PERFORMANCE CALCULATIONS OF THE SHADED-POLE MOTOR

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By

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PREFACE

Inherently the shaded-pole motor is of fractional horsepower rating. Since the fractional horsepower motor is a result of ventilating fan development an engineering result of the fan industry was the shaded-pole motor. Not long after the two-phase induction motor was developed by Tesla, engineers discovered that a motor would start and run, provided that the reactances and resistances of the winding coils were suitably proportioned, when both phases were connected to a single-phase power supply. The extension of this development has resulted in the well known split-phase motor, which has two parallel windings; one that is a low resistance type and the other a high resistance. During the starting period both windings are used in the motor in order to obtain sufficient starting torque. When the motor speed reaches a certain percentage of rated speed a switch removes the high resistance winding from the power supply and the motor continues to run with only the low resistance winding connected to the power supply.

It is sometimes difficult to build very small, i.e., fractional horsepower, motors that will operate satisfactorily on a single-phase supply. Therefore, it is considered desirable, many times, to leave the starting winding connected even during the running period. Because an outside resistance is used in series with the starting winding this type is sometimes called the resistance split-phase motor. These motors are preferable to the normal split-phase motors only in sizes below 1/100 horsepower, as a general rule. However, to avoid the cost of a centrifugal type cut-out switch they have been used for ratings as high as 1/20 horsepower.

From 1889 until about 1919, these two types of split-phase induction motors were used very extensively on fans and whatever other applications there were for fractional horsepower single phase motors.[3] There were, and still are, many commutator motors of the series type used along with the two mentioned types of split-phase motors. Their main advantage is the ability to operate either on alternating or direct current power supplies.

It was discovered during the latter part of the nineteenth century that a squirrel-cage motor with a salient pole single-phase field would run if a portion of the pole were short-circuited with a winding or coil. This, of course, eliminated the cost of not only the cut-out switch but also the cost of the high resistance parallel winding. The saving in the cost of this winding, compared with the distributed field winding of the split-phase type has given a tremendous popularity to the "shaded-pole" motor, until today, for ratings under 1/20 horsepower, it overshadows all other single-phase motor types.

ACKNOWLEDGEMENT

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CHAPTER I

THE THEORY AND HISTORY OF THE SHADED-POLE MOTOR

If a polyphase induction motor is running with no-load or a small load and some of the supply lines are disconnected from the motor so as to leave the equivalent of a single-phase, the motor will continue running, but at a somewhat slower speed. If the polyphase induction motor is a three-phase motor and one of the supply lines is disconnected, it will still have two of its three windings (providing it is a wye-wound motor) connected to the supply. It is obvious that a polyphase motor converted by this method into a single-phase induction motor will have less capacity than it had when running as a three-phase induction motor. But where the polyphase motor is self-starting, the single-phase motor has no starting torque. To make it self-starting, the single-phase motor must be provided with some type of device which will alter this undesirable characteristic during the starting period. From this statement, it is seen that during the starting period the motor must be changed to a type which is not a single-phase induction motor in the sense in which this definition is commonly used. It becomes a true single-phase motor only after the torque and speed have been raised to a point beyond which the device used to obtain the additional torque has been removed from the motor supply.[1] For these reasons it is necessary to clearly differentiate between the starting period of the single-phase motor and its running period.

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[1] Langsdorf, A. S. Theory of Alternating Current Machinery. p. 658

The single-phase motor has less desirable operating characteristics than the polyphase induction motor, but it has achieved a wide field of usefulness where a polyphase power supply is not available or economically feasible. Advances during the recent years have made the single-phase motor satisfactory in its smaller sizes so that several million of them are now in use in the commercial and domestic fields.[2]

As has been stated previously, one of the greatest disadvantages to the single-phase motor is its lack of starting torque. During the period from the discovery of the single-phase induction motor to the present time, scores of devices to overcome this undesirable characteristic have been devised. Although most of them have been proven economically or mechanically unpractical, some of them have been reliable and are in wide spread use today. Of these, the capacitor-start, the split-phase, the repulsion-start-induction run, and the shaded-pole motor are the most common.

To operate a fan near the desired speed at 133 cycles per second (at one time nearly all alternating current systems were operated at this particular frequency) it was necessary to use ten poles. The synchronous speed of a motor at this frequency would be 1596 rpm. The poles were concentrated and a coil was placed on alternate poles connected in series; the polarity on all of the coils being the same. This was called a consequent pole winding. A return path for the magnetic flux from the energized poles was formed by the poles without coils. To obtain a phase displacement of the flux in the pole, or otherwise cause a rotating effect of the flux to cause a starting torque, the trailing edge of the poles with windings was "shaded"; that is, a band of copper was placed around a portion of the pole. This displaced the flux in this section, while the flux in the remaining

[2] Puchstein and Lloyd. Alternating Current Machines. p. 229.

portion of the pole was in phase with the current in the field coils.[3]

About 1890 it was discovered that a squirrel-cage induction motor with a salient-pole single-phase field would run if a portion of the pole were short-circuited with a winding or coil.[4] Thus a shaded-pole motor is a single-phase induction motor provided with an auxiliary short-circuited winding or windings displaced in magnetic position from the main windings.[5]

The shaded-pole motor is usually the smallest and least efficient of the single-phase induction motors. The efficiency will vary with the size of the machine and is usually between the limits of 10 to 30 percent. The starting torque is very low and is usually in the range of 25 to 50 percent of the full-load torque. The ratings of the machine are usually chosen so that the maximum torque may be only 125 to 175 percent of the full-load torque. This ratio is lower on motors applied to fans and blowers because of the speed torque characteristics of the load. This gives little chance of overloading and pull-out, which is the desired effect. Also, when used for fan or blower operation, fairly stable operation at reduced speeds is possible by motor "breakdown"; or, a series impedance is used with the primary winding to reduce the applied voltage which results in speeds well below synchronism. This characteristic is possible because of the inherently high slip of the shaded-pole motor.[6]

As has been stated before, the simplest way of providing a single-phase

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[3] Denman, E. W. "Development of Fan Motor Windings," Electrical Journal, (June 1919).

[4] Trickey, P. H. "An Analysis of the Shaded-Pole Motor," Electrical Engineering, (September 1936).

[5] Veinott, C. G. Fractional Horsepower Motors, p. 297.

[6] Pender and Del Mar. Electrical Engineers Handbook, Sec. 9, p. 96.

induction motor with starting torque is to place a permanently short-circuited winding of high resistance in the stator at an electrical angle of about 30 to 60 degrees from the main winding. This winding usually consists of an uninsulated strip of copper encircling about one-third of a pole pitch. A current is induced in this coil by the portion of the main field flux linking it. The flux generated by this coil causes the main field flux in this portion of the pole to be smaller in magnitude and lag in time phase the flux in the unshaded portion of the pole. Thus, the air-gap field, or flux, will have two components; an undampened alternating flux, and a dampened flux ϕ_2 which is displaced 90 degrees in space and ϕ degrees in time. If each field is represented as the sum of two equal and oppositely revolving fields, the net field revolving forward (from the pole center toward the shading coil) is:

$$\phi_F = \phi_1 + \phi_2 \underline{/90 - \phi}$$ [7]

and the backward revolving field is:

$$\phi_B = \phi_1 + \phi_2 \underline{/90 + \phi}$$

Evidently the forward field is larger and a starting torque will be produced. The net torque produced is proportional to the difference of the squares of the two fields, or

starting torque = $4K\phi_1\phi_2 \sin \phi$ [8]

In practice ϕ seldom exceeds 15 degrees, ϕ_2 is of magnitude 1/3 ϕ_1, and ϕ_1 is much smaller than for a plain single-phase motor because of the space occupied by the shading-coil flux.[9]

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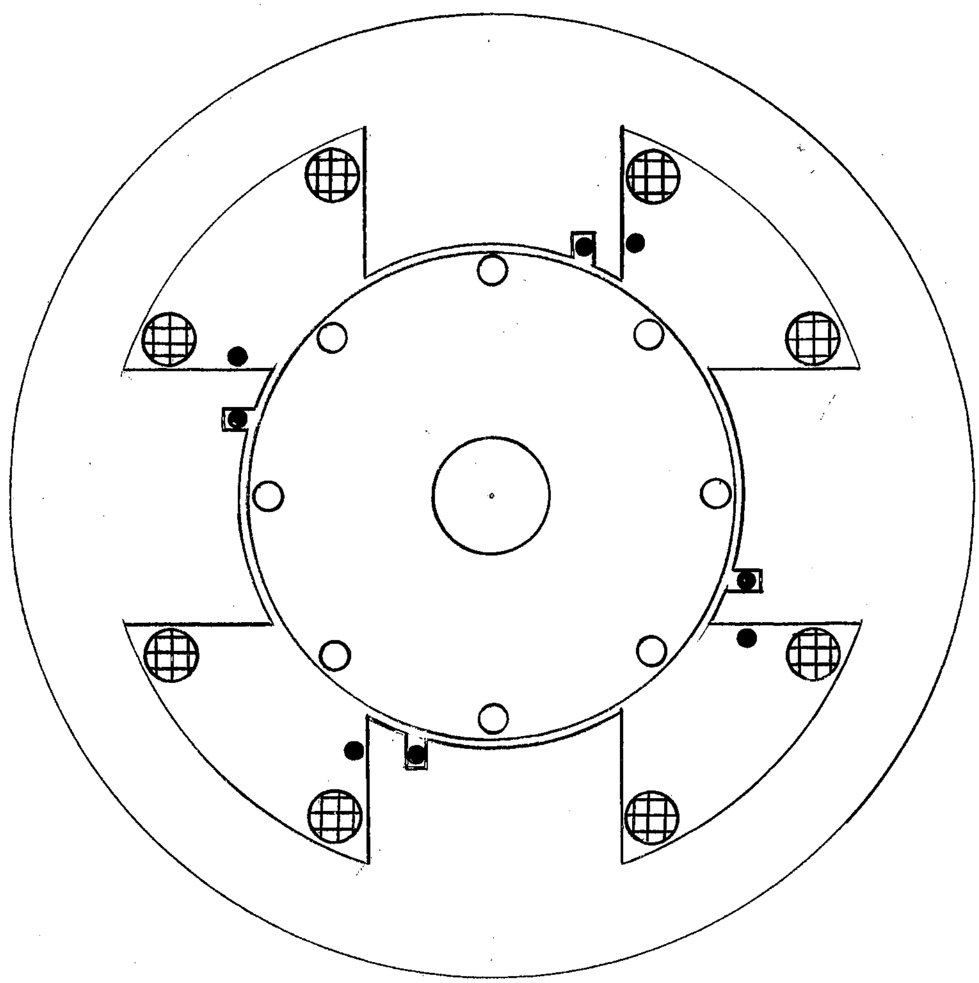
[7] Standard Handbook for Electrical Engineers, Sec. 7, p. 262.

[8] Ibid.

