on in separate communications. Since the methodology presented here has been in part justified by a calculation, the extension of the formalism to more complicated problems is of importance. The applications presented in this paper are aimed at nuclear physics, but there seems to be no obvious reason why the theory should not prove equally useful in atomic or molecular collisions.

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PHYSICAL REVIEW C VOLUME 6, NUMBER 4 OCTOBER 1972

K-Electron-Capture-to-Positron-Emission Ratio in the Decays of ^{15}O and ^{19}Ne

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The K/β^+ ratio in the decays of ^{19}Ne and ^{15}O have been measured as $(9.6 \pm 0.3)\times 10^{-4}$ and $(10.7 \pm 0.6)\times 10^{-4}$, respectively. A gas-flow proportional counter, operating in anticoincidence with the surrounding plastic scintillator, was used. Theoretical K/β^+ ratios for ^{19}Ne and ^{15}O were computed, using exchange-overlap corrections calculated by Vatai and, separately, exchange corrections extrapolated from the results of Bahcall for $14 \le Z \le 37$. The experimental results were found to be in better agreement with Vatai's calculations.

I. INTRODUCTION

Electron exchange and overlap corrections, Bk, for K capture to positron emission ratios have been calculated by Bahcall[1] for nuclides in the range $14 \le Z \le 37$ and by Vatai[2] for nuclides in the range $13 \le Z \le 37$. The effect of the exchange-overlap correction is to reduce the theoretical value for K/β^+ ratios by a factor of $(1 - Bk)$. The calculations of Bahcall and Vatai differ by about 7% in the value of Bk at $Z = 14$ and the discrepancy increases on extrapolation into the region $Z \le 14$, as shown in Table I for $Z = 8$ and $Z = 10$.

A recent measurement[3] of the K/β^+ ratio in the decay of ^{30}P is in agreement with Vatai's calculations. Ledingham *et al.*[3] also include a summary of other experimental measurements of K/β^+ ratios for $Z \le 15$, including the results given in this paper. We describe below measurements of K/β^+ ratios in the decays of ^{19}Ne and ^{15}O.

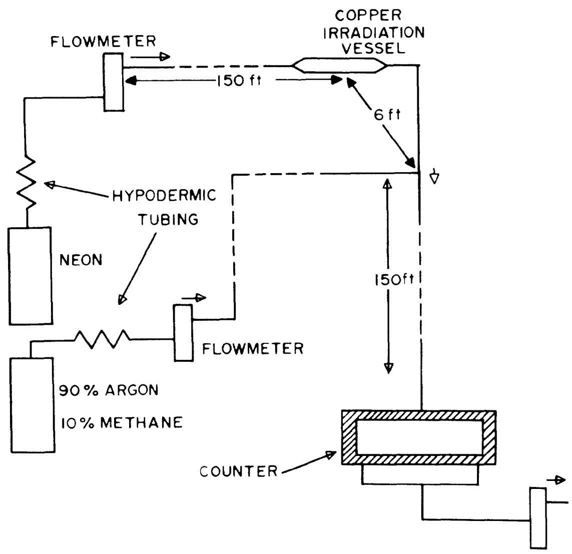

FIG. 1. Schematic outline of the gas-flow system.

The transition

$$^{19}_{10}Ne \rightarrow ^{19}_{9}F + e^{+} + \nu$$

is superallowed, with a half-life of 17.43 ± 0.06 sec[4] and a positron end-point energy of 2.217 ± 0.005 MeV.[5] The mean energy of the emitted radiations following K capture is about 700 eV.[6]

The half-life of the ^{15}O decay has been measured as 122.6 ± 1.0 sec.[7] The maximum energy of the emitted positrons is 1.737 ± 0.004 MeV,[5] and the mean K peak energy is about 400 eV.[6]

TABLE I. Exchange and overlap corrections Bk, and comparison of experimental and theoretical values for K/β^+ ratios in the decays of ^{15}O and ^{19}Ne.

