

1 **Assessing Practical Identifiability during Calibration and Cross-**  
2 **Validation of a Structured Model for High-Solids Anaerobic Digestion**

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4 Vicente Pastor-Poquet <sup>a,b,\*</sup>, Stefano Papirio <sup>c</sup>, Jérôme Harmand <sup>b</sup>, Jean-Philippe Steyer <sup>b</sup>,  
5 Eric Trably <sup>b</sup>, Renaud Escudie <sup>b</sup>, and Giovanni Esposito <sup>c</sup>

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7 <sup>a</sup> Department of Civil and Mechanical Engineering, University of Cassino and Southern  
8 Lazio, via Di Biasio 43, 03043 Cassino (FR), Italy

9 \* Corresponding author. E-mail: [vicente.pastor.poquet@gmail.com](mailto:vicente.pastor.poquet@gmail.com)

10 <sup>b</sup> LBE, Univ. Montpellier, INRA, 102 Avenue des Etangs, 11100, Narbonne, France

11 <sup>c</sup> Department of Civil, Architectural and Environmental Engineering, University of  
12 Napoli Federico II, via Claudio 21, 80125 Napoli, Italy

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14

15 **ABSTRACT**

16 High-solids anaerobic digestion (HS-AD) of the organic fraction of municipal solid  
17 waste (OFMSW) is operated at a total solid (TS) content  $\geq 10\%$  to enhance the waste  
18 treatment economy, though it might be associated to free ammonia (NH<sub>3</sub>) inhibition.

19 This study aimed to calibrate and cross-validate a HS-AD model for homogenized  
20 reactors in order to assess the effects of high NH<sub>3</sub> levels in HS-AD of OFMSW, but also  
21 to evaluate the suitability of the reversible non-competitive inhibition function to  
22 reproduce the effect of NH<sub>3</sub> on the main acetogenic and methanogenic populations. The  
23 practical identifiability of structural/biochemical parameters (i.e. 35) and initial  
24 conditions (i.e. 32) was evaluated using batch experiments at different TS and/or

25 inoculum-to-substrate ratios. Variance-based global sensitivity analysis and  
26 approximate Bayesian computation were used for parameter optimization. The  
27 experimental data in this study permitted to estimate up to 8 biochemical parameters,  
28 whereas the rest of parameters and biomass contents were poorly identifiable. The study  
29 also showed the relatively high levels of  $\text{NH}_3$  (i.e. up to 2.3 g N/L) and ionic strength  
30 (i.e. up to 0.9 M) when increasing TS in HS-AD of OFMSW. However, the  $\text{NH}_3$  non-  
31 competitive function was unable to capture the acetogenic/methanogenic inhibition.  
32 Therefore, the calibration emphasized the need for target-oriented experimental data to  
33 enhance the practical identifiability and the predictive capabilities of structured HS-AD  
34 models, but also the need for further testing the  $\text{NH}_3$  inhibition function used in these  
35 simulations.

36

37 **Keywords:** High-solids Anaerobic Digestion Model; Ammonia Inhibition; Ionic  
38 Strength; Global Sensitivity Analysis; Approximate Bayesian Computation.

39

## 40 1 INTRODUCTION

41 High-solids anaerobic digestion (HS-AD) of the organic fraction of municipal solid  
42 waste (OFMSW) is operated at total solids (TS)  $\geq 10\%$  to minimize the reactor volume,  
43 the water addition and the digestate dewatering (Pastor-Poquet et al., 2019a). HS-AD  
44 can also lead up to 80 % TS removal, easing the digestate post-treatment. However, HS-  
45 AD of OFMSW is usually associated to free ammonia ( $\text{NH}_3$ ) inhibition, resulting in  
46 volatile fatty acids (VFA) accumulation.  $\text{NH}_3$  affects both acetoclastic and  
47 hydrogenotrophic methanogens but also the rest of VFA degraders (acetogens) in  
48 anaerobic digestion (AD), being the inhibition related to the operative parameters (i.e.  
49 temperature and pH) and the biomass acclimation (Rajagopal et al., 2013).

50

51 Adding lignocellulosic materials to OFMSW permits to minimize the buildup of total  
52 ammonia nitrogen (TAN), while their low hydrolysis rates permit to increase the TS  
53 content and to counteract the VFA accumulation in HS-AD (Capson-Tojo et al., 2017;  
54 Pastor-Poquet et al., 2019a). However, including lignocellulosic waste in OFMSW  
55 depends on the season or the local waste management strategy. Whether or not a  
56 lignocellulosic co-substrate is used, understanding the effects of  $\text{NH}_3$  inhibition is  
57 crucial to optimize HS-AD of OFMSW.

58

59 A HS-AD model was recently developed for homogenized reactors to evaluate the  $\text{NH}_3$   
60 inhibition in HS-AD of OFMSW (Pastor-Poquet et al., 2018, 2019b). This structured  
61 model, based on the Anaerobic Digestion Model No. 1 (ADM1) (Batstone et al., 2002),  
62 gathers the main bio-physical-chemical mechanisms in HS-AD. In the HS-AD model,  
63 apparent (i.e.  $\text{kmol/kg H}_2\text{O}$ ) instead of global (i.e.  $\text{kmol/kg}$ ) concentrations determine

64 the effect of TS upon solutes, as a consequence of the low water content within HS-AD.  
 65 Meanwhile, an extended set of mass balances allows the simulation of the organic mass  
 66 removal from the biogas production. A liquid solution ‘non-ideality’ subroutine was  
 67 subsequently included in the model as a function of the ionic strength ( $I$ ), since ‘non-  
 68 ideality’ determines the pH, CO<sub>2</sub> liquid-gas transfer and NH<sub>3</sub> inhibition in HS-AD  
 69 (Pastor-Poquet et al., 2019b). As a preliminary assumption, the HS-AD model included  
 70 a reversible non-competitive inhibition function by NH<sub>3</sub> ( $I_{nh3}$ ) [Equation 1] in the  
 71 biochemical rates of acetogenic and methanogenic populations, a mathematical resource  
 72 commonly used in structured AD models (Astals et al., 2018).

$$I_{nh3} = \frac{K_{i,Snh3}}{K_{i,Snh3} + S_{nh3,App}} \quad (1)$$

74  
 75 Simulating the NH<sub>3</sub> inhibition at high TS with the HS-AD model requires an adequate  
 76 set of input parameters,  $\theta$ , to be estimated by calibration (Pastor-Poquet et al., 2018,  
 77 2019b).  $\theta$  includes both the structural/biochemical parameters,  $\theta_P$ , and the initial and  
 78 influent conditions,  $\theta_B$ :  $\theta = (\theta_P, \theta_B)$ . Nonetheless, calibration of structured AD models is  
 79 not trivial due to the equation complexity and large number of  $\theta$  involved (Dochain &  
 80 Vanrolleghem, 2001; Donoso-Bravo et al., 2011).

81  
 82 To calibrate a mathematical model,  $\theta$  must be structurally and practically identifiable,  
 83 instead of correlated. The  $\theta$  structural identifiability is theoretically assessed, assuming  
 84 noise-free experimental data and error-free model structure. Noteworthy, nearly all  $\theta$  in  
 85 ADM1 are (locally) structurally identifiable (Nimmegeers et al., 2017). This is a  
 86 prerequisite to assess the  $\theta$  practical identifiability and calibrate the HS-AD model using

87 ‘imperfect’ experimental data. Unfortunately, the reduced number of experimental data  
88 often available and/or the presence of experimental errors yield non-identifiable  
89 parameters; i.e. parameters that cannot be uniquely estimated. These are known as  
90 practical identifiability issues.

91

92 Calibration usually consists of minimizing an objective function,  $J(\theta)$ , that condenses  
93 the ‘goodness of fitting’ between the experimental data,  $y$ , and the model outputs,  $y^{sim}(\theta)$ ,  
94 being these a function of  $N$  input parameters,  $\theta$  (Dochain & Vanrolleghem, 2001;  
95 Flotats et al., 2010). Several  $J(\theta)$  can be used to calibrate AD models as the weighted  
96 sum of squares or any user-defined alternative (Donoso-Bravo et al., 2011; Ratto et al.,  
97 2001). Assuming the existence of a global minimum (optimum) for an objective  
98 function,  $J(\theta_{opt})$ , this value is reached using the optimal set of input parameters,  $\theta_{opt}$ .

99

100 Practical identifiability issues commonly translate into  $J(\theta)$  showing many local  
101 optimums and/or flat valleys, where the precise value of  $\theta$  cannot be easily determined  
102 (Guisasola et al., 2006; Rodriguez-Fernandez et al., 2006). Thus, only practical  
103 identification of a reduced  $\theta$  subset (i.e.  $N' < N$ ) is often possible for ADM1-based  
104 models (Nimmegeers et al., 2017). This is called model over-parameterization, where  
105 the modification of two individual  $\theta$ ,  $\theta_i$  (with  $i = 1, \dots, N$ ), can lead to a similar model  
106 output. Particularly, when using batch experiments – highly dependent on the initial  
107 conditions – to calibrate AD models, different sets of experimental conditions, including  
108 different inoculum-to-substrate ratios (ISR), are required to reduce the  $\theta$  correlation  
109 (Donoso-Bravo et al., 2011; Flotats et al., 2010).

110

111 Two main approaches can be used to calibrate complex models: the Bayesian and the  
112 frequentist. The frequentist approach searches for optimal  $\theta$  values,  $\theta_{opt}$ , whereas the  
113 Bayesian approach considers  $\theta_{opt}$  as probabilistic distributions conditioned on the  
114 experimental data,  $p(\theta_{opt}|y)$ , instead of single values (Ratto et al., 2001; Rodriguez-  
115 Fernandez et al., 2006; Saltelli et al., 2006). In both approaches, when facing over-  
116 parameterization, it must be assessed which  $\theta_i$  significantly influence  $y^{sim}(\theta)$  (sensitivity  
117 analysis) and need to be adequately calibrated.

118

119 ADM1-based models contain several  $\theta_P$  (i.e.  $\geq 35$ ) and  $\theta_B$  (i.e.  $\geq 24$ ).  $\theta_P$  might be  
120 obtained from literature, though a different model structure – from where these  $\theta_P$  were  
121 obtained – potentially influences the optimal  $\theta_P$  values/distributions (Pastor-Poquet et  
122 al., 2019b). On the other hand,  $\theta_B$  might not be easily determined due to the lack of  
123 experimental data or the difficulties to translate the data into adequate model units  
124 (Donoso-Bravo et al., 2011; Flotats et al., 2003).

125

126 Parameter inference based on the Bayes' theorem [Equation 2] is particularly suited to  
127 calibrate structured AD models since it can deal with complex  $J(\theta)$  showing several  
128 optima or flat geometries, where frequentist inference might not be well suited  
129 (Kennedy & O'Hagan, 2001; Toni & Stumpf, 2010). In Bayesian inference, the prior  
130 parameter distribution,  $p(\theta)$ , is sampled to obtain the posterior parameter distribution,  
131  $p(\theta|y)$ , conditioned on the experimental data,  $y$ , and the likelihood function,  $p(y|\theta)$ ,  
132 while  $p(y)$  can be considered as a normalizing constant. Importantly, any user-defined  
133  $J(\theta)$  arising from  $p(y|\theta)$  can be used in 'informal' statistical approaches [Equation 3], as

134 variance-based global sensitivity analysis (GSA) and approximate Bayesian  
 135 computation (ABC) (Donoso-Bravo et al., 2011; Nott et al., 2012).

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \quad (2)$$

$$J(\theta) = f(p(y|\theta)) \quad (3)$$

136

137 Variance-based GSA provides an appropriate assessment about the potentiality of  $\theta_i$  to  
 138 influence the model outputs and the correlations existing with the rest of  $\theta$ ,  $\theta_j$  (with  $j =$   
 139  $1, \dots, N$  and  $i \neq j$ ) (Kennedy & Petropoulos, 2017; Oakley & O'Hagan, 2004). Similarly,  
 140 ABC permits also to highlight practical identifiability issues yielding simultaneously the  
 141 most likely  $p(\theta|y)$  (Beaumont et al., 2009; Filippi et al., 2013; Toni & Stumpf, 2010).

142 As main disadvantage, Bayesian inference is often computationally intensive.

143

144 The mathematical performance of the HS-AD model was previously verified, though  
 145 the model was only validated for a single HS-AD batch experiment due to the elevated  
 146 number of  $\theta$  requiring calibration (i.e.  $> 30$ ) (Pastor-Poquet et al., 2018, 2019b). Instead,  
 147 this study aimed to fully calibrate and cross-validate the HS-AD model to simulate the  
 148 effect of high  $\text{NH}_3$  levels in HS-AD of OFMSW, while testing the non-competitive  $\text{NH}_3$   
 149 inhibition function [Equation 1] on the main acetogenic/methanogenic populations. In  
 150 particular, this study assessed the practical identifiability of 35  $\theta_P$  and 32  $\theta_B$ , by using  
 151 nine HS-AD batch digesters at different TS and/or ISR as a source of experimental data.  
 152 Identifiability was assessed by variance-based GSA and ABC permitting to approximate  
 153 also  $p(\theta|y)$ . Importantly, the proposed methodology can be easily readapted to account  
 154 for further HS-AD datasets (e.g. batch, continuous) and/or model configurations.

155

156

157 **2 METHODOLOGY**158 **2.1 Experimental Data**

159 To calibrate and cross-validate the HS-AD model, while further evaluating the effects of  
160 increasing the initial TS content on HS-AD simulations, four different batch

161 experiments were used at thermophilic (55°C) conditions from 10 to 30 % TS [Table 1].

162 The laboratory-scale reactor design volume ( $V_{\text{Reactor}}$ ) was either 160 or 280 mL for the  
163 different experiments. In all experiments, centrifuged inoculum was used to increase

164 simultaneously the initial TS and ISR. These small-scale digesters were manually

165 shaken when the biogas production was measured. The batch experiments are described

166 next, whereas a thorough description of these experiments and the bio-physical-

167 chemical analyses performed was reported elsewhere (Pastor-Poquet et al., 2019a).

168

169 Experiment 1 consisted of a sacrifice test for mono-digestion of OFMSW using ISR =

170 1.00 g VS/g VS. In this experiment, the main physical-chemical dynamics (i.e. biogas

171 production and composition, TS, VS, VFA, TAN, and mono-valent ions) were

172 evaluated at different operational times. In Experiments 2 to 4, the biogas production

173 and composition were measured at different experimental times, whereas the rest of

174 physical-chemical analyses (i.e. TS, VS, VFA, TAN and ions) were only performed

175 before starting and after ending each experiment. Non-sacrifice experiments included

176 mono-digestion of OFMSW using ISR = 1.50 g VS/g VS (Experiment 2) and ISR =

177 0.50 g VS/g VS (Experiment 3), but also co-digestion of OFMSW and beech sawdust

178 using ISR = 0.16 g VS/g VS (Experiment 4).

