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Reapportionment of the House of Representatives Under the 1950 Census

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REAPPORTIONMENT OF THE HOUSE OF REPRESENTATIVES UNDER THE 1950 CENSUS

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The Census of 1950 is completed and the results were transmitted by the President to Congress in January last. He reported, not only the populations of the forty-eight states, but also the number of Representatives to which each state will be entitled, under the present law, at the election in November, 1952, and thereafter until a new census results in a fresh reapportionment. The choice of our next President will also be affected by this report, since the electors for every state are equal to its Senators and Representatives combined. A good many newspaper readers may have wondered that the President and the Census Bureau knew how many Congressmen to give each state. They may have been puzzled by references to certain mathematical formulas with queer names.

Hence this article endeavors to give a simple explanation of the distribution of seats in the House of Representatives. It is not certain at the date of writing (in early April, 1951) whether Congress will abide by the scheme submitted to it last January, but a change is unlikely. At all events, this exposition will be applicable to whatever scheme goes into effect for the current decade. To those who still repeat with feeling the childish jingle,

> The rule of three, it puzzles me,
> And fractions drive me mad.

no promise can be made that the illustrative calculations in the text will be easy, but they require only a knowledge of arithmetic and a little figuring. This subject interests many different kinds of people—lawyers, mathematicians, students of government, politicians who carry on government, and common or garden citizens, and the essentials are here.[1]

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* See Contributors' Section, Masthead, page 688, for biographical data.

[1] Readers wishing to pursue the subject further may care to consult Chafee, *Congressional Reapportionment*, 42 HARV. L. REV. 1015 (1929), which examines legal as well as mathematical questions; and Chafee, *Reapportioning the House of Representatives under the 1940 Census*, 66 PROC. MASS. HISTORICAL SOC. 365 (1942), which considers how, after the First Census of 1790, seats would have been distributed by the chief mathematical methods. A more recent source is Note, *Apportionment of the House of Representatives*, 58 YALE L. J. 1360 (1949); I strongly dislike its proposal to increase the size of the House, but the mathematical portion is full and excellent.

Considerably different views are expressed by Willcox, *The Apportionment Problem and the Size of the House: A Return to Webster*, 35 CORNELL L. Q. 367 (1949). Although

I. The Present Law

The Apportionment Act of November 15, 1941,[2] still in force, provides for automatic reapportionment on the basis of the 1950 census, if Congress does nothing.

Within a week after the first day of the first regular session of the Eighty-second Congress (and every fifth Congress thereafter), it directs the President to transmit to Congress a statement showing the representative populations of each state, under the census just taken, "and the number of Representatives to which each State would be entitled under an apportionment of the then existing number of Representatives [now 435] by the method known as the method of equal proportions, no State to receive less than one Member."

This scheme becomes effective in the Eighty-third Congress unless a subsequent statute is passed. The Clerk of the House is to send to the Governor of each state the number of its seats, within fifteen calendar days after the receipt of the statement. The law goes on to provide for districting inside each state.

The law also determined the reapportionment based on the 1940 census, using the same method of equal proportions.

II. The Problems Raised by the Present Law

Two purposes of the 1941 Act are significant—the automatic reapportionment and the selection of a particular mathematical method.

In the first place, the Act provides for an automatic apportionment after future censuses, unless Congress takes specific action. This prevents the lamentable situation following the 1920 census, when the failure of Congress to reapportion the House allowed the populations of 1910 to govern the distribution of Representatives and Presidential Electors for over twenty years. Consequently, Congress adopted the policy of an automatic reapportionment in 1929[3] and has continued it in subsequent statutes. This article will assume the desirability of that policy.

It is true that in 1941 the automatic reapportionment was not allowed

disagreeing with many of his conclusions, I esteem my old friend Mr. Willcox, and his initiation of the first Modern Method used, too highly to engage in controversy with him, and shall largely direct this article to an affirmative presentation of my position on the mathematical problems.

For mathematicians, the prime source is Huntington, *The Apportionment of Representatives in Congress*, 30 Trans. Am. Mathematical Soc. 85 (1928). Official documents and other items are listed in 42 Harv. L. Rev. at 1015, n. 2.

2 55 Stat. 761, 2 U.S.C.A. (1949 Supp.) §§ 2a, 2b.

3 46 Stat. 26, § 22 (June 18, 1929).

to take effect, because Congress did enact a slightly different distribution of seats by changing the mathematical method from Major Fractions (used for the 1910 and 1930 censuses) to Equal Proportions.[4] The problem therefore arises: Should Congress repeat this course for the 1950 census, or should it simply accept automatic reapportionment this time?

The only wise reason for not abiding by the automatic reapportionment under the present law, it is respectfully submitted, would be the demonstrated superiority of a different mathematical method over the Method of Equal Proportions. The distribution of seats in the House and the Electoral College should be governed by a general and satisfactory principle, and not depend on pulling and hauling and jockeying after each census. The present law gives us such a satisfactory general principle. It would be well to stick to it, unless we can find a still better general principle.

Thus, the first problem is really the same as the next problem—is there a better mathematical method than the presently prescribed Method of Equal Proportions?

The adoption of that method is the other important feature of the 1941 Act. The main purpose of this article is to discuss that matter, showing why mathematics is involved in reapportionment of the House, explaining and comparing the various available methods, and reaching a conclusion that the method in the present law is the best.

III. Why Reapportionment Involves Mathematics

The original provisions of the Constitution (Art. I, sec. 2) are somewhat modified by the first sentence of section 2 of the Fourteenth Amendment, which reads:

> Representatives shall be apportioned among the several States according to their respective numbers, counting the whole number of persons in each State, excluding Indians not taxed.

