Loading and planform shape influence of the wing structural layout through topology optimization

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Topology optimization is a technique used to identify the optimal layout of a structure for a given objective and assigned boundary conditions. The evolution it has experienced over the last three decades made it a mature and reliable method for industrial applications. In this paper topology optimization is employed to investigate the influence of two wing planform features and of the loading condition on the wing internal structural configuration. The planform shape of the Common Research Model wing is used as a reference. Then the geometry is modified parametrically to alter the sweep angle and the aspect ratio. For the baseline shape, the optimization is also performed considering the aerodynamic loading induced by the pull-up manoeuvre. The results of topology optimization for all these cases are compared and the common patterns are used to identify some guidelines for the preliminary design of the wing primary structure.

I. Introduction

I_N this paper an application of topology optimization to the internal primary structure of a civil transport aircraft's wing is presented. The attention, in particular, is focused on the role played by the planform shape and ultimately by the loading conditions on the main features of the structural layout. As already happens for the aerodynamics, the main purpose here is the identification of common characters in the structural configuration that might be employed as a general trends in the wing design.

Topology optimization is suitable for this task. It is a well established technique for the conceptual design of structural elements and mechanical components [1–3]. As any other optimization approach it is based on an iterative process. It identifies the best material distribution for a certain objective, within a design volume, under given boundary conditions and manufacturing constraints [4].

It is especially indicated to support the work of engineers and to speed up the traditional approach consisting in the repetitive improvement of an initial design, until satisfactory global performances are obtained. Such a conventional method to determine the final design, which Eschenauer [5] defines as *Current Design World State*, cannot guarantee that an optimal structural configuration is actually selected.

Topology optimization received greater interest over the last three decades [6] and a big effort has been done in order to make it ready for industrial applications.

Historically it has been introduced by Mitchell [7] in 1904 which studied the optimal configuration of truss structures with respect to weight. The same problem was extended later in other works [8] and also [9]. However the research became extremely active after the papers written by Bendsøe and Kikuchi [10] and by Bendsøe [11]. Since these two seminal works, several methods have been proposed to implement topology optimization, among these the Solid Isotropic Material with Penalization (SIMP) [12] method is one of the most used.

The aerospace field has not been exempt by the potential improvements deriving by the implementation of topology optimization for the design of least-weight structures. An overview of these applications is given by the review article of Zhu *et al.* [13]. They have reviewed the main applications of topology optimization in the design of aerospace structures and classified them in four main categories. One of these, named by the authors -*standard material layout design for airframe structures*- includes the works which use topology optimization to identify the general airframe configuration, while the others are related to the design of subparts. Some examples are the design of an engine pylon [14], of a leading edge rib [15] or of the cockpit airframe [16]. Of course most of the interest has been addressed towards the design of the

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wing internal structure. It is crucial for the global aircraft performances, although it also presents several critical aspects related to the functional constraints to which the wing is subject.

One of the first examples of topology optimization used to investigate the factors influencing the wing structural design was proposed by Balabanov and Haftka [17]. They used topology optimization based on the *ground structure approach* to understand the effect of the fuselage deformation on the wing internal structure, whose parts were modeled as a truss elements. The total compliance was used as the objective. More recently, other works considered various aspects of the wing design and different test cases. Dunning *et al.* [18, 19] performed the topology optimization of a rectangular straight wing box implementing the Level Set Method and taking into account the aero-structural coupling. Walker [20] and Rao [21], used both the SIMP method to find the optimal material distribution for a unit-length wing segment and for a straight wing respectively. In both these last examples the aerodynamic loading was decoupled by structural optimization. In the same direction Aage *et al.* [22] performed topology optimization using again a density-based method for the layout optimization of the Common Research Model's wing. This work suggested the presence of curved elements in the optimized wing structure. The attention was also paid to the importance of an accurate mesh refinement in order to capture the presence of a wider range of length scales.