179



180 Within Experiments 2 and 4, different initial TS contents – dilutions – were evaluated,  
181 though all the initial batch conditions contained exactly the same amount of substrate  
182 and centrifuged inoculum. Briefly: 1) the inoculum was centrifuged; 2) the same  
183 amount of substrate and inoculum was added to each bottle; 3) distilled water was  
184 added to reach the different TS contents; and 4) each bottle was manually homogenized.  
185 This strategy was aimed to use the mass balances among the different initial TS  
186 conditions, as explained in Section 2.3.1, since soluble materials were partially removed  
187 when centrifuging the inoculum. In total, nine different HS-AD batch conditions were  
188 assessed at different TS, ISR and/or co-digestion ratios, subsequently named as “Batch  
189 No. 1 to 9” [Table 1].

190

## 191 **2.2 HS-AD Model**

192 The HS-AD model included the main biochemical rates of Pastor-Poquet et al. (2018),  
193 though some minor modifications were also implemented [Table 2]. Firstly, a reversible  
194 non-competitive  $\text{NH}_3$  inhibition function [Equation 1] was included in the valeric ( $S_{va}$ ),  
195 butyric ( $S_{bu}$ ), propionic ( $S_{pro}$ ), and hydrogen ( $S_{h2}$ ) uptake rates, similarly to the  $\text{NH}_3$   
196 ( $S_{nh3}$ ) inhibition on the acetate ( $S_{ac}$ ) uptake, aiming to simulate the VFA accumulation  
197 observed in HS-AD experiments likely consequence of the  $\text{NH}_3$  buildup (Pastor-Poquet  
198 et al., 2019a). Secondly, carbohydrates ( $X_{ch}$ ) were split into readily-biodegradable  
199 ( $X_{ch,fast}$ ) and slowly-biodegradable ( $X_{ch,slow}$ ) to simulate the relatively lower hydrolysis  
200 rates of sawdust and the longer methane production observed in co-digestion  
201 experiments (i.e.  $\geq 200$  d) [Table 1]. Importantly, the hydrolysis of both  $X_{ch,fast}$  and  
202  $X_{ch,slow}$  pooled into soluble sugars ( $S_{su}$ ).

203

## 204 **2.3 Model Calibration and Validation**

205 A common set of biochemical parameters was used for all HS-AD simulations at  
 206 different ISR and/or TS. All biochemical parameters for thermophilic (55°C) AD were  
 207 extracted from Batstone et al. (2002), though some of those needed to be calibrated (i.e.  
 208  $\theta_P$ ) to improve the model fitting. The initial conditions of the batch experiments were  
 209 predefined according to the experimental data available, as described by Pastor-Poquet  
 210 et al. (2018) and also mentioned next. Moreover, different ranges of initial biomass  
 211 concentrations were used (i.e.  $\theta_B$ ) to assess the potential interrelationship of  $\theta_B$  with  $\theta_P$   
 212 in batch experiments.

213

### 214 **2.3.1 Initial Conditions**

215 The initial conditions used for each batch simulation are shown in Table 3. Since the  
 216 same amount of substrate and inoculum was used along different initial TS contents in  
 217 Experiment 2, but also in Experiment 4, mass balances were used to reduce the number  
 218 of ‘unknown’ initial conditions [Table 1]. To use mass balances, the anaerobic  
 219 biodegradability (BD) of each substrate-inoculum mixture in Experiment 2 and  
 220 Experiment 4 was assumed constant, whereas the soluble (S) and particulate (X)  
 221 components were assumed proportional, among the different TS contents. With these  
 222 assumptions, mass balances permitted to extrapolate the molar and chemical oxygen  
 223 demand (COD) concentrations as a function of the initial reactor content mass ( $M_{Global}$ ),  
 224 reactor content specific weight ( $\rho_{Global}$ ) and reactor content volume ( $V_{Global}$ ). For  
 225 example, the concentration of acetoclastic methanogens ( $X_{ac}$ ) in Batch No. 3 was  
 226 approximated from Batch No. 2 as:  $X_{ac, Batch3} = X_{ac, Batch2} \cdot V_{Global, Batch2} / V_{Global, Batch3}$ .

227

228 The nitrogen content of soluble ( $S_i$ ) and particulate ( $X_i$ ) inert in each substrate-inoculum  
229 mixture ( $N_{i,subs}$ ) determined the initial protein ( $X_{pr}$ ) + amino-acid ( $S_{aa}$ ) content,  
230 according to the nitrogen balance, as shown elsewhere (Pastor-Poquet et al., 2018).  
231 Thus,  $N_{i,subs}$  was different for each batch experiment [Table 3]. On the other hand,  
232 despite the initial conditions of batch experiments were maintained for all the  
233 simulations, different biomass concentrations (i.e. degraders of sugars,  $X_{su}$ ; amino acids,  
234  $X_{aa}$ ; long-chain fatty acids,  $X_{fa}$ ; valerate,  $X_{c5}$ ; butyrate,  $X_{bu}$ ; propionate,  $X_{pro}$ ; acetate,  
235  $X_{ac}$ ; and hydrogen,  $X_{h2}$ ) were also assessed, as mentioned before. Since mass balances  
236 were used among different TS in Experiments 2 and 4, only the initial conditions of  
237 Batch No. 1, 2, 6 and 7 [Table 1] were evaluated.

238

### 239 **2.3.2 Biochemical Parameters, Biomass Concentrations and Calibration Ranges**

240 The modified biochemical parameters ( $\theta_P$ ) and modified biomass concentrations ( $\theta_B$ ) in  
241 this study, including their initial values and potential variability ranges, are shown in  
242 Table 4. In total, 35  $\theta_P$  and 32  $\theta_B$  were evaluated.  $\theta_P$  related to the hydrolysis, sugar  
243 fractioning ( $f_{su}$ ), maximum growth rate ( $k_m$ ) and half-saturation constant ( $K_S$ ), but also  
244 the pH,  $NH_3$  and  $H_2$  inhibition constants ( $K_i$ ), since all these  $\theta_P$  are simultaneously  
245 associated to the substrate under study, are correlated among themselves, and strongly  
246 regulate the biogas production/composition from solid substrates (Batstone et al., 2002;  
247 Garcia-Gen et al., 2015). On the other hand,  $\theta_B$  included all the initial biomass  
248 concentrations in the HS-AD simulations (i.e.  $X_{su}$ ,  $X_{aa}$ ,  $X_{fa}$ ,  $X_{c5}$ ,  $X_{bu}$ ,  $X_{pro}$ ,  $X_{ac}$  and  $X_{h2}$ ),  
249 as these concentrations might not only strongly influence the biogas production during  
250 the initial days of batch experiments, but might also be potentially interrelated among  
251 themselves and/or to the previous  $\theta_P$ .

252  
 253 The sugars uptake yields butyrate ( $f_{bu,su}$ ), propionate ( $f_{pro,su}$ ), acetate ( $f_{ac,su}$ ) and hydrogen  
 254 ( $f_{h2,su}$ ) as COD fractions in ADM1. Therefore, a maximum of three fractions can be  
 255 selected simultaneously to fulfill the COD balance:  $f_{bu,su} + f_{pro,su} + f_{ac,su} + f_{h2,su} = 1$ . In  
 256 this study,  $f_{bu,su}$ ,  $f_{ac,su}$  and  $f_{h2,su}$  were selected, while  $f_{pro,su}$  was recalculated:  $f_{pro,su} = 1 -$   
 257  $f_{bu,su} - f_{ac,su} - f_{h2,su}$ . Importantly, further structural parameters and initial conditions need  
 258 to be induced in the HS-AD model as, for example, the amino-acid (AA,  $S_{aa}$ )  
 259 fractioning and the biomass yield coefficients ( $Y_b$ ), though these were not assessed here  
 260 aiming to reduce the problem under study. In either case, the proposed methodology for  
 261 calibration/validation can easily include any further  $\theta$ .

262  
 263 Variability ranges for structural parameters are suggested in ADM1 (Batstone et al.,  
 264 2002). However, considerably wider ranges were assessed in this study to emphasize  
 265 the absence of prior knowledge about the optimal values. For simplicity, all  $\theta_P$  were  
 266 allowed to vary by 90 % from their initial values,  $\theta_{P,0}$ :  $(1 - 0.9) \cdot \theta_{P,0} \leq \theta_P \leq (1 +$   
 267  $0.9) \cdot \theta_{P,0}$ ; uniform  $p(\theta_P)$  [Table 4]. As the only exception, the lower pH threshold for  
 268 acetoclastic methanogens ( $pH_{LL,ac}$ ) was bounded between a pH value where  
 269 methanogenesis potentially collapses (i.e.  $\leq 5.0$ ) and the upper pH threshold for  
 270 acetoclastic methanogens ( $pH_{UL,ac}$ , i.e. 7.0), to maintain the suitability of the Hill  
 271 function to simulate the pH inhibition [Table 2].

272  
 273 Different methods are available to approximate the initial conditions (i.e. biomass  
 274 concentrations) in batch simulations as, for example, simulating a continuous reactor  
 275 fed with exactly the same substrate until reaching steady state, and then use these steady

276 conditions to initialize the batch simulations (Dochain & Vanrolleghem, 2001; Donoso-  
 277 Bravo et al., 2011). However, since the inoculum was centrifuged right before setting  
 278 the batch experiments in this study, absence of prior knowledge about the initial  
 279 biomass concentrations was preferred. To explore homogeneously different orders of  
 280 magnitude in the biomass concentrations (often  $0 < \theta_B < 1 \text{ kmol COD/m}^3$ ), the  
 281 logarithm-transformed  $\theta_B$  were allowed to vary by 50 % from their initial values,  $\theta_{B,0}$ :  $(1$   
 282  $+ 0.5) \cdot \log_{10}(\theta_{B,0}) \leq \log_{10}(\theta_B) \leq (1 - 0.5) \cdot \log_{10}(\theta_{B,0})$ ; uniform  $p(\log_{10}(\theta_B))$  [Table 4].

283

### 284 2.3.3 Objective Function

285 The weighted sum of squares between all the available experimental and the  
 286 corresponding simulated values was used as objective function,  $J(\theta)$  [Equation 4].  $J(\theta)$   
 287 was adapted from Flotats et al. (2003) to assess the model ‘goodness of fitting’, being:  $\theta$   
 288 the structural parameters and/or initial conditions implemented in the HS-AD model;  $R$   
 289 the number of batch simulations;  $D$  the number of experimental datasets;  $t_{exp}$  the  
 290 experimental time of each batch experiment;  $y_{i,j,t}$  the experimental measurements;  
 291  $y_{i,j,t}^{sim}(\theta)$  the simulated values; and  $w_{i,j}$  the weighting coefficients – calculated as a  
 292 function of the average experimental data,  $\bar{y}_{i,j}$  [Equation 5]. With this approach,  $J(\theta)$   
 293 was lower-bounded by the – preliminarily unknown – global minimum:  $J(\theta) \geq J(\theta_{opt})$ .

$$J(\theta) = \sum_{i=1}^R \sum_{j=1}^D \sum_{t=t_1}^{t_{exp}} w_{i,j} \left( y_{i,j,t} - y_{i,j,t}^{sim}(\theta) \right)^2 \quad (4)$$

$$w_{i,j} = \frac{1}{\sum_{t=t_1}^{t_{exp}} (y_{i,j,t} - \bar{y}_{i,j})^2} \quad (5)$$

294

295 Noteworthy, only 6 out of 9 experimental conditions were used to calculate  $J(\theta)$  in this  
 296 study: Batch No. 1, 3, 5, 6, 7 and 9 [Table 1]. Meanwhile, 3 out of 9 experimental

297 conditions were used for cross-validation: Batch No. 2, 4 and 8 [Table 1]. Other  
298 experimental combinations could be used for calibration/cross-validation as, for  
299 example, Batch No. 2, 4, 5, 7 and 8 for calibration and Batch No. 1, 3, 6 and 9 for cross-  
300 validation. However, the configuration used in this study permitted to include the most  
301 extreme HS-AD conditions for model calibration (i.e.  $TS \leq 10\%$ ,  $TS \geq 30\%$ ), while  
302 intermediate conditions (i.e.  $10 \leq TS \leq 30\%$ ) were used for cross-validation. This  
303 strategy also aimed to enhance the number of experimental data used for calibration and  
304 to ensure the most diverse operative conditions where  $\theta$  might be representative.  
305 Additionally, the proposed configuration included the most informative dataset (i.e.  
306 Batch No. 1) to increase the complexity of  $J(\theta)$  and remove identifiability issues related  
307 to the lack of bio-physical-chemical dynamics in the overall dataset, as further discussed  
308 in Section 3.1.3.

309

### 310 **2.3.4 Global Sensitivity Analysis**

311 GSA was aimed to highlight the most influential  $\theta$  to be calibrated with the available set  
312 of experimental data. For GSA, multiple  $\theta$  combinations were used to evaluate  $J(\theta)$   
313 [Equation 4]. Latin-hypercube sampling (LHS) served to explore the global  $\theta$  space  
314 (Solon et al., 2015). Subsequently,  $J(\theta)$  arrays and their corresponding  $\theta$  were assessed  
315 by the GSA engine of Kennedy and O'Hagan (2001) and Oakley and O'Hagan (2004).  
316 The GSA engine calculates the individual (IE) and global (GE) effects of  $\theta_i$  upon the  
317 global model output (e.g.  $p(y|\theta)$ ,  $J(\theta)$ ) variance. On the other hand, the GSA engine  
318 provides also all the double correlations between  $\theta_i$  and  $\theta_j$  (i.e.  $\theta_i \cdot \theta_j$ , being  $i \neq j$ ). Both  
319 IE and  $\theta_i \cdot \theta_j$  are expressed as a percentage of the global variance explained. Thus,  $\theta_i$   
320 showing a relatively low IE are associated to poor practical identifiability, since these  $\theta_i$

321 influence minimally the model output. Meanwhile, GE comprises IE, the double  
322 correlations  $\theta_i \cdot \theta_j$  and all the rest of multiple correlations for a single  $\theta_i$ . Therefore, GE  
323 itself should not be used to assess the identifiability. Instead, the overall correlation of  $\theta_i$   
324 with  $\theta$  can be inferred by the relative difference between IE and GE – though IE and GE  
325 are expressed in different units, as interactions can be repeated in the GE of two or more  
326  $\theta_i$  (Oakley & O'Hagan, 2004; Saltelli et al., 2006). Summarizing, both a low IE and a  
327 large IE to GE difference mean poor identifiability for a single  $\theta_i$ , though *a priori* no  
328 fixed threshold – neither for IE, nor for GE minus IE – permits to distinguish whether  $\theta_i$   
329 should be considered as non-identifiable.

