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Hugh MacColl after One Hundred Years

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Hugh MacColl's contributions to the *Educational Times*

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Résumé : Il y eut plus de 1900 contributeurs à la section mathématique du journal *Educational Times* durant ses 67 ans d'existence. Cette rubrique mensuelle contenait des problèmes et leurs solutions, de courts articles, et souvent de courts comptes rendus de l'assemblée la plus récente de la London Mathematical Society. Hugh MacColl (1837-1909), enseignant et tuteur, était l'un des contributeurs les plus prolifiques à cette section. Il soumettait ses contributions depuis Boulogne-sur-Mer, France. Son expertise portait essentiellement sur la probabilité géométrique, les probabilités classiques et la logique. Après un bref aperçu de l'histoire de la section mathématique du *Educational Times* et de la correspondance de MacColl avec son directeur de publication, W. J. C. Miller, nous étudierons les contributions de MacColl à ce journal.

Abstract: There were over nineteen hundred contributors to the mathematical department of the *Educational Times* during its sixty-seven-year existence. The monthly column contained problems and their solutions, brief articles, and often short accounts of the most recent meeting of the London Mathematical Society. Hugh MacColl (1837-1909), a teacher and tutor, was one of the more prolific contributors to the department. He submitted his contributions from Boulogne-sur-Mer, France. His expertise was mainly in geometric probability, standard probability, and logic. After a brief history of the mathematical department of the *Educational Times* and some of Carroll's correspondence with the editor, W. J. C. Miller, we highlight MacColl's contributions to the journal.

The *Educational Times*

The *Educational Times and Journal of the College of Preceptors* (*ET*) was a long-lived pedagogical journal published in London from 1847 to 1918. In the second year of its publication, a mathematical department was added to serve as an outlet for men and women to exhibit their mathematical skills. The section consisted of problems, solutions, and brief articles. William

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K. Clifford, professor of mathematics and mechanics at University College London, claimed that in the late nineteenth century, *ET* did more to encourage original mathematical research than any other European periodical [Miller 1897]. The first publications of the British mathematician Godfrey H. Hardy, the philosopher Bertrand Russell, and the Indian mathematician Srinivasa Ramanujan appeared as solutions to problems in *ET* [Tattersall 2008]. Among its contributors were some of the most esteemed mathematicians in the British Commonwealth, Europe, and America including John Couch Adams, Emile Borel, Eugène Catalan, Arthur Cayley, Ernest Césaro, Augustus De Morgan, Francis Galton, Charles Hermite, Felix Klein, Magnus Mittag-Leffler, James Clerk Maxwell, Simon Newcomb, Benjamin Pierce, and James J. Sylvester.

The College of Preceptors was incorporated by Royal Charter in London in 1849. Its main objectives were to promote sound learning, advance interest in education among the middle class, and provide means to raise the status and qualifications of teachers. In order to accomplish those goals, training was offered to those entering the teaching profession and periodic examinations for certification were administered to both teachers and students. The group aimed to establish education as a subject of study in colleges and universities. In addition, the organization strove to facilitate better communication between teachers and the public. *ET* contained notices of available scholarships, lists of successful candidates on examinations given by the College, notices of vacancies for teachers and governesses, numerous book reviews, and textbook advertisements. To many, the most singular feature of the monthly journal was the section devoted to mathematical problems and their solutions that was inaugurated in the winter of 1848.

From December 1848 to December 1915, more than 18,000 problems were posed in the pages of *ET*.[1] The first editors of the department of mathematical questions and solutions were Richard Wilson and James Wharton, both of whom matriculated at of St. John's College, Cambridge. When Stephen Watson of Haydonbridge and William John Clarke Miller, mathematical master and vice-principal of Huddersfield College in Yorkshire, assumed the editorship of the department in the late 1850s, the quality of the problems and their solutions rose dramatically.[2] Numbered problems first appeared in the August 1849 issue, before then about a dozen unnumbered problems had been published. Solutions were submitted to 81 percent of the problems with proposers submitting their own solutions 25.8 percent of the time. There were a remarkable number of contributions by women [Tattersall 2004]. The majority of contributors hailed from England (62.2%), India (7.7%), Ireland (7.2%), France (5.6%), and the United States (5.3%) [Tattersall 2007]. In