As in the article written by Aage, also in this study, topology optimization is applied to the whole wing design. The wing of the Common Research Model (CRM) [23] is also used as baseline geometry and the other variants are generated throughout a planform parametrization. However, the novel element is now represented by employing topology optimization to investigate how changes of two main wing planform metrics, the sweep angle Λ and the aspect ratio AR, affect the internal structural layout of the wing. The driving component is, ultimately, the aerodynamic loading which is evaluated both in cruise and manoeuvring conditions.

The interest in investigating these trends is due to the fact that in the early stages of the design process the wing planform shape is optimized with respect to the overall aircraft performances. The internal structure, on the contrary, remains almost unchanged, based on the traditional orthogonal assembly of spars, ribs and stiffeners. It is only subject to sizing optimization. To understand how the planform shape, affects the layout of the structural parts, could instead suggest the most appropriate structural design in relation to a given wing planform and guide towards a more efficient solution.

In the following section an overview of the optimization framework used to carry out both the aerodynamic analysis and topology optimization is provided. Then topology optimization is introduced and the SIMP method briefly described. Sections II.B, II.C are focused on the CAD and on the aerodynamic modules respectively. Then the problem formulation is explained in detail, and finally results obtained for the different test cases are discussed.

II. Optimization framework

In this paper topology optimization is used as tool for the preliminary design. Given the multidisciplinary character of the problem it is then part of a wider framework including also other elements. In particular, investigating the effect of two main planform parameters on the structural layout of the wing posed the problem of generating new geometries computing the aerodynamic field for each case. A possible way to tackle this recursive task is to automate the sequence by integrating topology optimization within a modular framework which for a given design vector automatically induces changes in the geometry and recomputes the aerodynamic loading for each new wing planform. Such a work flow requires at least three main modules. A parametric CAD, which takes the design vector in input, an aerodynamic solver
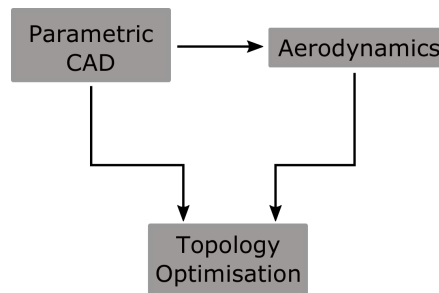

Fig. 1 *Overview of the optimization framework.*

to calculate the pressure distribution over the wing surface and finally the topology optimization module which uses the

new geometry from the CAD module and the loads from the aerodynamic solver and performs the optimization. The data-flow between the different modules is depicted in figure 1.

The one way arrow connecting the aerodynamic and the topology optimization blocks indicates that the aeroelastic coupling is not taken into account in this formulation. It is assumed that the deformation induced on the wing has a negligible influence on the final structural layout. For this reason the aerodynamic loads are computed only once for each geometry and then used as a boundary condition for the structural optimization. This approach is shared among other articles [20–22].

A. Topology optimization: the SIMP method

If topology optimization has acquired a central role in the conceptual design of structural elements is because it has received a great attention over the last three decades, viewing the methods available and their accuracy improving significantly. One of the strategies which has attracted most the attention of the researchers, is known as SIMP (Solid Isotropic Material with Penalization) method. It has been object of numerous publications. The *homogenization method* proposed by Bendsøe and Kikuchi [**?**] represented the first step towards the application of SIMP to real applications.

The SIMP method is a density-based approach relying on the finite element discretization. In this method a so called virtual or pseudo-density $\rho(x, y, z)$ [**?**] ranging from 0 to 1 is associated to each finite element and represents the fraction of total volume which can be used to distribute the material. In practice the value of the virtual density is used to modifie the mechanical properties of the material. In particular the Young's modulus is updated at each iteration of the optimization according to relation 1:

$$\bar{E}(x, y, z) = (\rho)^P E(x, y, z) \tag{1}$$

where the p exponent is the penalization factor, used to force the solution towards the extreme values. It is a integer value ranging from one to three and it is modified using a so called continuation method [**?**]. The bar symbol is used instead to indicate the modified property due to the virtual density.