330

331 A maximum  $\theta$  subset of  $N' = 30$  and/or 400 simulations of  $J(\theta)$  can be evaluated  
332 simultaneously with the GSA engine (Kennedy & Petropoulos, 2017). Therefore, to  
333 assess  $\theta$  interactions when  $N > 30$ , a combination strategy was followed. Firstly, 30  $\theta_i$   
334 were randomly selected and evaluated by GSA (i.e. GSA No. 1). From these  $\theta$ , only  
335 those showing the smallest IE (e.g.  $< 1\%$ ) were disregarded as non-identifiable,  
336 removed from the initial  $\theta$  subset, and not used for further GSA. Importantly, these non-  
337 identifiable  $\theta_i$  were fixed at their initial values [Table 4] for all subsequent model  
338 simulations, since non-identifiability implies that these  $\theta$  can be fixed at any value  
339 within  $p(\theta)$  (Dochain & Vanrolleghem, 2001; Guisasola et al., 2006). Then,  $\theta_P$  and/or  $\theta_B$   
340 non-previously-assessed by GSA were combined with the non-removed  $\theta$  subset, and a  
341 new GSA was performed (i.e. GSA No. 2). The GSA methodology was repeated until  
342 the last remaining  $\theta$  subset was considered as ‘potentially identifiable’,  $\theta'$ .

343

344 In total, seven GSA with different  $\theta$  combinations were progressively performed [Table  
 345 5]. In this study, the only criterion for non-identifiability was assumed as  $IE \leq 0.20 \%$ ,  
 346 which also coincided with a relatively low GE (i.e.  $< 4.5$ ), though this conservative  
 347 criterion could be modified as mentioned in Section 3.1.1. To enhance the GSA  
 348 representativeness in presence of a high number of  $\theta$  (i.e.  $20 \leq N' \leq 30$ ) and/or wide  
 349 variability ranges (i.e.  $\pm 50 \%$ ), each GSA was conducted in triplicate and the results  
 350 averaged. Finally, all  $J(\theta)$  arrays used for GSA were searched for the minimum  
 351 observed value,  $J_{min}(\theta)$  (i.e.  $\geq J(\theta_{opt})$ ), to be subsequently used in ABC.

352

### 353 **2.3.5 Approximate Bayesian Computation**

354 The  $\theta'$  posterior distribution,  $p(\theta'|y)$ , was assessed by ABC (Toni & Stumpf, 2010). In  
 355 short, multiple simulations were carried at different  $\theta'$  combinations sampled by LHS,  
 356 whereas relatively high  $J(\theta')$  values were discarded by a progressively stringent  
 357 criterion based in a tolerance coefficient,  $\epsilon$  (i.e.  $> 1.0$ ). In other words, only  $J(\theta') -$   
 358  $J_{min}(\theta') \leq \epsilon$  were accepted for posterior evaluation:  $p(\theta'|J(\theta') - J_{min}(\theta') \leq \epsilon)$ . With this  
 359 approach,  $\theta'$  identifiability was further assessed by the convergence in the confidence  
 360 range.

361

362 In this study,  $\epsilon$  was successively reduced from 2.50 to 1.05 (i.e. 2.50, 1.80, 1.30, 1.10  
 363 and 1.05). Within each explored  $J(\theta') - J_{min}(\theta') \leq \epsilon$  range, 400 simulations were used.  $\theta'$   
 364 were allowed to vary within the same range used for GSA [Table 4]. Meanwhile, the 5  
 365 to 95 % interquartile range of each  $\theta'$  was used as confidence range, but also as a  
 366 criterion for identifiability/convergence. The posterior mean, median, mode, Kurtosis,



367 Skewness and correlation matrix were also evaluated, as described in Martin and Ayesa  
368 (2010).

369

### 370 **2.3.6 Cross-Validation**

371 Cross-validation assesses the model ‘goodness of fitting’ in experiments not used for  
372 calibration (Bennett et al., 2013). In this study, the  $\theta'$  posterior mean was considered as  
373  $\theta_{opt}$ . Thus,  $\theta_{opt}$  were used to simulate all batch experiments, including the three  
374 experimental conditions selected for cross-validation: Batch No. 2, 4 and 8 [Table 1].

375

376

## 377 **3 RESULTS AND DISCUSSION**

### 378 **3.1 GSA – Selecting the Most Influencing Input Parameters for Calibration**

#### 379 **3.1.1 Preliminary Identifiability Assessment**

380 GSA results are summarized in Table 5. GSA was started with 30  $\theta_P$  and progressively  
381 led to only 14  $\theta'$ : 13  $\theta_P$  (i.e.  $K_{h,pr}$ ,  $K_{h,ch,slow}$ ,  $k_{m,fa}$ ,  $k_{m,c5}$ ,  $k_{m,c4}$ ,  $k_{m,pro}$ ,  $k_{m,ac}$ ,  $k_{m,h2}$ ,  $k_d$ ,  $pH_{LL,ac}$ ,  
382  $f_{bu,su}$ ,  $f_{ac,su}$ ,  $f_{h2,su}$ ) and 1  $\theta_B$  (i.e.  $X_{ac,Batch7}$ ). In this study, only the  $\theta_i$  showing  $IE \leq 0.20$  %  
383 were fixed at their initial values [Table 4] to enhance the capabilities of GSA and ABC  
384 for calibrating structured AD models, as mentioned in Section 2.3.4. The overall  $J(\theta)$   
385 variance explained by the GSA engine was around 70 % in all cases, confirming the  
386 validity of this methodology to assess the most influential  $\theta$  in the HS-AD model  
387 (Oakley & O'Hagan, 2004).

388

389 IE provides a relative measure of the  $\theta_i$  practical identifiability, while a high  $\theta_i$

390 correlation – high difference between IE and GE – suggests that  $\theta_i$  cannot be

391 independently calibrated with the available set of experimental data (Saltelli et al.,  
 392 2006). As an example, GSA No. 1 showed that 40.7 % out of 72.4 % of the global  $J(\theta)$   
 393 variance was explained by adding up the IE of 30  $\theta_P$  [Table 5], while the remaining  
 394 31.7 % variance was explained by correlations among these. For example,  $k_{m,c4}$   
 395 explained individually around 4.4 % of the global variance (i.e. IE), though showing GE  
 396 up to 17.9. This meant that the correlation of  $k_{m,c4}$  with other  $\theta$  was high, and any  
 397 improvement in  $J(\theta)$  obtained by solely modifying  $k_{m,c4}$  could be partially or totally  
 398 compensated by the modification of a correlated  $\theta_i$ . In this line, the  $k_{m,c4} \cdot f_{bu,bu}$   
 399 correlation in GSA No. 1 explained up to 2.8 % of the global variance (data not shown),  
 400 due to the influence of  $f_{bu,su}$  in  $J(\theta)$  (i.e. IE = 2.0 % and GE = 24.4) [Table 5].

401

402 In this study,  $\theta_i$  were disregarded by a single and low-demanding criterion (i.e. IE  $\leq$   
 403 0.20 %) [Table 5], since any chosen criterion for ‘potential identifiability’ would  
 404 influence the GSA results when  $N \geq 30$ . Meanwhile, GSA also depends on  $N'$  (i.e.  $\leq 30$ )  
 405 and/or the particular combination of  $\theta_P$  and  $\theta_B$  used. Thus, using a more demanding  
 406 identifiability criterion (e.g. IE  $\leq 0.50$  % instead of 0.20 %) might have led to discard  $\theta_i$   
 407 during preliminary GSA runs, which would be subsequently characterized as  
 408 ‘potentially identifiable’. For example, GSA No. 1 showed an IE = 0.22 % for  $k_{m,c5}$ ,  
 409 whereas GSA No. 7 eventually showed an IE = 0.67 % [Table 5]. To enhance  
 410 identifiability, a strategy to reduce the gap between IE and GE for each  $\theta_i$  is required  
 411 (Kennedy & Petropoulos, 2017). However, reducing the IE to GE gap in this study  
 412 would require to readapt the criterion used for ‘potential identifiability’. One strategy  
 413 might consist on fixing those  $\theta_i$  showing, for example, IE  $\leq 0.30$  % in GSA No. 7 (i.e.  
 414  $f_{ac,su}$ ), and then perform further rounds of GSA (i.e. GSA No. 8) until IE  $\sim$  GE for all  $\theta_i$ .

415  
416 With all the above, a second assessment for identifiability can be useful when using  
417 variance-based GSA for structured AD models. ABC is a well-suited tool in this regard,  
418 yielding also  $p(\theta|y)$ , in contrast to GSA. Importantly, both methodologies should yield  
419 equivalent results regarding the  $\theta_i$  identifiability, though ABC is much computationally  
420 intensive than GSA, as explained in Section 3.2.1.

421

### 422 **3.1.2. Importance of the $p(\theta)$ for Model Calibration**

423 It must be noted that any  $p(\theta)$  could be used to calibrate AD models, provided that  $\theta$  do  
424 not contradict biochemical laws (e.g.  $\theta_P \geq 0$  and  $\theta_B \geq 0$ ) or disrupt mathematical  
425 resources (e.g.  $\text{pH}_{\text{UL,ac}} > \text{pH}_{\text{LL,ac}}$ ) and the overall  $p(\theta)$  range is as densely and  
426 homogeneously sampled as possible. On the other hand, the  $p(\theta)$  distribution used (e.g.  
427 uniform, log-transformed uniform, normal) can be also crucial to determine  $p(\theta|y)$  and  
428 the overall  $\theta$  estimation.

429

430 In this study, using triplicates permitted to enhance the GSA representativeness (overall  
431 sampling) in presence of large  $N$  and  $p(\theta)$  ranges. Meanwhile, a uniform distribution  
432 with mostly a 90 % modification was predefined for  $\theta_P$ , whereas a log-transformed  
433 uniform distribution with a 50 % modification was allowed for  $\theta_B$  [Table 4], as  
434 mentioned in Section 2.3.2. These specific  $p(\theta)$  ranges were considered sufficiently  
435 wide for the objectives of this study, and were based mainly on experience and visual  
436 analysis of the overall model results. Particularly, the order of magnitude of many  $\theta_P$  are  
437 relatively well characterized in literature according to their corresponding bio-physical-  
438 chemical meaning. Nonetheless, the order of magnitude of  $\theta_B$  is highly unknown,

439 particularly after the inoculum centrifugation used in this study. Thus, using a uniform  
 440  $p(\theta)$  permitted to emphasize the precision over punctual  $\theta_P$  values within the  $p(\theta_P)$  range,  
 441 whereas the precision over the ‘unknown’ order of magnitude in the  $\theta_B$  range was  
 442 emphasized by using a log-transformed uniform distribution  $p(\log_{10}(\theta_B))$ .

443

444 With all the above, using a different  $p(\theta)$  range or distribution might alter the results of  
 445 variance-based GSA and the overall calibration of structured AD models. For example,  
 446 using considerably narrower  $p(\theta_P)$  ranges in GSA No. 1 (i.e.  $(1 - 0.3) \cdot \theta_{P,0} \leq \theta_P \leq (1 +$   
 447  $0.3) \cdot \theta_{P,0}$ ; *uniform*  $p(\theta_P)$ ) resulted in some  $\theta_P$  – which were highlighted as ‘potentially  
 448 identifiable’ in this study – being disregarded as non-identifiable (e.g.  $K_{h,pr}$  and  $k_{m,c4}$ ),  
 449 likely because part of their ‘optimal’ or ‘sub-optimal’  $\theta_P$  values were left outside the  
 450  $p(\theta_P)$  range [Supplementary Information]. On the other hand, using wider ranges (i.e.  
 451  $0.01 \cdot \theta_{P,0} \leq \theta_P \leq 3 \cdot \theta_{P,0}$ ; *uniform*  $p(\theta_P)$ ), the results were not substantially different; i.e.  
 452 only the  $k_{m,c5}$  identifiability was additionally disregarded by GSA No. 1. However,  
 453 using a log-transformed uniform distribution (i.e.  $(1 + 0.5) \cdot \log_{10}(\theta_{P,0}) \leq \log_{10}(\theta_P) \leq (1$   
 454  $- 0.5) \cdot \log_{10}(\theta_{P,0})$ ; *uniform*  $p(\log_{10}(\theta_P))$ ) relatively altered the results of GSA No. 1; i.e.  
 455 the order of magnitude of  $k_{m,c5}$  and  $k_{m,c4}$  was disregarded, while the order of magnitude  
 456 of  $K_{h,ch,slow}$  and  $K_{S,Xh2}$  has was highlighted as highly important for the HS-AD model  
 457 calibration. Therefore, for the correct calibration of structured AD models, both the  $p(\theta)$   
 458 range and distribution should be set accordingly to all the prior information available for  
 459 these  $\theta$ , minimizing the  $p(\theta)$  range explored alongside the number of simulations  
 460 required for the objectives of the study.

461

### 462 3.1.3 Importance of the Available Data for Model Calibration

463 Provided the  $\theta$  are structurally identifiable, practical identifiability relates to the quantity  
464 as well as the quality (i.e. experimental errors and/or the sampling frequency) of the  
465 experimental data available (Donoso-Bravo et al., 2011; Guisasola et al., 2006;  
466 Nimmegeers et al., 2017; Rodriguez-Fernandez et al., 2006). Particularly, a reduced  
467 number of experimental data associated to some model dynamics usually prevents  
468 practical identifiability of the  $\theta$  involved in these specific dynamics. For example,  $Y_b$   
469 might not be identifiable in AD models provided that the biomass concentration  
470 dynamics were measured (Bernard et al., 2001).

471

472 In this study, the hydrolysis constant of readily-biodegradable carbohydrates ( $K_{h,ch,fast}$ )  
473 and lipids ( $K_{h,li}$ ), but also the maximum growth rate of sugar ( $k_{m,su}$ ) and amino acid  
474 ( $k_{m,aa}$ ) degraders, showed a reduced influence in  $J(\theta)$  by GSA. These results suggest that  
475 either insufficient experimental data was available to calibrate  $K_{h,ch,fast}$ ,  $K_{h,li}$ ,  $k_{m,su}$  and  
476  $k_{m,aa}$ , or that the biogas production in the HS-AD batch experiments [Table 1] was  
477 mostly influenced by the VFA uptake – as the main limiting step. In the same line, due  
478 to the Monod properties,  $k_m$  and  $K_s$  might be correlated when using batch experiments  
479 for calibration (Dochain & Vanrolleghem, 2001; Flotats et al., 2010; Guisasola et al.,  
480 2006). Nevertheless, GSA showed negligible influence for all  $K_s$  in this study, likely  
481 due to using different batch experiments (i.e. ISR and TS) and/or a reduced number of  
482 experimental data to obtain  $J(\theta)$  [Equation 4].

483

484 The liquid-gas transfer coefficient (kLa) was also disregarded during the initial steps of  
485 GSA (i.e. IE = 0.07 % and GE = 0.39) [Table 5]. The kLa coefficient is related to the  
486 homogenization and mixing strategy, as well as other operational parameters in AD (e.g.

