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# The Hausdorff Edition

Walter Purkert et Erhard Scholz

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# The Hausdorff Edition

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**Résumé :** Nous présentons dans cet article la genèse du projet de l'Édition Hausdorff, ainsi que sa structure organisationnelle ; une discussion suit sur un des aspects centraux de l'œuvre de Hausdorff.

**Abstract:** A short overview is provided of the genesis of the Hausdorff Edition project, and of its organizational structure, followed by a discussion of a central aspect of Hausdorff's work.

## 1. Introduction and background

The aim of the Felix Hausdorff Edition is to completely edit Felix Hausdorff's (1868-1942) mathematical work and his philosophical and literary writings published during the lifetime under the pseudonym "Paul Mongré". The voluminous *Nachlass*, largely made up of mathematical manuscripts, is consulted by the editors in view of commentaries, and is partially, but very selectively, included in the edition.<sup>1</sup> Therefore the collection of roughly 26000 handwritten pages of Hausdorff's *Nachlass* had to be made accessible for a scientific evaluation. This was an indispensable precondition for starting the edition.

In 1964, Günter Bergmann, a former student of Hausdorff then working at the University of Münster, began to order and catalogue the disordered (and partially destroyed) collection of Hausdorff's manuscripts. Acting on behalf of Felix Hausdorff's daughter, Lenore König—who survived the Nazi genocide in clandestinity—Bergmann transferred the collection to the Bonn University Library in 1980. Personal documents were added from the Lenore König *Nachlass* in 1994. The surviving letters from F. Hausdorff to P. Alexandroff were added to Bonn's collection in 1996, with the assistance of Professor Friedrich

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*Philosophia Scientiæ*, 14 (1), 2010, 127–135.

1. This intention is expressed by the lengthy German title of the Collected Works, *Felix Hausdorff: Gesammelte Werke einschliesslich der unter dem Pseudonym Paul Mongré erschienenen philosophischen und literarischen Schriften und ausgewählter Texte aus dem Nachlaß*.

Hirzebruch (Bonn) and a generous donation of manuscripts to the university library by Professor Shiryaev (Moscow).

In 1992, the *Akademie der Wissenschaften Nordrhein-Westfalen* formed a commission consisting of mathematicians and historians of mathematics, with the task to prepare the edition of the collected works.<sup>2</sup> Between October 1993 and December 1995, Walter Purkert prepared a detailed inventory of the manuscript collection, with support from the *Deutsche Forschungsgemeinschaft* (DFG); Purkert's inventory was published electronically.<sup>3</sup>

A network of cooperating editors was subsequently formed by Egbert Brieskorn, with support from other members of the commission, and in parallel to the ongoing indexation and synthesis of the *Nachlass*. In October 1996, the editorial work began, after the principles of the edition were agreed upon by the cooperating editors. The first volume of the series, entitled *Analysis, Algebra and Number Theory*, was published in 2001. During this initial period the edition was generously supported by a second grant of the DFG.

The staff of the Hausdorff Edition is made up of a scientific coordinator and a part-time technical support for IT questions and  $\text{\TeX}$  typesetting. The coordinator is a historian of mathematics with research background in the history of set theory and expert knowledge of the *Nachlass*, gained during preparation of the finder's aid (W. Purkert). Along with the coordinator, E. Brieskorn, the initiator and *spiritus rector* of the edition project, has been at the intellectual center of the edition since 1996; they are supported by the other members of the editorial board (F. Hirzebruch, R. Remmert, E. Scholz).

Editorial work is performed in a decentralized fashion by cooperating editors, each with expertise in different fields, and generally without financial support from the edition project. The structure of the edition reflects a style of academic work widely spread until closely to the end of the 20<sup>th</sup> century, where contributing editors were not yet dependent on external funding as a central criterion for the "evaluation" of their academic achievements. In July 2002, the edition was taken over by the NRW Academy as a long-term enterprise with the same structure.

