Impact of precipitating electrons and magnetosphere-ionosphere coupling processes on ionospheric conductance

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Abstract. Modeling of electrodynamic coupling between the magnetosphere and ionosphere depends on accurate specification of ionospheric conductances produced by auroral electron precipitation. Magnetospheric models determine the plasma properties on magnetic field lines connected to the auroral ionosphere, but the precipitation of energetic particles into the ionosphere is the result of a two-step process. The first step is the initiation of electron precipitation into both magnetic conjugate points from Earth's plasma sheet via wave-particle interactions. The second step consists of the multiple atmospheric reflections of electrons at the two magnetic conjugate points, which produces secondary superthermal electron fluxes. The steady state solution for the precipitating particle fluxes into the ionosphere differs significantly from that calculated based on the originating magnetospheric population predicted by MHD and ring current kinetic models. Thus, standard techniques for calculating conductances from the mean energy and energy flux of precipitating electrons in model simulations must be modified to account for these additional processes. Here we offer simple parametric relations for calculating Pedersen and Hall height-integrated conductances that include the contributions from superthermal electrons produced by magnetosphere-ionosphere-atmosphere coupling in the auroral regions.

1. Introduction

As space weather models continue to improve, the accurate specification of ionospheric electrical conductivities becomes increasingly important. The effects of ionospheric conductivities on magnetohydrodynamic (MHD) codes has been examined by Raeder et al., (2001), Ridley et al. (2004), Wiltberger et al. (2009), and Lotko et al. (2014). In general, many of the discrepancies between MHD model results and observations are attributed to uncertainties in auroral conductivities. For example, Merkin et al. (2005) and Wiltberger et al. (2017) studied the effects on MHD modeling resulting from anomalous resistivity associated with the Farley-Buneman instability (Dimant and Oppenheim, 2011). The conductance enhancements produced differences in the modeled values of the cross polar cap potential of up to 20 percent. Sensitivity of auroral electrodynamic models to ionospheric conductances has also been demonstrated by Cousins et al. (2015) and McGranaghan et al. (2016).

Given that ionospheric conductances are critical to the accuracy of space weather models, it is important that they be accurately and self-consistently computed. Here we show that multiple atmospheric reflections of superthermal electrons (SE) can significantly alter the conductivities caused by precipitating particles resulting from pitch angle scattering in the magnetosphere. The conductance change via multiple atmospheric reflections of SE is comparable to the anomalous turbulent conductivities introduced by Dimant and Oppenheim (2011). Like enhanced conductances from instabilities, the process described here can reduce the calculated cross-polar cap potential, which is often overestimated by global MHD codes.

The magnetosphere and ionosphere are strongly coupled by precipitating magnetospheric electrons from the Earth's plasmasheet. Therefore, first principle simulations of precipitating electron fluxes are required to understand spatial and temporal variations of ionospheric conductances and related electric fields. As discussed by Khazanov et al. (2015 – 2017), the first step in such simulations is initiation of electron precipitation from Earth's plasma sheet via wave particle interactions into both magnetically conjugate points. The second step is to account for multiple atmospheric reflections of electrons between the ionosphere and magnetosphere at the two magnetically conjugate points. This paper focuses on the resulting height-integrated Pedersen and Hall conductances in the auroral regions produced by multiple atmospheric reflections. Specifically, our goal here is to present correction factors that can be used with standard techniques for calculating ionospheric conductances accounting for the effects of multiple reflection processes as they were introduced by Khazanov et al. (2015 – 2017). The correction factors are calculated using the formulas presented by Robinson et al. (1987), hereafter referred to as RB1987, that are commonly used in MHD and kinetic ring current models to calculate ionospheric conductance.