This in turn is reflected on stiffness matrix formulation which becomes:

$$\left[\bar{K}_e \right] = (\rho_e)^P \left[K_e \right] \tag{2}$$

And the global stiffness matrix is then defined as:

$$\left[\bar{K} \right] = \sum_{e=1}^{N} \left[\bar{K}_e \right]$$

The topology optimization problem can be formulated in many different ways [24]. In the literature it is usually written in the form of compliance minimization for a given volume fraction. The compliance is defined by 3:

$$C = \{U\}^T \cdot [K] \cdot \{U\} \tag{3}$$

and substituting the previous expression it becomes:

$$C = \sum_{e=1}^{N} \{U\}^T \cdot [K_e] \cdot \{U\}$$

The volume fraction V_f is defined as the ratio between the volume occupied by the material and the entire volume of the design region V_0.

$$V_f = \frac{V_m}{V_0} \tag{4}$$

Usually the volume fraction V_f is constrained to be smaller than a certain threshold, which will be indicated with V_f^*. A volume fraction $V_f^* = 0.4$, for example, indicates that at most the forty percent of the total design volume can be actually used to distribute the available material. Using this global constraint, the optimization algorithm modifies the virtual density in each finite element until a minimum of the compliance is found.

B. Geometry parametrization

As stated is section II, this work evaluates the influence of two main planform features on the wing internal structural layout. It is crucial to modify easily the shape and to obtain the new wing design for each value of sweep angle and aspect ratio.

$$AR = \frac{b^2}{S}$$

This task has been attained by defining a parametrization of the baseline planform geometry. The wing of the Common Research Model (CRM) is here used as a reference, and its geometric features are resumed in table **??**. The value of

S_{ref}	383.7 m^2
b	58.7 m
Λ	37 deg
AR	9

Table 1 *Geometric features of the CRM wing.*

sweep angle as well as the presence of a kink are both typical features of a transonic wing.

The parametrization consists in the introduction of three parameters which uniquely define a new sweep angle or aspect ratio. They are illustrated in figure 2 A 3D model, based on this parametrization, has then been implemented in a
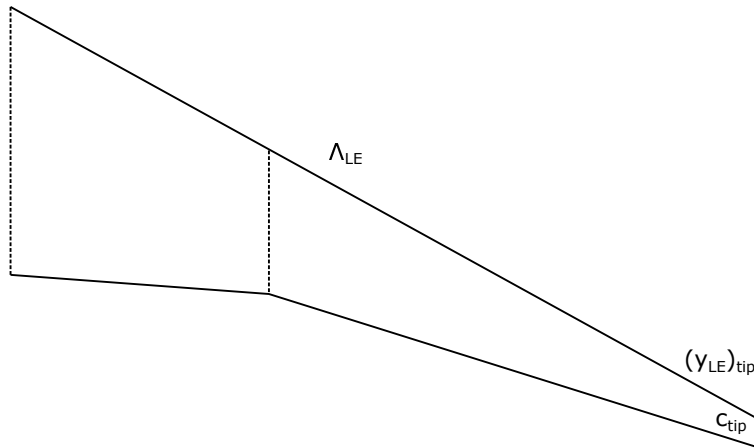

Fig. 2 *Parametrization of the wing planform.*

Python-based CAD module.

In principle one would consider the whole volume surrounded by the wing external surface. However, such a geometry would be characterized by complex geometric features, which during the generation of the mesh would require some precautions in order to guarantee a smooth convergence of the optimization. This, in turn, would results in a finer mesh and an increasing computational cost.