## 2. Structure of the edition and organization of work

In the preparation phase it was discussed whether the arrangement of the Hausdorff Edition should be by time order or by subject matter. It was decided to organize the volumes according to subject matter, and inside each block in time order. This choice makes it possible, perhaps even attractive, for readers

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2. Members of the commission in 1992 included E. Brieskorn, F. Hirzebruch, B. Korte, R. Remmert, C. J. Scriba, and E. Scholz.

3. <http://www.aic.uni-wuppertal.de/fb7/hausdorff/findbuch.asp>.

to buy and to read separate volumes of the edition according to their topical interests. Moreover, the editorial responsibility for the volumes is greatly facilitated by this decision. Twenty editors contribute to the edition, with expertise in mathematics and foundations (9+3), history of mathematics and astronomy (4+1), philosophy (1), and German literature (2).

The volume structure of the edition (with year of publication) is:

- I Biography, Early Work on Set Theory (2012)<sup>4</sup>
- II *Grundzüge der Mengenlehre* (2002)<sup>5</sup>
- III Descriptive Set Theory and Topology (2008)<sup>6</sup>
- IV Analysis, Algebra and Number Theory (2001)<sup>7</sup>
- V Astronomy, Optics and Probability Theory (2005)<sup>8</sup>
- VI Geometry, Space and Time (2011)<sup>9</sup>
- VII Philosophical Work (2004)<sup>10</sup>
- VIII Literary Work (2010)<sup>11</sup>
- IX Correspondence (2011)<sup>12</sup>

For several of the volumes and for the discussion of overarching subjects several smaller topical editorial workshops were organized by the office of the edition: *Hausdorff and Modernity*, Bonn, October 5-7, 1998; editorial conferences to separate volumes, *Grundzüge* (Vol. II), Königswinter, September 30-October 2, 1999; literature (Vol. VIII), Rauschholzhausen, February 12-15, 2003; space and time (Vol. VI), Rauschholzhausen, October 17-19, 2005; and diverse smaller working meetings on topology (Vols. II, III) at Bonn and Berlin.

During the work, the scientific coordinator accompanies all the volumes consecutively. Much of the day-to-day coordination between the editorial office at Bonn and the different locations of the contributing editors is done this way.

### 3. The reedition of Hausdorff's books

It was clear from the beginning on, that all published papers (mathematical papers, philosophical papers, essays in literary journals) will be included into

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- 4. Editors E. Brieskorn, U. Felgner, V. Kanovei, P. Koepke, W. Purkert.
  - 5. Editors E. Brieskorn, S. D. Chatterji, M. Epple, U. Felgner, H. Herrlich, M. Hušek, V. Kanovei, P. Koepke, G. Preuß, W. Purkert, E. Scholz.
  - 6. Editors U. Felgner, H. Herrlich, M. Hušek, V. Kanovei, P. Koepke, G. Preuß, W. Purkert, E. Scholz.
  - 7. Editors S. D. Chatterji, R. Remmert, W. Scharlau.
  - 8. Editors J. Bemelmans, Ch. Binder, S. D. Chatterji, S. Hildebrandt, W. Purkert, F. Schmeidler, E. Scholz.
  - 9. Editor M. Epple, supported by E. Brieskorn.
  - 10. Editor W. Stegmaier.
  - 11. Editors F. Vollhardt, U. Roth.
  - 12. Editors E. Brieskorn, W. Purkert.

the edition. Each such document is accompanied by expert commentary. But what was to be done with Hausdorff's books? Hausdorff wrote five books, the first three under his pseudonym Paul Mongré, and the last two under his own name. In 1897, a volume of aphorisms in Nietzschean style, entitled *Sant' Ilario: Gedanken aus der Landschaft Zarathustras* was published. One year later Hausdorff published *Das Chaos in kosmischer Auslese*, the main aim of which was to destroy metaphysics in all its guises. The third Mongré book was a volume of poems that appeared in 1900 under the title *Ekstasen*. Only *Das Chaos in kosmischer Auslese* was reprinted after the war, in 1976, by the philosopher Max Bense, strangely enough under a new title, namely *Zwischen Chaos und Kosmos oder Vom Ende der Metaphysik*. But this reprint did not appear in a well-known publishing house but in Bense's own very small private publishing firm "Agis Verlag Baden-Baden". The original editions of the three books mentioned above are very rare (Fig. 1 shows the cover page of *Sant' Ilario*). It appeared quite reasonable to reprint these works, to make them accessible to today's reader.

