TAO Model

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Turbulent Axial Odometer Model

Michael E. Olsen<sup>1</sup> Randolph P. Lillard<sup>2</sup>

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AIAA Aviation Forum

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Results
Conclusions

- High Reynolds number experiments expanded understanding
- Flat plate flowfields
  - - $(R_{ij}^+)$   $(R_{11}^+)$  increases with Re
- First two items easy, third not so much
- ullet Reynolds-stress models should benefit from better match of  $\underline{\mathsf{all}}\ R_{ij}$
- $T_{ijk}$  depend directly on  $R_{ij}$  fields (and their derivatives)
- Attached flow:  $R_{11}^+$  and  $R_{33}^+$  generally don't matter
- Separated flow:  $\partial_k T_{ijk}$  and full  $R_{ij}$  tensor do matter

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1	$c_f$	VS.	$Re_{\theta}$	 		. (6	%	revi	sio	n)						
										41	-51				4	

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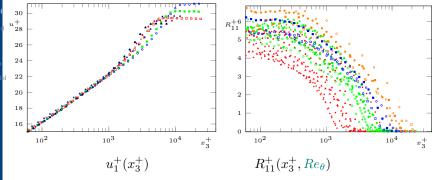


# Introduction/Motivation



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Results



- ① Recent Experiments (1990's on) :  $R_{ij}$  are not pure functions of  $u_{\tau}$   $R_{11}^+$  grows with  $Re_{\theta}$ , arguably  $R_{13}^+$  and  $R_{33}^+$  are pure  ${\sf F}(u_{\tau})$
- Challenge: reproduce this behavior in a RANS model

Notivation: Why do we care about  $R^+ = R^+$ ?

- ① Evidence for this behavior in  $R_{ij}$  for canonical separated flows
- ② Separated flows: all  $R_{ij}$  important (no longer a TSL Mohr's circle)
- 3 Modeling  $\partial_k(T_{ijk})$ : accurate  $R_{ij}$  predictions necessary

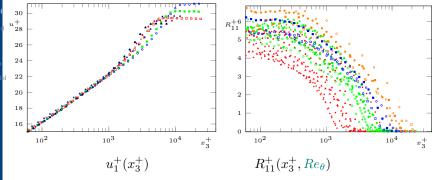


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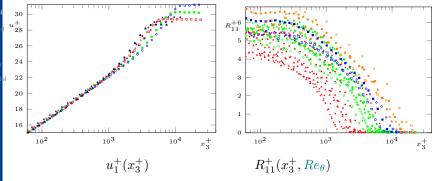


# Introduction/Motivation



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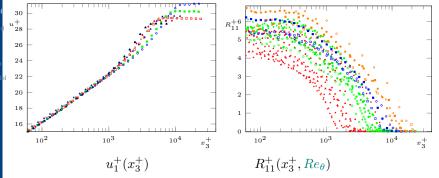


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TAO Model Mo

### odel Motivation: An Outer Scale in a Field Equation

If wishes were horses

Cebeci-Smith would still be among us  $(Re_{\theta} \text{ unavailable})$ 

- Physical phenomenon responsible very long structures seen in high Re
- Concept: How long has this streamline been in turbulent flow?

An Equation for streamline length  $l_m$  (An odometer)

$$\rho \partial_t(l_p) + \rho u_i \partial_i(l_p) = \rho(u_i u_i)^{\frac{1}{2}}; (l_p|_0 = 0)$$

Turn this length into a Reynolds number,  $R_0 = k^{\frac{1}{2}} l_n / \nu$ 

$$\partial_t(\rho R_o) + \partial_i(\rho u_i R_o) = \rho \sqrt{u_i u_i} \frac{\sqrt{\rho}}{dt}$$

Add boundary layer sync and laminar reset

$$\partial_t(\rho R_o) + \partial_i(\rho u_i R_o) = \rho \frac{\sqrt{u_i u_i} \sqrt{k}}{\nu} + \partial_i \left( (\mu + \sigma_t \mu_t) \partial_i R_o \right) - \frac{\rho \omega R_o}{(1 + R_T)}$$

Simple BCI Inflow:  $R_0|_0 = 0$  Wall:  $\partial R_0 = 0$ 

Olsen, I

TAO Equation





TAO Model

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$$\partial_t(\rho R_o) + \partial_i(\rho u_i R_o) = \rho \sqrt{u_i u_i} \frac{\sqrt{\rho}}{\nu}$$

$$\partial_t(\rho R_o) + \partial_i(\rho u_i R_o) = \rho \frac{\sqrt{u_i u_i} \sqrt{k}}{\nu} + \partial_i \left( (\mu + \sigma_t \mu_t) \partial_i R_o \right) - \frac{\rho \omega R_o}{(1 + R_T)}$$