Moreover, following some practical considerations, the volumes corresponding to the leading and trailing edges are intended for the high-lift devices and the secondary structure, so they can, in the first instance, be excluded from the design region. In conclusion, only the volume corresponding to the wing box is considered as design space for the optimization. This volume, according to the baseline geometry, is bounded by the front and rear spar of the original structure, at 10 and 70% of the chord respectively. By the upper and lower skins on the top and bottom. In total, four variations have been considered from the baseline. First, two test cases have been obtained assuming a sweep angle at 20° and 30°. Then two further variants characterized by and aspect ratio of 7 and 11 respectively.
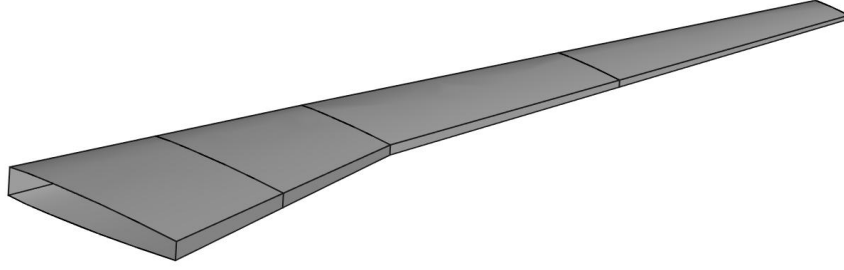
Fig. 3 *Parametric model of the wing box. Baseline planform.*

C. Aerodynamic loading

The multidisciplinary character of the problem considered in this work depends on the changes in the aerodynamic field due to the modifications induced in the planform shape. Aerodynamic loading is the discriminant boundary condition driving the optimization of the structural layout.

As Stanford [25] pointed out in his paper, the aeroelastic coupling for topology optimization is not always considered. In his article, he classified other relevant works in the literature in three main categories, depending on whether and how the aeroelastic effect is introduced. The first category, to which this paper belongs, neglects the effect of the wing deformation on the re-distribution of the aerodynamic forces. This means that for each geometry the aerodynamic loading is computed only once, at the beginning of the work flow described in figure 1, and then used into topology optimization as a boundary condition.

The aerodynamic field is calculated through the open-source tool AVL [26] which implements a vortex lattice model. The accuracy of its results is satisfactory if used during the early stage of the design process. An extension to higher-fidelity models would represent an interesting development towards a more accurate optimized design, however such an upgrade does not justify in this context, a higher computational cost and the increasing complexity of the whole framework because of the presence of other approximations.

The most important information required for the optimization is the pressure distribution over the wing surfaces. It is provided by AVL in the form of a differential pressure coefficient ΔC_p. The lift distribution, as also showed in other similar works [22], [20], cannot be approximated as a sequence of concentrated loads along the span. Such a discretization would force, in fact, topology optimization to localize most of the available material in correspondence of this loads. This, in turn, would prevent the formation of novel features and would automatically drive the solution towards the traditional rib-like elements.

The input file for AVL is written using again the same design vector describing the planform shape, and the resulting model for the baseline geometry is showed in figure 4. The lifting surface has been divided into 20 elements in the chordwise direction with a spacing following a cosine distribution, and 40 elements evenly distributed along the span.

The aerodynamic solution was obtained first, imposing the equilibrium in the horizontal flight in cruise conditions.

$$\frac{1}{2}\rho V^2 C_L S n = W_{MTOW} \tag{5}$$

where n is the load factor, assumed to be unitary. From equation 5 a value of $C_L = 0.5$ has been obtained, in agreement with the value suggested by Vassberg in his paper [23]. This value was then assumed as a target value in AVL and the corresponding c_l distribution for the CRM wing is reported in figure 5.