(This article will ignore these Indians and use "population" to mean the fignre employed in each state to determine its right to seats in the House.)

The original Constitution supplies two more requirements; but one is obsolete practically, that there must not be more than one Representative for every thirty thousand people. It is still very important that "each State shall have at least one Representative."

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[4] The events in Congress leading to this change and the displacement of the automatic Act of 1929 by a new statute of 1941 are narrated in 66 PROC. MASS. HISTORICAL SOC. at 401-7 (1942).

The members of the Philadelphia Convention of 1787 were probably too busy with other issues to realize the mathematical tangles they were creating. Everything would be very easy if state populations were in round numbers—100,000—200,000—and so on. But people don't multiply and migrate with such obliging regularity, as every census proves. The mathematical difficulties are due to the fact that an absolutely equal apportionment is impossible because a fraction of a Congressman cannot be assigned to a state. All that can be done, in the words of Daniel Webster; is to give each state the number of representatives required by its population *"as near as may be"*.[5]

Five ways of doing this have been tried, and many others suggested. The latest method actually adopted is the Method of Equal Proportions.

This article will endeavor to explain the difficulties and methods by an example involving only five hypothetical states. It seems simpler to use small populations, so that the constitutional minimum district of 30,000 will be disregarded. (If this disregard bothers anybody, just imagine that the given figures are multiplied by 100.)

TABLE 1

States	Populations
A	1,250
B	2,450
C	3,455
D	4,461
E	5,384
Total	17,000

Suppose, in Table 1, Congress decided to have a House with 17 seats. By any method, the minimum number of seats for these states would be A, 1; B, 2; C, 3; D, 4; E, 5. But that disposes of only 15 seats. The big question is—what states will get the other two seats? Obviously, you cannot give A an extra quarter of a Congressman, and so on. The root of the whole trouble is that Congressmen cannot be split up. Yet when the necessity of using whole human beings forces the assignment of the two last seats to only two states out of the five, e.g., giving B 3 members and 5 to D, substantial inequalities exist.

Whatever Congress does, it cannot possibly avoid inequalities. Therefore the true problem is: How can Congress best minimize the inevitable inequalities in representation?

[5] Webster is quoted in 1 STORY, COMMENTARIES ON THE CONSTITUTION 504n (5th ed. 1905); Chafee, *supra* note 1, at 1023.

IV. The Old-fashioned Methods

The instinctive approach to reapportionment is what was taken by Jefferson[6] and by Congress for over a century. First, you obtain an "ideal district" by dividing the total population of the nation by the number of proposed seats. Thus, in my example (see Table 2), the ideal district for 17 seats would be 1,000. Then you divide this number into the population of each state, getting a quotient with an integer and a fraction. Next, you assign seats for the integers. In my example, 15 seats would go in this way: A, 1; B, 2; C, 3; D, 4; E, 5.

Table 2
(17 seats; ideal district, 1000)

State	Population	Quotient
A	1250	1.25
B	2450	2.45
C	3455	3.455
D	4461	4.461
E	5384	5.384

All the trouble arose out of the fractions. In the early reapportionments, Congress ignored them. (You try for 17 seats and get only 15.) After the census of 1840, under Webster's influence, each fraction over one-half was counted as a whole number. (This would do no good in my example—Table 2.) From 1850 through 1900, under the Vinton Method, Congress gave the left-over seats to the states with the largest fractions until the desired size of House was attained. (In Table 2, D would get 5 members, and C four members.)

All sorts of perplexities and ill-feelings arose. Still, they got along well enough so long as there was room in the House for more chairs, so that they could keep adding a seat here and there for a disgruntled state. But by 1910 the House had reached its present capacity of 435 members. Henceforth any change meant that some states would lose seats, and nobody likes that. Consequently, it became important to have a plan of distribution that would hold up mathematically.

There were at least four objections to all these old methods which used the "ideal district":

(1) They produced queer results called paradoxes—startling fluctuations in the size of the House or the number of seats for a state, without any sensible relation to change in population. A state which had

[6] References about Jefferson's rather inadequate views on reapportionment are collected in Chafee, *supra* note 1, at 1022, n. 21; 66 Proc. Mass. Historical Soc. at 380, n. 3, 382, n. 1, 383, n. 1 (1942).

gained in population might lose a seat, while new seats went to other states which had gained relatively less in population or had even lost (the "Alabama paradox").

(2) Every time you assumed a different size for the House, you had to start the whole calculation all over again to see how seats would be distributed.

(3) After a scheme was finished, there was no satisfactory measure of inequalities. State A, which lost a seat by the scheme, would say: "Look at State B, which has a new seat; its average district is smaller than for A." State B could reply: "If you get the seat instead of us, your average district will be smaller than ours." Nobody knew how to end the argument conclusively. The only way to satisfy A and B was to give both of them the seats they demanded, but that violated the essential basis of the method used and might very well mean a larger House (and a correspondingly smaller "ideal district") than you set out to get at the start.

(4) The biggest objection of all was that the fractions don't mean anything really. Whenever one selected a slightly larger or smaller size for the House and hence a different "ideal district," the fractions all jumped up or down. Furthermore, these methods rested on an underlying assumption that the people in the fraction would not be represented at all in Congress unless their state got an extra seat. This assumption was plainly false. Every inhabitant of a state has *his* Congressman, whether he is a voter or not. For example, if C in Table 2 gets only 3 seats, this does not mean three districts of 1000 each and 455 inhabitants outside these three districts. The three Congressmen take care of everybody in state C. What does really matter is that if C has 3 seats and D has 5, the average district in C (1152) is much larger than in D (892). The people of C are less fully represented than those in D. This is the real thing to worry about; the size of the old-fashioned fractions has very little to do with it.

