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Disturbance Observer Based on Biologically Inspired Integral Sliding Mode Control for Trajectory Tracking of Mobile Robots

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ABSTRACT This paper proposes an integral sliding control system based on the nonlinear disturbance observer, aiming to the trajectory tracking of the mobile robot under the external disturbance. First, a kinematic model of mobile robot was built, besides, the position error signal was gained by the biological membrane potential model, and the problem of velocity oscillation was solved by the design of the backstepping controller. Then, an integral sliding control system was designed in accordance with the kinematic model of the mobile robot, meanwhile, a disturbance observer was designed in consideration of external disturbance to do the real-time observation on the disturbance occurring in the system with an addition of feedforward compensation and the observation error was converged by selecting the design parameters. The Lyapunov function was used to prove the stability of the system. Finally, the simulation of tracking circularity trajectory was utilized, with the comparison of trajectory without the use of jammer, to prove that this method can well overcome the nonlinear and uncertainty originated from external, thereby improving the control performance and increasing the robustness.

INDEX TERMS Mobile robots, trajectory tracking, disturbance observer, integral sliding mode variable structure, back-stepping control.

I. INTRODUCTION

In recent years, mobile robots have been widely used in military, industrial, and transportation. Thus, mobile robots have always been an important research object for researchers [1]–[4]. In the study of mobile robot control, trajectory tracking control is one of the most important aspects, high-precision trajectory tracking is the premise of mobile robot to complete the work. However, the non-linear of the mobile robots itself and the disturbance of the external environment make a great difficulty on the mobile robots trajectory tracking control. Based on the research of this problem, sliding mode variable structure control [5], neural network control [6], [7], adaptive control [8]–[11] and other methods have been applied to the trajectory tracking control of mobile robots. However, trajectory tracking control with the inflection point, the velocity jumps, and the good robustness of the desired trajectory remain unsolved.

At present, some achievements have been addressed in the motion control of mobile robots [12]–[14]. Back-stepping control [15],with the characteristics of simple design and easy implementation, is largely used in trajectory tracking of mobile robots and can be combined with other methods in many fields [16]. In [17], a globally asymptotically stable tracking controller is designed by using integral back-stepping, time-varying and an introduction of virtual feedback parameters. In [18], a proportional-integral-derivative based membrane controller is introduced to design the dynamic controller of wheeled mobile robots to make the actual velocity follow the desired velocity command. A globally stable system can be obtained via the methods mentioned above. However, the back-stepping is still inadequate, if a

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large inflection point or initial tracking error occurs on the desired trajectory, which may leading to an oscillation of velocity. In order to achieve the effect of track, a large driving torque support is required, which is more difficult to achieve in practice tracking. Therefore, the problem of unrealistic velocity oscillation in the controller is directly addressed via the error dynamics and analysis of controller. A bio-inspired variable structure suggests in [19] to improve the velocity control signal, ameliorating trajectory tracking. But in practical, it is not enough to only consider the kinematics model. The kinematics model can not guarantee the tracking of velocity signal, and the velocity error can directly affects the state error, affecting the result of trajectory tracking. Owing to this, a consideration of dynamics model is necessary, and so do the design of dynamic controller, with which the error between the tracking velocity and the actual velocity could be converged to zero. Hence, the effect of trajectory tracking can be achieved.

The sliding mode control includes good robustness, which is widely applied in robot dynamics control [20]–[22]. A lot of sliding mode control approaches have been proposed in recent years, such as terminal sliding model [23], global sliding model [24], neural sliding model [25]. A steady state error may occurs if a certain external disturbance in trajectory, ordinary sliding mode variable structure happens, leading to a condition that the required performance or trajectory tracking can not be reached in the system. And the designed trajectory has no robustness at the time interval before the sliding mode. In order to solve this problem, an integral variable structure sliding mode control is proposed [26], [27]. The advantages of this method include fast response, superior transient performance and robustness with regard to parameter variations, solving the tracking problem to a certain degree.In the practical tracking system, however, there will be much uncertainty factors or non-linearity, which may affect the performance of indicators and even lead the system unable to reach the steady state. In order to estimate the unknown disturbance, reduce the influence of external disturbance and improve the control precision and robustness, this paper designs a non-linear disturbance observer, which can estimate disturbance, in basis of feed-forward compensation with the characteristics of simple design and without disturbance model, so that there is no need to use additional sensors.

The rest part of this paper is organized as follows. Section II, the preliminaries of the robot model and the establishment of equations are illustrated. The system controller for trajectory tracking of mobile robot, and disturbance observer are introduced in Section III. The stability of the dynamics is proved by Lyapunov functions in section IV. Section V shows the simulation results and analysis of this paper. Finally, some concluding remarks are given in Section VI.

II. ROBOT MODEL AND FORMULATION

This paper studies on a two-wheeled mobile robot, as shown in Fig.1. M is the mass center of robot, where x, y are

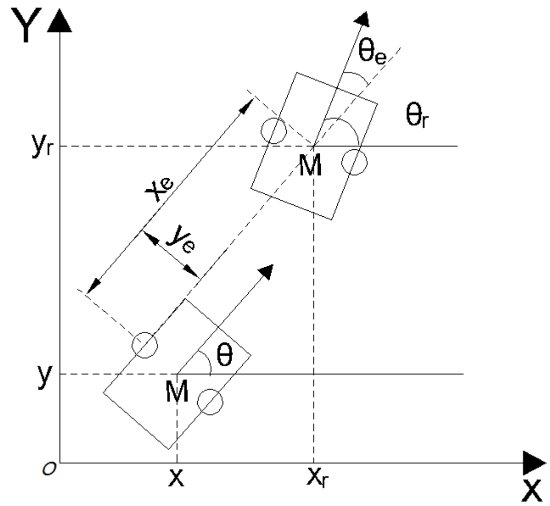

FIGURE 1. A nonholonomic two-wheeled mobile robot.