1. From 1915 to 1918, *ET* was published quarterly without a section devoted to mathematical problems. Problems numbered 18,139 to 18,702 were sent to individual subscribers on a monthly basis. Solutions to 63 percent of these problems appear in *Mathematical Questions with their Solutions from the Educational Times.*

2. For a comprehensive account of the early history of the *Educational Times* see [Delve 2003].

addition, there were contributors from Australia, Austria, Belgium, Bohemia, Canada, Ceylon, Dagestan, Germany, Hong Kong, Italy, Malaysia, Malta, Mauritius, The Netherlands, New Zealand, Russia, Scotland, South Africa, Spain, Sweden, Switzerland, and Wales. [3]

Under Miller's editorship, the standard directions for submission of problems were: to make your answers as short as possible, write each question and answer on a separate sheet of paper with your name at the top of each, and remember to pay the postage in full [Miller 1886]. Space in *ET* was at such a premium that normally less than a page and a half was devoted to the mathematics section. Various departments vied for space. On one occasion Miller commented, "Want of space necessitated the omission of several solutions this month" [Miller 1853a]. Soon thereafter, he wrote, "In consequence of the great pressure of advertisements and reports from the College, the mathematical matter is necessarily abridged this month" [Miller 1853b]. On several occasions the mathematics section was completely omitted. In 1864, Miller took action: problems and solutions that had appeared in *ET* were republished semiannually in *Mathematical Questions with Their Solutions from the "Educational Times" (MQ)* [Grattan-Guinness 1992]. More importantly, *MQ* contained solutions to problems whose solutions hadn't appeared in *ET*. In 1876 Miller, Fellow of the Royal Statistical Society and a member of the London Mathematical Society, migrated to London and became registrar, secretary and statistician to the General Medical Council. He served as editor of *MQ* and the mathematical department of *ET* until illness forced him to retire in 1897 [Finkel 1896]. During his tenure, he endeavored to publish solutions at most two months after the problems had appeared in print. Many nineteenth-century mathematical textbooks did not contain pages of diverse exercises as do their modern counterparts. It was customary for teachers and students to seek out or create their own applications of theory. *ET* proved to be an invaluable source of practice problems to anyone interested in mathematics.

Hugh MacColl was one of the more productive contributors to the *ET* mathematical department. His problems were well contrived, his solutions clever, and his articles innovative and informative. MacColl made 205 contributions to *ET*, ranking him 22$^{\text{nd}}$ in terms of output among the over one thousand eight hundred and fifty contributors to its mathematical department, placing him just ahead of Thomas Simmons, 25$^{\text{th}}$ Wrangler on the 1874 Cambridge Mathematical Tripos. MacColl posed 125 problems, solved 70 problems, and submitted articles on various aspects of analysis, logic, and probability. One article described short cut methods to evaluate $\int x^m(a+bx^n)^p dx$ [MacColl 1871f]. In another, he suggested a scientific notation whereby, for example, 23×10^7 was represented by 23_7 and 57×10^{-3} by 57_{-3} [MacColl 1880b]. Another dealt with paradoxes in symbolic logic that he found of interest [MacColl 1903]. Another involved the determination of the

3. The author has compiled a database listing the posers, solvers, and mathematical classification of the problems for all but 35 of the relevant 816 issues of *ET*.

limits of integration of multiple integrals using analytic methods without any reference to curves or curved surfaces [MacColl 1904a].

The problems that MacColl posed dealt with geometric probability (39), logic (34), probability (15), analysis (12), geometric expectation (6), algebra (6), geometry (5), integral calculus (4), arithmetic (2), spherical trigonometry (1), and applied mathematics (1). The problems he solved dealt with geometric probability (27), logic (21), analysis (12), probability (6), combinatorics (2), and algebra (2). Up to 1885 all of his submissions with few exceptions were under the appellation H. McColl. In the majority of exceptions, he is listed as H. M'Coll. Of the 125 problems that MacColl posed in *ET*, 53 went unsolved. Of the remaining 72 problems, he submitted solutions to 42 of them. In 34 of those problems, his was the only solution.

