

Cornell University School of Hotel Administration

The Scholarly Commons

Working Papers

School of Hotel Administration Collection

12-23-1997

Analysis of Senior-Subordinated Structures Backed by Private-Label Mortgages

Jarl Kallberg
New York University

Crocker H. Liu
Cornell University School of Hotel Administration, chl62@cornell.edu

A. R. Radhakrishnan
New York University

Follow this and additional works at: <https://scholarship.sha.cornell.edu/workingpapers>

 Part of the [Finance and Financial Management Commons](#), and the [Real Estate Commons](#)

Recommended Citation

Kallberg, J., Liu, C. H., & Radhakrishnan, A. R. (1997). *Analysis of senior-subordinated structures backed by private-label mortgages* [Electronic version]. Retrieved [insert date], from Cornell University, School of Hotel Administration site: <https://scholarship.sha.cornell.edu/workingpapers/41>

This Working Paper is brought to you for free and open access by the School of Hotel Administration Collection at The Scholarly Commons. It has been accepted for inclusion in Working Papers by an authorized administrator of The Scholarly Commons. For more information, please contact hotellibrary@cornell.edu.

If you have a disability and are having trouble accessing information on this website or need materials in an alternate format, contact web-accessibility@cornell.edu for assistance.

Analysis of Senior-Subordinated Structures Backed by Private-Label Mortgages

Abstract

This paper does a valuation analysis of senior-subordinated structure tranches backed by non-agency mortgages. The valuation is done using Monte Carlo simulation and employs the CIR interest rate process in conjunction with an empirical model estimated for non-agency mortgage prepayments and defaults. The sensitivity of the value of tranches to a number of variables are analyzed. We find that the interest rate process parameters significantly affect prepayments and defaults but not the relative value of the senior tranche. It is found that with the shifting of prepayments, the senior tranche does not dominate all the junior tranches at all interest rates. The shifting of prepayments has the unintended effect of providing stability to the junior tranches by making their cashflows less sensitive to prepayments. Our main conclusion is that while the shifting of prepayments increases protection from default to the senior tranche for a given level of subordination, it has the unwanted effect of lowering its value through increased contraction risk. This value loss should be taken into account in determining the optimum level of subordination.

Keywords

tranches, mortgages, prepayment, default rate, mortgage-backed securities

Disciplines

Finance and Financial Management | Real Estate

Comments

Required Publisher Statement

Copyright held by the authors.

Analysis of Senior-Subordinated Structures Backed by Private-Label Mortgages

Kose John, Crocker Liu, and A.R. Radhakrishnan*

Revision December 23, 1997

*Charles William Gerstenberg Professor of Banking and Finance, Associate Professor of Finance, and Doctoral Candidate in Finance respectively. All are from the Stern School of Business, New York University. We thank Michael Keenan, Pierluigi Balduzzi and Richard Levich for their comments. Address all correspondence to Crocker Liu, Department of Finance, Stern School of Business, New York University, MEC 9-190, 44 West 4th Street, New York, NY 10012, Tel: (212) 998-0353, Fax: (212) 995-4233, e-mail: cliu@stern.nyu.edu

Abstract

This paper does a valuation analysis of senior-subordinated structure tranches backed by non-agency mortgages. The valuation is done using Monte Carlo simulation and employs the CIR interest rate process in conjunction with an empirical model estimated for non-agency mortgage prepayments and defaults. The sensitivity of the value of tranches to a number of variables are analyzed. We find that the interest rate process parameters significantly affect prepayments and defaults but not the relative value of the senior tranche. It is found that with the shifting of prepayments, the senior tranche does not dominate all the junior tranches at all interest rates. The shifting of prepayments has the unintended effect of providing stability to the junior tranches by making their cashflows less sensitive to prepayments. Our main conclusion is that while the shifting of prepayments increases protection from default to the senior tranche for a given level of subordination, it has the unwanted effect of lowering its value through increased contraction risk. This value loss should be taken into account in determining the optimum level of subordination.

1 Introduction

While home mortgages have been in use in the U.S. for several decades, the creation of a secondary market, which enabled the mortgage industry to tap funds from a broad range of investors, dates back only to 1970. Prior to that, banks and savings and loans used to issue mortgage loans and carry these illiquid loans in their asset portfolios. This meant considerable risk for these institutions if the local economy deteriorated and the default rate rose. In 1970, Government National Mortgage Association (GNMA), a government-chartered private corporation, began issuing pass-through securities that were collateralized by pools of government-guaranteed mortgages. The full and timely payment of principal and interest were guaranteed by GNMA under the full faith and credit of the U.S. Government. This took the default risk away from mortgages and opened the mortgage market to a wide array of investors. The local banks now simply originate mortgage loans according to the agency guidelines and then sell them to GNMA or investment banks to be securitized and sold to individual investors. The local banks, however, continue to service these loans. GNMA was soon followed by other quasi-government agencies such as FHLMC (Freddie Mac) and FNMA (Fannie Mae) with their own guaranteed pass-throughs. Since then the secondary market for agency-guaranteed securities has grown from one with generic pass-throughs to one with complex Collateralized Mortgage Obligations (CMOs) comprising of PACs, POs, IOs etc and become a trillion dollar plus market. All the mortgage-backed securities that are guaranteed by the above agencies automatically acquire a AAA rating against default risk. But they are still subject to prepayment risk.

The Bank of America was the first to issue privately-issued, as opposed to government-guaranteed, mortgage-backed securities (MBS) in 1977. The growth of the privately-issued market was rather slow till 1987. Since then, however, it has shown tremendous growth with over \$90 billion of private-label MBS issued in 1992. This growth in privately-issued MBS arose as a result of several factors. For one, loans less than \$203,000, the statutory limit, and loans made to mortgagors with impeccable credit rating, low debt/income ratio, and full documentation are eligible for agency backing. The prosperity of the 1980's led to a large increase in the issuance of jumbo loans which are loans that exceed the preceding

statutory limit. The real estate downturn and recession of the early 1990's also created a large number of potential borrowers with poor credit and/or high debt-to-income ratios. These borrowers who would have been denied mortgages in the earlier days, found mortgage bankers willing to lend to them, albeit at higher interest rates and with substantial down payments. Both these factors, together with Resolution Trust Corporation's (RTC) need to quickly securitize the unusual mortgage assets of bankrupt thrifts following the S & L crisis, resulted in a large number of loans that would not qualify for agency guarantees. The private sector thus stepped in to pool these loans and securitize them.

Since these private label securities are not guaranteed by federal agencies, they are subject to default risk. Consequently, they have to be credit enhanced and rated by rating agencies before they are sold in the secondary market. Initially, most of these private-label MBSs were credit enhanced by corporate guarantees, letters of credit, surety bonds¹, pool insurance, spread accounts² and/or reserve funds. However, the relative scarcity of financial institutions with AAA rating meant that most of the deals credit-enhanced through corporate guarantees were structured with only a AA rating, limiting the potential base for these securities. Reserve funds required that the cashflows be invested at rates similar to fed rates, which are low, resulting in a high cost of carry. The rating agencies did not give much credit to spread accounts making them very expensive. Currently, the majority of private-label securities use a senior-subordinate credit enhancement structure. Subordination involves dividing the cashflows into two tranches, senior and subordinate. This senior/subordinate structure is designed to protect the senior bonds at the expense of the subordinate bonds by first allocating the losses to the subordinates. Only after the subordinate principal is exhausted will the senior bond be affected by credit losses. In 1992, a new SEC regulation helped credit enhancement through subordination to gain dominance from its limited use before that. Prior to that, SEC regulations did not allow the lower rated subordinate bonds to be included as a publicly traded security. As a result, the issuers had to retain them, thus affecting the capital reserve requirements due to the

¹A surety bond is a 100% guarantee on all scheduled payments of principal and interest. It differs from other types of insurance because there is no loss limit.

²Spread accounts use excess interest cash flow that is not allocated to other classes to offset credit losses.

riskiness of the subordinate bonds. However, the new regulation allowed subordinated tranches rated BBB and above to be included as a publicly traded security. Also, Standard and Poor's dramatically reduced the amount of subordination necessary to achieve a AAA rating in 1993. As a result, 85% of all private-label MBSs were credit-enhanced through subordination by 1993. (See Table 1 for amounts of non-agency issuance of MBS and the different types of credit enhancement types used.)

The agency-guaranteed MBSs are subject to prepayment (and interest rate) risk. Prepayment risk arises from the option that the mortgagors have to prepay the entire loan anytime during the life of the loan. This adversely affects the security holder if done for financial reasons i.e. when the interest rate is low enough to allow the mortgagor to refinance at a lower rate, since the prepaid principal will have to be reinvested at the lower interest rate. But prepayment would prove beneficial if done at high interest rates for non-financial reasons such as moving out of town, divorce etc.