the coordinates of the position, θ is the heading angle, and the position coordinates in the $x-o-y$ coordinate system are expressed as: $q = (x, y, \theta)^T$. Assuming that the mobile base satisfies the conditions of pure rolling and non-slipping, $u = (v, w)^T$ are the controlling inputs of the mobile robot under rolling conditions without slipping. The control variables of mobile robot are the linear velocity v and the angular velocity w. Where $w = \dot{\theta}$. The kinematic equation of two-wheeled mobile robot can be expressed as:

$$\dot{q} = A(q)u. \tag{1}$$

where

$$A(q) = \begin{bmatrix} A_1(q) & A_2(q) \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}.$$

The simplified dynamic model of the mobile robot is:

$$M\dot{u} = \tau + d \tag{2}$$

where $M = diag\,(m, I)$. m and I denote the mass and inertia of robot respectively, $\tau = (\tau_1, \tau_2)^T$ is the control torque of the mobile robot, $d = (d_1, d_2)^T$ is the uncertain external disturbance term.

The mobile robot moves from posture $q = (x, y, \theta)^T$ to posture $q_r = (x_r, y_r, \theta_r)^T$, the tracking error for Kinematics formation is obtained as:

$$q_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{3}$$

By a direct calculation, (3) yields:

$$\dot{x}_e = wy_e - v + v_r \cos\theta_e \tag{4}$$

$$\dot{y}_e = -wx_e + v_r sin\theta_e \tag{5}$$

$$\dot{\theta}_e = w_r - w \tag{6}$$

Assumption (i): The unknown parameters of the mobile robot are in known compact sets.

Assumption (ii): The disturbances under consideration are supposed to be harmonic with known frequencies but unknown amplitudes.

Lemma 1 (Barbalat) [28]: If the differentiable function $f(t)$ has a finite limit as $t \to \infty$, meanwhile $\dot{f}(t)$ is uniformly continuous, then $f(t) \to 0$ as $t \to \infty$.

III. SYSTEM CONTROLLER AND DISTURBANCE OBSERVER DESIGN

A. BACK-STEPPING MOTION CONTROLLER

This paper designs the following control inputs to achieve trajectory tracking (in many literature, for example [29] design to achieve trajectory tracking):

$$u_c = \begin{bmatrix} v_c \\ w_c \end{bmatrix} = \begin{bmatrix} k_1 x_e + v_r cos\theta_e \\ w_r + k_2 y_e v_r + k_3 v_r sin\theta_e \end{bmatrix} \quad (7)$$

where k_1, k_2, k_3 are pre-defined positive constants, from (7), we can see that the shortcoming is quite obvious, the virtual velocity control law is directly related to the state errors, so large velocities can be generated under large initial error conditions, and it will suffer from unrealistic velocity oscillation when the tracking errors suddenly change. This shows that the initial linear acceleration and angular acceleration are very large, that is to say, the force and torque of the follower are very large, which does not hold in practice. To resolve the velocity oscillation problem, a novel tracking controller has been proposed in this section by incorporating the bioinspired model with the back-stepping technique.

A typical biological neural model is the shunting model [30], and can be used to solve the problem of sudden velocity oscillation. The Shunting Neural Dynamic Model is described as:

$$\dot{\tilde{V}} = -A\tilde{V} + (B - \tilde{V})s^{+}(t) - (D + \tilde{V})s^{-}(t) \quad (8)$$

where $\tilde{V}$ is the membrane voltage in the neural network, the parameters A, B, and D are nonnegative constants representing the passive decay rate, the upper and lower bounds of the neural activity respectively, and the variables $s^{+}(t)$ and $s^{-}(t)$ are the excitatory and inhibitory inputs to the neuron.

By analyzing the back-stepping technology based on the tracking controller, it can be well known that the oscillation is indeed caused by the sudden changes in the tracking error. Inspired by the smooth neural dynamics of the shunting neural model, a biological tracking controller is proposed to solve the velocity oscillation problem. A biological neural model equation with the error in the longitudinal direction is obtained as:

$$\dot{V}_x = -AV_x + (B - V_x) f(x_e) - (D + V_x) g(x_e) \quad (9)$$

where A, B, D are the positive constants and setting the value of D and B be equal to each other, x_e is the biological model control input, V_x is the biological model control output voltage, so that $f(x_e) = max(x_e, 0)$ and $g(x_e) = max(-x_e, 0)$. According to [31], V_x is used to replace the tracking error x_e in the back-stepping model (7), so that a new back-stepping virtual control laws is obtain as:

$$u_c = \begin{bmatrix} v_c \\ w_c \end{bmatrix} = \begin{bmatrix} k_1 V_x + v_r cos\theta_e \\ w_r + k_2 y_e v_r + k_3 v_r sin\theta_e \end{bmatrix} \quad (10)$$

To prove that the trajectory tracking control system under the controller laws (10) is asymptotically stable and the tracking errors converge to zeros, we choose the following Lyapunov function:

$$V_1 = \frac{1}{2}\left(x_e^2 + y_e^2\right) + \frac{1}{k_2}(1 - cos\theta_e) + \frac{k_1}{2B}V_x^2 \quad (11)$$

It is obvious that $V_1 \geq 0$, and $V_1 = 0$ if and only if $x_e = 0$, $y_e = 0$, $\theta_e = 0$.