RB1987 derived height-integrated Pedersen and Hall conductances as a function of mean electron energy and total electron energy flux. These conductance formulas are widely used in the space science community in global models for magnetosphere-ionosphere processes (see for example recent papers by Wolf et al. [2017], Wiltberger et al. [2017] and Perlongo et al. [2017]). In deriving these conductances, RB1987 assumed Maxwellian distributions for the precipitating electrons. Here we assume Maxwellian and kappa distributions based on the results described by *McIntosh and Anderson* [2014]. They presented maps of auroral electron spectra characterized by different types using 8 years of particle spectrometer data from the Defense Meteorological Satellite Program (DMSP) suite of polar-orbiting spacecraft. The electron spectra, which were sampled from both hemispheres, were categorized as either diffuse or accelerated. Diffuse spectra were best-fit with Maxwellian or kappa distributions, while accelerated spectra were identified as displaying characteristics of either monoenergetic or broadband acceleration. A total of 30 million spectra were characterized, with 47.05% being best-fit with Maxwellian distributions, 31.37% being best-fit with kappa distributions, 12.20% as monoenergetic, and 9.38% as broadband. Spectra with Maxwellian or kappa distributions represent the region of the diffuse aurora. In this paper, we focus on Magnetosphere-Ionosphere Coupling (MIC) of precipitating electrons in the diffuse aurora, as the diffuse aurora accounts for about 75% of the auroral energy precipitating into the ionosphere (*Newell et al.* [2009]). Also, McIntosh and Anderson [2014] showed that Maxwellian and kappa distributions account for most of the electron energy input to Earth's atmosphere even during geomagnetically active periods. Thus, accurate quantification of energy fluxes and conductances in diffuse aurora is critical to studies of magnetosphere-ionosphere coupling associated with space weather events.

The RB1987 height-integrated conductance formulas are:

$$\Sigma_P = \frac{40\bar{E}}{16+\bar{E}^2}\Phi_E^{1/2}, \quad \frac{\Sigma_H}{\Sigma_P} = 0.45(\bar{E})^{0.85} \ , \qquad (1)$$

where Σ_P and Σ_H are the Pedersen and Hall conductances, $\bar{E}$ is the electron mean energy, and Φ_E is the electron energy flux. RB1987 showed that these formulas are relatively insensitive to the exact shape of the precipitating electron energy spectrum provided the mean energy is determined from

$$\bar{E} = \frac{\int_{E_{\min}}^{E_{\max}} E\Phi(E)\,dE}{\int_{E_{\min}}^{E_{\max}} \Phi(E)\,dE} \qquad (2)$$

with the integration limits E_{min}=500 eV and E_{max}=30 keV. E_{min} is the energy of electrons that penetrate to ionospheric heights of about 200 km. Lower energy electrons that deposit energy above 200 km do not contribute significantly to the height-integrated conductivities. The upper limit corresponds to the maximum energy of most satellite-based electrostatic analyzers. E_{max} can be set to higher values if data are available for higher electron energies. As noted in RB1987, the errors in using Equations 1 to estimate conductance for non-Maxwellian distributions are minimized provided the appropriate limits of integration are used in Equation 2 to calculate the mean energy. As we are here concentrating on the ratios of conductances with and without multiple electron scattering, the effects of these errors are further minimized.

It is important to emphasize that correction factors for the RB1987 formulas are needed because they were developed specifically for use with satellite-based measurements of electron fluxes at altitudes around 800 km. They are *not appropriate* to use with mean energies and energy fluxes calculated by MHD or electron ring current kinetic models, as those do not include the fluxes of backscattered superthermal electrons (SE) that can contribute to ionospheric height-integrated conductivities. For example, Wiltberger et al. (2017) assume Maxwellian distribution functions in the plasma reference frame (Pembroke et al., 2012) to estimate ionospheric conductances in MHD modeling using the Lyons-Fedder-Mobarry (LFM) code. Similarly, Perlongo et al. (2017) applied these equations to the ring current electron populations that were calculated based on the bounce-averaged kinetic approach by Liemohn et al. (1999).