487 temperature and pH) (Batstone et al., 2002). In this study, all the HS-AD digesters were  
488 manually shaken. However, increasing the TS content can hamper the liquid-gas  
489 transfer mechanisms in HS-AD (Pastor-Poquet et al., 2019a). Therefore, a calibration  
490 strategy should be specifically envisaged to correctly calibrate  $kLa$  (e.g. using different  
491 stirring velocities/devices for mixing), since the presence of several  $\theta$  in HS-AD models  
492 can prevent identifiability of  $kLa$  with the reduced number of experimental data usually  
493 available.

494

495 Importantly, both the  $NH_3$  and  $H_2$  inhibition parameters [Table 2] were shown as non-  
496 identifiable in this study, despite the strong influence of these parameters to regulate the  
497 biogas production in an ADM1-based model, as mentioned in Section 2.3.2. These  
498 results were associated to the reduced TAN and VFA dynamics in the experimental data,  
499 since only one single sacrifice experiment was available for calibration/validation.

500 Therefore, despite using different initial conditions (i.e. ISR and/or TS) for model  
501 calibration, the  $NH_3$  inhibition parameters in HS-AD of OFMSW cannot be assessed by  
502 using traditional batch experiments, where only the biogas production and composition  
503 are (usually) dynamically evaluated.

504

505 The above results condense the importance of an adequate sampling to enhance  
506 identifiability in AD models, but also to test hypotheses regarding the effects of  
507 inhibitory substances in HS-AD. Particularly, an extensive sampling for VFA, pH and  
508 TAN at different operational times during batch experiments is required to identify  
509 crucial parameters regarding the  $NH_3$  inhibition in HS-AD of OFMSW. Therefore,  
510 sacrifice experiments and/or any sampling technique for batch setups – allowing the

511 thorough characterization of the reactor content variables in dynamic mode – should be  
512 recommended to calibrate structured HS-AD models using batch experiments. On the  
513 other hand, including (semi-)continuous datasets when available might also alleviate  
514 these identifiability issues during calibration (Bennett et al., 2013; Nimmegeers et al.,  
515 2017).

516

### 517 **3.1.4 The Importance of Initial Conditions for Model Calibration**

518 Interestingly, all  $\theta_B$  except  $X_{ac, Batch7}$  were shown as non-identifiable in this study [Table  
519 5]. The reason presumably lies on the high  $k_m$  of all microorganisms ‘shading’ the effect  
520 of their initial concentration. For example,  $X_{pro}$  was associated to a maximum growth  
521 rate ( $k_{m, pro}$ ) around  $10 \text{ d}^{-1}$  [Table 4]. Thus,  $X_{pro}$  doubles within 1 h (i.e.  $\frac{\log(2)}{k_m} =$   
522  $\frac{\log(2) \cdot 24}{10} = 0.6 \text{ h}$ ), whereas the HS-AD batch experiments lasted considerably longer  
523 than 20 days [Table 1].

524

525  $X_{ac}$  and  $X_{h2}$  are important variables to avoid batch acidification during the initial 0 - 10  
526 days of HS-AD simulations, due to the rapid changes occurring in the bio-physical-  
527 chemistry during these days and the influence of these two microbial populations to  
528 define the buffering capacity (Batstone et al., 2002; Capson-Tojo et al., 2017).

529 Nonetheless, these biomass concentrations were also rapidly disregarded by GSA in this  
530 study [Tables 3 and 4], except the ‘potential identifiability’ of  $X_{ac, Batch7}$  suggested by  
531 GSA No. 7 which was likely explained by the influence of the order of magnitude of  
532 this particular biomass content to regulate the risk of acidification of the most extreme  
533 HS-AD condition in Experiment 7 (i.e. Batch No. 9, TS = 30 %) [Table 1].

534

535 With all the above, it is likely that only the initial biomass magnitude – not a precise  
 536 value – was needed to calibrate the HS-AD model based on batch experiments. In other  
 537 words, approximate biomass concentrations serve mainly to avoid acidification in HS-  
 538 AD batch simulations, since these might not influence significantly the model  
 539 calibration. The  $\theta$  influence on  $J(\theta)$  in this study was further assessed by ABC.

540

## 541 **3.2 Parameter Optimization**

### 542 **3.2.1 Second Identifiability Assessment**

543 Figure 1 shows  $p(\theta|y)$  using an  $\varepsilon = 1.05$ . The main statistics for these  $p(\theta|y)$  are  
 544 summarized in Table 6, including the confidence ranges (i.e. 5 - 95 % interquartile  
 545 range), since a reliable assessment of the  $\theta$  confidence range is as important as the value  
 546 of  $\theta_{opt}$  themselves (Guisasola et al., 2006). The correlation matrix is included as  
 547 Supplementary Information. Figure 2 shows the 5 - 95 % interquartile range vs.  $\varepsilon$ , since  
 548 reducing progressively  $\varepsilon$  permitted to assess the convergence of the  $\theta$ ' posterior as  
 549 second identifiability assessment.

550

551 Parameter identifiability is roughly associated to the ‘sharpness’ of the posterior  
 552 distribution,  $p(\theta|y)$  (Martin & Ayesa, 2010; Toni & Stumpf, 2010). In this line,  $K_{h,pr}$ ,  
 553  $K_{h,ch,slow}$ ,  $k_{m,fa}$ ,  $k_{m,c5}$ ,  $k_{m,c4}$ ,  $k_{m,ac}$ ,  $pH_{LL,ac}$  and  $f_{bu,su}$  showed relatively well-defined bell-  
 554 shaped distributions by ABC, suggesting an adequate identifiability [Figure 1].  
 555 Meanwhile,  $k_{m,pro}$ ,  $k_{m,h2}$ ,  $k_d$ ,  $X_{ac,Batch7}$ ,  $f_{ac,su}$  and  $f_{h2,su}$  showed a more uniform-like  
 556 distribution, suggesting a poorer identifiability. The substantial reduction observed in  
 557 the interquartile range for  $K_{h,pr}$ ,  $K_{h,ch,slow}$ ,  $k_{m,fa}$ ,  $k_{m,c5}$ ,  $k_{m,c4}$ ,  $k_{m,ac}$ ,  $pH_{LL,ac}$  and  $f_{bu,su}$  (i.e. 60  
 558 - 80 %) corroborated their adequate identifiability in this study, in contrast to  $k_{m,pro}$ ,



559  $k_{m,h2}$ ,  $k_d$ ,  $X_{ac,Batch7}$ ,  $f_{ac,su}$  and  $f_{h2,su}$  that showed a much constant interquartile range (i.e.  $\leq$   
 560 50 % reduction) [Figure 2]. The poor practical identifiability of these last  $\theta'$  is explained  
 561 by their high correlation with the rest of  $\theta'$ . For example, the  $f_{ac,su} \cdot f_{bu,su}$  correlation was -  
 562 0.82, while  $k_{m,pro} \cdot k_d$  was 0.72 – data not shown. As suggested in Section 3.1.4,  $X_{ac,Batch7}$   
 563 served mainly to counteract the potential acidification in Batch No. 9, since the poor  
 564 reduction in the interquartile range (i.e. 23 %) alongside the high correlation with other  
 565  $\theta'$  (i.e.  $pH_{LL,ac} \cdot X_{ac,Batch7} = 0.24$ ) indicated that only an approximate biomass content is  
 566 needed to calibrate structured HS-AD models based on batch experiments.

567

568 As expected, ABC supported the identifiability assessment previously performed by  
 569 GSA. In particular,  $\theta'$  showing  $IE < 1.5$  % in GSA No. 7 (i.e.  $k_{m,pro}$ ,  $k_{m,h2}$ , and  $f_{ac,su}$ )  
 570 [Table 5] were associated to a poor identifiability. However, some parameters showing  
 571 an  $IE \geq 1.5$  % in GSA No. 7 (i.e.  $k_d$ ,  $X_{ac,Batch7}$  and  $f_{h2,su}$ ) were also indicated as non-  
 572 identifiable by ABC in contrast to GSA, suggesting that ABC was a more sensitive  
 573 methodology for parameter identifiability in this study. With all the above, a more  
 574 restrictive IE threshold (i.e. 0.50 % instead of 0.20 %) could have been used in further  
 575 GSA rounds, once fixing poorly-identifiable parameters to any value within the prior, as  
 576 mentioned in Section 3.1.1.

577

578 ABC is computationally intensive due to the high level of  $J(\theta') - J_{min}(\theta') \leq \epsilon$  rejection,  
 579 particularly when using highly-demanding  $\epsilon$  (Filippi et al., 2013; Toni & Stumpf, 2010).  
 580 For example, the acceptance ratio was 0.129 when using  $\epsilon = 1.80$ , meaning that only 1  
 581 out of 8 simulations was accepted for posterior evaluation, whereas the acceptance ratio  
 582 was 0.004 when using  $\epsilon = 1.10$  – data not shown. Thus, ABC is not recommended to

583 assess identifiability in complex models with a large number of  $\theta$  (i.e.  $N \geq 30$ ). Different  
584 upgrades have been proposed to increase the ABC efficiency (Beaumont et al., 2009;  
585 Filippi et al., 2013; Toni & Stumpf, 2010), though the evaluation of these upgrades for  
586 calibrating structured AD models was out of the scope of this study. Conversely, the  
587 GSA engine relies upon a Bayesian emulator to speed up the analysis of model outputs  
588 (Kennedy & O'Hagan, 2001; Oakley & O'Hagan, 2004). Therefore, GSA can be an  
589 adequate tool to reduce the global computation required for parameter optimization, by  
590 preliminarily reducing the number of  $\theta'$  to be further assessed by ABC as shown in this  
591 study.

592

### 593 **3.2.2 Batch Simulations**

594 Using the  $\theta'$  mean as  $\theta_{opt}$  [Table 6] led to a good approximation of both the methane  
595 production [Figure 3] and the rest of variables at the end of all batch experiments  
596 [Figure 4] used either for calibration (i.e. Batch No. 1, 3, 5, 6, 7 and 9) or cross-  
597 validation (i.e. Batch No. 2, 4 and 8) [Table 1]. Therefore, the  $\theta'$  mean might be a good  
598 approximation of  $\theta_{opt}$ , particularly for those  $\theta'$  where practical identifiability was likely  
599 (i.e.  $K_{h,pr}$ ,  $K_{h,ch,slow}$ ,  $k_{m,fa}$ ,  $k_{m,c5}$ ,  $k_{m,c4}$ ,  $k_{m,ac}$ ,  $pH_{LL,ac}$  and  $f_{bu,su}$ ). Importantly, the HS-AD  
600 model was able to capture particularly well the TS and TAN contents, but also VS (data  
601 not shown), in all mono- and co-digestion experiments, confirming the suitability of the  
602 hypotheses used for the model development (Pastor-Poquet et al., 2018).

603

604 Some disagreements were also observed between the simulations and the experimental  
605 results. Particularly, the implementation of a reversible  $NH_3$  inhibition function  
606 [Equation 1] in all the VFA and  $H_2$  degrading populations [Table 2] was unable to

607 capture the VFA accumulation at the end of HS-AD experiments [Figure 4]. As the  
608 most noticeable example, calibration failed to represent the  $S_{bu}$  accumulation in Batch  
609 No. 6 [Figure 4e], yielding also a slight miss-adjustment in the methane production  
610 [Figure 3c]. More in general,  $S_{ac}$  and  $S_{bu}$  at the end of all experiments were poorly  
611 represented [Figure 4], despite the butyrate ( $k_{m,c4}$ ) and acetate ( $k_{m,ac}$ ) growth rates were  
612 adequately identified and the  $NH_3$  inhibition upon the acetate uptake is a relatively well-  
613 established strategy in structured AD models (Batstone et al., 2002).

614

615 Two main reasons might explain the VFA disagreement between the model simulations  
616 and the experimental data available. The first reason relates to the relatively low amount  
617 of experimental data hampering calibration, as mentioned in Section 3.1.3. In this line,  
618 the  $NH_3$  half-inhibition parameters in the VFA and/or  $H_2$  uptakes ( $K_{i,nh3}$ ) were  
619 disregarded as unimportant by GSA to represent the experimental data [Table 5], mainly  
620 because only Batch No. 1 contained the VFA, pH and TAN dynamics. The second  
621 reason relates to the poor suitability of the reversible non-competitive  $NH_3$  inhibition  
622 function [Equation 1] to explain the VFA accumulation in HS-AD simulations, as  
623 discussed in next section.

624

### 625 **3.3 Main Effects of Increasing the TS Content in HS-AD of OFMSW**

626 In this study, calibration/cross-validation served to further test the hypotheses used for  
627 model construction (e.g. mass balances), particularly regarding the TS and VS  
628 simulation. Noteworthy, the correct simulation of TS is crucial in HS-AD, as TS  
629 determines the apparent concentration of soluble compounds subsequently affecting all  
630 the HS-AD bio-physical-chemical dynamics (Pastor-Poquet et al., 2019b). For example,

631 TS = 20 % supposes approximately 20 % higher apparent concentrations (i.e. kmol/kg  
632 H<sub>2</sub>O) regarding the corresponding global concentrations (i.e. kmol/kg).

633

634 The HS-AD model was also calibrated/validated to assess the effects of increasing TS  
635 upon the NH<sub>3</sub> inhibition in HS-AD. Specifically, a high solute content – potentially  
636 associated to a high TS – exacerbates the solution ‘non-ideality’, affecting all the HS-  
637 AD dynamics (e.g. pH, NH<sub>3</sub> concentration, CO<sub>2</sub> transfer) (Pastor-Poquet et al., 2019b).  
638 More in detail, ‘non-ideality’ can lower  $S_{nh3}$ , serving as a potential source of NH<sub>3</sub>  
639 inhibition abatement in HS-AD of OFMSW. In this study,  $I$  ranged from 0.22 to 0.93 M  
640 [Figure 4c], emphasizing the need for an adequate ‘non-ideal’ bio-physical-chemical  
641 approach (Hafner & Bisogni, 2009; Solon et al., 2015). Importantly, despite the high  $I$   
642 observed,  $S_{nh3}$  reached up to 0.13 mol N/kg in this study [Figure 4d] – equivalent to  
643 0.16 mol N/kg H<sub>2</sub>O (i.e. 2.3 g N/L). These  $S_{nh3}$  were considerably high, since reactors  
644 operated at  $S_{nh3} \geq 1.0$  g N/L usually show an inefficient VFA conversion (Rajagopal et  
645 al., 2013).