Quite different is the editorial situation of Hausdorff's mathematical books *Grundzüge der Mengenlehre* (1914) and *Mengenlehre* (1927, second edition 1935). The *Grundzüge* were reprinted by Chelsea three times, most recently in 1978. These reprints are available in almost all mathematical libraries and in many private libraries. Nevertheless, we decided to incorporate the *Grundzüge* into our edition, devoting a volume of about 880 pages to this book.

Is such an effort justified? The *Grundzüge der Mengenlehre* is without any doubt Hausdorff's most influential work: it is his *opus magnum*. It was a milestone on the path from classical mathematics of the 18<sup>th</sup> and 19<sup>th</sup> centuries to the so-called modern mathematics of structures. Mathematical structures are abstract sets provided with axiomatically defined topological, algebraic or order properties. Hausdorff created in his book one of the fundamental structures or concepts of modern mathematics, the concept of topological space. He defined it by his famous system of neighbourhood axioms.

But Hausdorff did much more than introduce a new fundamental concept. Beginning with the most general case of topological spaces and specializing the spaces step by step by introducing additional separability axioms, he developed a new mathematical discipline which is now called general topology. In the frame of this theory Hausdorff also developed the theory of metric spaces; these spaces first appeared in Fréchet's dissertation from 1906, but more as a by-product of Fréchet's limit spaces. With Hausdorff's book, metric spaces found universal acceptance within the mathematical community. At the end of the hierarchy of spaces stood the Euclidean spaces. The Cantorian point-set theory of Euclidean spaces turned out to be a very special case of Hausdorff's general topology.

The *Grundzüge* contains numerous further innovations, including the concept of connectivity and the idea of decomposition of an arbitrary space into components of connectivity, the concept of a complete space, the concept of a

metric in the set of all bounded and closed subsets of a metric space (today called Hausdorff's metric), and Hausdorff's paradoxical decomposition of the sphere. With these contributions in mind, the principal aims of the *Grundzüge*-volume of the Hausdorff Edition became clear: to define the place of this work within the history of mathematics, to study its historical basis and presuppositions, and above all, to study its reception and its historical impact. Three forms of commentary were selected to achieve this aim: detailed historical introduction, line-based comments referring to passages of Hausdorff's text, and historical essays on special topics.

The historical introduction shows Hausdorff's unusual path to set theory via his philosophical interests. His book is compared with others on general set theory and point sets, including those of the Youngs, Shegalkin, Hessenberg, Schoenflies, Sierpiński, Fréchet, Fraenkel, and Kuratowski. The reception of Hausdorff's work is analyzed with respect to its influence on the Polish and Russian schools, on the Bourbaki group, and in the journal *Fundamenta Mathematicae*. The *Grundzüge* is shown, via this reception, to have incorporated the concepts of general topology into *Analysis situs* and geometric topology.

The line-based comments aim to explain remarkable points of Hausdorff's text or to put a certain part of the text into historical context. For instance, Hausdorff introduced (years before Kuratowski) a purely set-theoretical definition of an ordered pair, and gave purely set-theoretical definitions of the fundamental concepts of a map, and of a function. As an example how the line-based comments are organized, Fig. 2 shows the first one from the preface to *Grundzüge*: an allusion to Voltaire that one would not expect in a mathematical text. Mongré's work is filled with such allusions to literature, philosophy, painting, and drama.

