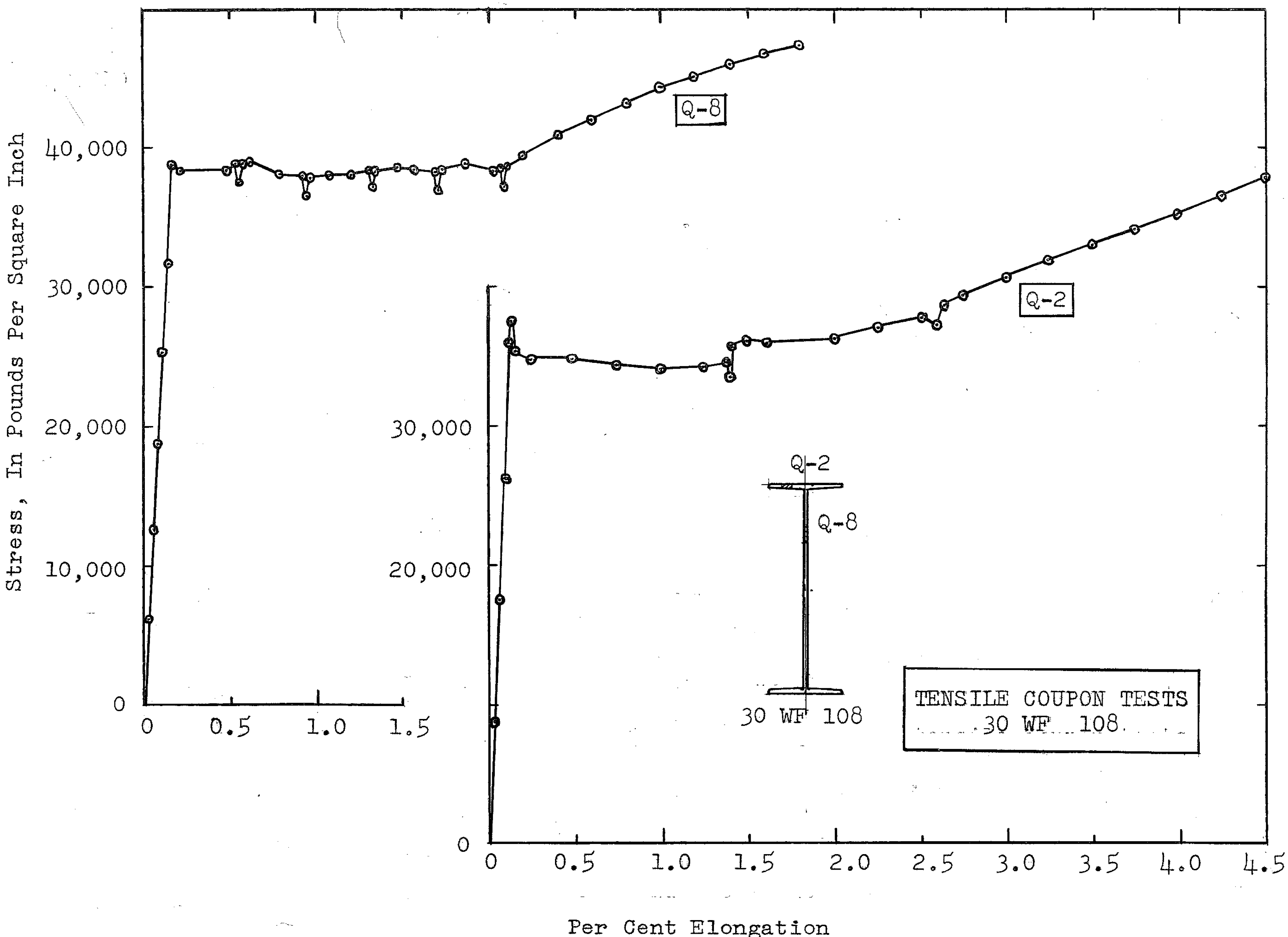
Section	Area of Section A in^2	Depth of Section d in.	Flange: Width b in.	Flange: Average Thickness t in.	Web Thickness w in.	Axis x-x: I in^4	Axis x-x: S in^3	Axis x-x: Z in^3	Axis x-x: f
30WF108									
Handbook	31.77	29.82	10.484	0.760	0.548	4461.0	299.2	345.5	1.16
Measured	30.46	29.93	10.460	0.740	0.518	4345.8	290.3	334.1	1.15
% Variation	-4.12	+0.37	-0.23	-2.63	-5.46	- 2.58	- 2.98	- 3.30	-0.86
14WF38									
Handbook	11.17	14.12	6.776	0.513	0.313	385.3	54.6	61.9	1.13
Measured	10.83	14.07	6.880	0.486	0.313	369.5	52.5	59.12	1.13
% Variation	-3.22	-0.35	+1.51	-0.526	0.000	- 4.10	-4.00	-3.85	0
12WF36									
Handbook	10.59	12.24	6.565	0.540	0.305	280.8	45.9	51.42	1.12
Measured	10.78	12.30	6.625	0.514	0.337	282.1	45.9	51.79	1.13
% Variation	+1.79	+0.49	+0.91	-4.81	+10.50	+ 0.46	0	+0.72	-0.89

TABLE 3

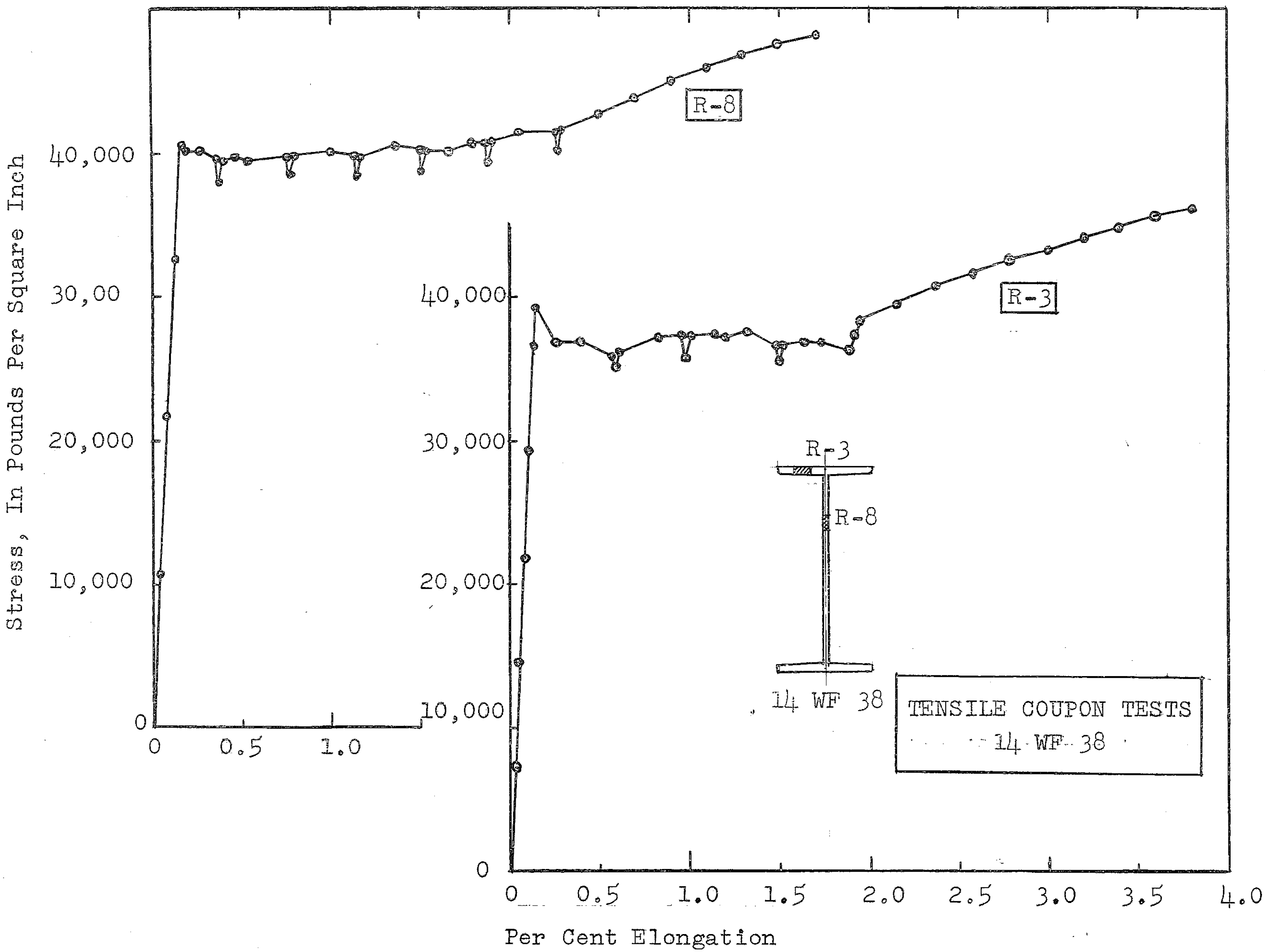
SUMMARY OF COUPON TEST RESULTS

Material				Average Coupon Test Results						
Coupon Number	Shape	Location	Type Coupon	σ_{yst} (ksi)	σ_{yu} (ksi)	σ_{yL} (ksi)	σ_{ult} (ksi)	ϵ_y (in/in)	ϵ_{st} (in/in)	E_{st} (ksi)
P-2		Flange		33.7	38.6	36.0	60.0	0.00129	0.0213	511
P-4	12WF36		Tension	34.2	37.8	36.2	59.5	0.00142	0.0190	436
P-9		Web		37.7	41.3	39.8	64.7	0.00142	0.0206	468
P-10				38.1	39.4	38.6	62.1	0.00137	0.0180	515
R-3	14WF38	Flange	Tension	35.6	39.2	36.0	61.2	0.00150	0.0190	512
R-8		Web		39.2	40.9	39.7	63.8	0.00175	0.0180	546
Q-2	30WF108	Flange	Tension	33.1	36.9	34.8	60.6	0.00137	0.0200	479
Q-8		Web		37.6	39.1	38.1	62.2	0.00175	0.0205	577

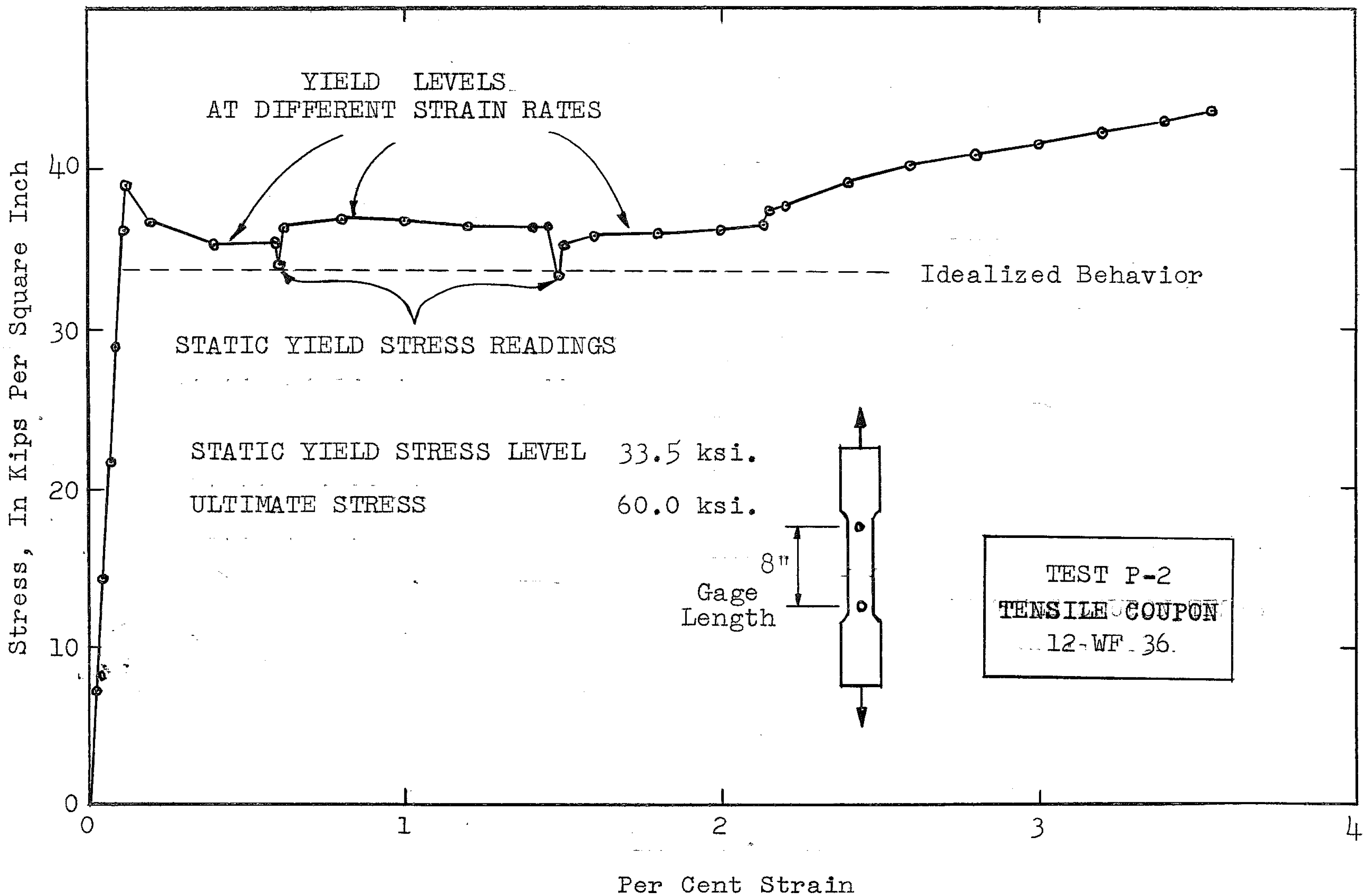

Stress, In Kips Per Square Inch
Per Cent Elongation
P-10
P-9
P-4
P-2
0
10
20
30
0.5
1.0
1.5
2.0
2.5
3.0
3.5
P-2 P-4
P-9
P-10
12 WF 36
TENSILE COUPON TESTS
12 WF 36


Q-8
Q-2
40,000
30,000
20,000
10,000
0
0
0.5
1.0
1.5
30,000
20,000
0
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
Stress, In Pounds Per Square Inch
Per Cent Elongation
Q-2
Q-8
30 WF 108
TENSILE COUPON TESTS
30 WF 108

