



Uncertainty Analysis of Experimental Discharge Coefficients in Additively Manufactured Liquid Injector Elements

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Outline

- Introduction
- Facility Overview
- Approach
 - Test Matrix
 - Hardware Description
 - Procedure
 - Analysis Methodology
- Uncertainty Results
 - Individual Test Articles
 - Common Design Variant Across all Manufacturers
 - All Design Variants for each Manufacturer
 - Design Variant Prediction Interval
- Conclusion

Additive Rocket Engine Components

- Advantages
 - Mass Reduction
 - Part count reduction
 - Reduced assembly time
 - Reduced manufacturing costs
 - Reduced lead time
- Challenges
 - Still a maturing technology
 - Small geometric features and passages/Large scale parts
 - Manufacturing imperfections
- Several successful test programs with AM parts.



Hot-fire testing of an Aerojet Rocketdyne RL 10C-X prototype engine with 3-D printed core components. [1]



Subscale Integral injector manufactured with SLM. [2]



SLM produced integrated nozzle film coolant ring designed and tested at NASA. [3]

1. “3-D Printed RL10C-X Prototype Rocket Engine Soars Through Initial Round of Testing,” Aerojet Rocketdyne Press Release, www.rocket.com/article/3-d-printed-rl10c-x-prototype-rocket-engine-soars-through-initial-round-testing, 2019.
2. Soller, S. et. al., “Design and Testing of Liquid Propellant Injectors for Additive Manufacturing,” 7th European Conference for Aerospace Sciences, 2017.
3. Gradl, P., et. al. “Additive Manufacturing of Liquid Rocket Engine Combustion Devices: A Summary of Process Developments and Hot-Fire Testing Results,” 54th AIAA/SAE/ASE Joint Propulsion Conference, 2018.

Objective

- Assess the manufacturer-to-manufacturer variability in flow discharge coefficients of identical parts with small flow passages.

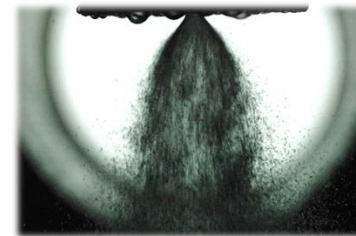
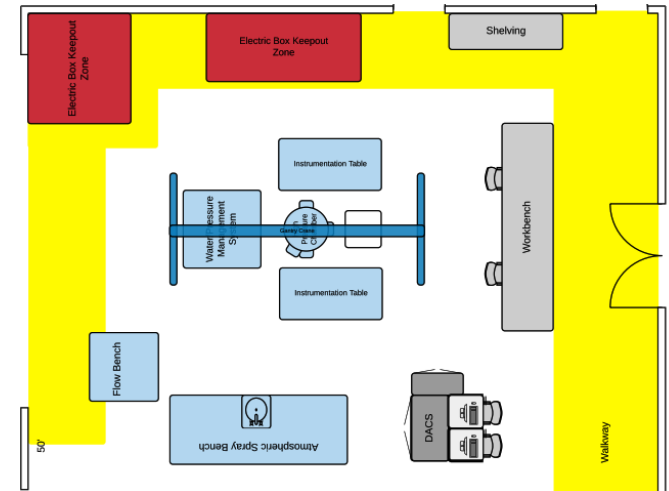
Scope:

- Investigate two internal geometries:
 - 1) Radial-Fed Annulus
 - 2) Cavitating Venturi
- Investigate subtractive (baseline) and additive manufacturing (11 vendors) .
- Cold flow (water) parts over relevant operating regimes
- Perform detailed measurement uncertainty analysis
- Determine and compare differences in discharge coefficients among the manufacturing methods

PRC Injector Spray Facility

Non-Reactive, cold flow environments for the study of injectors and injection processes in liquid injection devices.

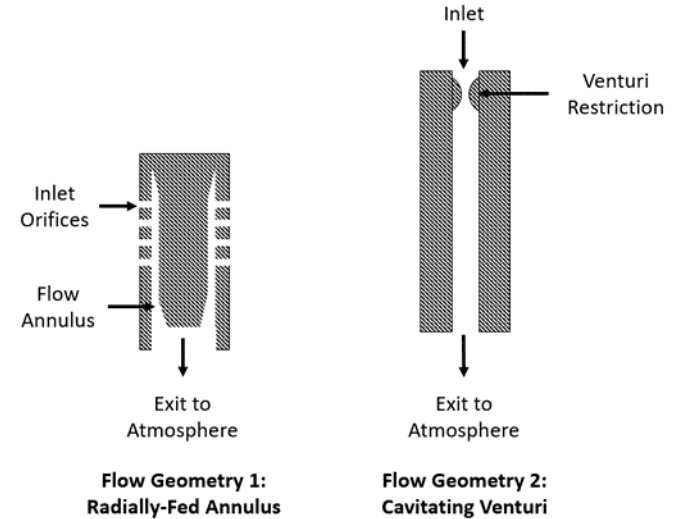
- Pressurized Spray Chamber
 - 18" Internal Diameter x 72" Tall
 - 500 psig Max Pressure
 - Four - 6" Diameter Optical Access Ports
- Atmospheric Spray Bench
- Flow Bench
- Liquid simulant flow rates up to 2 lbm/s (water and water based solutions)
- Gas flow rates up to 1 lb/s (Nitrogen/Compressed Air)
- Optical Diagnostic Access:
 - High Speed or Standard Video
 - Laser Diagnostics (PIV, PDPA)
- Common DAQ system
 - High Speed 1Ms/s
 - Integrated adjustable Low Pass Filtering
 - Temperature/Static Pressure (1000Hz)



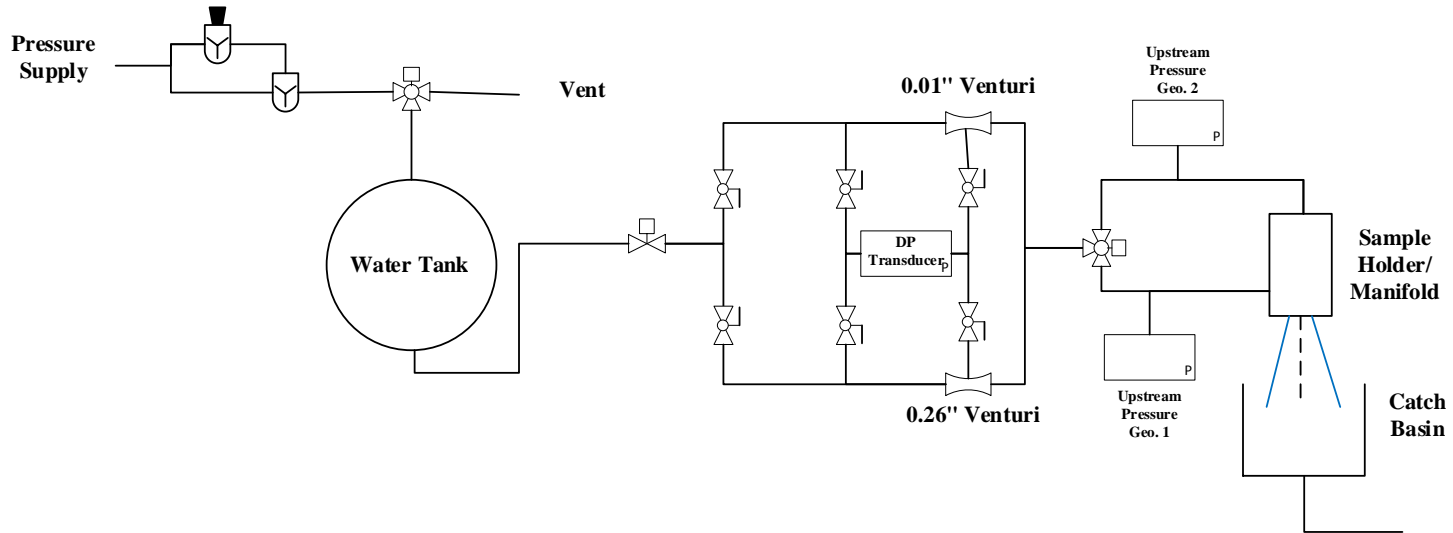
sure (left) and 500 psig (right)