Isotope	Bk [a]	Bk [b]	K/β^+ (theor)	K/β^+ (expt)
^{15}O	0.59	0.986	9.2×10^{-4}	$(10.7\pm0.6)\times10^{-4}$
^{19}Ne	0.80	0.987	9.7×10^{-4}	$(9.6\pm0.3)\times10^{-4}$

[a] Extrapolated from the calculations of Bahcall (Ref. 1).
[b] From the extrapolation given by Vatai (Ref. 2).

II. EXPERIMENTAL PROCEDURE

^{19}Ne was produced by the $^{20}Ne(\gamma, n)^{19}Ne$ reaction on neon gas which was allowed to flow through the x-ray beam of the Glasgow electron linear accelerator. The gas-flow system is shown in Fig. 1. The counter has been described in detail previously,[8,9] and consists of a central cylindrical proportional counter surrounded by a plastic scintillator. The scintillation and proportional counters were operated in anticoincidence, so as to reduce the intensity of low-energy signals from the proportional counter owing to positrons emitted near the wall of the counter. The associated electronic arrangement is outlined in Fig. 2 and is essentially a computerized version of the system described by Drever.[10] The positrons were detected as large amplitude pulses (Y analog-to-digital-converter overflow output) from the proportional counter.

In addition to ^{19}Ne, ^{15}O and ^{11}C were also produced by the accelerator x-ray beam. The ^{15}O was probably produced by the $^{20}Ne(\gamma, n\alpha)^{15}O$ reaction. The source of the ^{11}C is uncertain, but it

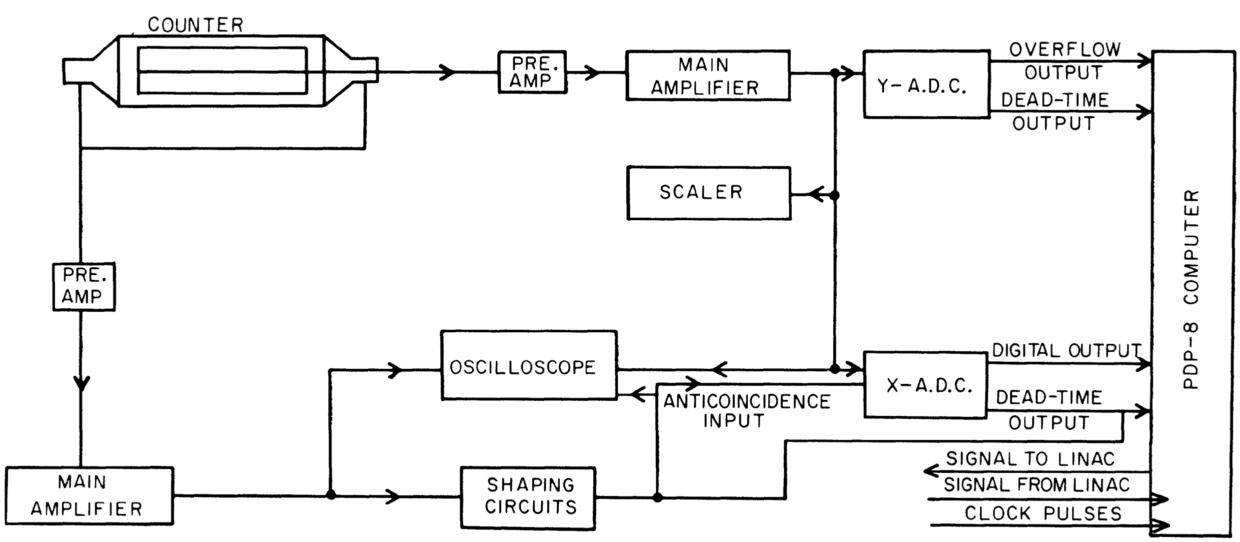

FIG. 2. Electronic arrangement.