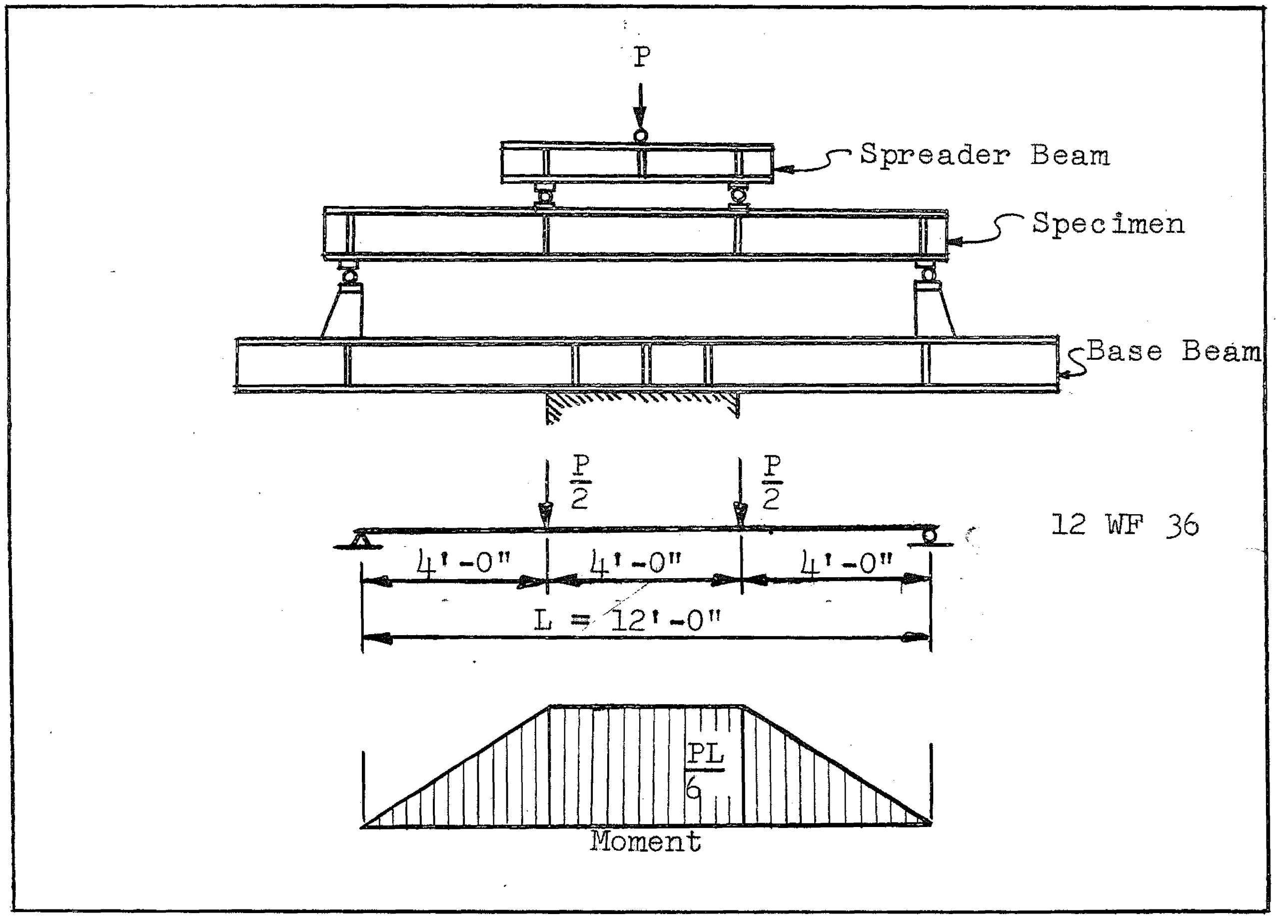
2



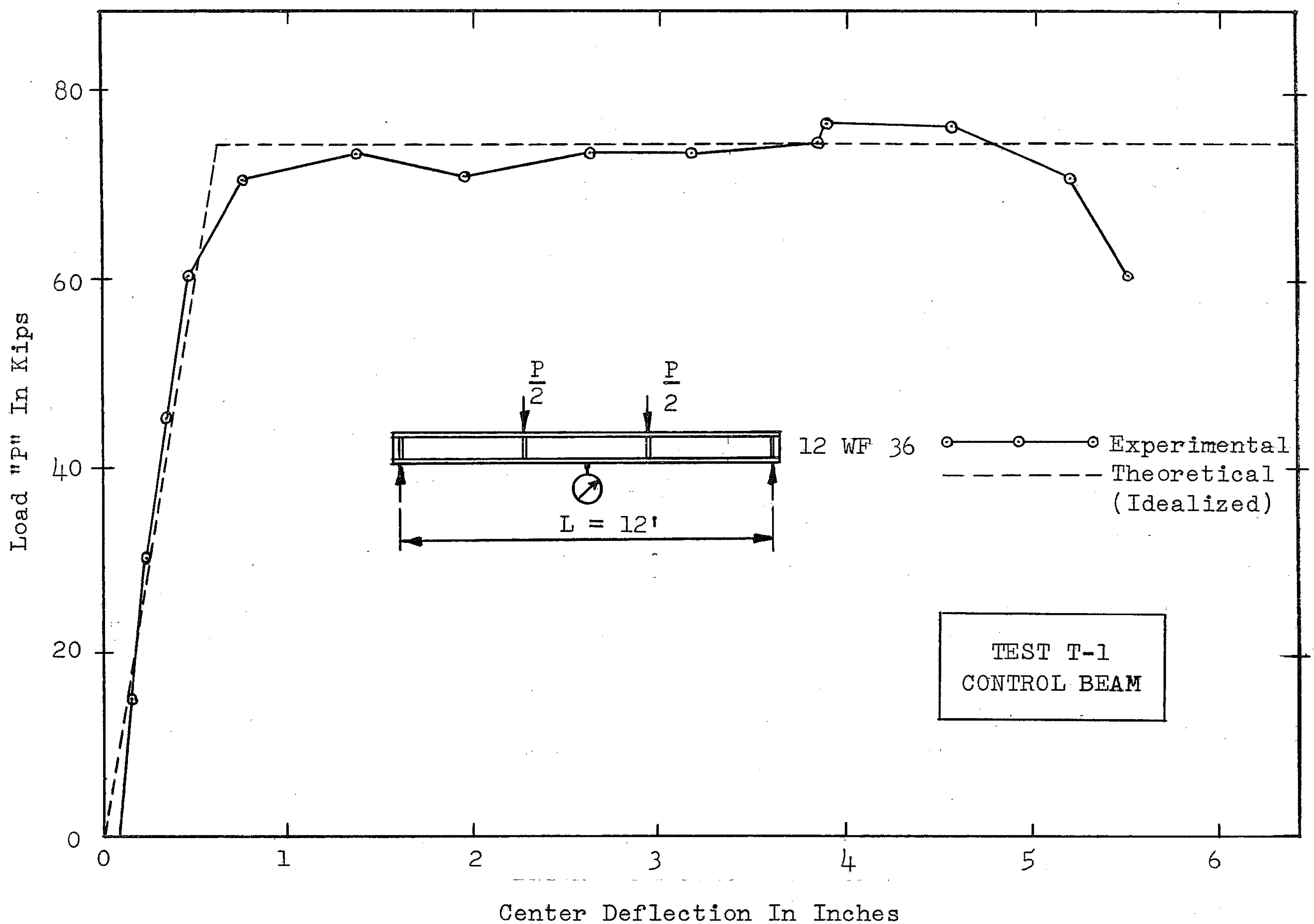
TENSILE COUPON TESTS
14 WF 38



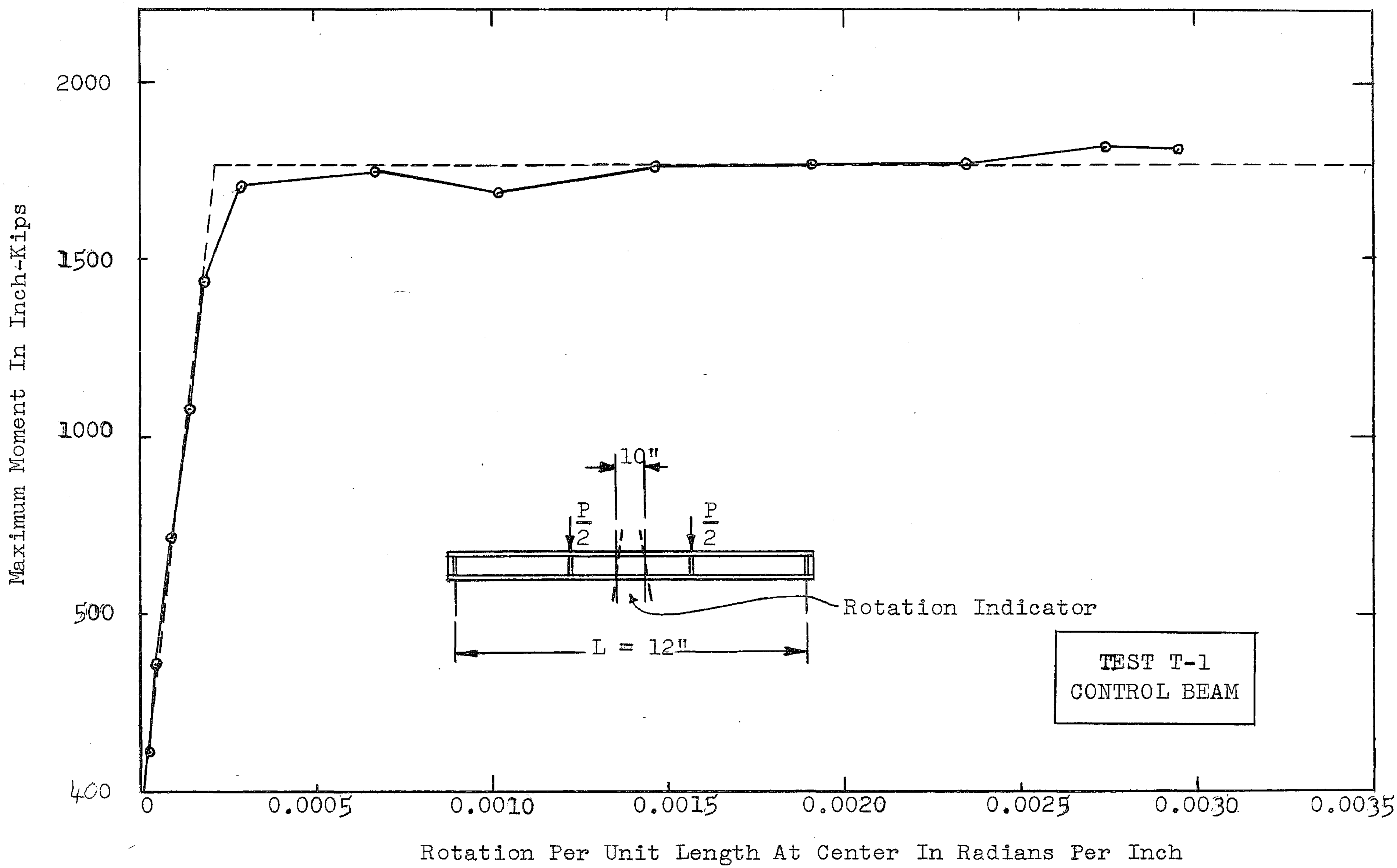
YIELD LEVELS
AT DIFFERENT STRAIN RATES
Idealized Behavior
STATIC YIELD STRESS READINGS
STATIC YIELD STRESS LEVEL 33.5 ksi.
ULTIMATE STRESS 60.0 ksi.
8"
Gage
Length
TEST P-2
TENSILE COUPON
12-WF 36
Stress, In Kips Per Square Inch
0
10
20
30
40
0
1
2
3
4
Per Cent Strain



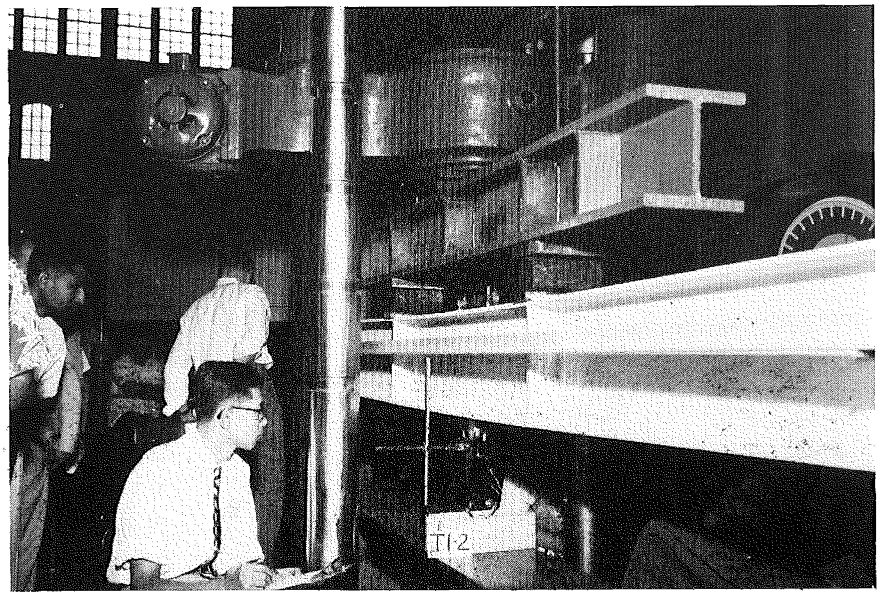
SETUP FOR CONTROL BEAM TEST (T-1)


80
60
40
20
0
Load "P" In Kips
0
1
2
3
4
5
6
Center Deflection In Inches
P/2
P/2
12 WF 36
L = 12'
Experimental
Theoretical (Idealized)
TEST T-1
CONTROL BEAM

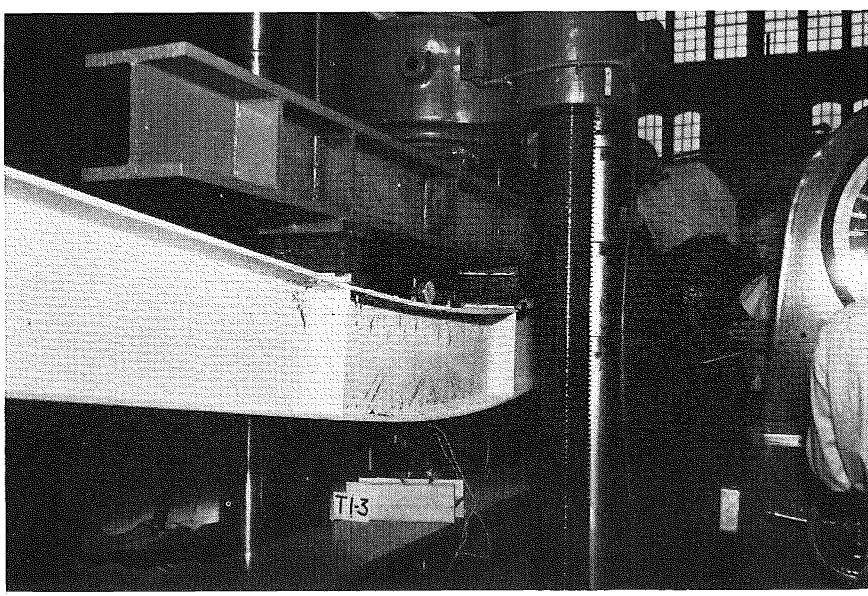
9



2000
1500
1000
500
400
Maximum Moment In Inch-Kips
0
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
0.0035
Rotation Per Unit Length At Center In Radians Per Inch
10"
P/2
P/2
Rotation Indicator
L = 12"
TEST T-1
CONTROL BEAM

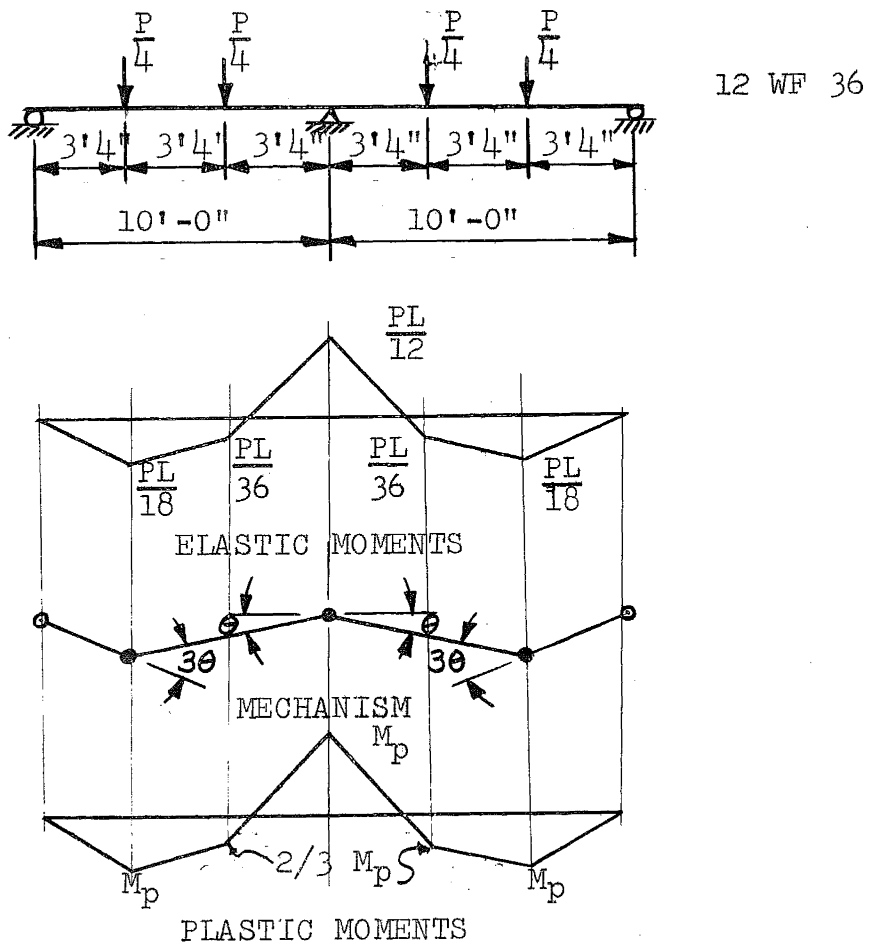


8



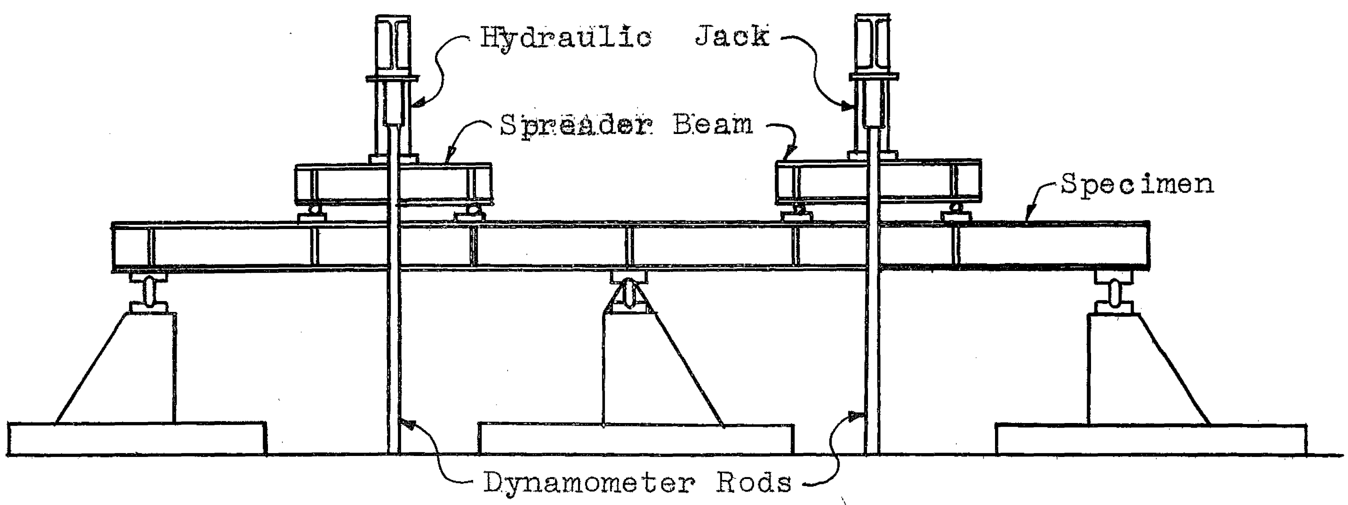
9

The control beam test showed how a plastic hinge is developed in a wide flange section.

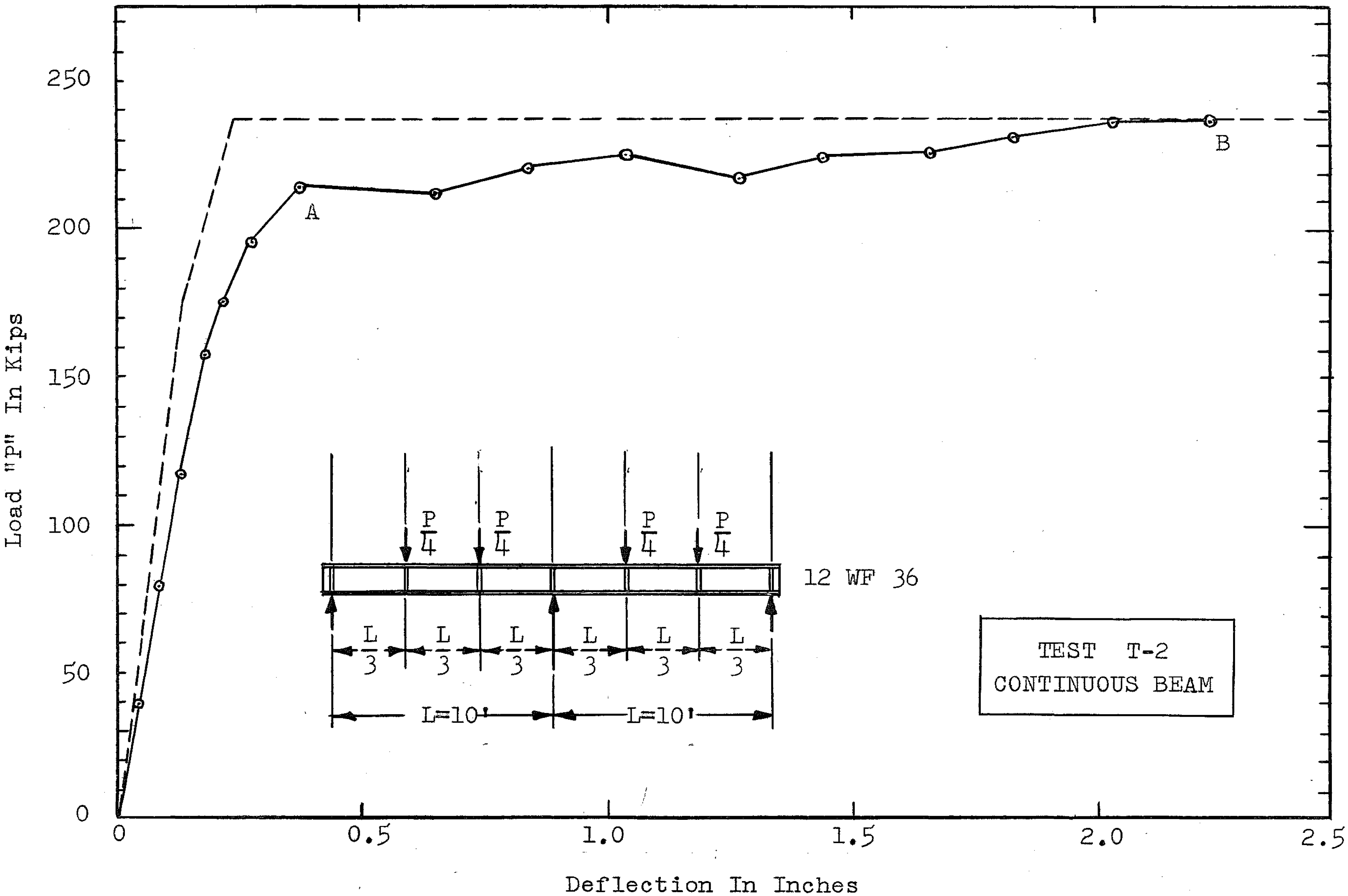


$$P_y = 12 \frac{M_y}{L}$$

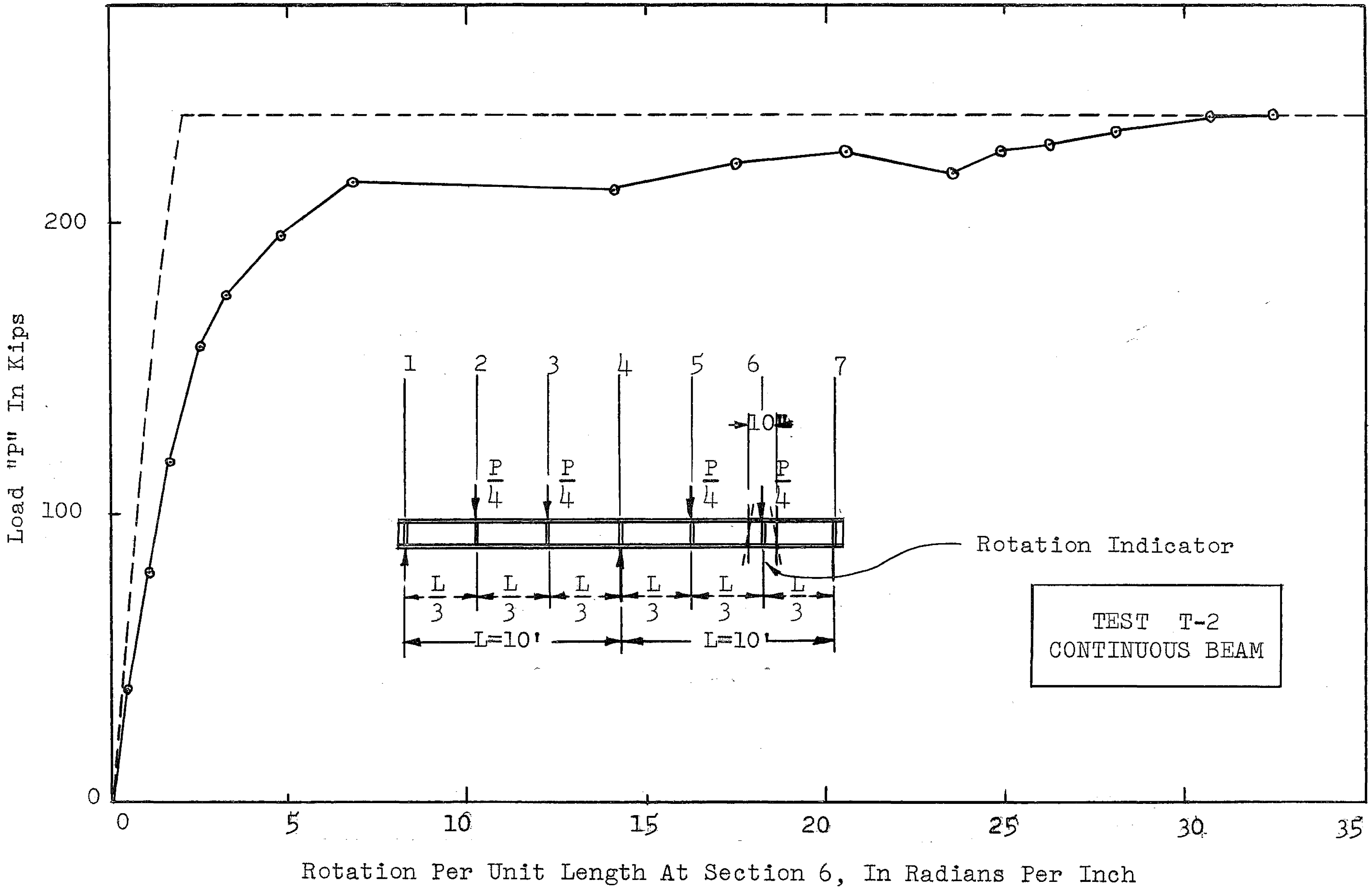
$$P_u = 16 \frac{M_p}{L}$$



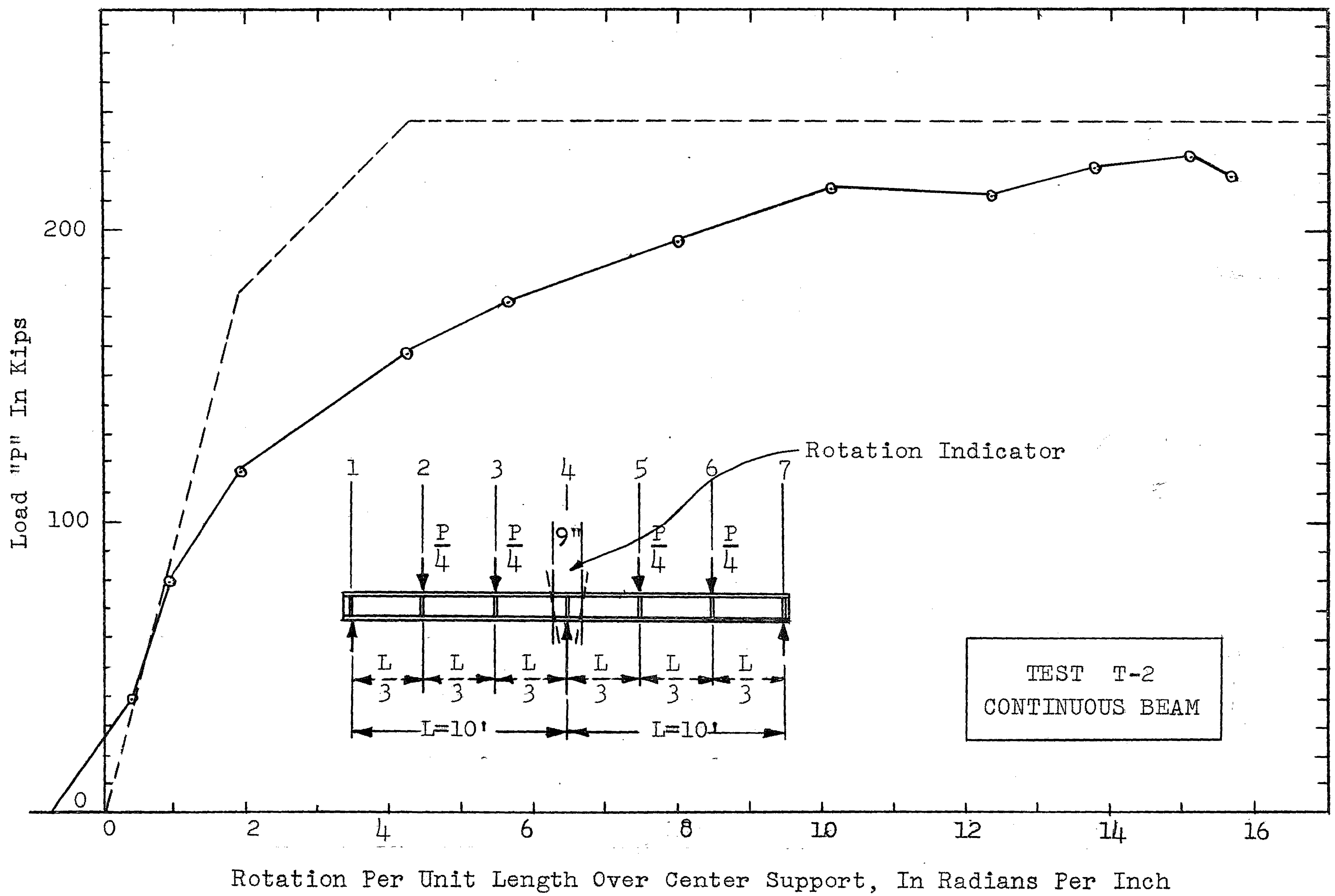
SETUP FOR CONTINUOUS BEAM TEST (T-2)



