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Abstract

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Keywords

mortgages, futures, market, treasury bills, GNMA, hedging

Disciplines

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Comments

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Effective Hedging of Mortgage Interest Rate Risk

by Robert W. Kolb, John B. Corgel, and
Raymond Chiang*

The recent increase in the volatility of mortgage rates emphasizes the importance of an institutional structure that builders, investors, and other mortgage market participants can use to manage their mortgage interest rate risk. This need has been met in large measure by the emergence of an interest rate futures market and by the development of the GNMA futures contract in particular.

Unfortunately, the hedging effectiveness of the GNMA futures market has been diminished by a lack of understanding of the selection of proper hedge ratios. This paper presents a derivation of the optimal hedge ratio for hedging interest rate risk with a GNMA futures contract. The hedge ratio is then applied to different hedging situations and the results of the traditional and newly derived hedging strategies are examined.

I. Introduction

The recent advent of interest rate futures markets has greatly enriched the hedging opportunities of mortgage market participants faced with undesired interest rate risk. The variety of futures contracts now spans a number of instruments with different risks, maturities, and coupon structures, including futures contracts on treasury bills and bonds, GNMA passthrough certificates, GNMA CDRs (collateralized depository receipts), and commercial paper. Yet, despite the diversity of

*Kolb: Emory University; Corgel and Chiang: University of Florida. The views expressed in this paper are those of the authors and are not intended to reflect the views or policy of the Federal Home Loan Bank Board.

instruments, not every mortgage market participant is able to hedge an instrument with exactly the same characteristics as an instrument for which futures contracts are traded. This lack of correspondence between the risk, maturity, and coupon structure of the hedged and hedging instruments is usually not fully addressed in practical guides to hedging with interest rate futures.¹ These practical guides, using what has been called the traditional theory [Ederington, 1979, p. 159], usually gives examples in which each dollar of the risky asset is hedged with one dollar of a futures contract. Consequently, this traditional approach fails to account for the different price sensitivities of the hedged and hedging instruments.

The need to adjust for the different price sensitivities of the hedged and hedging instruments has drawn the attention of both market professionals and academicians. Market professionals, such as Bass [1980], Stigum [1980], and Weissman [1980], have suggested that hedging strategies ought to adjust for differentials in risk and maturity. The academic research of McEnally and Rice [1979] and Ederington [1979] has attempted to deal with differing price sensitivities by following a "portfolio approach," in which the hedger is seen as buying and selling "futures for the same risk-return reasons as one buys other securities" [Ederington, 1979, p. 161]. The hedger, however, often is not adding a futures contract to a portfolio, but instead confronts a certain interest rate risk on a particular instrument over a fairly definite time horizon. In such instances, the portfolio approach is not easily adapted to the hedger's needs. Franckle [1980] has derived a superior hedge to that of Ederington by adjusting Ederington's hedge ratio for differences in maturities. Although an improvement, a hedge ratio that includes an adjustment for maturities, but ignores the durations of the underlying instruments, is not sufficiently precise.²

These limitations of the traditional and portfolio approaches emphasize the need for a strategy that can be applied to particular risk situations, while still taking account of different price sensitivities for the hedged and hedging instruments. Anytime the price sensitivities (depending on the maturity, coupon, risk, and term structure of the instruments) do not match, one is confronted with a situation of "cross-hedging." The difficulties of matching the price sensitivities of the hedged and hedging instruments are particularly acute when hedging with GNMA futures contracts. Since these contracts are written on certificates collateralized by a pool of government-insured and guaranteed mortgages, in which one obtains only a fractional interest, hedging with GNMA futures invariably involves cross-hedging. GNMA futures contracts are written on the assumption that the residential mortgages in the pool have an 8 percent coupon (or an equivalent number of mortgages in

¹See Bacon and Williams [1976], Chicago Board of Trade [1975], Chicago Board of Trade [1977], Chicago Board of Trade [1978], Loosigian [1980], and International Monetary Market [1977].