[9] Ibid.

TYPICAL MOTOR CONSTRUCTION



Main Field Windings

Rotor Windings

Shading Coils

FIGURE A

If there were no shading coil on the salient pole no rotational effect would be produced because the flux lines in that pole and the air gap surrounding it would simply alternate from positive to negative, that is, it would change the direction from out of the pole to into the pole. However, the action of the shading coil causes the flux in that portion of the pole to lag behind the flux in the unshaded portion of the pole by a small angle. As in any induction motor, this causes the points of maximum flux to progress around the motor and results in a small rotating field which draws the rotor around with it. The direction of rotation, of course, is from the main pole toward the shading coil.[10]

Figure A shows the most typical construction of a shaded-pole motor type. Motors using the shaded-pole principle have been developed in many forms and in greatly varying ratings as in any popular motor type. The rotor is almost invariably a simple squirrel-cage type.[11] The shading coils or windings are four coils (for a rotor speed of 1800 rpm) of one turn each of bare copper strap. It makes no difference in motor operation if these coils are short-circuited separately or short-circuited with all the coils in series. The stator core is usually composed of single-piece identical laminations held together by rivets near the periphery of the laminations. The main windings are, in most cases, preformed or wound and slid over the poles. As has been stated before there are many variations of the shaded-pole motor types but one example of a general type should be enough to set forth the main points of construction.

The performance characteristics of the shaded-pole motor are very similar to the resistance split-phase motor. As has been mentioned before it is very difficult to obtain a great amount of starting torque. However, the motor

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[10] Ibid.

[11] Ibid.

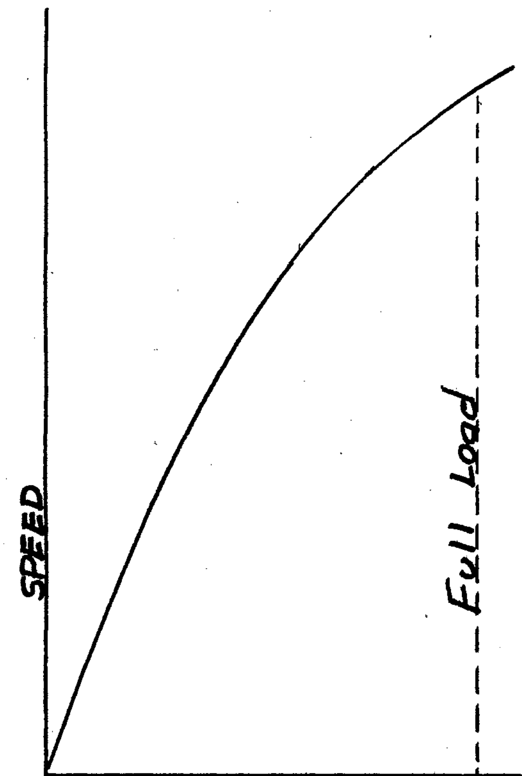

FIGURE B

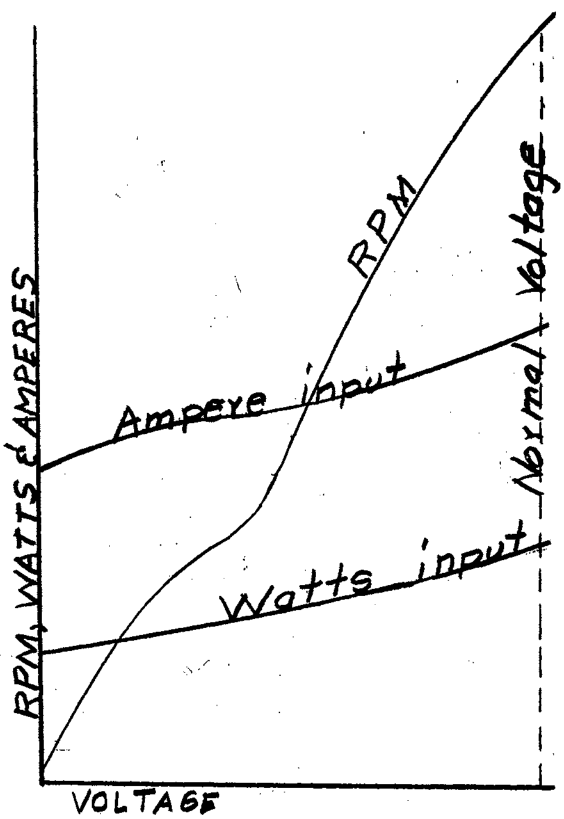

FIGURE C

is usually used for a type of duty that does not require an appreciable amount of starting torque (such as a household fan). The input power of this motor will be almost twice that required by a capacitor motor that does the same amount of work. Therefore it is not unreasonable to expect the power factor to be fairly high for such a small motor because of the very high inherent losses.

When the motors are used for fan duty they are usually operated very near to the breakdown point.[12] This can be seen by observing Figure B which is a typical speed-torque curve of the fan blade which is normally used with the shaded-pole motor. Operation of the motor at this point usually gives the best efficiency and the most power that the motor will develop. As would be expected the motor is noisy and is often subject to what is commonly known as "cogging," that is, variation of the magnitude of starting torque with different rotor positions. Since these are inherent objections to the motor several things are done to reduce these objections. Therefore, the shaded-pole motor usually has larger air gaps, a greater amount of skew, and lower inductions than split-phase motors of similar ratings.

To obtain speed control a resistor or reactor or "choke coil" may be used in series with the motor power supply. It is all the more necessary to operate the full load speed near to break down if speed change is desirable.

Very often the designers problem is simply to obtain a motor to drive a certain fan at a specified speed. Unless a really extensive and thorough job is necessary the usual design procedure is to drive the blade with several different motors and by varying the voltage obtain readings of speed, watts and amperes as shown in Figure C. The curves obtained by the above test are known as "fan saturation tests or curves." With the results of the several

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[12] Ibid.

motor tests known, the designer choses the motor that has the most desirable characteristics at the specified speed. This motor is then rewound or a new one built with the field coil turns changed in inverse proportion to the ratio of the test voltage and the normal voltage of the motor. This procedure is used rather than to calculate the performance of the motor because it is less cumbersome.

As may be seen from Figure C, the curve of speed versus volts has a typical saturation curve shape, that is, it has a high rate of change at first and the rate of change decreasing as the voltage is increased. The maximum value will be at a speed slightly less than synchronous. This curve has definite value in the selecting of a motor to be used for fan operation. If the motor is to be single speed a motor that operates on the flat part of the curve should be used. However, if the motor is to be used for multiple speed operation one should be chosen that will operate not too far above the knee of the curve so that too great a voltage change will not be necessary to obtain the desired speed change.

It is indeed unfortunate that the design procedure and the performance calculations of such a simple motor should be so lengthy and involved. Even so, the amount of work done to shorten this undesirable characteristic has been very slight in comparison to the work done on other types of motors. Most of the development of the performance calculations and design procedure has been done by P. H. Trickey. Yet even his papers on the motor are quite involved and lengthy but they are at least a step in the right direction.

It is the object of this thesis to try and develop a somewhat shorter method for the performance calculations of the shaded-pole motor. Time does not allow investigations into a design procedure for this motor.

CHAPTER II

ANALYTICAL SOLUTION OF MOTOR CONSTANTS

There has never been an adequate method of analysis available for the performance calculations of the shaded-pole motor. Efforts have been made by many writers but most of the work that has been presented so far is of design nature. In the smaller sizes of motor ratings (below 1/20 horsepower) this type of motor has surpassed all other single-phase motor types. Yet the other types of single-phase motors such as the capacitor, the resistance, split-phase and the repulsion-start-induction-run motors have all been analyzed so far as motor performance calculations are concerned. The object of this thesis is to attempt to devise a method for the performance calculations of the shaded-pole motor using three easily obtained laboratory tests.

Any induction motor is essentially a transformer. That is, the stator, or primary has a voltage applied to its windings and there is induced in the rotor, or secondary, a voltage that is proportional to the turns ratio of the two windings. The only difference in the induction motor and a real transformer is that the voltage of the secondary of the transformer is taken off as useful energy whereas in the induction motor this voltage causes a current to flow in the windings of the rotor which in turn reacts with the stator magnetomotive force. The reaction between these two fluxes causes a troque to be produced which makes the rotor revolve.

The two tests usually used for the determination of motor constants in any induction motor are the no-load, or running light test and the blocked rotor test. The blocked rotor test is used to determine the value of the primary and secondary reactances.

Thus,

$$Z_e = {}^{V_{BR}}/_{I_{BR}}$$

Since the primary resistance can be measured directly using the ammeter-voltmeter test this in effect determines the value of the rotor resistance. Since Z_e may be expressed as

$$Z_e = R_e + jX_e$$

and since

$$R_e = R_1 + R_2$$

where

$$R_e = {}^{W_{BR}}/(I_{BR})^2$$

then the value of X_e is

$$X_e = (Z_e^2 - R_e^2)^{\frac{1}{2}}$$

and the usual assumption is that

$$X_1 = X_2 = {}^{X_e}/2$$

where X_1 and X_2 are the stator and rotor reactance magnitudes.

To find the value of X_c, the reactive component of the core, it is the usual procedure to write that

$$X_c = {}^{(V_{NL} - I_{NL}Z_1)}/I_{NL} \sin \phi_{NL}$$

where

$$\cos \phi_{NL} = {}^{W_{NL}}/I_{NL}V_{NL}$$

The value of R_c, the resistive component of the core, is usually assumed to be

$$R_c = W_{NL} - R_1 \, I^2_{NL}/(I_{NL} \cos \phi_{NL})^2$$

The usual method of determining the motor parameters has been presented. It is understood, of course, that these values may be calculated using design data, but in doing so the procedure is very complicated and of little use in the calculation of motor performance.

The use of the no-load test is the source of considerable error in determining the motor constants. In the general single-phase motor the no-load speed of the motor is only about 97% of synchronous speed. The use of the no-load test makes the assumption that there is no current flow in the rotor circuit of the motor. By referring to Figure III it can be seen that the only variable in the rotor circuit is the resistive load component Z_R which varies as a function of the slip. The relationship is

$$Z_R = {}^{R_2\ S^2}/(1 - S^2)$$

where R is the rotor resistance and S is defined as the ratio,

$$\frac{\text{Rotor actual speed}}{\text{Motor synchronous speed}}$$

Therefore, the only condition when no current flows in the rotor is when the value of S becomes unity, that is, when the rotor is turning at synchronous speed. This may be illustrated in another way. When the rotor is turning at synchronous speed there is no voltage generated in the rotor windings because the rotor bars are cutting no lines of flux. Therefore, there can be no current flow in the rotor. If it is assumed that the rotor is turning at a speed of 97% of synchronous at no-load the value of Z_R at this speed becomes

$$Z_R = 16\ R_2 \text{ (approximately)}$$

Since this is a very finite number there must be a small amount of current flowing in the rotor which will introduce an appreciable error into the solution of motor constants.