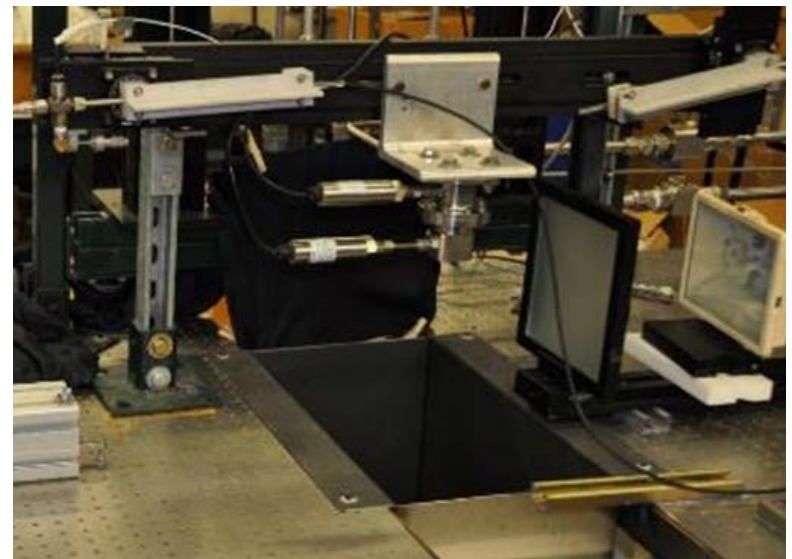
- 11 Manufacturers using SLM printers
- 4 Design Variants on the same build plate
- 45 Injectors
- Operating Conditions
 - Atmospheric Back Pressure
 - Flow Geometry 1
 - 75 psig to 550 psig
 - 0.5 lb/s to 1.6 lb/s
 - Flow Geometry 2
 - 50 psig to 1550 psig
 - 0.05 lb/s to 0.3 lb/s



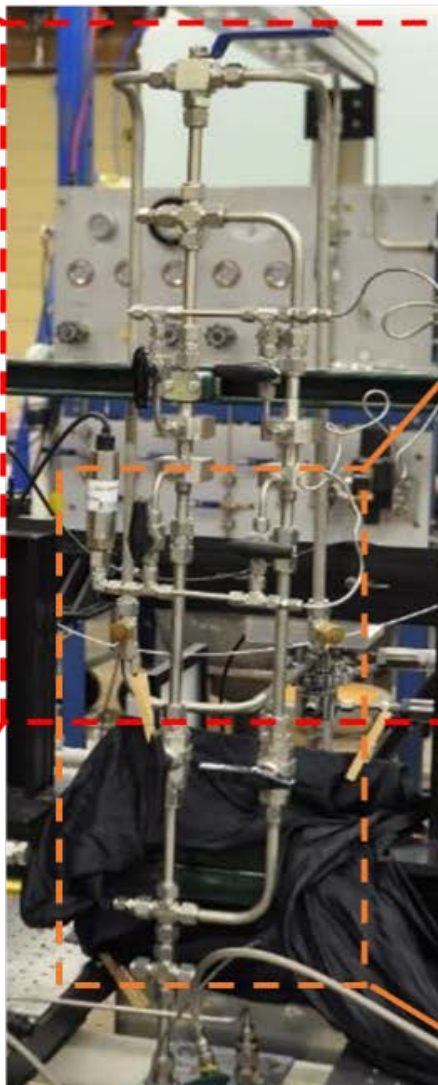
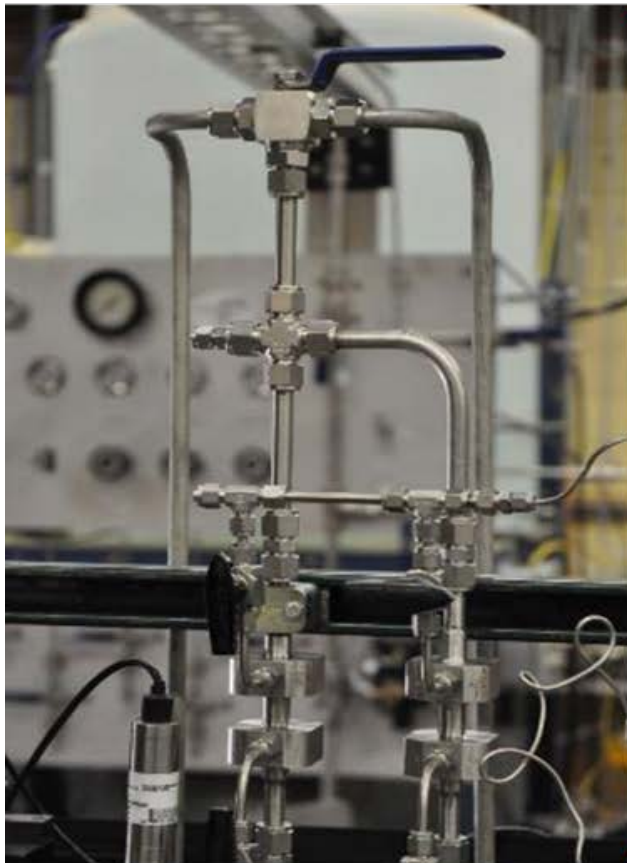
	Flow Geometry 1: Radially-Fed Annulus				Flow Geometry 2: Cavitating Venturi	
Design Variant	Radial Hole ID (% of Baseline)	# of Radial Holes	Annulus ID (% of Baseline)	Annulus OD (% of Baseline)	Flow Duct ID (% of Baseline)	Venturi ID (% of Baseline)
1	100	168	100	100	100	100
2	100	168	111.2	100	100	105.3
3	115.8	168	107.5	111.9	100	110.5
4	157.9	67	103.7	100	100	115.8
Baseline	100	168	100	100	100	100



- Venturis shared common DP Transducer
- Venturi manually selected based on Flow rate
- Pressures measured at 1000Hz and averaged over 7 seconds of steady state flow