A special form of commentary in the *Grundzüge*-volume are eleven historical essays. Each essay is devoted to one of the book's signal achievements or contributions, including the concept of function, the concept of cardinal number, Hausdorff's theory of  $\eta_\alpha$ -sets and its historical impact, the concept of topological space, separation axioms, connectedness, countability axioms, Hausdorff's metric and hyperspaces, completion and total boundedness, descriptive set theory, and measure and integration theory. Ulrich Felgner's essay on Hausdorff's  $\eta_\alpha$ -sets, for example, considers existing historiography of set theory to be one-sided. While the history of foundational questions, such as the set-theoretical paradoxes, attempts to avoid the contradictions, problems of axiomatization, and logical foundations have received significant attention, the mathematical substance added to Cantorian set theory in the generation after Cantor is less well known. Hausdorff is a leading figure in this respect: he developed the higher theory of ordered sets based on such fundamental concepts like cofinality, regular and singular numbers, element and gap characters, and partial order; most of this new theory appears in the *Grundzüge*.

An especially interesting part of Hausdorff's creations is the theory of  $\eta_\alpha$ -sets. Let  $\eta$  be the order type of the rational numbers in their natural order. Cantor showed that  $\eta$  is universal in the class  $T(\aleph_0)$  of all countable order types, such that if one takes an arbitrary countable order type, say  $\beta$ , then there exists a subset of  $\eta$  which is order-isomorphic to the given  $\beta$ . Hausdorff solved the hard task of finding a universal type for the class  $T(\aleph_\alpha)$  of all order types of a given arbitrary power  $\aleph_\alpha$ . He found such a universal type for any  $\aleph_\alpha$  using his general theory of ordered products and powers, and called them  $\eta_\alpha$ -sets (in analogy to Cantor's  $\eta = \eta_0$ ). His theory of  $\eta_\alpha$ -sets was largely ignored, however, and nearly forgotten, leading the eminent set theorist Abraham Fraenkel to write in the 1950s that Hausdorff's theory is quite unimportant. Felgner's essay shows how Hausdorff's theory later gained importance, in particular for model theory (saturated models), abstract algebra, and general topology.

Hausdorff's second mathematical book, *Mengenlehre*, was identified as a revised edition of the *Grundzüge*, but was in fact a completely new book. The historical significance of *Mengenlehre* lies in the fact that it was the first monographic presentation of a new discipline: descriptive set theory. For this reason, *Mengenlehre* was incorporated into Vol. III: Descriptive Set Theory and Topology.

## 4. Hausdorff's *Nachlass* in the Hausdorff Edition

Hausdorff's *Nachlass* is extensive, consisting of roughly 26000 folios. There are three categories of manuscripts in the *Nachlass*: letters, Hausdorff's lectures, and mathematical or philosophical papers.

The letters in Hausdorff's *Nachlass* are few, and most of the letters he received are considered to be lost. All surviving letters in the *Nachlass* will be included in Vol. IX, including the correspondence with the Russian mathematician Alexandroff mentioned above. But the main part of Volume IX will consist of Hausdorff's letters that we found in numerous archives around the world, for instance, letters to Hilbert, Engel, Pólya, Toeplitz, von Mises, Fraenkel, Lie, Riesz, Féjer, and also to Elisabeth Förster-Nietzsche, Peter Gast, Paul Lauterbach, Fritz Mauthner, Richard Dehmel, Paul Fechter, Moritz Schlick and others. As a guide for the composition of Vol. IX we regard the corresponding volumes of the Poincaré-edition.

Concerning the lecture notes we have in Hausdorff's case the quite rare circumstance that all of his lecture notes from nearly 40 years of academic teaching are preserved in the *Nachlass*. The series begins in 1895 with the equilibrium figure of rotating fluid bodies, and ends in 1933 with an introduction to algebraic topology. Hausdorff's lectures not only reflect his interests and scientific life but also in some sense the development of important parts

of mathematics in the first third of the 20<sup>th</sup> century. All the notes are from Hausdorff's own hand, and all lectures are fully elaborated, ready for printing. Strangely, Hausdorff never used his notes in his lectures. According to late Prof. Bergmann from Münster, who attended Hausdorff's lectures in the early 1930s, Hausdorff would arrive at the lecture hall, place a closed folder of notes on his desk, lecture for an hour and a half, collect the still-closed folder of notes, and exit the hall.