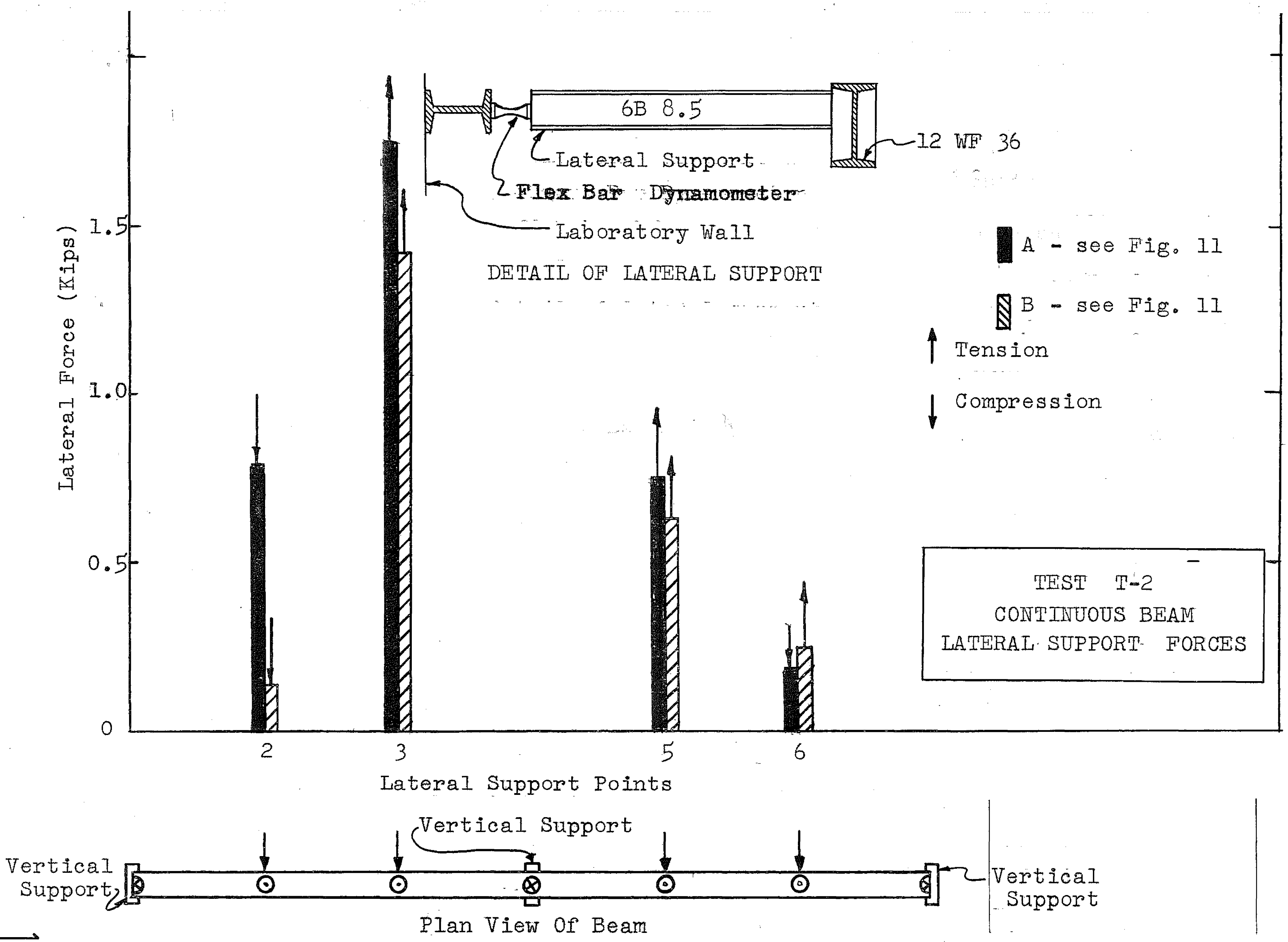
Load "P" In Kips
250
200
150
100
50
0
0
0.5
1.0
1.5
2.0
2.5
Deflection In Inches
A
B
P/4
P/4
P/4
P/4
12 WF 36
L/3
L/3
L/3
L/3
L/3
L/3
L=10'
L=10'
TEST T-2
CONTINUOUS BEAM


Load "P" In Kips
200
100
0
0
5
10
15
20
25
30
35
Rotation Per Unit Length At Section 6, In Radians Per Inch
1 2 3 4 5 6 7
10"
P/4 P/4 P/4 P/4
Rotation Indicator
L/3 L/3 L/3 L/3 L/3 L/3
L=10' L=10'
TEST T-2
CONTINUOUS BEAM

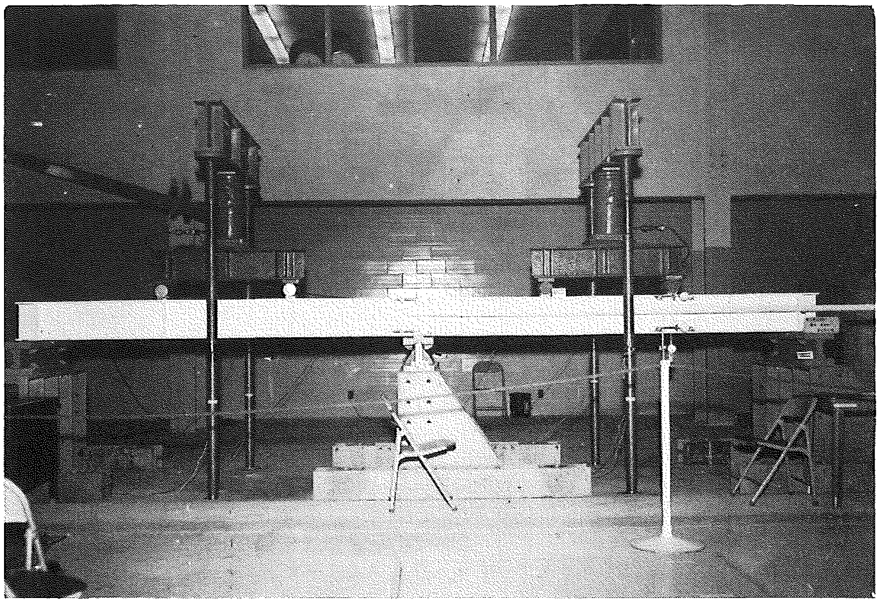
12



Load "P" In Kips
200
100
0
0
2
4
6
8
10
12
14
16
Rotation Per Unit Length Over Center Support, In Radians Per Inch
Rotation Indicator
1
2
3
4
5
6
7
P/4
P/4
9"
P/4
P/4
L/3
L/3
L/3
L/3
L/3
L/3
L=10'
L=10'
TEST T-2
CONTINUOUS BEAM

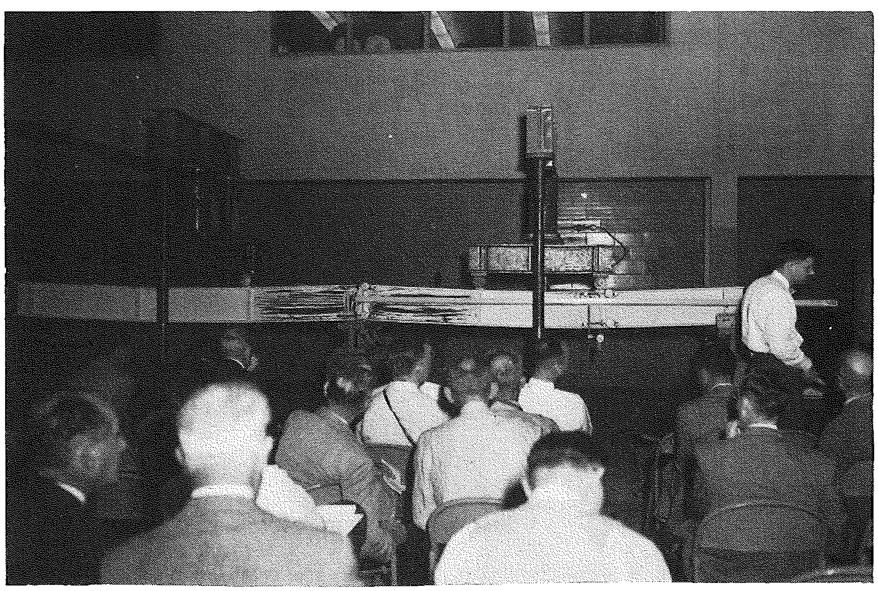

6B 8.5
12 WF 36
Lateral Support
Flex Bar Dynamometer
Laboratory Wall
DETAIL OF LATERAL SUPPORT
A - see Fig. 11
B - see Fig. 11
Tension
Compression
Lateral Force (Kips)
1.5
1.0
0.5
0
2
3
5
6
Lateral Support Points
TEST T-2
CONTINUOUS BEAM
LATERAL SUPPORT FORCES
Vertical Support
Vertical Support
Vertical Support
Plan View Of Beam

14



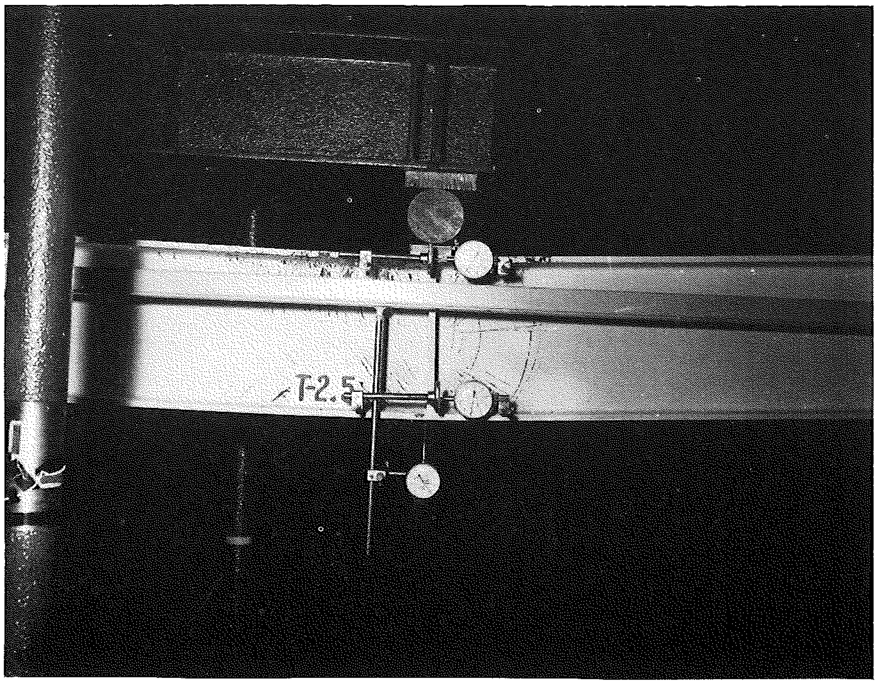
15

A continuous beam was tested to show how redistribution of moment affects the carrying capacity of an indeterminate structure.



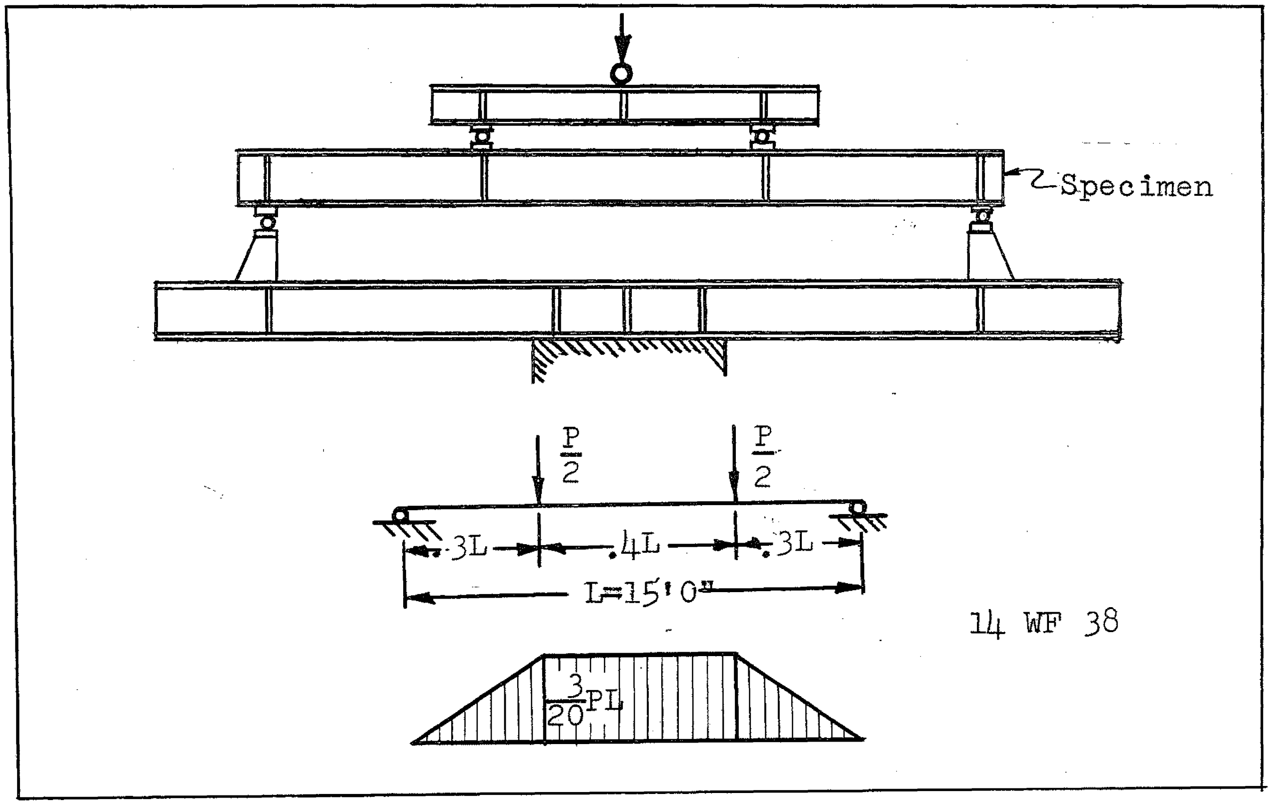
16

High shear stresses near the center support caused premature yielding and excessive deflection in the elastic range, but the predicted ultimate load was attained.



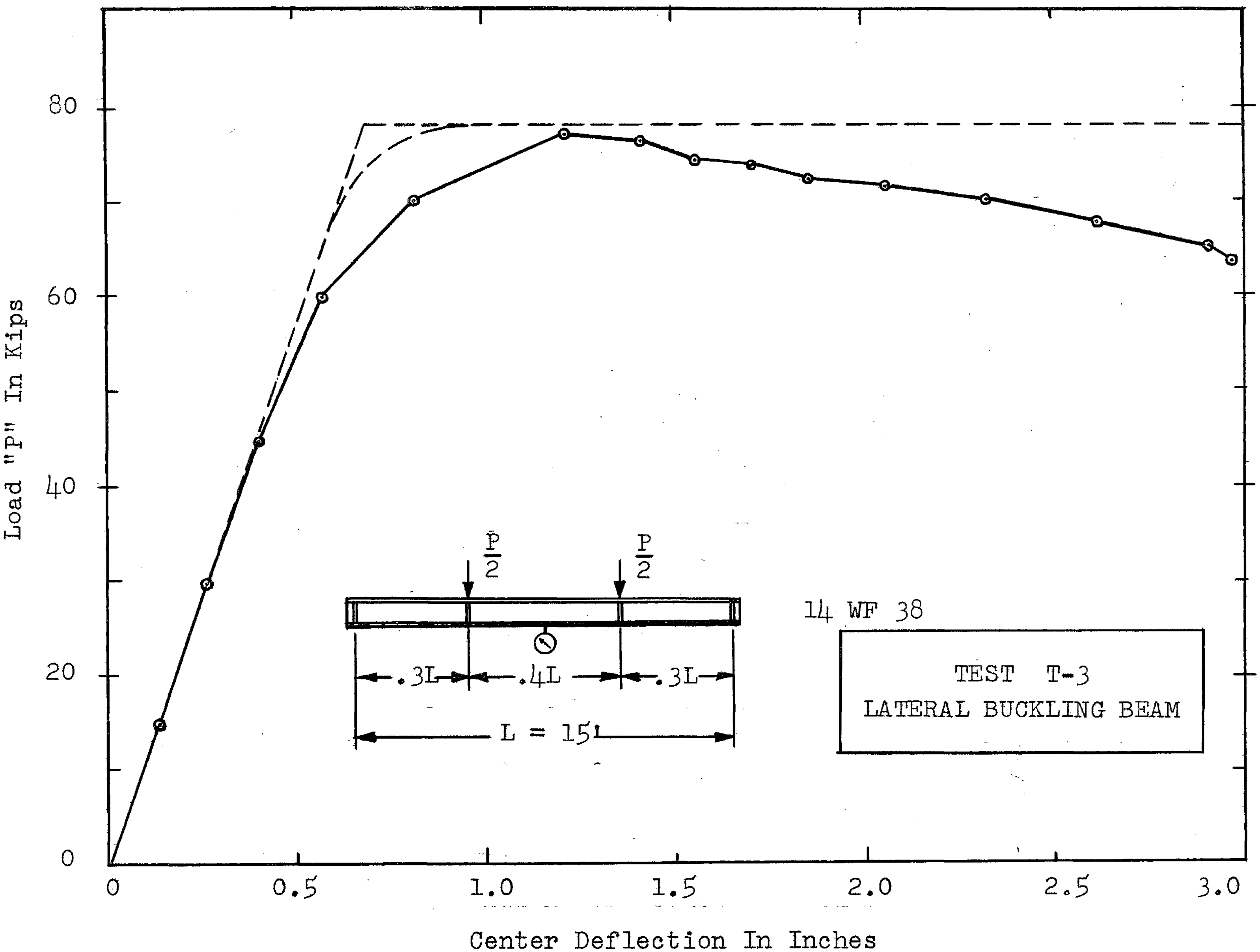
17

A rotation indicator was used to measure the rotation over a short length of the beam near the plastic hinge.