Very little is known about MacColl's early life and education. He was born in Strontian, Argyllshire in the Scottish Highlands in 1837, the youngest of six children of John and Martha MacColl. His father was a shepherd and tenant farmer who died at age 45 when Hugh was three years old [Astroh, Grattan-Guinness & Read 2001, 81]. The death of his father had a life-long effect on him. His older brother Malcolm assumed the responsibility of supporting Hugh through his elementary and secondary education. However, when Malcolm was dismissed from his clerical post, Hugh abandoned any hope of a university education. He worked for a time in England as a schoolmaster before marrying Mary Elizabeth Johnson of Loughborough, Leicestershire. In 1865, they migrated to fashionable and prosperous Boulogne-sur-Mer which had a sizable English community. While in Boulogne, Hugh and Elizabeth had five children, four girls and a boy. There he taught English and mathematics at the *Collège communal* until 1869. From then on, he supported his family privately by tutoring mathematics, classics, English, and logic mainly to students preparing to take university and military examinations. MacColl was undoubtedly able to peruse the latest issue of *ET* to keep up with pedagogical activities in Britain and sharpen his mathematical skills at Merridew's English Library in Boulogne.

The MacColl–Miller correspondence

In November of 1865, MacColl sent Miller revised solutions to problems numbered 1739 and 1761 and an article on the numerical solutions of equations that dealt with the determination of the number of real roots of polynomials over a given interval [MacColl 1866a].

> I have taken great pains on the solution of 1761, endeavoring to make it so complete and general as to be in itself sufficient for the purposes of numerical solution; and have been as brief as I could consistently with the accomplishment of this object. I could not make my solution shorter without sacrificing so much accuracy, clearness, and generality. [MacColl 1865a]

MacColl felt that his method of derived functions to solve polynomial equations was more efficient than Sturm's method using successive derivatives [Sturm 1835]. MacColl included a discussion and some examples of polynomials with nearly equal roots. In his reply, Miller praised MacColl's article.[4] MacColl replied that:

> I cannot sufficiently thank you for your kind letter. I had certainly bestowed great pains on the solutions which I sent last, but I was by no means prepared for so much praise of them as you have been good enough to give. I feel very pleased for your good opinion of my methods is sufficient to convince me that they are of some value, and that other mathematicians will in time see the use of them. [MacColl 1865b]

MacColl added that it is beyond his comprehension how Miller can know all the mathematics necessary to edit *ET* and *MQ*. Unbeknownst to MacColl, Miller had a group of mathematicians reviewing many of the problems and solutions before they were published. We also get a glimpse of the state of MacColl's mathematical background when he added:

> But as soon as I have left the subject of numerical solution, I must really apply myself to the study of mathematical subjects which I have not yet touched. Beyond analytical conic sections and the elements of differential and integral calculus I know very little. The theory of equations I only took up quite recently—about a year ago. Would it be too much of me (considering how little time you have) to ask your advice as to the course of study I ought to pursue, and the books I ought to read? [MacColl 1865b]

In reference to the article he had submitted on angular and linear notation [MacColl 1866b], which described a coordinate system based on the perpendicular distances from a point to a set of lines.

> If you find that my discovery really possesses all the importance that I myself attach to it, I will in another letter, give you the history, so far as I can recollect it, of the development in my mind [...] important discoveries and inventions do not *always* fall to the lot of the talented [...] there are many treasures slightly inserted under the surface of the field of science, which an accidental tread may lay bare [...] I believe that such a piece of good fortune has now happened to me. [MacColl 1866c]

In March, MacColl wrote that he had just taken up the subject of modern analytic geometry and confessed that the idea for the article had originated when he encountered some difficulty in explaining to a pupil the difference between Cartesian and trilinear coordinates as found in [Todhunter 1862, 32]. He mentioned that he had sent Sylvester a copy of his discovery and, after hearing back from Sylvester, wrote:

4. For a note on correspondence between Miller and contributors, see [Grattan-Guinness 1994].

> Professor Sylvester's opinion is rather less favorable. He sums up his (confessedly hasty) criticism in the following words: "The brochure on the whole appears to me creditable to Mr. M'Coll's ingenuity and clearness of understanding but, as far as I am able to judge, does not contain principles which will be productive of such important consequences as the author appears to contemplate. . . ". [MacColl 1866d]

MacColl realized that he had omitted to include a crucial definition that made the article unintelligible. He wrote about his plans to revise and improve the article. He added that complex analysis is beyond him and that there are a number of algebraic terms that are unfamiliar to him. He concluded by asking Miller's opinion as to whether his knowledge of analytical geometry and the calculus was sufficient to do relevant mathematical research. Perhaps in an effort to further his knowledge of applied mathematics he added:

> If you would kindly advise me as to the course of study I might pursue and the books I ought to read, I should feel very much obliged. I long to study physics—especially astronomy—of which at present I know next to nothing, but first I think it would be advised to make myself better acquainted with analytical geometry and the calculus. [MacColl 1866d]

The next month he wrote:

> When I first mark what I conceive (rightly or wrongly) to be an important discovery, I feel tempted, like many other mathematical tyros to adopt a tone of exultation which is hardly becoming. [MacColl 1866e]

His revised version of the article on angular and linear notation was published in *MQ* later that year [MacColl 1866f]. In a subsequent letter to Miller, he reiterated the disturbing comment in Sylvester's review of his article:

> The interpretation affixed by the author to trilinear or quadralinear coordinates is not, I think new; I scarcely remember the time when I was not acquainted with it. [MacColl 1866g]

In June, MacColl wrote to Sylvester to thank him for his frank criticism adding that he had lower his opinion of the importance of his article. However, after consulting several books on modern analytic geometry, he wrote Miller that:

> In spite of what Prof. Sylvester said about 'my interpretation of trilinear & quadrilinear coordinated not being he thought new', I still cannot help think it *is* new [MacColl 1866h].

Geometric probability

The *a priori* definition of the probability that an event will occur is the number ways that the event can occur divided by the number of all possibilities.

In geometric or local probability, the definition of the probability of an event is given by the area pertaining to the event divided by the total possible area. This is normally expressed as the quotient of two double integrals, where the limits of integration of the numerator delineate the area pertaining to event E and the limits of integration of the denominator the total possible area S as given by

$$P[E] = \frac{\iint\limits_E dxdy}{\iint\limits_S dxdy}$$

The history of geometric or local probability can be traced back to 1777 and Buffon's needle problem: Given a large plane area ruled with equidistant parallel lines and a slender needle to determine the probability that if thrown down the needle will fall across a line [Buffon 1777]. Interest in this type of problem was rekindled when the editor of *ET* asked readers to determine the probability that if a line segment is divided at random into three pieces that a triangle can be formed from the three segments [Miller 1859]. Soon thereafter, W. S. B. Woolhouse, a multitalented London actuary, asked readers on three occasions with three distinct hypotheses to determine the probability that three random points form an acute triangle [Woolhouse 1862; 1863a; 1863b]. Woolhouse's problem piqued the interest of several readers including J. J. Sylvester, professor of mathematics at the Royal Military Academy [Sylvester 1866a], and W. C. Crofton, an instructor at the Academy who would succeed Sylvester in 1870.

In the autumn of 1865, Crofton provided a clever solution to Woolhouse's problem when the points are taken on a sphere [Crofton 1866a]. He then posed a problem concerning the average distance between two given points within a circle [Crofton 1866b]. Sylvester devised a series of interesting geometric probability problems and discussed them at a meeting of the British Society for the Advancement of Science [Sylvester 1866b]. One of the problems he highlighted was: Given four points at random in the plane determine the probability that one of the points lies inside the triangle formed by the other three [Sylvester 1864]. The problem caused quite a stir among *ET* readers leading to six different interpretations of the phrase 'at random in the plane' that in turn led to six different answers [Pfeifer 1989]. It was Crofton who while working on the problem devised an ingenious method to attack geometric probability problems bypassing the integration techniques normally employed [Crofton 1867; 1868; 1885]. [5]

After retiring from teaching at the *Collège communal* and getting settled as a private tutor, MacColl became fascinated with geometric probability problems. His calculus skills were more than adequate to evaluate the multiple integrals that he encountered. His solutions to such problems are impressive.

5. For an account of Sylvester and Crofton's contributions to the subject, see [Kendall & Moran 1963], [Solomon1978], and [Senta, Parshall & Jongmans 2001].

