



Munich Personal RePEc Archive

# **Towards Analysis of Vertical Structure of Industries: a method and its application to U.S. industries**

Nishimura Sadao

15. December 2010

Online at <https://mpra.ub.uni-muenchen.de/27464/>

MPRA Paper No. 27464, posted 16. December 2010 14:19 UTC

# Towards Analysis of Vertical Structure of Industries: a method and its application to U.S. industries

Sadao Nishimura\*

## Abstract

In this paper we present a method to analyse vertical structure of industries. A product of an industry has a hierarchical value structure with layers, each of which consists of value added injected by various production stages (current and previous) of various industries. This vertical structure makes it possible to measure value added (VA) levels (vertical positions) of VA receivers (stages of use industries), while products of supply industries and their value added flow into these stages of industries which have respective VA levels. These flows are value added contributions by supply industries. By calculating each industry's VA contributions and corresponding target VA levels, it is possible to evaluate vertical structure of industries in the whole economy. We applied this method to 1998–2008 U.S. Input-Output Tables. Resulting average VA contribution levels and graphs of VA contributions are displayed.

**Keywords:** vertical structure, value added, supply, use.

**JEL Classification Codes:** C61, D57

---

\*Nanzan University, 18 Yamazato-cho, Showa-ku, Nagoya, 466-8673, Japan

# 1 Introduction

A phenomenon called vertical specialization has attracted a great deal of attentions for recent decades. This phenomenon is referred to as several different names such as fragmentation, disintegration of production, multi-stage production, intra-product specialization, and so on<sup>1</sup>.

Probably one of the reasons of many arguments raised on this problem is the controversy in the United States over “offshoring.” That is, on the one hand there are many who insist the offshoring is prone to exporting many domestic jobs abroad to other (more often than not, emerging or developing) countries, and have domestic workers lose their opportunities to work. On the other hand, many economists defend the offshoring as a variant of international trade, and therefore it is beneficial for both countries of job offers and receivers.

Obviously, among the factors which brought about offshoring is significant progress in information and communication technologies (ICT) in recent years. Typical services had been difficult to be traded between distant places as they needed to be produced just when and where they were consumed. However, recent conspicuous technological progress particularly in digitalization of information made it possible to exchange services between distantly separated firms and individuals. The UNCTAD, for example, prepared a chapter in its World Investment Report 2004 ([6]) to analyse this problem from various points of view using vast range of facts and data.

Arguments in the literature, however, do not seem to have converged into standard models or theories. One of the reasons for this situation is, according to my view, the lack of systematic methods in dealing with data. Data and documents are not in short supply. On the contrary, many researchers refer and cite a great deal of data, facts, and anecdotes in their studies.

One of the key to study offshoring is how to analyse the structure of industries. If we can show empirically the effects of offshoring on the industrial structure of an economy (e.g. American economy), theoretical discussion will probably become integrated into some consistent models based on actual data, and by this reason, empirical studies will also develop further.

The problem is not easy, however. It seems that offshoring phenomenon is, for the industries in question, essentially a microeconomic problem. Therefore, data related to offshoring may be also microscopic, which are essentially connected to individual firms engaging in offshoring. On the other hand, however, if offshoring has

---

<sup>1</sup>David Hummels, Jun Ishii, and Kei-Mu Yi [4].

some effects on the industries' structure at all, it will probably have to be clarified by examining the position of the industry in the whole economy, particularly by examining changes in their position in the vertical structure of total industry.

This paper is an attempt towards this research direction. We use the Input-Output data and analysis in order to investigate the vertical structure of an economy's industries. By making out the concept of 'vertical structure' of industries, we will become able to apply this concept in examining the effects of offshoring (or other phenomena) on the structure of industries.

The basic idea of this paper is as follows. The fundamental equation in input-output analysis is:

$$X = AX + Y,$$

where  $X$  is a vector of total production,  $Y$  of final demand, and  $A$  is a direct input requirement coefficient matrix.

By solving this equation for  $X$ , we have

$$X = LY = (I + A + A^2 + \dots)Y. \quad (1)$$

$L$  is the well-known Leontief inverse matrix, and  $(I + A + A^2 + \dots)$  is an expression in its expanded form. This expansion form is the starting point of our discussion.

The first term of the right-hand part of the equation ( $IY = Y$ ) indicates the products of the final production stage (stage-0) of the economy. To produce stage-0 products, the economy has to produce inputs which correspond to the second term ( $AY$ ), the products of stage-1. This process traces back to infinite stage numbers<sup>2</sup>.

The stage number  $\lambda$  of  $A^\lambda Y$  indicates how 'old' its production process is and how long ago the corresponding value added are embedded in these inputs. In other words, the various stages' value added are piled up to form the total value of final products  $Y$ . This forms a hierarchy structure, which somewhat resembles growth rings of trees. If we can analyse changes in this growth ring structure raised by some impacts, we can find the effects of the impacts on the structure of industries in question.

---

<sup>2</sup>In order to avoid confusions, we use the following terminology. All production stages are assigned a non-negative integer starting from 0. The last stage, which is not actually a production process, is absorption by final demand and assigned 0. One stage before stage-0 is stage-1, which produces stage-0 products that are absorbed by stage-0 activity (final demand), and uses stage-1 products as inputs, which are products of stage-2. In general, production stage- $\lambda$  uses stage- $(\lambda + 1)$  products as inputs and produces stage- $\lambda$  products.

The value added hierarchy structure of industries or products can also be utilized in somewhat different perspective. (1) shows production of  $X$ , which is larger than final demand  $Y$ , needs to be produced to fulfill the final demand. Then, total amount of each product is distributed to various stages (from stage-0 to stage- $\infty$ ) of various industries (products) in order to support production of various products. If an industry's product is mainly distributed into rather lower-numbered stages of production, we presume the industry is contributing in near final demand stages. That industry can be called a (relatively) downstream industry. If the other is the case, it can be called an upstream one.

A product of any industry is a heap of value added injected in various production stages of (use) industries. If we can evaluate the height of the value added within the total value added stratum of the product, then we can also evaluate the height of the contribution performed by (supply) industries sending their value added to destinations. Accordingly, it will become able to calculate the vertical position of each industry within the economy-wide structure of industries. This method presumes the stage-0 (producing final goods to final demand) is at the highest position in the vertical hierarchy, and the earliest stage- $\infty$  is at the lowest (ground)<sup>3</sup>.

In the next section, the formal model of vertical structural analysis is presented. It is applied to some industries of the United States, particularly those closely related to 'offshoring'.

## **2 A Method to Analyse Vertical Structure of Industries**

### **2.1 Value Added Hierarchy of Products of Use Industries**

Each industry produces its peculiar product and supplies it to many other industries and final demand. When our attention is centered on this aspect of an industry, we call it a supply industry. At the same time, each industry uses various intermediates supplied by other industries which are necessary for its production processes. When we pick up this aspect of an industry, we call it a use industry. That is, industries have two different relationships with

---

<sup>3</sup>Here is some obvious problem. For example, we include both consumption goods and investment goods in final goods by statistical convention. But investment goods (capital goods) can be considered as intermediate goods in a long roundabout production process. For now we want to concentrate our discussion on quantitative aspects of the problem.

others. In this section we will discuss the aspect of use industry<sup>4</sup>.

The total value of product of a use industry constitutes piled layers of the value added created in a myriad of production process chains which are in the end connected to the product of this use industry. Intermediates produced in preceding production processes are absorbed into some production stages, which, in turn, send their products as intermediates to later stages. Such production process chains intersect, get intermingled, and after a long travel reach their last stage. This is the final products absorbed by final demand.

Let us consider the composition of value added which a use industry's product fills with.

First, we assume there is no need to distinguish between a use industry and its product<sup>5</sup>. Each product is produced by only one use industry, and each industry produces only one product. Then, assume  $P$  is a column vector of product prices, and  $V$  is a column vector of direct value added of use industries. About product value construction we have the price model expression of input-output relations.

$$P' = P'A + V'.$$

The right-hand side of this equation is the cost of inputs per unit of output including value added. If we take the unit of each product equal to the amount the monetary unit can purchase, and  $v$  be the rate of value added in the product value, we have

$$u' = u'A + v'. \quad (2)$$

where  $u$  is a column vector of 1's ( $u = (1, 1, \dots, 1)'$ ). Using the expansion form of Leontief inverse matrix, we get the following expression.

$$u' = v'(I + A + A^2 + \dots). \quad (3)$$

The first term in the right-hand side parentheses  $I (= A^0)$  indicates that unit amount of each product is used by stage-0. The power 0 of  $A$  means the stage number of the process, which actually is not a production process but the activity of absorption (purchase) by final demand. The second term  $A$  represents intermediate inputs required by the production stage-1 which is to produce unit amount of each product to deliver to the final demand (stage-0). In general  $A^\lambda$  is intermediate inputs required by the production stage- $\lambda$  whose products are delivered to the next

---

<sup>4</sup>Usage of these terms might be somewhat confusing, but these some commonality with the words "supply" and "use" used in the commodity-by-industry approach in input-output analysis.