In this paper we introduce correction factors for those MHD and ring current kinetic calculations to account for the contributions from degraded and secondary electrons in the same flux tube. The correction factors to the RB1987 formulas account for the presence of two magnetically conjugate points on closed field lines and multiple SE atmospheric reflections. We define correction factors K_P and K_H for the Pedersen and Hall conductances as follows:

$$\Sigma^{K}_{PP} = K_P(\bar{E})\Sigma_P, \;\; \Sigma^{K}_{PH} = K_H(\bar{E})\Sigma_H \qquad (3)$$

where Σ^K_P and Σ^K_H are the Pedersen and Hall conductances produced by precipitating energetic electrons after multiple atmospheric reflections, calculated using the kinetic code STET developed by Khazanov et al. (2016), and Σ_P and Σ_H are conductances calculated using Equation 1 with mean energies and energy fluxes of electrons without multiple atmospheric reflections.

This paper proceeds with an example of the analysis of magnetosphere-ionosphere-atmosphere (MIA) coupling processes in the auroral region and describes how the conductances are changed by including the effects of multiple reflections (Section 2). Section 3 discusses the methodology for calculating the correction factors for the Pedersen and Hall conductances and presents analytic expressions based on the results of the calculations. We discuss and summarize the results and their application in Section 4.

2. Electron Spectra Resulting from Multiple Atmospheric Reflections

To demonstrate the effect of SE MIA coupling processes on the formation of electric conductances, we use the STET code developed by Khazanov et al. (2016). The STET model and physical scenario for SE coupling processes in the aurora used here is based on those recently developed and described by Khazanov et al. (2015, 2016, 2017). To avoid repetition, we refer the reader to those papers for full details, and provide here only a brief description of SE MIA coupling elements needed in this study. Because the major focus of this paper is electric conductance calculations in the presence of the SE multiple atmospheric reflection (and to be consistent with RB1987), we restrict ourselves by considering only precipitating magnetospheric electrons with energies greater than 500 eV, as lower energy electrons deposit their energy above 200 km altitude where currents transverse to the magnetic field are weak. The maximum energy in calculations presented below is selected to be 30 keV because most auroral energy flux is carried by electrons with energies below this value.

The methods for calculating precipitating electron fluxes in MHD and electron kinetic models are quite different. Most MHD models use methods similar to that introduced by Fedder et al. (1995), based on the linearized kinetic theory of loss-cone precipitation with allowance for acceleration by magnetic field-aligned, electrostatic potential drops (Knight, 1973; Fridman and Lemaire, 1980). Because electron dynamics are completely absent in MHD calculations, precipitation characterization is based on numerous but very reasonable physical assumptions that are discussed extensively by Wiltberger et al. (2009) and Zhang et al. (2015). On the other hand, estimating precipitation properties in the electron kinetic models is more straightforward and based on pitch-angle electron diffusion into the loss-cone via different wave-particle interaction processes in the magnetosphere.

Whistler mode chorus waves and/or electron cyclotron harmonic (ECH) waves interact with plasma sheet electrons and initiate their precipitation into both the northern and southern auroral ionospheres (Su et al., 2009; Ni et al., 2011; Khazanov et al., 2015, 2017), providing the *first step* in the calculation of magnetospheric electrons precipitated into the atmosphere. These high-energy auroral electrons backscatter via elastic collisional processes with the neutral atmosphere, and lose their energy due to non-elastic collisions and the production of secondary electrons. The auroral electrons with lower energies and the new secondary electrons are not lost to the ionosphere but escape to the magnetosphere from both magnetically conjugate regions. Khazanov et al. (2014) found that 15 ~ 40% of the total auroral energy returns to the magnetosphere and the conjugate ionosphere. Some of the escaping ionospheric electrons become trapped within the inner magnetosphere via Coulomb collision and/or wave-particle interaction, as described by Khazanov et al. (2017), and precipitate back to the atmosphere again via subsequent electron pitch-angle diffusion. Other escaping electrons can reach the conjugate ionosphere along the closed magnetic field lines and continuously ionize the upper atmosphere at the conjugate location. Electrons at the conjugate location can also be scattered back to the original ionosphere along closed field lines, continuing the collisional processes with the neutral atmosphere. These reflection processes can be repeated multiple times in both magnetically conjugate auroral regions and represent the *second step* in the calculation of magnetospheric electrons precipitated into the atmosphere. This second step is completely missing in the MHD and ring current electron kinetic models and lead to underestimation of the energy deposition into both magnetically conjugate atmospheres. The result of multiple reflections is that there is a fully self-consistent and steady-state solution for energetic electron fluxes within a closed magnetic flux tube connected to the magnetospheric equatorial plane where wave-particle interactions continuously fill the loss cone.