It has been shown that the no-load or running light test is in error for the purpose of determining the values of circuit parameters. At the same time it has been shown that for a better solution of circuit parameters that the synchronous load test should be used. It has also been the usual assumption that the values of primary and secondary reactances are equal. In the presentation that follows no assumption of that sort has been made.

For the calculation of the rotor resistance the assumption is made that the current through the shunting impedance branch, Z_c, remains constant from synchronous to blocked rotor condition. This is not exactly a true condition but it simplifies considerably the determination of R_2.

By observing Figure I and Figure II the following relationships may be written.

$$W_B = (I_B)^2R_1 + (I_s)^2R_c + R_2(I_B \rightarrow I_s)^2 \qquad (1)$$

and

$$W_s = (R_1 + R_c)\ (I_s)^2 \qquad (2)$$

Since W_B, W_s, I_s, I_B, and R_1 may be obtained from laboratory tests the values or expressions for R_c and R_1 may be solved for from equations (1) and (2).

Rewriting equation (2)

$$(I_s)^2R_c = W_s - (I_s)^2R_1 \qquad (3)$$

or,

$$R_c = (W_s - I_s^2R_1)/(I_s)^2 \qquad (4)$$

Knowing the value of R_c, the expression for R_2 may be solved for from equation (1). Solving for R_2 from equation (1)

$$R_2 = (W_B - I_B^2R_1 - I_s^2R_c)/\ (I_B \rightarrow I_s)^2 \qquad (5)$$

The expressions for R_c, R_2, and R_1 having been derived the analytical expressions for X_1, X_2, and X_c must be obtained.

From Figure I it can be seen that

$$Z_s \underline{/\ \phi_s} = V_s/I_s = R_s + jX_s = Z_1 + Z_c \qquad (6)$$

or

$$R_s + jX_s = R_1 + jX_1 + R_c + jX_c \qquad (7)$$

Since the quadrature components and the real components of each side

EQUIVALENT CIRCUIT UNDER SYNCHRONOUS CONDITION

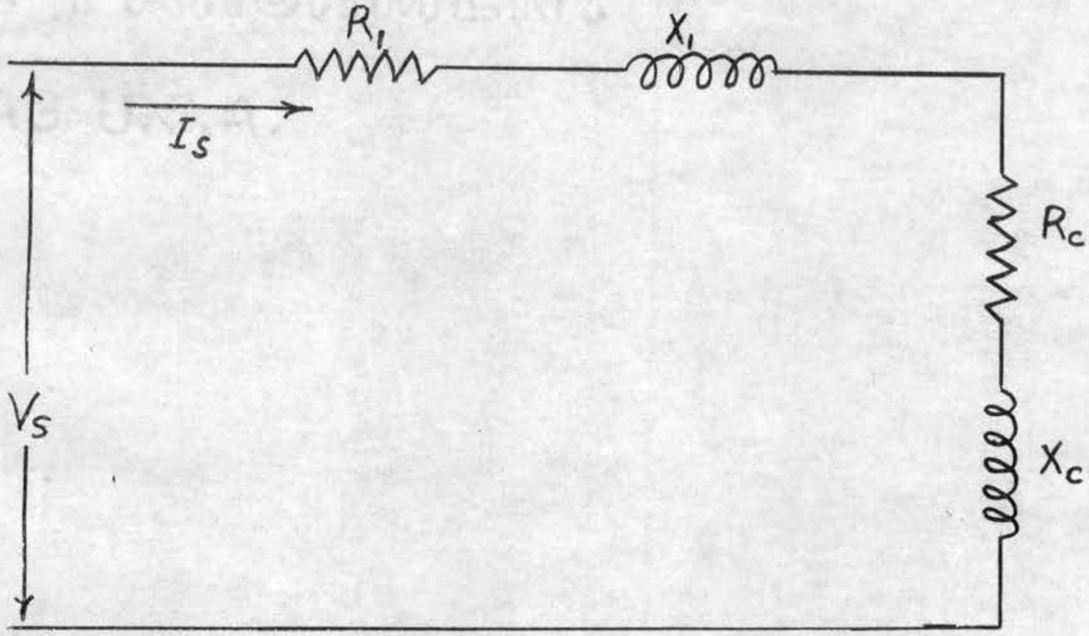

FIGURE I

X_1 Primary or stator reactance

R_1 Primary or stator resistance

X_c Core reactance

R_c Core resistance

EQUIVALENT CIRCUIT UNDER BLOCKED-ROTOR CONDITIONS

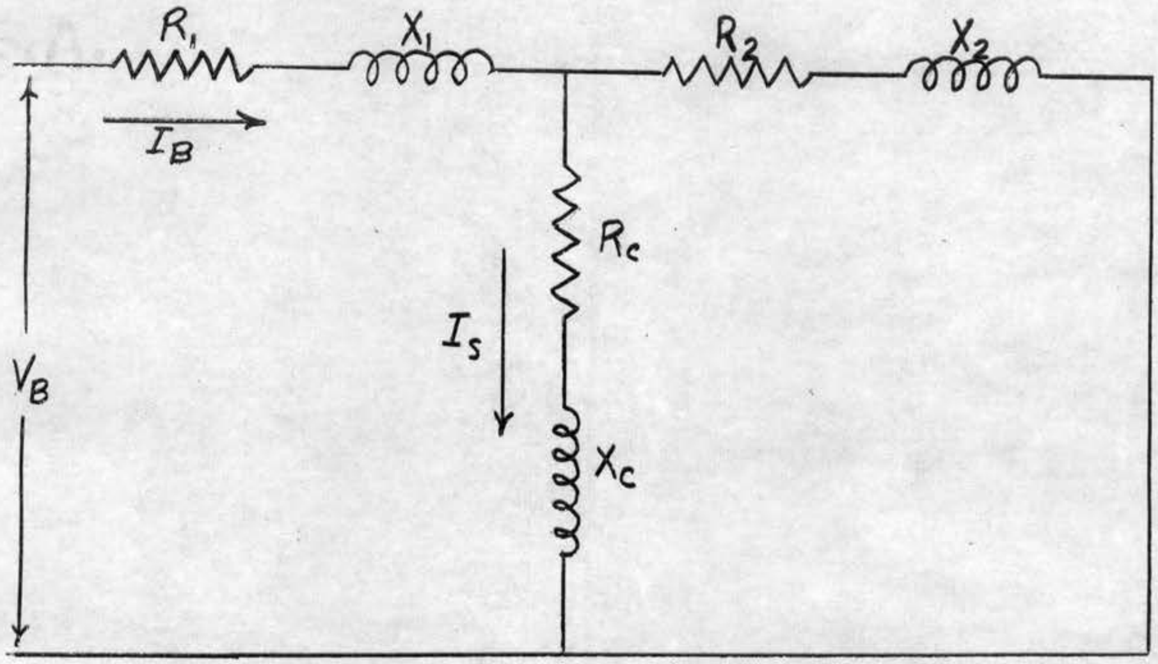

FIGURE II

X_1 Primary or stator reactance

R_1 Primary or stator resistance

X_c Core reactance

R_c Core resistance

X_2 Secondary or rotor reactance

R_2 Secondary or rotor resistance

of equation (7) must be equal the following expressions may be established. That is,

$$R_s = R_1 + R_c \tag{8}$$

and

$$X_s = X_1 + X_c \tag{9}$$

From Figure II it can be seen that

$$Z_B \underline{/-\phi_B} = R_B + jX_B = Z_1 + Z_2Z_c / (Z_2 + Z_c) \tag{10}$$

or,

$$R_B + jX_B = R_1 + jX_1 + \frac{(R_2 + jX_2)\,(R_c + jX_c)}{(R_2 + R_c) + j(X_2 + X_c)} \tag{11}$$

The analytical solution of equation (11) in the form that it is now in will be rather complicated. However, by that assumption that the value $(X_2 + X_c)$ is equal to $1.75X_s$ equation (11) is simplified so that an analytical solution is feasible.

Rewriting equation (11)

$$R_B + jX_B = R_1 + jX_1 + \frac{(R_2R_c - X_2X_c) + j(X_2R_c + X_cR_2)}{(R_2 + R_c) + j1.75X_s} \tag{12}$$

or,

$$R_B + jX_B = R_1 + jX_1 + \frac{A_0 + jB_0}{C_0 + j1.75X_s} \tag{13}$$

where $A_0 = (R_2R_c - X_2X_c)$

$B_0 = (X_2R_c + X_cR_2)$

$C_0 = (R_2 + R_c\ )$

Rationalizing equation (13)

$$R_B + jX_B = R_1 + jX_1 + \frac{(A_0 + jB_0)\,(C_0 - 1.75X_s)}{C_0{}^2 + 3.06X_s^2} \tag{14}$$

$$= R_1 + jX_1 + \frac{(A_0C_0 + 1.75B_0X_s) + j(B_0C_0 - 1.75X_sA_0)}{C_0^2 + 3{,}06X_s^2} \tag{15}$$

Since the real and quadrature components of both sides of equation (15) must be equal, equation (15) may be separated into two equations. One of these equations will contain the real components and the other will contain the quadrature components. Equation (15) becomes

$$R_B = R_1 + \frac{(A_0C_0 + 1.75B_0X_s)}{C_0^2 + 3.06X_s^2} \tag{16}$$

and

$$X_B = X_1 + \frac{(B_0C_0 - 1.75X_sA_0)}{C_0^2 + 3.06X_s^2} \tag{17}$$

From equations (16) and (17) the analytical expressions of X_1 and X_2 may be obtained.