In short, the instinctive approach of the "ideal district" turns out to be a mistake. Congress had to junk the whole conception of the "ideal district" and go at the matter in an altogether different way, based on the realities of the relation between Representatives and their constituents.

V. The Discovery of the Five Modern Methods

In the first decade of this century, mathematicians inside and outside the Bureau of the Census got to work. In 1910 Dr. Joseph A. Hill, Assistant to the Director of the Census, started the mathematical analysis of the problem. In 1911 Professor Walter F. Willcox devised the Method of Major Fractions, which was adopted by Congress in the apportionment of that year. In 1921 Professor Edward V. Huntington made the first extensive investigation of the mathematically possible methods, and recommended the Method of Equal Proportions as the best solution. His conclusions were unanimously endorsed by the Advisory Committee to the Director of the Census and by a specially appointed committee created by the National Academy of Sciences.

The great lesson from these investigations is that there are only five known methods which offer a satisfactory solution of the apportionment problem. Two of these five were rejected as "artificial" by the National Academy of Sciences. One (called the Method of Smallest Divisors) is extremely favorable to small states;[7] and the other (called the Method of Greatest Divisors), to large states. Neither offers a readily under-

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[7] *Mathematical footnote.* This Method of Smallest Divisors has received some support from Mr. Willcox, who names it the Method of Included Fractions. 35 CORNELL L. Q. at 370, 377-80. He attributes to it the merit of producing a less range than any other method between the largest average district and the smallest. Even if this were so, it would not offset the great disadvantages of its being extremely favorable to small states and providing no convenient way to measure inequalities. See 58 YALE L. J. at 1375.

Mathematicians, moreover, can prove that the alleged merit is not inherent in this method. Various calculations have shown me that as I alter the populations and the number of seats in the House, sometimes one method produces the minimum range and sometimes another, and it is not necessarily the Method of Smallest Divisors. Indeed, that method is capable of giving the worst possible result. For instance, with the populations given in Table 4, below, that method will give the 16th and 17th seats to A and B respectively. The range between A's average district of 625 (with 2 members) and C's of 1152 (with 3 members) will be 527, greater than the range by any other method. The minimum range is actually brought about by its opposite extreme, the Method of Greatest Divisors. Being very favorable to the large states, this will give the 16th and 17th members to E and D respectively. The range between D's average district of 892 (with 5 members) and A's of 1250 (with 1 member) is 358, and no other method can reduce this inequality.

The reason why Mr. Willcox's table of actual populations in the United States (35 CORNELL L. Q. at 380) seems to show that the Method of Smallest Divisors (Included Fractions) produces the minimum range is because Nevada in his table always has the smallest district. So mathematical forces have no chance to operate at the lower end of the range; Nevada gets its single seat from the Constitution, not from mathematics. Eliminate the three states (Delaware, Nevada, and Wyoming) which are really outside the mathematical scheme, and take only the 45 states which are affected by your choice of methods. Then you will surely find that Mr. Willcox's Method of Smallest Divisors (Included Fractions) can lose out sometimes. Indeed, this happens right in his own table (p. 380). The range between Vermont with the smallest district (179,615) under his proposed method (right-hand column) and South Carolina with the largest (316,634) is 137,019. But the Method of Equal Proportions (center column) gives New Hampshire the smallest district (245,762) and Vermont now the largest (359,231) with a range of only 113,469—nearly 24,000 less. Of course, this does not mean that Equal Proportions will always produce a minimum range. It is pretty much of an accident which method wins.

standable way to measure inequalities between two states which are competing for a seat. So this article will consider only the remaining three modern methods. These are:

(1) The Method of Equal Proportions, devised by Professor Huntington, recommended unanimously by the Census Advisory Committee and the National Academy of Sciences Committee, used in the apportionment of 1941 and specified in the present law for the coming reapportionment of 1951.

(2) The Method of Major Fractions, devised by Professor Willcox, used in the apportionments of 1911 and 1931.

(3) The Method of Harmonic Mean, which has a good deal in its favor, despite its repulsive name; it has never been used.

VI. The Central Principle of These Three Modern Methods

After 48 seats have been assigned to all the states, so that each has its constitutional minimum of one Representative, then each successive seat is given to the state with the strongest claim to that seat. The essence of these three modern methods is that they ascertain the strength of a state's claim, not by the size of accidental fractions (as in Table 2), but by measuring something that really matters. The measurements are based on the relations between the many human beings who live in the state and the few human beings who sit for that state in the House of Representatives. And, while thus measuring the strength of the claim to the next seat, we are also enabled to measure the inequalities between the state which gets the seat and any disappointed state.

In order to understand the two things that really matter and so are suitable for measuring inequalities, it is desirable to remember that the Constitution embodies two ideals of a reasonably equal apportionment. First, it contemplates approximately equal Congressional districts, that is, every representative should have as nearly as possible the same number of constituents. This ratio is found by dividing the population of a state by the number of its representatives. In that way we get the average district inside each state.

Secondly, the Constitution contemplates that every inhabitant, no matter in what state he lives, shall have as nearly as possible the same representation at Washington, that is, the same share in the attention of a Congressman. In other words, a million inhabitants, no matter where they live, should have the same number of Congressmen. This ratio is found by dividing the number of a state's representatives by its population. (The number of millions in its population is often used purely for statistical convenience, in order to avoid very small decimals.)