Figure 1 shows the scenario for our simulations as discussed in detail by Khazanov et al. (2015, 2016, 2017). The larger red and yellow arrows indicate the primary precipitating electron fluxes caused by wave-particle interactions and whistler waves (orange shading). These primary electron fluxes are reflected from the atmosphere back to the magnetosphere (smaller red and yellow arrows) possibly multiple times, and can precipitate into the conjugate region. The blue arrows indicate the fluxes of secondary electrons that escape from one hemisphere and precipitate into the conjugate hemisphere. The purple arrows indicate energy thermally conducted back to the ionosphere from particles trapped in the magnetosphere through collisions. The STET code self-consistently calculates the electron fluxes resulting from these processes on closed magnetic flux tubes. The results are irrespective of the exact mechanism causing the primary electron precipitation.

Figure 2 shows STET calculations for downward fluxes at an ionospheric altitude of 800 km that we take as the boundary between the ionosphere and magnetosphere. The calculations presented below assume that the loss cone is continuously fed by electrons with a Maxwellian distribution at the equatorial plane of the magnetosphere:

$$\Phi(E) = CEe^{-E/E_0} \tag{4}$$

where C is a normalization factor, and E_0 is the characteristic energy of magnetospheric electrons. The constant C is normalized for a total energy flux of 1 erg cm^{-2} s^{-1} at ionospheric altitude of 800 km with the assumption that the pitch angle distribution is isotropic in the atmospheric loss cone. Balancing the losses to the ionosphere with a continuous source of new electrons as well as including the effect of multiple reflections, the steady state electron energy distribution entering the ionosphere is shown by the dashed curves in Figure 2. The calculations were performed for four different characteristic energies: 1 keV (red), 3 keV (green), 7 keV (blue), and 20 keV (cyan). The solid curves show the energy spectra without multiple reflections from the atmosphere; i. e. the distribution of electrons at the equatorial plane of the magnetosphere in the atmospheric loss cone provided by magnetospheric processes. These solid lines represent the *first step* in the formation of auroral electron precipitation, and is the part of the electron flux that is approximated in MHD simulations and directly calculated in kinetic models, as we discussed earlier in this section. These fluxes are what are provided as the precipitating flux from MHD or electron ring current kinetic models. Kinetic models, like STET, can calculate from first principles the ultimate fluxes that include all the MIA coupling processes at the ionospheric conjugate points from scattering and reflection. These fluxes are shown in Figure 2 as dashed lines for each energy level, and they are the fluxes that are measured by a low Earth orbit satellite measuring precipitating fluxes. For this reason, the RB1987 formulas (Equation 1), which were developed specifically for use with satellite-based measurements of electron fluxes at altitudes around 800 km, are *not appropriate* to use with mean energies and energy fluxes calculated by MHD or electron ring current kinetic models. As we have demonstrated, such models include only *step 1* in their calculated fluxes and none of the *step 2* fluxes of backscattered SE that can significantly contribute to ionospheric height-integrated conductivities.

The fluxes shown in Figure 2 were calculated for an L value of 6, and are based on STET model parameters described by Khazanov et al. (2016). As indicated in the figure, the self-consistent energy fluxes into the atmosphere as a result of multiple reflections are enhanced by energy-dependent factors of 3 or more. The total energy flux is determined from integrating under the curves in Figure 2, and is significantly larger for the dashed curves. The mean energies are lower, owing to the cascading of energy from high to low values and the production of secondary electrons.

Table 1 lists the mean energies corresponding to the dashed curves in Figure 2 for each of the primary Maxwellian electron spectra shown by the solid curves. The energy flux assumed for the primary spectra is 1 erg/cm^{2}sec in all cases, with 15 different values of characteristic energies, E_o, selected between 400 eV to 30 KeV. For the mean energies calculated using the dashed lines we use notations $\overline{E}_{WMR}$, and those using the solid lines notations are $\overline{E}_{NMR}$, correspondingly. Lower indices in these notations correspond to the mean energies that are calculated with (WMR) and without (NMR) multiple atmospheric reflections of SE as we discussed above. The data presented in Table 1 are used in the next section to calculate coefficients K_P and K_H in formulas (2).