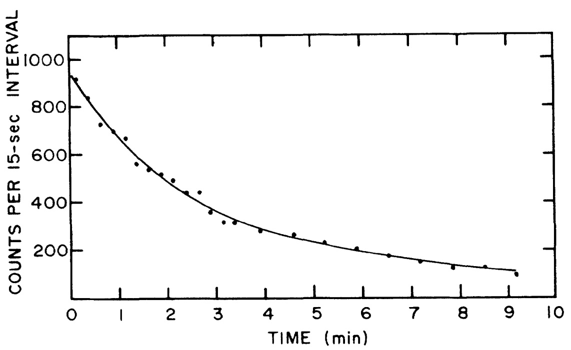

FIG. 3. Decay of activity trapped in the central counter following the measurement on ^{15}O.

may have been produced from organic contaminants in the neon gas, or by the $^{20}Ne(\gamma, n2\alpha)^{11}C$ reaction.

By varying the flow rate of the neon gas from the irradiation vessel to the counter to take advantage of the differences in half-lives of ^{19}Ne and ^{15}O, it was possible to obtain in the counter activities consisting mainly of either ^{19}Ne or ^{15}O. K/β^+ ratio measurements on both isotopes were then possible.

The amount of activity was estimated, for the ^{15}O runs, by closing off the counter and following the decay of the trapped activity (Fig. 3). Analysis of the decay curve indicated that about $(7.0\pm^{0.6}_{0.7})\%$ of the activity in the counter was due to ^{11}C. The percentage of ^{19}Ne activity present during the ^{15}O measurements was estimated from the variation

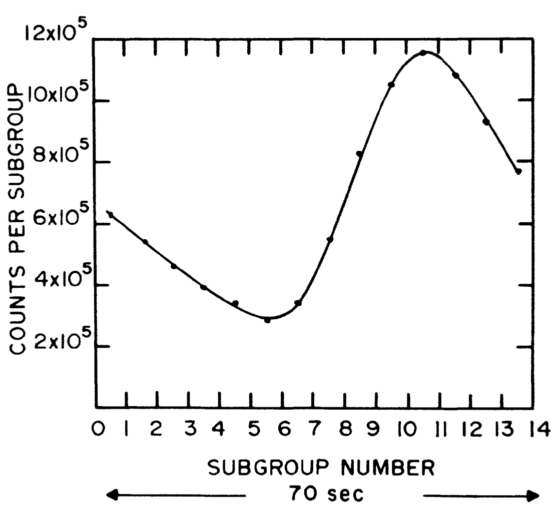

FIG. 4. Variation of activity in the central counter when the accelerator x-ray beam is on for 20 sec in every 70 sec. Gas-flow conditions were maintained as during the $^{19}Ne\,K/\beta^+$ measurements. The points on the curve are the experimental data. The smooth line was given by a computer simulation.

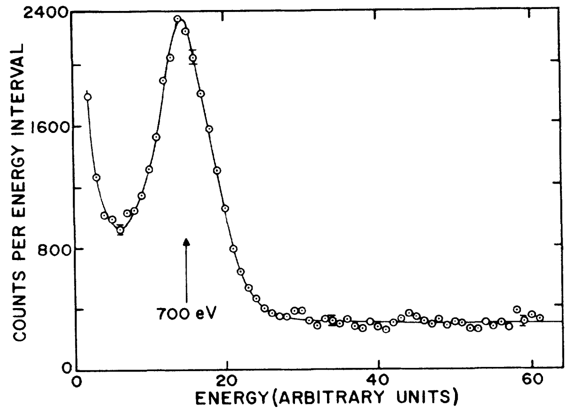

FIG. 5. Pulse-height spectrum from the central counter, in anticoincidence with the scintillation counter, from measurements on ^{19}Ne.

in activity obtained by putting a vessel of known volume into the flow lines. The ^{19}Ne contribution was calculated to be $(3.0\pm^{1.0}_{0.8})\%$ of the total activity in the counter.