They require much ingenuity in setting up the problem and a solid grasp of integral calculus to obtain the answer. Between February 1871 and February 1872, MacColl solved a dozen problems in *ET* including a succinct solution to the problem that Miller had posed:

> If a straight line is divided at random into three segments, show that the chance that these three segments will form an acute-angled triangle is $3ln2 - 2$ and three times out of four no triangle can be formed. [Miller 1859]

MacColl began by considering two points x and y with x less than y on the interval $[0, a]$. Thus, the three segments have lengths $x, y - x$, and $a - y$. In order for the three lengths to form a triangle, the triangle inequality must be satisfied. That is, the sum of two sides must be greater than the third side. Hence, $x < a/2, a/2 < y$, and $y - x < a/2$. Thus, the probability of a triangle being formed is given by:

$$\frac{\int_{a/2}^{a}\int_{y-a/2}^{a/2} dxdy}{\int_0^a\int_0^y dxdy} = \frac{\int_{a/2}^{a}(a-y)dy}{\int_0^a ydy} = \frac{a^2/8}{a^2/2} = 1/4$$

For a triangle with an obtuse angle, the square of one side must be greater than the sum of the squares of the other two sides. Hence, $x^2 > (y - x)^2 + (a - y)^2$ or equivalently $x > y - a + \frac{a^2}{2y}$. Thus, the probability is given by:

$$\frac{\int_{a/2}^{a}\int_{y-a+(a^2/2y)}^{a/2} dxdy}{\int_0^a\int_0^y dxdy} = \frac{\int_{a/2}^{a}\left(\frac{3a}{2} - y - \frac{a^2}{2y}\right)dy}{\int_0^a ydy} = \frac{\left(3a^2/8\right) - \frac{a^2}{2}ln2}{a^2/2} = 3/4 - ln2$$

Since the three cases in which an obtuse triangle is formed are mutually exclusive with similar answers, the probability that an obtuse triangle is formed is $3(3/4 - ln2) = 9/4 - 3ln2$. Thus, the probability that an acute triangle is formed is given by 1/4 (the probability that a triangle is formed) minus $(9/4) - 3ln2$ (the probability that an obtuse triangle is formed) yielding the desired answer, $3ln2 - 2$. Obtaining the solution required a clear knowledge of the mathematics involved, the skill to obtain an analytic description of the area under consideration, and the ability to evaluate double integrals [MacColl 1871e].

Below are several geometric probability problems posed by MacColl. The first was solved by MacColl and Stephen Watson; the second by MacColl and George Shoobridge Carr, author of *A Synopsis of Elementary Results in Pure Mathematics,* the most influential book in Ramanujan's early mathematical education; the third by James Heber Taylor, a private tutor in Cambridge. For the remainder, although interesting, no solutions were published in *ET* or *MQ*:

If P and Q be random points within a circle, what is the probability that the circle of which P is the centre and PQ the radius will lie wholly within the given circle? [MacColl 1871c]

Three random points P, Q, R are taken in the perimeter of a square. Find the respective chances of the straight lines PQ, QR, RP forming an acute-angled triangle, and obtuse-angled triangle, or no triangle. [MacColl 1885a]

Four random points P, Q, R, S are taken within a circle; find the chance that the straight lines PQ and RS (produced if necessary) will intersect within the circle. [MacColl 1885b]

A point is taken at random in the diameter of a square, and from this point three straight lines are drawn in random directions to meet the perimeter of the square; find the chance that these three lines can be the sides of a triangle. [MacColl 1891a]

Four points P, P', Q, Q' are taken at random within a circle. What is the chance that the circles of which P and Q are the centers and PP' and QQ' the radii will intersect? [MacColl 1891b]

If x, y,z be each taken at random between 0 and 1, show that the chance that the fraction $z(1-x-y)/(1-y-yz)$ will also be between 0 and 1 is $5/4 - ln2$. [MacColl 1899]

Given that x and y are each taken at random between 0 and 1, what is the chance that $2x + y - 3a$ is negative, the constant a being between 0 and ∞. [MacColl 1901a]

Given that $2 - x - y, 3y - 4x - 6, y + 2x - 7$ are all three negative. What is the chance that y is negative? [MacColl 1901b]

What is the chance that xy is greater than 5; firstly, on the assumption that x and y are each taken at random between 1 and 5; secondly, on the assumption that x is taken at random between 1 and 5; and then y at random between 1 and x? [MacColl 1904b]

