Turbulent Axial Odometer Model

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AIAA Aviation Forum

- High Reynolds number experiments expanded understanding
- Flat plate flowfields
  1. c_f vs. Re_θ .... (6% revision)
  2. $[\kappa, B^+]$ .... $([.41, 5] \mapsto [.385, 4.1])$
  3. R^+_{ij} .... (R^+_{11} increases with Re_θ)
- First two items easy, third — not so much
- Reynolds-stress models should benefit from better match of all R_{ij}
- T_{ijk} depend directly on R_{ij} fields (and their derivatives)
- Attached flow: R^+_{11} and R^+_{33} generally *don't matter*
- Separated flow: $\partial_k T_{ijk}$ and full R_{ij} tensor *do matter*

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TAO Model

Olsen, Lillard

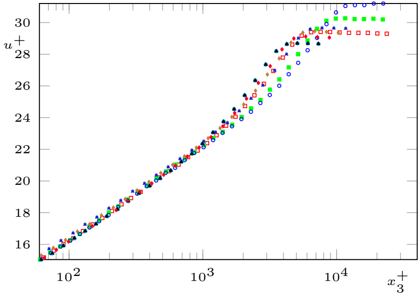
Introduction

TAO Equation

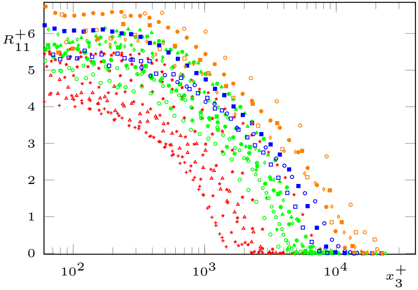
TAO in a model

Results

Conclusions



$u_1^+(x_3^+)$

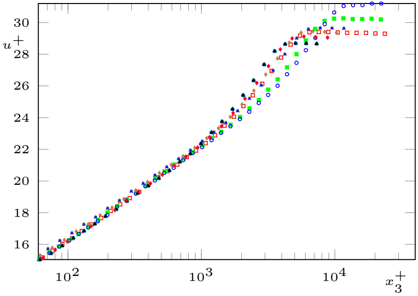


$R_{11}^+(x_3^+, Re_\theta)$

1. Recent Experiments (1990's on) : R_{ij} are not pure functions of u_τ
   R_{11}^+ grows with Re_θ, arguably R_{13}^+ and R_{33}^+ *are* pure F(u_τ)
2. Challenge: reproduce this behavior in a RANS model

Motivation: Why do we care about R_{11}^+ , R_{33}^+ ?

1. Evidence for this behavior in R_{ij} for canonical separated flows
2. Separated flows: all R_{ij} important (no longer a TSL — Mohr's circle)
3. Modeling $\partial_k(T_{ijk})$: accurate R_{ij} predictions necessary



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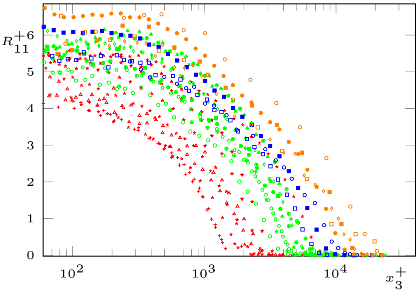

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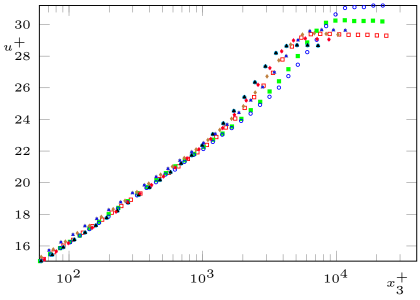

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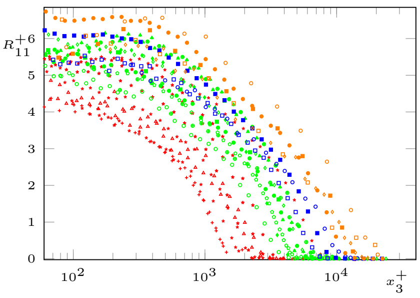