Substituting the values for A_0, B_0. amd C_0 into equation (16) and expanding it becomes

$$R_B = R_1 + \frac{(R_2R_c - X_2X_c)(R_2 + R_c) + 1.75X_s(X_2R_c + X_cR_2)}{(R_2 + R_c)^2 + 3.06X_s^2} \tag{18}$$

From equation (9) the expression

$$X_c = X_s - X_1 \tag{9$_a$}$$

may be written. Substituting equation (9_a) into equation (18) the expression for X_B becomes

$$R_B = R_1 + \left\{\left[R_2R_c - X_2(X_s - X_1)\right](R_2 + R_c) + 1.75X_s\left[X_2R_c + (X_s - X_1)R_2\right]\right\} / \left[(R_2 + R_c)^2 + 3.06X_s^2\right] \tag{19}$$

Letting $A_1 = (R_2 + R_c)^2 + 3.06X_s^2$

$B_2 = R_2R_c$

$C_2 = (R_2 + R_2)$

equation (19) becomes

$$R_B = \frac{A_1R_1 + \left[B_2 - X_2(X_s - X_1)\right]C_2 + 1.75X_s\left[X_2R_c + (X_s - X_1)R_2\right]}{A_1} \tag{20}$$

Expanding equation (20)

$$A_1R_B = A_1E_1 + B_2C_2 - C_2X_2X_s + C_2X_2X_1 + 1.75X_sX_2R_c + 1.75R_2X_s^2 - 1.75X_sX_1R_2 \tag{21}$$

Rearranging equation (21)

$$A_1R_B = A_1R_1 + B_2C_2 + X_2\ (C_2X_1 - C_2X_s + 1.75X_sR_c) + 1.75R_2X_s^2 - 1.75X_sX_1R_2 \tag{22}$$

Rearranging equation (22)

$$X_2(C_2X_1 - C_2X_s + 1.75X_sR_c) = A_1R_B - A_1R_1 - B_2C_2 - 1.75R_2X_s^2 + 1.75X_sX_1R_2 \tag{23}$$

Solving for X_2 from equation (23)

$$X_2 = \frac{A_1R_B - A_1R_1 - B_2C_2 - 1.75R_2X_s^2 + 1.75X_sX_1R_2}{C_2X_1 + 1.75X_sR_c - C_2X_s} \tag{24}$$

Thus an expression for X_2 in terms of X_1 has been derived. By observing equation (24) it will be noted that all of the components except those of X_1 and X_2 are known. Therefore the equation may be written as

$$X_2 = \frac{A_3 + C_3X_1}{C_2X_1 + B_3} \tag{25}$$

Where $A_3 = A_1R_B - A_1R_1 - B_2C_2 - 1.75X_s^2R_2$

$B_3 = 1.75R_cX_s - C_2X_s$

$C_3 = 1.75X_sR_2$

Rewriting equation (17)

$$X_B = X_1 + \frac{B_0C_0 - 1.75X_sA_0}{C_0^2 + 3.06X_s^2} \tag{17}$$

Substituting the values for the constants into equation (17) it becomes

$$X_B = X_1 + \frac{(X_2R_c + X_cR_2)C_2 - 1.75X_s(B_2 - X_2X_c)}{A_1} \tag{26}$$

Since $X_c = X_s - X_1$ this relationship may be substituted into equation (26).

The equation then becomes

$$X_B A_1 = X_1 A_1 + \left[R_c X_2 + (X_s - X_1)R_2\right] C_2 - 1.75X_s \left[B_2 - X_2(X_s - X_1)\right] \quad (27)$$

Expanding equation (27)

$$A_1 X_B = \left[R_c X_2 + R_2 X_s - R_2 X_1\right] C_2 - 1.75X_s \left[B_2 - X_2 X_s + X_1 X_2\right] + A_1 X_1 \quad (28)$$

Rearranging equation (28)

$$A_1 X_B = X_1 A_1 + C_2 R_c X_2 - C_2 R_2 X_s - C_2 R_2 X_1 - 1.75X_s B_2 + 1.75X_2 X_s^2$$
$$- 1.75X_1 X_2 X_s \quad (29)$$

Rearranging equation (29) it transforms to

$$A_1 X_B = X_1 A_1 - C_2 R_2 X_1 + C_2 R_c X_2 + 1.75X_2 X_s^2 - 1.75X_1 X_2 X_s + C_2 R_2 X_s$$
$$- 1.75X_s B_2 \quad (30)$$

or

$$A_1 X_B = (A_1 - C_2 R_2)X_1 + (C_2 R_c + 1.75X_s^2)X_2 - 1.75X_1 X_2 X_s$$
$$+ (C_2 R_2 X_s - 1.75X_s B_2) \quad (31)$$

All of the values of equation (31) are known but X_1 and X_2. For easier manipulation of the equation it may be rewritten as

$$A_1 X_B = A_4 X_1 + B_4 X_2 - 1.75(X_1 X_s)X_2 + C_4 \quad (32)$$

where

$$A_4 = A_1 - C_2 R_2$$
$$B_4 = C_2 R_c + 1.75X_s^2$$
$$C_4 = C_2 R_2 X_s - 1.75X_s B_2$$

Substituting equation (25) into equation (32) it becomes

$$A_1 X_B = A_4 X_1 + B_4 \frac{(A_3 + C_3 X_1)}{(C_2 X_1 + B_3)} - 1.75X_1 X_s \frac{(A_3 + C_3 X_1)}{(C_2 X_1 + B_3)} + C_4 \quad (33)$$

Expanding equation (33)

$$A_1 X_B = A_4 X_1 + \frac{B_4 A_3 + B_4 C_3 X_1}{C_2 X_1 + B_3} - \frac{1.75X_1 X_s A_3 - 1.75X_1^2 X_s C_3}{C_3 X_1 + B_3} + C_4 \quad (34)$$

or

$$A_1X_B(C_2X_1 + B_3) = A_4X_1\ (C_2X_1 + B_3) + B_4A_3 = B_4C_3X_1 - 1.75X_1X_sA_3 - 1.75X_1^2X_sC_3 + C_4(C_2X_1 + B_3) \quad (35)$$

Equation (35) may be rearranged further by grouping the X_1^2 terms, the X_1 terms and the constants together;

$$0 = X_1^2(A_4C_2 - 1.75X_sC_3) + X_1(B_3A_4 + B_4C_3 + C_4C_2 - 1.75X_sA_3 - A_1X_BC_2) + (B_4A_3 + B_3C_4 - A_1X_BB_3) \quad (36)$$

By inspection it is readily seen that equation (36) is a quadratic equation of form

$$ax^2 + bx + c = 0 \quad (36_a)$$

the solution of which is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (37)$$

or the solution of equation (36) is

$$X_1 = \frac{-b \pm \sqrt{b^2 - 4\ ac}}{2a} \quad (38)$$

where

$$a = A_4C_2 - 1.75X_sC_3$$

$$b = B_3A_4 + B_4C_3 + C_4C_2 - 1.75X_sA_3 - A_1X_BC_2$$

$$c = B_4A_3 - B_3A_1X_B + C_4B_3$$

and

$$A_1 = (R_2 + R_c)^2 + 3.06X_s^2$$

$$A_3 = A_1R_B - A_1R_1 - B_2C_2 - 1.75R_2X_s^2$$

$$A_4 = A_1 - C_2R_2$$

$$B_2 = R_2R_c$$

$$B_3 = 1.75R_cX_s - C_2X_s$$

$$B_4 = C_2R_c + 1.75X_s^2$$

$$C_2 = R_2 + R_c$$

$$C_3 = 1.75R_2X_s$$

$$C_4 = C_2R_2X_s - 1.75X_sB_2$$

The solution of equation (38) will yield two solutions from which the real answer must be ascertained.

The solution for the primary, core and the secondary reactances just presented was simplified by the assumption that the quantity $(X_2 + X_c)$ was equal to $1.75X_s$. The solution of these quantities will now be presented without making the assumption that the value $(X_2 + X_c)$ is equal to $1.75X_s$.

Rewriting equation (11) as

$$R_B + jX_B = R_1 + jX_1 + \frac{(R_2R_c - X_2X_c) + j(R_2X_c + R_cX_2)}{(R_2 + R_c) + j(X_2 + X_c)} \qquad (39)$$

Rationalizing equation (39)

$$R_B + jX_B = R_1 + jX_1 + \frac{(A_2 - X_2X_c) + j(R_2X_c + R_cX_2)\left[A_1 - j(X_2 + X_c)\right]}{A_1^2 + (X_2 + X_c)^2} \qquad (40)$$

where

$$A_1 = R_2 + R_c$$

$$A_2 = R_2R_c$$

Expanding equation (40)

$$R_B + jX_B = R_1 + jX_1 + \frac{A_1(A_2 - X_2X_c) + jA_1(R_2X_c + R_cX_2)}{A_1^2 + (X_2 + X_c)^2}$$

$$- \frac{j(A_2 - X_2X_c)(X_2 + X_c) + (R_2X_c + R_cX_2)(X_2 + X_c)}{A_1^2 + (X_2 + X_c)^2} \qquad (41)$$

Since the real and quadrature components of both sides or equation (41) are equal the equation may be separated into its real and quadrature components. Separating the equation into its real and quadrature parts;

$$R_B = R_1 + \frac{A_1(A_2 - X_2X_c) + (R_2X_c + R_cX_2)(X_2 + X_c)}{A_1^2 + (X_2 + X_c)^2} \tag{42}$$

and

$$X_B = X_1 + \frac{A_1(R_2X_c + R_cX_2) - (A_2 - X_2X_c)(X_2 + X_c)}{A_1^2 + (X_2 + X_c)^2} \tag{43}$$

Expanding equation (42)

$$(R_B - R_1)\left[A_1^2 + (X_2 + X_c)^2\right] = A_1(A_2 - X_2X_c) + (R_2X_c + R_cX_2)(X_2 + X_c) \tag{44}$$

or

$$(R_B - R_1)A_1^2 + (R_B - R_1)(X_2 + X_c)^2 = A_1A_2 - A_1X_2X_c + X_2R_2X_c$$
$$+ R_cX_2^2 + R_2X_c^2 + R_cX_cX_2 \tag{45}$$

Rearranging equation (45)

$$A_3 = (R_1 - R_B)(X_2^2 + 2X_cX_2 + X_c^2) - A_1X_2X_c + X_2R_2X_c + R_2X_c^2 + R_cX_cX_2 \tag{46}$$

where

$$A_3 = A_1^2(R_B - R_1) - A_1A_2$$

Since $X_c = X_s - X_1$ this relationship may be substituted into equation (46)

$$A_3 = A_4X_2^2 + 2X_2A_4(X_s - X_1) + A_4(X_s^2 - 2X_sX_1 + X_1^2) - A_1X_2(X_s - X_1)$$
$$+ X_2R_2(X_s - X_1) + R_cX_2^2 + R_2(X_s - X_1)^2 + R_cX_2(X_s - X_1) \tag{47}$$

Expanding equation (47)

$$A_3 = A_4X_2^2 + 2X_2A_4X_s - 2X_2A_4X_1 + A_4X_s^2 - 2A_4X_sX_1 + A_4X_1^2 - A_1X_2X_s$$
$$+ A_1X_2X_1 + X_2X_sR_2 - X_2X_1R_2 + R_cX_2^2 + R_2X_s^2 - 2R_2X_sX_1$$
$$+ R_2X_1^2 + R_cX_2X_s - R_cX_2X_1 \tag{48}$$

Rearranging equation (48)

$$A_3 - A_4X_s^2 - R_2X_s^2 = X_2^2(A_4 + R_c) + X_1^2(A_4 + R_2) + X_2(2A_4X_s - A_1X_s + R_2X_s + R_cX_s)$$
$$X_1(-2X_2A_4 - 2A_4X_s + A_1X_2 - X_2R_2 - 2R_2X_s - R_cX_2) \tag{49}$$

Equation (49) may be written as

$$A_5 = A_6X_1^2 + (X_2A_7 + A_8)X_1 + A_9X_2 + A_{10}X_2^2 \tag{50}$$

where
$$A_1 = R_2 + R_c$$
$$A_2 = R_2R_c$$
$$A_3 = A_1^2(R_B - R_1) - A_1A_2$$
$$A_4 = R_1 - R_B$$
$$A_5 = A_3 - A_4X_s^2 - R_rX_s^2$$
$$A_6 = A_4 + R_2$$
$$A_7 = A_1 - 2A_4 - R_2 - R_c = -2A_4$$
$$A_8 = 2X_s(A_4 + R_2) = -2X_sA_6$$
$$A_9 = X_s(2A_4 - A_1 + R_2 + R_c) = -X_sA_7$$
$$A_{10} = A_4 + R_c$$

Equation (50) may be solved for X_1 in terms of X_2 but the solution will yield two values or roots. The correct root cannot be ascertained by any known method.