Venturis

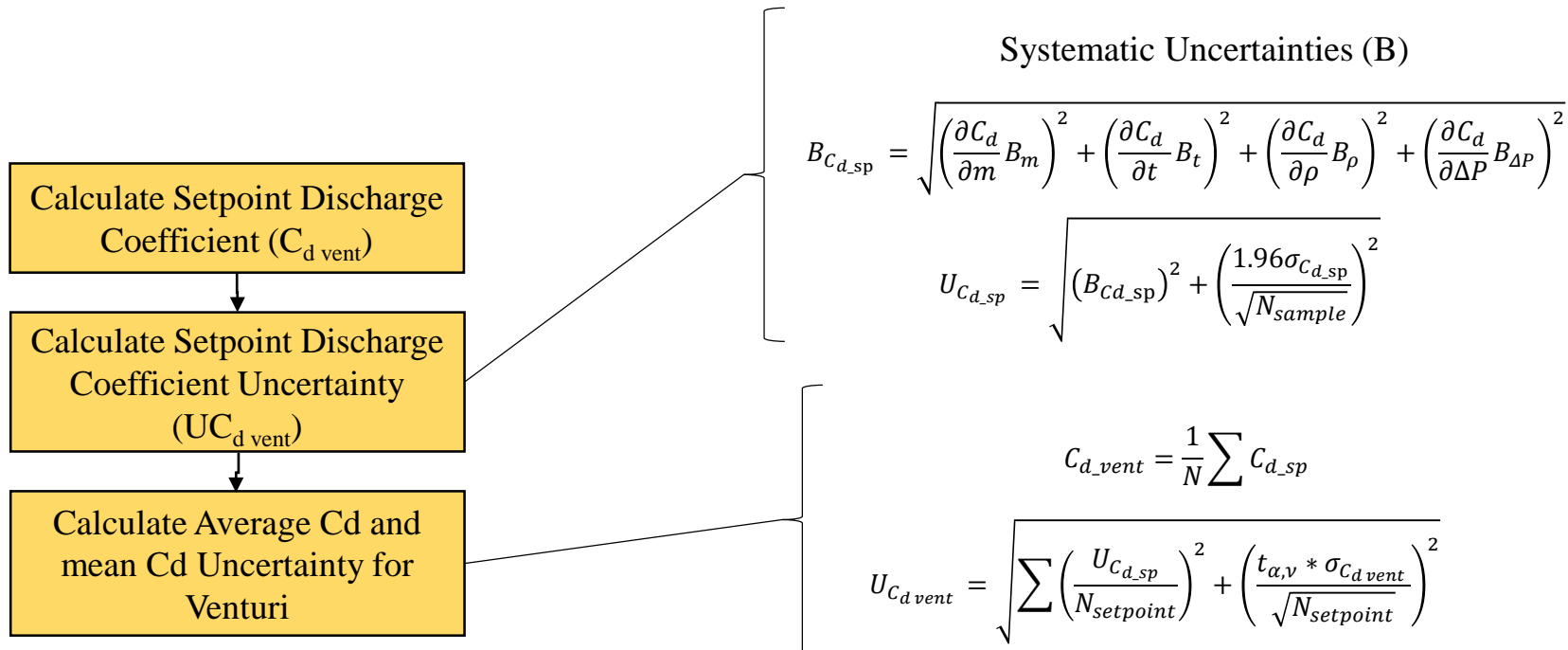


Establish single Discharge Coefficient, C_d , for each venturi

- In-line, end-to-end calibration
- Timed collection at steady flow
 - 18 setpoints (0.10 venturi)
 - 12 setpoints (0.26 venturi)

$$C_d = \frac{\frac{m}{t}}{\sqrt{\frac{2\rho\Delta P}{\frac{1}{\left(\frac{\pi d_t^2}{4}\right)^2} - \frac{1}{\left(\frac{\pi d_1^2}{4}\right)^2}}}}$$

m = mass of collected water
 t = collection time
 d_t = venturi throat diameter
 d_1 = venturi inlet diameter
 ρ = density
 ΔP = venturi pressure drop (inlet to throat)
 σ = standard deviation
 $t_{\alpha,v}$ = student t distribution