²See Section II of this paper.

the pool with other coupons) and mature in 30 years, but are, in fact, prepaid in 12 years.³ A firm seeking to hedge a commitment on a 20-year commercial mortgage, for example, should expect that a given change in interest rates will affect the value of the GNMA futures contract and the commercial mortgage differently. If the firm has naively hedged each dollar of the mortgage commitment with one dollar of the GNMA futures contract, then a change in interest rates will generate either a profit or a loss. However, it is possible to determine an optimal hedge ratio that controls for mismatches in the four factors mentioned above.

The need for such a technique has been exacerbated by extremely volatile interest rates that currently affect the operations of financial institutions, corporations, builders, and underwriters to a much greater extent than ever before. Furthermore, the recent development of the interest rate futures market, with an organized exchange and standardized contracts, has lowered the cost of hedging and increased the hedging opportunities of those facing undesired interest rate risk. The resulting rapid growth of these markets, and the GNMA futures market in particular, emphasizes the need for a hedging technique that deals with diverse price sensitivities, yet can be applied to hedge a particular risk.

The purpose of this paper is to provide a technique for determining the optimal hedge ratio and to show how it can be implemented in a practical context. Consequently, Section II derives the optimal hedge ratio and formulates more efficient hedging rules using GNMA futures. Section III provides practical examples demonstrating the efficiency of the hedging rules. Finally, Section IV concludes the paper with some final guides for the practical implementation of the strategy.

II. The Optimal Hedge Ratio and More Efficient Hedging Rules

Upon entering the futures market to hedge some interest rate risk, the hedger knows the maturity and coupon of both the hedged and hedging instruments. However, if the hedge is a cross-hedge, as is often the case with a GNMA futures hedge, the changes in the risk and term structures of the two instruments may differ during the life of the hedge. These last two features, pertaining to the risk and term structure of interest rates, really concern the size and pattern of interest rate changes. In formulating the hedge these features cannot be accounted for since they are unknown to the hedger. If the future course of interest rates could be determined, the prospective hedger would not hedge anyway. Instead, the portfolio would simply be altered to profit from the rate changes that were about to occur. Since the risk and term structure of the two instruments cannot be predicted with certainty at the time the hedge is instituted, it is impossible to guarantee in advance that the hedge will be perfect (a perfect hedge is one that leaves the hedger's

³Dunn and McConnell [1980] developed a complex pricing model for GNMA passthrough securities which incorporates random prepayment possibilities.

wealth unchanged). However, if changes in the term and risk structures are assumed known, it is possible to derive a hedge ratio that protects against interest rate risk due to a mismatch of maturity and coupon between the hedged and hedging instruments.

Any hedging strategy makes some implicit assumptions about the kinds of interest rate changes that will occur, and any hedging rule implies beliefs about the future course of interest rates. The derivation of this hedge ratio explicitly assumes: (1) The term structure for each instrument is flat over the life of the hedge, and (2) Changes in the risk-free rate drive changes in the rates on all other instruments. A flat term structure is assumed in order to make the mathematics tractable. Moreover, it is a convenient approximation to the case of a yield curve that maintains its shape, changing only its level. The risk-free rate is assigned a crucial role in order to provide a common variable against which changes in other rates can be measured.

With this in mind the basic strategy revolves around choosing some number, N , of futures contracts, j , to hedge one unit of asset, i , with the goal that over the life of the hedge:

$$\Delta P_i + \Delta P_j(N) = 0, \quad (1)$$

where P_i and P_j are, respectively, the values of the instrument to be hedged and the futures contract. Clearly, for any given interest rate shock, the size of ΔP_i and ΔP_j depends on the sensitivity of P_i and P_j to a change in interest rates. Our problem, then, is to choose the number of futures contracts (N) to trade to offset the differing interest rate sensitivities of i and j , and thereby to make equation (1) hold.

To find N , one must solve the equation:

$$\frac{\partial P_i}{\partial R_f} + \left(\frac{\partial P_j}{\partial R_f} \right) N = 0. \quad (2)$$

The values of instrument i and futures contract j are given by equations (3) and (4), respectively:

$$P_i = \sum_{t=1}^I \frac{C_{it}}{(R_i)^t}, \quad (3)$$

$$P_j = \sum_{t=1}^J \frac{C_{jt}}{(R_j)^t} - \sum_{t=1}^J \frac{C_{jt}}{(R_j^*)^t}, \quad (4)$$

where:

C_{it}, C_{jt} = the cash flows of i in the t^{th} period, and the cash flow of the asset underlying futures contract j in the t^{th} period.