The time derivative of V_1 is:

$$\dot{V}_1 = x_e\dot{x}_e + y_e\dot{y}_e + \frac{\dot{\theta}_e}{k_2}sin\theta_e + \frac{k_1}{B}V_x\dot{V}_x \quad (12)$$

Substituting (4), (5) (6) and (9) into (12), we have:

$$\begin{aligned} \dot{V}_1 &= x_e\dot{x}_e + y_e\dot{y}_e + \frac{\dot{\theta}_e}{k_2}\sin\theta_e + \frac{k_1}{B}V_x\dot{V}_x \\ &= x_e(-v + v_r\cos\theta_e) + \frac{\sin\theta_e}{k_2}(w_r - w + k_2 v_r y_e) \\ &\quad + \frac{k_1}{B}\left[-A - f(x_e) - g(x_e)\right]V_x^2 + k_1\left[f(x_e) - g(x_e)\right]V_x \end{aligned} \quad (13)$$

The tracking error x_e has two properties:

1) $x_e \leq 0$, under the parameter definition of (9), we can get

$$\begin{cases} f(x_e) = 0, \quad g(x_e) = -x_e \\ A + f(x_e) + g(x_e) > 0 \\ -f(x_e) + g(x_e) + x_e = 0 \end{cases} \quad (14)$$

2) $x_e > 0$, under the parameter definition of (9), we can get

$$\begin{cases} f(x_e) = x_e, \quad g(x_e) = 0 \\ A + f(x_e) + g(x_e) > 0 \\ -f(x_e) + g(x_e) + x_e = 0 \end{cases} \quad (15)$$

From the discussion above, we can get the following conclusion:

$$\begin{cases} A + f(x_e) + g(x_e) > 0 \\ -f(x_e) + g(x_e) + x_e = 0 \end{cases} \quad (16)$$

Substituting (10) into (13), and under analysis of the (8), (9), (13) yields:

$$\begin{aligned} \dot{V}_1 &= x_e\left(-k_1V_x - v_r cos\theta_e + v_r cos\theta_e\right) \\ &\quad \times \frac{sin\theta_e}{k_2}\left(w_r - w_r - k_2 y_e v_r - k_3 v_r sin\theta_e + k_2 v_r y_e\right) \\ &\quad + \frac{k_1}{B}\left[-A - f(x_e) - g(x_e)\right]V_x^2 \\ &\quad + k_1\left[f(x_e) - g(x_e)\right]V_x \\ &= -\frac{k_1}{B}\left[A + f(x_e) + g(x_e)\right]V_x^2 - \frac{\sin^2\theta_e}{k_2}k_3v_r \\ &\quad - k_1\left[-f(x_e) + g(x_e) + x_e\right]V_x \\ &= -\frac{k_1}{B}\left[A + f(x_e) + g(x_e)\right]V_x^2 - \frac{\sin^2\theta_e}{k_2}k_3v_r \leq 0 \end{aligned} \quad (17)$$

Thus, it can be proved that all the parameters of V_1 are bounded, which implies that $\dot{V}_1$ is uniformly continuous with a stable biological tracking controller. Furthermore, the Barbalat's Lemma 1 permits us to conclude $\lim_{t\to\infty} \dot{V}_1 = 0$, then $\lim_{t\to\infty} x_e = 0$, $\lim_{t\to\infty} y_e = 0$, $\lim_{t\to\infty} \theta_e = 0$, $\lim_{t\to\infty} V_x = 0$, so it can be concluded that the closed-loop system is stable.

B. SLIDING MODE SYSTEM DYNAMIC CONTROLLER DESIGN

Firstly, we consider the dynamic model (without disturbance d), from (2), we can get:

$$\dot{\tilde{u}} = M^{-1}(\tau - \lambda X) \tag{18}$$

where $\tilde{u} = u - u_c$ are the errors of tracking velocity, $u = (v, w)^T$ are the controlling inputs of the mobile robot under rolling conditions without slipping, the control variables of mobile robot are the linear velocity v and the angular velocity w. $u_c = (v_c, w_c)^T$ are the tracking velocity. $\lambda = diag[\dot{v}_c, \dot{w}_c]$ is the derivative of tracking velocity, $X = [m, I]^T$, m and I present the mass and inertia of the robot respectively, and $\lambda X = M\dot{u}_c$.

Based on (18), parameter estimators are designed with a projection operation. Define $\psi(\tilde{u}) = -\Upsilon\lambda^T\tilde{u}$. According to Assumption(i), there are X_{imax} and X_{imin} such that $0 < X_{imin} < X_i < X_{imax}$; let $\overline{X}_{imin} = \frac{2-\sqrt{2}}{2} X_{imin}$, $\overline{X}_{imax} = \frac{2-\sqrt{2}}{2} X_{imax}$. Adaptive law for $\hat{X}_i$, $i = 1, 2$, are chosen the following:

$$\dot{\hat{X}}_i = \begin{cases} \psi_i(\tilde{u}), \quad if\ X_{i\min} < \hat{X}_i < X_{i\max} \\ \psi_i(\tilde{u}) + (1 - \frac{\hat{X}_i}{X_{i\min}})^2 [1 + \psi_i{}^2(\tilde{u})], \\ \qquad if\ \hat{X}_i \le X_{i\min} \\ \psi_i(\tilde{u}) - (1 - \frac{\hat{X}_i}{X_{i\max}})^2 [1 + \psi_i{}^2(\tilde{u})], \\ \qquad if\ \hat{X}_i \ge X_{i\max} \end{cases} \tag{19}$$

where $\hat{X}$ is the estimate of X. The result could be divided into the following cases shown below, that is:
1) $\hat{X}_i$ is continuous;
2) Estimate $\hat{X}_i$ satisfies $\overline{X}_{imin} < \hat{X}_i < \overline{X}_{imax}$ for $t \ge 0$, if $\overline{X}_{imin} < \hat{X}_i(0) < \overline{X}_{imax}$
3)$-\tilde{X}^T(\Upsilon^{-1}\dot{\hat{X}} + \lambda^T\tilde{u}) \le 0$ for, $t \ge 0$, the detailed proof could see in [32].