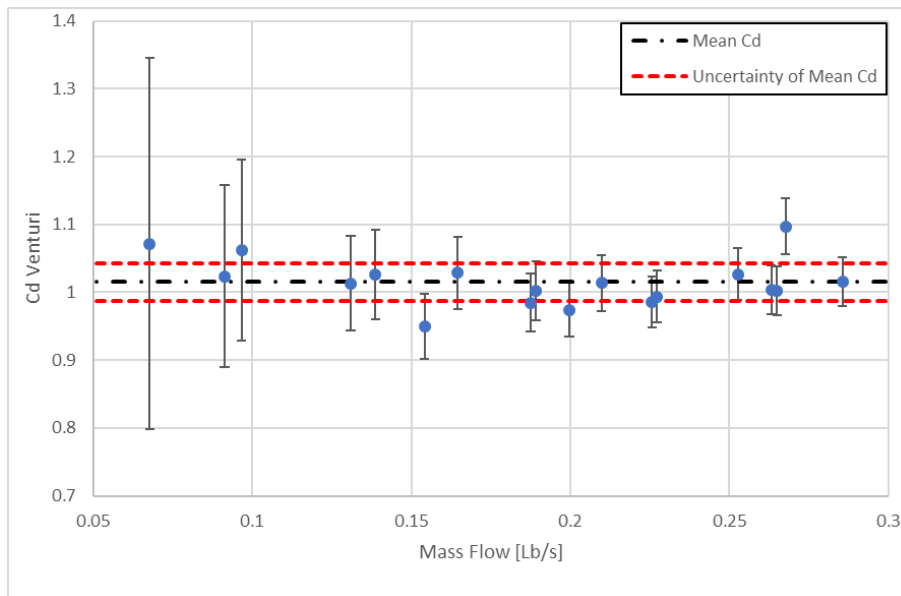


0.100" venturi

$$C_d = 1.015$$

$$U_{cd} = 2.7\%$$

0.02 Lb/s to 0.3 Lb/s

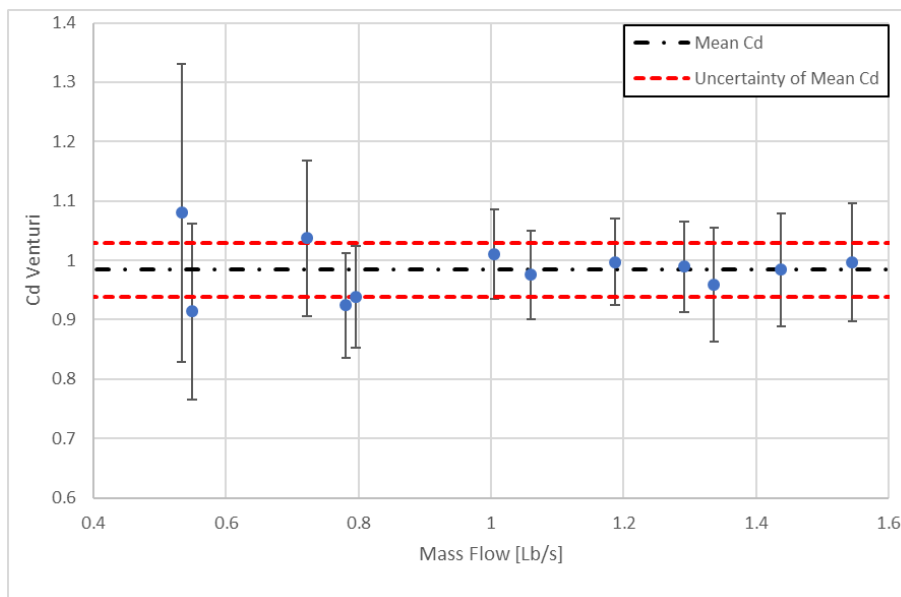


0.260" venturi

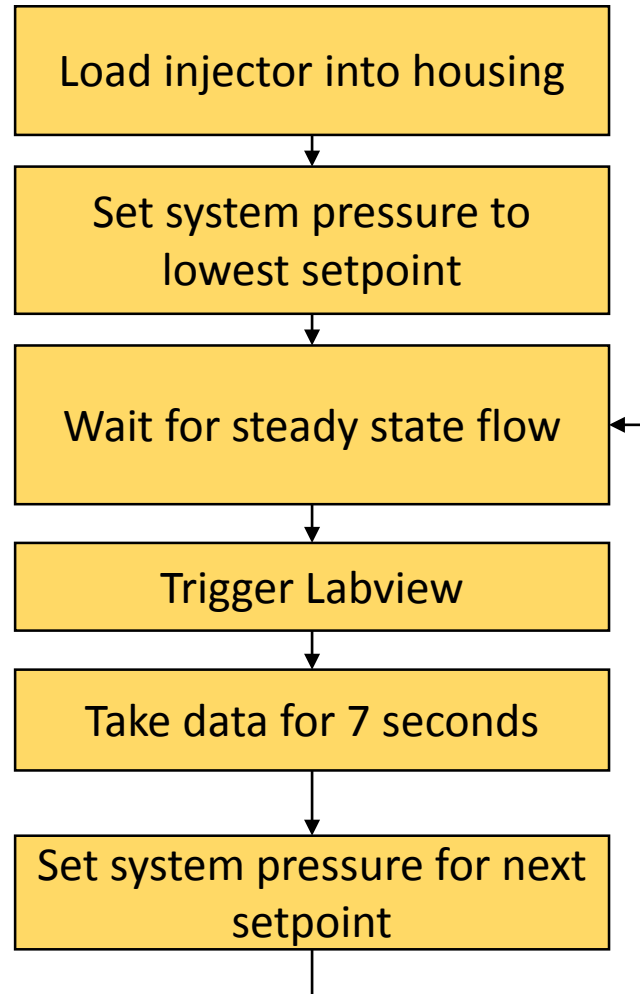
$$C_d = 0.984$$

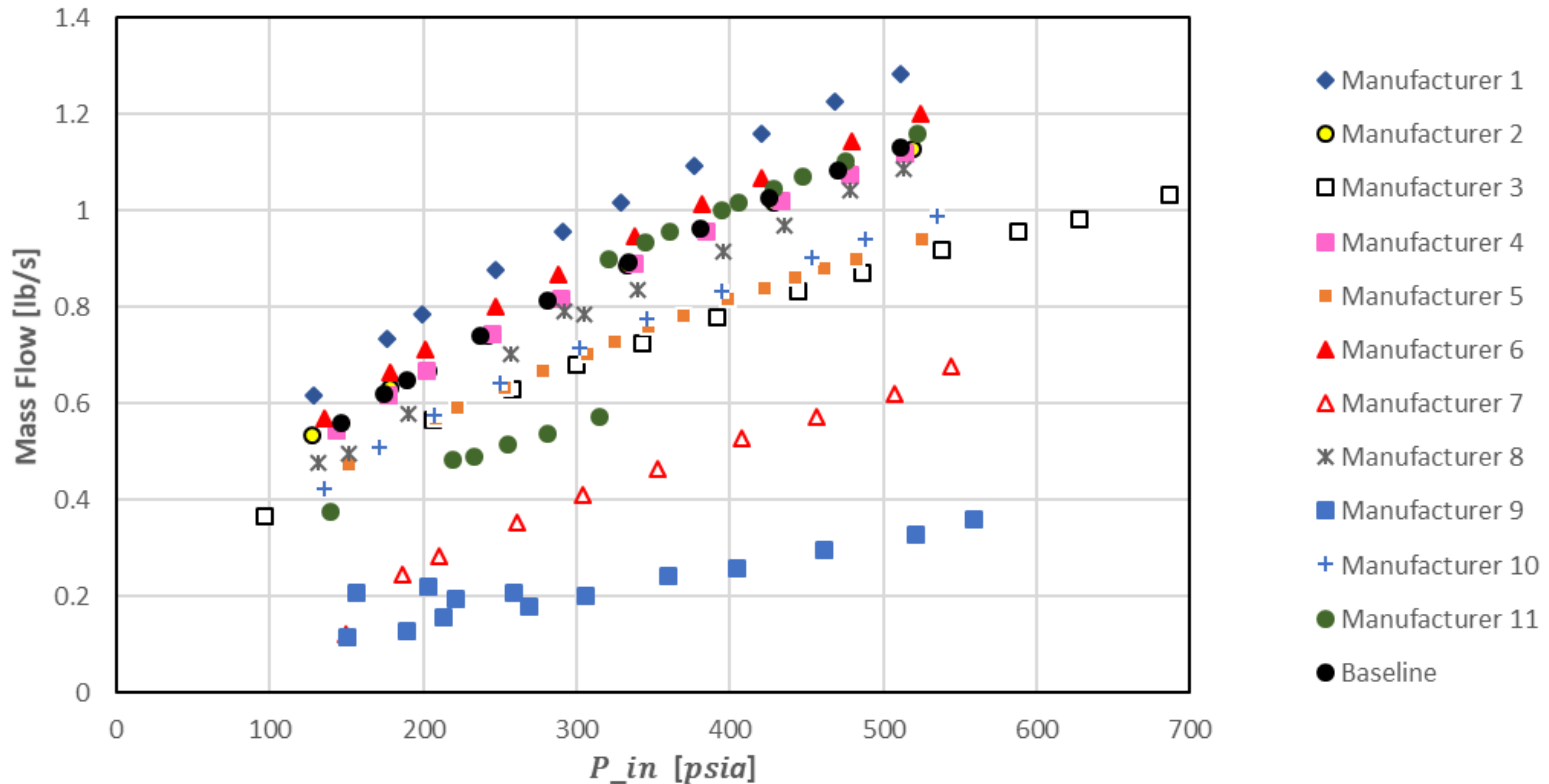
$$U_{cd} = 4.6\%$$

0.1 Lb/s to 2.1 Lb/s

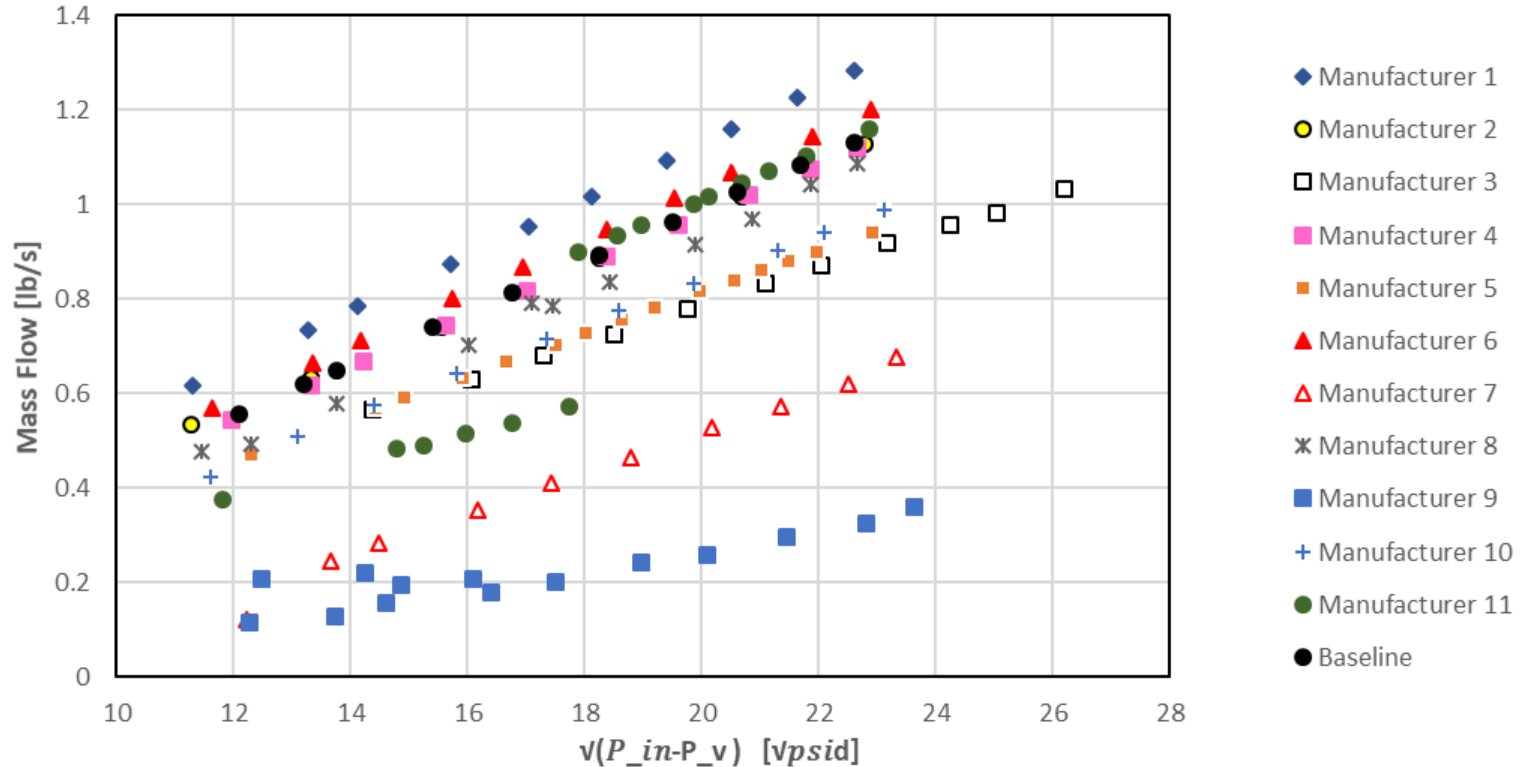


Larger uncertainty at low mass flow due to low ΔP





- Noted “Clustering”: 4 groups
- Wide range of mass flow variation at given inlet pressure
- 2 significantly lower performing test articles
- Discontinuous trend for Manufacturer 11