<sup>5</sup>If we adopt the commodity-by-industry approach to input-output analysis, we have to distinguish between the two.

production stage- $(\lambda - 1)$  as intermediate inputs. As the right-hand side of (3) is the sum of total amount of products premultiplied by  $v'$ , the equation means the sum of all value added poured into the unit of final product equals 1 for each product.

Let us take the product  $j$ , and examine the hierarchical structure of its value added. The value added in a unit amount of (final) product  $j$  injected by input  $i$  at production stage- $\lambda$  is shown by  $(i, j)$  element of  $v'A^\lambda$ , or  $v_i a_{ij}^{(\lambda)}$ <sup>6</sup>. Total value added injected by all inputs at the production stage  $\lambda$  is  $z_j(\lambda) = \sum_i v_i a_{ij}^{(\lambda)}$ . Then,

$$1 = z_j(0) + z_j(1) + \dots + z_j(\lambda) + \dots .$$

The larger the stage number  $\lambda$  is, the earlier is the production stage. That means, the larger  $\lambda$  is, the smaller is the value added accumulated in product  $j$  through successive stages up to stage- $\lambda$ .

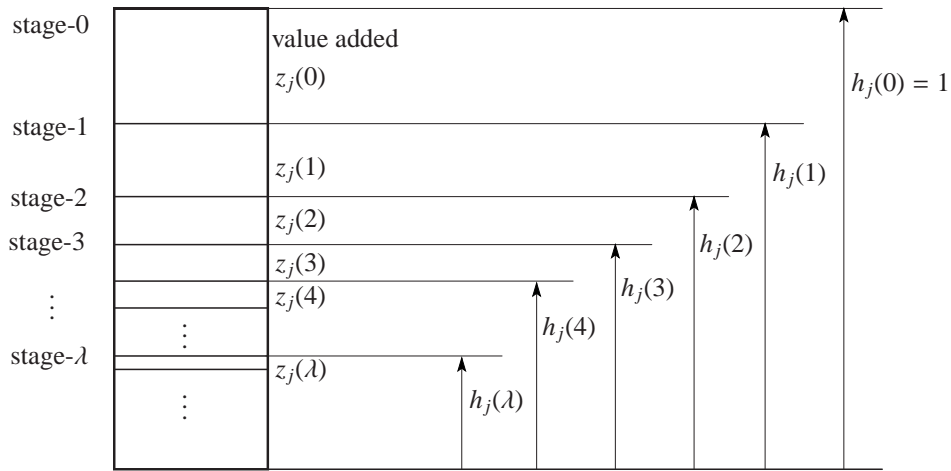


Figure 1: Vertical Value Added Structure of product  $j$

Figure 1 shows this hierarchical value added structure for product  $j$ . On the top of the layers lies the value added to the product  $j$  absorbed in the last stage of production. The second layer is the value added absorbed in the second last stage, and so on. In general, inputs to production stage- $\lambda$  have value equal to the sum of value added layer  $z_j(\lambda)$  and below  $(z_j(\lambda) + z_j(\lambda + 1) + \dots)$ , and the immediate upper layer  $z_j(\lambda - 1)$  is the value added newly created at the production stage- $\lambda$ .

We now define the **value added level** (or simply **VA level**) of a production stage. This is a concept of height of the value added layer corresponding to the stage in question. For example, the VA level of stage- $\lambda$ ,  $h_j(\lambda)$ , is

<sup>6</sup>  $a_{ij}^{(\lambda)}$  means the  $(i, j)$  element of  $A^\lambda$ .

expressed as

$$h_j(\lambda) = z_j(\lambda) + z_j(\lambda + 1) + z_j(\lambda + 2) \cdots ,$$

where  $h_j(\lambda) \geq h_j(\lambda + 1)$ ,  $h_j(0) = 1$  and  $h_j(\infty) = 0$ .

Let  $h(\lambda) = (h_1(\lambda), \dots, h_j(\lambda), \dots, h_n(\lambda))'$ , where  $n$  is the number of products or industries. Then we have

$$\begin{aligned} h'(\lambda) &= v'(A^\lambda + A^{(\lambda+1)} + A^{(\lambda+2)} + \dots) \\ &= v'(I + A + A^2 + \dots)A^\lambda \\ &= u'A^\lambda. \end{aligned} \tag{4}$$

That is, the VA level of stage- $\lambda$  of product  $j$ ,  $h_j(\lambda)$ , is the column sum of  $j$ -th column of matrix  $A^\lambda$ . If  $j$  is different,  $h_j(\lambda)$  is not generally the same. It is evident that  $h'(\lambda + 1) = h'(\lambda)A$ .

Then let our discussion concentrate on individual product or industry. Take a product (or industry)  $j$ . The VA level of the stage- $\lambda$  of product  $j$  is the  $j$ -th element of the row vector  $h'(\lambda) = u'A^\lambda$ , or  $\sum_i a_{ij}^{(\lambda)}$ . In this stage- $\lambda$ , an individual product  $i$  contributes in value added formation by the amount of  $v_i a_{ij}^{(\lambda)}$ . Let a hat over a vector  $\hat{x}$  denote a diagonal matrix with elements of  $x$  along the main diagonal. Then  $j$ -th column vector of  $\hat{A}^\lambda$  represents contributions of  $n$  (different) products in product  $j$ 's stage- $\lambda$ .

Using  $a_{ij}^{(\lambda)}$ 's ( $i = 1, \dots, n$  and  $\lambda = 0, \dots, \infty$ ), we construct the following matrix with infinite rows. This matrix is related to use industry  $j$ , and we can construct  $n$  kinds of similar matrices for different use industries.

$$\mathcal{A}(j) = \begin{pmatrix} 0 & 0 & \dots & 1 & \dots & 0 \\ a_{1j} & a_{2j} & \dots & a_{jj} & \dots & a_{nj} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{1j}^{(\lambda)} & a_{2j}^{(\lambda)} & \dots & a_{jj}^{(\lambda)} & \dots & a_{nj}^{(\lambda)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \tag{5}$$

The first row of  $\mathcal{A}(j)$  is the transpose of  $j$ -th column of identity matrix or  $A^0$ , the second is  $j$ -th column of  $A$ , the third is  $j$ -th column of  $A^2$ , and so on. In general the  $\lambda$ -th row is the transpose of the  $j$ -th column of  $A^\lambda$ . Elements of  $\mathcal{A}(j)$ , thus, exhaust direct and indirect inputs to use industry  $j$  to produce product  $j$ . Each element of  $\mathcal{A}(j)$ 's  $i$ -th column shows the amount of product supplied by industry  $i$  to various stages of industry  $j$ . Finally, the sum of  $\mathcal{A}(j)$ 's  $(\lambda + 1)$ -th row elements is VA level  $h_j(\lambda)$  of the stage- $\lambda$  ( $h_j(\lambda) = \sum_i a_{ij}^{(\lambda)}$ ).



Next, we postmultiply  $\mathcal{A}(j)$  by  $\hat{v}$  to get:

$$\mathcal{V}(j) = \begin{pmatrix} 0 & 0 & \dots & v_j & \dots & 0 \\ a_{1j}v_1 & a_{2j}v_2 & \dots & a_{jj}v_j & \dots & a_{nj}v_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{1j}^{(\lambda)}v_1 & a_{2j}^{(\lambda)}v_2 & \dots & a_{jj}^{(\lambda)}v_j & \dots & a_{nj}^{(\lambda)}v_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \quad (6)$$

Elements of  $\mathcal{V}(j)$  represent amounts of value added allocated to various stages of use industry  $j$  from various supply industries. For example,  $a_{ij}^{(\lambda)}v_i$ ,  $(\lambda + 1, i)$ -element of  $\mathcal{V}(j)$ , is the amount of value added conveyed from supply industry  $i$  to use industry  $j$ 's stage- $\lambda$ .

$\mathcal{V}(j)$  shows the most detailed vertical structure of value added embedded in use industry  $j$ 's product.