Both these ideals are important. The Congressman probably cares more about the size of his district—how many voters he must canvass before elections, how many letters and callers he can expect, etc. The inhabitant cares more about the size of his fractional share in his Congressman; it affects his chances of getting letters answered, his interests considered in Washington, etc.

TABLE 3
(17 seats; Method of Equal Proportions)

State	Poulation	Seats	Average District	Share in Congressman
A	1250	1	1250	1/1250
B	2450	3	817	1/817
C	3455	3	1152	1/1152
D	4461	5	892	1/892
E	5384	5	1077	1/1077

Return to our illustration. In Table 3 where seats have been assigned by Equal Proportions (as will be explained later), the Congressmen from D have smaller average districts than those from C (892 as against 1152); and a resident of D has 1/892 of the attention of a Representative while a resident of C has a substantially smaller share (1/1152).

Approximate equality in both the size of the average district and the individual's share in his Congressman is desirable. There is no inherent reason to prefer one of these ideals rather than the other. If an absolutely equal apportionment could be made, all the states would have the same average districts and the same representation per million; it would make no difference which of the two ratios was considered in the assignment of seats. But inevitable inequalities do, in fact, exist, and in measuring these we shall find that it may make a difference which kind of inequality you look at.

The Method of the Harmonic Mean minimizes inequalities in districts. The Method of Major Fractions minimizes inequalities in representation per million. One clear advantage of the Method of Equal Proportions, as will be shown later, is that it attains both ideals.

VII. How the Bureau of the Census Distributes Seats under the Method of Equal Proportions and What it Would Do under the Other Modern Methods

There are two stages in an apportionment: (1) the distribution of seats to the various states; (2) the measurement of inequalities under this distribution to show that they are reduced to a minimum. The Nineteenth Century Congresses spent all their time on the first stage, and did not know how to tackle the second stage at all. Now, the first stage is entirely taken care of by the Census Bureau, and Congressmen can devote all their attention to the second stage, in a very satisfactory way. There is no longer any need for Congressmen to talk about the "ideal district" (total number of seats divided by national population) or about the fractions. They aren't even part of the first stage. None of these devices was used by the Census Bureau for the current apportionment. The Bureau compiles what is called a "priority list" by using decimals as "multipliers," whether the method used be Equal Proportions or one of the other modern methods. What the Bureau does is complicated and hard to explain. The point is that Congressmen don't need to understand it. They do not have to bother about the way the Bureau distributes seats, because they can test the fairness of the result so easily. All it takes to measure inequalities is a little division or subtraction. A later section of this article will illustrate how this is done.

The rest of this section can consequently be skipped by readers who do not like mathematics. Still, some may want to know what actually happens in the Bureau of the Census. Certainly, nothing important in a democracy ought to be kept a mystery.

Let us begin with my simplified illustration, with only five states. Each state is given its constitutional minimum of one seat. The next ten seats will be distributed to the same states, whatever the mathematical method used. The allotment of the first fifteen seats has now been accomplished, with the following result:

TABLE 4
(15 seats; any method)

State	Population	Seats	Average District Now	Average District If This State Got 16th Seat
A	1250	1	1250	625
B	2450	2	1225	817
C	3455	3	1152	864
D	4461	4	1115	892
E	5384	5	1077	897
	17,000	15		

The question now is what state shall get the 16th seat? The way to decide this, under any modern method, is to see which state has the strongest claim to an added seat. How ought we to measure the strength of each claim—that is the real problem.

Conceivably, we might say that A has the largest district now and so ought to get another seat. (This would really be applying the Method of Smallest Divisors, decisively rejected for reasons stated above.)[8] Notice how the smallest state wins out. And is it fair that A should have two very small districts of only 625 people, while C has 1152 in each district?

So, let's go at it a different way. Which state, after getting a new Representative, would have the largest district? By his test E (see last column of Table 4) would get the 16th seat. (This is the method of Greatest Divisors, also repudiated.)[9] Notice how the largest state comes out ahead. And is it fair to favor E, which already has the smallest district of all five states?

Evidently, we should make a fresh start. It is not fair to measure the claims by the present situation alone, or to look only at the situation after the extra seat is assigned. We ought to have both ends of the process in mind at once, so far as is possible. That means choosing a test in between the number of seats a state has now, and the number (higher by 1) which it would have if given the coveted 16th seat.

For example, look at state A (in Table 4). Instead of dividing its population by 1, its present number of seats, as we did first, or by 2, the coveted number, as we did next, we ought to choose a divisor "midway" between 1 and 2. Similarly, we shall measure B's claim to the 16th seat by dividing its population by a fignre "midway" between 2 and 3. And so with the others. After that, the state which has the highest quotient possesses the strongest claim; it will get the 16th seat. If we want a 17th or 18th seat, we can tell in just the same way which state is entitled to it; and so on until we get as large a House as we desire.

What has been said thus far applies to each of the three significant modern methods. Now, let us see how they differ from each other.

What do we mean by "midway"? For instance, what amount is "midway" between 1 and 2 for A's claim? Offhand, one would say 1½ or 1.5. This is the arithmetic mean, and is in fact used (as we shall see) in the Method of Major Fractions. But there are two other conceptions of a "midway" point familiar to mathematicians, which are at least as useful as the *arithmetic mean.*

The *geometric mean* is also "midway" between two numbers. It is the square root of their product. Thus the geometric mean of 1 and 2

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8 See note 7 *supra.* Under the Method of Smallest Divisors, the claims to a second, third, fourth, etc., seat are measured by dividing the population of every state by 1, 2, 3, . . . The highest quotient gets the next seat to be assigned.