Other mathematical interests

MacColl introduced an innovative integral notion to handle certain geometric probability problems [MacColl 1871a; 1871b]. It was clever and useful but not a powerful as that of Crofton for handling intricate geometric probability problems. He let the symbol $\int f(x)px$ represent the limiting value of $\sum f(x)\Delta x$, as Δx is diminished without limit and the number of terms increased without limit, subject to two conditions: (1) the positive values of $f(x)$ greater than unity are to be considered as unity, and (2) any negative values of $f(x)$ are to be taken as zero. In essence, what he proposed was a real-valued function $f(x)$ defined on the open interval (a,b) that can be expressed as follows:

$$\int_a^b f(x)px = \begin{cases} 0 & f(x) \leq 0 \\ \int_a^b f(x)dx & 0 < f(x) < 1 \\ b - a & f(x) \geq 1 \end{cases}$$

For example,

$$\int_1^9 (5-x)px = (4-1) + \int_4^5 (5-x)dx + 0 = 3 + \left| 5x - \frac{x^2}{2} \right|_4^5 = 3\ 1/2$$

MacColl applied this novel notation to solve the following problem that, quite inexplicably, he would later propose:

> In the quadratic equation $ax^2 - bx + c$, the numerical values of the coefficients a, b, c are each taken at random between 1 and 10. What is the chance that the equation has two real roots between 1 and 10? [MacColl 1872]

He reasoned that according to the quadratic formula, the roots of the equation are real and unequal if $b^2 - 4ac > 0$ or equivalently $c < b^2/4a$. The probability of this is given by the length of the assigned limits of c over the length of the interval or $\frac{1}{9}\left(\frac{b^2}{4a} - 1\right)$. Treating b as the variable and using MacColl's notation, the average value of $\frac{1}{9}\left(\frac{b^2}{4a} - 1\right)$ as b goes from 1 to 10 is given by:

$$\frac{1}{9}\int_1^{10} \frac{1}{9}\left(\frac{b^2}{4a} - 1\right) pb$$

If a is less than $5/2$ we have that:

$$\frac{1}{9}\int_1^{10} \frac{1}{9}\left(\frac{b^2}{4a} - 1\right) pb = \frac{1}{9}(10 - \sqrt{40a}) + \frac{1}{9}\int_{\sqrt{4a}}^{\sqrt{40a}} \frac{1}{9}\left(\frac{b^2}{4a} - 1\right) db + 0 =$$
$$\frac{10}{9} - \frac{4}{243}\left(10^{\frac{3}{2}} - 1\right) a^{\frac{1}{2}} = f_1(a)$$

When a is greater than $5/2$ we obtain:

$$\frac{1}{9}\int_1^{10} \frac{1}{9}\left(\frac{b^2}{4a} - 1\right) pb = \frac{1}{9}\int_{\sqrt{4a}}^{10} \frac{1}{9}\left(\frac{b^2}{4a} - 1\right) db + 0 =$$
$$\frac{2}{243a}\left(125 + 2a^{\frac{3}{2}} - 15a\right) = f_2(a)$$

Treating a as the variable and taking it at random between 1 and 10 the final answer is given by:

$$\frac{1}{9}\left(\int_{1}^{\frac{5}{2}} f_1(a)pa + \int_{\frac{5}{2}}^{10} f_2(a)pa\right) = \frac{1500ln(2) + 160\sqrt{10} - 468}{6561}$$

[MacColl 1871d]

Another of MacColl's interests was inverse probability or Baysian analysis problems. According to [Dale 1999, 583], MacColl was one of the first, and perhaps the first, to devise a symbolic notation to represent conditional probability, denoting the probability of A given B first as A_B [MacColl 1880a] and later as $\frac{A}{B}$ [MacColl 1897]. MacColl's work in inverse probability is highlighted in [Dale 1999, 501–504]. Another innovative probability notation that MacColl devised dealt with series of events [MacColl 1871a; 1871b]. He defined $p(r)$ to denote the probability of the occurrence of the r^{th} event given in a specific ordering of possibilities and $p(:r)$ denoted the nonoccurrence of the r^{th} event. Hence, $p(r) + p(:r) = 1$. He denoted the probability of the occurrence of the m^{th} and n^{th} events and the nonoccurrence of the r^{th} event by $p(m.n:r)$ and extended it to $p(m_a.n_{:c}:r_e)$ which signifies the occurrence of the m^{th} and n^{th} and nonoccurrence of the r^{th} events imply the occurrence of the a^{th} and e^{th} and nonoccurrence of the c^{th} event.