It is relatively easy to model the 'optimal' prepayment³ rate based on stochastic interest rates. However, this rate does not account for mortgagors who do not refinance their mortgage when it is theoretically optimal to do so and prepayment for non-financial reasons. The prior financial literature has modeled the impact of prepayment on MBS pricing from one of two perspectives in general. Dunn and McConnell (1981a and 1981b) recognize non-financial prepayment together with 'optimal' prepayment in pricing MBSs. However, their model does not account for mortgagors who do not exercise their options when they are in the money. Dunn and Spatt (1986), Johnston and Van Drunen (1988), and Timmis (1985) include transaction costs payable on refinancing in their models that could account for the above under-exercise. It is clear that non-financial prepayment and non-prepayment when optimal are very difficult to be modeled theoretically since they depend on individual circumstances and the cost of refinancing. An alternative perspective is to estimate an empirical model with easily observable variables to predict prepayments using past prepayment history. An example of this perspective is Schwartz and Torous (1989). McConnell and Singh (1993) employ the empirical prepayment model of Schwartz

³'Optimal' prepayment assumes that mortgagors will exercise their option as soon as the gain from immediate exercise exceeds the value of the option of postponing prepayment

and Torous (1989) to value the various tranches of CMOs, as opposed to generic MBSs. Both perspectives model agency backed MBS and as such treat defaults as if they were prepayments.

The private-label securities are subject to default risk as well as prepayment risk. This adds another layer of difficulty to the valuation. In theory, the mortgagors have an incentive to default on their loans when the house value drops by a fixed amount below the present value of future mortgage payments. In practice however, it is found that most mortgagors choose not to default even when it is optimal to do so. This could be due to the negative effect default has on one's credit history, personal reputation etc. Here also it is possible to have an empirical default model based on past defaults. In fact, Schwartz and Torous (1993) employ Poisson regression to estimate the parameters of a proportional hazards model for prepayment and default decisions. However, their sample consists solely of agency-backed loans. Since whole-loans include a large number of jumbo loans which generally prepay faster than agency loans when interest rates decline, whole-loan prepayment behavior is different from that of agency-backed loans. A number of investment banks use a multiplier greater than one to convert agency prepayment values to whole-loan values (see Table 2).

The purpose of this paper is to employ a logit regression model to value senior-subordinate tranches backed by private-label mortgage pools and examine the effect of mortgage prepayments and defaults, and changes in interest rates on the value of these tranches. More specifically, we estimate empirical functions for prepayment and default of whole-loans through logit regression on a sample of pools securitized by Prudential Home Mortgage. We also study the effect of interest rate volatilities and the degree of subordination on tranche prices. The shifting prepayment structure, explained later, and the sequential retirement of principal from the senior to the lowermost tranche as a result of prepayments and from the the lowermost to the senior tranche as a result of defaults make the estimation of cashflows of the tranches very complex. The stochastic nature of these cashflows makes the application of backward-looking methods such as the finite-difference method to value these tranches impossible. Therefore, one has to resort to Monte Carlo simulation for pricing the senior-subordinate tranches.

Section two provides a detailed description of the senior-subordinate structure. Section

three describes the empirical model for prepayments and defaults. Cash flow estimation of the tranches is detailed in section four (The appendix provides a numerical example of how the cashflows are estimated). Valuation results are discussed in section five and the conclusion appears in the last section.

2 Senior-Subordinated Structure

Senior-subordinated structures are created to provide credit enhancement for the private-label mortgage pools. Credit enhancement of the senior class is achieved because the subordinated class stands in a first loss position for losses due to defaults and foreclosures on loans in the mortgage pool. The size of the senior portion is determined by the credit rating agencies so as to enable the entire senior portion to qualify for AAA rating. The rating agency analyzes the private-label pool and determines the amount of credit support (in other words, the size of the subordinated portion) based on collateral credit quality, collateral documentation, loan purpose, property type, geographical area of loans, expected future macro-economic conditions, etc. Even with AAA rating, many investors do not feel comfortable with the credit risk of the senior class. They believe that the rating agency's subordination requirements might not be sufficient. This has given rise to the super-senior structure whereby a layer is stripped from the senior class and is used as additional subordination. Since this stripped layer of senior bonds retain the original level of subordination, they generally maintain the same AAA rating.

The subordinated tranches are called 'B pieces' and can be further stratified based upon subordination; the most senior part of a B piece could be rated a single A, the next senior part could be rated a BBB and so on⁴. The size of each sub-tranche is once again a function of the collateral quality and the desired rating. The lower the collateral quality, the higher the credit protection required for a sub-tranche for a given rating. The lower the desired rating, the lower is the credit protection required for a sub-tranche for a given collateral pool. As an example, let us assume that for some collateral the AAA, AA, A and BBB credit protection levels are 10%, 6%, 4% and 3% respectively. Then the first 90%

⁴Bonds that are senior to some tranches, but junior to others are called mezzanine.

of the pool will be issued as AAA bonds, the next 4% (10%-6%) will be AA, the next 2% (6%-4%) will be A and the next 1% (4%-3%) will be BBB. The last 3% will be unrated or could achieve some lower rating. This process of stratifying the tranches based upon subordination is called credit tranching.

To further protect the senior bonds, senior-subordinated structures employ a 'shifting' interest structure for distributing the prepayments and recoveries from defaults. Shifting interest means that the prepayments and recoveries from defaults are not allocated pro-rata between the senior and subordinated tranches. In the most common shifting structure, the senior tranche receives a pro-rata share of interest, amortized principal, prepayments and default recoveries, and a specified percentage of the share of prepayments and default recoveries of the subordinate tranches. In a typical schedule, the percentages are 100%, 70%, 60%, 40%, 20% and 0% in years 1 through 5, 6, 7, 8, 9, 10 and after respectively. In other words, the subordinate tranches will not receive any prepayments for the first 5 years and will receive its pro-rata share of the cash flows after year 9. This accelerates the amortization of the senior tranche and increases the relative size of the subordinated bonds over time, thus increasing the level of protection for the senior bonds. In addition, the current level of subordination of each tranche is calculated every month to ensure that it is at least as high as the original level. If it is not, amortized principal meant for junior tranches is redirected to that tranche till the original level of subordination is restored. A by-product of this shifting structure is that the 'B pieces' get additional call protection from prepayments and their negative convexity and average life variability are significantly reduced.

3 Estimation of Empirical Model for Prepayments and Defaults

As mentioned in the introduction, whole-loans exhibit different prepayment behavior than agency-backed loans. Thus, we estimate separate empirical models for the prepayments and the defaults of whole-loan mortgages. Data from 23 whole-loan mortgage pools securitized by Prudential Home Mortgage between 1988 and 1993 are used for this purpose. The average pool size was \$314 million and 1747 months of data were available across all the pools. Monthly prepayment values were taken directly from monthly CPR prepayment data reported on Bloomberg for the pools and the monthly default values of the pools were estimated from data obtained from Prudential on individual loans that defaulted.

The variables that can be used in an empirical model for prepayments and defaults are limited by the fact that one should be able to simulate the future values of these variables in order to use the model in pricing securities. This restricts the number of macro-economic variables that can be used. Here, we restrict ourselves to variables that can be derived from two stochastic processes - the interest rate process and the housing value process - in addition to pool specific variables such as original loan-to-value ratio, coupon rate, the fraction of the pool still remaining, etc.⁵ The variables related to the interest rate process that we use are the refinancing rate - by far the most significant predictor of prepayment -, the monthly change in the refinancing rate, a measure of the volatility of the refinancing rate given by the sum of the squared changes in the refinancing rate over the previous three months and the slope of the term structure. Variables associated with the housing value process used in our study include the return on the repeat sale housing value index, cumulative returns on the housing value index since the inception of the pool, and a volatility measure given by the sum of the squared changes in the index over the previous three months. The burnout⁶ variable used is the log of the fraction of the pool remaining

⁵Unemployment rate and GNP growth rate may be very significant in explaining mortgage default values, but can be used in the prediction model only if one is prepared to assume different stochastic processes for each of them in addition to interest rate and housing value processes.

⁶Burn out is the tendency of prepayment from premium mortgage pools (i.e. pools with coupon rates

at any point in time.

Since the prepayment and default values are probabilities, a logit model is used to estimate them as a function of the above explanatory variables. Since we are dealing with proportions data, and not binary response data, a weighted least squares regression is used to estimate the minimum chi-squared estimates of the coefficients (see Greene (1993)). The regression results are reported in Table 3.

As mentioned, since the prepayments and defaults in each month are exogenously specified by the empirical model, the pricing of the mortgage-backed securities can not be done by any of the backward-looking methods such as the finite-difference method and the binomial model. This is because the fraction of the mortgage pool outstanding at any time will be a function of prepayments and defaults in prior periods and these depend upon prior period interest rates and housing index returns. To apply, say, the finite-difference method, the fraction of the pool or face value remaining at any time should be independent of prior period variable values. The net result is that only a forward-looking method such as Monte Carlo simulation can be used to price MBSs with empirical prepayments and defaults. Monte Carlo simulation requires the generation of a short-term interest rate as well as all the explanatory variables in the empirical prepayment and default models. The following discussion reveals how the variable values are generated.

