Compensation for phase distortions in nonlinear media by phase conjugation

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We demonstrate theoretically that the distortion-correction property of phase-conjugate beams propagating in reverse through aberrating media is also operative when the indices of refraction of the media depend on the intensity. A necessary condition is that the phase-conjugate mirror that generates the reflected beam possess a unity (magnitude) "reflection" coefficient.

The ability of nonlinear optical phase-conjugate mirrors (PCM's) to correct for linear refractive-index inhomogeneities has been demonstrated both theoretically[1,2] and experimentally.[2] Propagation media often possess intensity-dependent indices of refraction that cause severe propagation distortion, such as thermal blooming and self-focusing. These distortions are often manifested in high-intensity atmospheric propagation of laser beams, thermally induced lensing within intracavity laser gain media, or high-intensity transmission through optical fibers. In this Letter we extend the distortion-correction proof of Yariv[1,2] to media with intensity-dependent indices of refraction.[3]

A typical geometry is sketched in Fig. 1. We consider a monochromatic electromagnetic field at radian frequency ω, which propagates essentially in the $+z$ direction and is given by

$$E_1 = \psi(\mathbf{r})\exp[i(\omega t - kz)] + \text{c.c.}, \tag{1}$$

where $k = \omega/c$.

This field encounters a region of space described by an intensity- and spatial-dependent permittivity, ϵ. After passage through this medium, the resultant field is incident upon a PCM, which gives rise to a (conjugate) field given by

$$E_2 = f(\mathbf{r})\exp[i(\omega t + kz)] + \text{c.c.} \tag{2}$$

The total field is thus

$$E = [\psi(\mathbf{r})\exp(-ikz) + f(\mathbf{r})\exp(ikz)]\exp(i\omega t) + \text{c.c.} \tag{3}$$

We next investigate the nature of the propagation of the *total* field E in the medium. It is clear that, because of the dependence of the dielectric constant ϵ on the intensity, we cannot separate the propagation problem to that of ψ (or f) alone, and both must be considered simultaneously.

The dielectric constant of the isotropic medium is taken to have an arbitrary dependence on the local intensity,

$$\epsilon = \epsilon_0(\mathbf{r}) + \sum_{n=1}^{\infty} \epsilon_{2n}(\mathbf{r})|E|^{2n}, \tag{4}$$

where the first term describes the linear, spatially dependent refractive-index inhomogeneities and the following terms depict the nonlinear (intensity-dependent), spatially dependent refractive-index contributions.

In the analysis that follows, we retain only the first-order nonlinear index term in the above power series. It can be easily shown that the analysis will yield similar results for all successive intensity-dependent terms of ϵ.

Assuming "slow" variations of ϵ, i.e., $1/\epsilon|d\epsilon/dx|\lambda \ll 1$, the wave equation in cgs units is given by

$$\nabla^2 E - \frac{\mu}{c^2}[\epsilon_0(\mathbf{r}) + \epsilon_2(\mathbf{r})|E|^2]\frac{\partial^2 E}{\partial t^2} = 0. \tag{5}$$

Substitution of the field of Eq. (3) into Eq. (5) yields

$$\left\{\nabla^2\psi(\mathbf{r}) + \left[\frac{\omega^2\mu}{c^2}\epsilon_0(\mathbf{r}) - k^2\right]\psi(\mathbf{r}) + \frac{\omega^2\mu}{c^2}\epsilon_2(\mathbf{r})|E|^2\psi(\mathbf{r}) - 2ik\frac{\partial\psi(\mathbf{r})}{\partial z}\right\}\exp(-ikz) + \left\{\nabla^2 f(\mathbf{r}) + \left[\frac{\omega^2\mu}{c^2}\epsilon_0(\mathbf{r}) - k^2\right]f(\mathbf{r}) + \frac{\omega^2\mu}{c^2}\epsilon_2(\mathbf{r})|E|^2 f(\mathbf{r}) + 2ik\frac{\partial f(\mathbf{r})}{\partial z}\right\}\exp(+ikz) = 0. \tag{6}$$

Using Eq. (3), we can express the squared magnitude of the field as

$$|E(\mathbf{r})|^2 = |\psi(\mathbf{r})|^2 + |f(\mathbf{r})|^2 + \psi f^* \exp(-2ikz) + \psi^* f \exp(2ikz).$$