9 See p. 649 *supra.* Under the Method of Greatest Divisors, the claims to a second and successive seats are measured by using as divisors 2, 3, 4, . . .

is the square root of 2. This is as much larger than 1 as 2 is larger than it. Similarly

$$\frac{\sqrt{2 \times 3}}{2} \text{ equals } \frac{3}{\sqrt{2 \times 3}},$$

and so on. The geometric mean is chosen for the divisors in the Method of Equal Proportions.

The last "midway" point which would be useful in apportionments is the *harmonic mean.* For 1 and 2, the harmonic mean is 1.333. . . It is one-third of the way from 1 and two-thirds of the way from 2. The harmonic mean of 2 and 3 is 2.40. In general, it is a point so situated between the two given numbers that its distances from each are proportioned to the lower number and the higher number, respectively. The harmonic mean is calculated by dividing twice the product of the two numbers by their sum. For instance, the harmonic mean of 4 and 5 is

$$\frac{2 \times 4 \times 5}{4 \text{ plus } 5} \text{ equals } \frac{40}{9} \text{ equals } 4.444\ldots$$

This lies .444. . . from 4 and .555. . . from 5. That kind of "midway" point is used in the Method of the Harmonic Mean.

The next task is to show how the Method of Equal Proportions is worked in the Census Bureau, through the use of the geometric mean. Then we shall examine briefly the operation of the other two methods.

(*a*) *Distribution of Seats by Method of Equal Proportions*

In our simplified illustration, the claim of each state to a 2d seat is to be measured by dividing its population by the geometric mean between 1 and 2; to a 3d seat, by using the geometric mean between 2 and 3 as a divisor, and so on. Therefore the Bureau would first obtain a series of divisors comprising the successive geometric means: 1.414, 2.449, 3.4641, 4.4721, 5.477, . . . In effect, it would divide each state's population by these several divisors, thus getting quotients. Any particular quotient does not represent the average district or anything else *per se.* Its value is just to serve as a rank index for measuring the claim of that state to an added seat. Each rank index, with the name of the state and the number of the seat claimed, is put on a card. For our hypothetical five states claiming the 16th seat in the House, the situation would be as in Table 5 (although the cards would not read exactly as here presented).

The next step is to take all these rank indexes and arrange them in

TABLE 5
(16 and 17 seats; Method of Equal Proportions)

State	Population	Seats Already	Seats Claimed	Divisor	Rank Index
A	1250	1	2	1.414	884.0
B	2450	2	3	2.449	1000.4
C	3455	3	4	3.4641	997.378
D	4461	4	5	4.4721	997.51
E	5384	5	6	5.477	983.02

a single series in order of size, beginning with the largest rank index and working down to the smallest. This forms the priority list.

In our illustration the relevant portion of the priority list, under Equal Proportions, would look somewhat as in Table 6.

TABLE 6
(16th to 20th seat; Method of Equal Proportions)

Seat in House	Rank Index	State	Cumulative No. of Seats for States
16	1000.4	B	3
17	997.51	D	5
18	997.37	C	4
19	983.02	E	6
20	884.0	A	2

Therefore, B has the strongest claim to the 16th seat, under this method. D will get the 17th seat. And you can go on and fill 20 seats from this part of the priority list if you desire.

To pass to the real situation in the United States—Given a set of census populations and a particular method, e.g., the Method of Equal Proportions, the Bureau can by a single process of calculations prepare for Congress a statement showing the distribution of seats for a House of 434—435—436—437 members and so on, to cover any size of House which may possibly be desired. It is no longer necessary, as it was under the older methods of the last century, to start the calculation over again for every different number of members. The Bureau prepares a "priority list" long enough to cover the largest number of Representatives conceivably desired; the names of the states are arranged on this list in a series determined by the particular method, the states appearing more or less frequently according to their size. The only significant difference from the above simplified explanation is that the Bureau of the Census does not actually use divisors. It reaches the same result by taking the reciprocals of the appropriate divisors and using them as "multipliers." The reason is that on many calculating machines it is much easier to multiply than to divide. For example, in measuring the claim of a state to a 2d seat, the Bureau does not divide by 1.414 (as in Table 5). Instead, it multiplies the population of the state by .7072 which equals $\frac{1}{1.414}$.

After the priority list is compiled, one goes down the list giving a seat to each state every time that its name appears, and stopping when the required number of seats have been filled. Still other states below stand ready to take successively any additional seats, should these be provided. The priority list somewhat resembles a line of passengers standing in the corridor of a dining car waiting for places. Each is given a seat in turn, and one can tell at a glance who are next entitled to seats if seats become available.

Just to show how easy it is to understand a priority list, here is a sample section of such a list worked out under the 1920 census according to the Method of Equal Proportions. This gives the order of rank for the 433d to 438th seat in the House.

Seat No. in House	Rank Index	State	Cumulative No. of Seats for States
433	245,659	Pennsylvania	36
434	245,136	Ohio	24
435	244,771	Illinois	27
436	244,375	New York	43
437	244,003	North Carolina	11
438	243,410	Virginia	10

Notice that if the normal House of 435 were increased by one number, New York would gain its 43d seat; further additions would benefit first North Carolina and then Virginia.

The first advantage of these modern methods is that they avoid all the paradoxes and uncertainties of the Nineteenth Century methods.

The second advantage is that members of Congress are relieved of all the burden of calculating possible apportionments over and over as they used to do. All the necessary calculations in distributing seats can be made by experts and electric machines in the Bureau of the Census. We can leave the mathematics in the Census Bureau where it belongs.