We assume that the instantaneous riskless rate follows the square-root mean-reverting process as specified by Cox, Ingersoll and Ross (1985)⁷:

$$dr = k(m - r) + \sigma\sqrt{r}dz_t \quad (1)$$

where

k is the speed of mean reversion,

m is the steady-state mean and

σ^2 is the instantaneous variance.

The above parameters are estimated using a trial and error process by assuming initial above the refinancing rate) to slow down over time, all other things being equal.

⁷With no loss of generality, we can assume that the Local Expectations Hypothesis holds and therefore, regard the interest risk factor as having been absorbed into the values of other interest parameters

parameter values, estimating the yield curve, and then calculating the sum of squared errors of the deviations from the actual yield curve. The parameter values are changed until the sum of squared deviations is minimized.

Given the above process, the interest rate for any maturity, T , can be calculated as a function of the short-term risk-less rate (r), k , m and σ (see Cox, Ingersoll and Ross (1985)). This allows us to calculate the 10-year interest rate as a proxy to the fixed-rate mortgage refinancing rate⁸. Other interest rate-related variables such as the slope of the term structure, the volatility of the refinancing rate (This is simply the sum of the squared changes) and the change in the refinancing rate are easily estimable.

The mortgage house value dynamics is assumed to be given by

$$dH = (\mu - b)Hdt + \sigma_H dz_H \quad (2)$$

with

$$(dz_r)(dz_H) = \rho dt \quad (3)$$

where

μ is the instantaneous expected housing rate of return,

b is the housing payout rate (rent, for example),

σ_H^2 is the instantaneous variance of housing returns and

ρ is the instantaneous correlation coefficient between the

increments to the standardized Wiener processes dz_r and dz_H .

dH/H gives us a proxy for the housing index return. As is generally done in this literature (see Schwartz and Torous (1992) and Cunningham and Hendershott (1984)), μ is assumed to be 0.1, b to be 0.05 and σ_H to be 0.05.

The next section reveals how the cashflows of each tranche are estimated given these rates. The price of each tranche is simply the discounted present value of these cashflows.

⁸10-year rate is a slightly better predictor of the fixed-rate mortgage rates than the 30-year rate. A regression of the mortgage rate against the 10-year rate gives an R^2 of 0.87 as opposed to 0.84 for the 30-year rate. Based on the regression, we have, mortgage rate = $2.495 + 0.9195R_{10-yr}$

4 Cashflow Estimation of the Tranches

We assume that the private-label pool backing the senior-subordinated bonds consists of a large number of equally sized, fixed rate, fully prepayable, fully amortizing mortgages. Let $F_G(0)$ be the face value of the pool of mortgages at origination, c be the fixed monthly coupon rate on the mortgages and let T be the term to maturity in months. Let A , the scheduled payment, be the amortized principal and interest paid out in every month. If no loans are prepaid and there are no defaults, then A is given by

$$A = \frac{cF_G(0)}{(1 - e^{-cT})} \quad (4)$$

and the principal remaining at time t if no loans are prepaid and defaulted is

$$F_G(t) = \frac{A}{c}(1 - e^{-c(T-t)}) \quad (5)$$

But the prepayments and defaults reduce the fraction of loans outstanding in each month and thereby reduce the scheduled payment in each month over time. Let $y(t)$ be the fraction of the pool outstanding, $\pi(t)$ be the prepayment rate, $\gamma(t)$ be the default rate in month t and α be the constant default recovery rate⁹. Then,

$$y(t) = y(t-1)(1 - \pi(t) - \gamma(t)) \quad (6)$$

The scheduled payment in month t now becomes,

$$A(t) = \frac{cF_G(0)y(t)}{(1 - e^{-c(T-t)})} \quad (7)$$

the remaining principal is,

$$F_G(t) = \frac{A(t)}{c}(1 - e^{-c(T-t)}) \quad (8)$$

and the pool cash flow in month t is,

$$CF(t) = A(t) + F_G(t-1)(\pi(t) + \alpha\gamma(t)) \quad (9)$$

⁹When a loan is defaulted and the property is foreclosed, a fraction of the remaining principal of the loan is recovered, net of all legal and foreclosure costs. This fraction is called the recovery rate. Here we assume that the default recovery is made immediately. In reality, it takes several months. But this should not make any significant difference to our results.

Assume that there is one senior tranche, with initial face value, $F_s(0)$, and coupon, c_s , and three subordinated tranches with initial face values $F_{j1}(0)$, $F_{j2}(0)$ and $F_{j3}(0)$ and with coupons c_{j1} , c_{j2} and c_{j3} respectively. The shifting interest structure means that the senior tranche gets a percentage, $p(t)$, of the junior tranches' share of prepayment in the first 9 years.

$$p(t) = \begin{cases} 1.00, & \text{when } 0 \leq t \leq 60 \\ 0.70, & \text{when } 60 \leq t \leq 72 \\ 0.60, & \text{when } 72 \leq t \leq 84 \\ 0.40, & \text{when } 84 \leq t \leq 96 \\ 0.20, & \text{when } 96 \leq t \leq 108 \\ 0.00, & \text{when } t > 108 \end{cases} \quad (10)$$

So in any month t , the senior tranche would receive a fraction, $p_s(t)$, of the prepayment and default recoveries,

$$p_s(t) = \frac{F_s(t) + p(t) \sum_k F_{jk}(t)}{F_s(t) + \sum_k F_{jk}(t)} \quad (11)$$

No coupon payments are made on the principal that is prepaid and recovered from default between months t and $t - 1$. This means that in order to calculate the coupon payment due on each tranche, one has to calculate the principal remaining on each tranche after the prepayments and default recoveries have been deducted, but before the amortized principal is subtracted. Let this be $F_s^*(t)$ and $F_{jk}^*(t)$, ($k = 1, 2, 3$) respectively for the senior and junior tranches. Then,

$$F_s^*(t) = F_s(t - 1) - F_G(t - 1)(\pi(t)p_s(t) + \gamma(t)Min(p_s(t), \alpha)) \quad (12)$$

$$F_{j3}^*(t) = F_{j3}(t - 1) - F_G(t - 1)(\pi(t)(1 - p_s(t)) + \gamma(t)Max(0, (\alpha - p_s(t)))) \\ \frac{F_{j3}(t - 1)}{\sum_k F_{jk}(t - 1)} - \gamma(t)(1 - \alpha) \quad (13)$$

$$F_{jk}^*(t) = F_{jk}(t - 1) - F_G(t - 1)(\pi(t)(1 - p_s(t)) + \gamma(t)Max(0, (\alpha - p_s(t)))) \\ \frac{F_{jk}(t - 1)}{\sum_k F_{jk}(t - 1)}, k = 2, 3 \quad (14)$$

Once we have $F^*(t)$ s, we can calculate $F(t)$ s by subtracting the amortized principal. For the whole pool, the amortized principal payment in month t is,

$$AP(t) = A(t) - F_G(t - 1)(1 - \pi(t) - \gamma(t))c \quad (15)$$

Then,

$$F_s(t) = F_s^*(t) - AP(t) \frac{F_s^*(t)}{F_s^*(t) + \sum_l F_{jl}^*(t)} \quad (16)$$

$$F_{jk}(t) = F_{jk}^*(t) - AP(t) \frac{F^* - jk(t)}{F_s^*(t) + \sum_l F_{jl}^*(t)}, k = 1, 2, 3 \quad (17)$$

Cash flows for the various tranches in month t will be

$$CF_s(t) = F_s^*(t)c_s + AP(t) \frac{F_s^*(t)}{F_s^*(t) + \sum_k F_{jk}^*(t)} + F_G(t-1)(\pi(t)p_s(t) + \gamma(t)Min(p_s(t), \alpha)) \quad (18)$$

$$\begin{aligned} CF_{jk}(t) = & F_{jk}^*(t)c_{jk} + AP(t) \frac{F_{jk}^*(t)}{F_s^*(t) + \sum_k F_{jk}^*(t)} + F_G(t-1)(\pi(t)(1 - p_s(t)) \\ & + \frac{F_{jk}(t-1)}{\sum_k F_{jk}(t-1)}Max(0, (\alpha - p_s(t))))), k = 1, 2, 3 \end{aligned} \quad (19)$$

If prepayment rates are moderately high during the mortgage life, the senior tranche will be retired before maturity. As a result, the cash flows and the outstanding principal values will undergo changes to reflect the absence of cash flows to the senior tranche. All the prepayments and default recoveries will be shared pro-rata by all the junior tranches. But tranche 3 will continue to bear all the losses from defaults. If the default rate is high enough, it is likely that tranche 3 will be retired. In that case, tranche 2 will start to take on the default losses. In the unlikely event that all junior tranches are retired, the senior tranche will be hit with the default losses. Also, as mentioned before, each tranche is subjected to a ‘subordination test’ in each month. If the tranche fails the test, prepayments due to the lower tranches are redirected to this tranche until this condition is satisfied.¹⁰

5 Valuation Results

We consider a senior-subordinated structure backed by a pool of 30-year fixed-rate private-label mortgages with a coupon rate of eight percent¹¹. The structure is assumed to have four tranches; one senior, S, with a face value equal to 90% of the pool face value and

¹⁰A simple example in appendix demonstrates the allocation of the cash flows to the various tranches and the principle of shifting of prepayment structure.