3. Conductance Dependence on Multiple Atmospheric Reflections

In this section, we present correction factors for Equations 1 to account for multiple reflections of SE. The correction factors account for the change in energy flux and mean energy of precipitating electrons caused by superthermal electrons produced by multiple reflections. Here we use the relations from RB1987, which were derived using Maxwellian electron distributions. However, as pointed out by RB1987, the relations are approximately valid for other distributions provided the energy flux and mean energy are calculated by integrating over the appropriate energy range. In particular, since we are here only concerned with the ratio of conductances with and without multiple reflections, we expect errors in the calculations will be minimized and the correction factors will apply generally to most auroral energy distributions. That is, the percentage error in conductance when the RB1987 formulas are used for non-Maxwellian distributions is approximately the same with and without multiple reflections.

The following methodology is used to calculate the modification of ionospheric conductances due to SE multiple atmospheric reflections. First, we run two cases of the STET code as described above in Section 2. One of these cases solves the STET kinetic equation along the magnetic field line without taking into account multiple reflection processes in both magnetically conjugate atmospheres (solid line in Figure 2), while the other case (dashed lines) includes them. We will find the correction factor, $K = K(\overline{E})$, to the conductances derived from the RB1987 formulas given by Equation 1.

For the results that are presented below, we will use the approach developed by Khazanov et al. (2016). As in the prior study, we introduce the boundary conditions for precipitating magnetospheric electron fluxes at 800 km. We calculate the differential electron energy fluxes from 500 eV to 30 keV, assuming their distribution function is isotropic in pitch angle at the equator, and that they represent the contribution of precipitated electrons driven by unspecified magnetospheric processes from the plasma sheet to the loss cone. To be applicable to the majority of electron spectra commonly observed in the auroral oval, we perform the calculations for Maxwellian and Kappa distributions. For Kappa distributions, the electron spectra are given by

$$\Phi(E) = CE\left(1+\frac{E}{\kappa E_0}\right)^{-\kappa-1} \tag{5}$$

where C is a normalization factor, E_0 is the characteristic energy of magnetospheric electrons, and κ is the kappa index. Similar to the formula (4) that represent the Maxwellian distribution function, the constant C is normalized for a total energy flux of 1 erg cm^{-2} s^{-1} at ionospheric altitude of 800 km with the assumption that the pitch angle distribution is isotropic in the atmospheric loss cone. For the Kappa distributions that were selected for these calculations we used $\kappa = 3.5$, consistent with the THEMIS (Time History of Events and Macroscale Interactions during Substorms) satellite energetic electron observations in the inner magnetosphere (Runov et al. [2015]).

In order to calculate the factors K_P and K_H in the relations of (3) we used the original formulas (1) developed by RB1987 with their definition of the mean energy provided by Equation (2). In terms of the mean energies and electron energy fluxes that are calculated *with* and *without* electron multiple atmospheric reflection, the correction factors for height-integrated Pedersen and Hall conductances are:

$$K_P = \frac{\Sigma_P^K}{\Sigma_P} \equiv \frac{\Sigma_P^{WMR}}{\Sigma_P^{NMR}} = \frac{\bar{E}_{WMR}}{\bar{E}_{NMR}} \cdot \frac{\left(16+\bar{E}_{NMR}^2\right)}{\left(16+\bar{E}_{WMR}^2\right)} \cdot \sqrt{\frac{\Phi_E^{WMR}}{\Phi_E^{NMR}}}$$

(6)

$$K_H = \frac{\Sigma_H^K}{\Sigma_H} \equiv \frac{\Sigma_H^{WMR}}{\Sigma_H^{NMR}} = \frac{\Sigma_P^{WMR}}{\Sigma_P^{NMR}} \cdot \left(\frac{\bar{E}_{WMR}}{\bar{E}_{NMR}}\right)^{0.85} .$$

Here for the Maxwellian distribution (4), electron energy flux is calculated based on the data presented in Figure 2 and the mean energies are taken from Table 1. Notations *WMR* and *NMR* represent electron fluxes plotted in Figure 2 as dashed and solid lines, respectively. Similar calculations were performed (not shown here) for the Kappa distribution (5).