## 2.2 Value Added Contributions by Supply Industries

Next, we deal with the other aspect of inter-industry relationship: supply of value added by supply industries. Every use industry produces its particular product by directly and indirectly absorbing value added created in various supply industries. Then it sells a part of its output to final demand and the rest to various use industries as their intermediate inputs. This is the role of supply industry each industry performs in the economy's input-output network.

Let  $X_i$  ( $i = 1, \dots, n$ ) be the total amount of production in (supply) industry  $i$ , or product  $i$ , and  $X$  be the column vector of those outputs ( $X = (X_1, \dots, X_i, \dots, X_n)'$ ). Similarly, final demand vector is denoted by  $Y = (Y_1, \dots, Y_n)'$ . Then we have the following basic equation of input-output analysis:

$$X = AX + Y,$$

where  $A$  is the direct requirement coefficient matrix as before. This equation can be expressed as  $\hat{X}u = A\hat{X}u + \hat{X}y$ , where  $y = \hat{X}^{-1}Y$ . Then substituting  $B = \hat{X}^{-1}A\hat{X}$  for  $A$ , we obtain:

$$u = Bu + y. \quad (7)$$

$(i, j)$  element of  $B$ ,  $b_{ij} = \frac{1}{X_i}a_{ij}X_j$ , represents the fraction of product  $i$  allocated to use industry  $j$  in the total output of product  $i$ . As (7) shows, the sum of these fractions including that allocated to final demand ( $y$ ) is 1. We call  $B$  a

**direct allocation matrix**<sup>7</sup>. Note that  $B^\lambda = \hat{X}^{-1}A^\lambda\hat{X}$ .

We obtain the following from (7):

$$u = (I - B)^{-1}y = (I + B + B^2 + \dots)y. \quad (8)$$

$u$  represents total outputs of industries normalized to 1's. The first term of the right-hand side  $Iy$  ( $= y$ ) is the part of the total output allocated to final demand. The second term  $By$  is the part of the output allocated to the stage-1 of use industries. In general, the  $(\lambda + 1)$ -th term  $B^\lambda y$  is the part of the output allocated to the stage- $\lambda$  of use industries. (8) shows the allocation of total output of supply industries among the different stages of different use industries.

Pay attention to the difference between (3) and (8). (3) shows the value added hierarchical structure of use industries, so focus is on use industries. (8) shows the allocation of supply industries' products, here focus is on supply industries.

All elements of the right-hand side of  $i$ -th row of (8) is allocations of the same product  $i$ . But, the rate of (direct) value added is the same for all outputs produced in industry  $i$ , the allocating fractions of value added is the same as those of quantity of product  $i$ . Thus the allocation matrix  $B$  of outputs is also an allocation matrix of value added.

One point needs to be discussed. Usually direct requirement coefficients  $a_{ij}$ 's are considered technologically constant. Then Leontief inverse matrix  $(I - A)^{-1} = (I + A + A^2 + \dots)$  is also a constant matrix. However, since  $X = (I + A + A^2 + \dots)Y$  and final demand  $Y$  must be considered a variable,  $X$  cannot be considered constant. That, in turn, means that  $B = \hat{X}^{-1}A\hat{X}$  cannot be considered a constant matrix, either. This issue is known as the "joint stability" problem. If  $A$  is constant,  $B$  in general cannot remain constant. But this issue will not exert damaging effect on our analysis. A purpose of our analysis is to clarify the existing vertical structure of industries, not to seek structural characteristics invariant over time. We are more interested in changes in coefficients  $b_{ij}$  of  $B$  than in their constancy.

As in the previous section, we decompose the expression (8) to make another form of matrix like (5). Consider the right-hand side of (8). Decompose  $(\lambda + 1)$ -th vector,  $B^\lambda y$ , by substituting  $\hat{y}$  for  $y$  to build a matrix  $B^\lambda \hat{y}$ . Then extract its  $i$ -th row. Denote this row vector by  $c_i^{(\lambda)} (= (b_{i1}^{(\lambda)} y_1, b_{i2}^{(\lambda)} y_2, \dots, b_{in}^{(\lambda)} y_n))$ . Corresponding to  $\lambda = 0, 1, \dots, \infty$

---

<sup>7</sup>R. E. Miller and P. D. Blair [5] calls this matrix a direct-output coefficient matrix.

we have infinite such row vectors. By arranging these row vectors we construct the following matrix  $C(i)$ .

$$C(i) = \begin{pmatrix} c_i^{(0)} \\ c_i \\ \vdots \\ c_i^{(\lambda)} \\ \vdots \end{pmatrix} = \begin{pmatrix} 0 & 0 & \dots & y_i & \dots & 0 \\ b_{i1}y_1 & b_{i2}y_2 & \dots & b_{ii}y_i & \dots & b_{in}y_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{i1}^{(\lambda)}y_1 & b_{i2}^{(\lambda)}y_2 & \dots & b_{ii}^{(\lambda)}y_i & \dots & b_{in}^{(\lambda)}y_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \quad (9)$$

This decomposition shows the allocation of industry  $i$ 's total output. Its  $j$ -th column corresponds to use industry  $j$ , and  $(\lambda + 1)$ -th row to stage- $\lambda$  of various use industries. Thus,  $(\lambda + 1, j)$  element of  $C$ , which is denoted by  $c_{ij}^{(\lambda)}$  ( $= b_{ij}^{(\lambda)}y_j$ ), shows the portion of supply industry  $i$ 's output which is put into stage- $\lambda$  of use industry  $j$ . Or  $(\lambda + 1, j)$  element also shows the portion of industry  $i$ 's value added which is put into stage- $\lambda$  of industry  $j$ . Therefore,  $C(i)$  shows total and most detailed allocation of product  $i$  or of value added created in supply industry  $i$ . In other words,  $C(i)$  shows value added contributions of supply industry  $i$  to use industries in producing their outputs. In this sense, we call  $C(i)$  supply industry  $i$ 's **value added contribution** or **VA contribution** matrix.

As we can see in (8), the sum of all elements of  $C(i)$  is 1 ( $\sum_j \sum_\lambda c_{ij}^{(\lambda)} = 1$ ). So, each element of  $C(i)$  is called **value added allocation segment** or VA allocation segment.

## 2.3 Vertical Structure of Industries

So far, we have constructed two basic components necessary to evaluate vertical structure of industries.

1. Each product provided by each supply industry enters into several stages of several use industries. Along with this product circulation, value added created by each supply industry is distributed among various use industry stages. The size of value added entering industry stages is measured by the value added allocation segment.
2. Each use industry absorbs various products and value added into its various stages. From early stages to late ones their valued added absorbed constitutes a hierarchical structure inside the use industry's product. This hierarchical structure of value added offers layers of destinations with varrious vertical heights to which supply industries's products flow in. We called this height as value added level (VA level).

Combining these two components we build a method to evaluate each industry's (average) valued added level (vertical height) in total industries.

“Upstream” industries or “downstream” industries are frequently used popular terms. Mining or mining-related industries, for example, extract ores etc. from the ground and somehow process them. But products of these industries are usually demanded by and absorbed into various industries many of which are largely far from final stages. In our terms products and their value added of these (supply) industries tend to flow into production stages of lower VA levels. That is the meaning of “upstream.”

On the other hand, industries are usually referred to as “downstream” ones when their products and value added largely flow into final or near-final stages of use industries, or are directly delivered to final demand. Products and value added of these industries largely flow into production stages of higher VA levels.

However, the same product produced by a supply industry mostly diverges into multitudes of different stages of different use industries. We need, therefore, some notion of average in order to evaluate an industry's overall vertical position in a whole system of industries. Let us discuss about it.

As explained in 2.1, VA levels for various industries' stage- $\lambda$  is shown by a vector  $h(\lambda)$ . But (5) shows the  $i$ -th element of  $h(\lambda)$  is the sum of  $\mathcal{A}(i)$ 's  $(\lambda + 1)$ -th row elements ( $h_i(\lambda) = \sum_i a_{ij}^{(\lambda)}$ ). So we make row sums of  $\mathcal{A}(i)$  to get

$$l_j = \mathcal{A}(j) u = \begin{pmatrix} h_j(0) \\ h_j(1) \\ \vdots \\ h_j(\lambda) \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 \\ \sum_i a_{ij} \\ \vdots \\ \sum_i a_{ij}^{(\lambda)} \\ \vdots \end{pmatrix}. \quad (10)$$

This  $l_i$  is a column vector that lines up all of the VA levels of use industry  $i$ . Then, let  $\mathcal{L} = (l_1, l_2, \dots, l_n)$ . This is a matrix with infinite rows (stages) and  $n$  columns (industries).  $\mathcal{L}$  exhausts VA levels of all (use) industries and all

production stages. We name  $\mathcal{L}$  VA level matrix.

$$\mathcal{L} = \begin{pmatrix} h_1(0) & h_2(0) & \dots & h_i(0) & \dots & h_i(0) \\ h_1(1) & h_2(1) & \dots & h_i(1) & \dots & h_i(1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_1(\lambda) & h_2(\lambda) & \dots & h_i(\lambda) & \dots & h_i(\lambda) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \quad (11)$$

In (5) we constructed  $\mathcal{A}(j)$ , whose elements are direct and indirect coefficients of input requirement by industry  $j$ .  $(\lambda + 1, j)$  element  $h_j^{(\lambda)}$  of (11) is the sum of  $(\lambda + 1)$ -th row elements of (5).