To find the other equation that will satisfy the solution for the motor constants equation (43) will now be used.

Expanding equation (43)

$$(X_B - X_1)\left[A_1^2 + (X_2 + X_c)^2\right] = A_1(R_2X_c + R_cX_2) - (A_2 - X_2X_c)(X_2 + X_c) \tag{51}$$

Substituting $X_c = X_s - X_1$ into the equation (51)

$$X_BA_1^2 + X_B\left[X_2^2 + 2X_2(X_s - X_1) + X_s^2 - 2X_sX_1 + X_1^2\right] - X_1A_1^2 +$$
$$-X_1\left[X_2^2 + 2(X_s - X_1)X_2 + X_s^2 - 2X_sX_1 + X_1^2\right]$$
$$= A_1\left[R_2(X_s - X_1) + R_cX_2\right] - \left[A_2 - X_2(X_s - X_1)\right]\left[X_2 + X_s - X_1\right] \tag{51}$$

Expanding equation (51)

$$X_BA_1^2 + X_B(X_2^2 + 2X_2X_s - 2X_2X_1 + X_s^2 - 2X_sX_1 + X_1^2) - X_1A_1^2 +$$

$$- X_1 (X_2^2 + 2X_2X_s - 2X_1X_2 + X_s^2 - 2X_sX_1 + X_1^2)$$

$$= A_1(R_2X_s - R_2X_1 + R_cX_2) - (A_2 - X_2X_s + X_1X_2)(X_2 + X_s - X_1) \tag{52}$$

Expanding equation (52)

$$X_BA_1^2 + X_BX_2^2 + 2X_2X_sX_B - 2X_2X_sX_B + X_BX_s^2 - 2X_sX_1X_B + X_BX_1^2 - X_1A_1^2 +$$

$$-2X_1X_2X_s + 2X_1^2X_2 - X_1X_s^2 + 2X_sX_1^2 - X_1^3$$

$$= A_1R_2X_s - A_1R_2X_1 + A_1R_cX_2 - (2A_2X_2 + A_2X_s - A_2X_1 - X_2^2X_s -$$

$$X_2X_s^2 + X_1X_2X_s + X_1X_2^2 + X_1X_2X_s - X_1^2X_2) \tag{53}$$

Which may be arranged into the following form:

$$X_BA_1^2 + X_BX_s^2 - A_1R_2X_s + A_2X_s$$

$$= X_1^3 + \left[-X_2 - (X_B + 2X_s)\right] X_1^2 + \left[X_2(2X_B) + 2X_sX_B + A_1^2 + X_s^2 - A_1R_2 + A_2\right] X_1 +$$

$$(A_1R_c + X_s^2 - A_2 - 2X_sX_B)X_2 + (X_s - X_B)X_2^2 \tag{54}$$

For simplification the equation may be rewritten as

$$B_7 = X_1^3 + (-X_2 + B_2)X_1^2 + (2X_BX_2 + B_3)X_1 + B_4X_2 + B_5X_2^2 \tag{55}$$

where

$$A_1 = R_2 + R_c$$

$$A_2 = R_2R_c$$

$$B_2 = -(X_B + 2X_s)$$

$$B_3 = 2X_sX_B + A_1^2 + X_s^2 - A_1R_2 + A_2$$

$$B_4 = A_1R_c + X_s^2 - A_2 - 2X_sX_B$$

$$B_5 = X_s - X_B$$

$$B_7 = X_BA_1^2 + X_BX_s^2 - A_1R_2X_s + A_2X_s$$

The two equations necessary for the exact solution of the motor reactances have been derived. Unfortunately they are somewhat unwieldy in any form into which they are put. There are two methods available in order to obtain a solution of the two equations. One method is to take the two roots of equation (50) (since it is a quadratic equation), substitute these roots into equation (55) and solve for the unknown. The other method is to plot the two equations and obtain the solution graphically. The latter method will probably be the easier of the two suggested.

The procedure used in most cases for the determination of motor constants is to assume that the primary and secondary reactances are equal. This is an approximate assumption at best and in most cases will not be true. The derivation presented in this thesis was made with as few assumptions as possible. In the first derivation one assumption was made and in the second derivation none were made. However, the curcuit used for the motor equivalent is only the approximate equivalent. The difference in the two circuits is not extensive and will not cause much error in the final result. The tests necessary for the determination of the parameters are simple and can be made in the laboratory with little trouble. The tests include that of synchronous, and blocked rotor conditions, that is, the power input, the input current and the applied voltage. The running-light or no-load speed must be determined. These are the tests necessary for the calculation of motor performance.

CHAPTER III

SOLUTION OF THE EQUIVALENT CIRCUIT

The solution of Figure III for the performance of the motor will now be presented. Since all of the motor constants have been determined analytically this may be done in terms of actual designations of the constants. The values to be determined are the input current, the input power, the power factor, the output power and the efficiency of the motor. These quantities are standard in the calculation of any motor performance.

Referring to Figure III it can be seen that the impedance of the rotor circuit is

$$Z_2 = (R_2 + Z_R) + jX_2 \tag{63}$$

and the impedance of the shunting impedance is

$$Z_c = R_c + jX_c \tag{64}$$

The equivalent impedance of the rotor and the shunting impedance is

$$Z_3 = Z_2 Z_c \; / \; (Z_2 + Z_c) \tag{65}$$

and since the stator impedance is in series with the equivalent impedance of the rotor and shunting impedances the total impedance of the motor is

$$\begin{aligned} Z_T &= R_1 + jX_1 + Z_3 \\ &= Z_1 + Z_3 \\ &= Z_T \angle \phi_T \end{aligned} \tag{66}$$

EQUIVALENT CIRCUIT UNDER RUNNING CONDITIONS

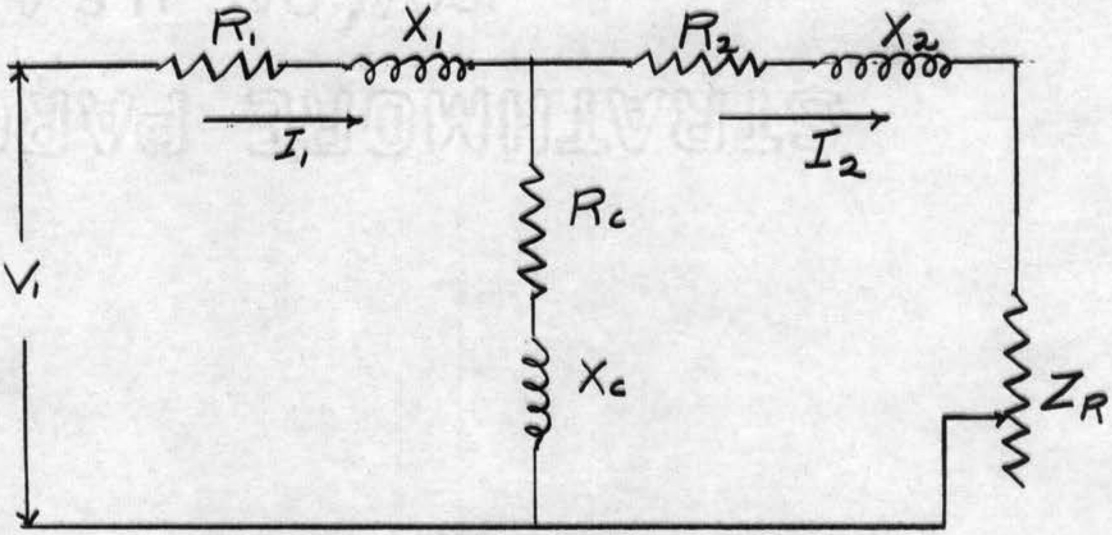

FIGURE III

X_1 Primary or stator reactance

R_1 Primary or stator resistance

X_c Core reactance

R_c Core resistance

X_z Secondary or rotor reactance

R_2 Secondary or rotor resistance

Z_r Equivalent load resistance

Since the ratio of applied voltage to the impedance presented to the voltage is current, the input current of the motor is

$$I_{in} = V_1 / Z_T \angle \phi_T$$

$$= V_1 \angle -\phi_T / Z_T \qquad (67)$$

The power factor angle is the angle of lead or lag of the current (with the voltage vector as the reference vector). Therefore, the power factor angle of the motor is $-\phi_T$ and the power factor of the motor is

$$PF = \cos(-\phi_T)$$

$$= \cos \phi_T \qquad (68)$$

The input power supplied to the motor is the product of the applied voltage, the input current and the power factor. To state the previous statement analytically

$$P_{in} = V_1 X I_{in} X \cos \phi_T \qquad (69)$$

Thus the power input, the power factor and the input current have been determined in terms of the circuit constants. The output power and the efficiency of the motor must be determined.

The primary (stator) impedance and the equivalent impedance Z_3 are in series, therefore the voltage drop across Z_3 is

$$V_{Z_3} = I_{in} X Z_3 \angle \phi_3 - \phi_T \qquad (70)$$

Since the same voltage is effectively across the rotor impedance Z_2 the current flowing through Z_2 is

$$I_2 = V_{Z_3} / Z_2 \qquad (71)$$

The output resistance of the motor is represented by the symbol Z_R and is a function of the slip of the motor. The analytical expression is

$$Z_R = R_2 S^2 / (1 - S^2) \qquad (72)$$

$$\text{where } S = \frac{\text{actual rotor speed}}{\text{motor synchronous speed}}$$

The power dissipated in the output resistance is

$$W_{Z_R} = (I_2)^2 \, Z_R \tag{73}$$

It is usually assumed that the friction and windage of a motor remains constant for the purpose of motor performance calculations. However, in the shaded pole motor the speed is a highly variable characteristic and for that reason the friction and windage cannot be assumed to remain constant. For the performance calculations in this paper the friction and windage will be assumed to vary as the speed varies and hence as the slip. It must be noted that the difference between the power input at synchronous speed and the power input at no-load is almost one-half of the power output of the motor at full-load. For this reason the friction and windage of the machine will be taken as this difference in power. This value will be assumed to vary as the function $S^{2.5}$. Now if one-half of the rotor resistance contributes power to the load of the motor then the power output of the motor is

$$P_{out} = W_{Z_R} + \tfrac{1}{2}(I_2)2_{R_2} - (F + W)(S)^{2.5} \tag{74}$$

The efficiency of the motor is the ratio of the power output to the power input and may be expressed by

$$\text{Eff} = P_{out}/P_{in} \tag{75}$$

Thus the analytical solution of the equivalent circuit has been presented. Since in any study the analytical solution should be followed by an example of the motor performance in numerical values this will not be presented.