Repeating this procedure for the ^{19}Ne runs resulted in large errors in the estimation of the amount of ^{19}Ne due to its short half-life. A different technique had to be adopted. The electron accelerator was interfaced with the PDP-8 computer so that the accelerator could be operated under program control. The block of computer memory which was used for storing the proportional-counter spectrum and the total number of positrons emitted was extended into 14 subgroups of 64 channels. These 14 subgroups were used for a cyclic sequential analysis of the activities produced in the counter when the accelerator was turned on by the PDP-8 computer for 20 sec in every 70 sec, with the flow rates in the gas-flow system set to the same value as in the ^{19}Ne K/β^+

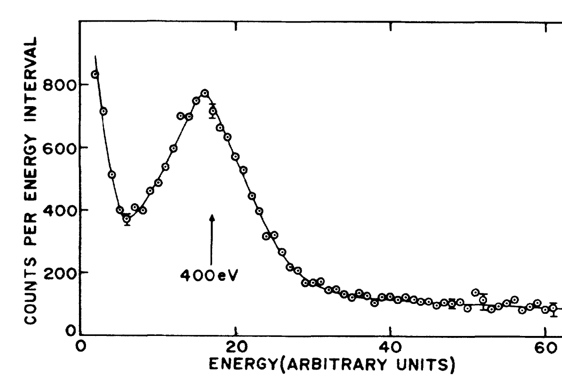

FIG. 6. Pulse-height spectrum from the central counter, in anticoincidence with the scintillation counter, from measurements on ^{15}O.

measurement. The number of positrons detected in the proportional counter as a function of subgroup number is shown in Fig. 4. The fractions of the total activity in the proportional counter due to ^{15}O and ^{11}C were estimated from a computer simulation of the sequential analysis. The simulated spectrum is given by the continuous line in Fig. 4.

The weighted means of the results from 13 cyclic sequential analyses were calculated as $(4.9\pm 0.4)\%$ of ^{15}O and $(2.0\pm 0.2)\%$ of ^{11}C.

III. ANALYSIS OF EXPERIMENTAL RESULTS

A. ^{19}Ne Analysis

A preliminary estimate of the K/β^+ ratio in the decay of ^{15}O was made based on the assumption that the peak obtained in the low-energy spectrum from the ^{15}O measurements consisted entirely of ^{15}O K capture events. This estimate gave a result of 1.05×10^{-3}. The K/β^+ ratio in the decay of ^{11}C was known.[9] Using the measured values for the amounts of ^{15}O and ^{11}C activity in the proportional counter during the ^{19}Ne K/β^+ measurement, the number of K capture events in the ^{19}Ne low-energy spectrum (Fig. 5) due to ^{11}C and ^{15}O were calculated. Poisson shapes of corresponding area at 200 and 400 eV were subtracted from the ^{19}Ne low-energy spectrum.

A computer program was written to subtract Poisson shapes corresponding to the ^{19}Ne K capture peak from what remained of the low-energy spectrum after subtraction of the ^{15}O and ^{11}C K capture peaks, background spectrum, and a straight line extrapolated from a least-squares best fit to points on the spectrum above the main peak. The remaining spectrum was examined visually for any peaks or valleys. Remaining spectra which showed a significant lack of smoothness were rejected.

The statistical errors in the computation of the final result arose from the estimation of the amounts of ^{11}C and ^{15}O and in the process of separating out the ^{19}Ne peak. These were added orthogonally to give an experimental result of $(9.6\pm 0.3)\times 10^{-4}$ for the K/β^+ ratio in the decay of ^{19}Ne.

B. ^{15}O Analysis

The ^{15}O low-energy spectrum is shown in Fig. 6. The analysis procedure was similar to that for ^{19}Ne. In this case, Poisson distributions corresponding to the ^{19}Ne and ^{11}C contributions to the low-energy spectrum were subtracted off. The smoothness criterion was then applied to the remaining spectrum on further subtraction of a range of ^{15}O K capture peaks. The final experimental result was obtained as $(10.7\pm 0.6)\times 10^{-4}$ for the K/β^+ ratio in the decay of ^{15}O, a correction of $(1.2\pm 1)\%$ having been made for positrons entering the proportional counter from the insensitive regions at each end of the counter.