The results were also corrected to take into account the effect of the compressibility through the Prandtl-Glauert factor:

$$1/B = \frac{1}{\sqrt{1 - M_\perp^2}}$$

where the wing-perpendicular Mach number $M_\perp$ was used for the assigned sweep angle.
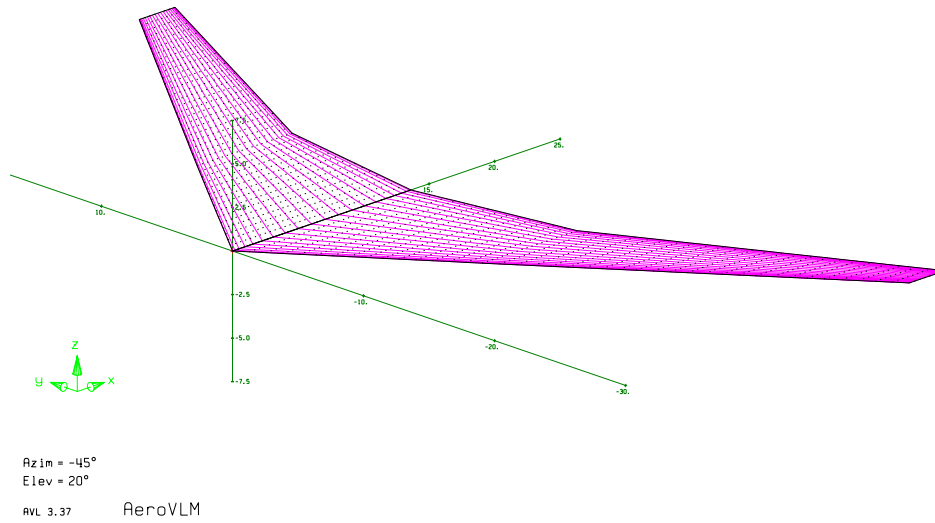
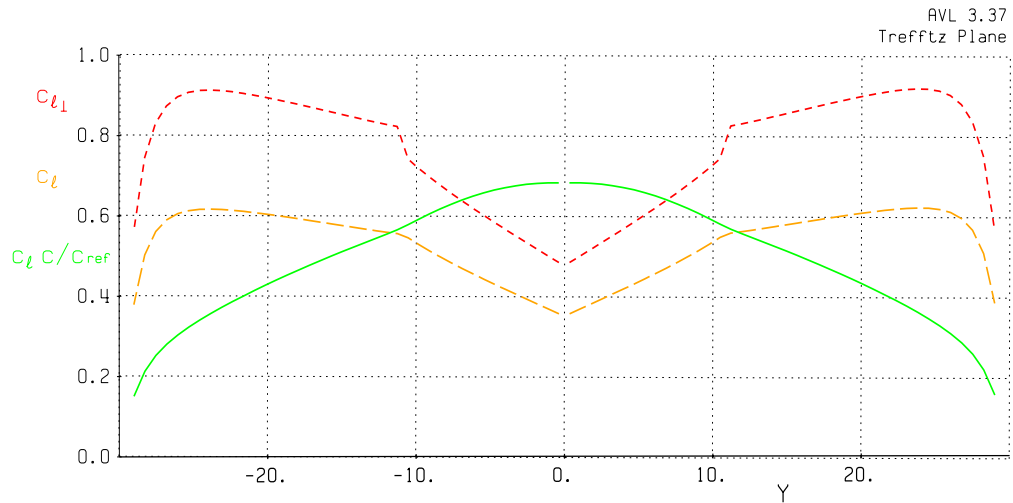**Fig. 4** *Aerodynamic model of the CRM wing.*

Fig. 5 *Lift distribution along the span. Cruise condition.*

The cruise flight does not represent, however, the most demanding condition for the structural integrity. From the experience it is known that the pull-up manoeuvre, for example, is a more critical situation. In order to assess the effect of the aerodynamic loading on the wing structural configuration and to induce higher stress in the finite element model, the pull-up manoeuvre was computed and used in alternative to the cruise condition. They are both summarized in table **??**.

	Cruise	Pull-up
Mach	0.85	0.65
z	10000 m	sea level
n	1	2.5

Table 2 *Flight conditions assumed to calculate the aerodynamic loading.*

The load factor assumed for the manoeuvre, descends from the FAR 25 [**?**] regulation, which for the category of

aircraft representative of the CRM model, suggests a value of $n = 2.5$. In the first instance, this value of load factor was used directly in equation 5, to derive the corresponding total lift coefficient in the case of an untrimmed aircraft.