R_i, R_j = $1 +$ the yields of asset i and the instrument underlying futures contract j .

R_j^* = $1 +$ the yield implied by the futures price on the instrument underlying futures contract j .

R_f = $1 +$ the risk-free rate

I, J = time to maturity of instrument i , and the asset underlying

futures contract j , respectively.

Equation (3) is simply the standard bond-pricing formula.

Note that equation (4) is not the futures price, but the *value* of a futures contract. When one enters a futures contract one agrees to pay the futures price, FP_j , at the maturity of the contract, in exchange for the series of cash flows, C_{jt} , associated with the deliverable instrument. Equation (5) expresses the futures price, FP_j , as a function of C_{jt} for the deliverable instrument, and the futures yield, R_j^* . Once one enters a futures contract only one R_j^* will make equation (5) hold. Consequently, the futures yield is fixed.

$$FP_j = \sum_{t=1}^j \frac{C_{jt}}{(R_j^*)^t} \quad (5)$$

Over the life of the futures contract, interest rates may vary, which would cause a change in the futures price in the market. This, of course, generates profits or losses on the futures contract. If one entered into contract at yield R_j^* , and yields change subsequently, then equation (4) expresses the profit or loss associated with the futures contract. It is in this sense that futures contracts have value.

Upon entry, if the market is efficient, a futures contract has no value, since the futures yield must equal the market rate on the deliverable instrument. In terms of equation (4), $R_j^* = R_j$, and $P_j = 0$ at the time the futures contract is instituted. Later, yields may change so that $R_j^* \neq R_j$, and $P_j \neq 0$.

Substituting equations (3) and (4) into equation (2) gives:

$$\frac{\partial \sum_{t=1}^i \frac{C_{it}}{(R_i)^t}}{\partial R_i} \frac{\partial R_i}{\partial R_f} + \frac{\partial \left[\sum_{t=1}^j \frac{C_{jt}}{(R_j)^t} - \sum_{t=1}^j \frac{C_{jt}}{(R_j^*)^t} \right]}{\partial R_j} \frac{\partial R_j}{\partial R_f} N = 0, \quad (6)$$

from which it follows:

$$\frac{1}{R_i} \sum_{t=1}^i \frac{-t C_{it}}{(R_i)^t} \frac{\partial R_i}{\partial R_f} + \frac{N}{R_j} \sum_{t=1}^j \frac{-t C_{jt}}{(R_j)^t} \frac{\partial R_j}{\partial R_f} = 0, \quad (7)$$

Solving for N yields:

$$N = - \frac{R_j \sum_{t=1}^i \frac{t C_{it}}{(R_i)^t} \frac{\partial R_i}{\partial R_f}}{R_i \sum_{t=1}^j \frac{t C_{jt}}{(R_j)^t} \frac{\partial R_j}{\partial R_f}} \quad (8)$$

Equation (8) appears quite formidable, but it can be simplified and rendered more intelligible by recalling that MacCaulay [1938] developed a measure of a financial asset's interest rate sensitivity called duration,⁴ and defined it as:

⁴For a detailed reference list on the duration literature, see Bierwag, Kaufman, and Khang [1978].

$$D_k = \frac{\sum_{t=1}^k \frac{t C_{tk}}{(R_k)^t}}{\sum_{t=1}^k \frac{C_{tk}}{(R_k)^t}} \quad (9)$$

Using equations (3) and (9) and substituting into equation (8) gives:

$$N = - \frac{R_i P_i D_i}{R_i F P_j D_j} \frac{\partial R_i}{\partial R_F} \quad (10)$$

Note that, in equation (10), P_i , D_i , $F P_j$, and D_j are all the values that are expected to obtain on the planned termination date of the hedge. The expectation is formed on the initiation date of the hedge. Since the yield curve is assumed to be flat, these are the prices and durations that will obtain, assuming no change in rates.