From above, we can deduce the control law τ as:

$$\tau = M\dot{\tilde{u}} + \lambda\hat{X} \tag{20}$$

where $\Upsilon = \vartheta E$, ϑ is positive constant, $E = diag[1, 1]$ is a R^2 matrix.

Considering that the dynamic model function (18) is the first order non-linear function, the sliding model system dynamic controller was selected, and the tracking errors shall be designed as $e_u = (e_v, e_w)^T$, where $e_u = -\tilde{u} = u_c - u$, $e_v = v_c - v$, $e_w = w_c - w$.

If we use the conventional sliding surfaces uncoupled state variables to control inputs, a sliding surface controlled could be given as: $s(t) = e_u$, which would bring steady-state errors when there is disturbance acting on the system, in order to reduce the steady state errors, we add the integral term of variable upper limit $\mu \int_0^t e_u dt$, to form following sliding function:

$$s(t) = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = e_u + \mu \int_0^t e_u dt \tag{21}$$

where $\mu = diag(\mu_1, \mu_2)$ is integral slip coefficient, and μ_1, μ_2 are both positive constants. When the system state is in sliding mode $s(t) = \dot{s}(t) = 0$, so that:

$$\begin{cases} e_u + \mu \int_0^t e_u dt = 0 \\ \dot{e}_u + \mu e_u = 0 \end{cases} \tag{22}$$

By solving equations (22), it can be obtained that:

$$\begin{aligned} e_v &= e_0 e^{-\mu_1 t} \\ e_w &= e_1 e^{-\mu_2 t} \end{aligned} \tag{23}$$

where e_0, e_1 are both positive constants. According to the Function (23), we can then find that e_w, e_v will finally converge to zero, which means that the stability of model can be guaranteed.

According to the above definition $e_u = -\tilde{u} = u_c - u$, substituting it to (22), we can get the new Equation:

$$\dot{\tilde{u}} + \mu\tilde{u} = 0 \tag{24}$$

Multiplied by M on the left side of (24), one has

$$M\dot{\tilde{u}} + \mu M\tilde{u} = 0 \tag{25}$$

By substituting (20) to (25), the control law τ can be rewritten as:

$$\tau = \lambda\hat{X} - M\mu\tilde{u} \tag{26}$$

Here, we choose the Lyapunov function as follows:

$$V_2 = V_1 + \frac{1}{2}\tilde{X}^T\Upsilon^{-1}\tilde{X} \tag{27}$$

where $\tilde{X} = X - \hat{X}$. Therefore, the derivative of V_2 is:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \tilde{X}^T\Upsilon^{-1}\dot{\tilde{X}} \\ &= -\frac{k_1}{B}\left[A + f(x_e) + g(x_e)\right]V_x^2 - \frac{\sin^2\theta_e}{k_2}k_3 v_r \\ &\quad - k_1\left[-f(x_e) + g(x_e) + x_e\right]V_x \\ &\quad - \tilde{X}^T(\Upsilon^{-1}\dot{\hat{X}} + \lambda^T\tilde{u}) \le 0 \end{aligned} \tag{28}$$

The control of law τ can keep the system on the sliding surface, however, the influence of external uncertainty should be considered, due to this, the symbolic function $sgn(s)$ shall then be introduced. Then control law τ can be rewritten as:

$$\tau = \lambda\hat{X} - M\left[\mu\tilde{u} + \alpha sgn(s)\right] \tag{29}$$

where $\alpha = diag[\alpha_1, \alpha_2]$ is the coefficient of symbolic function and α_1, α_2 are both positive constants. $sgn(s)$, a switch function, has been defined as follows:

$$sgn(s) = \begin{cases} -1, & s < 0 \\ 0, & s = 0 \\ 1, & s > 0 \end{cases}$$

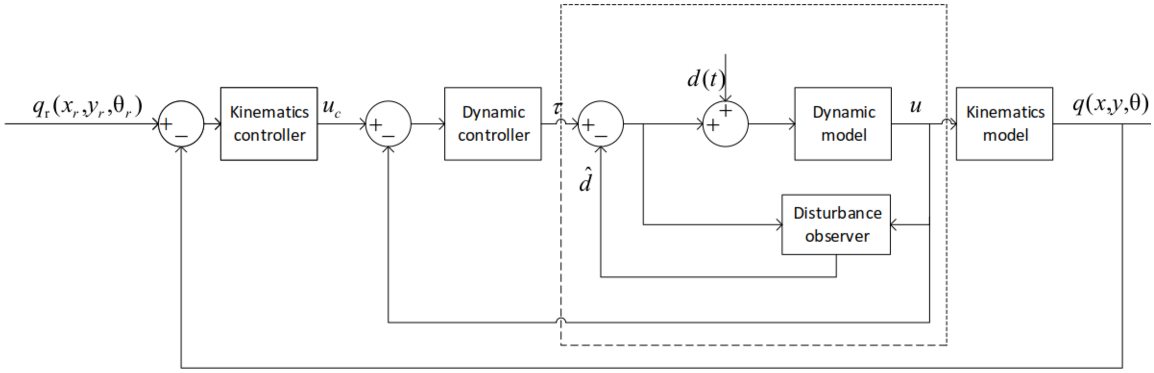

FIGURE 2. The control block diagram of closed-loop system based on disturbance observer.

Since the $sgn(s)$ function is intermittent at $s = 0$, it will lead the system to switch frequently and rise the fighting. In order to reduce the impact of chattering on system trajectory tracking, we divide it into the following two stages of the trajectory tracking.