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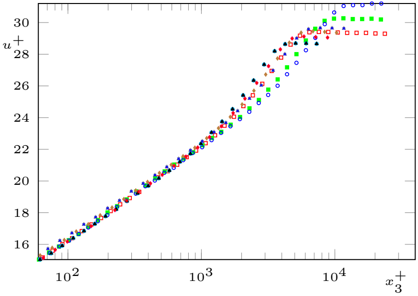

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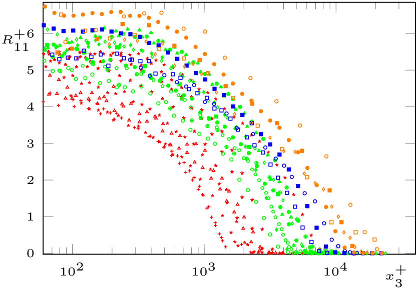

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Motivation: An Outer Scale in a Field Equation

- If wishes were horses
  Cebeci-Smith would still be among us (Re_θ unavailable)
- Physical phenomenon responsible — very long structures seen in high Re
- Concept: How long has this streamline been in turbulent flow?

An Equation for streamline length l_p (An odometer):

$$\rho\partial_t(l_p) + \rho u_i \partial_i(l_p) = \rho(u_i u_i)^{\frac{1}{2}}; (l_p|_0 = 0)$$

Turn this length into a Reynolds number, $R_o = k^{\frac{1}{2}} l_p/\nu$:

$$\partial_t(\rho R_o) + \partial_i(\rho u_i R_o) = \rho\sqrt{u_i u_i}\frac{\sqrt{k}}{\nu}$$

Add boundary layer sync and laminar reset:

$$\partial_t(\rho R_o) + \partial_i(\rho u_i R_o) = \rho\frac{\sqrt{u_i u_i}\sqrt{k}}{\nu} + \partial_i\left((\mu + \sigma_t \mu_t)\partial_i R_o\right) - \frac{\rho\omega R_o}{(1 + R_T)}$$

(Simple BC! Inflow: $R_o|_0 = 0$, Wall: $\partial R_o = 0$)

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The Turbulent Axial Odometer(TAO) equation

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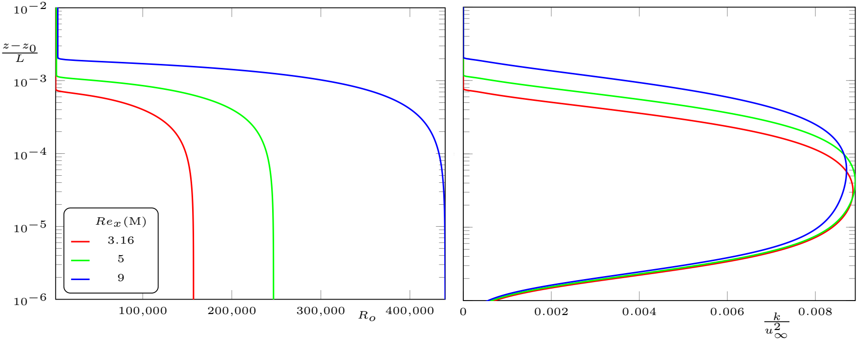


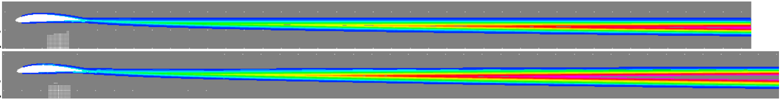
R_o k

Simplest flowfield: (and it works as designed)

- Extremely small away from the turbulent flow
- $\partial_3 R_o \approx 0$ in log layer (linearly proportional to x_1)
- So far - So good, but for a vehicle?