Assuming $\partial R_i / \partial R_F$ and $\partial R_j / \partial R_F$ can be estimated, those estimates should be calculated in the computation of N for improved hedging performance. However, if both i and j are risk free, then equation (10) becomes:

$$N = - \frac{R_i P_i D_i}{R_i F P_j D_j} \quad (11)$$

The R_i is the rate on the hedged instrument, i , that is anticipated to obtain on the planned termination date of the hedge. Since the term structure is assumed to be flat, R_i in equation (11) is the currently prevailing rate on i .⁵ P_i and D_i are, respectively, the price and duration of i expected to obtain on the planned termination date of the hedge.⁶

The next section illustrates applications of the hedge ratios in two situations: (1) Where one assumes both instruments i and j are risk free, thus the hedge ratio to be used is equation (11); and (2) where $\partial R_i / \partial R_j$ is estimated statistically, and the appropriate hedge ratio is equation (10).

III. Applications of the Hedging Rules

The hedging rules presented in the previous section are illustrated with the aid of two simplified examples of cross-hedging opportunities encountered in housing finance.⁷ In the first case, the hedged and

⁵The current rate is assumed to be the best estimate of the future rate. A more sophisticated approach would use the corresponding forward rate as an estimate of the futures rate to prevail on the planned termination date of the hedge.

⁶The unavoidable fact that hedging is tied to a certain time horizon, even for the "portfolio approach," is demonstrated by Franckle [1980]. The ratio $(\partial R_i / \partial R_F) / (\partial R_j / \partial R_F)$ in equation (10) is similar to the b term in Ederington's hedge ratio, whereas the balance of the right side of equation (10) is an adjustment for the timing of cash flows. This adjustment is superior to simply adjusting by maturities as in Franckle [1980].

⁷For more complex examples, see Schwartz [1979, pp. 179-191]. His examples, however, follow naive hedging rules.

hedging instruments are assumed to respond alike to changes in R_f . This assumption is relaxed in the second example.

Case 1: Hedging a Tax-Exempt Housing Bond Issue

Consider in the first instance, a state housing bond manager who learns on March 1 that the state plans to issue \$25 million, 10-year tax-exempt housing bonds at par on June 1 to provide funds for low-interest, single-family mortgage loans. Current yields on housing bonds are 10 percent. Since home mortgage rates are already high and rising rapidly, the manager recognizes the urgency of locking in the current tax-exempt bond yield. Otherwise, even below-market mortgage rates will be beyond the affordability limits of most home buyers. One approach available to the manager for transferring this interest rate risk is to use the GNMA futures market.

TABLE 1
HYPOTHETICAL RATES FOR GNMA
FUTURES AND TAX-EXEMPT HOUSING BONDS

Date	GNMA Futures	Housing Bonds
March 1	12.00%	10.00%
June 1	14.00%	12.00%

The rates assumed for this example are presented in Table 1. The price (P_i) of bonds is expected to be \$1,000 on June 1, and the duration (D_i) is 6.5590 years. The price of the underlying GNMA 8% certificate (FP_j) on June 1 is \$75,776.37 and its duration (D_j) is 6.8511 years. Determining the appropriate hedge ratio when hedging housing bonds (i) with GNMA futures (j) becomes a rather straightforward matter of assigning numerical values to equation (11).

$$N = - \frac{(\$1,000) (6.5590) (1.12)}{(\$75,776.37) (6.8511) (1.10)} = - .01286$$

An interpretation of N is that .01286 GNMA futures contracts should be traded (sold) for each bond issued. Since the manager expects the size of the bond issue to be \$25 million and the price of each bond to be \$1000, the size of the issue in terms of the number of bonds is 25,000. Consequently, the manager should sell 321.50 (.01286 \times 25,000) GNMA futures contracts to hedge the interest rate risk on the issue.

With the changes in rates shown in Table 1, the price of the bond on June 1 is actually \$885.29 instead of the expected price of \$1,000. Since the bonds are issued on June 1, there is an opportunity loss of \$2,867,750 on the issue (\$114.71 \times 25,000). Assuming an average mortgage loan of \$50,000, the opportunity loss in terms of single-family home purchases

would be approximately 57 homes, ignoring all transaction costs.