III. Model description and results

The CAE suite Altair HyperWorks [24]has been used to set up the finite element model and run the optimization. In particular, HyperMesh to generate the mesh and introduce the boundary conditions, OptiStruct to run the analysis and the SIMP-based optimization. In the design volume, represented by the box in figure 3, two sub-regions were defined. The boundary skin was treated as a non-design component, while the inner volume was the actual design space, subject to the material optimization. This approach is common to most of the relevant works and it is motivated by the need to preserve both the external shape, for the aerodynamics, and an enclosed volume destined to the fuel storage. The mesh generated for the two regions was also different. The non-design skin was discretized using 2D quadrilateral elements while the inner volume divided into tetrahedral elements. A thickness property was assigned to the skin in order to withstand part of the loads. In total the number of elements N was in the order of 10^7. The mesh size was dictated on the one hand by the need to capture structural elements across a wider range of length scales on the other was limited by computational considerations. This point was also put in evidence by Aage *et al.* [22] who highlighted how the main obstacle in applying topology optimization to large structures is represented by the ability to run high-resolution models.

As anticipated in subsection II.A, the problem in OptiStruct was formulated to minimize the compliance for a constrained volume fraction.

$$
\begin{aligned}
\min_{\rho} \quad & C(\rho) \\
s.t. \quad & V_f \leq V_f^* \\
& 0 \leq \rho_e \leq 1 \qquad e = 1, \cdots, N
\end{aligned}
\tag{6}
$$

where V_f^* is the threshold value assigned to the volume fraction. Realistic values of volume fraction relative to the primary structure are in the order of $0.15 \div 0.20$. In this study an initial value of 0.3 was used. A little larger than the reference, in order to facilitate the convergence of the optimization and to ensure that there was enough material to form the main structural elements. The root section was fully clamped, which is the only physical constrain introduced in the model. For what concern instead the loading, as discussed in the previous section, it was given in the form of a pressure distribution over the upper surface of the wing. The distribution of DC_p from AVL was then linearly interpolated in the chordwise and spanwise direction. Both to the skin and to the inner volume were associated isotropic material properties with a density equal to $2.78 \cdot 10^3 kg/m^3$ and an elastic modulus $E = 73 GPa$. Each simulation has been run in parallel on 16 CPUs for approximately 12 hours.

A. Effect of the planform shape

The objective of this study is the identification of common trends in the wing structural layout when the planform shape is modified. In particular two main design variables, the sweep angle and the aspect ratio were considered. Apart from the baseline geometry, topology optimization was performed for sweep angles, measured at the leading edge, of 20° and 30°. Similarly, also two values of aspect ratio were tested, $AR = 7$ and $AR = 9$. The two variables were modified one at a time.

The first results to be presented are those relative to the baseline planform, corresponding to the CRM wing. The reference case was also tested for a smaller value of volume fraction threshold $V_f^* = 0.2$. Figure 6 shows the results relative to the baseline wing planform. A plane, normal to the z-direction was used to visualize the material distribution in the design volume. As expected, the dominance of the bending moment determines, as a primary effect, the thickening of the skins, in order to increase the associated inertia moment. Part of the material is also used along a segment of the front and rear edges which behave like spars. The most unconventional features appear however in the inner region. Close to the root a rib-like element extends from the rear edge, goes towards the leading edge and presents a strong curvature. In the outer part of the wing, from the kink section towards the tip, the material is distributed longitudinally suggesting the presence of a beam-like component located in an intermediate chordwise position, which tends to join the leading edge at the extreme outer section. Although the results of topology optimization cannot be directly used but they require a certain idealization, all these features are in agreement with the results published by Aage *et al.* [22]. In that study the author also recognized the presence of curved/diagonal ribs close to the root section as important element of novelty and suggested the use of traditional straight components moving towards the tip of the wing.