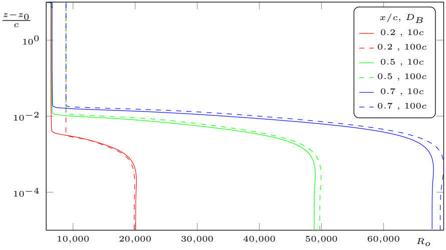
TAO Equation Solutions: Isolated Airfoil

10C
100C

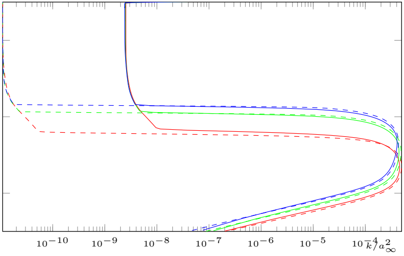


A more representative case, and it works as designed

- Two different solutions with Farfield Boundary 10C or 100C away
- Boundary layer solutions insensitive to farfield boundary distance—



R_o



k

$$\partial_t\left(R_{ij}\right)+\partial_k\left(u_k R_{ij}\right)=-R_{jk}\partial_k\overline{U_i}-R_{ik}\partial_k\overline{U_j}-\partial_k T_{ijk}+\nu\partial_k\partial_k R_{ij}$$
$$+\Pi_{ij}-2\nu\overline{\partial_k(u_i')\partial_k(u_j')}$$

$$\partial_t\left(T_{ijk}\right)+\partial_l\left(u_l T_{ijk}\right)=-T_{ijl}\partial_l\overline{U_k}-T_{jkl}\partial_l\overline{U_i}-T_{kil}\partial_l\overline{U_j}$$
$$+R_{ij}\partial_l R_{kl}+R_{jk}\partial_l R_{il}+R_{ki}\partial_l R_{jl}$$
$$+\nu\partial_l\partial_l T_{ijk}$$
$$+\Pi_{ijk}-\partial_l(Q_{ijkl})-\varepsilon_{ijk}$$

$$\Pi_{ij}=\frac{1}{\rho}\left[\overline{u_j'\partial_i(p')}+\overline{u_i'\partial_j(p')}\right]$$
$$\Pi_{ijk}=\frac{1}{\rho}\left[\overline{u_i'u_j'\partial_k(p')}+\overline{u_j'u_k'\partial_i(p')}+\overline{u_k'u_i'\partial_j(p')}\right]$$
$$Q_{ijkl}=\overline{u_i'u_j'u_k'u_l'}$$
$$\varepsilon_{ijk}=2\nu\left(\overline{u_i'\partial_l(u_j')\partial_l(u_k')}+\overline{u_j'\partial_l(u_k')\partial_l(u_i')}+\overline{u_k'\partial_l(u_i')\partial_l(u_j')}\right)$$

$$
\begin{aligned}
\partial_t(\rho k) + \partial_l(\rho u_l k) &= \rho\left[R_{ij}S_{ij} - \beta^* k\omega\right] + \partial_l\left((\mu + \sigma_k \mu_T)\partial_l k\right) - A_4 \partial_l (\rho \\
\partial_t(\rho\omega) + \partial_l(\rho u_l \omega) &= \alpha\rho S^2 - \beta\rho\omega^2 + \partial_l\left((\mu + \sigma_\omega \mu_T)\partial_l \omega\right) \\
\partial_t(\rho R_{ij}) + \partial_l(\rho u_l R_{ij}) &= A_0 \rho\omega\left(R_{ij}^{(eq)} - R_{ij}\right) \\
\partial_t(\rho T_{ijk}) + \partial_l(\rho u_l T_{ijk}) &= A_0 \rho\omega\left(T_{ijk}^{(eq)} - T_{ijk}\right) \\
\partial_t(\rho R_o) + \partial_i(\rho u_i R_o) &= \rho\frac{\sqrt{u_i u_i}\sqrt{k}}{\nu} + \partial_i\left((\mu + \sigma_t \mu_t)\partial_i R_o\right) - \frac{\rho\omega R_o}{(1 + R_T)}
\end{aligned}
$$