¹¹The servicing fee is assumed to be 0.5% of the principal outstanding per year.

three subordinate tranches, namely, J1, J2 and J3 with face values equal to 5%, 3% and 2% of the mortgage pool respectively. All four tranches are assumed to have a coupon rate of seven percent. This equal-coupon assumption makes it easy to compare the values of the tranches, but makes no material difference to the results. As previously mentioned, a shifting interest structure is employed to distribute the cashflows from prepayments and defaults. But the default losses are deducted from J3 until it is exhausted and then from J2 and so on. The excess coupon payments from the mortgage pool are assumed to go to a 'residual' tranche.

The values of the tranches are determined by averaging the values from 10,000 Monte Carlo trials. Each trial traces a random interest rate path based on the CIR interest rate process and a random repeat sales housing index value path based on the previously specified diffusion process.

Refinancing Rates

Table 4 shows the values of the different tranches as a percentage of the face value of the respective tranches for various values of the interest (refinancing) rates (see Figure 1 also). The generic MBS value is the value of a simple MBS backed by the mortgage pool and with the same 7% coupon as the senior-subordinate tranches. In other words, it is the value of the senior tranche with no subordination. The effect of subordination on the senior tranche can be measured by comparing its value with the value of the MBS. The first thing to note is that the S tranche is valued at a premium to the MBS, justifying the subordination. J3 is valued comparatively much lower than the other tranches since it bears almost all the default losses. J2, which is next-in-line for the default losses, is only slightly affected by defaults.

A fall in refinancing rates results in an increase in prepayments as can be seen in Panel B, but this effect is overwhelmed by the discounting effect of lower interest rates. As a result, tranches S, J1, and J2 are valued at far above their face values when low interest rates prevail. If all the mortgages are prepaid at the lower interest rates, this would not be the case. But since only a fraction of the mortgages prepay when the rates are low, these tranches continue to enjoy the high, original coupon rates. Also, S has, rather counter-intuitively, lower values than the riskier J1 and J2 tranches at lower interest rates. This is

due to the fact that S is retired earlier than the other tranches (see the weighted ages in Panel B) and therefore, can not enjoy the high coupons in a low-interest environment as long as the other tranches. In addition, the default losses hardly affect J1.

Prepayment primarily affects tranche S. Its weighted average life triples as the interest rate goes from 6.5 to 10%, whereas those of J1 and J2 go up by a little more than 10% (see figure 2). This is clearly due to the shifting of prepayment structure, whereby tranche S receives the bulk of the pool prepayments in the first 10 years. The increase in defaults (as interest rates go up) is sufficient to decrease the age of J3 by almost half. The overall result is that the senior tranche is subject to severe contraction risk, whereas the junior tranches 1 and 2 provide relatively far more stable cashflows regardless of the prepayment rates as can be clearly seen in Figure 2.

Another important observation is that the S tranche value is lower than that with no subordination (generic MBS value) at all interest rates.

Interest Rate Volatility

Table 5 looks at the effect of interest rate volatility on the senior-subordinated structure. The volatility that we consider here is the volatility parameter, σ_r , used in the CIR interest rate process to generate the interest rate path for simulation. We find that the total prepayments increase by more than 20% as the parameter value is increased from 0.04 to 0.06. This is to be expected because higher volatility would result in bigger jumps in both downward and upward directions and therefore, lead to higher prepayments since higher interest rates have relatively small impact on prepayments. But surprisingly, the total defaults almost double as the volatility parameter is increased. The only explanation is that the *mtgvar* variable acts as a proxy for an excluded variable in the default regression.

Mean Reversion Parameter

Mean reversion parameter is the speed at which the short-term interest rate reaches the long run mean and is represented by k in the CIR interest rate process. The lower this speed, the higher the volatility. Table 6 reports the results when k is increased from 0.02 to 0.08. The total prepayments almost halve, but unlike Table 3, the senior tranche value is unaffected in comparison to J1 value. This is because here the prepayments occur gradually over the mortgage maturity whereas in Table 3 a significant portion of it was

immediate due to the low current refinancing rate.

Shifting Prepayment Structure

Under the shifting of prepayments (or shifting interest structure) a significant portion of the pro-rata share of prepayment and default recoveries due to the subordinates is diverted to the senior tranche during the first ten years. This accelerates the amortization of the senior tranche and increases the degree of subordination (face value of subordinates over that of senior) over time, protecting the senior tranche investor. In order to study the effect of the shifting of prepayments, we consider the case with no shifting and re-estimate the values in Table 4. The results are shown in Table 7.

A comparison of panel B in Tables 4 and 7 reveals that the relative age of the senior tranche is shorter with shifting (see Figure 3). At lower interest rates, which implies high prepayments, the subordinate tranches are retired twice as fast with no-shifting (see Figure 4) and J1, especially, has no longer any protection from contraction risk. Even at high interest rates, the weighted age of the subordinate tranches are 30% lower. This clearly shows that in the absence of prepayment shifting, the subordinate tranches are retired 2-3 times faster and as a result, the senior tranche is left more vulnerable to default losses. But an inspection of panel A in Tables 4 and 7 reveals that the incorporation of shifting interest structure does very little to increase the senior tranche value. In fact, at low interest rates and at current coupon the senior tranche exhibits higher value with no shifting (see Figure 5). In addition to this, the senior tranche has higher values than J1 and generic MBS for all interest rates, discount and premium. This is because the potential loss from defaults, even without the protection from shifting, is very small and the longer weighted age of the senior tranche in the no-shifting case allows it to enjoy the higher coupons at the lower interest rates for a longer time. In addition, the requirement that the senior tranche be paid its pro-rata share of the total principal upon default ensures that tranche J1 will stay alive longer on average than tranche S and act as a buffer against losses.

With no shifting of prepayments, J1 and J2 loses value at lower interest rates (see Figure 6) since they no longer have protection from contraction risk. But in the absence of shifting, the value of the J3 increases at most interest rates since more of its principal gets retired through prepayments rather than through default losses. **Degree of Subordination**

Here we look at the values of the senior tranche as the degree of subordination is increased from 0 to 10% with the refinancing rate constant and equal to the coupon rate (par coupon). This analysis is analogous to choosing the optimal degree of subordination at the time of issuance of the tranches. We would like to see if the increased protection for tranche S results in an increased value for that tranche. It is seen in Table 8 and in Figure 7 that as the subordination is increased, the value of the senior tranche increases, reaches a peak, and then decreases. The highest value for the senior tranche is realized for a degree of subordination between four and five percent. So if subordination is increased beyond this optimum, the value loss from contraction risk more than offsets the value of enhanced protection from default. This value loss is caused by the fact that a greater degree of subordination results in larger shifting of prepayments and earlier retirement or contraction of the senior tranche. Thus, the senior tranche gets additional protection; but this is not reflected in the senior tranche values. Obviously, the protection from additional subordination adds little value to the tranche since the threat of loss from default is already extremely small.

6 Conclusion

This paper values the tranches of a senior-subordinated structure backed by a pool of whole-loan mortgages through Monte Carlo simulation. Empirical models for whole-loan prepayments and defaults are estimated using monthly data on a sample of 30-year, fixed rate, whole-loan mortgage pools. The valuation uses these empirically estimated prepayment and default functions and the one-factor CIR interest rate process. Values of a senior tranche and three subordinate tranches are analyzed by changing the refinancing rate and other parameters. Interest rate volatility and the mean reversion parameter affect both prepayments and defaults rather significantly. But the relative value of the senior tranche is unaffected by these variables. The senior tranche, as well as all other tranches, are affected by refinancing rates, but mostly via the discounting effect rather than the prepayment effect. It is seen that the senior tranche has a lower value than the seniormost junior tranche at lower interest rates. This is because as a result of shifting of prepayments the

senior tranche suffers from increased contraction risk at lower interest rates. It is retired earlier and therefore, has to be reinvested at lower rates.

This means that the shifting interest structure adds value to the senior tranche value only for discount coupons. Even though it provides enhanced protection from default to the senior tranche, this additional protection seems to have a negative net value for par and also premium coupons due to the additional contraction risk. The structure, in fact, serves to increase the value of the seniormost junior tranche by giving it protection from contraction risk.

Those who securitize non-agency mortgages want to offer as large a fraction of the face value of the pool as possible as the senior tranche with protection from default loss. For a given level of subordination, protection to the senior tranche can be enhanced through the use of the shifting of prepayments feature. However, the previously mentioned cost of the shifting of prepayments has to be taken into account in the design of the senior-subordinated structure, possibly by reducing the magnitude of the transfer of prepayments within this feature.