1) The first phase, choose the exponential function approach law:

$$\dot{s}_1 = -\epsilon_1 sgn(s_1) - \gamma_1 s_1, \quad e_v > k_{11} \tag{30}$$

$$\dot{s}_2 = -\epsilon_2 sgn(s_2) - \gamma_2 s_2, \quad e_w > k_{22} \tag{31}$$

where k_{11}, k_{22} are the coefficients of approach law selections, and they are both positive constants. Then control law τ is rewritten as:

$$\tau = \lambda\hat{X} - M\left[\mu\tilde{u} + \epsilon sgn(s) + \gamma s\right] \tag{32}$$

where $\epsilon = diag(\epsilon_1, \epsilon_2)$, $\gamma = diag(\gamma_1, \gamma_2) \in R^2$are the coefficients of approaching law, and $\epsilon_1, \epsilon_2, \gamma_1, \gamma_2$ are both positive constants.

2) The second phase, we choose the variable velocity one and has been shown below:

$$\dot{s}_1 = -\varphi_1\rho_1 |e_v| sat(s_1), \quad e_v \le k_{11} \tag{33}$$

$$\dot{s}_2 = -\varphi_2\rho_2 |e_w| sat(s_2), \quad e_w \le k_{22} \tag{34}$$

where $\rho = diag(\rho_1, \rho_2)$, $\varphi = diag(\varphi_1, \varphi_2) \in R^2$, and $\rho_1, \rho_2, \varphi_1, \varphi_2$ are all pre-defined positive constants,

$$sat(s) = \begin{cases} sgn(s), & s > \varphi \\ s/\varphi, & s \le \varphi \end{cases} \tag{35}$$

(35) is a saturation function, under which, control law τ is rewritten as:

$$\tau = \lambda\hat{X} - M\left[\mu\tilde{u} + \varphi\rho |e_u| sat(s)\right] \tag{36}$$

C. DISTURBANCE OBSERVER DESIGN

In the actual trajectory tracking, owing to some uncertainty and external disturbances, the steady-state error will make the system performance index to decrease. To tackle this issue, this paper designs the following disturbance observer to estimate the the external uncertainty and non-linear disturbances [33], and adds feed-forward compensation to achieve the elimination of the disturbance. The error could become smaller by adjusting the gain to obtain a high-precision estimation value and a condition of perturbation, leading to a more realistic tracking trajectory, and definitely a good robustness could be obtained. The control block diagram of the closed-loop system base on disturbance observer is shown in Fig.2.

The amplitude of disturbances are not necessarily required for the observer design and analysis. However, in order to get an accurate estimation, the frequencies of harmonic disturbances have to be known, but amplitude and phase should be unknown rather than keep constant. It is supposed that the disturbances are generated by the following exogenous system.

$$\begin{cases} \dot{\kappa} = \Lambda\kappa \\ d = C\kappa \end{cases} \tag{37}$$

where $\kappa = [\kappa_1; \kappa_2]$, $\Lambda = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Basic harmonic non-linear disturbance observer can be written as follows:

$$\begin{cases} \dot{z} = \left(\Lambda - LM^{-1}C\right)z + \Lambda Lu - L(M^{-1}CLu + M^{-1}\tau) \\ \hat{\kappa} = z + Lu \\ \hat{d} = C\hat{\kappa} \end{cases} \tag{38}$$

where z is the internal state of the non-linear observer, $L = diag(L_1, L_2) \in R^2$ are the non-linear disturbance observer gain, $\hat{d}$ is the estimate of the actual disturbance d, and the estimate error is defined as:

$$e_\kappa = \hat{\kappa} - \kappa \tag{39}$$

$$e_d = \hat{d} - d = C(\hat{\kappa} - \kappa) \tag{40}$$

the derivative of e_κ is:

$$\begin{aligned} \dot{e}_\kappa &= \dot{\hat{\kappa}} - \dot{\kappa} \\ &= z + L\dot{u} - \Lambda\kappa \\ &= \left(\Lambda - LM^{-1}C\right)z + \Lambda Lu - \Lambda\kappa \\ &\quad - L\left(M^{-1}Lu + M^{-1}\tau\right) + L\left(M^{-1}\tau + M^{-1}d\right) \end{aligned}$$

$$
\begin{aligned}
&= \left(\Lambda - LM^{-1}C\right) z + \Lambda L u - \Lambda \kappa \\
&\quad - L\left(M^{-1}Lu + M^{-1}\tau\right) + L\left(M^{-1}\tau + M^{-1}C\kappa\right) \\
&= \left(\Lambda - LM^{-1}C\right)(\hat{\kappa} - \kappa) \\
&= \left(\Lambda - LM^{-1}C\right) e_\kappa \qquad (41)
\end{aligned}
$$

Suppose that the relative degree from the disturbance to the output. This implies that $LM^{-1}C \neq 0$ for v, w. Without loss of generality, suppose that $LM^{-1}C > 0$, in the following analysis, which implies that $LM^{-1}C$ can be divided as:

$$LM^{-1}C = K\left[\alpha_0 + \alpha_1(u)\right] \qquad (42)$$

So (51) can be written as:

$$\dot{e}_\kappa = [\Lambda - K[\alpha_0 + \alpha_1(u)]]e_\kappa \qquad (43)$$

$$LM^{-1}C = \alpha_0 + \alpha_1(u) \qquad (44)$$

The estimation $\hat{d}$ yielded by harmonic non-linear disturbance observer (38) converges to the disturbance d globally exponentially if there exists a gain K such that the transfer function:

$$H(s) = C(SI - \bar{\Lambda})^{-1}K \qquad (45)$$

is asymptotically stable and strictly positive real where $\bar{\Lambda}=\Lambda - K\alpha_0 C$.