Since it is desirable in many cases to have a predetermined procedure for the calculation of motor performance a preliminary calculations sheet and a calculation sheet have been included in this thesis. The calculation sheet is a straight forward solution of the equivalent circuit of the motor.

PRELIMINARY CALCULATIONS

W_s = _____ I_s = _____ V_s = _____

W_{nl} = _____ I_{nl} = _____ V_{nl} = _____

W_B = _____ I_B = _____ V_B = _____

N_{nl} = _____ $\cos \phi_s = W_s / V_s I_s$ = _____ ϕ_s = _____

$\cos \phi_B = W_B / V_B I_B$ = _____ ϕ_B = _____

R_1 = _____ $Z_s = V_s / I_s$ = _____ $Z_B = V_B / I_B$ = _____

$(F \quad W) = W_{nl} - W_s$ = _____

$Z_s \angle \phi_s = R_s + jX_s$ = _____ + j_____

$Z_B \angle \phi_B = R_B + jX_B$ = _____ + j_____

R_c = _____ use equation (4)

R_2 = _____ use equation (5)

X_1 = _____ use equation (38)

X_2 = _____ use equation (25)

$X_c = X_s - X_1$ = _____

CALCULATION SHEET

(1) S

(2) $R_2S^2/(1 - S^2)$

(3) $(2) + R_2$

(4) $\sqrt{(3)^2 + X_1^2}$

(5) $\phi_2 = \arctan X_2/(3)$

(6) R_c

(7) X_c

(8) $(6) + (3)$

(9) $(7) + X_2$

(10) $\sqrt{(8)^2 + (9)^2}$

(11) $a = \arctan (9)/(8)$

(12) $\sqrt{(6)^2 + (7)^2}$

(13) $b = \arctan (7)/(6)$

(14) $(4)(12)/(10)$

(15) $c = (5) + (13) - (11)$

(16) $(14)\cos(15)$

(17) $(14) \sin (15)$

(18) $(16) + R_1$

(19) $(17) + X_1$

(20) $\sqrt{(18)^2 + (19)^2}$

(21) $d = \arctan (19)/(18)$

(22) $PF = \cos (21)$

(23) $I_{in} = V_1/(20)$

(24) $P_{in} = V_1(23)(22)$

(25) $(23)(14)$

(26) $(25)/(4)$

(27) $(26)^2(2)$

(28) $\frac{1}{2}(26)^2(R_2)$

(29) $(F + W)(S)^{2.5}$

(30) $P_{out} = (27) + (28) - (29)$

(31) $Eff = (30)/(24)$

(32) $HP_{out} = (30)/746$

CHAPTER IV

DETERMINATION OF MOTOR CONSTANTS AND EXAMPLE OF PERFORMANCE CALCULATIONS FROM LABORATORY TEST RESULTS

The following values were obtained from tests made upon the motor in the laboratory.

R_1 = 10.1 ohms

W_s = 42 watts	I_s = .86 amps	V_s = 115 volts
W_{nl} = 50.5 watts	I_{nl} = .92 amps	V_{nl} = 115 volts
W_B = 104 watts	I_B = 1.56 amps	V_B = 115 volts

(F + W) = 8.5 watts

Determination of motor constants

From equation (4)

$$R_c = (W_s - I_s^2 R_1)/(I_s)^2 \qquad (4)$$

$$= (42 - .86^2 \text{ X } 10.1)/\ .86^2$$

$$= \underline{46.8 \text{ ohms}}$$

From equation (5)

$$R_2 = (W_B - I_B^2\ R_1 - I_s^2\ R_c)/(I_B \rightarrow I_s)^2 \qquad (5)$$

$$= (104 - 1.56^2 \text{ X } 10.1 - .86^2 \text{ X } 46.8)/(1.56)\underline{/-54.5^\circ} - .86\underline{/-64.8^\circ})$$

$$= \underline{83.4 \text{ ohms}}$$

The blocked rotor impedance is

$$Z_B = V_B\ /\ I_B\ \underline{/-\phi_B}$$

$$= 115\ \underline{/\ 0^\circ}\ /\ 1.56\ \underline{/-54.5^\circ}$$

$$= 73.6\ \underline{/54.5^\circ} = \underline{42.7 + j60}$$

The synchronous impedance is

$$Z_s = V_s \,/\, I_s \,\underline{/ - \phi_s}$$

$$= 115 \,\underline{/ 0^o} \,/\, .86 \,\underline{/ -64.8^o}$$

$$= 133.8 \,\underline{/ 64.8^o} = 56.9 = j121$$

Therefore

$$X_B = 60 \text{ ohms}$$

$$X_s = 121 \text{ ohms}$$

$$R_B = 42.7 \text{ ohms}$$

$$R_s = 56.9 \text{ ohms}$$

From equation (36) the following constants are defined. The analytical expressions will not be shown.

$$A_1 = (83.4 + 46.8)^2 + 3.06 \text{ X } 121^2$$

$$= 6.17 \times 10^4$$

$$A_3 = 6.17 \times 10^4 (42.7 - 10.1) - .39 \text{ X } 10^4 \text{ X } 130.2 - 1.75 \text{ X } 83.4 \text{ X } 121^2$$

$$= - 62.8 \text{ X } 10^4$$

$$A_4 = (6.17 \text{ X } 10^4 - 83.4 \text{ X } 130.2)$$

$$= 5.084 \text{ X } 10^4$$

$$B_2 = 83.4 \text{ X } 46.8$$

$$= .390 \text{ X } 10^4$$

$$B_3 = 121.0 \text{ X } 46.8 \text{ X } 1.75 - 130.2 \text{ X } 121$$

$$= -.584 \text{ X } 10^4$$

$$B_4 = 130.2 \text{ X } 46.8 + 1.75 \text{ X } 121^2$$

$$= 3.17 \text{ X } 10^4$$

$$C_2 = 83.4 + 46.8$$

$$= 130.2$$

$$C_3 = 1.75 \text{ X } 83.4 \text{ X } 121$$

$$= 1.764 \text{ X } 10^4$$

$$C_4 = 130.2 \text{ X } 83.4 \text{ X } 121 - 1.75 \text{ X } 121 \text{ X } .39 \text{ X } 10^4$$

$$= 48.9 \text{ X } 10^4$$

From equation (38) the following constants are defined. The analytical expressions will not be shown.

$$a = 5.085 \text{ X } 10^4 \text{ X } 130.2 - 1.75 \text{ X } 121 \text{ X } 1.764 \text{ X } 10^4$$

$$= 290.2 \text{ X } 10^4$$

$$b = 5.085 \text{ X } 10^4 \text{ X } (-.584 \text{ X } 10^4) = 3.17 \text{ X } 10^4 \text{ X } 1.764 \text{ X } 10^4 - 1.75 \text{ X } 121 \text{ X } (-62.8 \text{ X } 10^4) + 130.2 \text{ X } 48.9 \text{ X } 10^4 - 6.17 \text{ X } 10^4 \text{ X } 60 \text{ X } 130.2$$

$$= -.231 \text{ X } 10^8$$

$$c = (3.17 \text{ X } 10^4 \text{ X } -.62.8 \text{ X } 10^4) - (60 \text{ X } 6.17 \text{ X } 10^8 \text{ X } -.584) + (48.9 \text{ X } - .584 \text{ X } 10^8)$$

$$= -11.5 \text{ X } 10^8$$

$$X_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (38)$$

$$= \frac{.231 \text{ X } 10^8 \pm \sqrt{(.231 \text{ X } 10^8)^2 - 4 \text{ X } 290.2 \text{ X } -11.5 \text{ X } 10^{12}}}{2 \text{ X } 290.2 \text{ X } 10^4}$$

$$= 1.408 \text{ X } 10^8 / .580 \text{ X } 10^7$$

$$= \underline{24.3 \text{ ohms}}$$

Since

$$X_c = X_s - X_1$$

$$= 121 - 24.3$$

$$= \underline{96.7 \text{ ohms}}$$

From equation (25) the value of X_2 may be determined.

$$X_2 = (A_3 + C_3 X_1) / (C_2 X_1 + B_3) \qquad (25)$$

$$= \frac{-62.8 \text{ X } 10^4 + 1.764 \text{ X } 10^4 \text{ X } 24.3}{130.2 \text{ X } 24.3 - .584 \text{ X } 10^4}$$