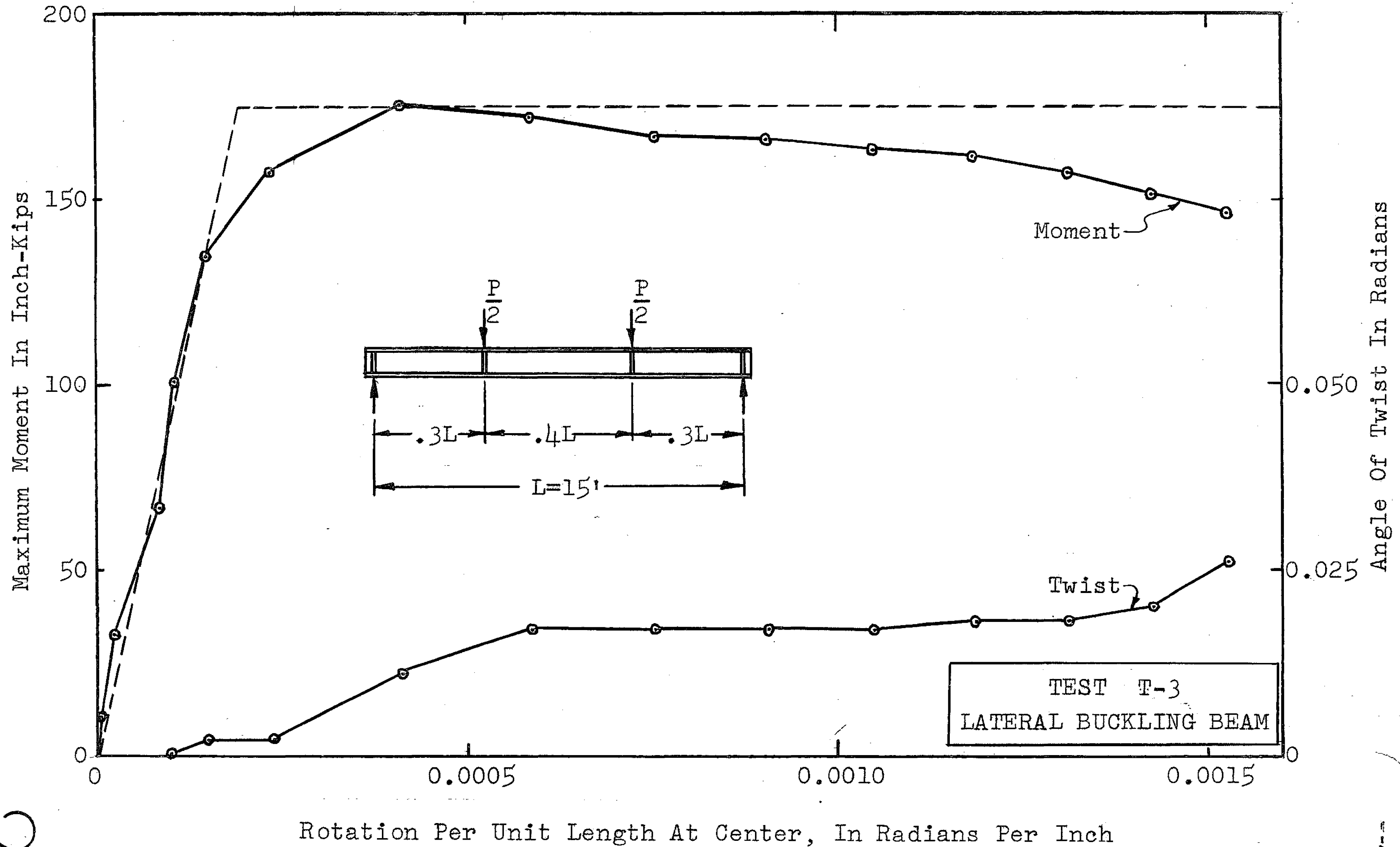


SETUP FOR LATERAL BUCKLING BEAM TEST (T-3)

18

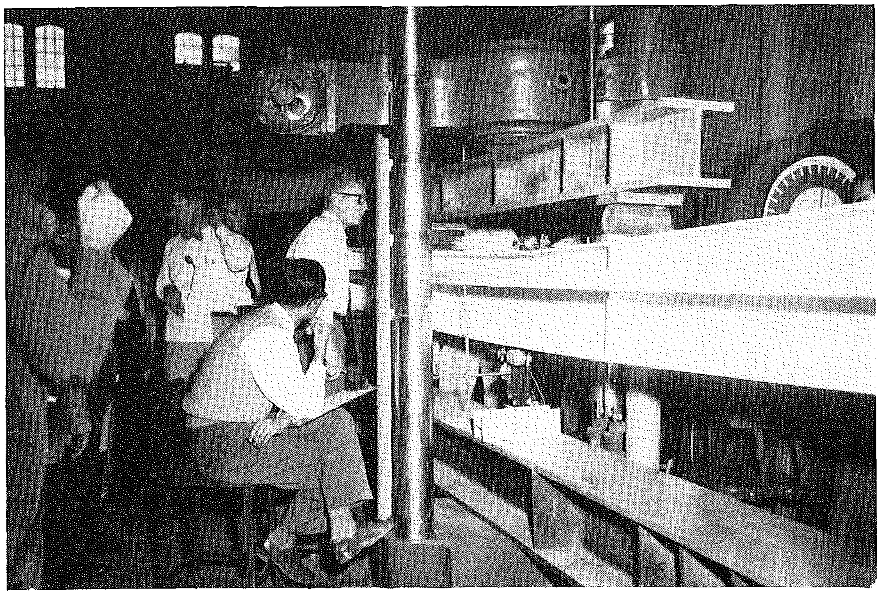

80
60
40
20
0
Load "P" In Kips
0
0.5
1.0
1.5
2.0
2.5
3.0
Center Deflection In Inches
P/2
P/2
14 WF 38
.3L
.4L
.3L
L = 15'
TEST T-3
LATERAL BUCKLING BEAM

19



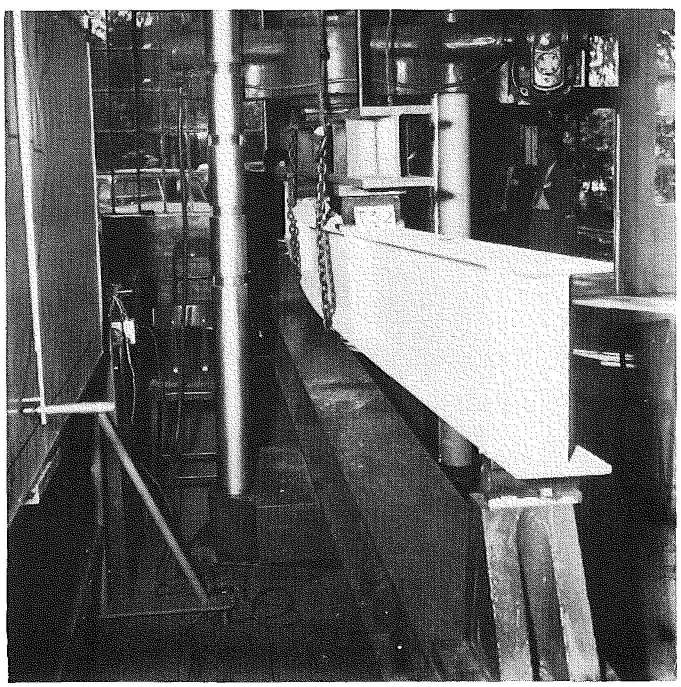
Maximum Moment In Inch-Kips
200
150
100
50
0
Angle Of Twist In Radians
0.050
0.025
0
0
0.0005
0.0010
0.0015
Rotation Per Unit Length At Center, In Radians Per Inch
Moment
Twist
P/2
P/2
.3L
.4L
.3L
L=15'
TEST T-3
LATERAL BUCKLING BEAM

20

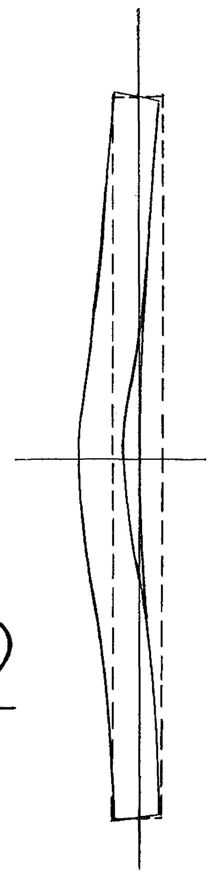


21

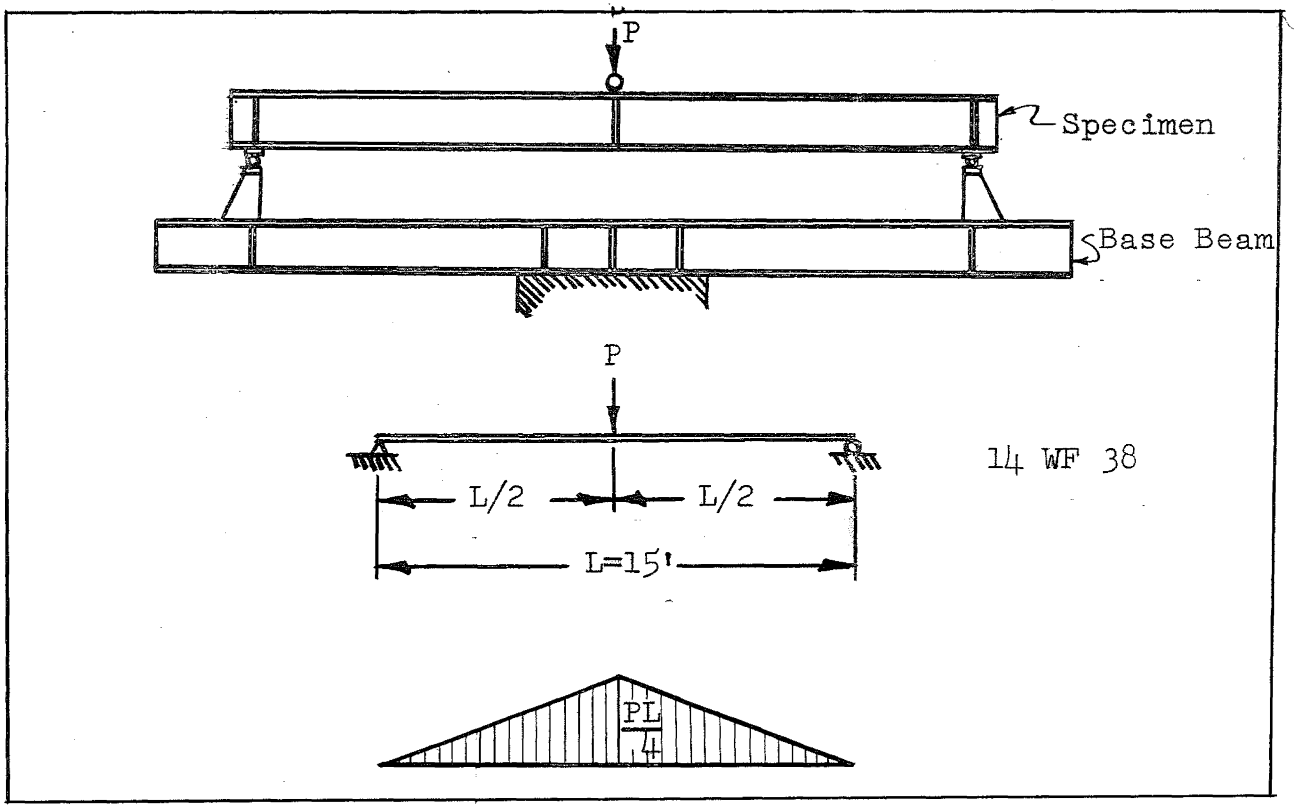
Applying load over an unsupported span too long to prevent premature buckling.........



22



...... caused severe lateral deformation of a beam.



SETUP FOR CENTRALLY-LOADED BEAM TEST (T-5)

23

$$M = \frac{PL}{4}$$

$(M_p = 61.5 \times 36 = 2215)$

$P = 46.75$

$M_p = 2100$

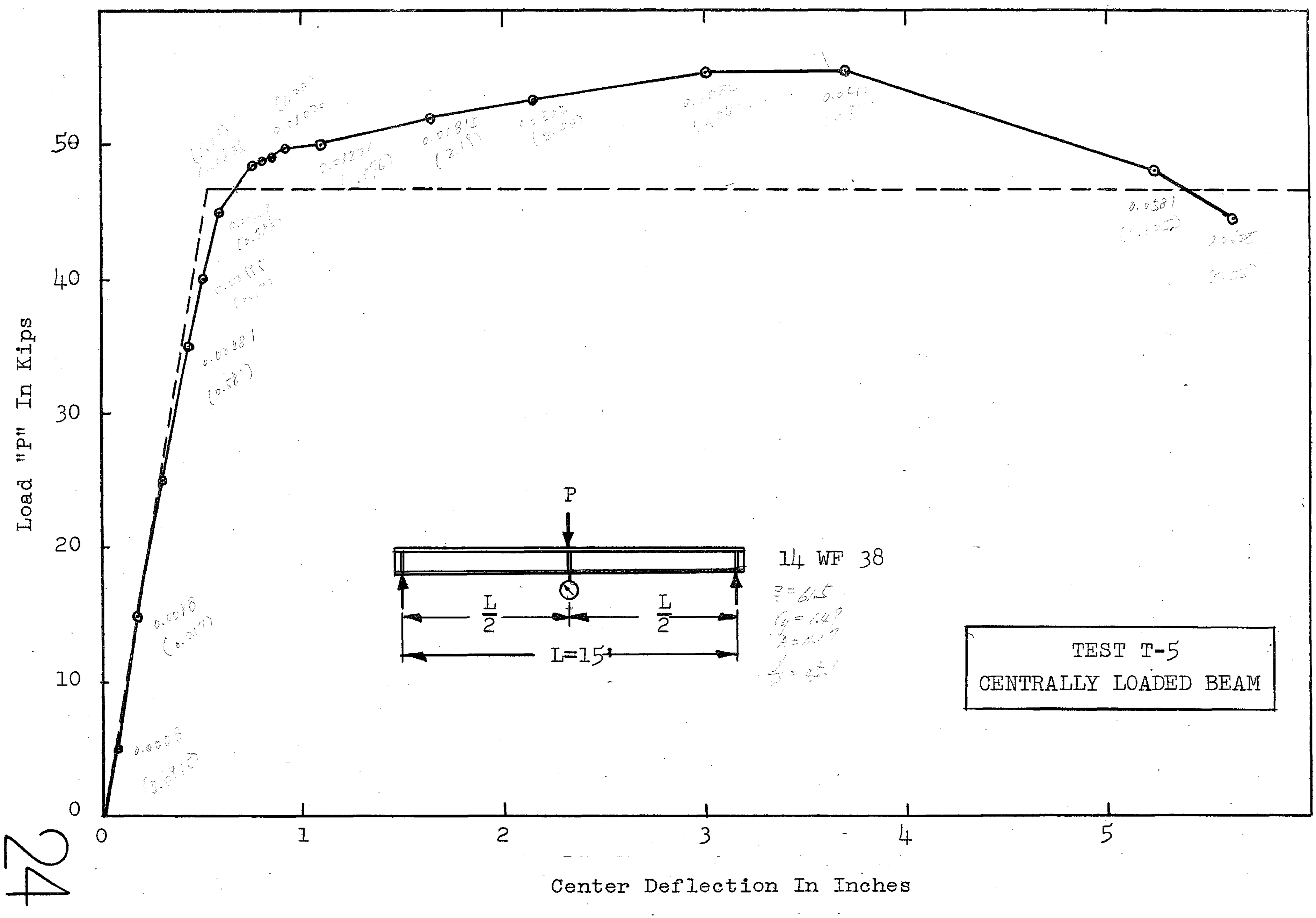
$$\frac{M}{M_p} = \frac{\frac{PL}{4}}{\frac{46.75L}{4}} = \frac{P}{46.75}$$

$$\delta_p = \frac{PL^3}{48EI} = \frac{46.75 \times 180^3}{48 \times 29.6 \times 385.3} =$$

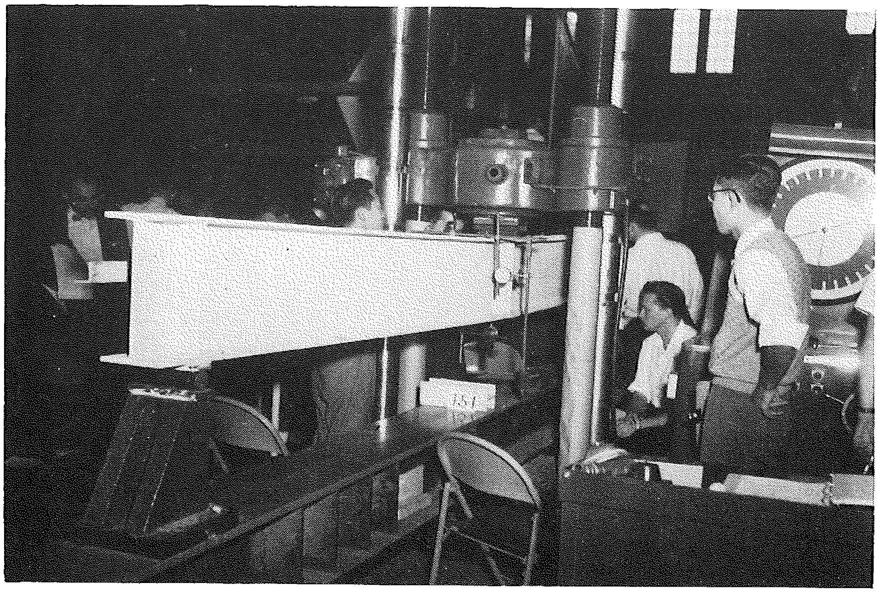
$\delta_p = 0.526 \quad (I = 364)$

$$\theta_p = \frac{M_p L}{4EI} = \frac{2100 \times 180}{4 \times 29.6 \times 10^3 \times 385.3} = 0.00829$$

(if using $I = 364$. $\theta_p = 0.00877$)

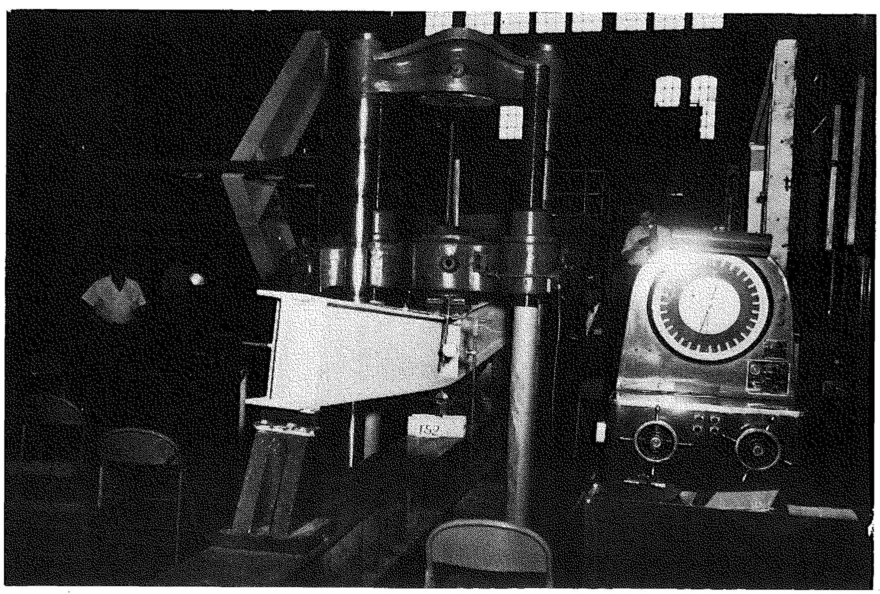


Load "P" In Kips
Center Deflection In Inches
0
10
20
30
40
50
0
1
2
3
4
5
P
14 WF 38
L/2
L/2
L=15'
TEST T-5
CENTRALLY LOADED BEAM

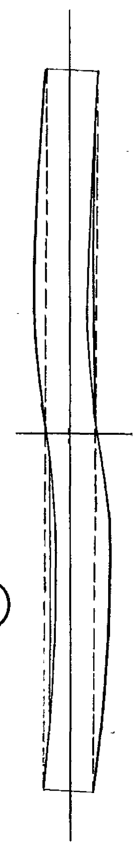


25

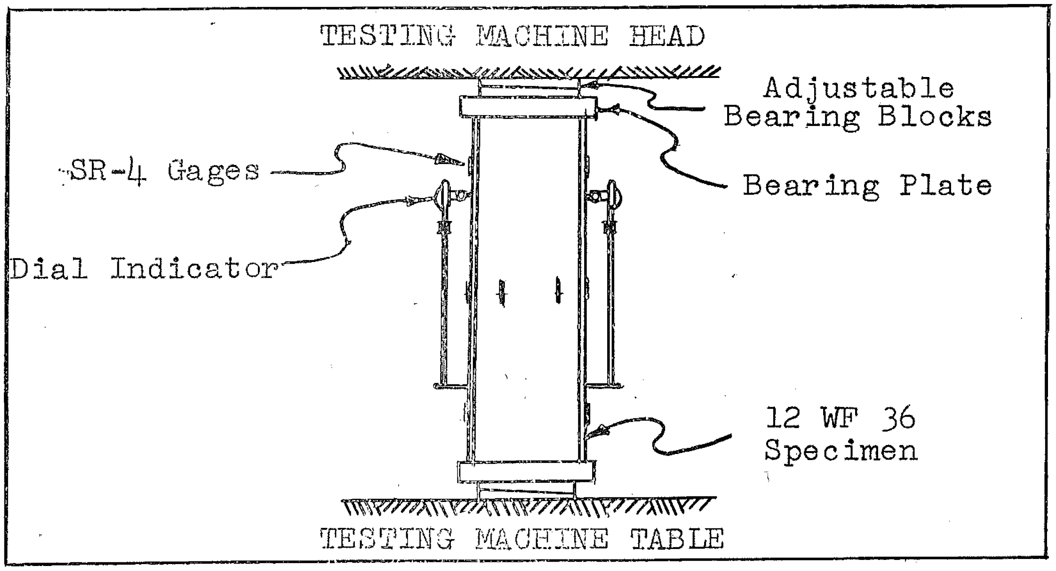
When a central concentrated load was applied to a beam of the same size as the lateral buckling beam.......