TAO Equation

TAO in a model



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TAO Equation

TAO in a model

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# The Turbulent Axial Odometer(TAO) equation



TAO Model

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Turn this length into a Reynolds number,  $R_o = k^{\frac{1}{2}} l_p / \nu$ :

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(Simple BC! Inflow:  $R_o|_0=0$ , Wall:  $\partial R_o=0$ )

Olsen, L

TAO Equation

Results Conclus

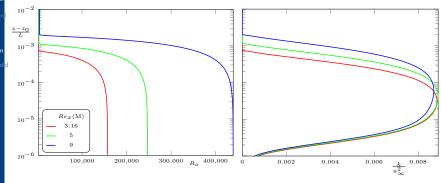
### TAO Equation Solutions: Flat Plate



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k





Simplest flowfield: (and it works as designed)

Extremely small away from the turbulent flow

 $R_{o}$ 

- $\partial_3 R_o \approx 0$  in log layer (linearly proportional to  $x_1$ )
- So far So good, but for a vehicle?

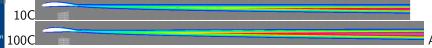


### TAO Equation Solutions: Isolated Airfoil



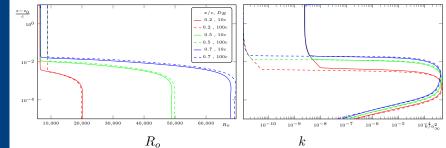
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more representative case, and it works as designed

- Two different solutions with Farfield Boundary 10C or 100C away
- Boundary layer solutions insensitive to farfield boundary distance—





# Exact Equations – Incompressible

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TAO Mode

Introduction

TAO in a mod

$$\partial_{t} (R_{ij}) + \partial_{k} (u_{k}R_{ij}) = -R_{jk}\partial_{k}\overline{U_{i}} - R_{ik}\partial_{k}\overline{U_{j}} - \partial_{k}T_{ijk} + \nu\partial_{k}\partial_{k}R_{ij}$$

$$+ \Pi_{ij} - 2\nu\overline{\partial_{k}(u'_{i})\partial_{k}(u'_{j})}$$

$$\partial_{t} (T_{ijk}) + \partial_{l} (u_{l}T_{ijk}) = -T_{ijl}\partial_{l}\overline{U_{k}} - T_{jkl}\partial_{l}\overline{U_{i}} - T_{kil}\partial_{l}\overline{U_{j}}$$

$$+ R_{ij}\partial_{l}R_{kl} + R_{jk}\partial_{l}R_{il} + R_{ki}\partial_{l}R_{jl}$$

$$+ \nu\partial_{l}\partial_{l}T_{ijk}$$

$$+ \Pi_{ijk} - \partial_{l}(Q_{ijkl}) - \varepsilon_{ijk}$$

$$\begin{split} \Pi_{ij} &= \frac{1}{\rho} \left[ \overline{u'_j \partial_i(p')} + \overline{u'_i \partial_j(p')} \right] \\ \Pi_{ijk} &= \frac{1}{\rho} \left[ \overline{u'_i u'_j \partial_k(p')} + \overline{u'_j u'_k \partial_i(p')} + \overline{u'_k u'_i \partial_j(p')} \right] \\ Q_{ijkl} &= \overline{u'_i u'_j u'_k u'_l} \\ \varepsilon_{ijk} &= 2\nu \left( \overline{u'_i \partial_l(u'_j) \partial_l(u'_k)} + \overline{u'_j \partial_l(u'_k) \partial_l(u'_i)} + \overline{u'_k \partial_l(u'_i) \partial_l(u'_j)} \right) \end{split}$$

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TAO in a mod

$$\partial_{t} (\rho k) + \partial_{l} (\rho u_{l} k) = \rho \left[ R_{ij} S_{ij} - \beta^{*} k \omega \right] + \partial_{l} ((\mu + \sigma_{k} \mu_{T}) \partial_{l} k) - A_{4} \partial_{l} (\rho \lambda_{t} (\rho \lambda_{t}) + \partial_{l} (\rho u_{l} \lambda_{t})) + \partial_{l} (\rho \lambda_{t}) + \partial_{l} (\rho$$

$$\partial_t \left( \rho T_{ijk} \right) + \partial_l \left( \rho u_l T_{ijk} \right) = A_0 \rho \omega \left( T_{ijk}^{(eq)} - T_{ijk} \right)$$