646

647 The inefficient VFA conversion in HS-AD experiments was not well simulated by the  
648 reversible NH<sub>3</sub> inhibition function, as mentioned in Section 3.2.2. To understand the  
649 poor VFA simulation, it is necessary to consider the relatively flat inhibition described  
650 by Equation 1 but also the COD fluxes in the HS-AD model. Briefly,  $I_{nh3}$  is 0.50 when  
651  $S_{nh3,App} = K_{i,Snh3}$ , whereas  $I_{nh3}$  is 0.33 when  $S_{nh3,App} = 2 \cdot K_{i,Snh3}$  [Equation 1]. In other  
652 words, a non-competitive reversible inhibition by NH<sub>3</sub> might be far too ‘blunt’ to  
653 describe the actual effect of NH<sub>3</sub> upon the anaerobic biomass (Astals et al., 2018). On  
654 the other hand, due to the COD fractioning used in this study, approximately 54 % of

655 the COD from  $S_{su}$  [Table 6] and 26 % of the COD from  $S_{aa}$  flowed through the butyrate  
656 pathway, whereas up to 80 % of the COD either from  $S_{su}$ ,  $S_{aa}$  and/or long-chain fatty  
657 acids ( $S_{fa}$ ) flowed through the acetate pathway (Batstone et al., 2002). Meanwhile, a  
658 considerable proportion of the initially biodegradable COD (i.e. 75 - 85 %) was  
659 assigned to  $X_{ch} + S_{su} + X_{pr} + S_{aa}$  [Table 3].

660

661 With all the above, the ‘blunt’ definition of the  $NH_3$  inhibition function, alongside the  
662 high COD flowing through the butyrate and acetate pathways, presumably favored the  
663  $X_{bu}$  and  $X_{ac}$  growth even at considerably high  $S_{nh3}$  (i.e. up to 2.3 g N/L) during  
664 simulations. Summarizing, the high substrate content counterbalanced the effect of the  
665  $NH_3$  inhibition, preventing the correct simulation of  $S_{bu}$  and  $S_{ac}$  accumulation at the end  
666 of the HS-AD experiments. Therefore, the reversible non-competitive  $NH_3$  inhibition  
667 function [Equation 1] in the VFA and/or  $H_2$  uptakes [Table 2] requires further testing to  
668 represent the VFA accumulation observed in HS-AD of OFMSW. To this particular aim,  
669 using extensive data regarding the VFA, pH and TAN dynamics in HS-AD simulations  
670 is strongly recommended.

671

672 To end up, the HS-AD model (Pastor-Poquet et al., 2018) is a suitable platform to  
673 understand the inner mechanisms of HS-AD, whereas further model developments  
674 and/or model configurations (i.e. inhibition functions) should be also tested to enhance  
675 our understanding about the VFA accumulation within HS-AD of OFMSW. Similarly,  
676 additional experimental data is needed to thoroughly understand the role of  $NH_3$   
677 inhibition in HS-AD simulations. Specifically, extensive data regarding the main  
678 species driving ‘non-ideality’ (i.e. VFA, pH, TAN) and/or further bio-physical-chemical

679 mechanisms (i.e. precipitation) in HS-AD seem to be crucial, due to the outstanding  
680 importance of ‘non-ideality’ for the biochemical parameter optimization (Pastor-Poquet  
681 et al., 2019b). In this scheme, the relatively simple calibration/validation methodology  
682 presented in this study can serve to test the  $\theta$  practical identifiability and confidence  
683 ranges in presence of any set of experimental data and/or HS-AD model structure.

684

685

#### 686 **4 CONCLUSIONS**

687 Nine different batch conditions were used to calibrate and cross-validate the HS-AD  
688 model. For parameter optimization, variance-based GSA in tandem with ABC served to  
689 evaluate the practical identifiability of 35  $\theta_P$  and 32  $\theta_B$ . Among all these, mostly 8  $\theta_P$   
690 were correctly identified with the available data, as corroborated by the convergence of  
691  $p(\theta|y)$ . The study also showed that HS-AD may be operated at  $S_{\text{nh}_3} \geq 2.3$  g N/L and  $I \geq$   
692 0.9 M, whereas a reversible non-competitive  $\text{NH}_3$  inhibition function was not able to  
693 explain the VFA accumulation in HS-AD of OFMSW. Therefore, further datasets about  
694 the VFA, pH and TAN dynamics are required to enhance the  $\theta$  practical identifiability,  
695 whereas further model configurations should be tested to enhance the simulation of  $\text{NH}_3$   
696 inhibition in HS-AD.

697

698

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## 708 REFERENCES

709

- 710 Astals, S., Peces, M., Batstone, D.J., Jensen, P.D., Tait, S. 2018. Characterising and  
711 modelling free ammonia and ammonium inhibition in anaerobic systems. *Water*  
712 *Res.*, **143**, 127-135.
- 713 Batstone, D.J., Keller, J., Angelidaki, I., Kalyuzhnyi, S.V., Pavlostathis, S.G., Rozzi, A.,  
714 . . . Vavilin, V.A. 2002. The IWA Anaerobic Digestion Model No. 1 (ADM1).  
715 *Water Sci. Technol.*, **45**(10), 65-73.
- 716 Beaumont, M.A., Cornuet, J.-M., Marin, J.-M., Robert, C.P. 2009. Adaptive  
717 approximate Bayesian computation. *Biometrika*, **96**(4), 983-990.
- 718 Bennett, N.D., Croke, B.F.W., Guariso, G., Guillaume, J.H.A., Hamilton, S.H.,  
719 Jakeman, A.J., . . . Andreassian, V. 2013. Characterising performance of  
720 environmental models. *Environ. Model. Software*, **40**, 1-20.
- 721 Bernard, O., Hadj-Sadok, Z., Dochain, D., Genovesi, A., Steyer, J.P. 2001. Dynamical  
722 model development and parameter identification for an anaerobic wastewater  
723 treatment process. *Biotechnol. Bioeng.*, **75**, 424-438.
- 724 Capson-Tojo, G., Trably, E., Rouez, M., Crest, M., Steyer, J.P., Delgenès, J.P., Escudié,  
725 R. 2017. Dry anaerobic digestion of food waste and cardboard at different  
726 substrate loads, solid contents and co-digestion proportions. *Bioresour. Technol.*,  
727 **233**, 166-175.
- 728 Dochain, D., Vanrolleghem, P. 2001. *Dynamical Modelling and Estimation in*  
729 *Wastewater Treatment Processes*. IWA Publishing, London, UK.
- 730 Donoso-Bravo, A., Mailier, J., Martin, C., Rodríguez, J., Aceves-Lara, C.A., Vande  
731 Wouwer, A. 2011. Model selection, identification and validation in anaerobic  
732 digestion: A review. *Water Res.*, **45**(17), 5347-5364.
- 733 Filippi, S., Barnes, C.P., Cornebise, J., Stumpf, M.P. 2013. On optimality of kernels for  
734 approximate Bayesian computation using sequential Monte Carlo. *Stat. Appl.*  
735 *Genet. Mol. Biol.*, **12**(1), 87-107.
- 736 Flotats, X., Ahring, B.K., Angelidaki, I. 2003. Parameter identification of thermophilic  
737 anaerobic degradation of valerate. *Appl. Biochem. Biotechnol.*, **109**(1-3), 47-62.
- 738 Flotats, X., Palatsi, J., Fernandez, B., Colomer, M.A., Illa, J. 2010. Identifying  
739 anaerobic digestion models using simultaneous batch experiments. *Environ.*  
740 *Engineer. Manag. J.*, **9**(3), 313-318.
- 741 Garcia-Gen, S., Sousbie, P., Rangaraj, G., Lema, J.M., Rodriguez, J., Steyer, J.P.,  
742 Torrijos, M. 2015. Kinetic modelling of anaerobic hydrolysis of solid wastes,  
743 including disintegration processes. *Waste Manage.*, **35**, 96-104.

- 744 Guisasola, A., Baeza, J.A., Carrera, J., Sin, G., Vanrolleghem, P.A., Lafuente, J. 2006.  
745 The influence of experimental data quality and quantity on parameter estimation  
746 accuracy: Andrews inhibition model as a case study. *Educ. Chem. Eng.*, **1**(1),  
747 139-145.
- 748 Hafner, S.D., Bisogni, J.J. 2009. Modeling of ammonia speciation in anaerobic  
749 digesters. *Water Res.*, **43**(17), 4105-4114.
- 750 Kennedy, M.C., O'Hagan, A. 2001. Bayesian calibration of computer models. *J. Roy.*  
751 *Stat. Soc. Ser. B. (Stat. Method.)*, **63**(3), 425-464.
- 752 Kennedy, M.C., Petropoulos, G.P. 2017. GEM-SA: The gaussian emulation machine for  
753 sensitivity analysis. in: *Sensitivity Analysis in Earth Observation Modelling*,  
754 (Ed.) G.P. Petropoulos, Elsevier. Amsterdam, Netherlands.
- 755 Martin, C., Ayesa, E. 2010. An Integrated Monte Carlo Methodology for the calibration  
756 of water quality models. *Ecol. Model.*, **221**(22), 2656-2667.
- 757 Nimmegeers, P., Lauwers, J., Telen, D., Logist, F., Impe, J.V. 2017. Identifiability of  
758 large-scale non-linear dynamic network models applied to the ADM1-case  
759 study. *Math. Biosci.*, **288**, 21-34.
- 760 Nott, D.J., Marshall, L., Brown, J. 2012. Generalized likelihood uncertainty estimation  
761 (GLUE) and approximate Bayesian computation: What's the connection? *Water*  
762 *Resour. Res.*, **48**(12).
- 763 Oakley, J.E., O'Hagan, A. 2004. Probabilistic sensitivity of complex models: A  
764 Bayesian approach. *J. Roy. Stat. Soc. Ser. B. (Stat. Method.)*, **66**(3), 751-769.
- 765 Pastor-Poquet, V., Papirio, S., Steyer, J.-P., Trably, E., Escudié, R., Esposito, G. 2018.  
766 High-solids anaerobic digestion model for homogenized reactors. *Water Res.*,  
767 **142**, 501-511.
- 768 Pastor-Poquet, V., Papirio, S., Trably, E., Rintala, J., Escudié, R., Esposito, G. 2019a.  
769 High-Solids Anaerobic Digestion requires a tradeoff between Total Solids,  
770 Inoculum-to-Substrate Ratio and Ammonia Inhibition. *Int. J. Environ. Sci.*  
771 *Technol.*, 1-15.
- 772 Pastor-Poquet, V., Papirio, S., Steyer, J.-P., Trably, E., Escudié, R., Esposito, G. 2019b.  
773 Modelling Non-Ideal Bio-Physical-Chemical Effects on High-Solids Anaerobic  
774 Digestion of the Organic Fraction of Municipal Solid Waste. *J. Environ.*  
775 *Manage.*, **238**, 408-419.
- 776 Rajagopal, R., Masse, D.I., Singh, G. 2013. A critical review on inhibition of anaerobic  
777 digestion process by excess ammonia. *Bioresour. Technol.*, **143**, 632-641.
- 778 Ratto, M., Tarantola, S., Saltelli, A. 2001. Sensitivity analysis in model calibration:  
779 GSA-GLUE approach. *Comput. Phys. Commun.*, **136**, 212-224.
- 780 Rodriguez-Fernandez, M., Egea, J.A., Banga, J.R. 2006. Novel metaheuristic for  
781 parameter estimation in nonlinear dynamic biological systems. *BMC*  
782 *Bioinformatics*, **7**, 483.
- 783 Saltelli, A., Ratto, M., Tarantola, S., Campolongo, F. 2006. Sensitivity analysis  
784 practices: Strategies for model-based inference. *Reliab. Eng. Syst. Saf.*, **91**(10-  
785 11), 1109-1125.
- 786 Solon, K., Flores-Alsina, X., Mbamba, C.K., Volcke, E.I., Tait, S., Batstone, D., . . .  
787 Jeppsson, U. 2015. Effects of ionic strength and ion pairing on (plant-wide)  
788 modelling of anaerobic digestion. *Water Res.*, **70**, 235-245.
- 789 Toni, T., Stumpf, M.P. 2010. Simulation-based model selection for dynamical systems  
790 in systems and population biology. *Bioinformatics*, **26**(1), 104-110.  
791



792 **TABLE CAPTIONS**

793 **Table 1.** Batch experiments and initial conditions used for HS-AD model calibration  
794 and cross-validation

795 **Table 2.** Biochemical rates used for the HS-AD model in this study

796 **Table 3.** Initial conditions used for all batch simulations in this study

797 **Table 4.** Main input parameters ( $\theta$ ) used for global sensitivity analysis (GSA), including  
798 the initial values, lower and upper thresholds

799 **Table 5.** Global sensitivity analysis (GSA) of input parameters ( $\theta$ ): Individual (IE) and  
800 global (GE) effects upon the objective function,  $J(\theta)$ , variance

801 **Table 6.** Calibration of potentially-identifiable input parameters ( $\theta'$ ): Prior and posterior  
802 distributions

803

804

805 **FIGURE CAPTIONS**

806

807 **Figure 1.** Posterior parameter distribution using 400 simulations and  $\varepsilon = 1.05$ 808 **Figure 2.** Interquartile range (percentiles 5 to 95 %) of the posterior parameter809 distribution using  $\varepsilon \geq 1.05$  and  $\varepsilon \leq 2.50$ 810 **Figure 3.** Methane production with mono-digestion of dried OFMSW at a)  $ISR = 1.00$ ;811 b)  $ISR = 1.50$ ; and c)  $ISR = 1.00$ ; and co-digestion of dried OFMSW and sawdust at d)812  $ISR = 0.16$ . Dots represent experimental data, while lines represent simulated values.813 **Figure 4.** Main variables at the end of the four batch experiments: a) Total solids (TS);814 b) total ammonia nitrogen (TAN,  $S_{in}$ ); c) ionic strength ( $I$ ); d) free ammonia nitrogen815 ( $NH_3$ ,  $S_{nh3}$ ); e) acetic acid ( $S_{ac}$ ); f) propionic acid ( $S_{pro}$ ); g) butyric acid ( $S_{bu}$ ); and h)816 valeric acid ( $S_{va}$ ). Crosses represent experimental data, while geometries represent

817 simulated values

818

819

**Table 1.** Batch experiments and initial conditions used for HS-AD model calibration and cross-validation

Substrate	Exp. No.	Batch No.	ISR (g VS/g VS)	Initial TS (%)	No. Replicates	Design Volume (mL)	Exp. Time (days)	Calibration or Validation
OFMSW	1	1	1.00	15.0	15	280	92	C
		2		10.8	3			V
	2	3	1.50	13.4	3	160	100	C
		4		16.4	3			V
		5		19.6	3			C
3	6	0.50	28.3	2	280	99	C	
OFMSW + Sawdust	4	7	0.16	10.0	3	280	284	C
		8		15.0	2			V
		9		30.2	1			C