We plan to review all the lecture notes in Volume I in an essay on Hausdorff as an academic teacher. In this essay, a list of all his lectures will be provided, along with brief evaluations of content and originality. Hausdorff's 1925 lectures on the theory of limits, for example, entitled "Divergente Reihen", concerned a field to which he made important contributions in the years 1921-1923. Some procedures which serve to attach a generalized limit to divergent series are called today Hausdorff procedures. There was no monograph on limit theory before Hardy's *Divergent Series* of 1948, a chapter of which is devoted to Hausdorff procedures. Hausdorff's lectures were situated at the forefront of research; had he published them, they would have constituted the first monograph in this field.

Four lecture series are to be edited *in extenso* in different volumes. The first of these is on set theory, delivered in 1901 for the benefit of three auditors; this was one of the first courses anywhere on the topic (printed in Vol. I). The second is a 1904 course for a more general audience entitled "Raum und Zeit", in which we find philosophical reflections connected with mathematical considerations. This lecture will be included into Volume VI "Geometry, Space and Time". The third course (in Vol. V), delivered in 1923, is on probability theory. Probability is defined there by the axioms of a normed measure, and many parts of this lecture resemble a course on measure theory, ten years before Kolmogoroff's *Grundbegriffe der Wahrscheinlichkeitsrechnung*. The 1923 lecture also contains very original ideas and proofs; one of these stimulated its editor to find a new treatment of the central limit theorem.<sup>13</sup> The fourth course, on algebraic topology, provides an interesting view of this topic, just when the field was taking shape (1933, in Vol. III).

From an editorial standpoint, editing the *Nachlass* lecture notes is a much simpler task than that of editing the ten thousand pages of mathematical manuscripts. A selection of these papers will be published, according to a classification scheme with three categories.

The first category concerns manuscript material that can be exploited in the annotation of published works. An example is Hausdorff's paper *Erweiterung einer Homöomorphie*, that appeared in *Fundamenta Mathematicae* in 1930. Several manuscripts show how Hausdorff worked on this problem over the years, how he improved and generalized the results step by step, up to the final published result.

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13. S. D. Chatterji, Lindeberg's Central Limit Theorem à la Hausdorff, *Expositiones Mathematicae* 25 (2007), 215-233.



The second category concerns self-contained writings, to be edited *in extenso*. In this group of manuscripts we find research notes with early (unpublished) findings, improvements or supplements to published papers, and documents of particular mathematical or historical importance. For example, in 1915 Hausdorff discovered the so-called long line (Faszikel 223). The long line is an important counterexample in topology, and also very interesting for function theory. Faszikel 223 is reprinted twice, in Volume IV with a long commentary from the point of view of complex analysis, and again in Volume III, with a detailed topological commentary. The long line was later discovered and published by Alexandroff (1924); giving rise to “Alexandroff’s long line”. Faszikel 166 deals with universal spaces. It is easy to find a nonseparable metric space that is universal for all separable metric spaces.<sup>14</sup> But to construct a separable universal metric space is quite difficult. Hausdorff found such a construction in 1924 but did not publish it. Eighty years later, in 2004, Vershik, a Russian mathematician discovered this construction again, of course without any knowledge of Hausdorff’s work, and showed the importance of this idea for probabilistic metric spaces. Prof. Hušek from Prague, the editor of this part, gave a talk on Hausdorff’s idea at a conference in Israel, and Vershik was most surprised to learn he had a predecessor.