Figure 3 presents the ratios K for height-integrated Pedersen and Hall conductances as functions of the characteristic E_0 and mean energies $\overline{E}$ for Maxwellian and Kappa distributions. The results show that the correction factors are the same for both types of distributions for mean energies above about 8 keV. The ratios that are presented in Figure 3 have simple analytical fits as functions of characteristic and/or mean energies. These analytical functions are:

For the Maxwellian Distribution Function

$$\begin{gathered} K_P\left(\overline{E}\right) = 2.16 - 0.87\exp\left(-0.16 \bullet \overline{E}\right); \quad K_P\left(E_0\right) = 2.10 - 0.78\exp\left(-0.34 \bullet E_0\right); \\ K_H\left(\overline{E}\right) = 1.87 - 0.54\exp\left(-0.16 \bullet \overline{E}\right); \quad K_H\left(E_0\right) = 1.83 - 0.49\exp\left(-0.35 \bullet E_0\right). \end{gathered} \tag{7}$$

For the Kappa Distribution Function

$$\begin{gathered} K_P\left(\overline{E}\right) = 2.33 - 0.82\exp\left(-0.08 \bullet \overline{E}\right); \quad K_P\left(E_0\right) = 2.11 - 0.50\exp\left(-0.35 \bullet E_0\right); \\ K_H\left(\overline{E}\right) = 1.96 - 0.37\exp\left(-0.06 \bullet \overline{E}\right); \quad K_H\left(E_0\right) = 1.85 - 0.16\exp\left(-0.20 \bullet E_0\right). \end{gathered} \tag{8}$$

In these formulas as well as in Figure 3, the mean energy $\overline{E}$ corresponds to the $\overline{E}_{NMR}$, i.e. the mean energy of precipitated electrons that is calculated without multiple atmospheric reflections. As mentioned in the introduction, these correspond to the values that are typically computed by global MHD and electron ring current models that do not include the fluxes of backscattered SE that contribute to ionospheric height-integrated conductivities. In this case, in order to simulate variations of ionospheric conductances and related electric fields, one can calculate conductances using RB1987 and then apply the correction factors from Equations 7 or 8, depending on whether either the Maxwellian or Kappa distributions best represent the primary electron spectra. The correction factors presented here may also be used with any other technique that calculates conductances from the primary energetic electron fluxes in the atmospheric loss cone at the magnetic equator provided that the electron energy spectra are similar to the Maxwellian or Kappa distributions dealt with here. As shown by RB1987, the analytic formulas for Hall and Pedersen conductance are accurate to within 25 percent for non-Maxwellian distributions. These differences are largely minimized in the calculation of the correction factors, which are the ratios between conductances calculated with and without multiple scattering.

4. Discussion and Conclusion

Accurate specification of ionospheric conductances associated with auroral precipitation is critical to space weather modeling of the geospace system. When empirical models of conductances are used in MHD or electron ring current simulations, there is no guarantee that the regions of enhanced conductance are consistent with the location of auroral activity resulting from the calculations. The same problem occurs if conductances are derived from observations that are independent of the model simulations. The optimum specification of auroral conductances is to calculate them self-consistently with the magnetospheric properties determined from MHD or ring current models. Thus, it is important to fully account for the ionospheric conductances resulting from the primary particle populations in the magnetosphere.

As has been discussed by Khazanov et al. (2015, 2016, 2017) and demonstrated in this paper again, the calculation of auroral electron precipitation into the atmosphere requires a *two-step process*. The first step is the initiation of electron precipitation from the Earth's plasma sheet via wave particle interaction or acceleration processes into both magnetically conjugate points. The second step is to account for the effects of multiple atmospheric reflections of electron fluxes formed at the boundary between the ionosphere and magnetosphere of the two magnetically conjugate points.