Several years later, MacColl proposed yet another probability notation [MacColl 1877; 1878]. In 1906, he published a handbook on symbolic logic [MacColl 1906] highlighting this notation and promoting the use of "+" for "or", juxtaposition for "and", a colon for implication, and a prime for negation. Hence, $(a + bc' : d)$ symbolized the logical statement that a or, b and not c, imply d. Some of the logic problems in *ET* posed by MacColl are:

> Suppose it to have been ascertained by observation (1) that, whenever the events A and B happen together, they are invariably followed by the event C, and also by either the event D or the event E; and (2) that, whenever the events D and E both happen, they have invariably been preceded by the event A, or else by both the events B and C. When may we conclude (from the occurrence or non-occurrence of the events A, B, C, D)—(1) that E will certainly happen; (2) that E will certainly *not* happen? [MacColl 1879]
>
> Given (1) that whenever the statements a, b, x are either all three true or all three false, then the statement c is false and y is true, or else c is true and y is false; (2) that whenever d, e, y are either all three true or all three false, then the statement a is false and x true, or a is true and x false, When may we infer from these premises that either x or y is true? [MacColl 1880c]

> When, from the three implications, (1) $a'b+ab' : dx$, (2) $ax+by : c$, (3) $cd : y$, may we conclude that either x or y is true, but not both? [MacColl 1880d]
>
> Show that fifteen *valid* syllogisms of the common logic (excluding the four defective syllogisms, *Darapti, Felapton, Fesapo, Bramantip.*) may be converted into mathematical syllogisms as follows: Considering each letter as representing a mathematical quantity (positive or negative, integral or functional, real or imaginary), for every proposition of the form "all α is β" substitute "α is a multiple of β" (that is "α/β is a real integer") for "no α is a β", put "$\alpha\beta$ is integer"; to "some α is not β," put "α is not a multiple of β"; and for "some α is a β" put "$\alpha\beta$ is not an integer". What is the weakest premise that must be added to the two given premises in each of the four defective syllogisms (in their logical or mathematical form) to make them also valid? (Definition x is said to be the stronger and y the weaker proposition when x implies y but x is not implied by y). [MacColl 1887]

MacColl submitted solutions to the first problem, the last two went unsolved. The second was solved by Elizabeth Blackwood who, making use of MacColl's notation expressed the premises as: $(abx + a'b'c' : c'y + cy')(dey + d'e'y' : a'y + ay')$. Hence, $x'y' : (a + b + c)(d + e + a)$ or equivalently $x'y' : a + (b + c)(d + e)$. Thus, by contraposition $a'(b'c' + d'e') : x + y$. Among the geometric probability problems that Blackwood proposed and MacColl solved are:

> A point is taken at random in a window consisting of nine equal square panes, and through this point a line is drawn in a random direction; find the respective chances of the line so drawn cutting one, two, three, four, or five panes. [Blackwood 1871a]
>
> A point is taken at random in a given quadrant of a circle, and a random line is drawn through it. Find the chance of its cutting the arc. [Blackwood 1871b]
>
> If A, B, C, D are random points in the perimeter of a square; find the chance that the straight lines AB, CD will intersect within the square. [Blackwood 1877]

Blackwood's origin, educational background, and profession remain a mystery. She contributed to *ET* from London from 1872 to 1874, from New York from 1875 to 1876, and from Boulogne-sur-Mer from 1877 to 1898. Her name does not appear in Boulogne city directories. She may have been using her maiden name when contributing to *ET*. There were other contributors from Boulogne; Miss Myra Greaves sent in solutions to problems from there from 1871 to 1877, and Mr. J. J. Sides, a teacher, contributed from 1871 to 1879.

In the summer of 1900, MacColl, renewing his interest in the subject, solved a logic problem that had been proposed by Charles L. Dodgson (Lewis Carroll). Namely,

> It is given that (1) if C is true, then, if A is true, B is not true; and (2) if A is true, B is true. Can C be true? What difference in meaning, if any exists between the following propositions? — (1) A, B, C cannot all be true at once; (2) if C and A are true, B is not true; (3) if C is true, then if A is true, B is not true. [Dodgson 1899]

An analysis problem that MacColl proposed and solved went as follows:

> Let $f(x)$ be any function of x, and let $f(a,b)$ express (according as a is greater or less than b) a superior or inferior limit to $f(x)$ for all values of x between a and b; show that if $f(a,b)$ has the same sign as $b-a$, no real root of the equation $f(x) = 0$ exists between a and b. [MacColl 1865c]

MacColl's proof is straightforward. He assumes the hypotheses and supposes that $f(a,b)$ and $b-a$ are positive; then since $a < b$, $f(a,b)$ is an inferior limit of $f(x)$ for all values of x between a and b. Thus, when x is thus restricted $f(x) > f(a,b) > 0$. Let $f(a,b)$ and $b-a$ be negative. Since $b < a$, $f(a,b)$ is a superior limit of $f(x)$ for all values of x between a and b. Thus, when x is thus restricted $f(x) < f(a,b) < 0$. Hence, the result is established.