An example of a document improving upon earlier publications comes from Hausdorff’s copy of *Mengenlehre*, where we found some autograph notes with improvements of proofs, and a few supplements. These notes are printed and commented immediately after the reprint of *Mengenlehre* in Volume III. Similarly, for Hausdorff’s paper *Gestufte Räume*, where Hausdorff posed a problem he could not solve; three years later he solved his problem (Faszikel 700), but never published his solution. Faszikel 700 is also included into Volume III.

In the early 1920s, Hausdorff was very interested in metrization theorems, and proved a quite general theorem, which he probably planned to publish. In 1924, Alexandroff and Urysohn visited Hausdorff in Bonn, and Urysohn explained his own metrization results. Some days later, Alexandroff and Urysohn travelled to France’s western seaboard. On August 17, 1924, Urysohn drowned in the Atlantic. Hausdorff decided not to publish his results, and he encouraged Alexandroff to publish Urysohn’s theorems. Hausdorff’s manuscript has historical interest, and is to be published in Vol. III.

Another document of historical interest, analyzed by E. Scholz in Vol. III, concerns algebraic topology. Most notably, in a manuscript from March 1933 on homomorphisms of homological groups, Hausdorff employed commutative diagrams (see figure 3). The document in question represents one of the first known uses of this modern tool.

The third and final category of mathematical manuscript from the *Nachlass* to be considered concerns summary essays. One example is an essay on dimension theory, published in Vol. III. In fact, during the 1920s, Hausdorff

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14. A metric space is universal if for any separable metric space  $X$  there is a subset in the universal space which is isometric to  $X$ .

wrote over twenty studies in this field, as noted by G. Preuss in his essay “Hausdorff’s studies on dimension theory”.

At present (early 2010) work is nearly finished on the literary work (Vol. VIII) and the volume on space and time (Vol. VI) is well progressing. After these volumes are finished, volumes I and IX (Correspondence) will be at the center of ongoing work. Vol. I will contain Hausdorff’s work on ordered sets and a detailed introduction to the biography and the work of our author.

If everything goes as planned, the edition will be completed in 2012.