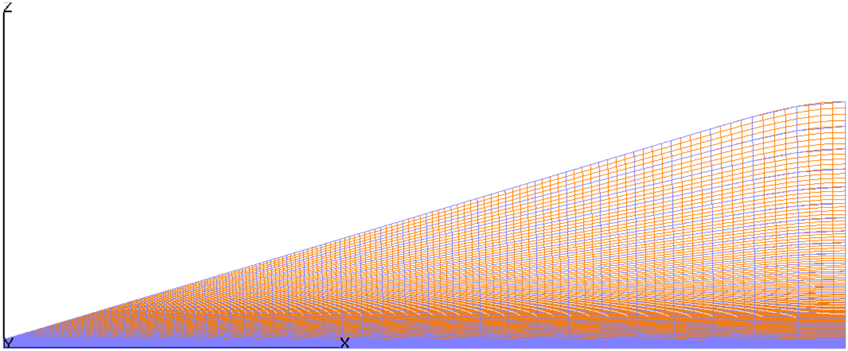
where:

$$
{R_{ij}}^{(eq)} = \frac{2}{3}k\delta_{ij} - \frac{A_1}{\omega}\left(\mathcal{P}_{ij} - \frac{1}{3}\bar{\mathcal{P}}\delta_{ij}\right) + \ldots
$$

$$
\psi = \max(\Psi_L, \Psi_R \ln(1 + R_o/R_m)) \qquad A_6 = \frac{2}{3}\frac{1 + \psi^2}{R_{NN} + \psi} \qquad A_1 = \psi/A_6
$$

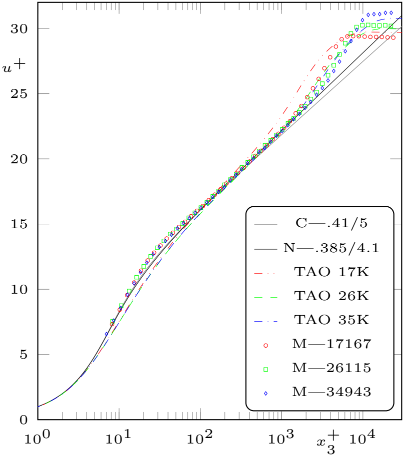
$$
\mathcal{K} = \frac{1 + \psi^2}{A_6} \qquad \beta^* = \psi/A_6 \qquad \beta = \beta^*/n_D \qquad \sigma_\omega = \frac{\beta/A_6^2 - \alpha}{\mathcal{K}A_8^2}
$$

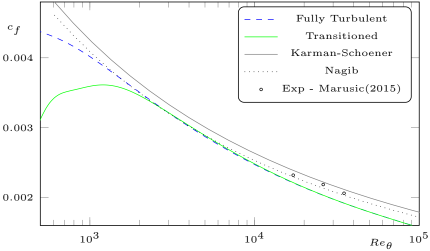
Flat Plate Solution



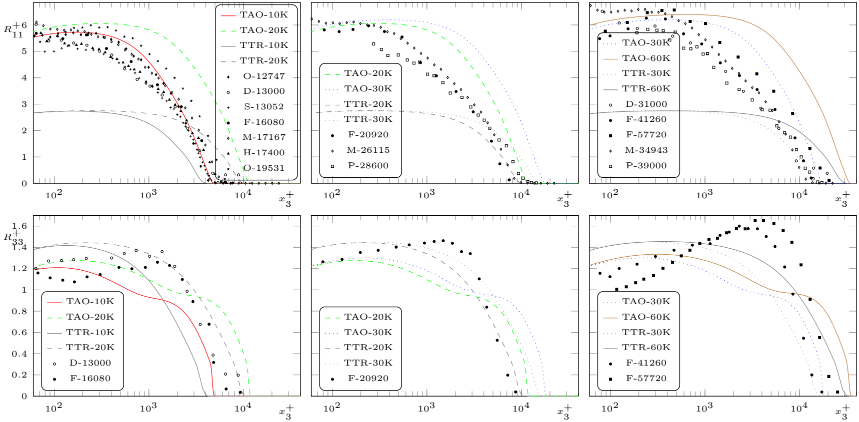
- $M_\infty = 0.2$
- $Re_L = 100^6$
- "Flight" freestream turbulence
- 513×513 grid (Grid convergence checked)