The third great advantage is, that once the distribution of seats by a particular method has been submitted by the Bureau to Congress, the inequality between any two states may be easily measured and proved to be the smallest amount possible under the principle of that method. As no apportionment can be absolutely exact because Congressmen can't be cut into pieces, there will always be some states which are relatively under-represented and others which are relatively over-represented. Whenever such an under-represented state complains that it ought to get one more seat instead of some other state, which denies this claim with equal vigour, the dispute can be quickly settled by a little simple arithmetic.

That process will be explained shortly, but first let us look at the effect of the other two significant modern methods.

(b) Distribution of Seats by Method of Major Fractions

In this method the Census Bureau would employ the same process of obtaining rank indexes through the use of multipliers, but here these would be derived from a series of arithmetic means as divisors—1.5, 2.5, 3.5, . . . The results which interest us in the previous illustration are in Table 7.

TABLE 7
(16th and 17th seats; Method of Major Fractions)

State	Population	Seats Already	Seats Claimed	Divisor	Rank Index
A	1250	1	2	1.5	833.33
B	2450	2	3	2.5	980.00
C	3455	3	4	3.5	987.1
D	4461	4	5	4.5	991.33
E	5384	5	6	5.5	978.91
	17,000	15			

Here D will lead in the priority list, which the reader can easily construct, and get the 16th seat. C, being next highest, gets the 17th. He fares better than in Equal Proportions, and B fares worse.

(c) Distribution of Seats by Method of Harmonic Mean

Again the same process would be followed, except that the basic divisors would be a series of harmonic means—1 1/3, 2 2/5, 3 3/7, 4 4/9, 5 5/11. For the sake of convenience, these divisors are stated as decimal fractions in Table 8.

When measured by this method, B has the strongest claim to the 16th seat; C again gets the 17th. This time, it is D which loses out.

When you compare the results of the three methods, you see (what can be mathematically demonstrated) that the Method of the Harmonic Mean tends to favor the smaller states somewhat and the Method of Major Fractions has a corresponding slight bias toward the larger states,

TABLE 8
(16th and 17th seats; Method of Harmonic Mean)

State	Population	Seats Already	Seats Claimed	Divisor	Rank Index
A	1250	1	2	1.33...	937.50
B	2450	2	3	2.40	1020.8
C	3455	3	4	3.428	1007.8
D	4461	4	5	4.44..	1003.8
E	5384	5	6	5.4545	987.2

while the Method of Equal Proportion falls in between. This is an additional advantage.

Another thing to notice is that the three methods which have been discussed run very closely to each other. Populations for the hypothetical states were deliberately chosen so that each would produce a different result from the others in a House of 17, but with actual populations two methods might coincide, or even three. Indeed, in this illustration the death of one man in D or his removal into any other state would, under the Method of Equal Proportions, shift D's last seat to C and produce exactly the same distribution of 17 seats as the Method of the Harmonic Mean. In the census of 1930, 435 seats were assigned in just the same way under both Equal Proportions and Major Fractions; and in that of 1940 they diverged as to only one seat, which Equal Proportions eventually gave to Arkansas while Major Fractions, again favoring larger states slightly, would have given to Michigan. And a similar result has occurred under the 1950 census, as we shall see. Returning to the illustration, we find that in a House of 18 instead of 17 the last three seats will go to B, C, and D, no matter which of the three significant modern methods is used.

VIII. THE MEASUREMENT OF INEQUALITIES IN CONGRESS

We can now forget all about the complex mathematics in the Census Bureau. All Congress needs to consider is the finished product submitted to it by the Bureau. Then it will be easy to remove any doubts about the fairness of the reported distribution of seats. Inequalities do exist; they are inevitable. The point is that still greater inequalities will be produced by any alternatives to the Bureau's scheme.

Here is the process for settling a dispute between state X, which has received a seat, and state Y which thinks it ought to have that seat. State X is now over-represented and state Y is under-represented. That is, an average district is smaller in X than in Y; and a resident of X has, on the average, a larger share in a Congressman than if he lived in Y. First, we measure the inequality as it is under the Bureau distribution of seats. Then we imagine the disputed seat transferred from X to Y, and measure what inequality would *then* occur. It always appears that the inequality will be increased by the shift. Therefore, the Bureau's distribution of the seats is the best possible. Q.E.D.

One qualification must, however, be added. The Census Bureau has assigned the seats by using a particular mathematical method. So all the above-described process can hope to prove is that the Bureau's scheme is the best possible *under that method.* And this means that we have to measure the inequalities (before and after the shift) in accordance with the principle of that method. There is more than one way to measure inequalities satisfactorily, as will be soon shown; we have to choose the right way for the given method.

(*a*) *How Congress Can Measure Inequalities under the Method of Equal Proportions*

In the illustration, the Method of Equal Proportions, which is prescribed by the existing law, will enable the Census Bureau to give Congress the following information (probably not in just this form). For convenience, the fractional share in a Congressman is expressed in decimals.

TABLE 9
(17 seats; Method of Equal Proportions)

State	Population	Seats	Average District	Av. Share in Representative
A	1250	1	1250	.0008
B	2450	3	816.66	.001225
C	3455	3	1151.66	.000868
D	4461	5	892.2	.00112
E	5385	5	1077	.000928

Then, a little more information (in Table 10) will be supplied to make it possible to measure the inequalities if C has been given a 4th seat, instead of the 3 assigned it by the Bureau. This seat will have to be taken away from either B or D, which are over-represented in the prescribed scheme (Table 9).