Next, we turn to supply industries' value added contributions. This is already done. That is,  $C(i)$  defined in (9) is a supply industry  $i$ 's matrix whose elements show its value added contributions to stage- $\lambda$  of use industry  $j$ . We named an element of this matrix VA contribution segment.

Between  $\mathcal{L}$  and  $C(i)$ , we can build an exact one-to-one, element by element correspondence, that is, between  $h_j(\lambda)$  and  $c_{ij}^\lambda = b_{ij}^\lambda y_j$ . Here,  $h_j(\lambda)$  is VA level of stage- $\lambda$  of use industry  $j$ , and to that level a portion of value added of supply industry  $i$ ,  $c_{ij}^\lambda$ , flows in.

There seems to be several ways to examine vertical structure of industries using  $\mathcal{L}$  and  $C(i)$ . We show here a method to evaluate a supply industry's overall vertical position in the whole economy.

As shown above, the sum of all the elements of  $C(i)$  is 1.

$$\sum_{j=1}^n \sum_{\lambda=0}^{\infty} c_{ij}^{(\lambda)} = \sum_j \sum_{\lambda} b_{ij}^{(\lambda)} y_j = 1.$$

Because of this nature, we can use segments as weights for averaging procedure. Values to be averaged are elements of  $\mathcal{L}$ , that is, VA levels of destinations to which all of value added from supply industry  $i$  flows in. Averaging weights are VA allocation segments of supply industry  $i$ . The value obtained in this averaging calculation is the averaged VA level of supply industry  $i$ 's VA contributions. More concretely we calculate the following:

$$\begin{aligned} L_i &= \sum_{j=1}^n \sum_{\lambda=0}^{\infty} c_{ij}^{(\lambda)} h_j(\lambda) = \sum_j \sum_{\lambda} h_j(\lambda) b_{ij}^{(\lambda)} y_j \\ &= \mathcal{L}' C(i). \end{aligned} \quad (12)$$

There is another way to show the vertical structure of industry more diagrammatically. First, line up supply industry  $i$ 's VA contribution segments according to their corresponding destinations' VA levels in ascending order.

These contribution segments are distribution shares in total value added, so from these lined VA contribution segments, we make cumulative shares of VA contributions in total value added sent out from industry  $i$ . We plot each pair of corresponding VA level and cumulative VA share as a point on diagram, and connect these points to get a line. This line starts from the origin, moving towards the northeast with drawing a kinked line, and finally reaches the point (1, 1).

Let us explain the procedure more concretely.

Before that, however, we must deal with the fact that both VA level matrix  $\mathcal{L}$  and VA contribution matrix  $C(i)$  have infinitely many elements. Of course, we cannot actually carry out calculations with infinitely many elements. But this is not an essential problem. Our method is based on the power series approximation of Leontief inverse matrix. According to R. E. Miller and P. D. Blair [5], empirically power series approximation up to 7th or 8th power of direct requirement matrix will have the calculation result close enough to the Leontief inverse. Therefore, we do not have to deal with matrices with infinitely many rows. In the empirical application shown later, we engaged in the power calculation up to 10th power.

Then following is the actual procedure of calculation for depicting a graph. (We have already limited the number of elements finite.) First rearrange all elements ( $h_j(\lambda)$ 's) of  $\mathcal{L}$  (whose rows are limited to  $q$ ) in ascending order:

$$h_1^*, h_2^*, \dots, h_k^*, \dots, h_q^*$$

where  $h_1^* \leq h_2^* \leq \dots \leq h_k^* \leq \dots \leq h_q^* = 1$ . These are rearranged VA levels.

Then line up all elements ( $c_{ij}^{(\lambda)}$ 's) of  $C(i)$  in the corresponding order:

$$c_{i1}^*, c_{i2}^*, \dots, c_{ik}^*, \dots, c_{iq}^*.$$

These are rearranged VA contribution segments, and  $c_{ip}^* \geq 0$  for any  $p$ . Define, then, cumulative VA contribution shares  $s_{ik}^*$ 's as follows:

$$\begin{aligned} s_{ik}^* &= c_{i1}^* + c_{i2}^* + \dots + c_{ik}^* \\ &= \sum_{k=1}^q c_{ik}^*. \end{aligned}$$

Finally, we make pairs of coordinates ( $c_{ik}^*, h_k^*$ ) for  $k = 1, \dots, q$ , and draw a kinked line by connecting these points in a diagram. Some graphs base on very simple numerical examples are shown in the next section 2.4.

We will show some examples based on actual U.S. input-output data in section 4.

## 2.4 Simple Numerical Examples

In this stage discussion, it would be better to show some small numerical examples before we go on to the next methodological problem: imports.

**Example 1** This is a very simple example with only 4 industries. The direct requirement coefficients matrix  $A$ , rate of value added  $v$ , and final demand  $Y$  are given as follows:

$$A = \begin{pmatrix} 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.8 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad v = \begin{pmatrix} 1 \\ 0.6 \\ 0.4 \\ 0.2 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad Y_1 = \begin{pmatrix} 0.2 \\ 0.4 \\ 0.7 \\ 1 \end{pmatrix}.$$

The rows and columns indicate supply and use industries respectively. As the values of matrix  $A$ 's elements show, this economy has a very simple industry structure, particularly when final demand is  $Y$ , and we can call it a vertical linear structure. Industry 1's product uses only its own primary resources without receiving any intermediates, then industry 2 uses industry 1's product as intermediate with adding its own value added, and industry 3 uses that product as intermediate and sends it forth to industry 4 with its own value added laid on it. Finally industry 4 uses industry 3's product as intermediate with its own value added laid on it to produce final product which is absorbed by final demand.

Given these data, the vertical structure of industries are shown in a diagram 2-a (left). The ordinate shows value added levels of various stages of each industry. The abscissa, correspondingly, shows cumulative value added contributions by the same stages. In this example, the vertical structure of industries are straight, and the industries' hierarchical structure shown in the diagram is very simple. Industry 1 is the lowest, upstream industry, and industry 4 is the highest, downstream industry. Industries 2 and 3 are located between them for all value added contribution shares.

In Figure 2-b (right), the vector of final demand is changed to  $Y_1$ . In this example, some portions of all industries' products are sent to final demand. Accordingly depicted graphs show some complex patterns, though

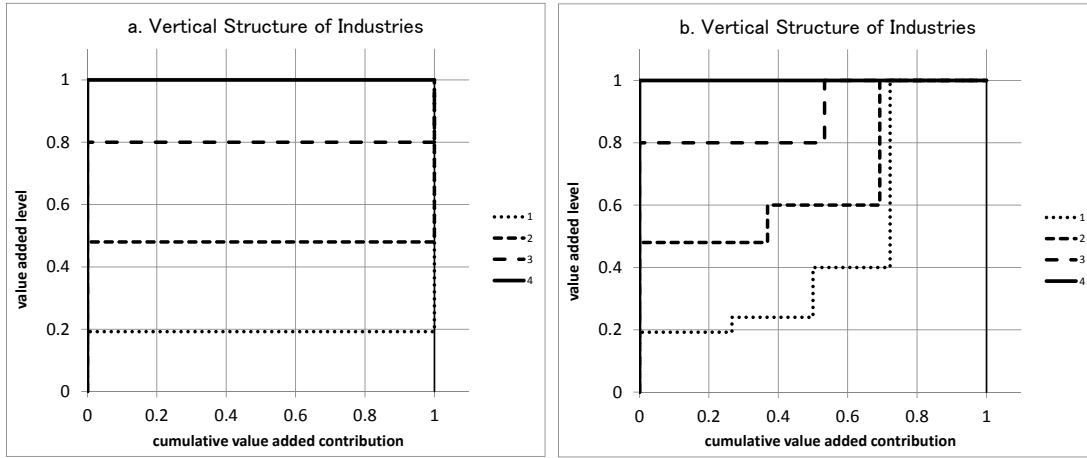


Figure 2: Example 1

the order of industries' value added levels are almost the same as the left figure. (If we change the values of final demand vector a little, intersections of graphs will appear.)