A comparison is now made between the naive hedging strategy and the price sensitivity (PS) strategy. In accordance with the naive strategy, the manager sells one dollar of face value in the futures market for each dollar of face value in the bond issue. Thus, 250 GNMA contracts would be sold according to the naive strategy versus 321.50 following the PS strategy. An increase in rates on GNMA futures from 12.00 percent to 14.00 percent causes the price of a GNMA futures contract to fall by \$8,943.12. Since the bond manager sold futures contracts on June 1, this change in price creates a realized gain from covering the short position.⁸ Table 2 shows the results from completing this hedge following both strategies.

TABLE 2

COMPARISON OF HEDGING STRATEGIES:
TAX-EXEMPT HOUSING BOND EXAMPLE

<i>Opportunity Loss on Bond Issue</i>	<i>Realized Gain on GNMA Futures Transactions</i>	
	<i>Naive Hedge</i>	<i>PS Hedge</i>
\$2,867,750	\$2,235,780	\$2,875,213
ERROR	-\$631,970	+\$7,463

While neither hedge is perfect, following the PS strategy hedges 99.74 percent of the opportunity loss while the naive hedge covers only 77.96 percent. Similarly, the error from the naive hedge is 84.68 times the error resulting from implementing the PS strategy. The PS strategy would have had a zero error had the changes in interest rates been infinitesimal, rather than discrete.⁹

Case 2: Hedging a Market Rate Commitment

Consider in this second case a builder (owner) who, on March 1, secures a \$5 million forward commitment for permanent financing on a planned commercial/residential project that he plans to "take down" on September 1. The mortgage has a 20-year term, with payments made semiannually, and a stated interest rate of 9.89 percent. Until recently, builders have ordinarily entered into mortgage commitment contracts with a stated rate of interest. The current trend, however, is toward market- or floating-rate mortgage commitment contracts.¹⁰ With a

⁸See Bacon and Williams [1976, p. 38] for the GNMA pricing model. The model uses the conventional twelfth year prepayment assumption.

⁹When interest rate changes are discrete, one can *construct* examples in which the naive hedge would give superior results. However, in general, the PS strategy would outperform the naive hedge over the long run.

¹⁰See *The Wall Street Journal*, April 11, 1980, p. 44.

market-rate commitment, the builder incurs the risk of mortgage rate movements that he/she is presumably unwilling to assume.

Selling GNMA futures contracts is one approach available to the builder for transferring interest rate risk, as it was for the bond manager. Thygeson [1978], however, has cited reasons against using the GNMA futures market to hedge with conventional mortgages in the case of a lender assuming the interest rate risk on a fixed-rate commitment. One issue involves the absence of symmetry between futures and commitment contracts, in that the commitment contract is only legally binding upon the lender. This problem fortunately vanishes with a market rate commitment since the borrower essentially commits to "taking down" the funds by electing to sell GNMA futures, otherwise incurring unabated monetary losses on the futures market transaction if rates rise.

The second issue is more vexatious. Ostensibly, yields on GNMA's and conventional mortgages in the cash market experienced movements in opposing directions at two points during the recession of the mid-1970s, thus raising the possibility of pronounced losses resulting from concurrent declines in mortgage yields and prices on GNMA's. While this evidence clearly indicates difficulties for cross-hedgers in periods of abnormal economic decline, the prevailing situation in a normal economic environment is for conventional mortgage and GNMA yields to be highly correlated (positive). Plant [1976], for example, has estimated the long-run, zero-order correlation coefficient to be .84.

This second example uses the fully developed hedge ratio given as equation (10), which reflects the estimation of $\partial R_i/\partial R_j$. Using monthly data for commercial mortgage commitments and GNMA futures contracts,¹¹ $\partial R_i/\partial R_j$ is estimated as being equal to γ_1 in the following OLS regression equation:¹²

$$R_{it} = \hat{\gamma}_0 + \gamma_1 R_{jt} + \epsilon_{it} \quad (12)$$

The error term, ϵ_{it} , is assumed to follow a first-order autoregressive process. Thus, with a correction for serial correlation, the estimated form of equation (12) is¹³

$$R_{it} = 6.2714 + .4211 R_{jt} \quad R^2 = .4252$$

(9.37)	(6.37)	SE = .2994	(13)
		D.W. = 1.23	
		t - statistics in parentheses	

¹¹These data are for the period January 1976 through September 1980. Although GNMA futures contracts were traded as early as October of 1975, the January 1976 starting date was selected to allow for market seasoning and more reliable data. Mortgage commitment rates are from the American Council of Life Insurance, 1981 and the yields on GNMA futures contracts are taken from *The Wall Street Journal*.