TABLE 10
(17 seats; Alternative Possibilities)

State	Population	Seats	Average District	Av. Share in Representative
B	2450	2	1225	.000816
C	3455	4	863.75	.001157
D	4461	4	1115.25	.000896

First, what inequality do we choose to measure? There are two kinds, in size of districts and in shares of a Congressman. With Equal Proportions, we can measure either kind, whichever we please.

Second, just how do we measure any inequality? There are two different ways commonly used. For example, suppose that we are comparing Mr. Roosevelt's popular vote in 1936 (27,476,673) with Mr. Landon's (16,679,583). We can use subtraction to reach the absolute difference, and say that Mr. Roosevelt's vote was larger by about 10,800,000. But it is just as informative to use division and discover the relative difference; then we say that Mr. Roosevelt's vote was 65 per cent larger than Mr. Landon's. All the time in practical affairs we measure inequalities in this way by percentages, for example, Dow-Jones averages and steel production. That is the way inequalities are measured in the Method of Equal Proportions.

Let us measure the inequalities in the size of districts in this way. If the "amount of disparity" in districts between any two states can be decreased by the transfer of a seat from one state to the other, then this transfer should be made; otherwise not. Suppose C claims a 4th seat and want to take it from B's 5.

TABLE 11
(Equal Proportions)

State	Seats Now	Av. District Now	Av. Dist. after Shift	New No. of Seats
B	3	816.66	1225	2
C	3	1151.66	863.75	4
Excess		141.02%	141.8%	

We obtain the ratio of an average district in the over-represented state to that in the under-represented state. Before the shift it is barely 141%. After the claimed transfer, it will be somewhat increased, to 141.8%. Therefore, the present scheme is fairer.

Now measure the inequalities in representation between these two states in the same way.

Here again the disparity is *increased* by the change, so the Census

TABLE 12
(Equal Proportions)

State	Seats Now	Share in Rep. Now	Share After Shift	New No. of Seats
B	3	.001225	.000816	2
C	3	.000868	.001157	4
Percentage excess		141.1%	141.7%	

Bureau scheme is right. (The percentages would be exactly the same as for districts in Table 11, if the decimals were carried further out.)

In just the same way, a Congressman can easily prove that C cannot get a 4th seat fairly by taking it from D, which has 5. (Tables 9 and 10 give all the data needed for the calculation; it is just a matter of division.)

Thus, in the Method of Equal Proportions, you use division (percentages) to obtain relative differences (ratios), and can then measure either the size of districts or the share in a Congressman. Thereby you prove that, under the principle of this Method, the Census Bureau has submitted the fairest possible distribution of seats. Inequalities are reduced to a minimum.

(*b*) *How Congress Can Measure Inequalities under the Method of the Harmonic Mean*

Here only one kind of inequality can be measured, that in size of districts. How is it done? By subtraction, which shows the *absolute* difference between the average districts in the two competing states.

In our illustration, Table 13 shows the distribution of 17 seats by Harmonic Mean and Table 14 gives the rest of the needed information.

TABLE 13
(17 seats; Method of Harmonic Mean)

State	Population	Seats	Average District	Av. Share in Representative
A	1250	1	1250	.0008
B	2450	3	816.66	.011225
C	3455	4	863.75	.001157
D	4461	4	1115.25	.000896
E	5385	5	1077	.000928

TABLE 14
(17 seats; Alternative Possibilities)

State	Population	Seats	Average District	Av. Share in Representative
B	2450	2	1225	.000816
C	3455	3	1151.66	.000868
D	4461	5	892.2	.00112

D is obviously under-represented, while B and C are over-represented (Table 13). Suppose D wants to get a 5th seat, which will give one of these two other states their situation in Table 14. Again we measure the disparity in districts before the shift, but by subtraction this time. Next, we see whether this disparity will be decreased by the transfer.

TABLE 15
(Harmonic Mean)

State	Seats Now	Av. District Now	Av. Dist. after Shift	New No. of Seats
C	4	863.75	1151.66	3
D	4	1115.25	892.2	5
Difference		251.50	259.46	

Since the disparity will be increased by the transfer, the present scheme is right, by this method. Furthermore, if D should seek to obtain its extra seat from state B, the disparity would rise from 298 to 332.

However, if you try to measure inequalities in representation by subtraction (using the last columns in Table 13 and 14) you will get into trouble. For instance, the shift of one seat from C to D *diminishes* the inequality from .000361 to .000252. That shows how the Method of the Harmonic Mean does not enable one to measure that kind of inequality.

So far as districts go, a Congressman may find it a little simpler to handle the Method of the Harmonic Mean, because it is easier to subtract than to divide. This slight merit has never led anybody to give serious support to the Harmonic Mean. Moreover, it is slightly more favorable to smaller states, which are already well treated in the Senate.

(c) *How Congress Can Measure Inequalities under the Method of Major Fractions*

By this method the Census Bureau would distribute seats with the results in Table 16.

TABLE 16
(17 seats; Major Fractions)

State	Population	Seats	Average District	Av. Share in Representative
A	1250	1	1250	.0008
B	2450	2	1225	.000816
C	3450	4	863.75	.001156
D	4461	5	892.2	.00112
E	5385	5	1077	.000928

Suppose B, which is obviously under-represented, wants to get a 3d member and says D should give it to him, leaving 4 seats for D.

In the same way, we can show that B ought not to get his 3d seat from C.