**Example 2** The number of industries remains 4. In this case positions of all 4 industries are, in a sense, symmetrical. Industry 2 receives intermediates from industry 1, industry 3 from industry 2, industry 4 from industry 3, and industry 1 from industry 4. All of the direct requirement coefficients are the same (0.3 in the left, and 0.7 in the right). However, final demands are biased (those of later number industries are larger than earlier ones).

$$A = \begin{pmatrix} 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0.3 \\ 0.3 & 0 & 0 & 0 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0.7 \\ 0.7 & 0 & 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}.$$

The graphs depicted on these data are shown in Figure 3-c and -d. When the direct requirement coefficient is  $A$  (left diagram), 70 percent of all of the value added is added in the last production stage, the graphs of value added levels jump at one step from 0.30 to 1.00. Accordingly, segments of final stage (producing final products) in total products are large without exception. That is, the top plateau parts, which show the portions absorbed to final demand, are relatively large for all industries (compar with the right diagram).

However, since there is a bias in final demands, quantities of final products are larger in later number industries. Thus, by and large later number industries tend to show higher vertical position (higher value added



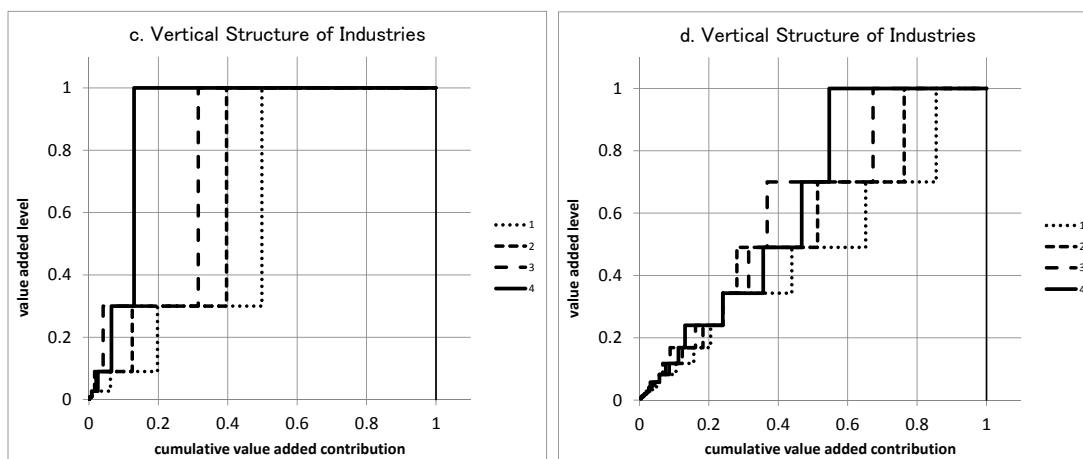


Figure 3: Example 2

level).

In the right diagram direct requirement coefficients only are changed from 0.3 to 0.7 (see the matrix  $A_1$ ). Thus, value added levels are equally lower (graphs are biased towards right) relatively to the left graphs. In other words, all industries become biased towards relatively upstream industries.

By making these graphs with actual input-output table data, we can evaluate each industries' vertical position in the whole economy.

Though we do not show, matrices such as  $\mathcal{A}(j)$  in (5) or  $\mathcal{V}(j)$  in (9) for industry  $j (= 1, \dots, 4)$  are also easily calculated.

### 3 Dealing with Imports

When we are to apply our method to analyse vertical structure of industries using actual input-output tables, we have one issue to make an adjustment for. This is how to deal with imports.

As discussed above, two factors are crucial for our analysis to evaluate an industry's vertical position (VA level). One is the destination industry-stages' VA levels, to which the supply industry's value added flows in. The other is the amount of value added contributions by the supply industry allocated to various industry-stages in the economy.

The former issue is not affected by existence of imports. Even if imported value (or value added) is included

in use industries' products, we should not change the evaluation of various stages' VA levels. For, imported values surely have contributed as an ingredient in constituting products values.

On the other hand, it is not desirable that a supply industry's value added contribution has any imported value within it. That is, we cannot use direct allocation matrix  $B = \hat{X}^{-1}A\hat{X}$  for evaluating supply industries' VA contributions.. Let us consider this further.

$(i, j)$  element of  $B$  represents what fraction of a unit of product  $i$  is (directly) allocated to product  $j$  or use industry  $j$ . If this economy has intermediate imports, direct requirement coefficient matrix is divided into two parts: domestic and import direct requirement coefficient matrices,  $D$  and  $M$  ( $A = D + M$ ). Then, since  $B = \hat{X}^{-1}A\hat{X} = \hat{X}^{-1}D\hat{X} + \hat{X}^{-1}M\hat{X}$ , direct allocation matrix  $B$  contains allocation of imported inputs. But imported inputs are supplied not by domestic industries but by foreign industries, whose value added should be excluded from domestic industries' contributions.

We need somewhat detailed discussion, however. Since  $A = D + M$ , we have  $X = AX + Y = DX + (MX + Y)$ , where  $Y$  is final demand. This final demand  $Y$  is usually defined as

$$Y = E + E_X - I_M,$$

where  $E$  is domestic expenditure on final products,  $E_X$  exports, and  $I_M$  imports. The problem is that  $I_M$  contains both final imports  $F_M$  and intermediate imports  $I_M^i$ . If  $Y$  should be to represent demand for domestic final products, imports to be deducted from  $E$  should be imports of final products. But conventionally not final imports but total (both final and intermediate) imports are deducted from  $E$  to obtain  $Y$ . In that sense,  $F = Y + I_M^i = E + E_X - F_M$  seems more appropriate than conventional  $Y$  as final demand per se. Let us call  $F$  genuin final demand.

Since  $I_M^i = MX$ , we have

$$X = DX + (MX + Y) = DX + F.$$

This is solved for  $X$  as

$$X = (I - D)^{-1} F = (I + D + D^2 + \dots)F. \quad (13)$$

Denoting  $R = \hat{X}^{-1}D\hat{X}$  and  $f = \hat{X}^{-1}F$ , we obtain

$$u = (I + R + R^2 + \dots + R^l + \dots) f. \quad (14)$$

$R$  is different from  $B$ , but is another allocation matrix. Since  $A \geq D$ , we also have  $B \geq R$ . Thus,  $I + B + B^2 + \dots \geq I + R + R^2 + \dots$ , which corresponds to and is consistent with the fact  $Y \leq F$  (or  $y \geq f$ ). (These inequalities will virtually hold in the strict sense.)

In calculations with actual input-output data, we sometimes find industries whose final demand values are negative. It is difficult to imagine what negative final demand means, and what kind of production it will induce. But usually the cases when we have negative final demand seem ones both with large intermediate imports and with small “genuine final demand” in the sense of our terminology,  $F$ .

In order to check this argument I examined 1998–2008 U.S. I-O annual data. In this data series, industries and commodities are classified into 65 (excluding two ‘commodities’: 66 Scrap, and 67 Noncomparable imports). In 2008 table, for example, 5 commodities among 65 show negative “final uses” values<sup>8</sup>. But by adding intermediate imports, all of these 5 negative values turned into positive. That is, all of  $F$  values showed positive. The same is true for almost all years from 1998 through 2008, except 2000 and 2002. (In these two years, ‘211 Oil and gas extraction’ showed negative ‘genuine final demand’ values. The reason is not clear.)

Then, (14) shows proportionate allocations of domestically produced value added. In the right-hand side, the  $(\lambda + 1)$ -th term  $R^\lambda f$  is a column vector of value added produced by  $n$  domestic supply industries that are injected into stage- $\lambda$  of various use industries. We decompose the expression (14) like  $C(i)$  in section 2.2. The matrix obtained, supply industry  $i$ ’s **domestic value added (VA) contribution** matrix, is:

$$\mathcal{S}(i) = \begin{pmatrix} s_{ij}^{(\lambda)} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \dots & f_i & \dots & 0 \\ r_{i1}f_1 & r_{i2}f_2 & \dots & r_{ii}f_i & \dots & r_{in}f_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{i1}^{(\lambda)}f_1 & r_{i2}^{(\lambda)}f_2 & \dots & r_{ii}^{(\lambda)}f_i & \dots & r_{in}^{(\lambda)}f_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \quad (15)$$

where  $r_{ij}^\lambda$  is the  $(i, j)$  element of  $R^\lambda = \hat{X}^{-1}D^\lambda\hat{X}$ ,  $f_j$   $j$ -th element of vector  $f$ , and  $s_{ij}^{(\lambda)} = r_{ij}^\lambda f_j$ . We have to use this  $\mathcal{S}(i)$  to evaluate domestic supply industries’ VA contributions.