Considering that the influence of non-linear disturbance d exists in system, the control law (32), (36) could be rewritten:

1) $\tau = \lambda\hat{X} - M\left[\mu\tilde{u} + \epsilon sgn(s) + \gamma s\right] - \hat{d}$

2) $\tau = \lambda\hat{X} - M\left[\mu\tilde{u} + \varphi\rho\left|e_u\right| sat(s)\right] - \hat{d}$

IV. DYNAMICS STABILITY ANALYSIS

Choose the following Lyapunov function:

$$V_3 = \frac{1}{2}s^T s \qquad (46)$$

The derivative of V_3 is:

$$\dot{V}_3 = s^T \dot{s} \qquad (47)$$

1) The first phase, substituting (30) and (31) into (47), it yields:

$$
\begin{aligned}
\dot{V}_3 &= s^T(-\epsilon sgn(s) - \gamma s) \\
&= (s_1, s_2)\begin{pmatrix} -\epsilon_1 sgn(s_1) - \gamma_1 s_1 \\ -\epsilon_2 sgn(s_2) - \gamma_2 s_2 \end{pmatrix} \\
&= -\epsilon_1 s_1 sgn(s_1) - \gamma_1 s_1^2 - \epsilon_2 s_2 sgn(s_1) - \gamma_2 s_2^2 \\
&= -\gamma_1 s_1^2 - \gamma_2 s_2^2 - \epsilon_1\left|s_1\right| - \epsilon_2\left|s_2\right| \leq 0 \qquad (48)
\end{aligned}
$$

It is easy to know $\dot{V}_3 \leq 0$. Thus, it can be proved that all the parameters of V_3 are bounded.

2) The second phase substituting (33) and (34) into (47), it yields:

(i) for $s > \varphi$:

$$
\begin{aligned}
\dot{V}_3 &= s^T \dot{s} \\
&= s^T(-\varphi\rho\left|e_u\right| sat(s)) \\
&= (s_1, s_2)\begin{pmatrix} -\varphi_1\rho_1\left|e_v\right| sat(s_1) \\ -\varphi_2\rho_2\left|e_w\right| sat(s_2) \end{pmatrix} \\
&= -\rho_1\left|e_v\right|\left|s_1\right| - \rho_2\left|e_w\right|\left|s_2\right| \leq 0 \qquad (49)
\end{aligned}
$$

(ii) for $s \leq \varphi$:

$$
\begin{aligned}
\dot{V}_3 &= s^T \dot{s} \\
&= s^T(-\varphi\rho\left|e_u\right| sat(s)) \\
&= (s_1, s_2)\begin{pmatrix} -\varphi_1\rho_1\left|e_v\right|\dfrac{s_1}{\varphi_1} \\ -\varphi_2\rho_2\left|e_w\right|\dfrac{s_2}{\varphi_2} \end{pmatrix} \\
&= -\rho_1\left|e_v\right|\frac{s_1^2}{\varphi_1} - \rho_2\left|e_w\right|\frac{s_2^2}{\varphi_2} \leq 0 \qquad (50)
\end{aligned}
$$

Thus, it can be proved that all the parameters of V_3 are bounded, which implies that $\dot{V}_3$ is uniformly continuous. Therefore, the Barbalat's Lemma 1 permits us to conclude $\lim_{t\to\infty}\dot{V}_3 = 0$, then $\lim_{t\to\infty}\dot{s} = 0$, $\lim_{t\to\infty}s = 0$. Hence, the system states will converge to the sliding surface $s = 0$ asymptotically, and the tracking errors will asymptotically reach to zero.

Next, we demonstrate that the estimated disturbance converge to the actual one in an asymptotical way. According to strictly positive real Lemma, transfer function (42) being stable and positive real implies that there exists a positive definite matrix P such that

$$\bar{\Lambda}P + P\bar{\Lambda} < 0 \qquad (51)$$

and $PK = C^T$.

Based on the analysis above, a lyapunov candidate function for observation error dynamics is defined in following way:

$$V_4 = e_\kappa^T P e_\kappa \qquad (52)$$

The derivative of V_4 is:

$$
\begin{aligned}
\dot{V}_4 &= 2e_\kappa^T P\dot{e}_\kappa \\
&= 2e_\kappa^T P\left[\Lambda - K\left[\alpha_0 + \alpha_1(u)\right]\right] e_\kappa \\
&= e_\kappa^T(\bar{\Lambda}^T P + P\bar{\Lambda})e_\kappa - 2e_\kappa^T PKCe_\kappa\alpha_1(u) \\
&< -\varpi e_\kappa^T e_\kappa - 2e_\kappa^T C^T Ce_\kappa\alpha_1(u) \qquad (53)
\end{aligned}
$$

where ϖ is a small positive scalar depending on (51). Since the relative degree from disturbance to output is uniformly well defined, it follows from (44) that $\alpha_1(u) > 0$ regardless of u. Noting that $e_\kappa^T C^T Ce_\kappa \geq 0$.

It is easy to know that $\forall\ \dot{V}_3 \leq 0$, $\forall\ \dot{V}_4 \leq 0$. Thus, it can be proved that all the parameters of V_3, V_4 are bounded. Thus, it can be concluded that the dynamic sliding surface control system is stable, the detailed proof could be seen in [33].