Flat Plate Velocity and Skin Friction

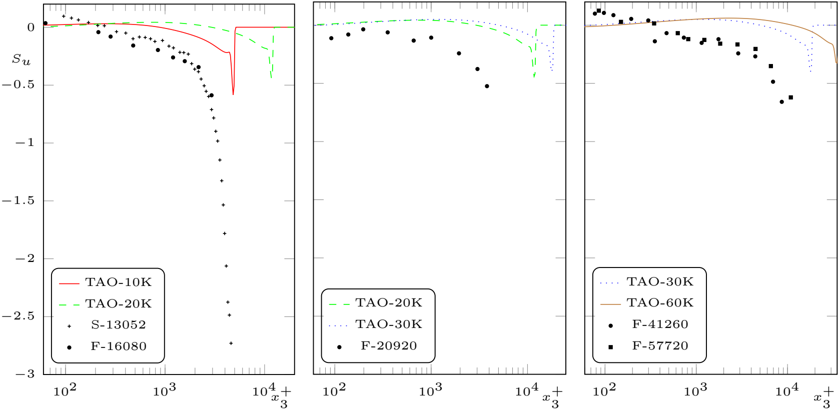




- Retained law of the wall axial velocity distribution
- Able to get good $c_f(Re_\theta)$ predictions
- –Did no harm–

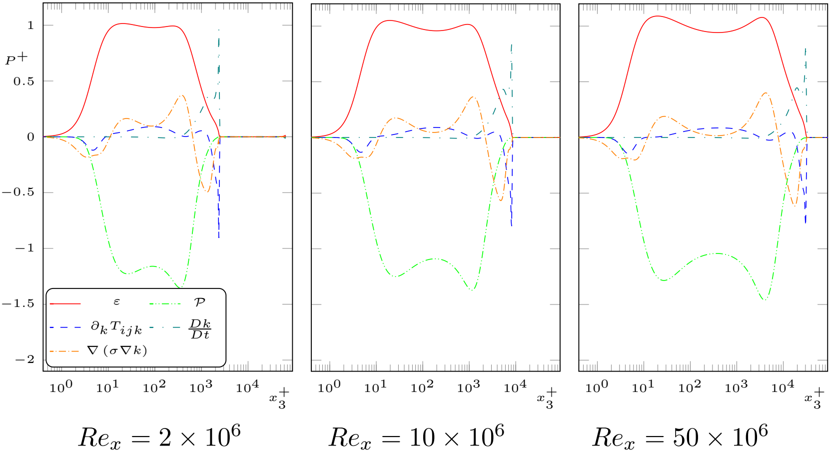


- Much improved R_{11}^+ predictions
- Better R_{33}^+ predictions
- Overall much improved R_{ij} prediction behavior (Mohr's circle)



- $Su = T_{111}/R_{11}^{1.5}$
- Not the most important transport term, but checkable
- Overall prediction encouraging (low in log region, high at edge)
- B.L. Edge position not identical in CFD/experiment

Given $\partial_k(T_{ijk})$ plausible, what does the tke balance look like?



- Transport Small, except at BL edge
- $\mathcal{P} = \varepsilon$ dominant balance in log layer

Conclusions

- R_o works as a turbulent odometer/outer scale
- Much improved R_{ij} predictions obtained
- T_{ijk} consistent with experiment (depends on R_{ij} predictions)

Future directions

- Matching/tuning more experiments (esp those with T_{ijk})
  - Junction Flow Experiment
  - Driver CS0, Spinning Cylinder
- Separated flows (promising results with earlier versions)
  - Junction Flow Experiment
  - Johnson Bump

Acknowledgements

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