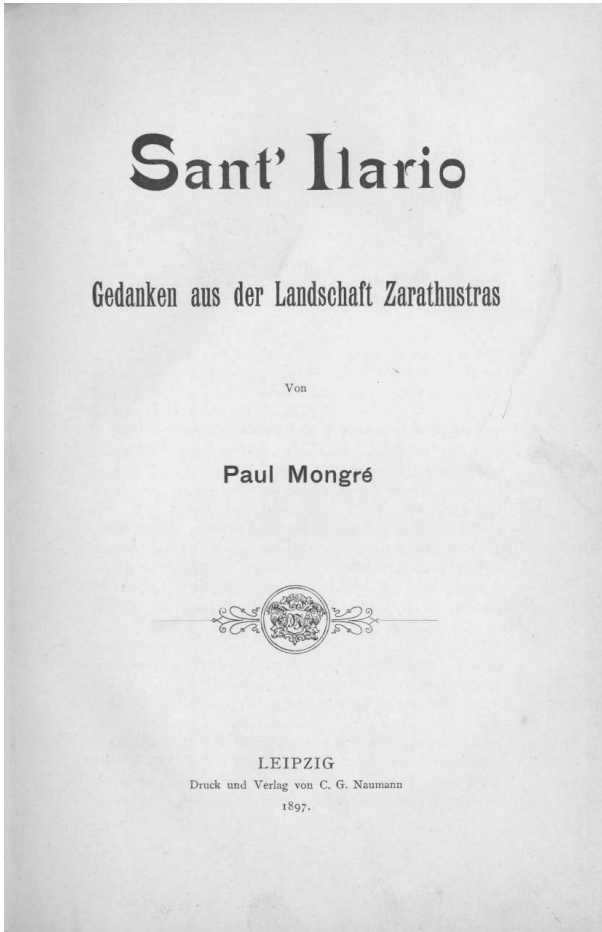


FIGURE 1: Cover page of “Sant’ Ilario”. Gedanken aus der Landschaft Zarathustras

## Vorwort.

Das vorliegende Werk will ein Lehrbuch und kein Bericht sein: es versucht die Hauptsachen der Mengenlehre ohne Voraussetzung höherer Vorkenntnisse mit vollständig ausgeführten Beweisen darzustellen und verzichtet dafür auf Vollständigkeit des behandelten Stoffes. Hinsichtlich der thematischen Begrenzung, bei der ja übrigens die Mitwirkung subjektiver Gründe nicht auszuschalten ist, habe ich der Mengenlehre selbst vor ihren Anwendungen den Vorzug gegeben; infolgedessen wird man vielleicht finden, daß den geordneten Mengen zuviel und etwa den reellen Funktionen einer reellen Variablen zuwenig Platz eingeräumt worden sei. Was die Tendenz anbelangt, immer zu beweisen und niemals bloß zu referieren, so ist mir die in dem bekannten Voltaireschen Worte bezeichnete Gefahr nicht entgangen; aber in einem Gebiet, wo schlechthin nichts selbstverständlich und das Richtige häufig paradox, das Plausible falsch ist, gibt es außer der lückenlosen Deduktion kaum ein Mittel, sich und den Leser vor Täuschungen zu bewahren. Ich habe, um von dem menschlichen Privileg des Irrtums einen möglichst sparsamen Gebrauch zu machen, nichts ungeprüft übernommen und manches von der Wiedergabe ausgeschlossen, was mir nur auf persönlichen Kredit hin glaubwürdig erschien; aber selbst fertige und im ganzen einwandfreie Darstellungen, die ich meinem Plane einzufriedern hatte, mußte ich häufig einer gründlichen Umarbeitung unterziehen, bis sie sich den mir vorschwebenden Forderungen an Präzision fügten. [1]

## Anmerkungen der Herausgeber

[1] (S. V.) *Voltaire*  
 HAUSDORFF bezieht sich hier möglicherweise auf die 1738 geschriebene Vorrede zu VOLTAIRES Komödie *L'Enfant prodigue* (Der verlorene Sohn) von 1736. Dort heißt es: „Tous les genres sont bons, hors le genre ennuyeux.“ (Alle Arten [von Kunst] sind gut, außer der langweiligen). In BÜCHMANNS *Geflügelte Worte* (Erstauflage 1864) findet man dieses Zitat, und BÜCHMANN bemerkt (S. 171), daß der Ausspruch von GOETHE (*Theater-Reden*, Epilog vom 11.6.1792) und auch von WIELAND paraphrasiert worden sei. Bei WIELAND heißt es am Ende des zweiten Briefes „An einen jungen Dichter“ (aus dem Jahre 1784):

Meiner Meinung nach kann ein Mann von Talenten in allen Gattungen schätzbare Werke hervorbringen, und (wenn ich Voltären hier eine Wendung abborgen darf) die einzige Gattung, die ich aus unserer Literatur verbannt zu sehen wünsche, ist – *die langweilige*. [C. M. WIELAND, *Sämtliche Werke*, 6. Supplement-Band, Leipzig, Göschen 1798, S. 295–296].

BÜCHMANNS *Geflügelte Worte* sind seit 1864 immer wieder aufgelegt worden und waren auch schon zu HAUSDORFFS Zeiten weit verbreitet. HAUSDORFF selbst schreibt im *Sant' Ilario* ([H 1897b]), S. 394: „Beweise sind, bei allen Philosophen, das Verdächtige und überdies Langweilige. [...]“ W.P./U.F.

FIGURE 2: The first line commentary to “Grundzüge der Mengenlehre”

Homomorphismen der Homologiegruppen kompakter Räume.