26

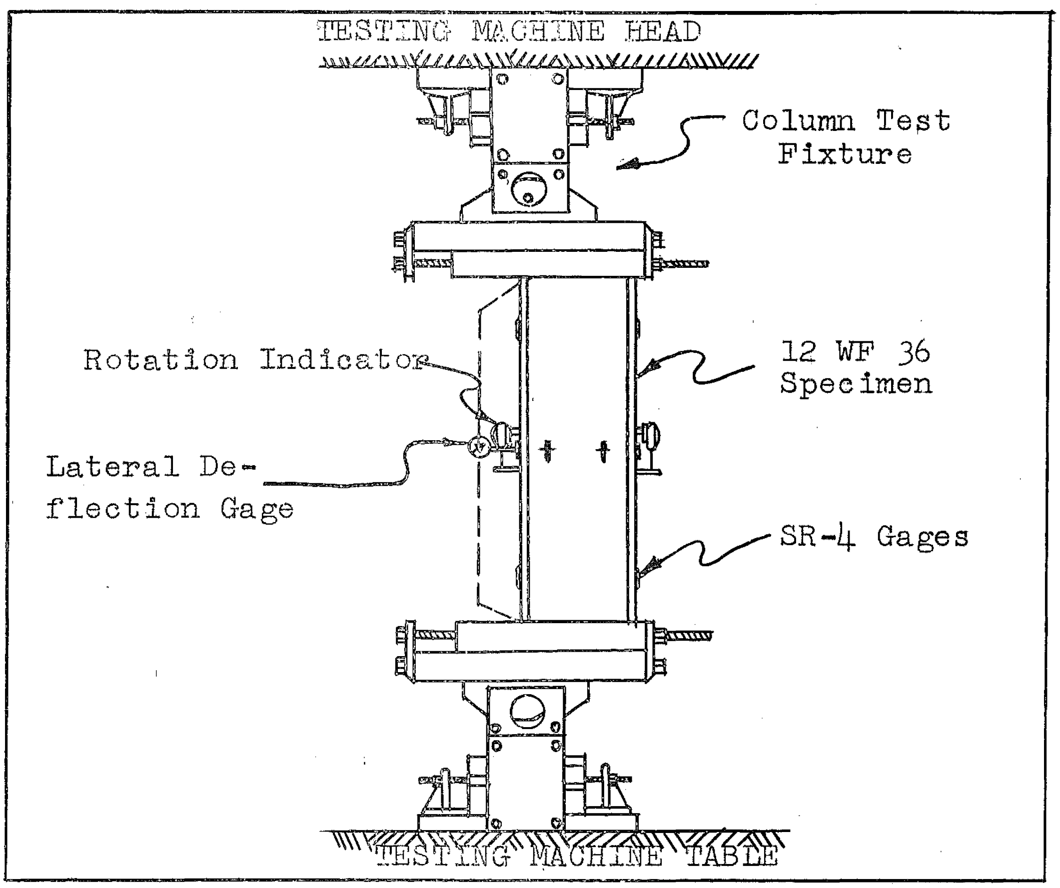


...... the steep moment gradient allowed the load to exceed the predicted ultimate load as a result of strain hardening.



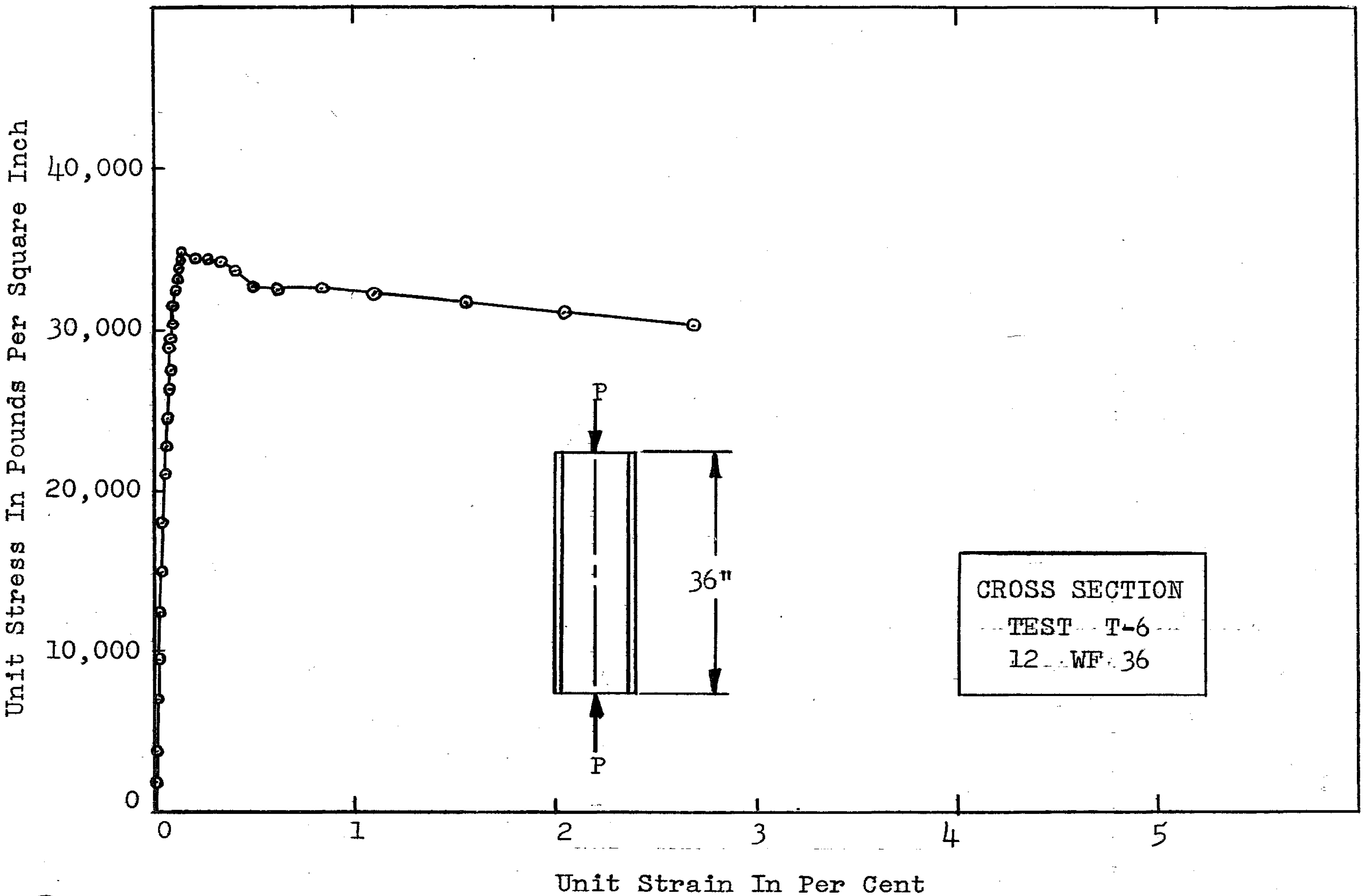
CROSS SECTION TEST SETUP (T-6)

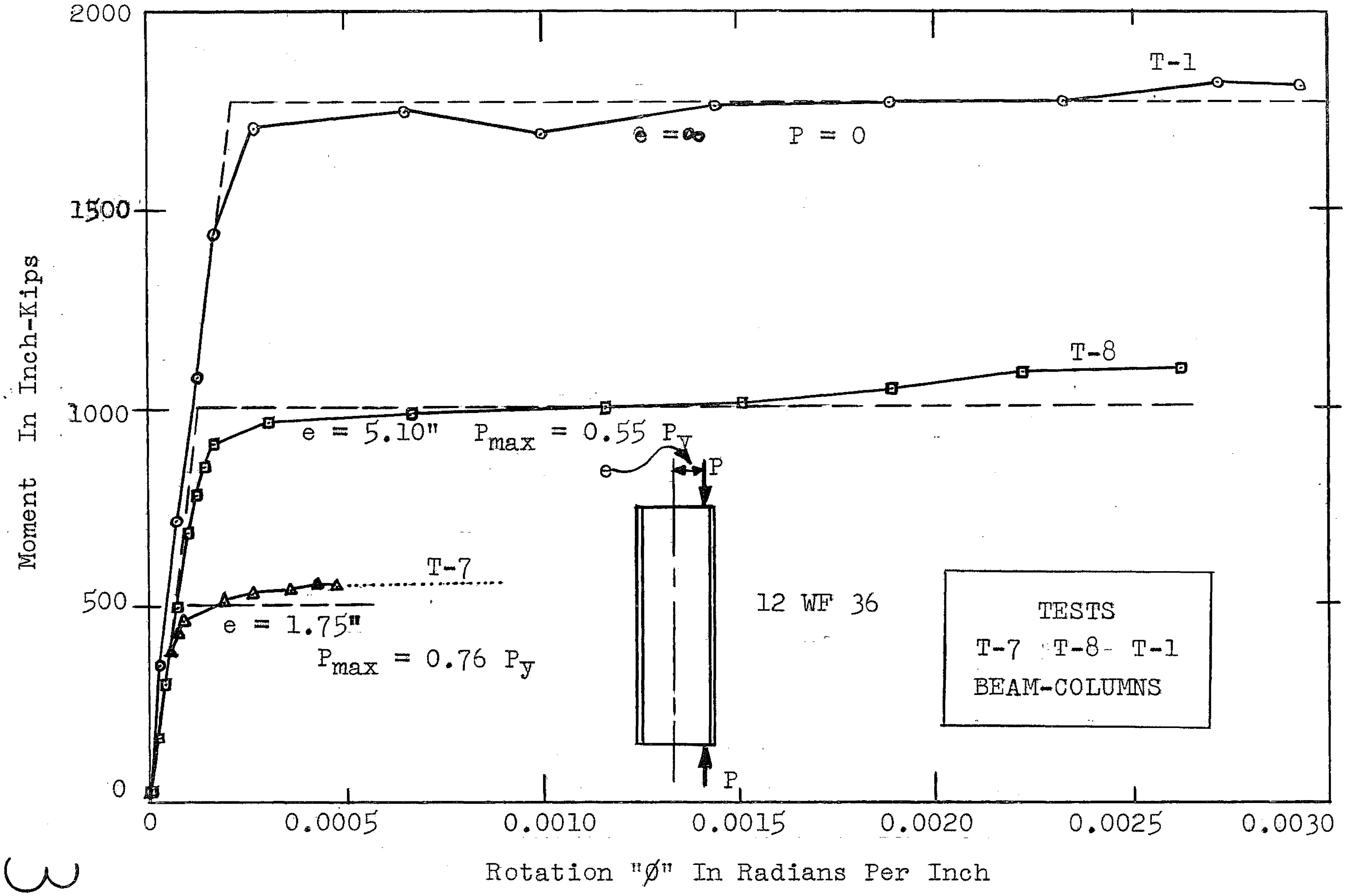
27



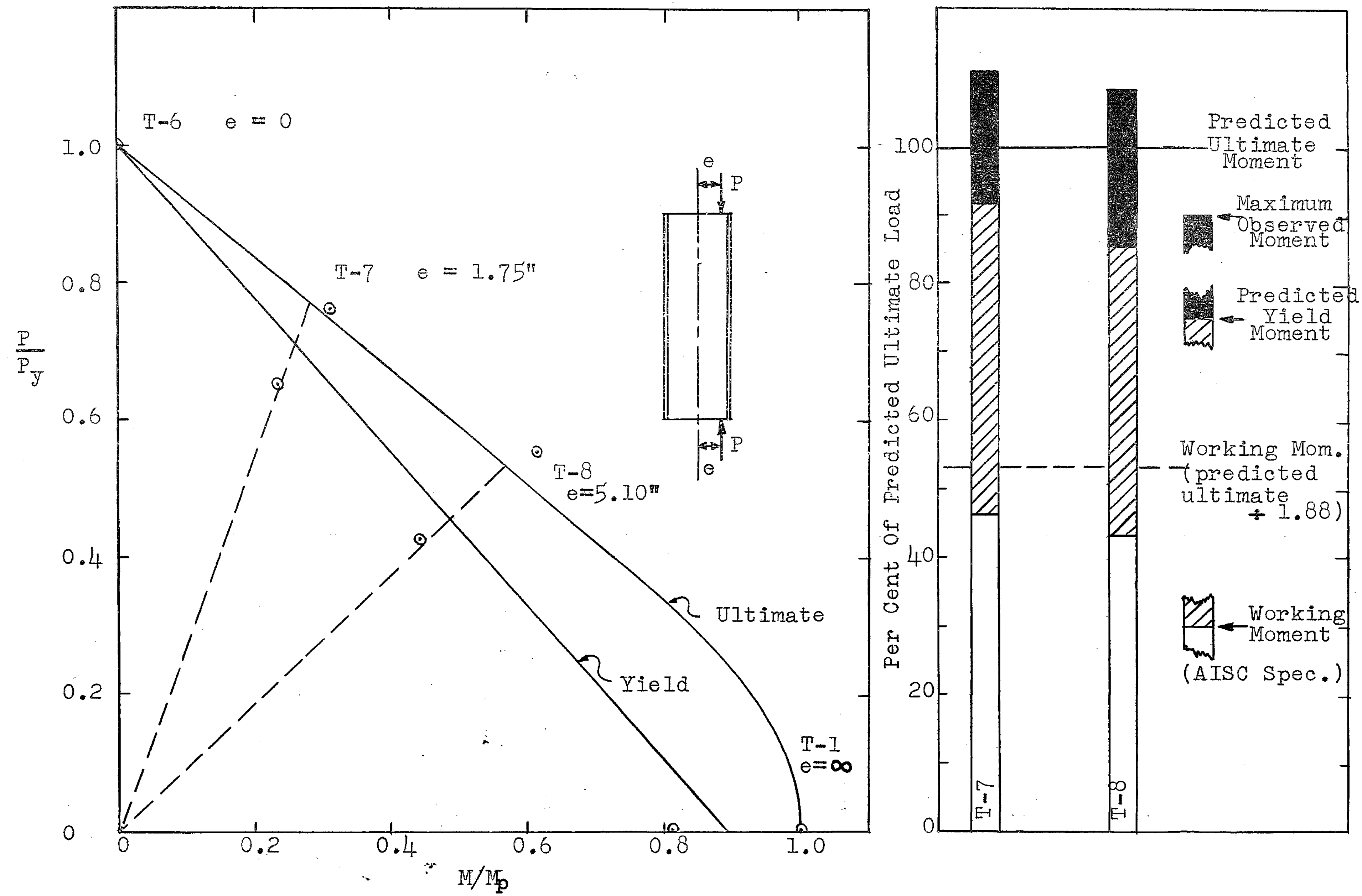
BEAM-COLUMN TEST SETUP (T-7, T-8)

28

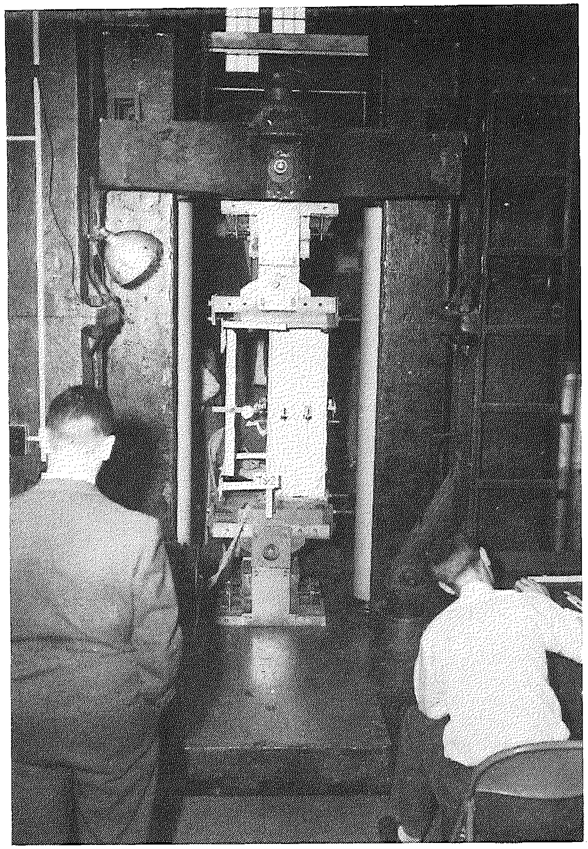

Unit Stress In Pounds Per Square Inch
40,000
30,000
20,000
10,000
0
0
1
2
3
4
5
Unit Strain In Per Cent
P
P
36"
CROSS SECTION
TEST T-6
12 WF 36


2000
1500
1000
500
0
Moment In Inch-Kips
T-1
e = ∞
P = 0
T-8
e = 5.10"
Pmax = 0.55 Py
P
T-7
e = 1.75"
Pmax = 0.76 Py
12 WF 36
P
TESTS
T-7 T-8 T-1
BEAM-COLUMNS
0
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
Rotation "Ø" In Radians Per Inch

30

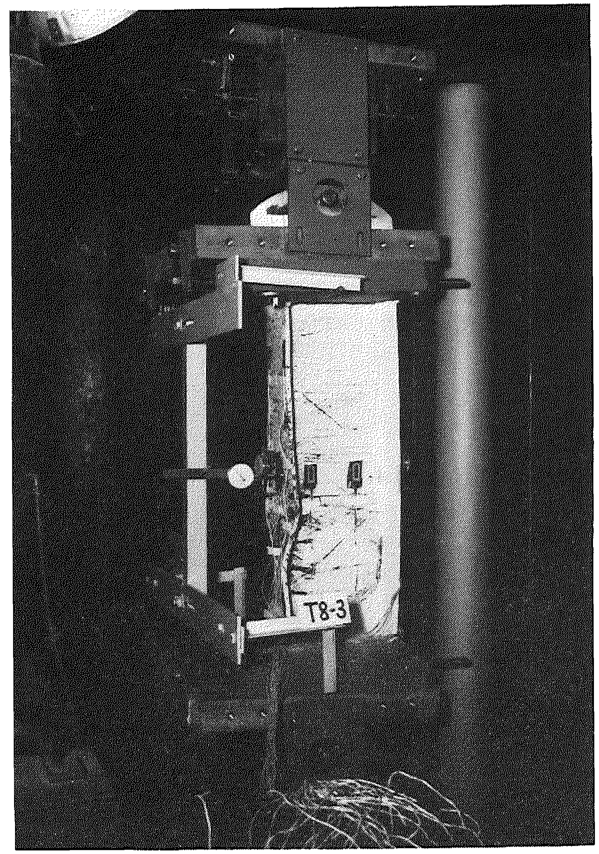

T-6 e = 0
T-7 e = 1.75"
T-8 e=5.10"
T-1 e=∞
Ultimate
Yield
e
P
P/Py
M/Mp
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
Per Cent Of Predicted Ultimate Load
100
80
60
40
20
0
Predicted Ultimate Moment
Maximum Observed Moment
Predicted Yield Moment
Working Mom. (predicted ultimate ÷ 1.88)
Working Moment (AISC Spec.)
T-7
T-8

31



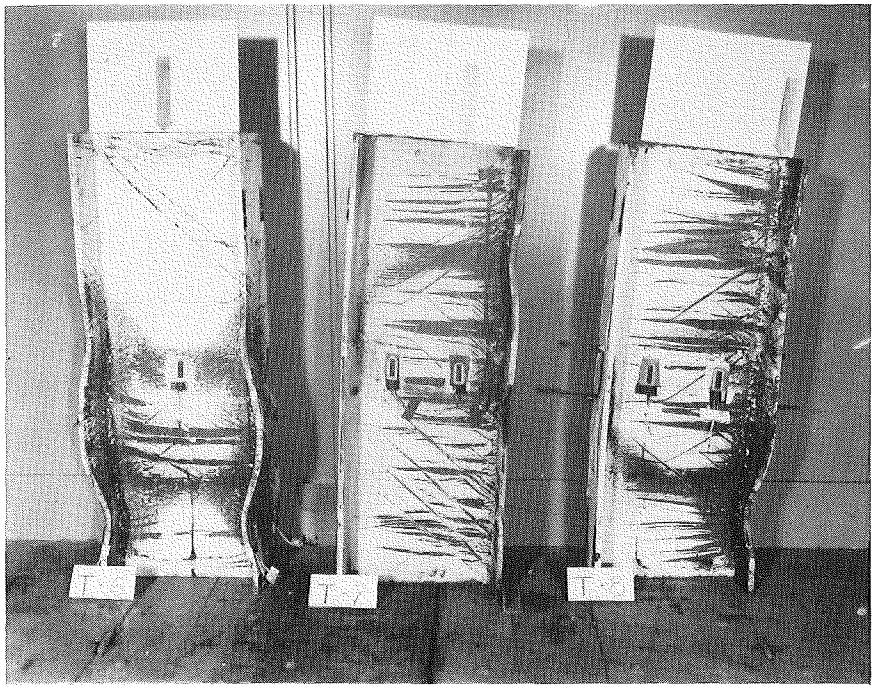
By using special column testing fixtures, an eccentric compression load was applied to a short section of 12WF36.