IV. DISCUSSION

The theoretical values of the K/β^+ ratios in the decays of ^{15}O and ^{19}Ne were calculated using K electron wave functions derived from relativistic Hartree-Fock-Slater solutions of the Dirac equation.[11,12] These wave functions were obtained for a nucleus of finite size and a diffuse surface and correspond, to an accuracy of better than 1%, with wave functions given by Suslov.[13] The results of these calculations, without exchange or overlap corrections, are given in Table I. As these corrections would reduce the theoretical result, it can be seen that the differences between theory and experiment would only be increased by the application of an exchange-overlap correction, and that the experimental results are in better agreement with the calculations of Vatai than the results extrapolated from the work of Bahcall.

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PHYSICAL REVIEW C VOLUME 6, NUMBER 4 OCTOBER 1972

Application of the Burnett-Kroll Soft-Proton Theorem to Nucleon-Nucleon Bremsstrahlung*

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We give an explicit analytic formula for the leading terms in the square of the matrix element for nucleon-nucleon bremsstrahlung in the relativistic, model-independent, soft-photon theory of Nyman. Such a formula should simplify direct and detailed comparisons of this soft-photon theory with experiment and with the predictions of other models. These comparisons have been difficult because Nyman in his original calculation of $NN \to NN\gamma$ was forced by the extremely complicated matrix element to use entirely numerical procedures. To obtain an analytic result we have used the Burnett-Kroll theorem, which makes it possible to bypass many of the complications of a straightforward algebraic calculation. We also emphasize the general utility of the Burnett-Kroll theorem as a calculational tool for obtaining in an almost trivial fashion the leading terms in the squares of very complicated radiative matrix elements.

I. INTRODUCTION

A number of years ago it was suggested[1] that an investigation of the nucleon-nucleon bremsstrahlung process $N+N \to N+N+\gamma$ would give information about the off-mass-shell components of the nucleon-nucleon scattering amplitude. More recently, however, it has become clear, as a result of various low-energy theorems[2-6] that the leading terms in an expansion of the $NN\gamma$ amplitude in powers of the photon momentum k depend only on the on-mass-shell parts of the NN amplitude. Thus one must work harder than was originally supposed to get information about off-mass-shell terms. However, experimental developments since the original suggestion have made it quite feasible to examine in detail regions where higher-order terms might be important, so nucleon-nucleon bremsstrahlung remains a very interesting process.

There have been a large number of calculations of nucleon-nucleon bremsstrahlung, including, for example, those of Refs. 7–11. Typical ones include a number of calculations based on primarily nonrelativistic potential models,[8] the relativistic one-boson exchange model of Baier, Kühnelt, and Urban[9] and the completely relativistic calculation of Nyman[10] which uses soft-photon techniques. This latter calculation is of particular interest because it is both relativistic and model-independent. By "model-independent" we mean that the radiative amplitude depends only on the on-mass-shell NN amplitude through the leading two orders, $O(k^{-1})$ and $O(k^0)$, in the photon momentum k. This result[2-6,12] follows directly from gauge invariance and certain smoothness and analyticity assumptions.[12] Within the definition of model-independent there exists the freedom to choose different continuations of the NN amplitude to the slightly unphysical point corresponding to the radiative process or to choose different sets of kinematic variables to describe the NN amplitude. One can show however, given the above assumptions, that this freedom alters the result only in order k.[3,12-14] Thus within the region of strict applicability of Nyman's calculation, i.e., when the $O(k)$ terms are truly negligible, all other calculations must agree with it (provided of course that they reproduce the same NN scattering amplitude and satisfy the necessary smoothness conditions[12]). Thus the soft-photon theory provides a benchmark against which to check more elaborate models.

From a practical point of view however, the region of strict applicability of the soft-photon theory is significantly smaller than the region which has been explored experimentally. The ex-