**Table 2.** Biochemical rates used for the HS-AD model in this study

Process	Rate ( $r_i$ , kgCOD $m^{-3} d^{-1}$ )
<b>Hydrolysis of Fast Biodegradable Carbohydrates</b>	$k_{h,ch,fast} \cdot X_{ch,fast}$
<b>Hydrolysis of Slow Biodegradable Carbohydrates</b>	$k_{h,ch,slow} \cdot X_{ch,slow}$
<b>Hydrolysis of Proteins</b>	$k_{h,pr} \cdot X_{pr}$
<b>Hydrolysis of Lipids</b>	$k_{h,li} \cdot X_{li}$
<b>Sugars Uptake</b>	$k_{m,su} \cdot S_{su,App} / (K_{S,Xsu} + S_{su,App}) \cdot X_{su} \cdot I_{pH} \cdot I_{in}$
<b>Aminoacids Uptake</b>	$k_{m,aa} \cdot S_{aa,App} / (K_{S,Xaa} + S_{aa,App}) \cdot X_{aa} \cdot I_{pH} \cdot I_{in}$
<b>LCFA Uptake</b>	$k_{m,fa} \cdot S_{fa} / (K_{S,Xfa} + S_{fa}) \cdot X_{fa} \cdot I_{pH} \cdot I_{in} \cdot I_{h2}$
<b>Valerate Uptake</b>	$k_{m,c5} \cdot S_{va,App} / (K_{S,Xc5} + S_{va,App}) \cdot X_{c5} \cdot I_{pH} \cdot I_{in} \cdot I_{h2} \cdot I_{nh3}$
<b>Butyrate Uptake</b>	$k_{m,c4} \cdot S_{bu,App} / (K_{S,Xc4} + S_{bu,App}) \cdot X_{c4} \cdot I_{pH} \cdot I_{in} \cdot I_{h2} \cdot I_{nh3}$
<b>Propionate Uptake</b>	$k_{m,pro} \cdot S_{pro,App} / (K_{S,Xpro} + S_{pro,App}) \cdot X_{pro} \cdot I_{pH} \cdot I_{in} \cdot I_{h2} \cdot I_{nh3}$
<b>Acetate Uptake</b>	$k_{m,ac} \cdot S_{ac,App} / (K_{S,Xac} + S_{ac,App}) \cdot X_{ac} \cdot I_{pH} \cdot I_{in} \cdot I_{nh3}$
<b>Hydrogen Uptake</b>	$k_{m,h2} \cdot S_{h2,App} / (K_{S,Xh2} + S_{h2,App}) \cdot X_{h2} \cdot I_{pH} \cdot I_{in} \cdot I_{nh3}$
<b>Sugar Degraders Decay</b>	$k_d \cdot X_{su}$
<b>Aminoacids Degraders Decay</b>	$k_d \cdot X_{aa}$
<b>LCFA Degraders Decay</b>	$k_d \cdot X_{fa}$
<b>Valerate Degraders Decay</b>	$k_d \cdot X_{c5}$
<b>Butyrate Degraders Decay</b>	$k_d \cdot X_{c4}$
<b>Propionate Degraders Decay</b>	$k_d \cdot X_{pro}$
<b>Acetate Degraders Decay</b>	$k_d \cdot X_{ac}$
<b>Hydrogen Degraders Decay</b>	$k_d \cdot X_{h2}$

with

$$I_{in} = S_{in,App} / (K_{i,Sin} + S_{in,App})$$

$$I_{h2} = K_{i,Sh2} / (K_{i,Sh2} + S_{h2,App})$$

$$I_{pH} = K_{pH} \cdot N_{pH} / (K_{pH} \cdot N_{pH} + S_{h+} \cdot N_{pH})$$

$$I_{nh3} = K_{i,Snh3} / (K_{i,Snh3} + S_{nh3,App})$$

**Table 3.** Initial conditions used for all batch simulations in this study

Name	Mono-digestion						Co-digestion			Units
	ISR = 1.00	ISR = 1.50				ISR = 0.50	ISR = 0.16			
	TS=15.0%	TS=9.5%	TS=13.5%	TS=16.5%	TS=19.4%	TS=28.3%	TS=10.0%	TS=15.0%	TS=30.2%	
S <sub>su</sub>	9.761	6.920	8.776	10.861	13.245	6.201	1.800	2.779	6.496	kg COD m <sup>-3</sup>
S <sub>aa</sub>	3.187	5.856	7.346	9.099	11.201	7.679	0.972	1.503	3.571	kg COD m <sup>-3</sup>
S <sub>fa</sub>	2.610	1.656	2.186	2.702	3.184	1.467	0.377	0.579	1.300	kg COD m <sup>-3</sup>
S <sub>va</sub>	0.791	1.015	1.282	1.582	1.936	1.061	1.467	2.269	5.314	kg COD m <sup>-3</sup>
S <sub>bu</sub>	0.500	0.195	0.244	0.302	0.370	1.518	0.230	0.355	0.831	kg COD m <sup>-3</sup>
S <sub>pro</sub>	2.059	0.877	1.109	1.368	1.674	2.565	1.367	2.115	4.952	kg COD m <sup>-3</sup>
S <sub>ac</sub>	0.103	0.035	0.044	0.054	0.066	0.871	0.058	0.090	0.210	kg COD m <sup>-3</sup>
S <sub>h2</sub>	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	kg COD m <sup>-3</sup>
S <sub>ch4</sub>	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	kg COD m <sup>-3</sup>
S <sub>ic</sub>	0.029	0.014	0.017	0.021	0.026	0.037	0.008	0.013	0.030	kmol C m <sup>-3</sup>
S <sub>in</sub>	0.186	0.125	0.157	0.194	0.238	0.229	0.033	0.051	0.120	kmol N m <sup>-3</sup>
S <sub>i</sub>	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	kg COD m <sup>-3</sup>
S <sub>i,subs</sub>	34.706	29.233	37.076	45.883	55.954	90.351	27.565	42.559	99.486	kgCOD m <sup>-3</sup>
X <sub>ch,fast</sub>	29.283	13.840	17.553	21.722	26.490	31.003	5.400	8.337	19.489	kg COD m <sup>-3</sup>
X <sub>ch,slow</sub>	-	-	-	-	-	-	27.360	42.242	98.743	kg COD m <sup>-3</sup>
X <sub>pr</sub>	28.680	11.713	14.692	18.197	22.402	38.393	5.834	9.016	21.425	kg COD m <sup>-3</sup>
X <sub>g</sub>	18.271	3.312	4.373	5.405	6.367	5.866	2.637	4.053	9.102	kg COD m <sup>-3</sup>
X <sub>su</sub> (*)	0.050	0.050	0.063	0.078	0.095	0.150	0.005	0.008	0.018	kg COD m <sup>-3</sup>
X <sub>aa</sub> (*)	0.050	0.050	0.063	0.078	0.095	0.060	0.005	0.008	0.018	kg COD m <sup>-3</sup>
X <sub>fa</sub> (*)	0.010	0.020	0.025	0.031	0.038	0.030	0.001	0.001	0.002	kg COD m <sup>-3</sup>
X <sub>c5</sub> (*)	0.005	0.010	0.013	0.016	0.019	0.030	0.001	0.001	0.002	kgCOD m <sup>-3</sup>
X <sub>c4</sub> (*)	0.001	0.050	0.063	0.078	0.095	0.030	0.001	0.001	0.002	kg COD m <sup>-3</sup>
X <sub>pro</sub> (*)	0.005	0.020	0.025	0.031	0.038	0.030	0.001	0.001	0.003	kg COD m <sup>-3</sup>
X <sub>ac</sub> (*)	0.024	0.150	0.190	0.234	0.286	0.100	0.003**	0.005	0.011	kg COD m <sup>-3</sup>
X <sub>h2</sub> (*)	0.050	0.200	0.253	0.312	0.382	0.090	0.003	0.005	0.011	kg COD m <sup>-3</sup>
X <sub>i</sub>	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08	kg COD m <sup>-3</sup>
X <sub>i,subs</sub>	86.765	73.083	92.689	114.706	139.885	225.877	68.914	106.398	248.714	kg COD m <sup>-3</sup>
S <sub>cat</sub>	0.100	0.060	0.075	0.091	0.109	0.166	0.040	0.059	0.120	kmoleq m <sup>-3</sup>
S <sub>an</sub>	0.051	0.040	0.050	0.060	0.073	0.069	0.020	0.030	0.060	kmoleq m <sup>-3</sup>
M <sub>Global</sub>	37.12	29.92	24.02	19.80	16.47	23.45	138.23	93.13	46.13	g
ρ <sub>Global</sub>	1078	1059	1075	1093	1113	1128	1088	1134	1316	kg m <sup>-3</sup>
TS	15.0	10.8	13.4	16.4	19.6	28.3	10.0	15.0	30.2	%
VS	12.4	9.1	11.4	13.9	16.6	24.0	9.6	14.2	28.6	%
N <sub>i,subs</sub>	0.0010	0.0012	0.0012	0.0012	0.0012	0.0010	0.0004	0.0004	0.0004	kmol N kg COD <sup>-1</sup>
V <sub>Reactor</sub>	280	160	160	160	160	280	160	160	160	mL

\* These values were assessed by Global Sensitivity Analysis (GSA). \*\* This value was also assessed by Approximate Bayesian Computation (ABC). The inoculum-to-substrate ratio (ISR) is expressed in g VS/g VS

**Table 4.** Main input parameters ( $\theta$ ) used for global sensitivity analysis (GSA), including the initial values, lower and upper thresholds

Model Parameter	Units	ADM1	Initial Value	Lower Threshold	Upper Threshold	Initial Concentration	Units	Initial Value*	Lower Threshold*	Upper Threshold*
$K_{h, ch, fast}$	$d^{-1}$	10	0.120	0.012	0.228	$X_{su, Batch1}$	kg COD $m^{-3}$	-1.301	-1.952	-0.651
$K_{h, pr}$	$d^{-1}$	10	0.050	0.005	0.095	$X_{su, Batch2}$	kg COD $m^{-3}$	-1.301	-1.952	-0.651
$K_{h, li}$	$d^{-1}$	10	0.080	0.008	0.152	$X_{su, Batch6}$	kg COD $m^{-3}$	-0.824	-1.236	-0.412
$K_{h, ch, slow}$	$d^{-1}$	-	0.012	0.001	0.023	$X_{su, Batch7}$	kg COD $m^{-3}$	-2.301	-3.452	-1.151
$K_{m, su}$	$d^{-1}$	70	70	7	133	$X_{aa, Batch1}$	kg COD $m^{-3}$	-1.301	-1.952	-0.651
$K_{m, aa}$	$d^{-1}$	70	70	7	133	$X_{aa, Batch2}$	kg COD $m^{-3}$	-1.301	-1.952	-0.651
$K_{m, fa}$	$d^{-1}$	10	10	1	19	$X_{aa, Batch6}$	kg COD $m^{-3}$	-1.222	-1.833	-0.611
$K_{m, e5}$	$d^{-1}$	30	8	1	15	$X_{aa, Batch7}$	kg COD $m^{-3}$	-2.301	-3.452	-1.151
$K_{m, e4}$	$d^{-1}$	30	13	1	25	$X_{fa, Batch1}$	kg COD $m^{-3}$	-2.000	-3.000	-1.000
$K_{m, pro}$	$d^{-1}$	20	10	1	19	$X_{fa, Batch2}$	kg COD $m^{-3}$	-1.699	-2.548	-0.849
$K_{m, ac}$	$d^{-1}$	16	16	2	30	$X_{fa, Batch6}$	kg COD $m^{-3}$	-1.523	-2.284	-0.761
$K_{m, h2}$	$d^{-1}$	35	20	2	38	$X_{fa, Batch7}$	kg COD $m^{-3}$	-3.301	-4.952	-1.651
$k_d$	$d^{-1}$	0.040	0.040	0.004	0.076	$X_{e5, Batch1}$	kg COD $m^{-3}$	-2.301	-3.452	-1.151
$K_{S, Xsu}$	kg COD $m^{-3}$	1.00	1.00	0.1	1.9	$X_{e5, Batch2}$	kg COD $m^{-3}$	-2.000	-3.000	-1.000
$K_{S, Xaa}$	kg COD $m^{-3}$	0.30	0.30	0.03	0.57	$X_{e5, Batch6}$	kg COD $m^{-3}$	-1.523	-2.284	-0.761
$K_{S, Xfa}$	kg COD $m^{-3}$	0.40	0.40	0.04	0.76	$X_{e5, Batch7}$	kg COD $m^{-3}$	-3.222	-4.833	-1.611
$K_{S, Xe5}$	kg COD $m^{-3}$	0.40	0.40	0.04	0.76	$X_{e4, Batch1}$	kg COD $m^{-3}$	-3.000	-4.500	-1.500
$K_{S, Xe4}$	kg COD $m^{-3}$	0.40	0.40	0.04	0.76	$X_{e4, Batch2}$	kg COD $m^{-3}$	-1.301	-1.952	-0.651
$K_{S, Xpro}$	kg COD $m^{-3}$	0.30	0.30	0.03	0.57	$X_{e4, Batch6}$	kg COD $m^{-3}$	-1.523	-2.284	-0.761
$K_{S, Xac}$	kg COD $m^{-3}$	0.30	0.30	0.03	0.57	$X_{e4, Batch7}$	kg COD $m^{-3}$	-3.222	-4.833	-1.611
$K_{S, Xh2}$	kg COD $m^{-3}$	5.00E-05	5.00E-05	0.000005	0.000095	$X_{pro, Batch1}$	kg COD $m^{-3}$	-2.301	-3.452	-1.151
$pH_{LL, ac}$	-	6.00	5.80	5.00	6.90	$X_{pro, Batch2}$	kg COD $m^{-3}$	-1.699	-2.548	-0.849
$K_{i, Snh3, Xe5}$	kmol N $m^{-3}$	-	0.0070	0.0007	0.0133	$X_{pro, Batch6}$	kg COD $m^{-3}$	-1.523	-2.284	-0.761
$K_{i, Snh3, Xe4}$	kmol N $m^{-3}$	-	0.0100	0.0010	0.0190	$X_{pro, Batch7}$	kg COD $m^{-3}$	-3.097	-4.645	-1.548
$K_{i, Snh3, Xpro}$	kmol N $m^{-3}$	-	0.0100	0.0010	0.0190	$X_{ac, Batch1}$	kg COD $m^{-3}$	-1.620	-2.430	-0.810
$K_{i, Snh3, Xac}$	kmol N $m^{-3}$	0.0110	0.0040	0.0004	0.0076	$X_{ac, Batch2}$	kg COD $m^{-3}$	-0.824	-1.236	-0.412
$K_{i, Snh3, Xh2}$	kmol N $m^{-3}$	-	0.0150	0.0015	0.0285	$X_{ac, Batch6}$	kg COD $m^{-3}$	-1.000	-1.500	-0.500
$f_{bu, su}$	kg COD kg COD $^{-1}$	0.130	0.500	0.050	0.950	$X_{ac, Batch7}$	kg COD $m^{-3}$	-2.523	-3.784	-1.261
$f_{ac, su}$	kg COD kg COD $^{-1}$	0.270	0.290	0.029	0.551	$X_{h2, Batch1}$	kg COD $m^{-3}$	-1.301	-1.952	-0.651
$f_{h2, su}$	kg COD kg COD $^{-1}$	0.190	0.100	0.010	0.190	$X_{h2, Batch2}$	kg COD $m^{-3}$	-0.699	-1.048	-0.349
$K_{i, Sh2, Xfa}$	kg COD $m^{-3}$	3.00E-05	1.00E-05	1.00E-06	1.90E-05	$X_{h2, Batch6}$	kg COD $m^{-3}$	-1.046	-1.569	-0.523
$K_{i, Sh2, Xe5}$	kg COD $m^{-3}$	3.00E-05	3.00E-05	3.00E-06	5.70E-05	$X_{h2, Batch7}$	kg COD $m^{-3}$	-2.523	-3.784	-1.261
$K_{i, Sh2, Xe4}$	kg COD $m^{-3}$	3.00E-05	3.00E-05	3.00E-06	5.70E-05	-	-	-	-	-
$K_{i, Sh2, Xpro}$	kg COD $m^{-3}$	1.00E-05	1.00E-05	1.00E-06	1.90E-05	-	-	-	-	-
$k_{t, a}$	$d^{-1}$	200	200	20	380	-	-	-	-	-