32



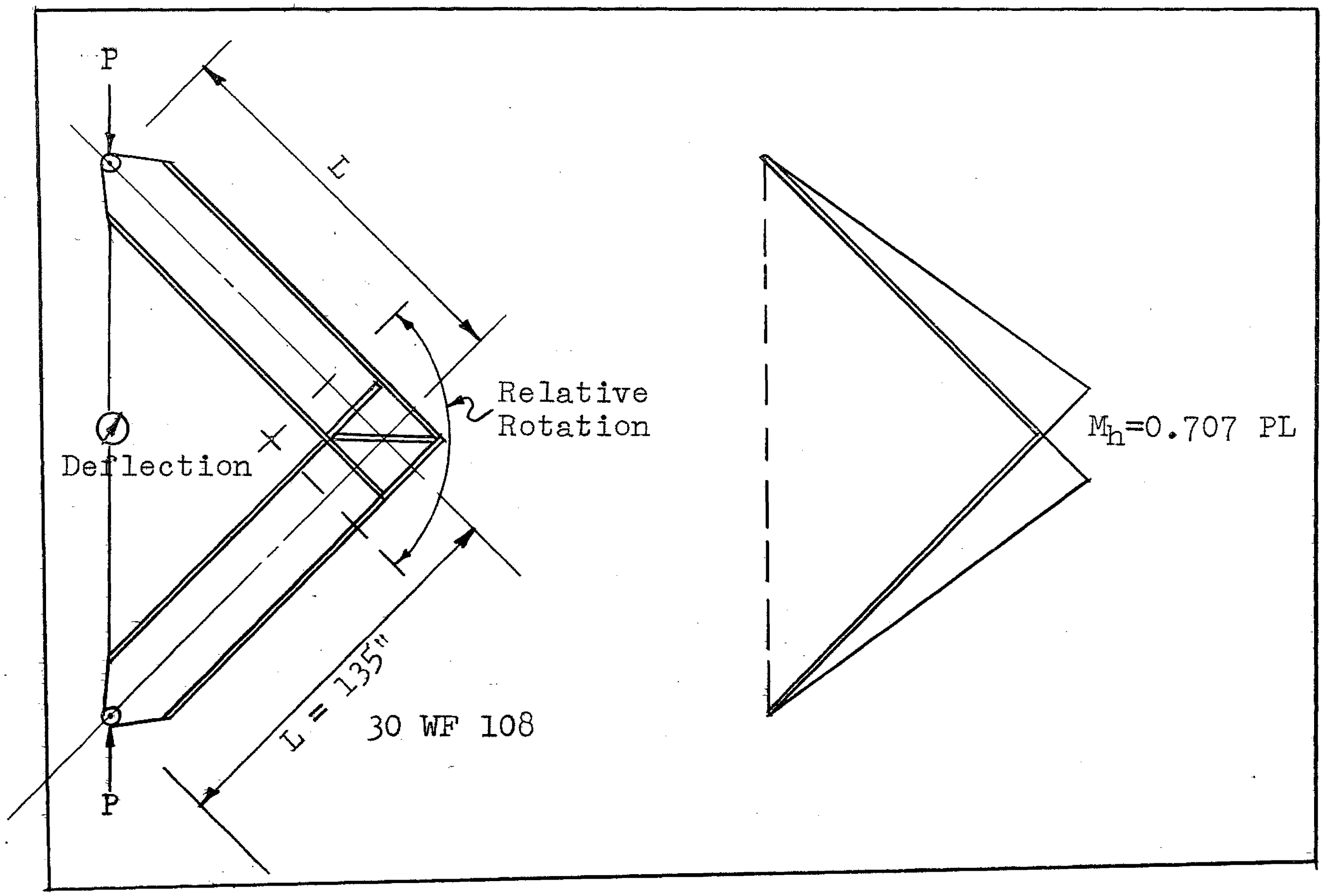
The results of the test showed the effect of axial load on plastic bending moment.

33



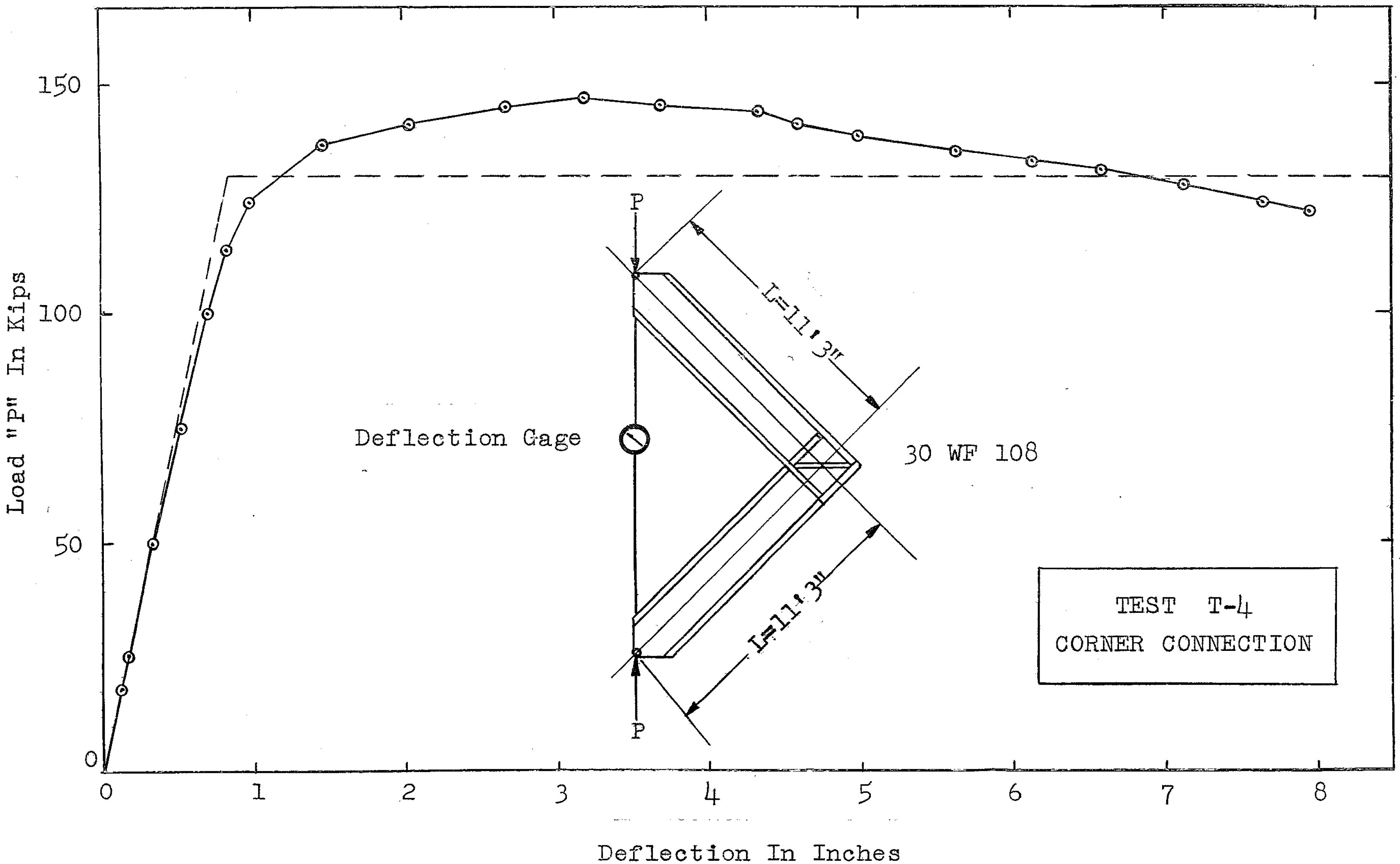
32A

An axial cross section test of a 12WF36 section was made to obtain the compressive properties, and two members were loaded eccentrically to show the effect of axial load in reducing the plastic bending moment.



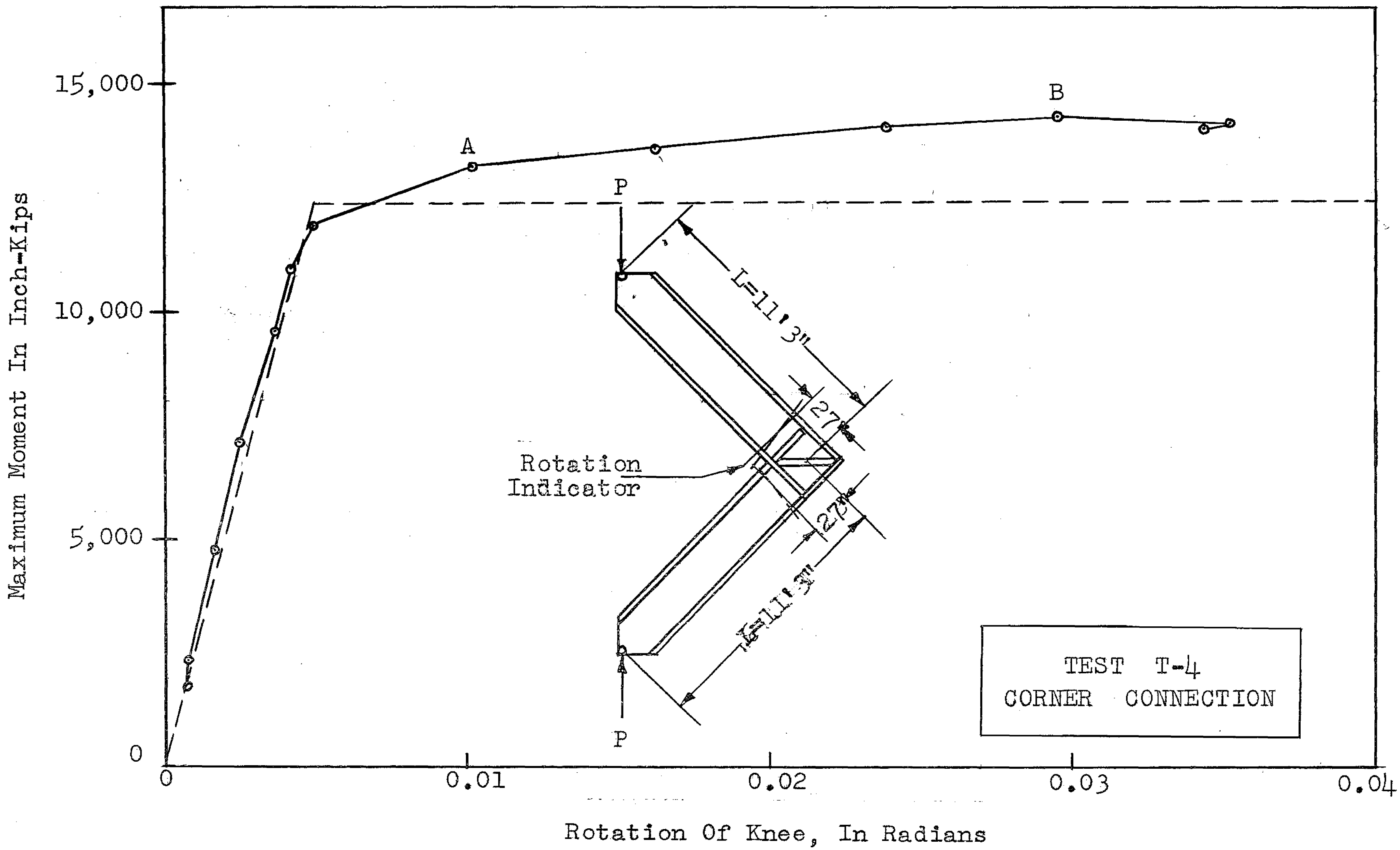
SCHEMATIC DIAGRAM OF CONNECTION TEST SETUP (T-4)

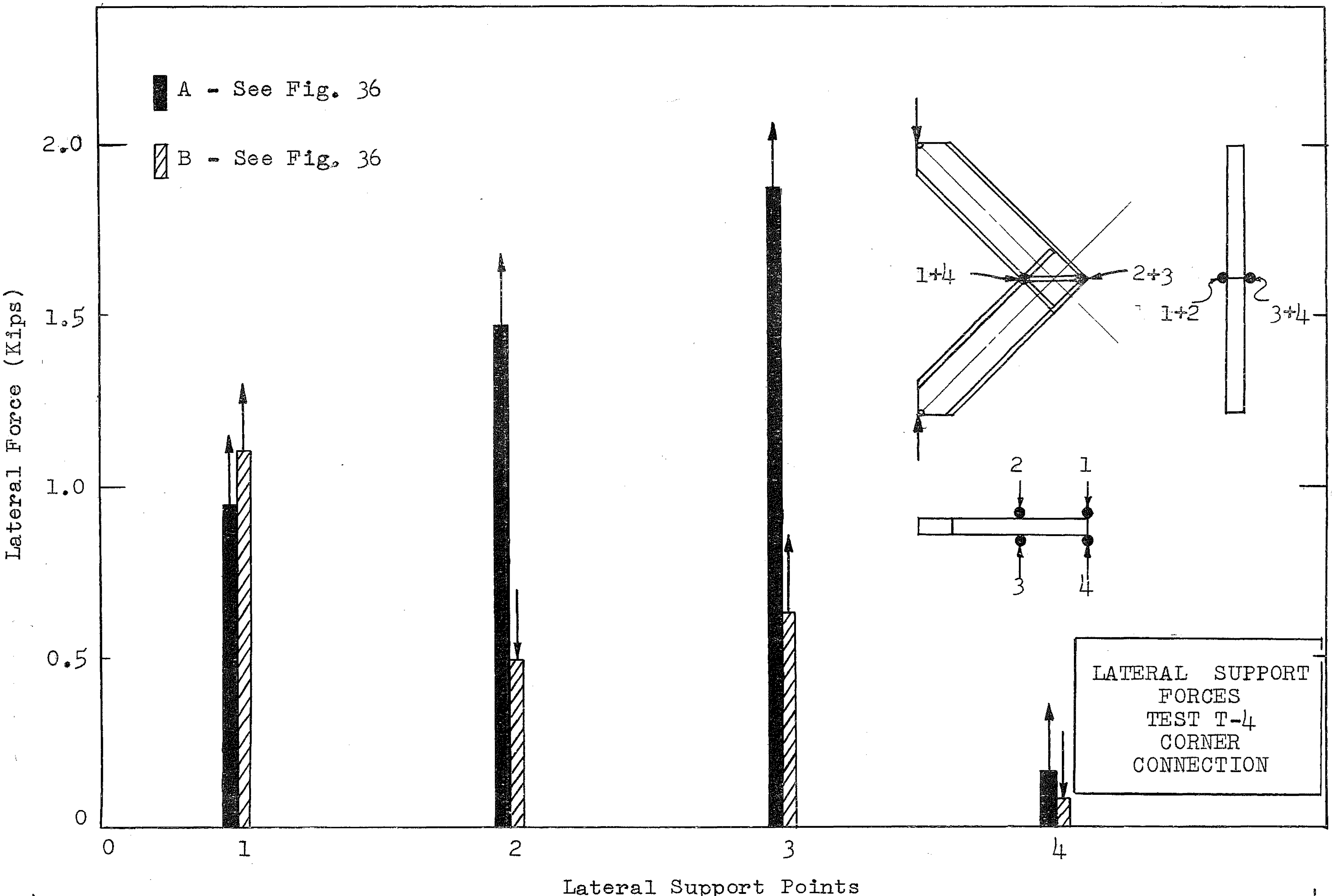
34



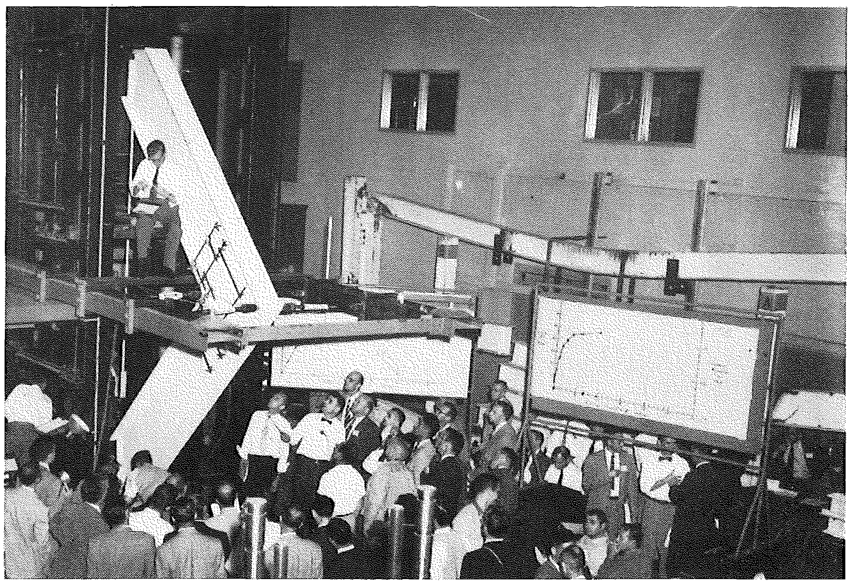
150
100
50
0
Load "P" In Kips
0
1
2
3
4
5
6
7
8
Deflection In Inches
P
L=11'3"
Deflection Gage
30 WF 108
L=11'3"
P
TEST T-4
CORNER CONNECTION

35


15,000
10,000
5,000
0
0
0.01
0.02
0.03
0.04
Maximum Moment In Inch-Kips
Rotation Of Knee, In Radians
A
B
P
P
L=11'3"
27"
27"
Rotation Indicator
TEST T-4
CORNER CONNECTION


A - See Fig. 36
B - See Fig. 36
2.0
1.5
1.0
0.5
0
Lateral Force (Kips)
0
1
2
3
4
Lateral Support Points
1+4
2+3
1+2
3+4
2
1
3
4
LATERAL SUPPORT FORCES TEST T-4 CORNER CONNECTION

37

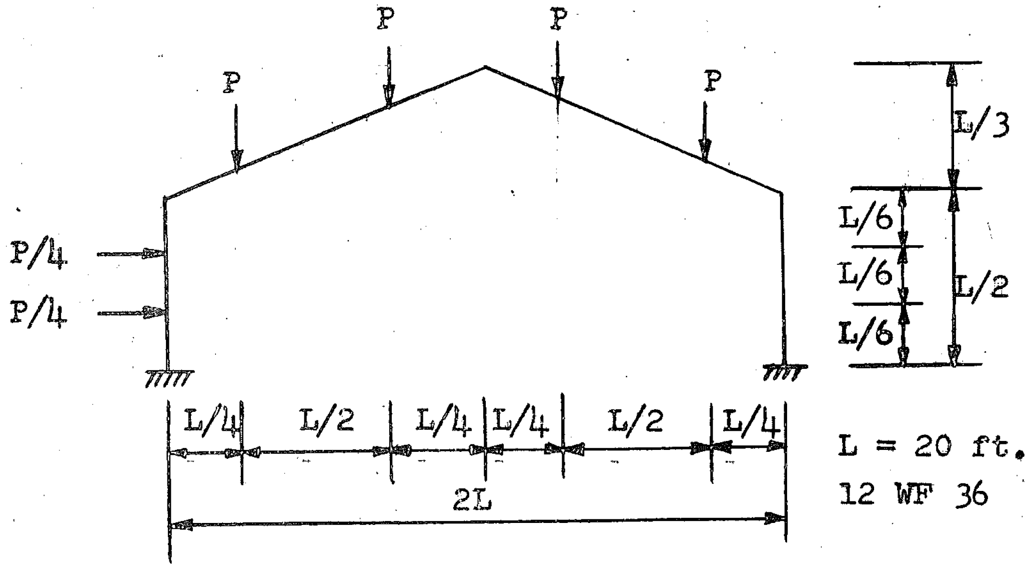


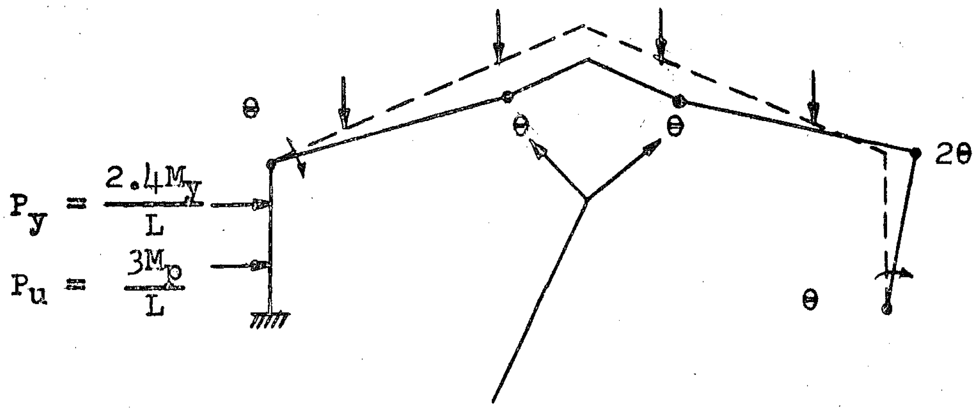
38

Application of load to the legs of the 30WF108 corner connection caused a combination of thrust shear and bending moment at the knee.

39

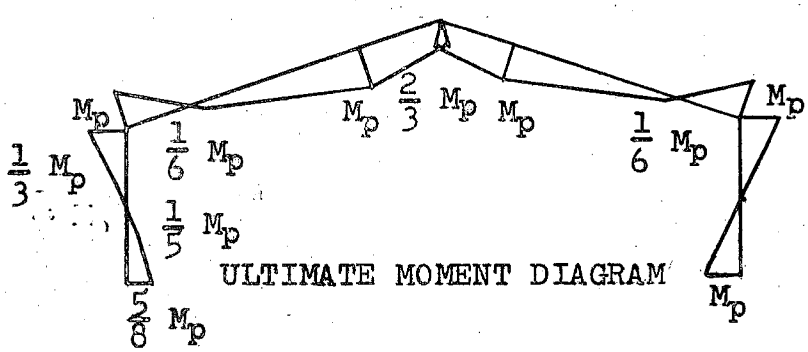
After the ultimate load on the connection was reached, local buckling appeared in the web and compression flange.