¹²We would like to thank an anonymous referee for pointing out an error in the regression in an early version of the paper.

¹³Ederington [1979] interpreted the R^2 from a similar regression as a measure of the effectiveness of the hedge.

Table 3 presents the rates assumed for this example. These rates are chosen to correspond to their actual long-run market relationship over the estimation period for the regression.

TABLE 3
RATES FOR GNMA FUTURES
AND CONVENTIONAL MORTGAGE COMMITMENTS

Date	GNMA Futures	Conventional Mortgage
March 1	9.00%	10.06%
September 1	11.00%	10.90%

To hedge this interest rate risk, the builder could sell 50 GNMA futures contracts in accordance with the naive strategy. Alternatively, the PS strategy could be followed in its expanded form as indicated in equation (10). With the initial futures rate of 9 percent, the futures price (FP_j) is \$92,802.75, and the duration of the underlying instrument (D_j) is 7.3564 years. The value of the mortgage anticipated to hold on September 1 (P_j) is \$5,000,000, with a duration (D_i) of 7.1819 years.

The hedge ratio is calculated below.

$$N = - \frac{(\$5,000,000) (7.1819) (1.09)}{(\$92,802.75) (7.3564) (1.1006)} .4211 = -21.94.$$

Following the PS strategy, the builder sells 21.94 contracts to hedge the interest rate risk on the commitment. With an increase in the mortgage commitment rate from 10.06 percent to 10.90 percent, the builder suffers an opportunity loss of \$274,080. It is this loss the builder is attempting to offset with futures market transactions. Table 4 presents the results from implementing two possible strategies for offsetting the loss.

TABLE 4
COMPARISON OF HEDGING STRATEGIES:
MORTGAGE COMMITMENT EXAMPLE

<i>Opportunity Loss on Mortgage Commitment</i>	<i>Realized Gain on GNMA Futures Transactions</i>	
	<i>Naive Hedge</i>	<i>PS Hedge</i>
\$274,080	\$594,294	\$260,734
ERROR	+\$320,214	-\$13,346

As in the previous example, neither hedge is perfect. Following the appropriate PS strategy, however, results in an error of less than five percent (4.87), while an error of approximately 116 percent (116.83) of the opportunity loss occurs when adopting the naive strategy. The error from the naive strategy is nearly 24 times larger.

IV. Conclusion

The development of an organized futures market in GNMA passthrough certificates has given those operating in the mortgage markets a new and varied set of opportunities for managing the risk associated with price movements in mortgages. With the wide diversity of interests found among mortgage market participants, cross-hedging transactions have become more the rule than the exception in the GNMA futures market. Thus, a need is seen for a hedging strategy that explicitly accounts for the price sensitivities of hedged and hedging mortgage instruments to facilitate efficient cross-hedging.

The PS strategy presented in this paper provides a rational approach to the problem of cross-hedging. This strategy accounts for differences in the maturities and coupon structures of the instruments involved, and when assuming a flat yield curve and infinitesimal changes in interest rates, the PS strategy results in a perfect hedge.

Since, in practice, the term structure is seldom flat and interest rates change constantly by differing amounts, the PS strategy cannot be expected to result in a perfect hedge. Yet, as the examples in Section III demonstrate, the PS strategy will produce substantially smaller hedging errors than a naive hedge will. In each instance the error from the PS strategy is less than 5 percent, whereas the naive strategy results in errors in excess of 20 percent.

One final note on applying the hedge ratio. Since rates vary over time, hedging performance is improved by periodic rebalancing of the hedge. The PS strategy is designed to hedge against a single interest-rate shock. With constantly fluctuating rates, hedging performance can be improved by periodic recalculation of N and by adjusting the hedge accordingly. The frequency of rebalancing will depend upon the size of the position, transaction costs of changing the hedge, and the anticipated volatility of interest rates.

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