References

- [1] Boyle, P., 'Options: A Monte Carlo approach', *Journal of Financial Economics*, 4 (1977), 323-328.
- [2] Cox J., J. Ingersoll and S. Ross, 'An intertemporal general equilibrium model model of asset prices', *Econometrica*, 53 (1985), 363-385.
- [3] Cunningham, D. and P. Hendershott, 'Pricing FHA mortgage default insurance', *Housing Finance Review*, 3 (1984), 373-392.
- [4] Dunn, Kenneth D. and John J. McConnell, 'Valuation of GNMA mortgage-backed securities', *Journal of Finance*, 36 (1981), 599-617.
- [5] Dunn, Kenneth D. and Chester S. Spatt, 'The effect of refinancing costs and market imperfections on the optimal call strategy and the pricing of debt contracts', Working paper, Carnegie-Mellon University (1986)
- [6] Greene, William H., *Econometric Analysis*, Second Edition, Prentice Hall, 1993. 7 Johnston, E. and L. Van Drunen, 'Pricing mortgage pools with heterogenous mortgagors: empirical evidence', Working paper, University of Utah (1988).
- [7] McConnell, John J. and Manoj Singh, 'Valuation and analysis of collateralized mortgage obligations', *Management Science*, 39 (1993),692-709.
- [8] Schwartz, Eduardo and Walter Torous, 'Prepayment and valuation of mortgage-backed securities', *Journal of Finance*, 44 (1989), 375-392.
- [9] Schwartz, Eduardo and Walter Torous, 'Prepayment, default and the valuation of mortgage pass-through securities ', *Journal of Business*, 65 (1992), 221-239.
- [10] Schwartz, Eduardo and Walter Torous, 'Mortgage prepayment and default decisions: A Poisson regression approach', *Journal of the American Real Estate and Urban Economics Association*, 21 (1993), 431-449. 12 Timmis, G.C., 'Valuation of GNMA

mortgage-backed securities with transaction costs, heterogenous households and endogenously generated prepayment rates', Working paper, Carnegie-Mellon University (1985).

Appendix

This section demonstrates the cash flow allocations to the four tranches upon prepayment and default by means of a simple example. There are ten mortgages of \$1,000 each that form a pool of size \$10,000. The pool is divided into four tranches with the following sizes: S = \$9,000, J1 = \$500, J2 = \$300 and J3 = \$200. The coupon rate for the mortgages as well as the tranches is assumed to be 10% and all payments are yearly. The maturity of the loans is 10 years. There is a shifting of prepayments in the first five years to protect the senior tranche. The structure is as follows. 100% of the prepayments and default recoveries due to the junior tranches in the first two years is shifted to the senior tranche, 50% in the next two years, 25% in the next year and none in the next 5 years. Two of the loans are assumed to prepay in the second year, one in the fourth, one in the fifth, one in the sixth and one in the seventh. One of the loans is assumed to default in the fourth year and another in the seventh. The default recovery is 60%. The cash flows and end-of-year balances are as follows.

				S		J1		J2		J3	
year	annual paymt	prepayment	default	cf	bal	cf	bal	cf	bal	cf	bal
1	1627	0	0	1465	8435	81	469	49	281	33	187
2	1627	1736	0	3201	6078	81	434	49	260	33	174
3	1302	0	0	1139	5546	81	396	49	238	33	158
4	1302	709	709	2229	3872	104	332	62	65	41	0
5	976	617	0	1459	2798	112	253	22	50	0	0
6	814	516	0	1200	1877	109	170	21	33	0	0
7	651	405	405	1195	810	86	0	17	0	0	0
8	325	0	0	325	565	0	0	0	0	0	0
9	325	0	0	325	296	0	0	0	0	0	0
10	325	0	0	325	0	0	0	0	0	0	0

Annual paymt is the amortized annual payment for the pool, *cf* is the tranche cash flow for the year and *bal* is the tranche principal balance at the end of the year.

Note that in the second year, the junior tranches do not receive anything out of \$1736 prepaid. In year 4, the default of one loan means that there is a loss of $709(1 - 0.6) = \$284$. This loss is enough to wipe out all the principal of the tranche J3 and part of J2. Default in year 7 results in a loss of $405(1 - 0.6) = \$162$ and this wipes out tranches J1 and J2. This example clearly shows that but for the shifting of prepayment in the first 5 years, a large portion of the default loss in year 7 would have been born by the senior tranche.

Table 1
Non-Agency MBS Issuance and Credit Enhancement Types

year	1987	1988	1989	1990	1991	1992	1993
issuance	11.1	15.9	14.2	24.4	49.7	89.5	96.6
subordination	30	79	62	49	51	48	85
reserve fund	0	0	0	0	19	17	1
pool insurance	0	4	10	23	10	28	11
letter of credit	0	0	13	15	5	1	0
surety bonds	18	4	2	4	0	2	2
corp guarantee	52	13	12	3	0	0	0

Issuance is in billions of dollars per year

Credit enhancement types are in percentages

Table 2
Multipliers for Non-Agency Mortgage Prepayments Relative to Agency Mortgage Prepayments

Issuer	Multiplier
Capstead	1.45
Chase	1.90
Citibank	0.82
Prudential	1.58
Residential Funding	1.40
Ryland	1.18

Source: Bear Stearns and Co., "Prepayment of Whole-Loan Securities," June 4, 1993.

Table 3
Logit Regression Results of Prepayments and Defaults of Whole-loans

Regression results of prepayments and defaults using monthly data on 23 whole-loan pools issued by Prudential Home Mortgage are reported here. The pools, all issued between 1988 and 1993, are backed by 30-year fixed-rate mortgages. The regressions are carried out using the model $\text{logit}(p) = \beta x$ where x is the set of explanatory variables, β , the set of regression parameters and p is the probability of prepayment or default.

Variables	Prepayments	Defaults
intercept	-10.617 (2.214)	-9.082 (5.324)
refdel	3.262 (0.255)	0.261 (0.130)
mtgdiff	1.353 (0.239)	1.188 (0.522)
mtgvar	1.230 (0.400)	3.662 (0.936)
logfac	0.199 (0.072)	-0.099 (0.053)
slope	0.687 (0.053)	0.110 (0.13)
hret	2.926 (1.044)	-0.225 (0.283)
hcret	-0.782 (0.502)	-1.795 (0.417)
hvar	-0.567 (2.776)	6.721 (3.123)
seas	0.427 (0.076)	0.224 (0.188)
ltv	4.107 (3.032)	1.583 (0.808)

where *refdel* is the difference between the coupon rate and the refinancing rate if this difference is positive and zero otherwise, *mtgdiff* is the change in the refinancing rate from the previous month, *mtgvar* is the refinancing rate variance calculated as the sum of the squared changes in the rates over the previous three months, *logfac* is the log of the fraction of the original loans remaining, *slope* is the slope of the yield curve calculated as the difference between the 30-year treasury rate and the 90-day T-bill rate, *hret* is the monthly percentage change in the average resale value of existing homes, *hcret* is the cumulative return on the above housing index since the inception of the pool, *hvar* is the housing index variance calculated as the sum of the squared changes in the index over the previous three months, *seas* is a dummy variable that has a value of one during months May-October and zero otherwise and *ltv* is the original average loan-to-value ratio of the pool.

The standard errors are given in brackets.

Table 4
Effect of Refinancing Rate on the Value of Tranches and on Prepayment and Default

Whole-loan mortgage pool: coupon = 8%, maturity = 30yrs, servicing fee = 0.05%,
 Default recovery rate = 0.4, loan-to-value ratio = 0.80
 Face value of the tranches/face value of the pool: $S = 90\%$, $J1 = 5\%$, $J2 = 3\%$, $J3 = 2\%$
 All tranches have the same coupon rate of 7%.
 CIR interest rate parameters: $k = 0.08$, $m = 0.1$, $\sigma = 0.05$
 Housing value dynamics parameters: $\mu = 0.10$, $b = 0.05$, $\sigma_H = 0.05$

Panel A:

	refin rate	Value of tranches				Generic MBS
		S	J1	J2	J3	
Par	8.0	106.26	107.86	98.56	35.80	104.69
Premium	7.0	108.83	117.19	113.13	54.83	108.29
	6.5	106.20	119.95	119.01	77.15	106.69
Discount	9.0	99.99	99.11	88.99	30.85	98.22
	10.0	93.79	91.26	81.46	29.85	91.99

Panel B:

	refin rate	Value-weighted age of the tranches in years ¹					Gen MBS	prepayment ²	default ³
		S	J1	J2	J3				
Par	8.0	11.98	17.49	15.41	4.94	12.22	47.87	4.69	
Premium	7.0	7.89	17.05	16.18	7.10	8.58	64.57	3.27	
	6.5	3.91	15.32	15.12	8.89	4.92	80.51	1.95	
Discount	9.0	12.70	17.58	15.07	4.47	12.85	44.33	5.15	
	10.0	12.96	17.59	14.85	4.48	13.08	42.91	5.42	

Generic MBS is the security if the pool is sold as one single tranche, in other words, it is senior tranche with no subordination.