Next issue to be considered is the value added contribution by imports. As discussed above, the height of VA

---

<sup>8</sup>These are 2. Forestry, fishing, and related activities, 3. Oil and gas extraction, 8. Wood products, 9. Nonmetallic mineral products, and 10. Primary metals.

levels to which imported intermediates flow in is not affected by existence of imports, except an issue discussed shortly.

On the other hand, value added contributions by imports have some crucial points to be considered. First total production  $X$  is shown as (13). Then intermediate imports vector is:

$$MX = M(I + D + D^2 + \dots)F.$$

(This does not contain final good imports.) Obviously, this is directly required imports to produce total output  $X$ , not containing ‘indirect’ imports. For there is no such thing as ‘indirect’ import. If some product is imported, its earlier production stages are basically carried in foreign countries, and earlier stages does not have any effect on this country’s inputs requirement<sup>9</sup>.

But this discussion evokes one probably important point. When a country imports a product, it brings into this country not only value added of its final production stage but also all the value added created and piled in all the previous stages.

This has something in relation to the value added levels mentioned above. As shown in (2),  $u' = u'A + v'$ . That is, each product’s value is equal to its intermediates’ value plus value added created by the use industry. Dividing  $A$  into  $D$  and  $M$ , we obtain

$$u' = u'(D + M) + v'.$$

From this equation, the following expression is obtained:

$$\begin{aligned} u' &= u'M(I - D)^{-1} + v'(I - D)^{-1} \\ &= u'M(I + D + D^2 + \dots) + v'(I + D + D^2 + \dots). \end{aligned} \tag{16}$$

The first term  $u'M(I + D + D^2 + \dots)$  of the right-hand side is the value of intermediate imports that are necessary to produce one units of each domestic product. What is important is that the amounts are expressed in product value, not value added. The second term  $v'(I + D + D^2 + \dots)$  is the amount of value added as recognized as  $v'$  is multiplied. In other words, though the term does not contain  $v'$ ,  $u'M(I + D + D^2 + \dots)$  is *the* value added that imports contribute in forming a unit value of each product.

---

<sup>9</sup>This statement does not hold, if some of foreign countries which export to this country use imported intermediates from this country in exports’ production processes. But we do not consider this case, though it seems getting important.

This can be shown in a somewhat different way. (3) shows each industry product's value added composition. Since this is equal to (16), we have

$$u' M(I + D + D^2 + \dots) = v'(I + A + A^2 + \dots) - v'(I + D + D^2 + \dots).$$

The difference between total value added (first term) and domestically produced value added (second term), that is, value added injected by imported products, is equal to  $u' M(I + D + D^2 + \dots)$ .

This is a point we should keep in mind when we want to evaluate the contribution of imports in VA level formation.

## 4 Some Empirical Application

In this section we make an attempt to apply our method to actual input-output data. We pick up 1998–2008 annual I-O Tables constructed by U.S. Bureau of Economic Analysis<sup>10</sup>. These tables consist of 65 industries and 67 commodities (including “66 Scrap, used and second hand goods” and “67 Non-comparable imports and rest-of-the-world adjustment”<sup>11</sup>). When we want to analyse individual industries relatively in detail, finer classification of industries is desirable. U.S. Benchmark detailed tables are released every some 5 years and have more than 400 industries classified, but its industry (commodity) classification system has been changed by Benchmark years. That makes it difficult to compare different years.

One point we need to discuss and have yet to do is U.S. I-O tables are produced by the commodity-by-industry approach. There have been raised a lot of problems and discussions in the literature about how to produce commodity-by-commodity or industry-by-industry tables from these original commodity-by-industry tables<sup>12</sup>. We used U.S. annual tables of “After Redefinition”, where redefining procedures have made the concept of industries much clearer than original tables. U.S. B.E.A. also provide three kinds of tables: 1. Commodity-by-commodity (CxC) total requirements (coefficients) tables, 2. industry-by-industry (IxI) total requirements (coefficients) tables, and 3. Industry-by-commodity (IxC) total requirements (coefficients) tables.

<sup>10</sup>U.S. Bureau of Economic Analysis [1]. This file (a set of several files) is downloadable from BEA's website.

<sup>11</sup>We could use OECD input-output tables. If we used OECD tables, we could compare different countries' vertical structures of industries. But, OECD tables are composed of somewhat small numbers of industries (commodities). OECD tables are also downloaded from its website.

See also N. Yamano and N. Ahmad [8] and Wixted, B., N. Yamano and C. Webb, [7].

<sup>12</sup>See J. Guo, A. M. Lawson and Mark A. Planting [2] for example, and many articles cited in this paper.

For analytical purposes, it is more convenient to use tables with formats of commodity-by-commodity or industry-or-industry. In order to transform data (data vectors and matrices) from one format to the other, we need a transformation matrix  $W$  which transforms commodity vector to industry vector. Fortunately, this can be obtained by calculating with total requirements tables produced in the two (CxC and IxI) formats. The direct method to calculate transformation matrix  $W$  is explained in Chapter 12 of Karen J. Horowitz and Mark A. Planting [3] (so-called US I-O manual). But we obtained the transformation by calculating in reverse order, that is, by deduction from completed CxC and IxI tables.

In the original tables, values of final demands are given in commodity units. This is a reason why our calculation is carried out basically on CxC tables.

**Average VA contribution levels** First, we calculated average VA contribution levels of (supply) industries. The larger and closer to 1 the contribution level calculated is, the more closely positioned to final demand is the industry. In other words, industries with higher VA contribution levels are relatively ‘downstream’ industries. Those with lower VA contribution levels are relatively ‘upstream’ ones.

Average VA contribution levels of 65 industries (shown by commodity base) are calculated. The results are shown in Appendix A, “Average VA contribution levels of U.S. Industries”. Industries (commodities) are sorted rearranged by 2008 contribution levels in ascending order. Industries placed in upper part of the table are relatively ‘upstream’ industries, and those in lower part relatively ‘downstream’ industries.

**Graphs of VA contribution levels** Next, we tried to show VA contribution level structure of each (supply) industry by depicting graphs according to the method explained in 2.3. Our procedure of calculations is as follows. (Our calculation of power matrix series was carried up to 10th power, which is apparently enough.)

First we calculated value added levels of industries. This calculation was based on (4),  $h'(\lambda) = u'A^\lambda$ . Calculation results were rearranged by each supply industry  $i$  to obtain matrices of  $\mathcal{A}(i)$ 's. Then, we calculated value added contribution segments of supply industries to get the VA level matrix  $\mathcal{L}$ , which was rearranged again in VA level's ascending order. Finally cumulative value added contribution shares were calculated. Then value added level and value added contribution shares of each supply industry-stage were paired to depict graphs. Of course, the needed adjustments for imports are carried out.

Obtained 65 U.S. industry graphs (commodity-base) are shown in the Appendix A. Each industry has three (kinked) curves for years 1998, 2003, and 2008. All curves in the graphs start from origin and reach (1.1). But there are little which move along the diagonal. Some curves are biased towards lower right, and others towards upper left.

In general, industries whose curves are biased towards lower left may be called upstream industries. These include 211. Oil and gas extraction, 331. Primary metals, 323. Printing and related support activities, and so on. 5412OP Miscellaneous professional, scientific, and technical services is included in this category too.

On the other hand, industries whose curves are biased towards upper left can be called downstream industries. Many industries are in this category. They tend to have larger segments of stage-0 (final demand) in total product allocation. But we should be cautious about looking at the segments of final demand. That is, the two main categories of final demand are consumption and investment. Though investment is conventionally and for convenience classified as final demand, it can be regarded as intermediate in the longer roundabout production processes. 213. Support activities for mining and 333. Machinery are probably its typical cases. Large parts of their products are absorbed into investments, which makes their average VA contribution levels high or their graphs biased towards upper left, it might not match our common sense.

Making these graphs are, of course, only one step towards vertical structure analysis of industries. If we want to analyse relationships between individual industries, we need an analysis peculiar to that relationship. But even in that case, knowing what is individual industries' vertical position in the whole economy will be helpful and sometimes indispensable.