- Linear trends with square root of pressure

Calculate mass flow (\dot{m}) at each setpoint

$$\dot{m} = \frac{\pi}{4} C_{d \text{ vent}} d_t^2 \sqrt{\frac{2\rho\Delta P}{1 - \left(\frac{d_t}{d_1}\right)^4}}$$

\dot{m} = mass flow rate
 d_t = venturi throat diameter
 d_1 = venturi inlet diameter
 ρ = density
 ΔP = venturi pressure drop (inlet to throat)
 $C_{d \text{ vent}}$ = venturi discharge coefficient
 P_{in} = inlet pressure
 P_{cav} = vapor pressure

Calculate mass flow Uncertainty at each setpoint ($U_{\dot{m}_{setpoint}}$)

$$U_{\dot{m}_{setpoint}} = \sqrt{\left(\frac{\partial \dot{m}}{\partial C_{d \text{ vent}}} U_{C_{d \text{ vent}}}\right)^2 + \left(\frac{\partial \dot{m}}{\partial \rho} U_{\rho}\right)^2 + \left(\frac{\partial \dot{m}}{\partial \Delta P} U_{\Delta P}\right)^2 + \left(\frac{1.96\sigma_{\dot{m}_{sample}}}{\sqrt{N_{\dot{m}_{sample}}}}\right)^2}$$

Calculate Cd for injector at each setpoint (C_d)

$$C_d = \frac{\dot{m}}{A_{throat} \sqrt{2\rho(P_{in} - P_{cav})}} \quad \begin{matrix} P_{in} = \text{inlet pressure} \\ P_{cav} = \text{vapor pressure} \end{matrix}$$

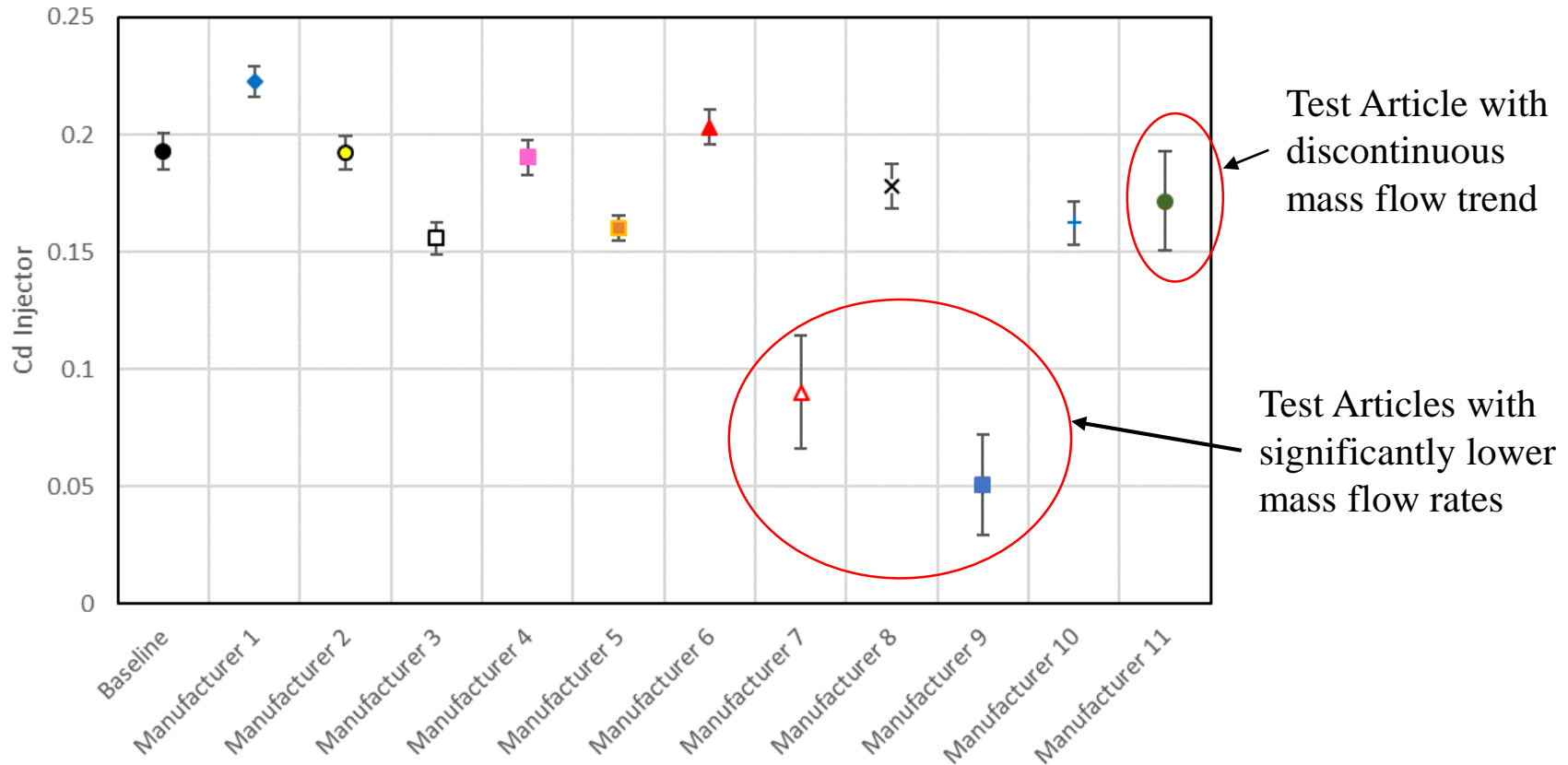
Calculate Cd for injector Uncertainty at each setpoint ($U_{C_{d_sp}}$)

$$U_{C_{d_sp}} = \sqrt{\left(\frac{\partial C_{d \text{ inj}}}{\partial \dot{m}} U_{\dot{m}}\right)^2 + \left(\frac{\partial C_{d \text{ inj}}}{\partial \rho} B_{\rho}\right)^2 + \left(\frac{\partial C_{d \text{ inj}}}{\partial P_{in}} B_{P_{in}}\right)^2 + \left(\frac{1.96\sigma_{C_d}}{\sqrt{N_{C_d}}}\right)^2}$$