¹ The principal paid off in each month is multiplied by the age of the loan in that month, aggregated, and divided by the total value of principal to create a value-weighted age of the tranches. This measure is superior to the actual age of the tranches because tranches can linger on for quite some time with very small amounts of principal.

² Prepayment is the total principal prepaid through maturity as a percentage of the face value of the pool.

³ Default is the total principal defaulted (but not the actual loss) through maturity as a percentage of the face value of the pool.

10,000 simulation runs were conducted for each valuation. Values are calculated at time 0.

Table 5

Effect of Interest Rate Volatility on the Value of Tranches and on Prepayment and Default

Whole-loan mortgage pool: coupon = 8%, maturity = 30yrs, servicing fee = 0.05%,
 current refinancing rate = 8%, loan-to-value ratio = 0.80, default recovery rate = 0.4
 Face value of the tranches/face value of the pool: $S = 90\%$, $J1 = 5\%$, $J2 = 3\%$, $J3 = 2\%$
 All tranches have the same coupon rate of 7%. CIR interest rate parameters: $k = 0.08$, $m = 0.1$
 Housing value dynamics parameters: $\mu = 0.10$, $b = 0.05$, $\sigma_H = 0.05$

Panel A:

Volatility ¹	Value of tranches				
	S	J1	J2	J3	Generic MBS
0.04	106.52	107.68	103.69	41.99	105.20
0.05	106.26	107.86	98.56	35.80	104.69
0.06	105.50	105.05	87.66	31.46	103.37

Panel B:

Volatility	Value-weighted age of the tranches in years ¹					prepayment ²	default ³
	S	J1	J2	J3	Gen MBS		
0.04	12.92	17.97	17.12	6.10	13.16	43.46	3.65
0.05	11.98	17.49	15.41	4.94	12.22	47.87	6.92
0.06	10.61	15.70	12.24	3.87	10.78	53.82	7.42

¹ The principal paid off in each month is multiplied by the age of the loan in that month, aggregated, and divided by the total value of principal to create a value-weighted age of the tranches. This measure is superior to the actual age of the tranches because tranches can linger on for quite some time with very small amounts of principal.

² Prepayment is the total principal prepaid through maturity as a percentage of the face value of the pool.

³ Default is the total principal defaulted through maturity as a percentage of the face value of the pool.

10,000 simulation runs were conducted for each valuation.

Table 6

Effect of Interest Rate Mean Reversion Parameter on the Value of Tranches and on Prepayment and Default

Whole-loan mortgage pool: coupon = 8%, maturity = 30yrs, servicing fee = 0.05%, current refinancing rate = 8%, loan-to-value ratio = 0.80, default recovery rate = 0.4
 Face value of the tranches/face value of the pool: $S = 90\%$, $J1 = 5\%$, $J2 = 3\%$, $J3 = 2\%$
 All tranches have the same coupon rate of 7%. CIR interest rate parameters: $k = 0.08$, $m = 0.10$, $\sigma = 0.05$
 Housing value dynamics parameters: $\mu = 0.10$, $b = 0.05$, $\sigma_H = 0.05$

Panel A:

Mean reversion	Value of tranches				
	S	J1	J2	J3	Generic MBS
0.02	102.12	103.81	96.49	59.73	101.12
0.05	105.19	109.15	98.11	42.94	103.89
0.08	106.26	107.86	98.56	38.80	104.69

Panel B:

Mean reversion	Value-weighted age of the tranches in years ¹					Gen MBS	prepayment ²	default ³
	S	J1	J2	J3				
0.02	2.95	7.29	6.11	2.93	3.27	88.89	3.70	
0.05	7.66	14.04	11.87	4.46	8.04	68.95	4.72	
0.08	11.98	17.49	15.41	4.94	12.22	47.87	4.69	

where *mean reversion* is the mean reversion parameter in the CIR interest rate process.

¹ The principal paid off in each month is multiplied by the age of the loan in that month, aggregated, and divided by the total value of principal to create a value-weighted age of the tranches. This measure is superior to the actual age of the tranches because tranches can linger on for quite some time with very small amounts of principal.

² Prepayment is the total principal prepaid through maturity as a percentage of the face value of the pool.

³ Default is the total principal defaulted through maturity as a percentage of the face value of the pool.

10,000 simulation runs were conducted for each valuation.

Table 7

Effect of Shifting Prepayment Structure on the Value of Tranches (Table 4 Repeated with no Shifting of Prepayments to the Senior Structure

Whole-loan mortgage pool: coupon = 8%, maturity = 30yrs, servicing fee = 0.05%,
 default recovery rate = 0.4, loan-to-value ratio = 0.80

Face value of the tranches/face value of the pool: $S = 90\%$, $J1 = 5\%$, $J2 = 3\%$, $J3 = 2\%$

All tranches have the same coupon rate of 7%.

CIR interest rate parameters: $k = 0.08$, $m = 0.1$, $\sigma = 0.05$

Housing value dynamics parameters: $\mu = 0.10$, $b = 0.05$, $\sigma_H = 0.05$

Panel A:

	refin rate	Value of tranches				
		S	J1	J2	J3	Generic MBS
Par	8.0	106.44	105.61	92.29	43.03	104.69
Premium	7.0	109.64	109.12	99.10	59.45	108.29
	6.5	107.55	107.30	101.14	75.12	106.69
Discount	9.0	99.95	98.95	85.69	38.22	98.22
	10.0	93.61	92.44	80.20	37.39	91.99

Panel B:

	refin rate	Value-weighted age of the tranches in years ¹					Gen MBS	prepayment ²	default ³
		S	J1	J2	J3				
Par	8.0	12.49	12.43	9.69	3.33	12.22	47.87	4.69	
Premium	7.0	8.76	8.74	6.86	2.59	8.58	64.57	3.27	
	6.5	5.02	4.99	3.89	1.62	4.92	80.51	1.95	
Discount	9.0	13.15	13.02	9.97	3.26	12.85	44.33	5.15	
	10.0	13.39	13.14	9.99	3.33	13.08	42.91	5.42	

¹ The principal paid off in each month is multiplied by the age of the loan in that month, aggregated, and divided by the total value of principal to create a value-weighted age of the tranches. This measure is superior to the actual age of the tranches because tranches can linger on for quite some time with very small amounts of principal.

² Prepayment is the total principal prepaid through maturity as a percentage of the face value of the pool.

³ Default is the total principal defaulted through maturity as a percentage of the face value of the pool.

10,000 simulation runs were conducted for each valuation.

Table 8
Effect of Degree of Subordination on the Value of Tranches

Whole-loan mortgage pool: coupon = 8%, maturity = 30yrs, servicing fee = 0.05%,
 default recovery rate = 0.4, loan-to-value ratio = 0.80

Face value of the tranches/face value of the pool: $S = F_s\%$, $J1 = 0.5(100 - F_s)\%$, $J2 = 0.3(100 - F_s)\%$,
 $J3 = 0.2(100 - F_s)\%$

All tranches have the same coupon rate of 7%.

CIR interest rate parameters: $k = 0.08$, $m = 0.1$, $\sigma = 0.05$

Housing value dynamics parameters: $\mu = 0.10$, $b = 0.05$, $\sigma_H = 0.05$

Subordination %	Value of tranches				Generic MBS
	S	J1	J2	J3	
0	104.69	-	-	-	104.69
1	105.58	26.25	16.18	11.30	104.69
2	106.10	55.35	24.90	10.36	104.69
3	106.30	79.86	37.52	11.96	104.69
4	106.34	94.65	51.17	14.55	104.69
5	106.34	101.97	63.87	17.63	104.69
6	106.33	105.27	74.67	21.07	104.69
8	106.30	107.34	89.99	28.28	104.69
10	106.26	107.86	98.56	35.80	104.69

All calculations are for par coupon i.e. with a refinancing rate of 8%
 10,000 simulation runs were conducted for each valuation.