$G_1, G_2, \dots$  sei eine Folge von Gruppen; es sei für jedes  $n$  ein Homomorphismus  $\varphi_n$  von  $G_{n+1}$  in  $G_n$  gegeben, d.h. jedem Element  $A_{n+1}$  von  $G_{n+1}$  ein Element  $A_n = A_n \varphi_n$  von  $G_n$  (additiv isomorph) zugeordnet. (Es ist besser, die Funktionszeichen  $\varphi_n$  nachzuschreiben). Schreiben wir auch  $A_{n+1} \rightarrow A_n$ .  
 Eine Folge  $A = (A_1, A_2, \dots)$  mit  $A_{n+1} \rightarrow A_n$  für  $n=1, 2, \dots$  heißt eine Fundamentalfolge, und die von der Fund.folge gebildete Gruppe

$$G = (G_1, G_2, \dots)$$

heißt die  $G_1, G_2, \dots$  eine result. Folge mit den Homomorphismen

$$\varphi'_n \text{ von } G_{n+1}' \text{ in } G_n'; \quad G' = (G_1', G_2', \dots)$$

zuzugewiesen für jedes  $n$  ein Homomorphismus  $\omega_n$  von  $G_n$  in  $G_n'$  gegeben,

mit folgender Eigenschaft:  $\varphi_n \omega_n = \omega_{n+1} \varphi_{n+1}'$  d.h.

$$\begin{array}{ccc} A_{n+1} & \xrightarrow{\varphi_n} & A_n \\ \omega_{n+1} \downarrow & & \downarrow \omega_n \\ A_{n+1}' & \xrightarrow{\varphi_n'} & A_n' \end{array}$$

wenn man von  $A_{n+1}$  über  $A_n$  nach  $A_n'$  gelangt, so gelangt man auch über  $A_{n+1}'$  nach denselben  $A_n'$ .

Oder: die Gesamtheit  $\omega$  der Homomorphismen

$\omega_n$  verwendet eine Fundamentalfolge  $A = (A_1, A_2, \dots)$  in einer Fundamentalfolge  $A' = (A_1', A_2', \dots)$ : Damit ist also ein Homomorphismus von  $G$  in  $G'$  definiert.

FIGURE 3: Hausdorff *Nachlass* Fasz. 571 (March 1933) with early usage of commutative diagrams

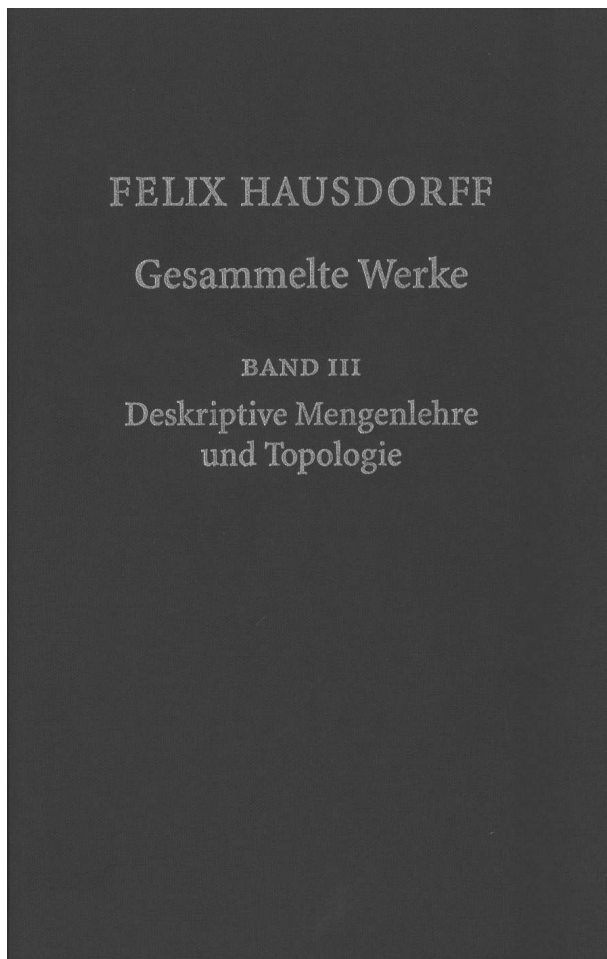


FIGURE 4: Volume III of the Hausdorff Edition