Calculate Average Cd and mean Cd Uncertainty for each test article

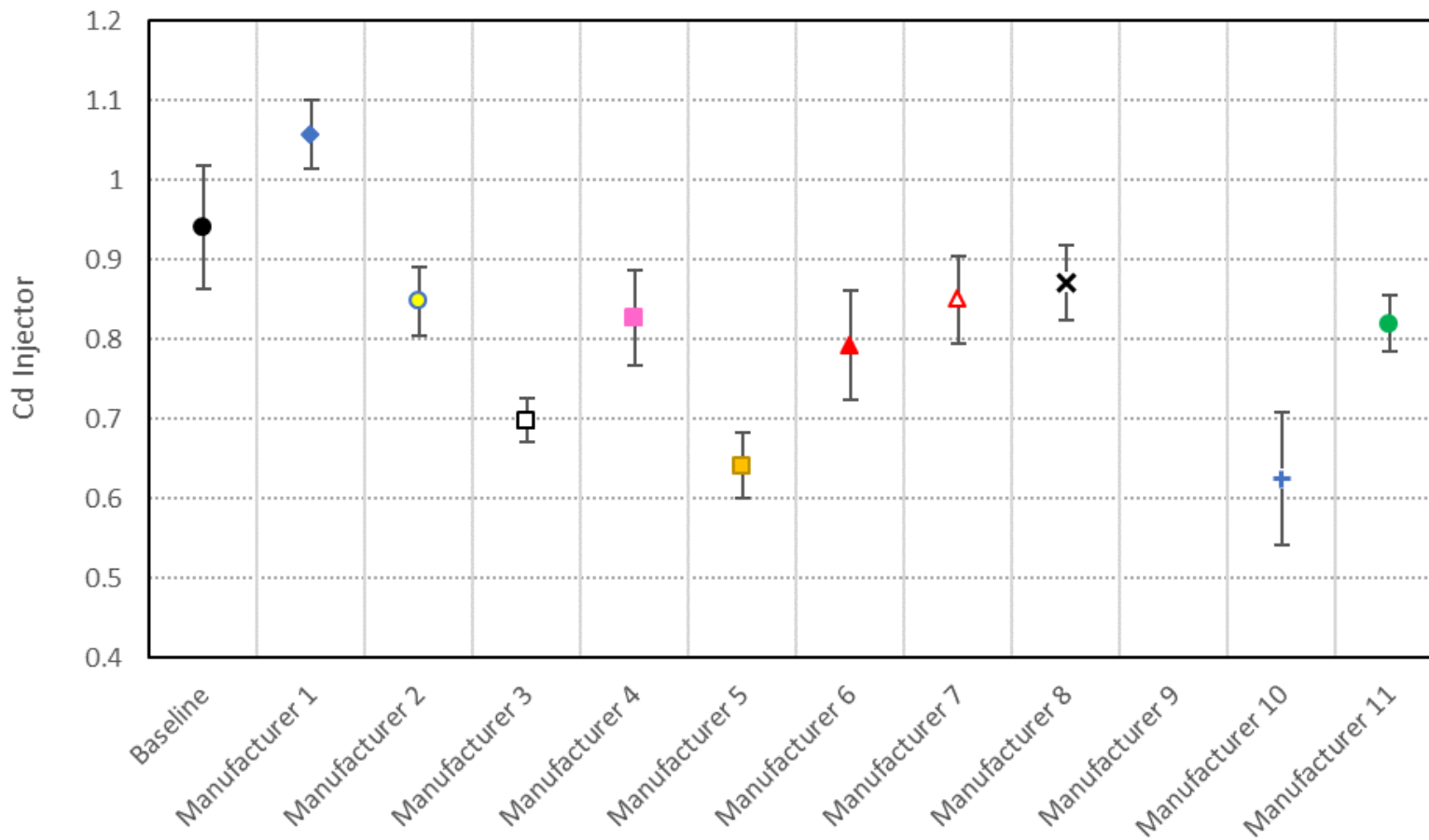
$$U_{C_{d \text{ inj}}} = \sqrt{\sum \left(\frac{U_{C_{d \text{ inj setpoint}}}}{N_{setpoint}}\right)^2 + \left(\frac{t_{\alpha,v} \sigma_{C_{d \text{ inj setpoint}}}}{\sqrt{N_{setpoint}}}\right)^2}$$

$$C_{d \text{ inj}} = \frac{1}{N} \sum C_{d \text{ sp}}$$



- Uncertainty ranged from 3% to 6% for 9 of the test articles
- 12%-53% Uncertainty for 3 of the test articles

Flow Geometry 2 Design Variant 1



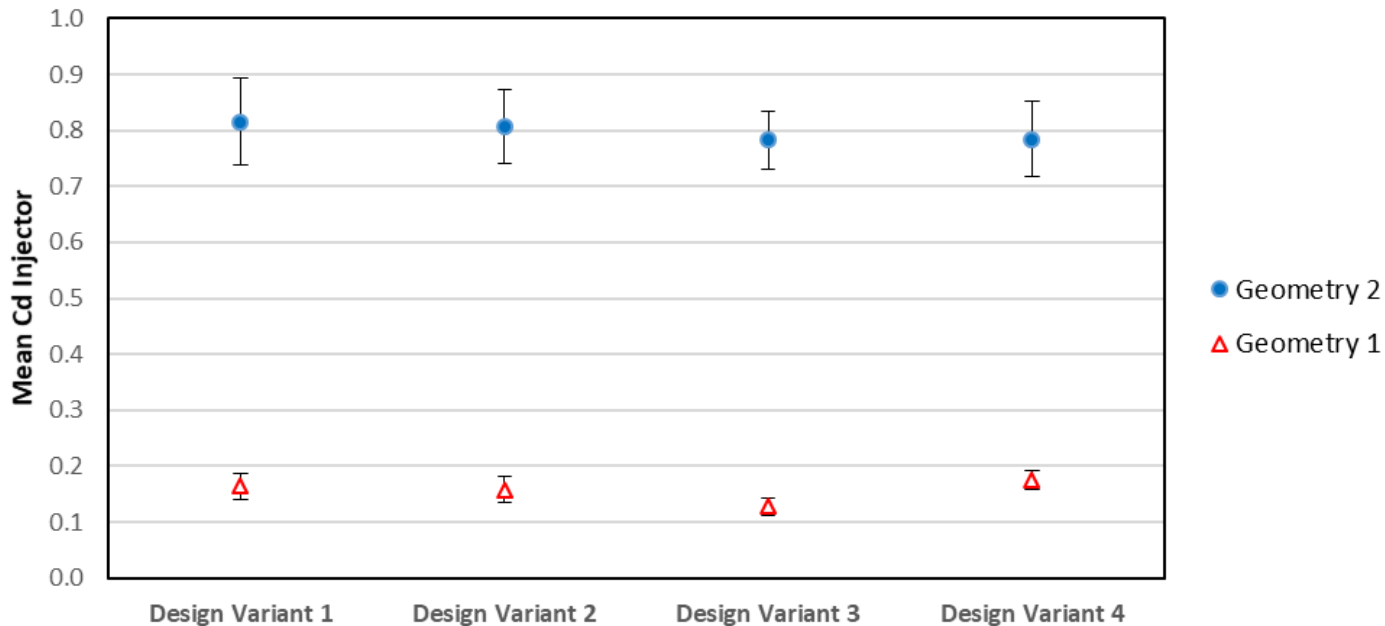
Uncertainty Range: 4%-13%

Calculate average Cd for all manufactures for a given design variant and flow path

Calculate uncertainty of average Cd for all manufactures for a given design variant and flow path

$$C_d = \frac{1}{N} \sum C_{d_inj}$$

$$U_{C_{d_inj}} = \sqrt{\sum \left(\frac{U_{C_{d_inj\ setpoint}}}{N_{setpoint}} \right)^2 + \left(\frac{t_{\alpha, \nu} \sigma_{C_{d_inj\ setpoint}}}{\sqrt{N_{setpoint}}} \right)^2}$$



- Geometry 1 uncertainty ranged from 10% to 14%
- Geometry 2 uncertainty ranged from 6% to 10%

Calculate average C_d for all design variants for a given manufacturer and flow path



Calculate uncertainty of average C_d for all design variants for a given manufacturer and flow path