Figure 1
Effect of Refinancing Rate on the Value of Tranches (with Shifting of Prepayments)

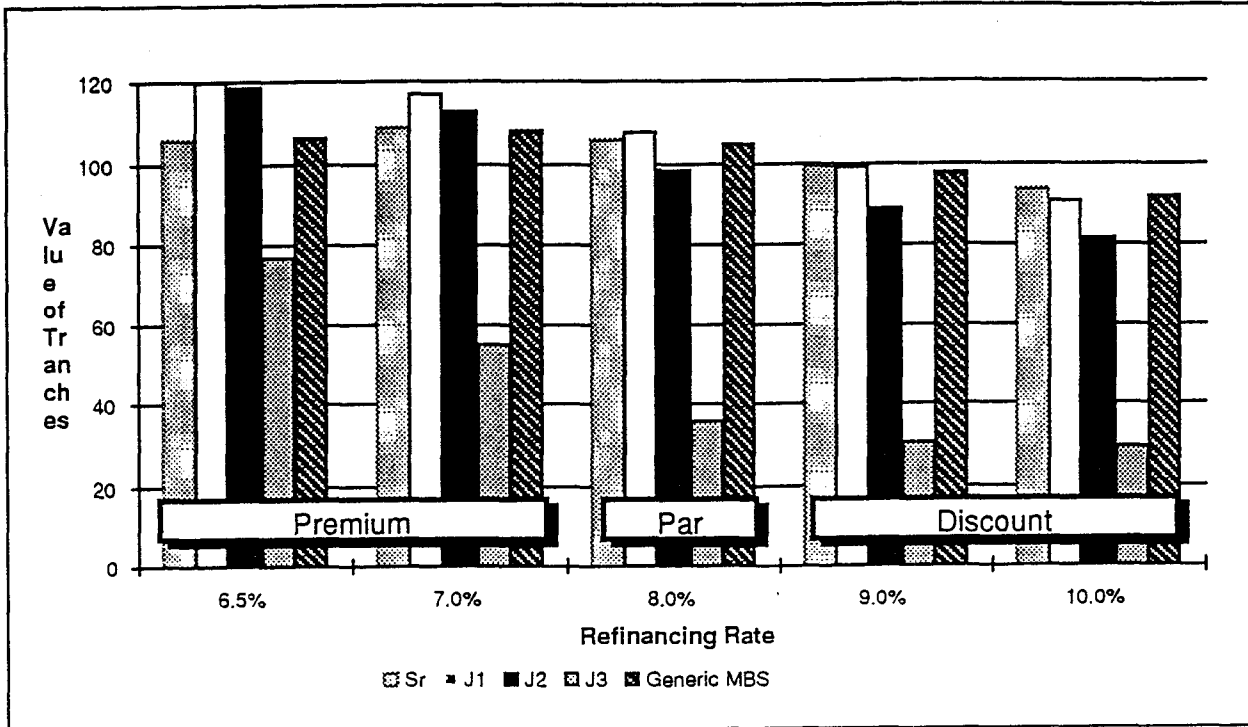


Figure 2
Effect of Refinancing Rate on the Value-Weighted Age of Tranches (with Shifting of Prepayments)

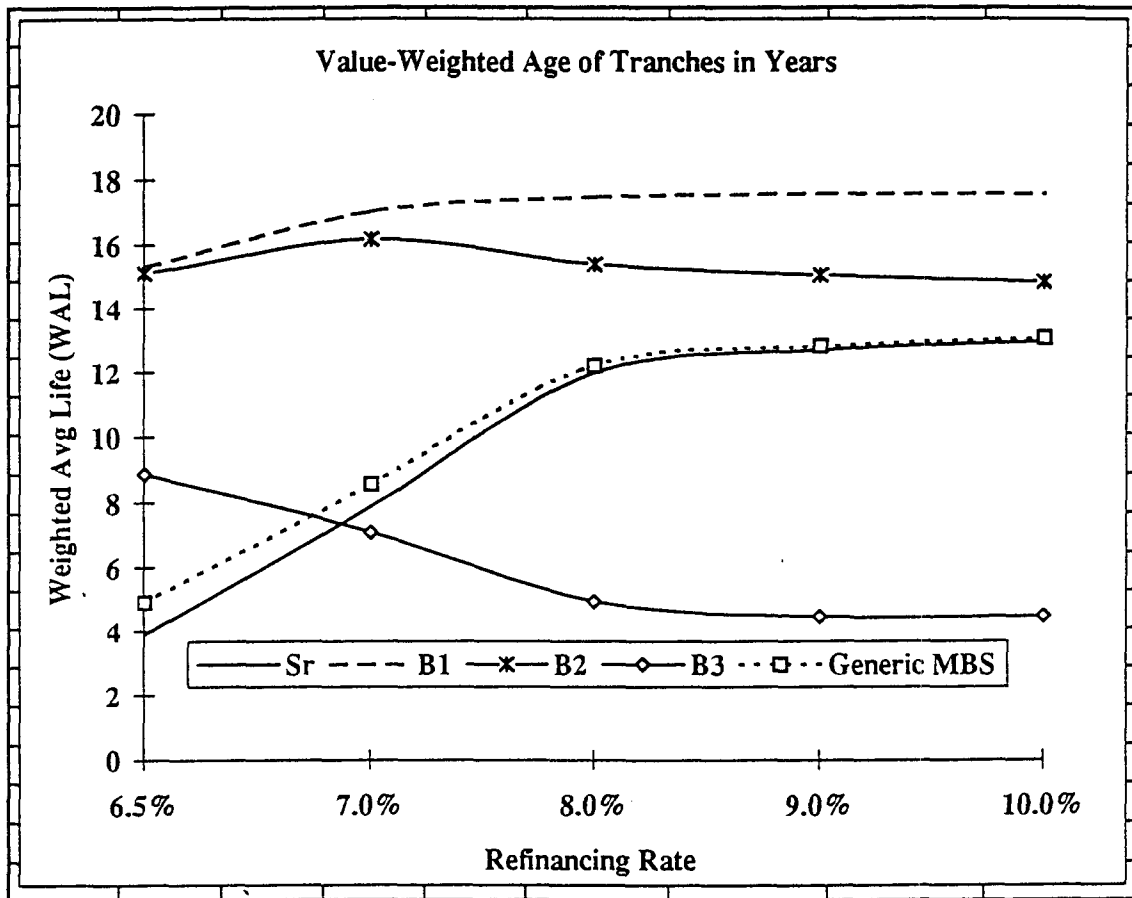


Figure 3
Effect of Shifting of Prepayments on the Value-Weighted Age of Senior Tranche

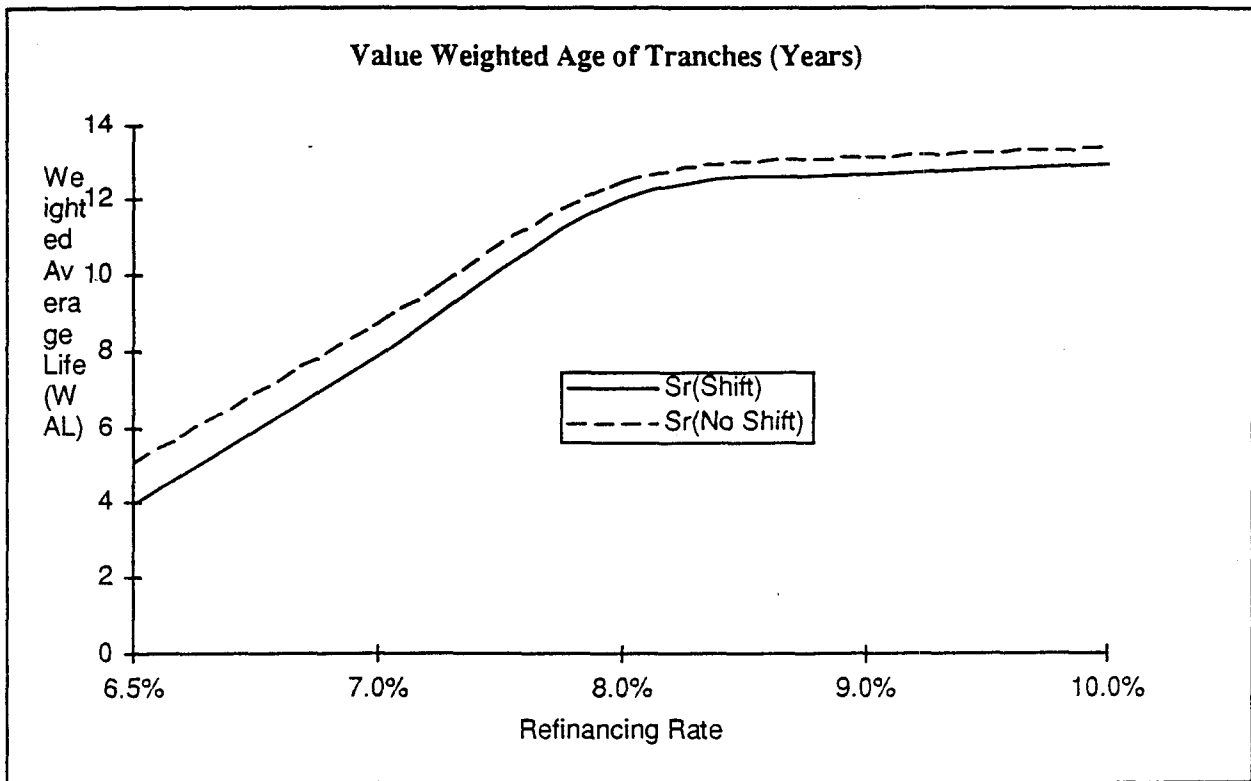


Figure 4
Effect of Shifting of Prepayments on the Value-Weighted Age of Junior Tranches