\* *Logarithm-transformed:  $\log_{10}(\theta_B)$*

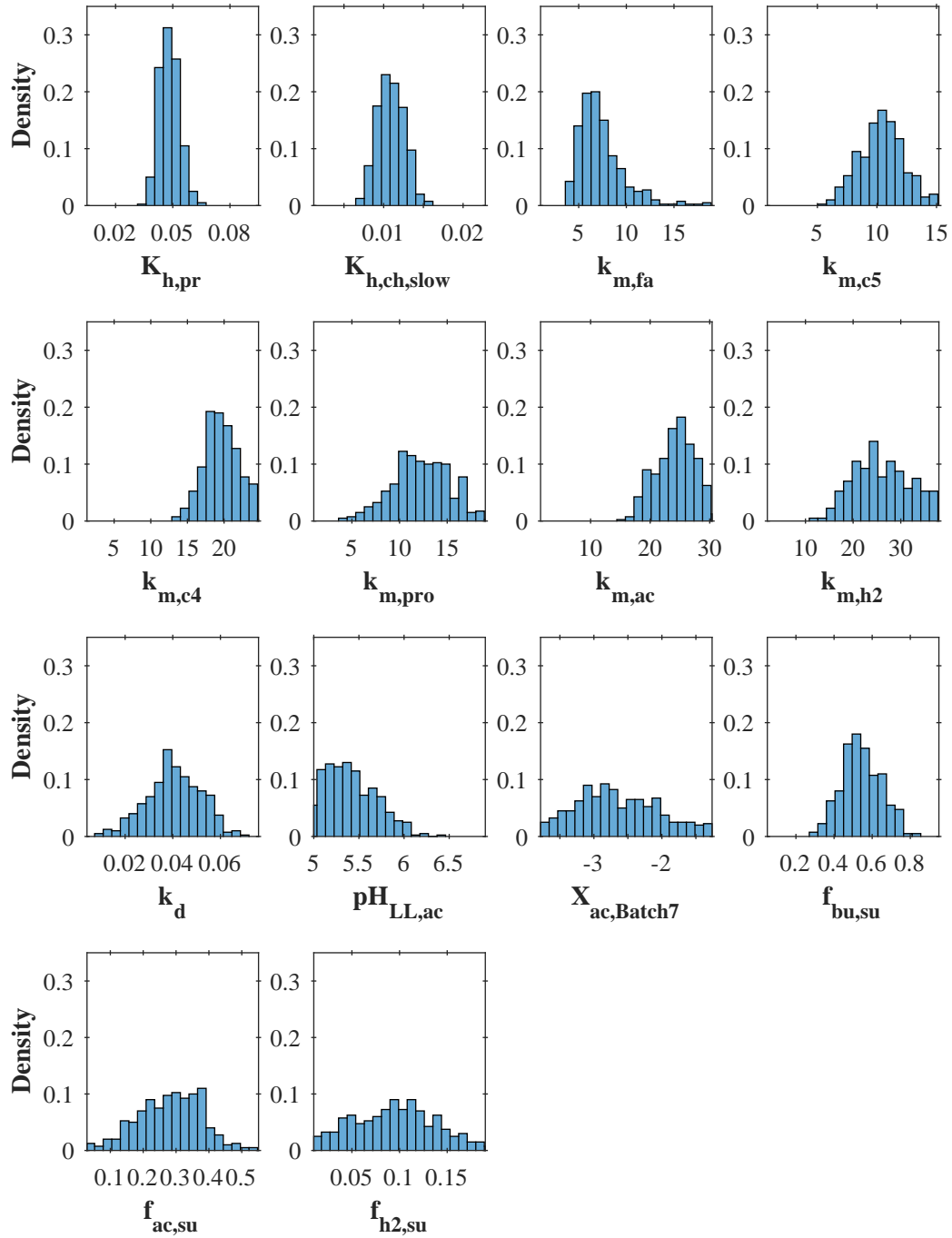
**Table 5.** Global sensitivity analysis (GSA) of input parameters ( $\theta$ ): Individual (IE) and global (GE) effects upon the objective function,  $J(\theta)$ , variance

GSA No.1			GSA No.2			GSA No.3			GSA No.4			GSA No.5			GSA No.6			GSA No.7		
$\theta$	IE (%)	GE	$\theta$	IE (%)	GE	$\theta$	IE (%)	GE	$\theta$	IE (%)	GE	$\theta$	IE (%)	GE	$\theta$	IE (%)	GE	$\theta$	IE (%)	GE
$K_{h, ch, fast}$	0.08	0.63	$K_{h, pr}$	1.75	5.23	$K_{h, pr}$	2.75	8.62	$K_{h, pr}$	2.21	7.33	$K_{h, pr}$	1.01	5.42	$K_{h, pr}$	2.00	10.44	$K_{h, pr}$	1.97	7.72
$K_{h, pr}$	2.01	7.56	$K_{h, li}$	0.25	0.69	$K_{h, li}$	0.26	2.54	$K_{h, li}$	0.11	2.85	$K_{h, ch, slow}$	4.28	14.61	$K_{h, ch, slow}$	4.07	14.47	$K_{h, ch, slow}$	4.43	17.86
$K_{h, li}$	0.29	2.07	$K_{h, ch, slow}$	5.87	16.21	$K_{h, ch, slow}$	5.83	14.32	$K_{h, ch, slow}$	5.22	16.59	$k_{m, fa}$	0.46	4.14	$k_{m, fa}$	1.15	5.66	$k_{m, fa}$	1.11	8.95
$K_{h, ch, slow}$	6.45	17.27	$k_{m, aa}$	0.07	0.43	$k_{m, fa}$	1.21	7.25	$k_{m, fa}$	0.56	4.81	$k_{m, c5}$	0.22	1.63	$k_{m, c5}$	1.59	8.34	$k_{m, c5}$	0.67	5.72
$k_{m, su}$	0.16	3.82	$k_{m, fa}$	0.81	5.97	$k_{m, c5}$	0.80	3.12	$k_{m, c5}$	0.73	3.31	$k_{m, c4}$	1.38	11.18	$k_{m, c4}$	1.50	13.79	$k_{m, c4}$	2.39	15.94
$k_{m, aa}$	0.53	2.43	$k_{m, c5}$	0.33	1.49	$k_{m, c4}$	2.23	17.17	$k_{m, c4}$	2.47	19.12	$k_{m, pro}$	1.77	8.66	$k_{m, pro}$	1.23	11.37	$k_{m, pro}$	0.72	9.24
$k_{m, fa}$	0.91	4.34	$k_{m, c4}$	2.12	12.96	$k_{m, pro}$	0.51	8.59	$k_{m, pro}$	0.75	10.21	$k_{m, ac}$	3.56	12.26	$k_{m, ac}$	5.11	17.49	$k_{m, ac}$	4.13	17.22
$k_{m, c5}$	0.22	3.64	$k_{m, pro}$	0.44	10.49	$k_{m, ac}$	7.18	17.17	$k_{m, ac}$	4.67	11.62	$k_{m, h2}$	3.72	13.46	$k_{m, h2}$	4.13	15.65	$k_{m, h2}$	1.46	12.35
$k_{m, c4}$	4.39	17.87	$k_{m, ac}$	3.16	10.71	$k_{m, h2}$	4.07	10.68	$k_{m, h2}$	3.57	11.84	$k_d$	5.16	15.78	$k_d$	3.37	9.47	$k_d$	4.43	13.37
$k_{m, pro}$	0.62	8.62	$k_{m, h2}$	2.60	8.32	$k_d$	3.67	9.00	$k_d$	3.22	8.23	$K_{S, Xh2}$	0.13	1.09	$pH_{LL, ac}$	10.24	24.61	$pH_{LL, ac}$	11.69	29.21
$k_{m, ac}$	4.88	16.35	$k_d$	2.86	7.83	$K_{S, Xh2}$	0.40	1.76	$K_{S, Xh2}$	0.22	0.91	$pH_{LL, ac}$	11.04	25.72	$f_{bu, su}$	3.34	26.74	$f_{bu, su}$	1.89	24.59
$k_{m, h2}$	1.60	11.78	$K_{S, Xh2}$	0.71	2.03	$pH_{LL, ac}$	11.09	22.10	$pH_{LL, ac}$	9.69	18.95	$K_{i, Snh3, Xh2}$	0.21	3.16	$f_{ac, su}$	0.96	13.49	$f_{ac, su}$	0.22	9.58
$k_d$	1.65	5.26	$pH_{LL, ac}$	11.48	25.57	$K_{i, Snh3, Xh2}$	0.32	2.61	$K_{i, Snh3, Xh2}$	0.39	1.91	$f_{bu, su}$	4.03	30.77	$f_{h2, su}$	5.00	14.87	$f_{h2, su}$	6.37	19.92
$K_{S, Xsu}$	0.05	0.34	$K_{i, Snh3, Xpro}$	0.20	1.79	$f_{bu, su}$	2.72	20.48	$f_{bu, su}$	2.93	24.05	$f_{ac, su}$	0.81	15.33	$X_{ac, Batch7}$	1.89	11.52	$X_{ac, Batch7}$	2.73	13.98
$K_{S, Xaa}$	0.15	1.12	$K_{i, Snh3, Xac}$	0.16	1.55	$f_{ac, su}$	0.41	10.59	$f_{ac, su}$	0.48	12.46	$f_{h2, su}$	7.05	17.03	$X_{h2, Batch7}$	0.00	0.00	-	-	-
$K_{S, Xfa}$	0.15	3.83	$K_{i, Snh3, Xh2}$	0.24	1.70	$f_{h2, su}$	6.02	13.39	$f_{h2, su}$	5.83	13.35	$K_{i, Sh2, Xc4}$	0.03	0.40	-	-	-	-	-	-
$K_{S, Xc5}$	0.07	0.63	$f_{bu, su}$	3.56	30.53	$K_{i, Sh2, Xc4}$	0.33	4.00	$K_{i, Sh2, Xc4}$	0.16	4.36	$X_{ac, Batch7}$	1.63	7.72	-	-	-	-	-	-
$K_{S, Xc4}$	0.00	0.00	$f_{ac, su}$	0.67	17.63	$K_{i, Sh2, Xpro}$	0.17	1.55	$K_{i, Sh2, Xpro}$	0.13	2.88	$X_{h2, Batch2}$	0.07	1.00	-	-	-	-	-	-
$K_{S, Xpro}$	0.00	0.00	$f_{h2, su}$	7.20	16.29	$X_{su, Batch6}$	0.06	0.69	$X_{c4, Batch2}$	0.06	0.28	$X_{h2, Batch6}$	0.00	0.00	-	-	-	-	-	-
$K_{S, Xac}$	0.13	1.13	$K_{i, Sh2, Xfa}$	0.00	0.00	$X_{aa, Batch6}$	0.02	0.38	$X_{c4, Batch6}$	0.06	0.33	$X_{h2, Batch7}$	0.40	3.15	-	-	-	-	-	-
$K_{S, Xh2}$	0.79	4.53	$K_{i, Sh2, Xc5}$	0.03	0.88	$X_{aa, Batch7}$	0.00	0.00	$X_{c4, Batch7}$	0.00	0.00	-	-	-	-	-	-	-	-	-
$pH_{LL, ac}$	6.73	16.94	$K_{i, Sh2, Xc4}$	0.22	2.74	$X_{fa, Batch1}$	0.03	0.10	$X_{pro, Batch1}$	0.05	0.44	-	-	-	-	-	-	-	-	-
$K_{i, Snh3, Xc5}$	0.02	0.29	$K_{i, Sh2, Xpro}$	0.25	2.26	$X_{fa, Batch2}$	0.00	0.00	$X_{pro, Batch2}$	0.00	0.00	-	-	-	-	-	-	-	-	-
$K_{i, Snh3, Xc4}$	0.16	2.64	$k_{La}$	0.07	0.39	$X_{fa, Batch6}$	0.00	0.00	$X_{pro, Batch6}$	0.05	0.64	-	-	-	-	-	-	-	-	-
$K_{i, Snh3, Xpro}$	0.33	4.71	$X_{su, Batch1}$	0.05	0.47	$X_{fa, Batch7}$	0.07	0.69	$X_{pro, Batch7}$	0.20	1.12	-	-	-	-	-	-	-	-	-
$K_{i, Snh3, Xac}$	0.99	6.99	$X_{su, Batch2}$	0.03	0.28	$X_{c5, Batch1}$	0.00	0.01	$X_{ac, Batch1}$	0.15	1.09	-	-	-	-	-	-	-	-	-
$K_{i, Snh3, Xh2}$	0.23	3.55	$X_{su, Batch6}$	0.22	3.38	$X_{c5, Batch2}$	0.16	0.72	$X_{ac, Batch2}$	0.14	0.38	-	-	-	-	-	-	-	-	-
$f_{bu, su}$	1.97	24.39	$X_{su, Batch7}$	0.04	0.29	$X_{c5, Batch6}$	0.03	0.14	$X_{ac, Batch6}$	0.16	1.89	-	-	-	-	-	-	-	-	-
$f_{ac, su}$	0.56	13.69	$X_{aa, Batch1}$	0.05	0.60	$X_{c5, Batch7}$	0.07	0.26	$X_{ac, Batch7}$	1.99	7.48	-	-	-	-	-	-	-	-	-
$f_{h2, su}$	4.53	12.54	$X_{aa, Batch2}$	0.00	0.00	$X_{c4, Batch1}$	0.06	0.76	$X_{h2, Batch1}$	0.18	0.74	-	-	-	-	-	-	-	-	-
$\Sigma$ IE (%) =	40.67	-	-	45.46	-	-	50.46	-	-	46.38	-	-	46.94	-	-	45.57	-	-	44.21	-
Total (%) =	72.37	-	-	75.17	-	-	78.67	-	-	74.74	-	-	72.81	-	-	70.49	-	-	68.79	-