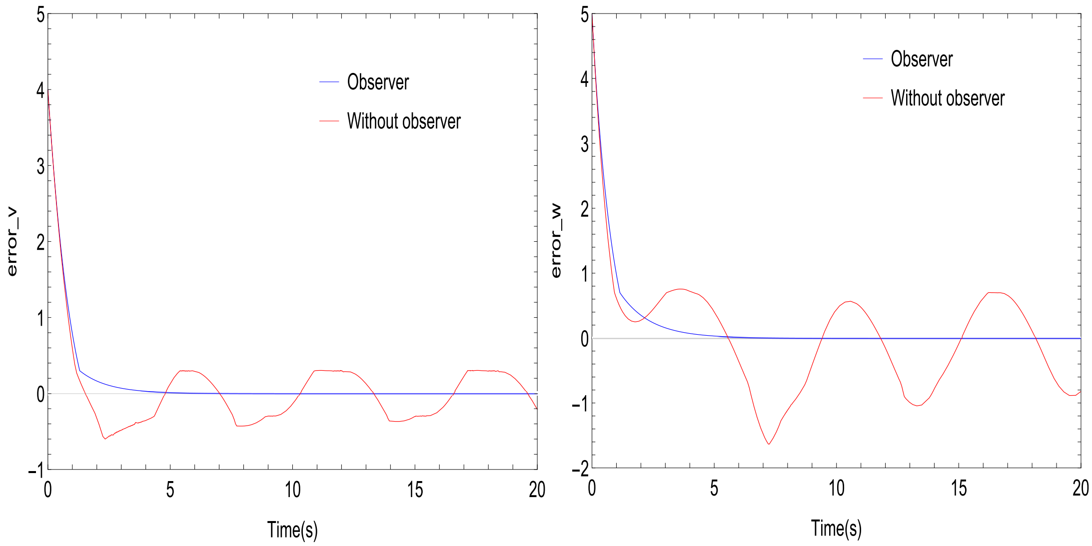

FIGURE 3. Errors of velocity tracking.

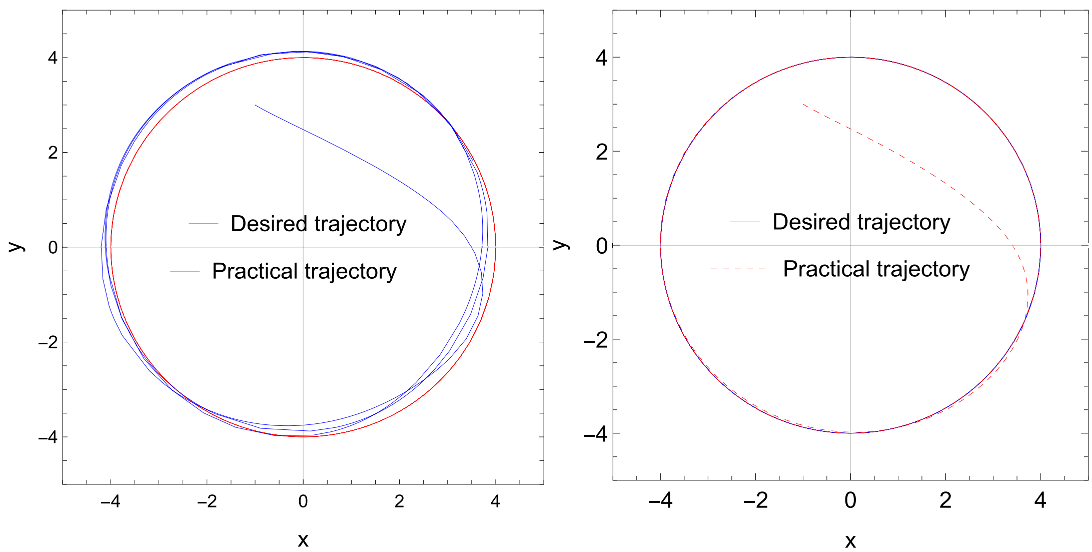

FIGURE 4. The comparison of trajectory tracking between the method using observer and non-observer's method.

V. SIMULATION AND ANALYSIS

In this section, a simulation is carried out on the trajectory-tracking of a circle to verify the effectiveness of the design. The mobile robot's parameters are given as: $m = 3\ kg$, $I = 2\ kg{\cdot}m^2$. The parameters of kinematics are chosen as: $k_1 = 3$, $k_2 = 1$, $k_3 = 2$. The parameters of biological heuristic model are chosen as: $A = 5$, $B = 3$. The parameters of adaptive model are given as: $\Upsilon = 0.5$, $\vartheta = 0.5$. The parameters of sliding mode are chosen as: $\mu_1 = 1$, $\mu_2 = 1$.

The parameters of dynamics are chosen as:

$$\varphi_1 = 0.8, \quad \varphi_2 = 0.6, \ \epsilon_1 = 0.8, \ \epsilon_2 = 0.8$$
$$\gamma_1 = 0.5, \quad \gamma_2 = 0.5, \ \rho_1 = 0.7, \ \rho_2 = 0.5$$

The parameters of approach law are chosen as: $k_{11} = 0.3$, $k_{22} = 0.7$. The parameters of observer are given as: $L_1 = 10$, $L_2 = 10$. The reference input errors of initial position and orientation are given as: $(x_e, y_e, \theta_e) = (1, 1, 0)$.

Else the disturbances are chosen as: $\kappa_1 = 2sint$, $\kappa_2 = 2cost$.

Then the expected trajectory of circle can be concluded as:

$$\begin{cases} x_r = 4cosw_rt \\ y_r = 4sinw_rt \\ \theta_r = w_rt \end{cases}$$

The expected values of the linear velocities are generated as $4m/s$, and the expected values of the angular velocities is $1rad/s$.