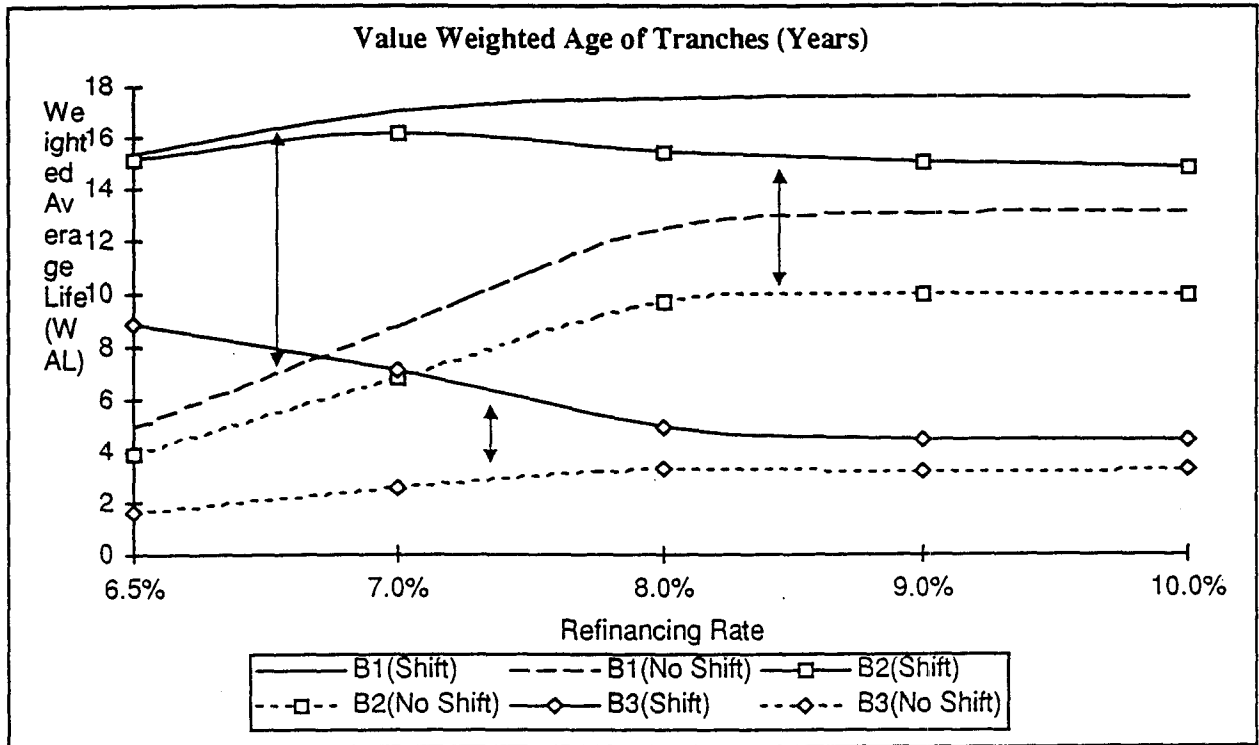


Figure 5
Effect of Shifting of Prepayments on Senior Tranche Value

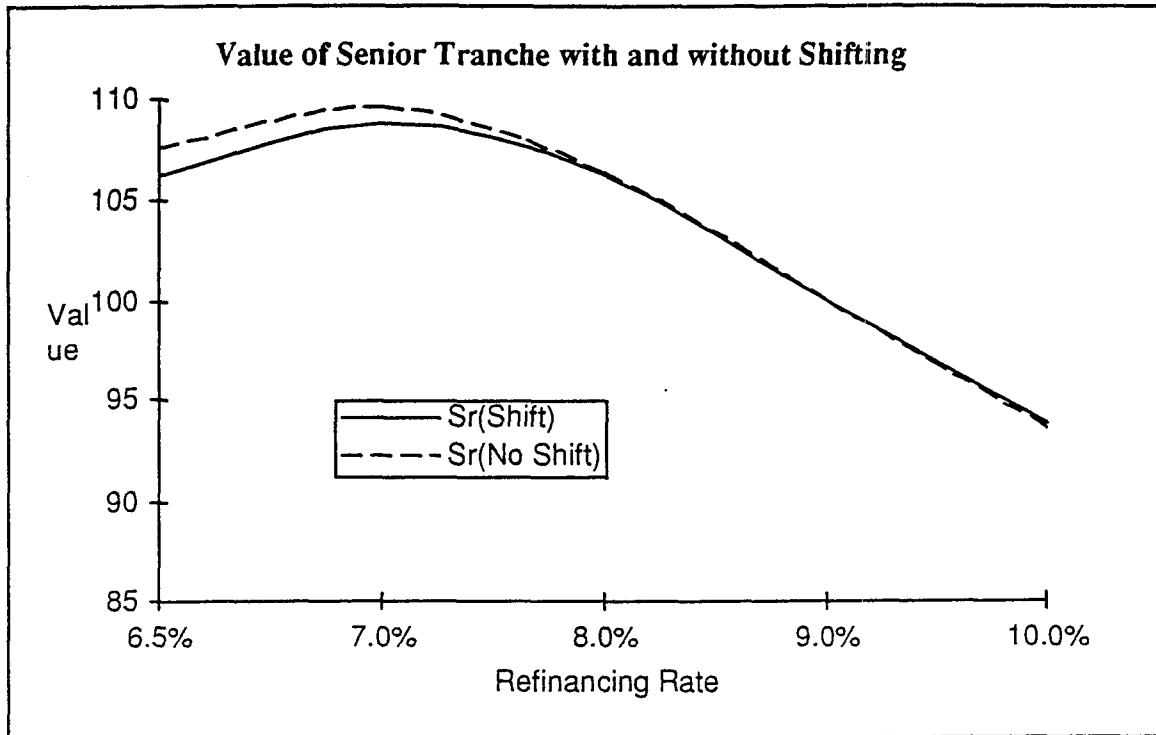


Figure 6
Effect of Shifting of Prepayments on Junior Tranche Values

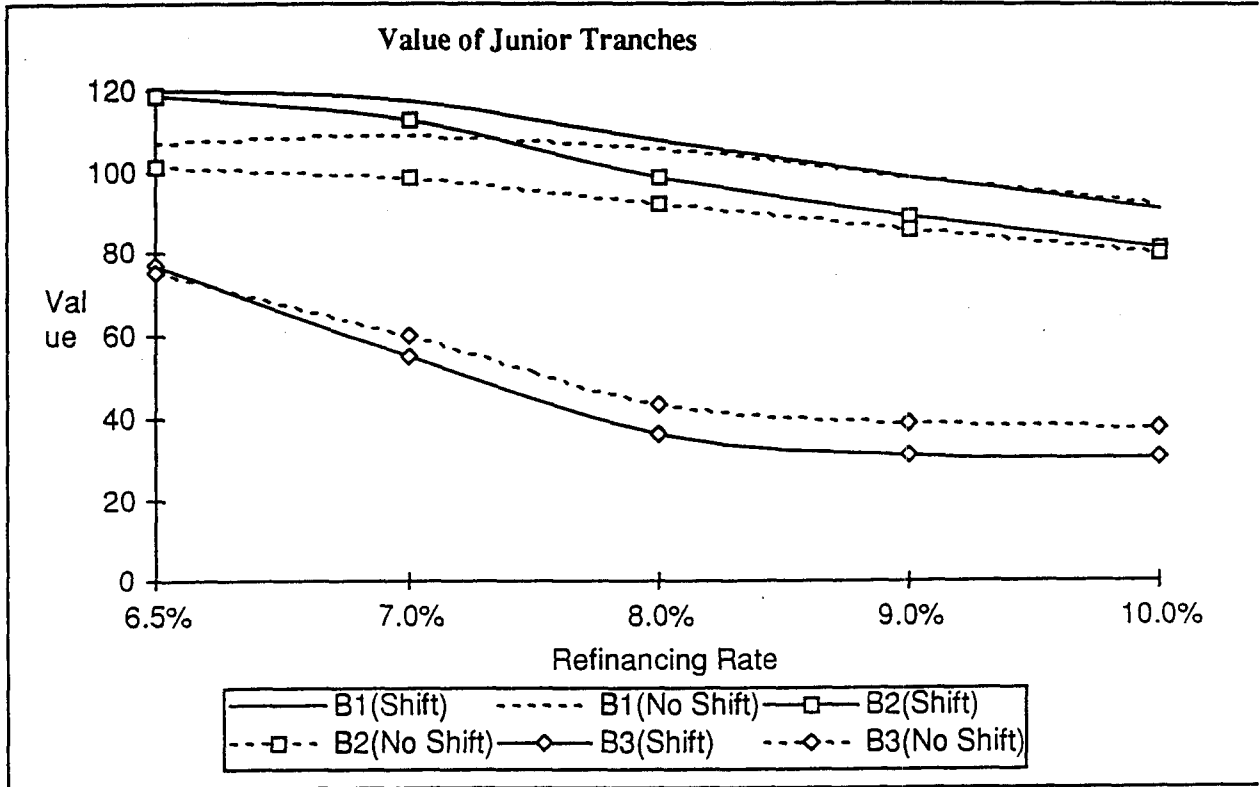


Figure 7
Effect of Degree of Subordination on Senior Tranche Value

Assumptions:

- Degree of subordination is increased from 0% to 10%
- Refinancing rate is constant and equal to the coupon rate (par coupon)