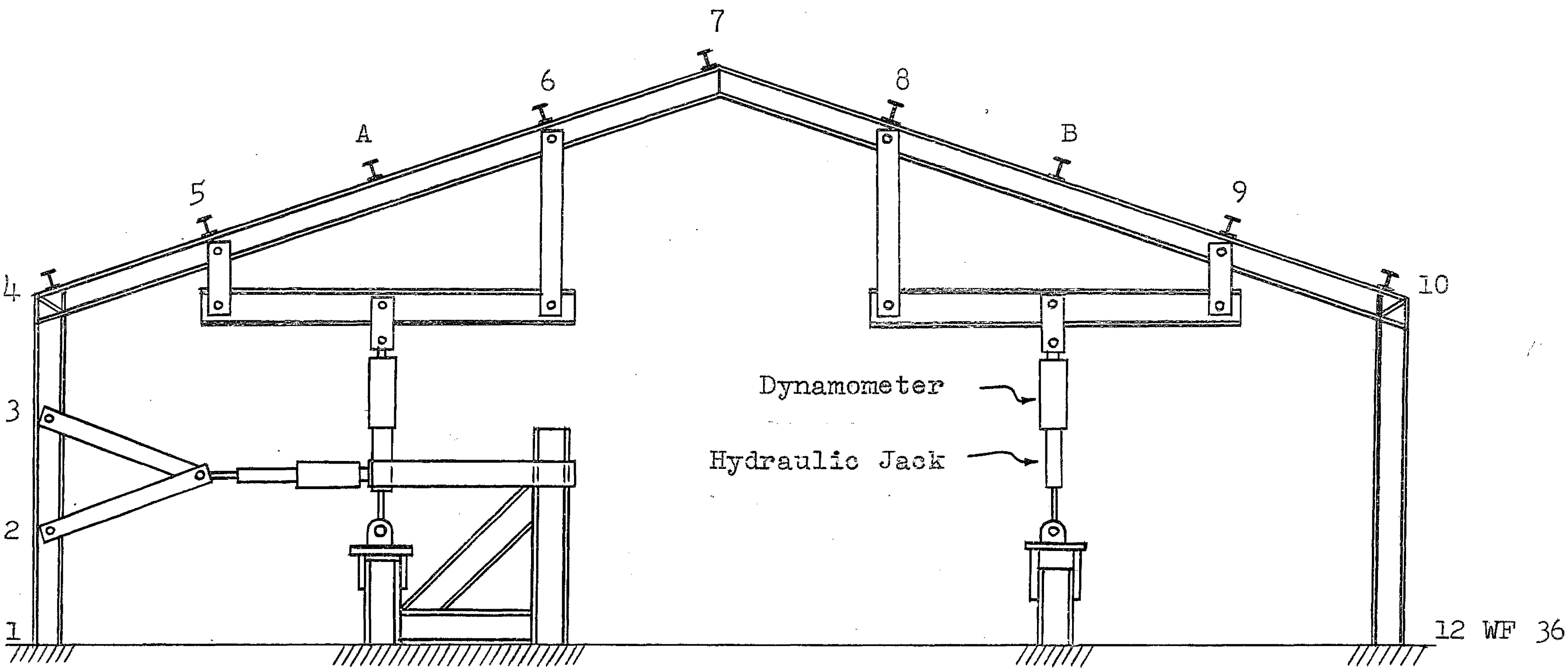
SIMULTANEOUS FORMATION
OF PLASTIC HINGES

MECHANISM



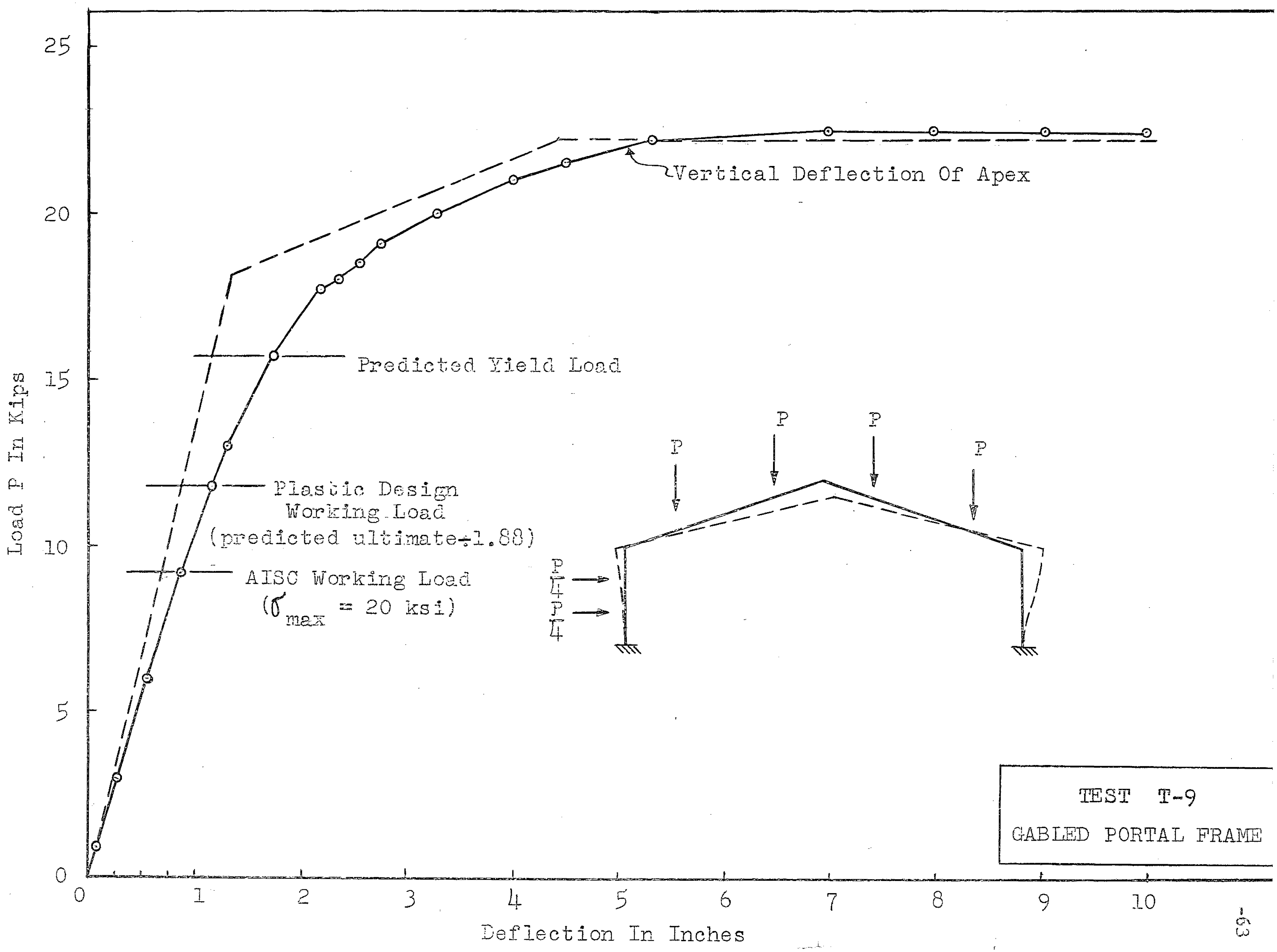
ULTIMATE MOMENT DIAGRAM

SCHEMATIC DIAGRAMS FOR GABLED PORTAL FRAME (T-9)

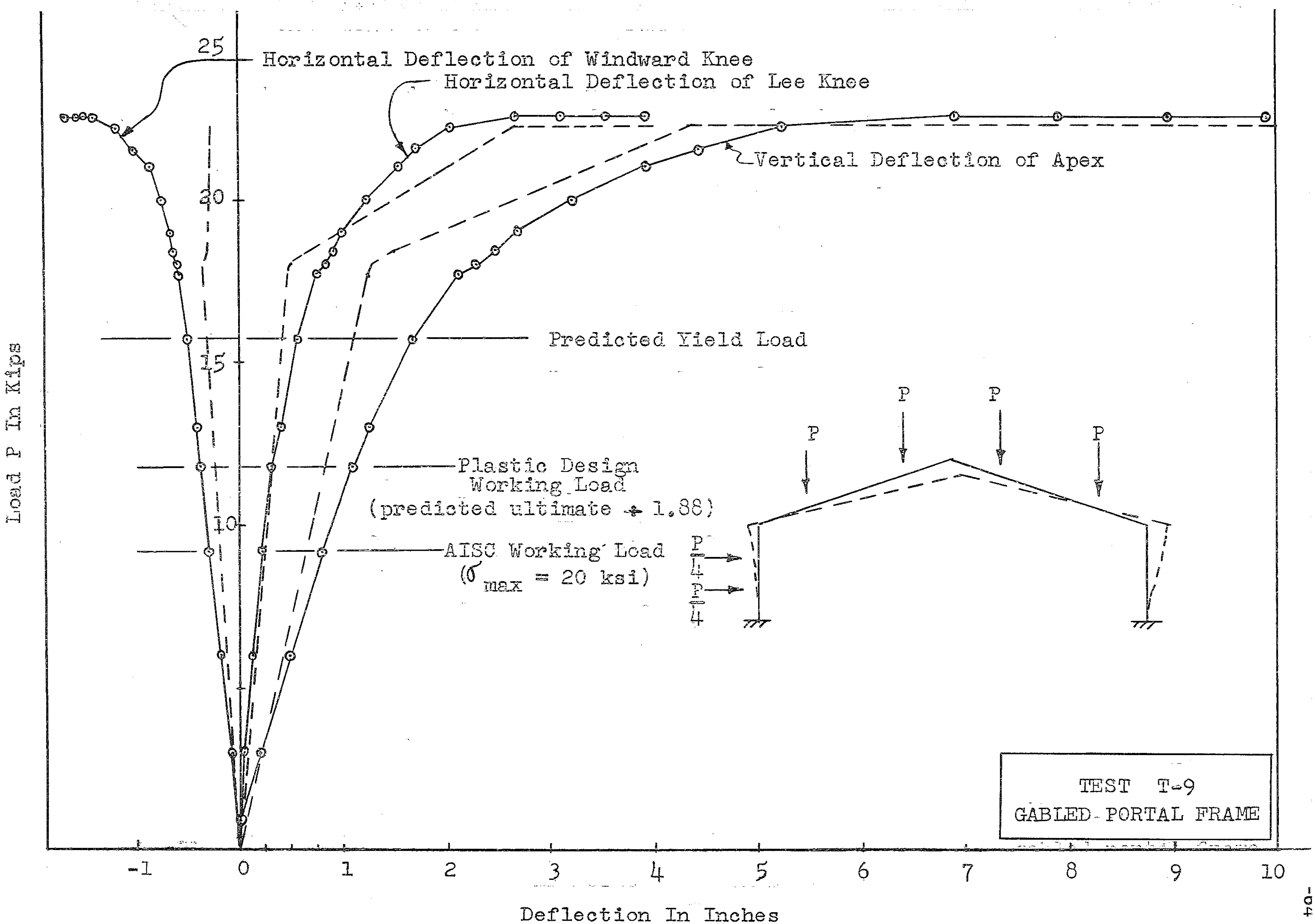


SETUP FOR GABLED PORTAL FRAME TEST (T-9)

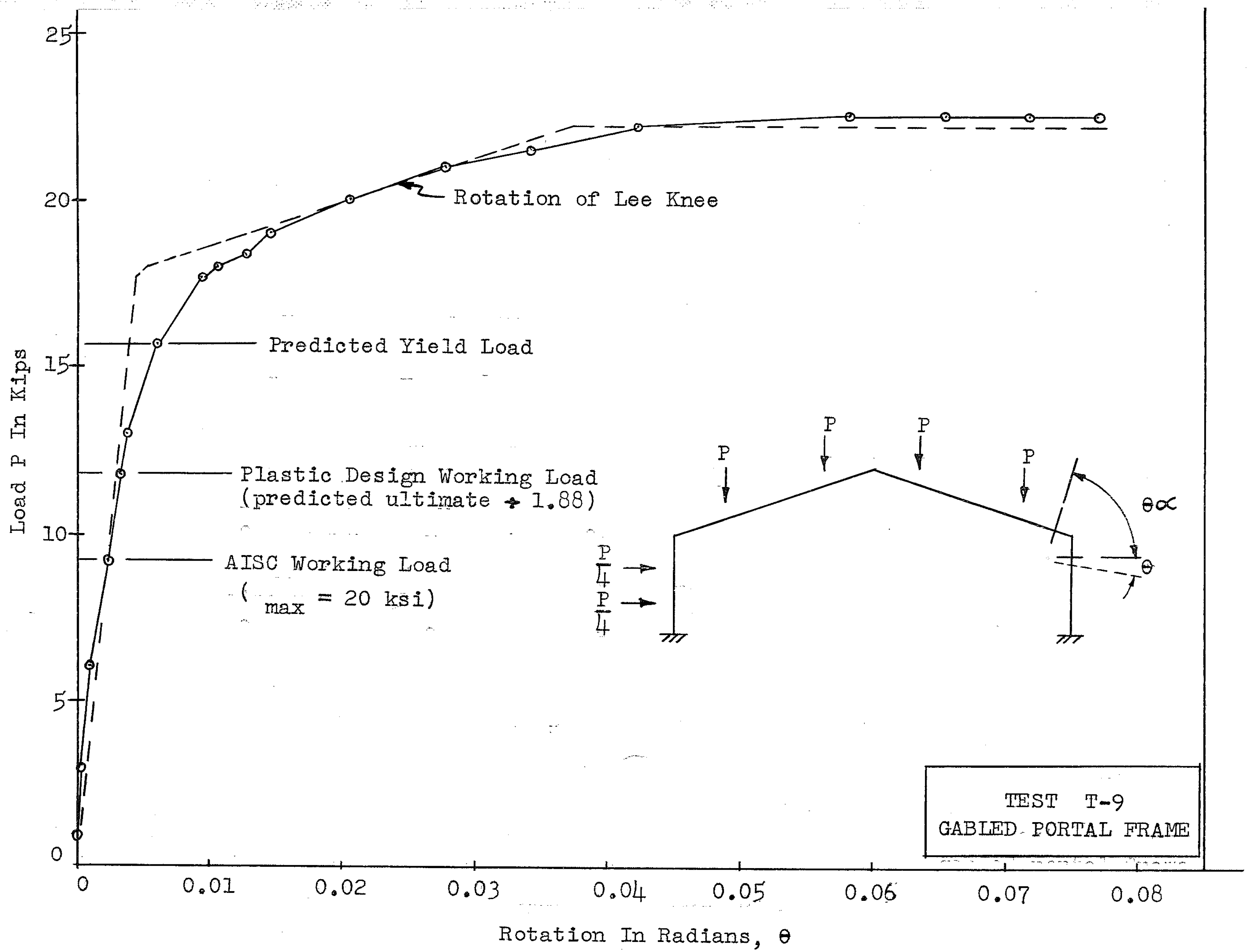
41



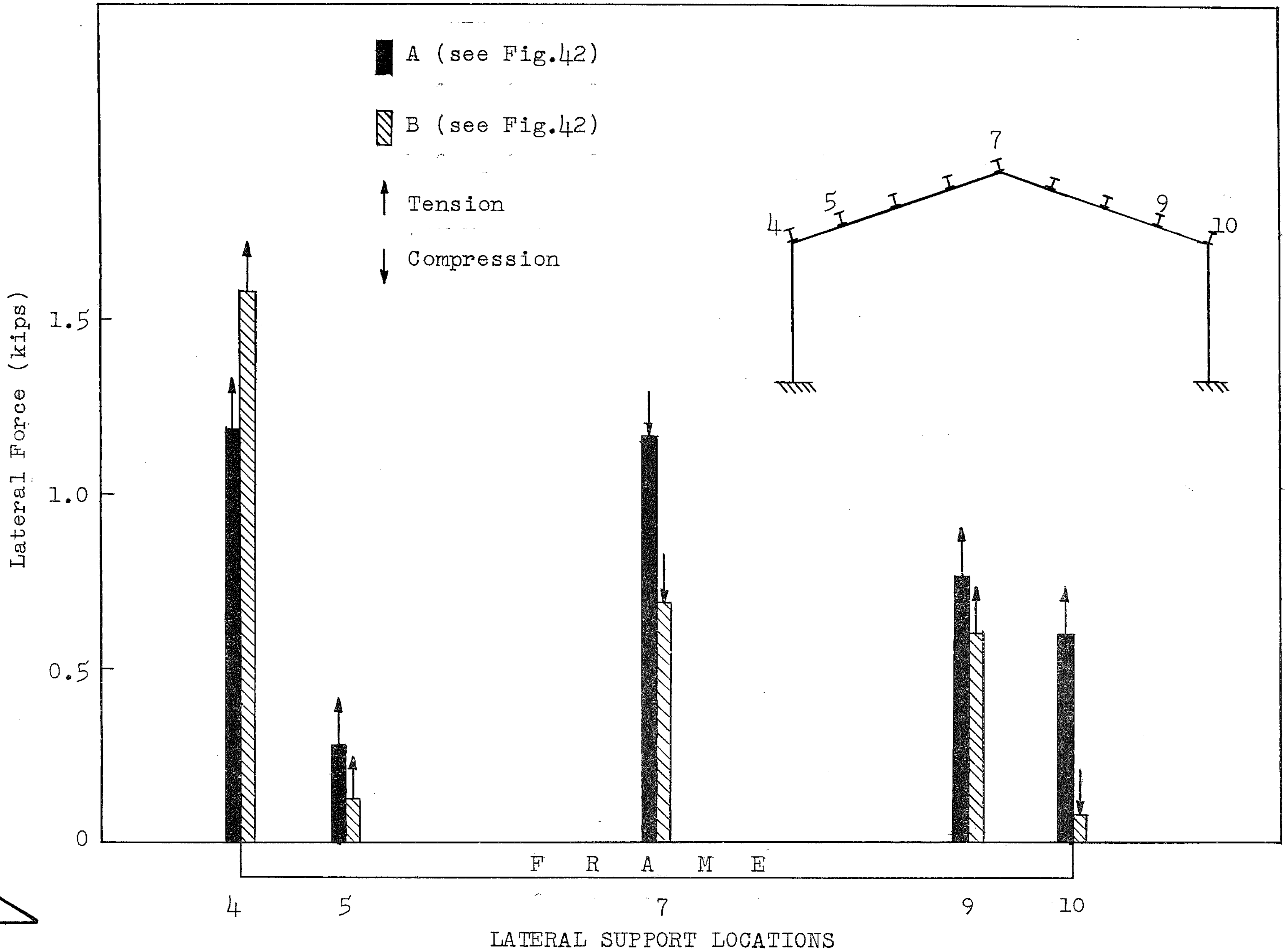
25
20
15
10
5
0
Load P In Kips
0
1
2
3
4
5
6
7
8
9
10
Deflection In Inches
Vertical Deflection Of Apex
Predicted Yield Load
Plastic Design
Working Load
(predicted ultimate÷1.88)
AISC Working Load
(σ_{max} = 20 ksi)
P
P
P
P
P/4
P/4
TEST T-9
GABLED PORTAL FRAME



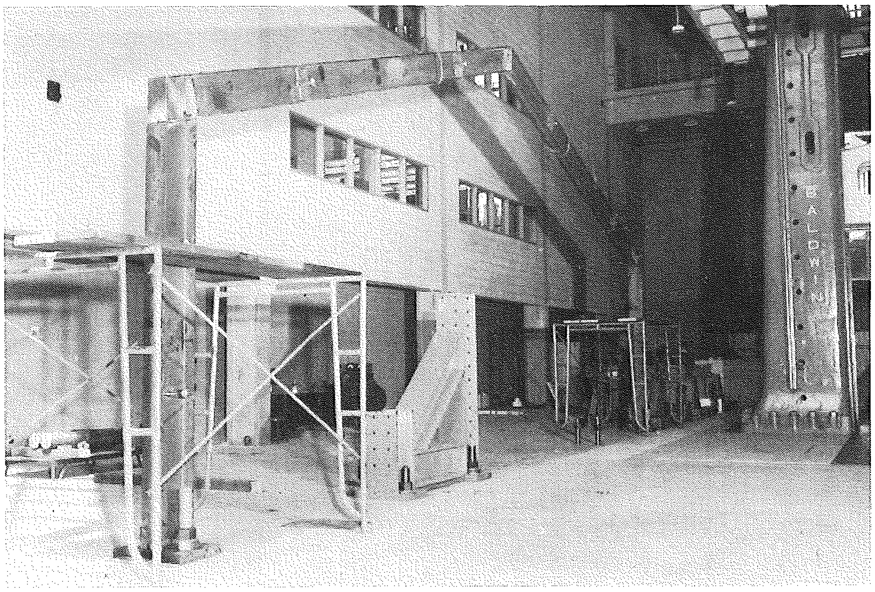
TEST T-9
GABLED PORTAL FRAME



Load P In Kips
Rotation In Radians, θ
Rotation of Lee Knee
Predicted Yield Load
Plastic Design Working Load
(predicted ultimate ÷ 1.88)
AISC Working Load
(max = 20 ksi)
P
P/4
θα
θ
TEST T-9
GABLED PORTAL FRAME

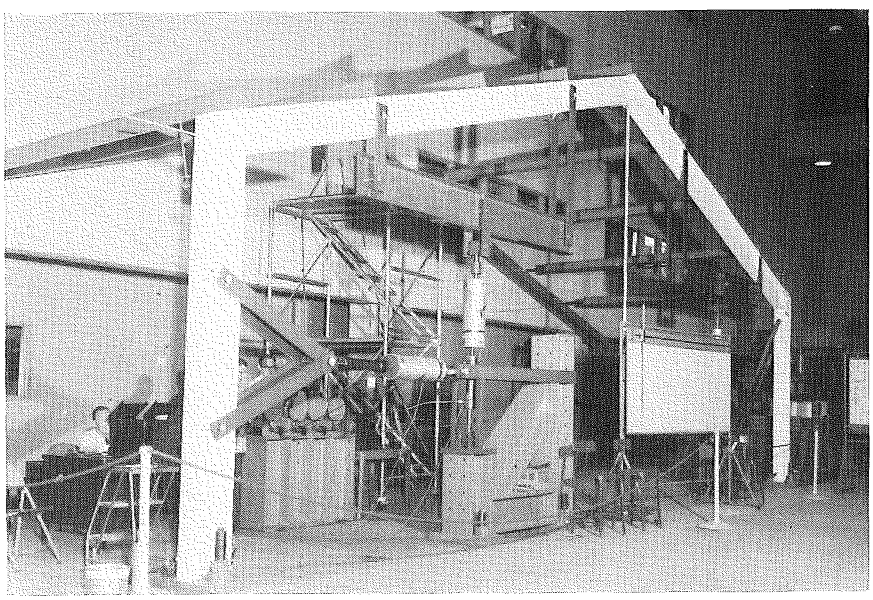

A (see Fig.42)
B (see Fig.42)
Tension
Compression
7
5
9
4
10
1.5
1.0
0.5
0
Lateral Force (kips)
F R A M E
4
5
7
9
10
LATERAL SUPPORT LOCATIONS

45



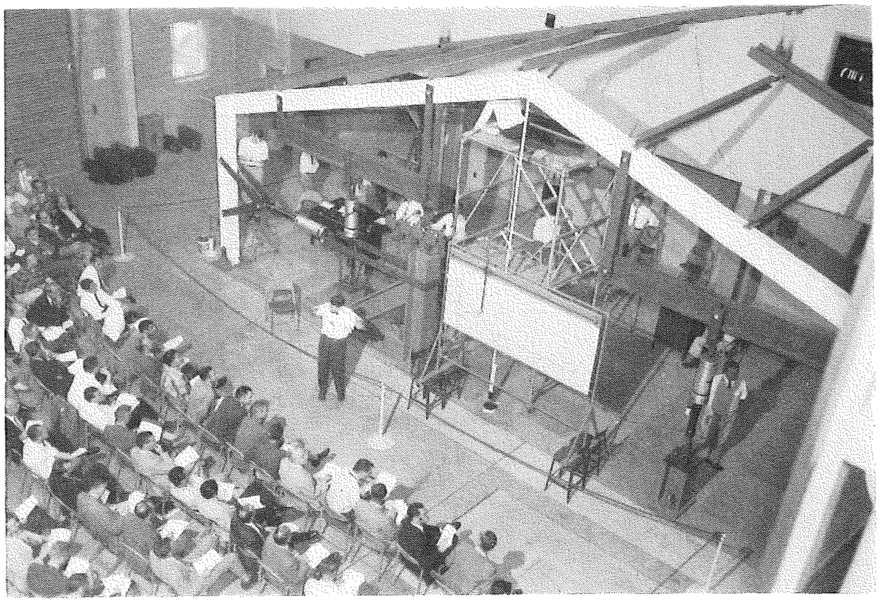
46

A gabled portal frame of 40 foot span was fabricated of 12WF36 members and erected on the laboratory test floor ....