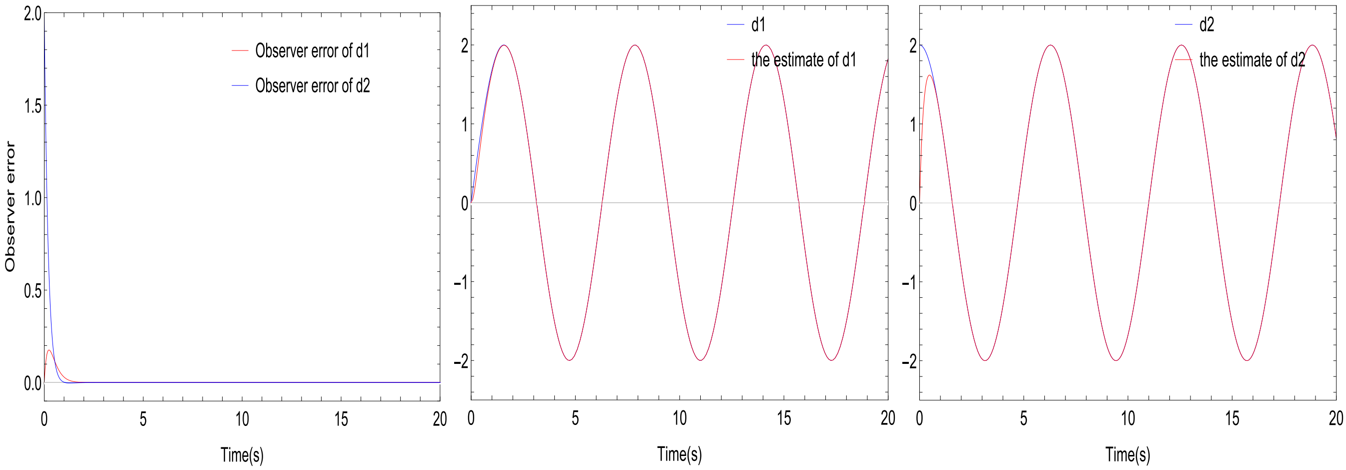

FIGURE 5. **Disturbance.**

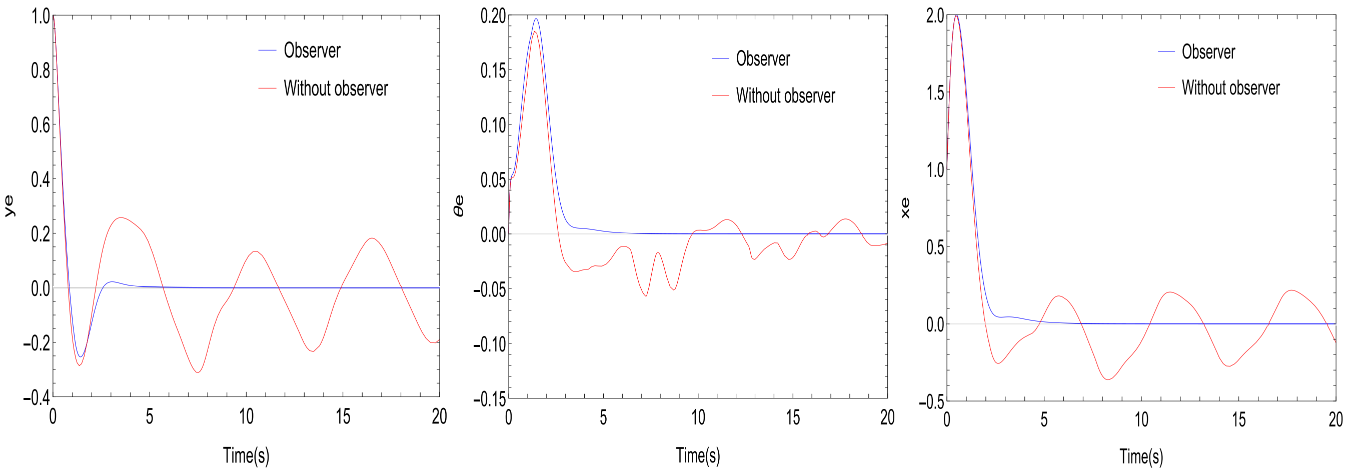

FIGURE 6. **Tracking errors of posture.**

Fig.3 shows the difference of the velocity tracking errors over time in the case of using observer or non-observer. We can clearly discover that the velocity errors fluctuate greatly without observer, as time goes by, the velocity errors always fluctuate around zero and does not eventually converge to zero, thus, an accurate tracking velocity is unable to gain. Then let's see the diagram of velocity change in the case of observer, after five seconds, the errors of velocity tracking starts to converges asymptotically to zero, the convergence process is smooth and there's no chattering, therefore the stability of sliding mode control is verified, and the error of tracking can be arbitrarily small by regulating L_1 and L_2.

In Fig.4, we can see that by employing the disturbance observer, the tracking result of circle is great and there is little error between the actual trajectory and the desired one, but a big error occurs in trajectory tracking without observer.

Fig.5 shows the estimation of disturbance and feed-forward compensation, it is obvious that the estimate errors of the disturbance reach zero asymptotically, which means that the external disturbances are compensated by using disturbance observer, therefore, the effectiveness of the disturbance observer is verified.

From Fig.6, we can clearly see that the errors of tracking posture are unable to converge to zero without disturbance observer. As shown in the pictures, all of (x_e, y_e, θ_e) converge to zeo. In consequence, the robustness of robot system could be guaranteed.

VI. CONCLUSION

In this paper, in order to investigate the problem of the velocity oscillation in back-stepping control, a biological membrane voltage model had been proposed, solving the problem of velocity oscillation effectively. Also an integral sliding mode variable structure torque controller is applied to reduce steady-state error of the system, the controller is designed to eliminate the matching disturbance and minimize the unmatched one. Finally, a non-linear disturbance observer with feed-forward compensation is designed to estimate the external disturbances, which improves the control precision and robustness of the system. The advantages of fast response and superior transient performance, could be seen in the designed control system.

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