An elegant Euclidean geometry problem that MacColl proposed in *ET* for a secondary school geometry class is as follows:

> Show that in any triangle the bisector of the less[er] of two angles is greater that the bisector of the greater. [MacColl 1898]

The next examples show that MacColl was not averse to the use of poetry to pose problems. The first describes a 3-dimensional geometric problem:

> Some shapeless solids lie about,
> No matter where they be;
> Within such solid or without,
> Let's take a centre C.
> From Centre C, in countless hosts,
> Let random radii run,
> And meet a surface, each at P,
> Or, maybe, meet with none.
> Those shapeless solids, far and near,
> Their total prove to be
> The average volume of the sphere
> Whose radius is CP.
> The sphere, beware, is positive,
> When *out* at P they fly;
> But, changing sign, 'tis negative,
> If *entrance* there you spy.
> One caution more, and I have done,
> The sphere is nought, if P there's none.
> [MacColl 1884]

The problem being: From the center C of a shapeless solid, straight lines are drawn to the surface meeting it at P, the number of such rays increasing without limit. If we consider CP as the radius of a sphere, the problem, which went unsolved, was to determine the average volume of all the spheres. In another, he posed an intriguing applied mathematics problem.

To put a question somewhat queer,
Suppose the earth an *airless sphere,*
Through which is bored, from pole to pole,
A bottomless cylindrical hole;
Right o'er the centre let there fall
A smooth and polished marble ball.
Dear Reader, may I ask you *when*
That marble would come back again?
[MacColl 1889]

A solution to the second problem was submitted by Daniel Biddle, a London physician, Fellow of the Royal Statistical Society, who in 1898 replaced Miller as editor of the *ET* mathematical department.

Conclusion

MacColl subscribed to *MQ* from 1865 until his death in 1909. He submitted problems to the editor of the *ET* mathematical department on an irregular basis. He contributed a set of eight analysis and algebraic problems that, beginning in the spring of 1865, appeared semi-regularly over the following eighteen months. He submitted sixteen problems that dealt mainly with geometric probability, geometric average, and probability problems that, beginning in the fall of 1870, appeared monthly over the next two and a half years. A smaller set of similar problems appeared in the spring of 1877, the fall of 1877, the fall of 1884, and the spring of 1890. A set of thirty-eight geometric probability, logic, and regular probability problems that MacColl submitted began appearing monthly in June of 1896. The last set of ten problems dealt mainly with logic and appeared from March 1902 to January 1910.

Before 1870, his solutions were mainly to analytic problems, from 1870 to 1872 to geometric probability problems, and after 1900 to logical problems. Little or no contributions from MacColl were published from 1866 to 1870, 1872 to 1876, 1886 to 1889, and 1891 to 1896. Adjusting to life in Boulogne and his responsibilities at the *Collège communal* may have been the cause of the first hiatus. From 1872 to 1876, he was concentrating on his studies for the University of London external examination for a BA degree in mathematics, which he completed in 1876. The third break could very well have been caused by the death of his first wife and his marriage three years later to Hortense Lina Marchal in 1887. Finally, the fourth break occurred during the period when MacColl was more interested in literature than mathematics and logic.

The mathematical proficiency, logical innovations, and number of contributions exhibited by MacColl in *ET* were impressive. From his intricate solutions to problems in geometric probability, inverse probability, analysis, and logic, we see that he was more than just an average mathematician and logician. While his problems and articles were clever, well-thought out, and often innovative, they were not of the caliber of Sylvester, 2nd Wrangler on the 1837 Cambridge mathematical tripos, or Crofton who had taken the highest mathematical honors when graduating from Trinity College, Dublin in 1848. MacColl had remarkable mathematical potential, and if fate had been kinder and offered him the opportunity of a normal university education, he may well have become one of the leading mathematicians of the late nineteenth century.

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