## **5 Concluding Remarks**

The main purpose of this paper is to propose a method to analyse vertical structure of industries. When our attempt of this kind is to show industries' positions in the whole economy, analysis based on input-output data seems inevitable.

Our method of analysis, particularly the one displayed by graphs of value added contribution levels, is, in a sense, an integration of quantity model and price model of input-output tables. The concept of value added level (height) is derived from the price model aspect of input-output models, on the one hand, and that of value added

contributions is derived from the quantity aspect, on the other.

In this context, proper dealing with imports seems very important, as discussed earlier. Value added brought in by imports are not created in this country. But accurate domestic direct requirements tables cannot be obtained without accurate import requirements tables. Unfortunately, usually (competitive) import data are not gathered separately from domestic data. Import requirements matrix is constructed with several approximation methods. I do not know how import requirements data are accurate or inaccurate, but I wonder a lot of analytical possibilities in this field might be lost for this reason. Even so, the method shown here may, we hope, have some value to advance research in this field.



## References

- [1] Bureau of Economic Analysis, U.S. Department of Commerce. 1998-2008 Annual I-O Tables. U.S. Bureau of Economic Analysis's website: [http://www.bea.gov/industry/io\\_annual.htm](http://www.bea.gov/industry/io_annual.htm), 2010.
- [2] Jiemin Guo, Ann M. Lawson, and Mark A. Planting. From Make-Use to Symmetric I-O Tables: An Assessment of Alternative Technology Assumptions. Paper presented at the 14th International Conference on Input-Output Techniques, October 10-15, 2002, Montreal, Canada, October 10-15 2002.
- [3] Karen J. Horowitz and Mark A. Planting. Concepts and Methods of the Input-Output Accounts. website of U.S. Bureau of Economic Analysis of the U.S. Department of Commerce, September 2006.
- [4] David Hummels, Jun Ishii, and Kei-Mu Yi. The Nature and Growth of Vertical Specialization in World Trade. *Journal of International Economics*, Vol. 54, pp. 75–96, 2001.
- [5] Ronald E. Miller and Peter D. Blair. *Input-Output Analysis: Foundations and Extensions*. Cambridge Univ. Pr., Cambridge U.K., 2nd ed. edition, 2009.
- [6] UNCTAD. *The Offshoring of Corporate Service Functions: The Next Global Shift?*, chapter 4, pp. 147–80. 2004.
- [7] B. Wixted, N. Yamano, and C. Webb. Input-Output Analysis in an Increasingly Globalised World: Applications. *OECD Science, Technology and Industry Working Papers*, Vol. 2006/7, .
- [8] N. Yamano and N. Ahmad. The OECD Input-Output Database: 2006 Edition. *OECD Science, Technology and Industry Working Papers*, Vol. 2006/8, , 2006.

## Appendix A.

### Average VA contribution levels of US industries

	code	commodity	1998	2003	2008
1	493	Warehousing and storage	0.3278	0.3206	0.3460
2	561	Administrative and support services	0.3462	0.3511	0.3599
3	211	Oil and gas extraction	0.3350	0.3368	0.3713
4	323	Printing and related support activities	0.3386	0.3367	0.3745
5	GFE	Federal government enterprises	0.3930	0.3686	0.3790
6	486	Pipeline transportation	0.3297	0.3691	0.3955
7	5412OP	Miscellaneous professional, scientific, and technical services	0.3991	0.3929	0.4133
8	113FF	Forestry, fishing, and related activities	0.3913	0.4063	0.4187
9	321	Wood products	0.5082	0.4503	0.4346
10	562	Waste management and remediation services	0.4251	0.4262	0.4396
11	55	Management of companies and enterprises	0.4355	0.4528	0.4778
12	487OS	Other transportation and support activities	0.4306	0.4343	0.4816
13	212	Mining, except oil and gas	0.4067	0.3871	0.4840
14	331	Primary metals	0.4494	0.4437	0.4878
15	521CI	Federal Reserve banks, credit intermediation, and related activities	0.5382	0.4877	0.4878
16	327	Nonmetallic mineral products	0.5011	0.4739	0.4937
17	332	Fabricated metal products	0.5030	0.5013	0.5074
18	322	Paper products	0.4550	0.4780	0.5151
19	326	Plastics and rubber products	0.5162	0.5050	0.5285
20	482	Rail transportation	0.5085	0.5091	0.5467
21	514	Other information services	0.4057	0.5278	0.5546
22	5411	Legal services	0.5636	0.5650	0.5808
23	532RL	Rental and leasing services and lessors of intangible assets	0.5949	0.5798	0.6100
24	524	Insurance carriers and related activities	0.6194	0.6178	0.6128
25	324	Petroleum and coal products	0.5926	0.5905	0.6151
26	512	Motion picture and sound recording industries	0.5944	0.6258	0.6219
27	325	Chemical products	0.5817	0.6215	0.6222
28	711AS	Performing arts, spectator sports, museums, and related activities	0.6263	0.6126	0.6226
29	523	Securities, commodity contracts, and investments	0.6147	0.5543	0.6444
30	513	Broadcasting (except internet) and telecommunications	0.6066	0.6224	0.6466
31	111CA	Farms	0.6242	0.6231	0.6517
32	22	Utilities	0.6348	0.6416	0.6594
33	484	Truck transportation	0.6281	0.6277	0.6626
34	313TT	Textile mills and textile product mills	0.6120	0.6892	0.7166
35	5415	Computer systems design and related services	0.7435	0.7332	0.7235
36	42	Wholesale trade	0.7116	0.7279	0.7322
37	721	Accommodation	0.7265	0.7283	0.7343
38	485	Transit and ground passenger transportation	0.6782	0.7017	0.7365
39	335	Electrical equipment, appliances, and components	0.6918	0.7277	0.7387
40	531	Real estate	0.8072	0.7931	0.7786
41	81	Other services, except government	0.7586	0.7945	0.8014
42	339	Miscellaneous manufacturing	0.7666	0.8025	0.8016
43	481	Air transportation	0.7630	0.7570	0.8056
44	334	Computer and electronic products	0.7889	0.8110	0.8093
45	311FT	Food and beverage and tobacco products	0.7940	0.7985	0.8143
46	337	Furniture and related products	0.8683	0.8366	0.8199
47	3364OT	Other transportation equipment	0.8142	0.8291	0.8425
48	GSLE	State and local government enterprises	0.8398	0.8428	0.8448
49	315AL	Apparel and leather and allied products	0.8121	0.8904	0.8546

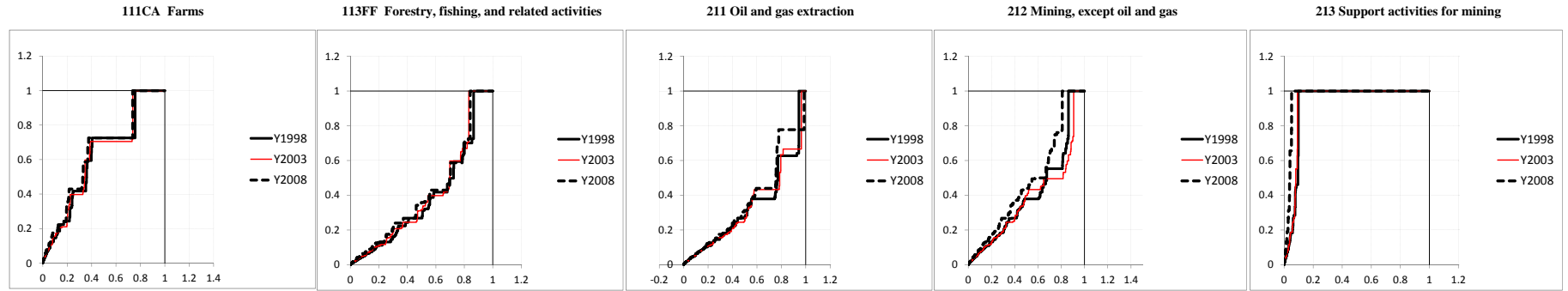
### Average VA contribution levels of US industries---continued