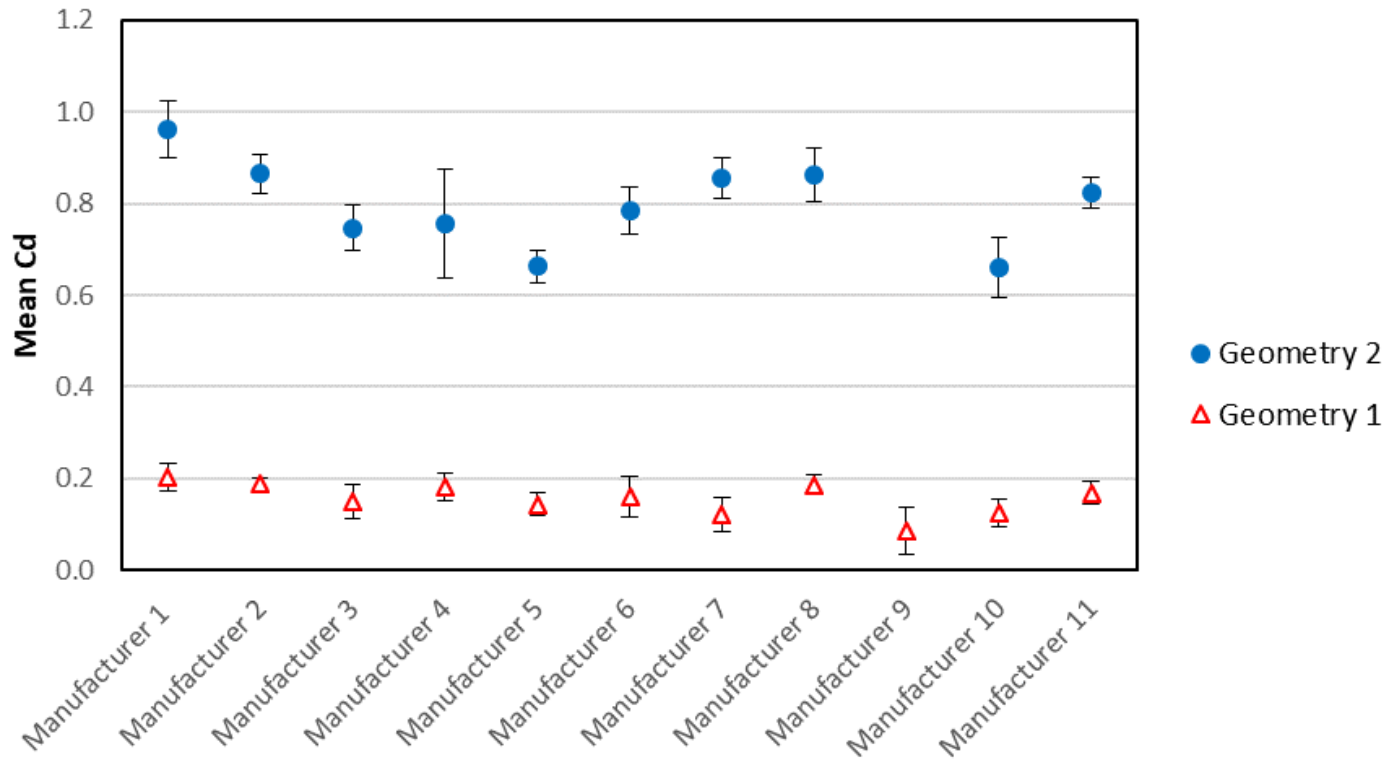


Normalize injector C_d for both flow paths

$$C_d = \frac{1}{N} \sum C_{d_inj}$$

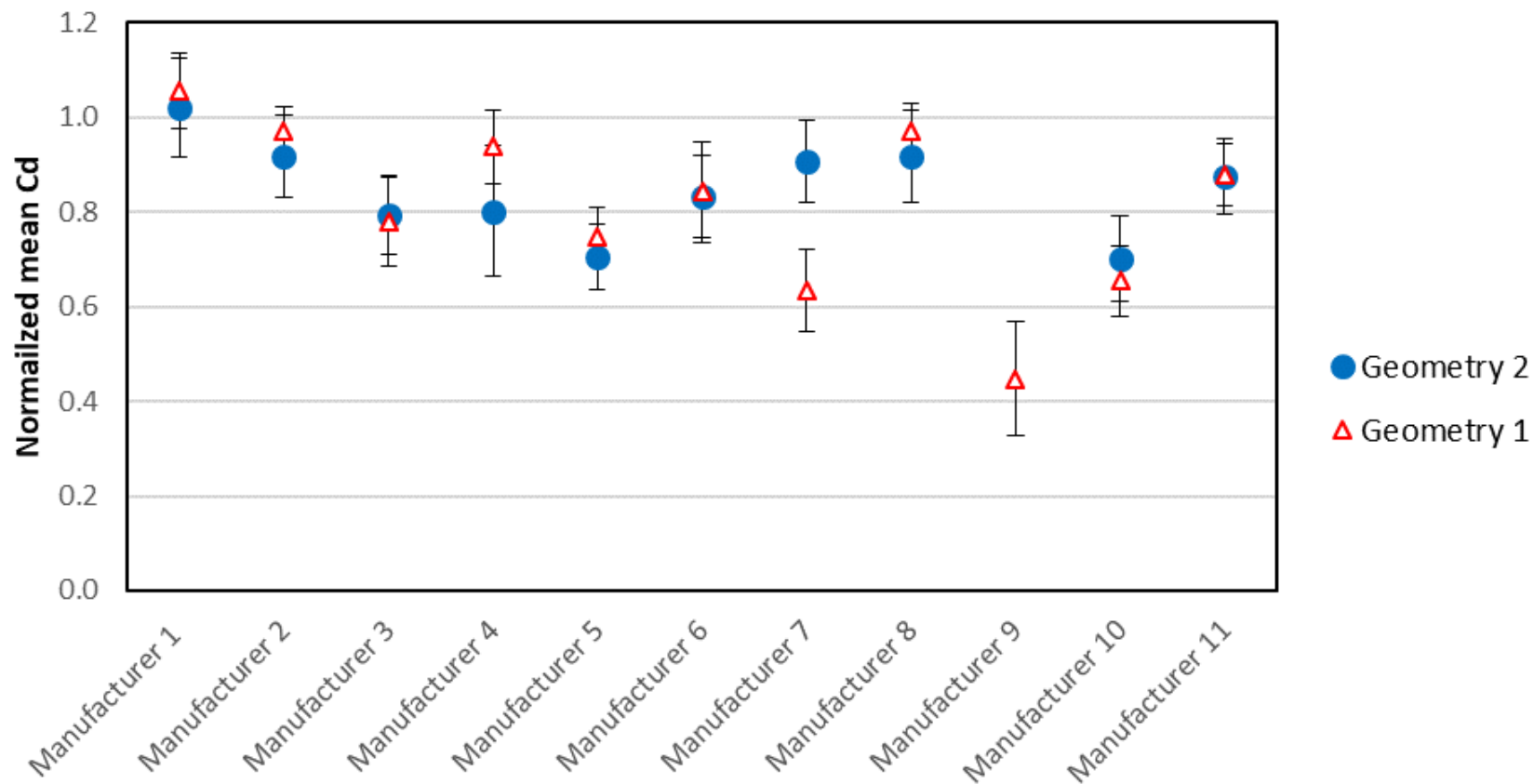
$$U_{C_{d\ inj}} = \sqrt{\sum \left(\frac{U_{C_{d\ inj\ setpoint}}}{N_{setpoint}} \right)^2 + \left(\frac{t_{\alpha, \nu} \sigma_{C_{d\ inj\ setpoint}}}{\sqrt{N_{setpoint}}} \right)^2}$$

$$C_{d\ norm} = \frac{C_{d\ inj}}{C_{d\ baseline}}$$



- Flow geometry 1
 - 10 manufacturers ranged from 7%-30%
 - 1 manufacturer was 61 %
- Flow geometry 2
 - 9 manufacturers ranged from 4%-10%
 - 1 manufacturer was 61 %

Normalized Results by Manufacturer



Calculate average Cd for all manufacturers for a given flow path and geometry

$$\bar{C}_d = \frac{1}{N} \sum C_{d_inj}$$

Calculate uncertainty of average Cd for all manufacturers for a given flow path and geometry

$$U_{C_{d_inj}} = \sqrt{\sum \left(\frac{U_{C_{d_inj\ setpoint}}}{N_{setpoint}} \right)^2 + \left(\frac{t_{\alpha,\nu} \sigma_{C_{d_inj\ setpoint}}}{\sqrt{N_{setpoint}}} \right)^2}$$