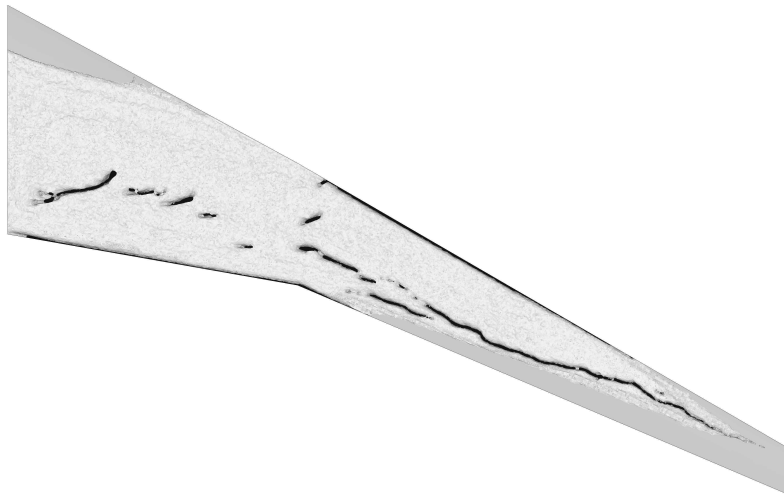**Fig. 6** *Lift distribution along the span. Cruise condition.*

Fig. 7 *Optimal layout for the modified geometry at* $\Lambda = 30°$.

Reducing the sweep angle to 30° 7 the same general behavior can be observed, but the presence of the curved component close to the root becomes less evident. This is further confirmed for the smallest value of 20° 8. In this second case, the material previously used to reinforce the rear edge tends to form a single longitudinal component running from the root to the tip along the semi-span. Overall the material shifts forward, as also confirmed by the presence of a void region at the rear edge, close to the root.

Pictures 9 and **??** show the results for two different values of aspect ratio. Again the bending has the consequence of increase the thickness of the skins. In this case however, the most evident effect is observed in the curvature of the beam component that has its origins at the root and runs almost until the tip section. From the curved spar some branches run diagonally in the direction of the leading edge.
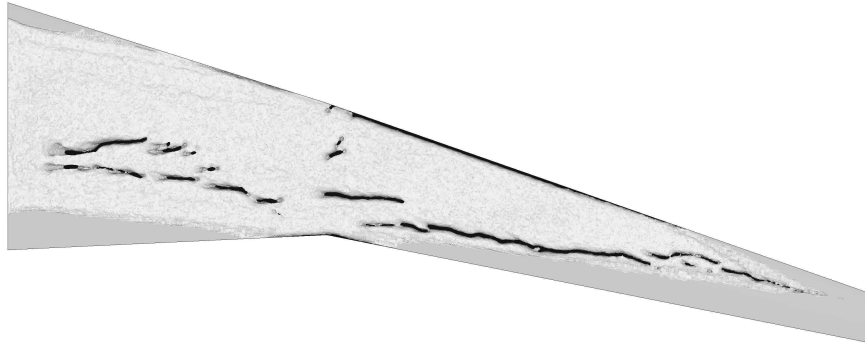
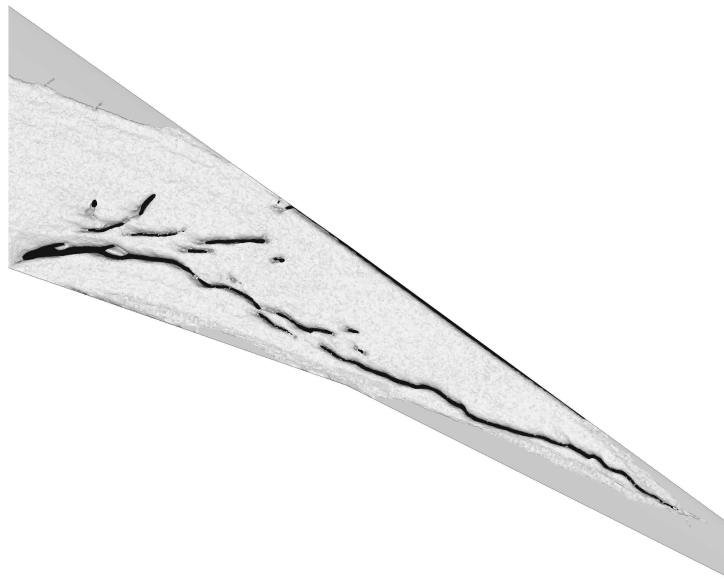**Fig. 8** *Optimal layout for the modified geometry at* $\Lambda = 20°$.