Here we offer simple parametric relations (7) and (8) for calculating Pedersen and Hall height-integrated electrical conductances that account for superthermal electron coupling in the auroral regions by calculating correction factors to the conductances calculated using the RB1987 formulas. The correction factors K account for SE MIA multiple reflection processes. The factors presented by formulas (7) and (8) are derived in the form of ratios for corresponding parameters as functions of the mean and characteristic energies of precipitated electrons and take into account magnetically conjugate points and multiple atmospheric reflections as described by Khazanov et al. (2015, 2016, 2017) and in Section 2 of this paper.

These parameters should only be used when there is a need to estimate electrical conductances from first principle simulations of the mean energy and energy flux of precipitating electron fluxes. In this case, depending on the most likely shape of the distribution function, one can use the traditional approach developed by RB1987 for calculation of ionospheric conductance (Equation 1) and multiply them by correction factors from the formulas given by Equations 7 and 8 to account for the conjugate ionosphere and MIA coupling processes.

Application of the correction factors presented here result in conductances a factor of two or more greater than those calculated without the effects of multiple elastic scattering. In calculating auroral electric fields, underestimating conductances causes erroneously large fields.

As pointed out by Dimant and Oppenheim (2011), underestimation of auroral conductances may explain the overestimation of cross polar cap potential calculated in MHD or ring current models. The correction factors derived here are similar to those found by Dimant and Oppenheim (2011). Therefore, we expect they will have comparable effects on calculations of cross polar cap potential drop and other electrodynamic parameters.

Relations that we derived in this paper for the correction of ionospheric conductance are mostly applicable for the regions of diffuse aurora where observations show Maxwellian or kappa distributions in 80% of all cases (*McIntosh and Anderson* [2014]). Overall, the diffuse aurora accounts for about 75% of the auroral energy precipitating into the ionosphere (*Newell et al.* [2009]).

The results of *McIntosh and Anderson* [2014] may also be used to determine where to use the correction factors for Maxwellian or Kappa distributions. Their results show the relative likelihood of Maxwellian or Kappa distributions as a function of magnetic latitude and local time over six different levels of magnetic activity. Within each magnetic latitude, magnetic local time, and Kp bin, they show the fraction of the total number of points of each type of distribution function. Given the mean energy, energy flux, and spectral shape, the RB1987 formulas, along with the correction factors given by Equations 7 and 8, can be used to accurately estimate auroral conductances.

Acknowledgments. This work was supported by NASA grants NNH14ZDA001N-HSR MAG14_2-0062, NASA Heliophysics Internal Scientist Funding Models (HISFM18-0006 and HISFM18-0009), NASA Van Allen Probes Project, and NASA LWS Program.

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Figure Captures

Figure 1. Illustration of ionosphere–magnetosphere exchange processes included in our model: wave-particle interactions (orange) from ECH and whistler waves causes primary precipitation of electrons (large red and yellow arrows), which can be reflected by the atmosphere back through the magnetosphere (small red and yellow arrows), perhaps multiple times, and precipitate into the conjugate region. Secondary−electron fluxes can also escape (blue) and precipitate into the conjugate region. Particles trapped in the magnetosphere deposit energy through collisions, which is thermally conducted (purple) back to the ionosphere.

Figure 2. Energy distributions of precipitating electrons obtained at 800km altitude at local midnight at L=6.0 with and without multiple atmospheric reflections in the magnetically conjugate points.

Figure 3. The ratios for the height-integrated Pederson and Hall conductances as the function of the mean and characteristic energies for Maxwellian and Kappa distribution function and their analytical fits presented by Equations 7 and 8.

Table 1. Mean energies

Eo, keV	0.4	0.8	1.0	2.0	3.0	5.0	7.0	10	15	20	30
$\bar{E}_{NMR}$, keV	1.08	1.79	2.17	4.10	6.06	9.59	12.12	14.43	16.34	17.30	18.24
$\bar{E}_{WMR}$, keV	1.06	1.72	2.05	3.75	5.43	8.39	10.47	12.35	13.92	14.72	15.51