Using this last expression in Eq. (6) and equating to zero the sum of all the terms with $\exp(ikz)$ and, separately, those of $\exp(-ikz)$ dependence, we obtain

$$\nabla^2\psi + \left[\frac{\omega^2\mu\epsilon_0(\mathbf{r})}{c^2} - k^2\right]\psi + \frac{\omega^2\mu\epsilon_2(\mathbf{r})}{c^2}(|\psi|^2 + 2|f|^2)\psi - 2ik\frac{\partial\psi}{\partial z} = 0, \tag{7}$$

and

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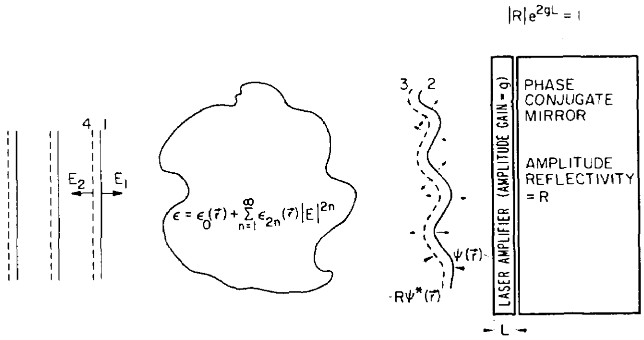

Fig. 1. Typical geometry for a PCM in order to compensate for nonlinear phase distortions, ϵ. Solid (dashed) curves correspond to the incident (conjugate, or time-reversed) equiphase surfaces. Arrows indicate local propagation vectors. If $|R| < 1$, then the use of a conventional laser amplifier satisfying $|R|\exp(2gL) = 1$ is required to ensure proper nonlinear phase compensation.

$$\nabla^2 f + \left[\frac{\omega^2\mu\epsilon_0(\mathbf{r})}{c^2} - k^2\right] f + \frac{\omega^2\mu\epsilon_2(\mathbf{r})}{c^2}(|f|^2 + 2|\psi|^2)f + 2ik\frac{\partial f}{\partial z} = 0, \quad (8)$$

where nonsynchronous $\exp(\pm i3kz)$ terms have been dropped. Taking the complex conjugate of Eq. (7) yields

$$\nabla^2\psi^* + \left[\frac{\omega^2\mu\epsilon_0(\mathbf{r})}{c^2} - k^2\right]\psi^* + \frac{\omega^2\mu\epsilon_2(\mathbf{r})}{c^2}(|\psi|^2 + 2|f|^2)\psi^* + 2ik\frac{\partial\psi^*}{\partial z} = 0. \quad (9)$$

Comparing Eqs. (8) and (9), we note that the conjugate amplitude of the forward wave, $\psi^*(\mathbf{r})$, and $f(\mathbf{r})$, the amplitude of the backward wave, obey the same propagation equation, provided that $|\psi(\mathbf{r})| = |f(\mathbf{r})|$. It follows immediately by a self-consistent argument that, if at some plane z_0 we have $\psi^*(x,y,z_0) = f(x,y,z_0)$, then

$$f(\mathbf{r}) = \psi^*(\mathbf{r}) \quad (10)$$

for all $z < z_0$. In other words, the reflected field is the complex conjugate of the incident field everywhere in the distorting medium and outside it, even when the distorting medium is nonlinear and both waves exist simultaneously. It follows that at the input plane the wave is healed of any distortion caused by the propagation through the distorting nonlinear medium.

It is interesting to note in Eqs. (7) and (8) that the forward ψ and backward f waves couple to each other by a four-wave mixing process in the distorting medium. We can view this as a process whereby ψ and f "write" a hologram that Bragg-scatters ψ in the backward direction of f and vice-versa. It is exactly this coupling that ensures the distortion correction.

Referring to Fig. 1, we note that the condition $f = \psi^*$ is achieved at the input plane of the phase-conjugate mirror by adjusting its reflectivity so that $|R| = 1$. This is achievable in a four-wave mixing process[4,5] in which gains well in excess of unity have been demonstrated.[6,7] Thus four-wave mixing as well as other conjugation schemes may be used in conjunction with a linear optical amplifier to obtain $|R| = 1$. The PCM can compensate for *linear* phase distortions within the amplifier.

For magnitude $R = 1 \pm \eta$, the compensation will take place in the manner described above if the accumulated phase error (in the length l of the distorting medium) $\eta\epsilon_2|\psi|^2 l$ is very small compared to unity.

If the distorting medium is lossy, or possesses gain, the condition $|\psi| = |f|$ cannot be maintained over all space, and the compensation does not take place.

The distortion-compensation property described above also applies to the case of pulsed propagation in which ψ and f do not overlap in time in the distorting medium. All that is required is that the pulses be long enough so that the above monochromatic compensation scheme is valid and that $|R| = 1$.

In conclusion, we have shown that nonlinear propagation distortion that is due to an intensity- and spatial-dependent dielectric constant can be compensated for by the use of a PCM. Thus undesirable distortions, such as atmospheric thermal blooming and even intracavity (circulating laser intensity) induced lensing effects, can be corrected through the use of a PCM; in the latter case one can use a PCM to replace one (or both) of the mirrors comprising a laser cavity.[8] We note that *catastrophic* self-focusing, which can lead to other nonlinear effects, such as beam breakup, stimulated Brillouin scattering, stimulated Raman scattering, or optical damange, obviously *cannot* be corrected by the scheme discussed here.

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