$$\partial_t(\rho R_o) + \partial_i(\rho u_i R_o) = \rho \frac{\sqrt{u_i u_i} \sqrt{k}}{v_i} + \partial_i((\mu + \sigma_t \mu_t) \partial_i R_o) - \frac{\rho \omega R_o}{(1 + R_o)}$$

where:

$$R_{ij}^{(eq)} = \frac{2}{3}k\delta_{ij} - \frac{A_1}{\Omega}(\mathcal{P}_{ij} - \frac{1}{3}\bar{\mathcal{P}}\delta_{ij}) + \dots$$

$$\psi = \max(\Psi_L, \Psi_R \ln(1 + R_o/R_m)) \quad A_6 = \frac{2}{3} \frac{1 + \psi^2}{R_{NN} + \psi} \quad A_1 = \psi/A_6$$

$$\mathcal{K} = \frac{1 + \psi^2}{A_c} \quad \beta^* = \psi/A_6 \qquad \beta = \beta^*/n_D \qquad \sigma_\omega = \frac{\beta/A_6^2 - \alpha}{KA_5^2}$$

 $\beta = \beta^*/n_D$ 

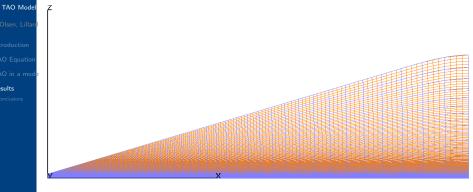
# Flat Plate Solution



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Results



- $M_{\infty} = 0.2$
- $Re_L = 100^6$
- "Flight" freestream turbulence
- $513 \times 513$  grid (Grid convergence checked)

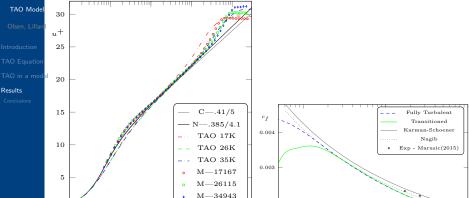


 $10^{0}$ 

## Flat Plate Velocity and Skin Friction



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0.002

103

Retained law of the wall axial velocity distribution

 $x_2^+ 10^4$ 

ullet Able to get good  $c_f(Re_{ heta})$  predictions

 $10^{2}$ 

–Did no harm–

 $10^{1}$ 



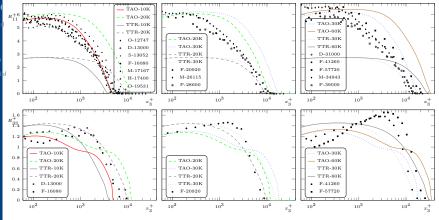
## Flat Plate Reynolds-stresses



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- Much improved R<sub>11</sub><sup>+</sup> predictions
- Better  $R_{33}^+$  predictions
- Overall much improved  $R_{ij}$  prediction behavior (Mohr's circle)

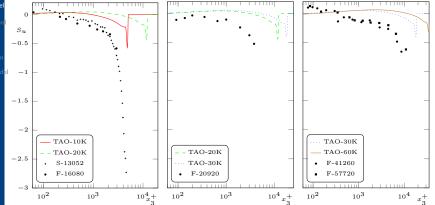


## Flat Plate Turbulent Transport



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- $\bullet$   $Su = T_{111}/R_{11}^{1.5}$
- Not the most important transport term, but checkable
- Overall prediction encouraging (low in log region, high at edge)
- B.L. Edge position not identical in CFD/experiment

# Flat Plate Turbulent Energy Balance

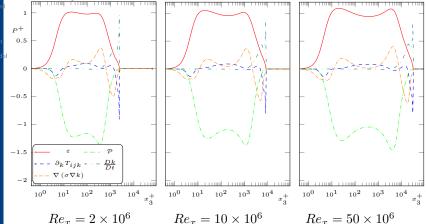




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Results

Given  $\partial_k(T_{ijk})$  plausible, what does the tke balance look like?



- Transport Small, except at BL edge
- $\mathcal{P} = \varepsilon$  dominant balance in log layer



### Conclusions/Future Directions

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Conclusions

- ullet  $R_o$  works as a turbulent odometer/outer scale
- Much improved  $R_{ij}$  predictions obtained
- ullet  $T_{ijk}$  consistent with experiment (depends on  $R_{ij}$  predictions)

#### Future directions

- ullet Matching/tuning more experiments (esp those with  $T_{ijk}$  )
  - Junction Flow Experiment
  - Driver CS0, Spinning Cylinder
- Separated flows (promising results with earlier versions)
  - Junction Flow Experiment
  - Johnson Bump



# Acknowledgements

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