$$= -20.1 \text{ X } 10^4 / -.268 \text{ X } 10^4$$

$$= \underline{75.0 \text{ ohms}}$$

EQUIVALENT CIRCUIT AS DETERMINED BY

LABORATORY TEST RESULTS

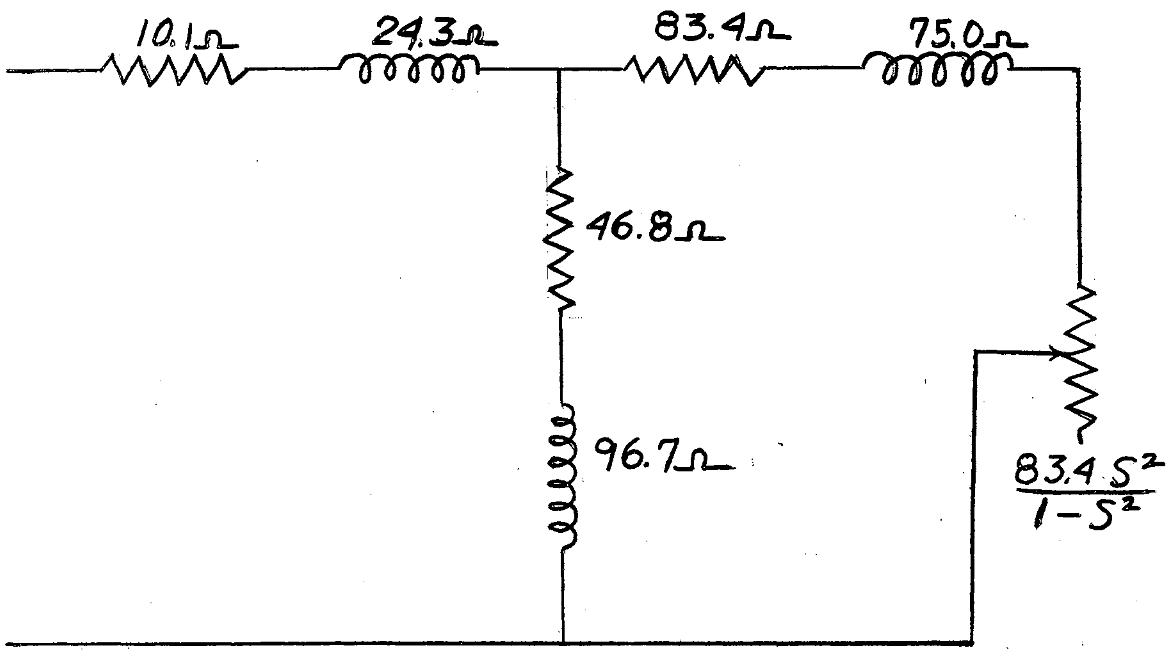

FIGURE IV

X_1 = 24.3 ohms

R_1 = 10.1 ohms

X_c = 96.7 ohms

R_c = 46.8 ohms

X_2 = 75.0 ohms

R_2 = 83.4 ohms

Z_r = 83.4 $S^2/(1 - S^2)$ ohms

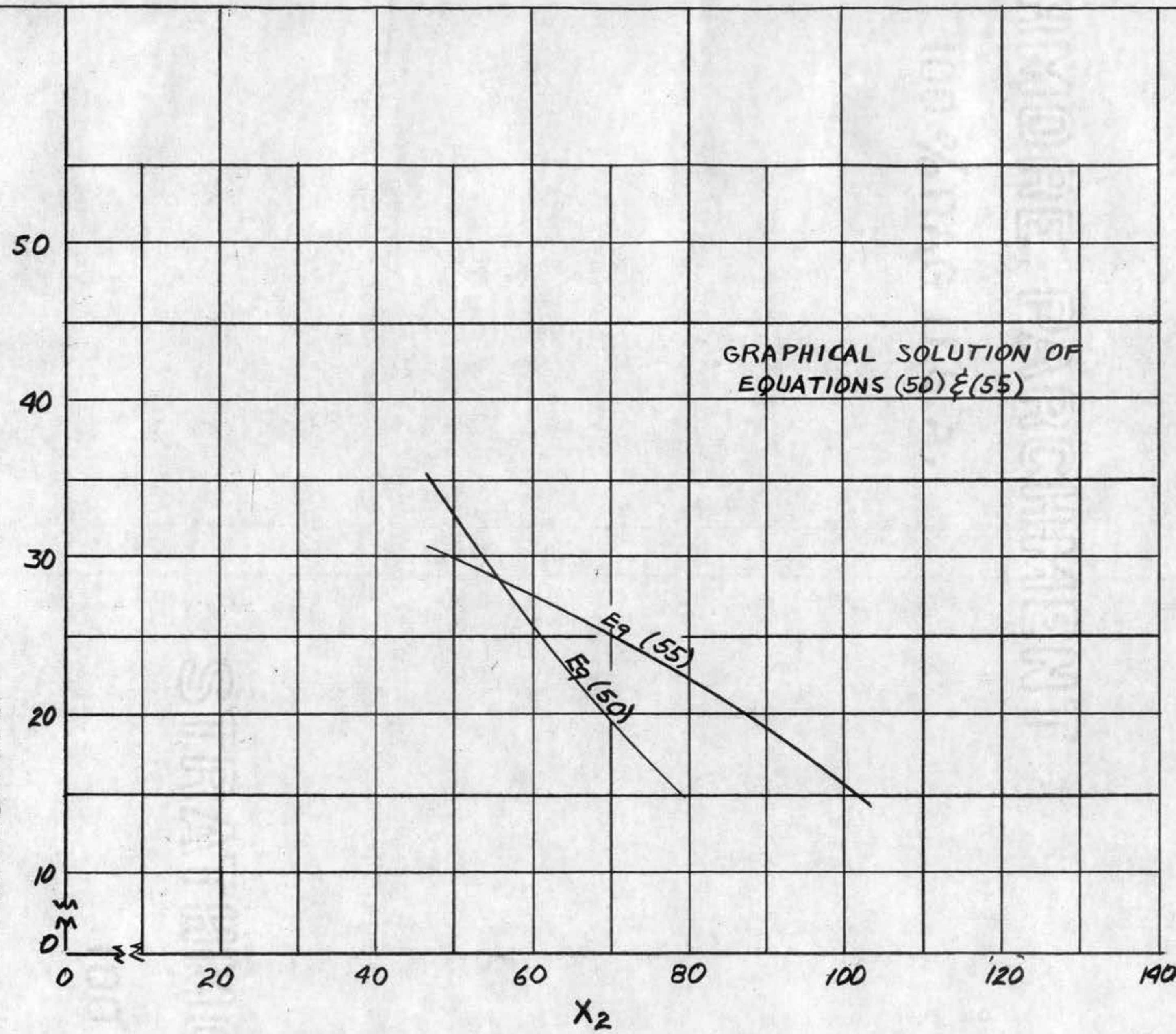
GRAPHICAL SOLUTION OF
EQUATIONS (50) & (55)
Eq (55)
Eq (50)
X_1
X_2
50
40
30
20
10
0
0
20
40
60
80
100
120
140

FIGURE V

Example of Motor Performance Calculations

The output of the motor is a function of the slip of the motor. Therefor let

$$S = .80$$

then $$S^2 = .64 \quad \text{and } 1 - S^2 = .36$$

The impedance of the rotor circuit is

$$Z_2 = (Z_R + R_2) + jX_2$$

$$= (83.4)(.64)/(.36) + 83.4 + j75.0$$

$$= 243\underline{/17.95^\circ} \text{ ohms}$$

The impedance of the shunting impedance is

$$Z_c = R_c + jX_c$$

$$= 46.8 + j96.7$$

$$= 107.5\underline{/64.2^\circ} \text{ ohms}$$

The sum of the shunting and rotor impedance is

$$Z_c + Z_2 = 279.2 + j171.7$$

$$= 327\underline{/31.6^\circ} \text{ ohms}$$

The magnitude of the equivalent impedance of the shunting and rotor impedance is

$$Z_3 = Z_2 Z_c/(Z_2 + Z_c)$$

$$= 243\ (107.5)\underline{/64.2^\circ + 17.95^\circ - 31.6^\circ}\ /327$$

$$= 79.9\underline{/50.55^\circ} \text{ ohms}$$

or expressed in coordinate form

$$= 50.6 + j61.6$$

The total impedance of the motor is the sum of Z_3 and Z_1 is

$$Z_T = Z_1 + Z_3$$

$$= 10.1 + j24.3 + 50.6 + j61.6$$

$$= 60.7 + 85.0$$

and expressed in polar form is

$$Z_T = 104.5\underline{/54.5^{\circ}} \text{ ohms}$$

The total impedance of the motor has been found and from this magnitude the remaining quantities of the motor performance may be obtained.

The input current is the ratio of the applied voltage to the impedance presented to this voltage. Therefor the input current is

$$I_{in} = V_1/Z_T$$

$$= 115/104.5$$

$$= \underline{1.100 \text{ amps}}$$

The power factor of the motor is defined as the cosine of the angle between the voltage and current. Since this angle is determined by the impedance of the circuit the power factor angle of the motor is the negative of the impedance angle. Therefore the power factor is

$$PF = \cos(-\phi_T)$$

$$= \cos(\phi_T)$$

$$= \cos(54.5)$$

$$= .580$$

The power input of the motor is the product of the applied voltage, the input current and the power factor and is

$$P_{in} = V_1 I_{in} \cos(\phi_T)$$

$$= 115\,(1.100)(.580)$$

$$= \underline{73.6 \text{ watts}}$$

The voltage drop across the impedance Z_3 is

$$V_{Z_3} = Z_3 I_{in}$$

$$= 79.9\,(1.100)$$

$$= 87.9 \text{ volts}$$

Since the same voltage is impressed across the rotor impedance the current flowing in the rotor circuit is

$$VI_2 = V_{Z_3} / Z_2$$

$$= 87.9 / 243$$

$$= .362 \text{ amps}$$

The power taken by the output resistance is

$$W_{Z_R} = (I_2)^2 Z_R$$

$$= (.362)^2 (148)$$

$$= \underline{19.30 \text{ watts}}$$

The total power output of the motor is from equation (74)

$$P_{out} = 19.3 + \tfrac{1}{2}(83.4)(.362)^2 - (9.5) \text{ X } (.8)^{2.5}$$

$$= \underline{19.29 \text{ watts}}$$

The efficiency of the motor is the ratic of the output power of the motor to the input power of the motor and is

$$Eff = 19.29 / 73.6$$

$$= 26.2\%$$

Comparison of Laboratory Tests and Calculated Results

	Calculated	Laboratory Tests	% Error
I_{in}	1.100	1.140	3.45
P_{in}	73.6	74.5	1.2
PF	.5800	.590	1.6
Eff	26.2	26	.9
&RPM	1440	1400	2.7

The greatest error obtained was the calculated speed of the motor. However, these errors increase in magnitude somewhat as the horsepower output of the motor is decreased. This is not an uncommon occurrence in motor performance calculations.

The example shown was the performance calculation of a Robins and Myers 1/40 H P Shaded Pole Motor. The entire calculation is shown on the calculation sheet for the 1/40 H P motor.

CALCULATION SHEET FOR ROBINS AND MYERS 1/40 H P MOTOR

		1	2	3	4	5	6	7
1	S	.80	.825	.850	.875	.900	.925	.956
2	$R_2S^2/1 - S^2$	148	178	217	273	356	493	896
3	$2 + R_2$	221.4	261.4	300.4	356.4	439.4	576.4	979.4
4	$\sqrt{3^2 + X_2^2}$	243	272	210	364	445	581	980
5	$\phi_2 = \tan^{-1} X_2/3$	17.95	16.0	14.0	11.9	9.7	7.4	4.9
6	R_c	46.8	46.8	46.8	46.8	46.8	46.8	46.8
7	X_c	96.7	96.7	96.7	96.7	96.7	96.7	96.7
8	6 + 3	279.2	308.2	347.2	403.2	486.2	613.2	1026.2
9	$7 + X_2$	171.7	171.7	171.7	171.7	171.7	171.7	171.7
10	$\sqrt{8^2 + 9^2}$	327	353	387	439	516	640	1040
11	$a = \tan^{-1} 9/8$	31.6	29.0	26.3	23.0	19.4	15.6	9.5
12	$\sqrt{6^2 + 7^2}$	107.5	107.5	107.5	107.5	107.5	107.5	107.5
13	$b = \tan^{-1} 7/6$	64.2	64.2	64.2	64.2	64.2	64.2	64.2
14	4 X 12/10	79.9	82.9	86.2	89.4	92.9	101.3	97.6
15	c = **5** + 13 - 11	50.55	51.2	51.9	53.1	54.5	56.0	59.6
16	14 cos 15	50.6	51.9	53.2	53.6	54.0	54.6	51.3
17	14 sin 15	61.6	64.5	67.8	71.5	75.6	81.0	87.5
18	$16 + R_1$	60.7	62.0	63.3	63.7	64.1	64.7	61.4
19	$17 + X_1$	85.0	88.7	92.1	95.8	99.9	104.4	110.9
20	$\sqrt{18^2 + 19^2}$	104.5	108	111.5	115	118.8	123	126.7
21	$\tan^{-1} 19/18$	54.5	55.0	55.4	56.3	57.3	58.2	61.0
22	PF = cos 21	.580	.572	.566	.553	.538	.526	.485
23	$I_{in} = V_1/20$	1.100	1.064	1.031	1.000	.969	.935	.910
24	$P_{in} = V_1$ X 23 X 22	73.6	70.0	67.3	63.5	59.8	56.6	50.6
25	23 X 4	87.9	88.3	89.0	89.4	89.9	91.3	92.2
26	25/4	.362	.324	.287	.245	.202	.157	.0943
27	26^2 X 2	19.30	18.70	17.9	16.7	14.55	12.13	7.97
28	$\frac{1}{2}$ X 26^2 X R_2	5.44	4.36	3.41	2.51	1.70	1.028	.371
29	$(F + W)(S)^{2.5}$	5.45	5.87	6.34	6.80	7.30	7.84	8.49
30	P_{out} = 27 + 28 - 29	19.29	17.19	14.97	12.41	8.95	5.32	-.149
31	Eff = 30/24	26.2	24.5	22.2	19.6	15.0	9.40	-.294
32	H P_{out} = 30/746	.0258	.0230	.020	.01665	.0120	.00715	≈ 0