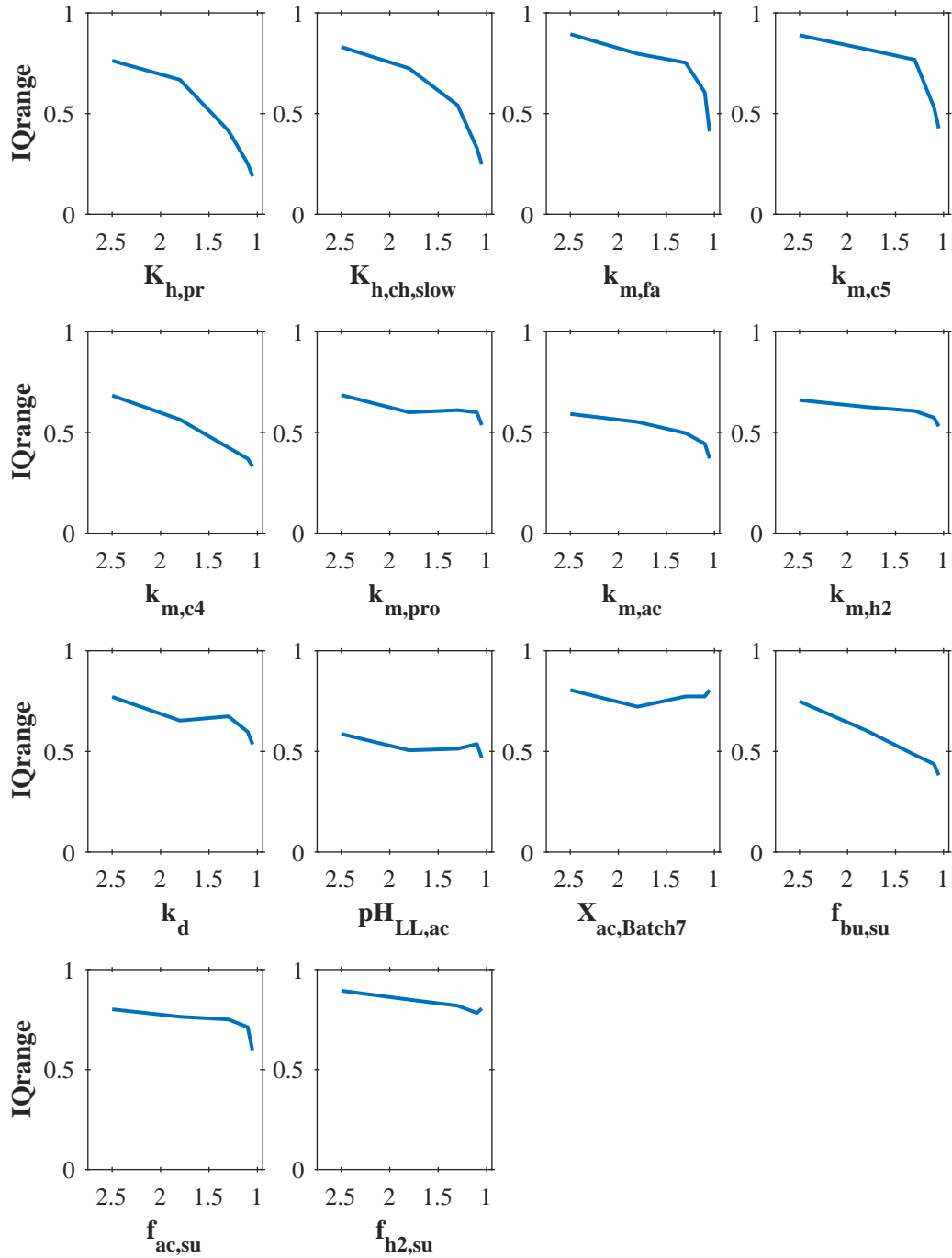
**Table 6.** Calibration of potentially-identifiable input parameters ( $\theta'$ ): Prior and posterior distributions

Parameter	Units	Initial Value	Initial Evaluation Range		Posterior Parameter Distribution						
			Minimum	Maximum	Mean	Median	Mode	5% Percentile	95% Percentile	Skewness	Kurtosis
$K_{h,pr}$	$d^{-1}$	0.050	0.005	0.095	0.048	0.048	0.044	0.040	0.057	0.217	2.83
$K_{h,ch,slow}$	$d^{-1}$	0.0120	0.0012	0.0228	0.0109	0.0108	0.0102	0.0084	0.0137	0.171	2.74
$k_{m,fa}$	$d^{-1}$	10.0	1.0	19.0	7.3	6.8	6.6	4.6	12.0	1.663	6.93
$k_{m,e5}$	$d^{-1}$	8.0	0.8	15.2	10.4	10.3	8.6	7.2	13.4	0.084	2.75
$k_{m,e4}$	$d^{-1}$	13.0	1.3	24.7	19.6	19.5	20.2	15.8	23.6	0.001	2.52
$k_{m,pro}$	$d^{-1}$	10.0	1.0	19.0	12.2	12.2	13.0	7.1	16.7	-0.148	2.66
$k_{m,ac}$	$d^{-1}$	16.0	1.6	30.4	24.2	24.5	23.2	18.7	29.4	-0.242	2.32
$k_{m,h2}$	$d^{-1}$	20.0	2.0	38.0	26.0	25.5	25.1	17.0	36.1	0.077	2.19
$k_d$	$d^{-1}$	0.040	0.004	0.076	0.040	0.040	0.045	0.020	0.059	-0.155	2.83
pH <sub>Lac</sub>		5.8	5.0	6.9	5.4	5.4	5.2	5.0	5.9	0.589	2.73
$X_{ac,Batch7}$	kg COD $m^{-3}$	-2.523	-3.784	-1.261	-2.648	-2.713	-2.853	-3.574	-1.543	0.303	2.39
$f_{bu,su}$	kg COD kg COD $^{-1}$	0.50	0.05	0.95	0.54	0.54	0.49	0.38	0.72	0.161	2.67
$f_{ac,su}$	kg COD kg COD $^{-1}$	0.29	0.03	0.55	0.28	0.29	0.23	0.12	0.43	-0.180	2.73
$f_{h2,su}$	kg COD kg COD $^{-1}$	0.10	0.01	0.19	0.09	0.09	0.09	0.02	0.17	0.060	2.31

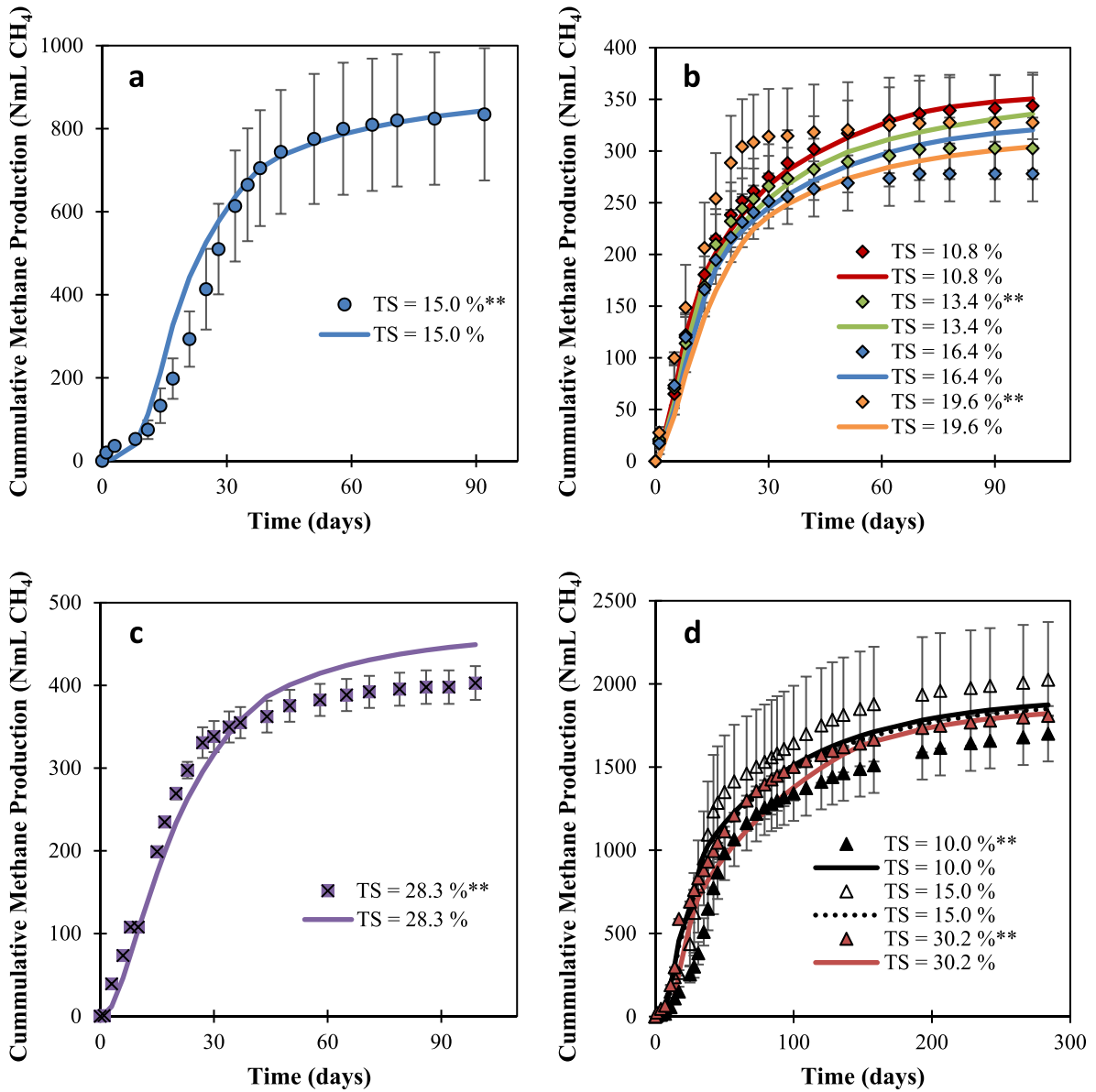




**Figure 1.** Posterior parameter distributions using 400 simulations and  $\epsilon = 1.05$

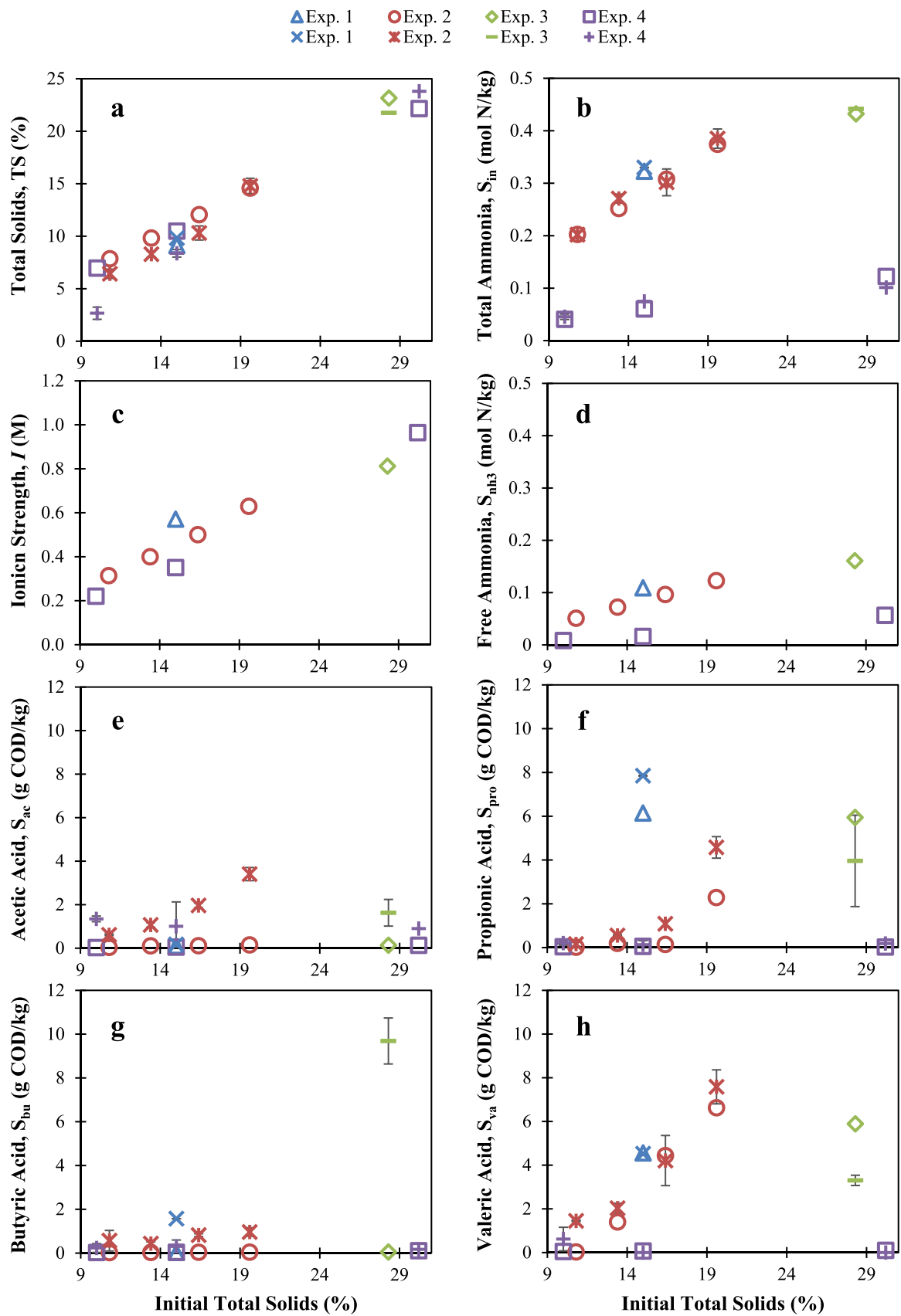


**Figure 2.** Interquartile range (percentiles 5 to 95 %) of the posterior parameter distributions using  $\epsilon \geq 1.05$  and  $\epsilon \leq 2.50$



**Figure 3.** Methane production with mono-digestion of dried OFMSW at a)  $ISR = 1.00$ ; b)  $ISR = 1.50$ ; and c)  $ISR = 1.00$ ; and co-digestion of dried OFMSW and sawdust at d)  $ISR = 0.16$ . Dots represent experimental data, while lines represent simulated values.

\*\*Experimental data used for calibration



**Figure 4.** Main variables at the end of the four batch experiments: a) Total solids (TS); b) total ammonia nitrogen (TAN,  $S_{in}$ ); c) ionic strength ( $I$ ); d) free ammonia nitrogen ( $NH_3$ ,  $S_{nh3}$ ); e) acetic acid ( $S_{ac}$ ); f) propionic acid ( $S_{pro}$ ); g) butyric acid ( $S_{bu}$ ); and h) valeric acid ( $S_{va}$ ). Crosses represent experimental data, while geometries represent simulated values