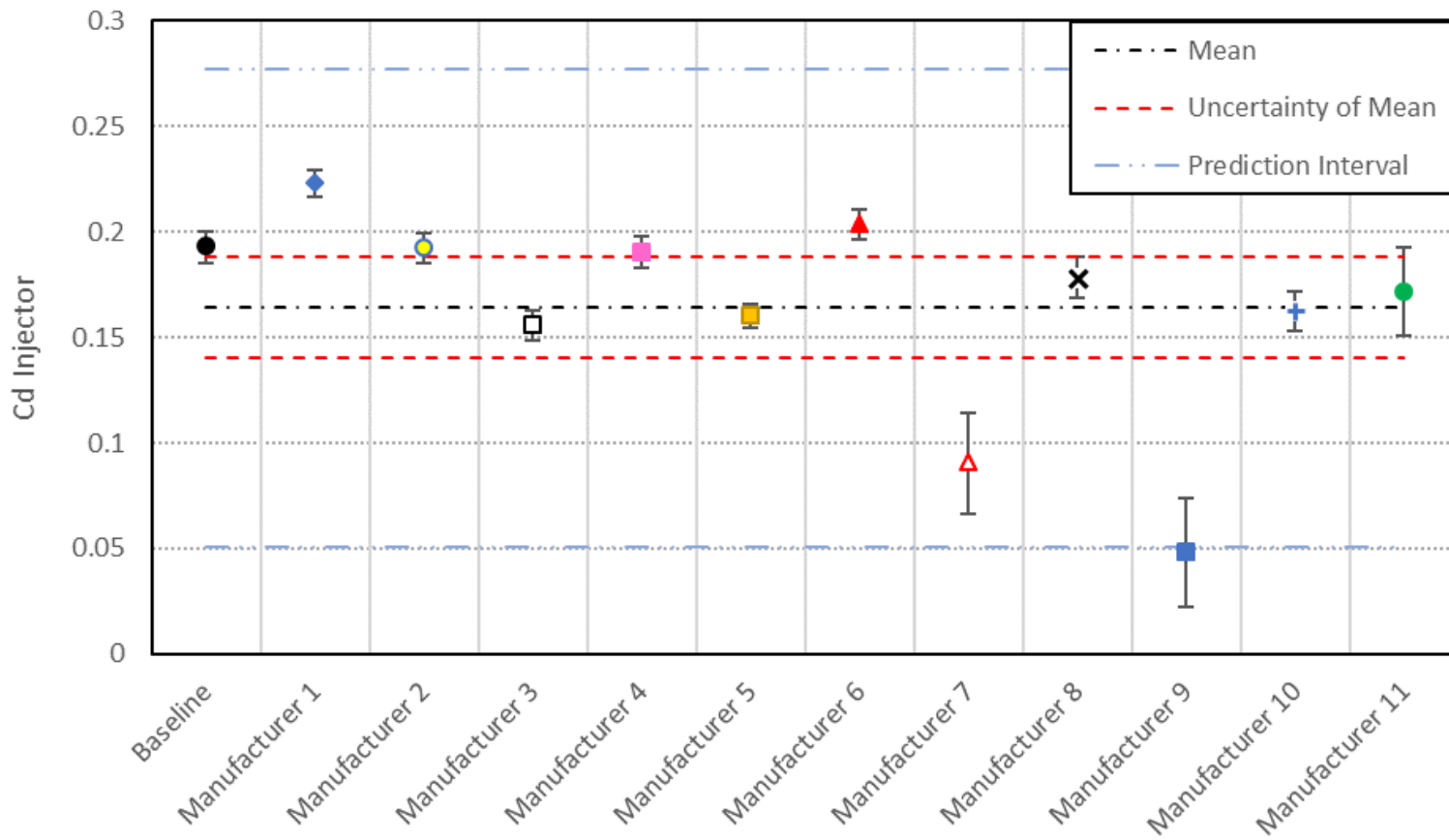
Calculate the predicted Cd for a new injector from the same population

$$C_{d_predicted} = \bar{C}_d \pm U_{95} \sqrt{1 + \frac{1}{n}}$$

Calculate the prediction interval for the new injector

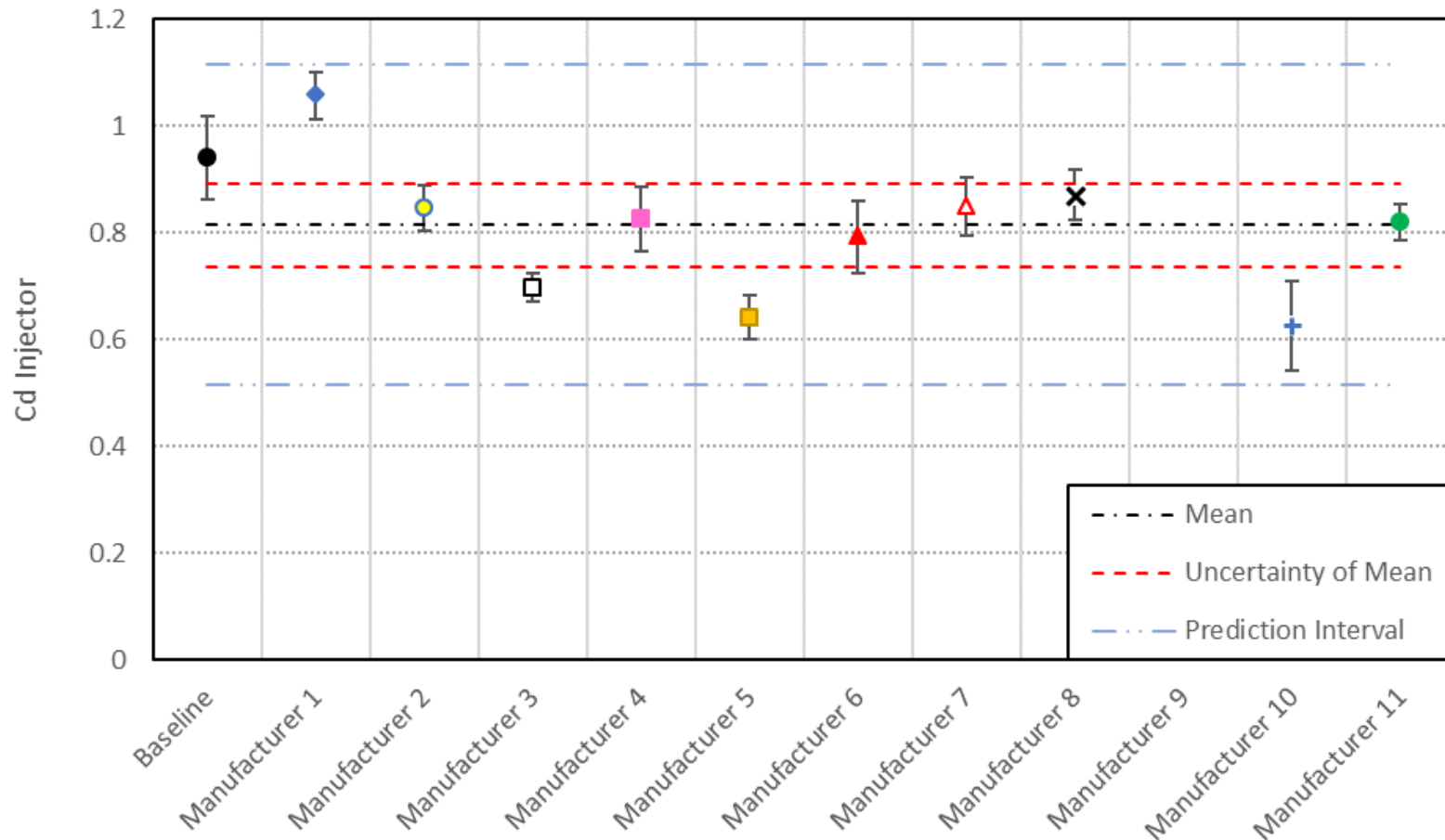
$$U_{95} = \sqrt{\sum \left(\frac{U_{C_{d_inj}}}{N} \right)^2 + (t_{\alpha,\nu} \sigma_{C_{d_inj}})^2}$$

Flow Geometry 1 Design Variant 1



Flow Geometry 2

Design Variant 1



- Flow Geometry 1:
 - The mean CD of all injectors was 0.16
 - Significant mean CD variability depending on the manufacturer
 - Additive CD's generally lower than subtractive baseline
- Flow Geometry 2:
 - The mean CD of all injectors was 0.80
 - Significant mean CD variability depending on the manufacturer
 - Additive CD's generally lower than subtractive baseline
- Differences among CD of all injectors are generally well beyond the uncertainty bars of the CD results.
- Manufacturer is more important than slight changes in geometry