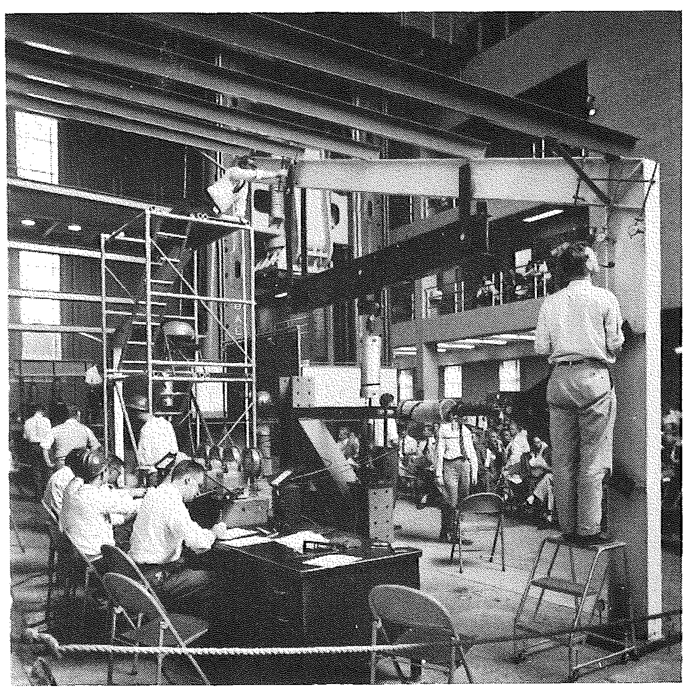
47

......after which loading equipment and instruments were added to the setup.



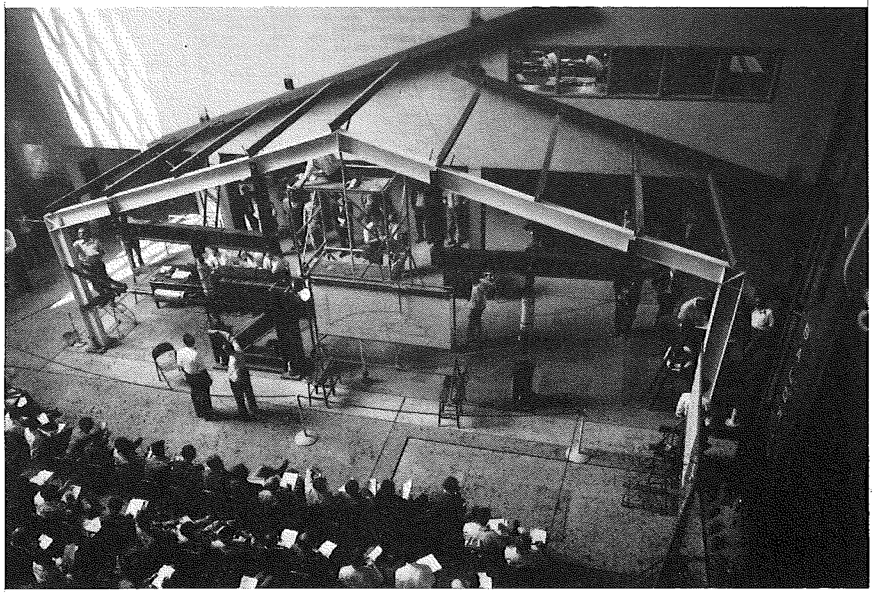
48

A narrator described the application of loads to the structure as the test commenced.



49

Meanwhile, at the rear of the test setup a crew manned the loading pumps and measuring instruments.

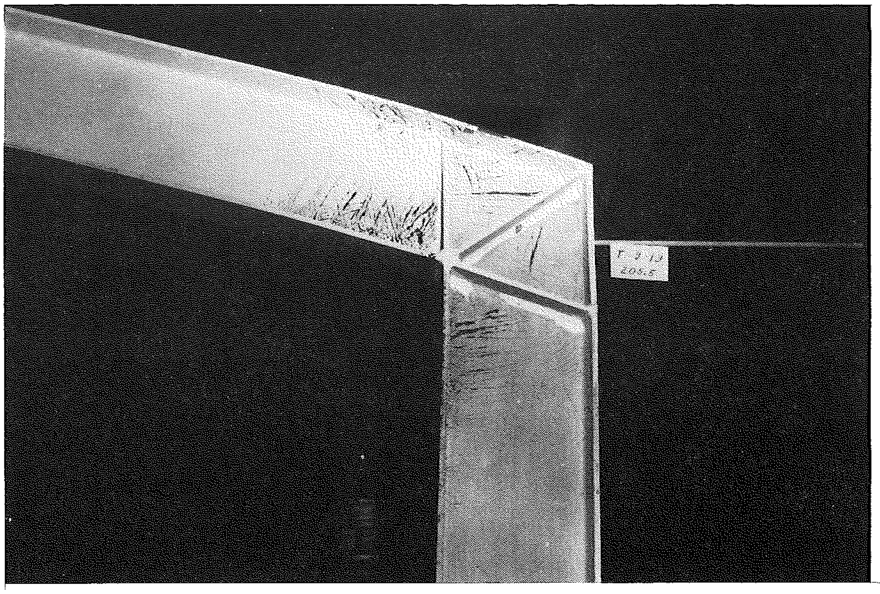


50

As the load neared ultimate load, yielding could be noted at the knees, bases and center load points.

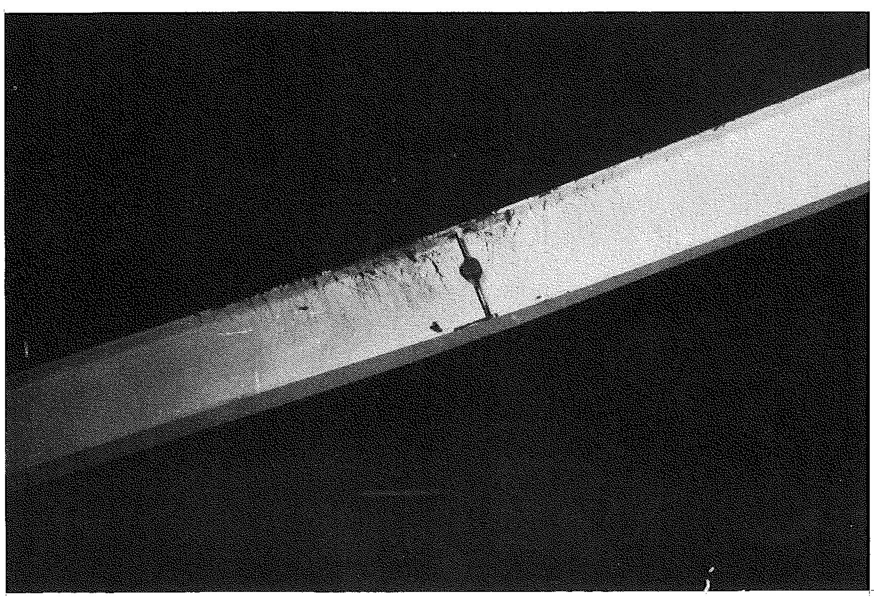
51

At ultimate load, with the mechanism fully formed, deflections were visible but not excessive.



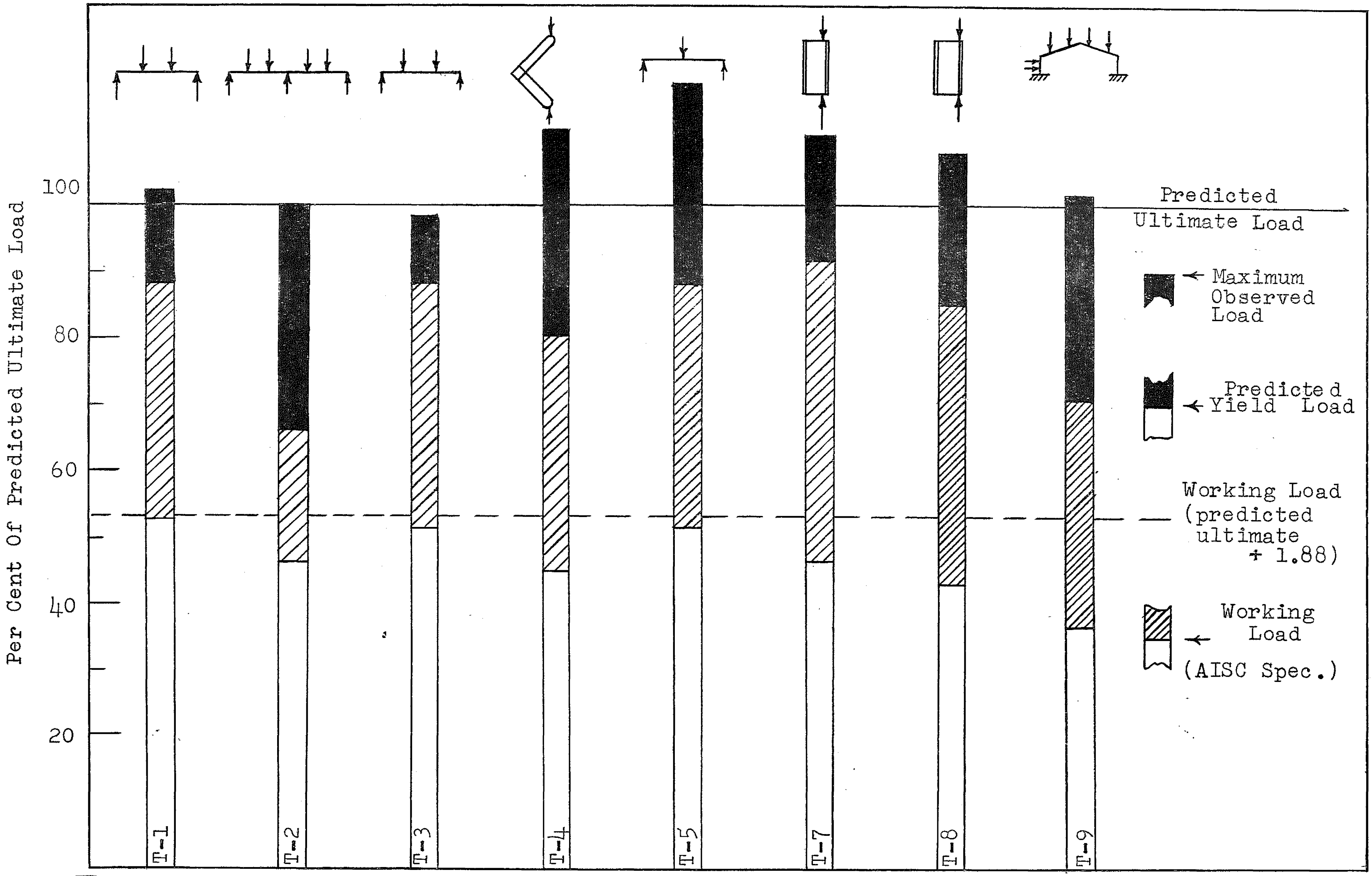
52

After the conclusion of the test, the first plastic hinge in the leeward knee showed considerable yielding but no buckling.

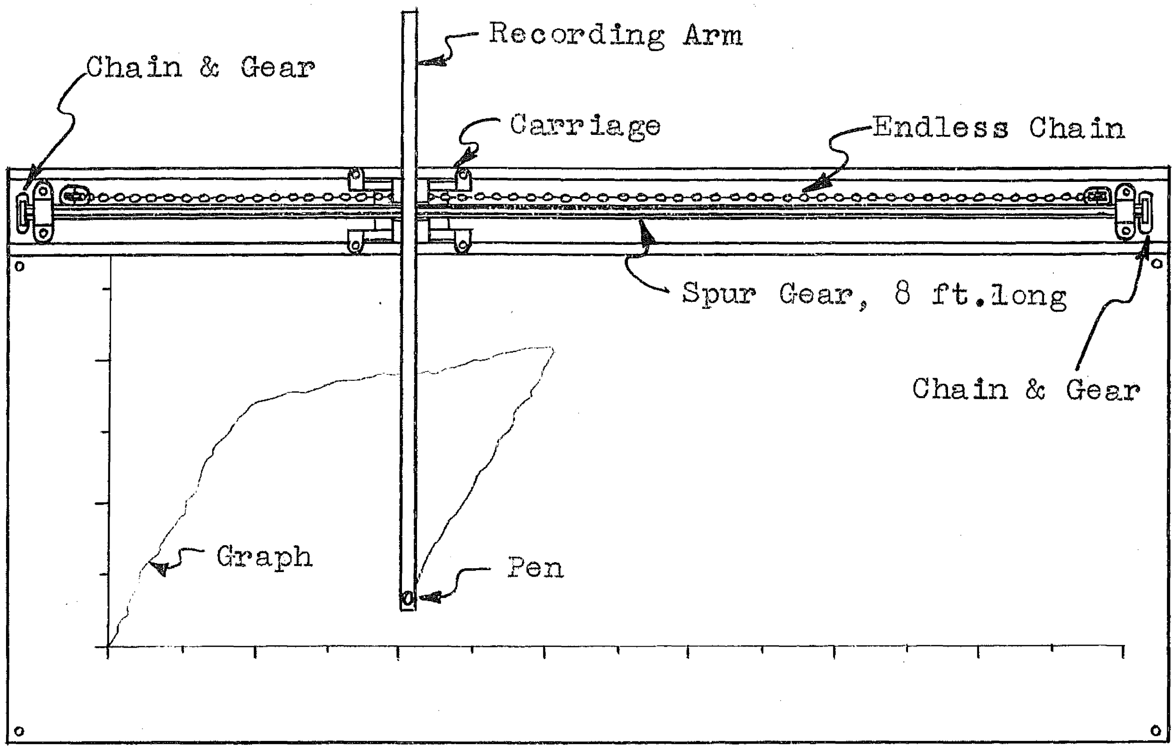


53

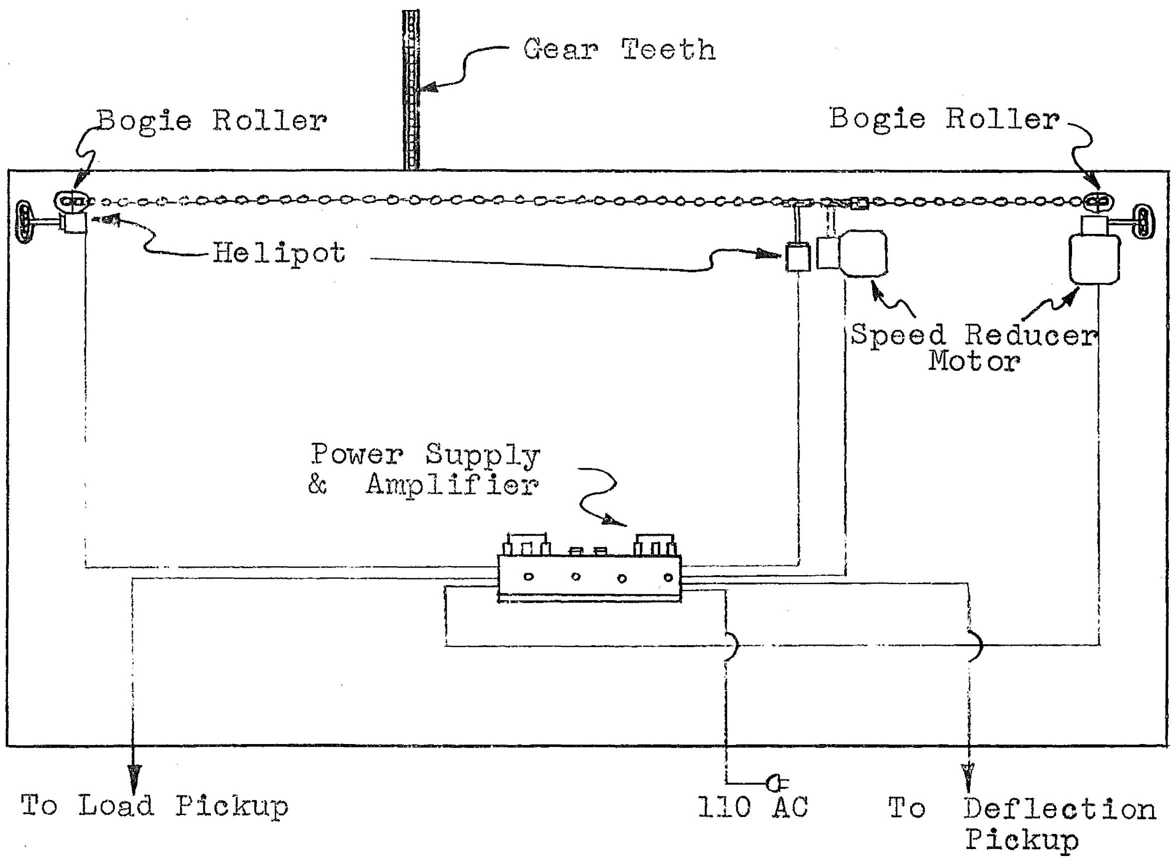
Pronounced yielding was also visible at the last plastic hinge which formed at the windward center load point.



SUMMARY OF TEST RESULTS



FRONT VIEW



BACK VIEW

AUTOMATIC PLOTTING BOARD

9. APPENDIX

Instruments Used in Tests

1. Automatic Plotting Board

The automatic plotting board is a device designed to plot a curve of load versus deflection electro-mechanically to large scale while a test is in progress. The board was designed and built by Mr. Ivan J. Taylor, Instruments Associate of Fritz Laboratory, especially for the Summer Course on Plastic Design.

A sketch of the plotting board is shown in Fig. 55. The experimental curve is drawn by a brush pen on a replaceable paper chart 8 ft. by 4 ft. in size. The pen is carried on a recording arm which is mounted to slide vertically on the carriage. The carriage has four wheels resting on tracks which allow it to be pulled horizontally across the panel. Horizontal motion is imparted to the carriage by an endless chain driven by a speed-reducer motor. The recording arm is driven vertically by the rotation of an 8-foot long spur gear coupled to a second speed-reducer motor.

The amount of motion of the driving motors is controlled by two bridge circuits, each containing two linear potentiometers, a voltage-controlled reversing switch, and a power supply. Each circuit is connected in such a way that if there is an unbalance between the two potentiometers, current will be directed to the motor, causing it to run either forward or backward. The motor is coupled by gear and chain drive to the second potentiometer as well as to its component of the recorder. This allows the motor to drive the second potentiometer until it balances the first, and thus cuts off the current to the motor.

The first or detecting potentiometer in each circuit is actuated either by a motion of the test specimen or a motion of a testing machine load indicating mechanism. This sets up the original circuit unbalance which causes the motor to drive the recording mechanism. Gear drive ratios are set so that a desired large motion on the chart is caused by whatever range of load or motion occurs on a given specimen.

Calibration of the plotting board is accomplished preliminary to actual testing by moving the detecting elements through several measured increments and marking the coordinate axes of the chart to correspond to these increments.

2. Dial Indicator

A dial indicator is a mechanical instrument used to measure small motions accurately by means of gear magnification. It is useful in combinations with level systems for many types of measurements. Dial Indicators and some of their applications are described in Reference 5.

3. Rotation Indicator

Relative rotations between two cross sections in a bending member are measured by a rotation indicator. From these rotation measurements, the curvature of the member may be calculated. Rotation indicators are described in Reference 4, and an example is shown in Fig. 17.

4. Dynamometer

A dynamometer is a load-measuring device. Dynamometers using SR-4 strain gages and strain indicators for measurement and indication are described in Reference 3.

measured increments and marking the coordinate axes of the chart to correspond to these increments.

5. Twist Indicator

A twist indicator is used to measure the twist of a beam as an indication of lateral buckling. The twist indicator consists of a weighted pendulum which has as its pivot a circular-wound linear potentiometer which may be connected electrically to a remote reading dial indicator. The unit may be fastened to the compression flange of a beam. Any twist of the beam will change the setting of the potentiometer, causing a change in reading of the indicator. This is calibrated in terms of angles of rotation.