TABLE 17
(17 seats; Alternative Possibilities)

State	Population	Seats	Average District	Av. Share in Representative
B	2450	3	816.66	.001225
C	3455	3	1151.66	.000868
D	4461	4	1115.25	.000896

TABLE 18
(Major Fractions)

State	Seats Now	Av. Share in Reps. Now	After Shift	New No. of Seats
B	2	.000816	.001225	3
D	5	.00112	.000896	4
Difference		.000304	.000329	

But the inequality in districts would be *decreased* by the transfer of a seat to B from either C or D. Therefore, if Congress is interested in inequalities in districts, as seems probable, there is very little to be said for the Method of Major Fractions. It does slightly favor the larger states, which are under-represented in the Senate; but it seems like warping the Great Compromise of 1787 to use this as argument. The only other argument is historical—its use after two censuses (1910 and 1930), whereas Equal Proportions has been used once (1940) before the current reapportionment. This does not amount to much.

Three other arguments have been made against the Method of Equal Proportions—all of them, in the author's opinion, unsound. First, the arithmetic mean is said to be easier for a Congressman to understand than the geometric mean. But it is no easier for a mathematician or a calculating machine in the Census Bureau, where either sort of mean is reduced to multipliers of long decimals which look much alike. A Congressman does not have to understand the geometric mean at all; he can test the results of the Method of Equal Proportions by fifth-grade long division, as has been shown.

Next, Webster's great name is invoked on behalf of the Method of Major Fractions, because it bears a superficial resemblance to the method used in 1840[10] under his influence.[11] However, his method did

10 See p. 647 *supra*.

11 *Mathematical footnote*. By starting with an "ideal" district and then varying this divisor up and down, you can give it a value which, when divided into the state populations, will attain the desired size of the House if every integer and every fraction over half is awarded seats. 35 CORNELL L. Q. at 371-4. The result is really the Method of

not work well and was never used again by Congress.[12] It rested on an entirely different principle from all the Modern Methods, because it employed the "ideal divisor," now abandoned, and had no comprehension whatever of that very satisfactory device, the priority list. Methods used before 1911 are of about as little use in getting the best apportionment today as anti-trust decisions before 1911 (*i.e.*, the Standard Oil case) are in deciding how to deal with big monopolies. Webster's real contribution to the reapportionment problem was, not his unworkable method, but his wise restatement of the Constitution so as to fit the stubborn facts of irregular populations and direct that Representatives are to be apportioned to the states "according to their respective numbers, *as near as may be*."[13]

Finally, it has been argued that the Method of Major Fractions is somehow more constitutional than any other. To a lawyer, this conception of degrees of constitutionality is meaningless. Either a statute is constitutional or it is not. All the mathematical methods which have been used, before 1910 or afterwards, are surely constitutional, though some of them worked much worse than others. Nor could the members of the Convention of 1787 have intended any particular method, because they were unaware that they were creating a mathematical problem. It is for us, their successors, to fill this unexpected hole in the Constitution by a method which fits well into the general pattern of that great document as *we* understand it.[14] Since there are two ideals for a good apportionment, approximate equality of districts and approximate equality of each citizen's share in a Representative,[15] and the Method of Equal Proportions attains both these important ideals, it certainly fits into the Constitution as well as the Method of Major Fractions, which considers only equality of representation and has to ignore equality in the size of Congressional districts.

Major Fractions. But Mr. Willcox's sliding divisor is a far less convenient device than the priority list for Major Fractions, which does the same thing without any resemblance to the Method of 1840. Moreover, the sliding divisor corresponds to nothing. It is not the "ideal district" of anything else but a tool; the priority list is a much better tool. See Huntington, 30 TRANS. AM. MATH. SOC. at 96-7.

12 The same method was adopted in Rhode Island in 1842. R. I. CONST. ART. V, § 1 (1930), where it still remains. It has proved unworkable. In order to have a House of the required size, the rule of "one representative for a fraction exceeding half the ratio" has to be modified into something like the Vinton Method, *supra* p. 647. See also the abstracts of the constitutional provisions in Missouri, Ohio, and Oklahoma in INDEX DIGEST OF STATE CONSTITUTIONS 869, 872 (N.Y. CONST. CONVENTION COMM., 1915).

13 See note 5 *supra*.

14 See Chafee, *The Disorderly Conduct of Words*, 41 COL. L. REV. 381, 398-404 (1941).

15 See p. 650 *supra*.

IX. Conclusion

Therefore, no alternative mathematical method seems plainly superior to the Method of Equal Proportions. We have a very satisfactory method in the present law. Why should we abandon it? Congress seems unlikely to do so in the current reapportionment, for a very practical reason. California is going to gain 7 seats under this statutory method. She would gain still another seat under the Method of Major Fractions, 8 in addition to her 23 existing seats; this would make Kansas lose one of her present 6 Representatives. That is the solitary difference in results between the two methods. So it seems highly probable that Congress will leave well enough alone.

The nation would indeed have cause for satisfaction if the problem of inequalities among the states, which has vexed Congress ever since the First Census, can now be regarded as settled, thus leaving the House and Senate free to consider Mr. Truman's recommendation[16] for remedying the much greater inequalities now existing among Congressional districts inside a state.*

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16 The President's Message on Reapportionment is in N. Y. Times, Jan. 10, 1951, p. 22, col. 2. See, *The Reapportionment of Congress, Report of Special Committee of American Political Science Assn.*, 45 Am. Pol. Sci. Rev. 153 (1951).

* The discussion of this subject will be continued in a *Symposium on Legislative Reapportionment* to appear in an early number of Law & Contemporary Problems. Professor F. W. Willcox will contribute a paper on Methods of Congressional Reapportionment to the symposium.