Fig. 9 *Optimal layout for the modified geometry with* $AR = 7$.

B. Effect of the aerodynamic loading

The pull up manoeuvre has been applied to the baseline planform with the main purpose to investigate the effect of the loading intensity. The results are presented in figure **??**. They are almost identical to the baseline case.

This similarity is actually not surprising. Topology optimization distributes the material following the load path determined by the loading condition. The aerodynamic load for the pull up manoeuvre was calculated for the untrimmed aircraft which results in a pressure distribution which is only scaled with respect to the cruise condition.

IV. Conclusions

The sweep angle Λ and the aspect ratio AR are two of the principal metrics used in the preliminary design of a transonic wing. The aerodynamic sensitivities with respect to these two features are known and used to drive the design of the wing planform. The conceptual design of the internal structure, on the contrary, is based on a more conventional approach. According to a well established configuration which relies on the assembly of straight structural parts which take specific loading components.
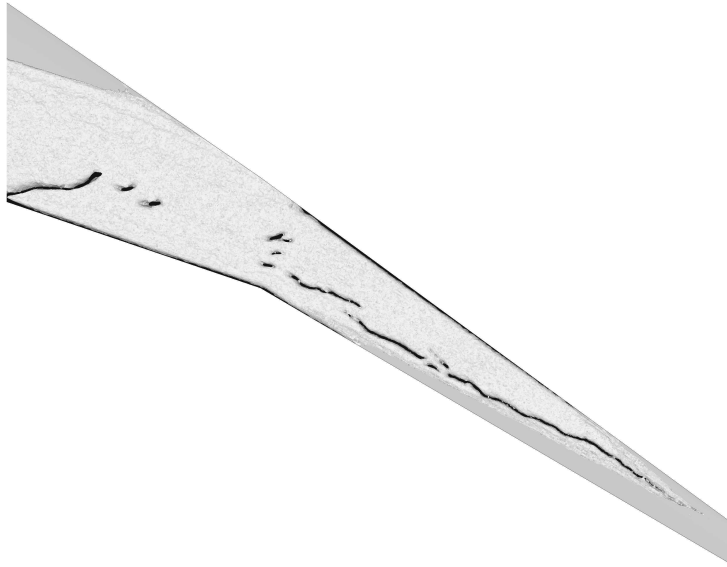
Fig. 10 *Lift distribution along the span. Cruise condition.*

The element of novelty, introduced by this paper, is represented by the use of topology optimization within a multidisciplinary framework as a tool to support the conceptual design and to investigate the existence of a relationship between the shape of the wing planform, and ultimately of the loading condition, and the layout of its structural components. The study is also characterized by some approximations that still require a further improvement. In particular the aerodynamic pressure can be computed more accurately. The design volume can be extended to the entire wing, provided that also the computational resources should be increased.

On the one hand, the influence of the loading intensity did not showed significant changes in the material distribution, no substantial changes were observed between the results obtained for the cruise conditions and those for the pull up manoeuvre.

On the other, some interesting trends could be associated to the variation of the geometric parameters. The results obtained modifying the sweep angle, suggested an effect on the curvature of the transversal component located in the inboard wing, between the root and the kink. Also the chordwise position of a spar-like element was sensible to this metric. Changes of the aspect ratio also induced a certain curvature on the beam component which ran along the span, while the effect on the transversal direction appeared of minor importance.

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