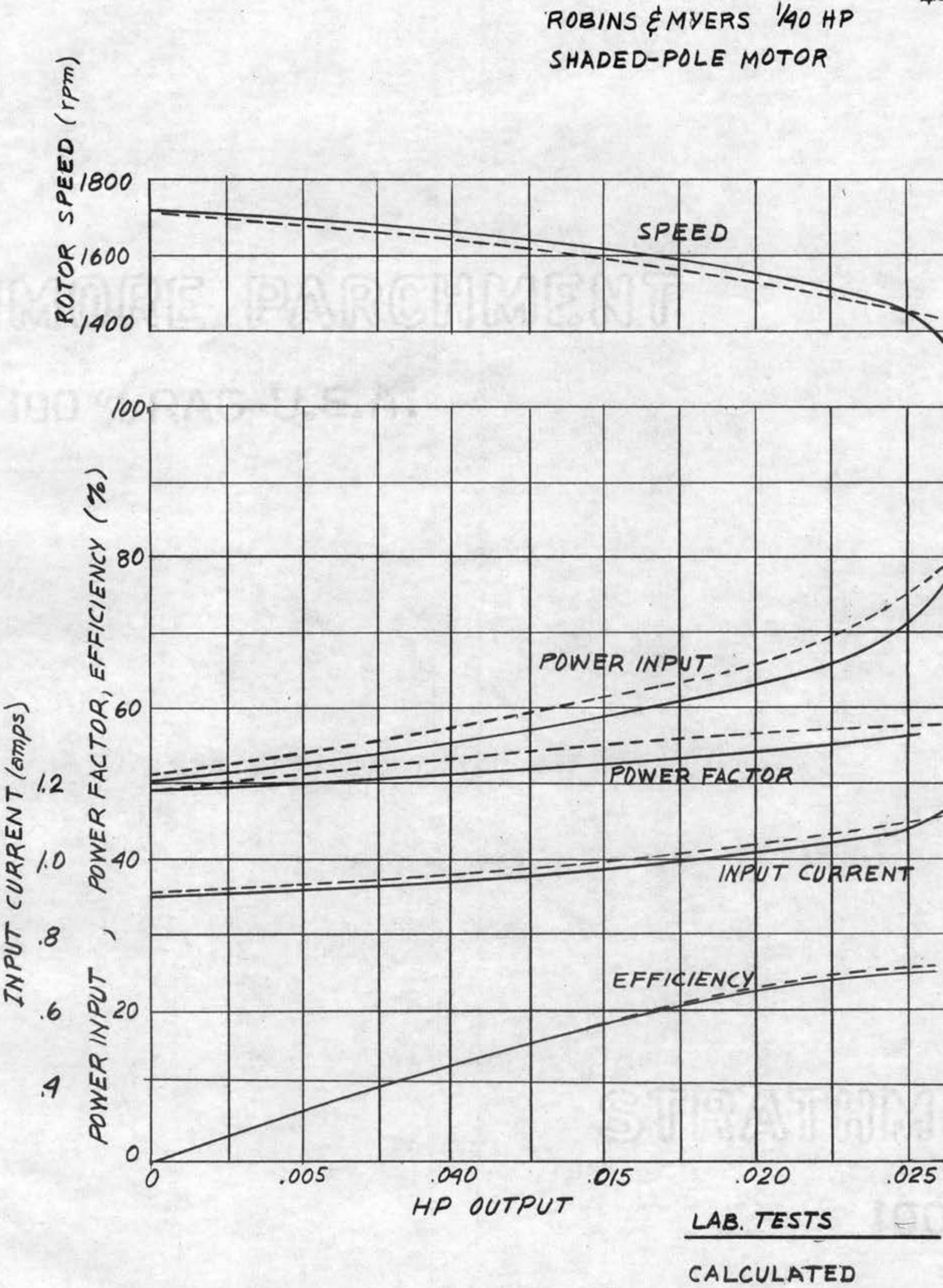
ROBINS & MYERS 1/40 HP
SHADED-POLE MOTOR
ROTOR SPEED (rpm)
1800
1600
1400
SPEED
100
80
60
40
20
0
1.2
1.0
.8
.6
.4
INPUT CURRENT (amps)
POWER INPUT, POWER FACTOR, EFFICIENCY (%)
POWER INPUT
POWER FACTOR
INPUT CURRENT
EFFICIENCY
0
.005
.010
.015
.020
.025
HP OUTPUT
LAB. TESTS
CALCULATED

FIGURE VI

CONCLUSIONS

The theory and operation of the shaded-pole motor is different from any other type of single-phase motor. It is therefore expected that the method of analyzing its performance should be different. When the writer first attempted to analyze this motor it was thought that perhaps one of the commonly used methods of analyzing a single-phase motor would be used. However, by using the standard laboratory tests, it was soon realized that:

1. The rotating field theory was very inadequate because of the high slip of the motor.
2. The cross field theory cannot be used because of this same high slip.

The most error, it is believed, arises from using the no-load or running light test of the motor for calculating the motor parameters. It was therefore necessary to use some other test to determine some of the motor constants and one of the tests ultimately used was the synchronous speed test or synchronous load test of the motor. From the blocked rotor and synchronous load tests of the motor two equivalent circuits were set up and from these circuits the analytical expressions for the resistance and reactances were determined. In the calculation of the motor reactances one assumption was made and using this assumption a relatively simple solution of the circuits was obtained. After obtaining this solution the original assumption was disregarded and the circuits were then solved analytically for the exact solution. However, the exact solution was so complicated that it is necessary for the average mathematician to graph the two final equations to determine a numerical solution.

This was done in this thesis in order to compare the results of the two methods. The results of the two solutions are tabulated below.

	Graphical Solution of Equations (50) and (55)	Solution of Equations (25) and (38)
X_1	28.6 ohms	24.3 ohms
X_2	55.5 ohms	75.0 ohms
X_c	92.4 ohms	96.7 ohms

Using either of the two solutions for the calculation of motor performance will yield approximately the same results and for that reason the motor performance was calculated using only the values calculated from equation (25) and equation (38). In any case it is far easier to use equations (25) and (38) to determine the motor constants than to obtain a graphical solution of equations (50) and (55).

Because of the highly variable speed of the motor it was soon apparent that the usual procedure of subtracting a constant value of friction and windage from the rotor output would not work. It was necessary to use a variable value, not necessarily the measured value of friction and windage, to be subtracted from the rotor output. In the motor used to determine this method the difference between the synchronous load test and no load test of the motor (in watts) was almost 50% of the motor output at full load. This value was used as the "friction and windage" of the motor and was allowed to vary as a function of the slip of the motor. It is realized that this is an unorthodox procedure but the results seem to justify it.

Various methods were used in order to determine the performance characteristics of this motor. As would be expected one of the first methods that was thought of was the circle diagram. However, after plotting the current locus of the two motors available it was soon determined that the current locus of the motors were not circles. By using various methods of trying to determine the nature of the current locus it was found that an ellipse fitted this locus. If the ellipse determined would have had its major and minor axes parallel to the direct and quadrature current axes the analytical solution could be obtained quite readily. Unfortunately this current locus ellipse had the major and minor axes tilted to the direct and quadrature current axes which makes a very complex problem to be solved. This method was discarded after this fact was determined. There was one conclusion made from this study and that was that again the synchronous load test must be used instead of the no-load test to locate one point on the ellipse if any solution is to be obtained to this problem.

The results of the performance calculation of the 1/40 horsepower motor was satisfactory. The greatest errors obtained were present in the calculated power input and the power factor. Both of these calculated values were higher than the laboratory tests indicated. The efficiency, input current and the rotor speed agree quite well with the laboratory tests.

Unfortunately, the proposed method cannot be used to determine the performance characteristics of the larger sizes of motors. However, it is believed that the method is practicable for motors of ratings less than 1/30 horsepower. Time does not permit the writer to obtain a solution to this problem, but it is hoped that a solution can be obtained and presented some time in the near future.

One of the greatest difficulties encountered in a study of this type is the determination of accurate laboratory tests. The motor used in this study had a current input variation of approximately two tenths of an ampere between no-load and full load. The instruments available in the average laboratory have an inherent error that will cause inaccurate results even if the utmost care is used in taking readings. For this reason there are errors present in both the calculated and laboratory results. In connection with the determination of laboratory tests it is well to mention a curious result that was noticed. When the blocked rotor test was taken the rotor was turned a small amount in both directions and as the rotor was turned it was observed that the current and power input varied a small amount. It is thought this variation was caused by some action of the shading coil as different parts of the rotor were passed. In any case the maximum value of current input should be used for the blocked rotor values.

It is hoped that this thesis has resulted in some advancement for the performance calculations of the shaded-pole motor. Very much remains to be done on this type of motor and it is hoped that this thesis will serve as a guide to those interested in the subject.

BIBLIOGRAPHY

Denman, E. W. "Development of Fan Motor Windings," Electrical Journal, (June, 1919).

Langsdorf, A. S. Theory of Alternating Current Machinery. New York: McGraw-Hill Book Company.

Pender and Del Mar. Electrical Engineers Handbook, 4th ed. New York: John Wiley and Sons.

Puchstein and Lloyd. Alternating Current Machinery. New York: McGraw-Hill Book Company.

Standard Handbook for Electrical Engineers, 8th ed. New York: McGraw-Hill Book Company.

Trickey, P. H. "An Analysis of the Shaded-Pole Motor," Electrical Engineering, (September, 1936).

Veinott, C. G. Fractional Horsepower Motors. New York: McGraw-Hill Book Company.

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