50	511	Publishing industries (includes software) and internet broadcasting	0.8108	0.8596	0.8565
51	333	Machinery	0.8347	0.8410	0.8594
52	3361MV	Motor vehicles, bodies and trailers, and parts	0.8338	0.8397	0.8595
53	722	Food services and drinking places	0.8589	0.8504	0.8617
54	23	Construction	0.9031	0.8964	0.9002
55	483	Water transportation	0.8345	0.8859	0.9364
56	44RT	Retail trade	0.9409	0.9392	0.9486
57	525	Funds, trusts, and other financial vehicles	0.9674	0.9433	0.9516
58	713	Amusements, gambling, and recreation industries	0.9784	0.9410	0.9517
59	61	Educational services	0.8893	0.9373	0.9531
60	213	Support activities for mining	0.9232	0.9281	0.9641
61	621	Ambulatory health care services	0.9678	0.9748	0.9758
62	624	Social assistance	0.9633	0.9885	0.9943
63	622HO	Hospitals and nursing and residential care facilities	0.9923	0.9957	0.9979
64	GFG	Federal general government	1.0000	1.0000	1.0000
65	GSLG	State and local general government	1.0000	1.0000	1.0000

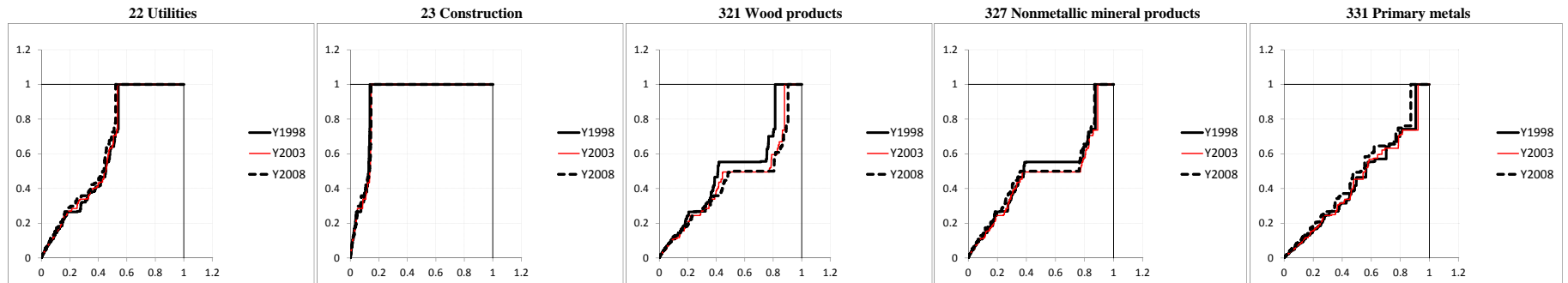
Industries are arranged in ascending order of 2008 contribution levels

Appendix B. Vertical Structure of U.S. Industries (commodity base) for 1998, 2003 and 2008

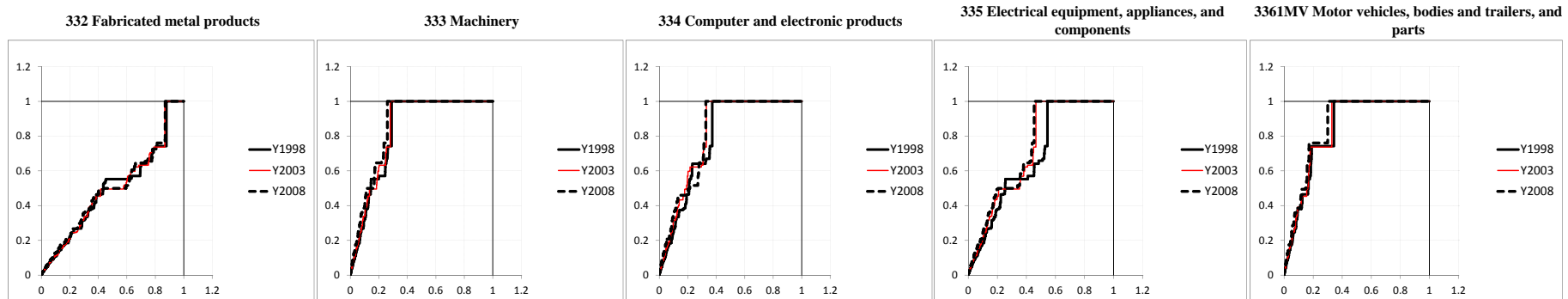
5



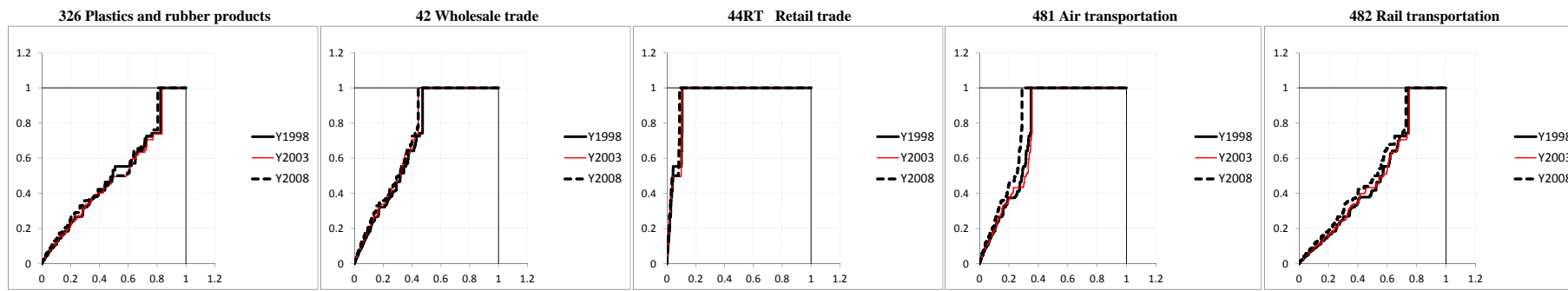
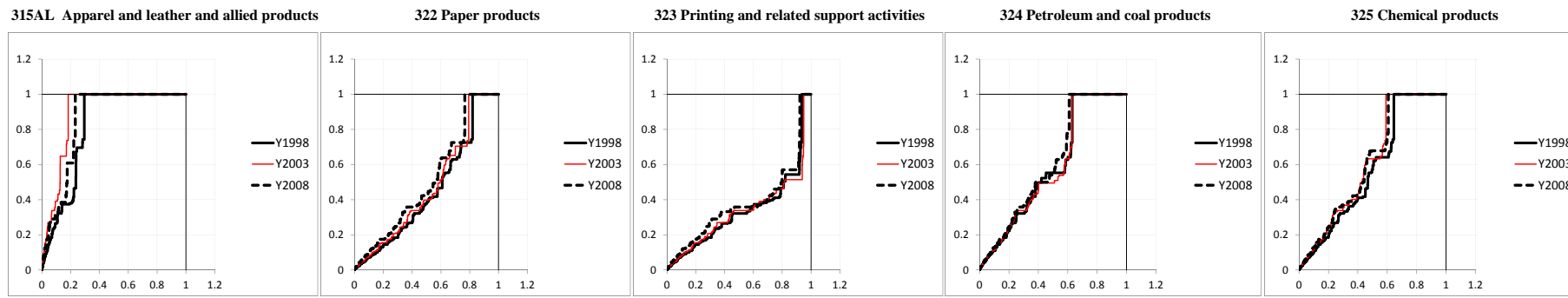
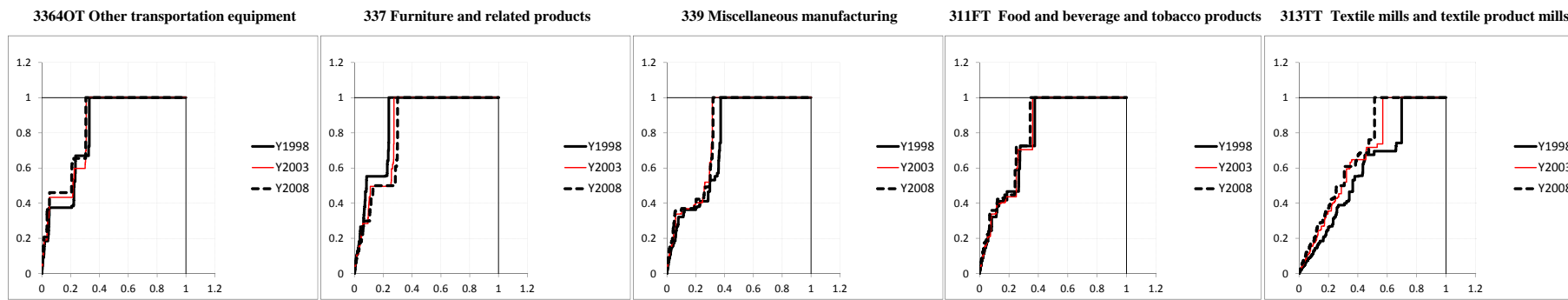
10



15

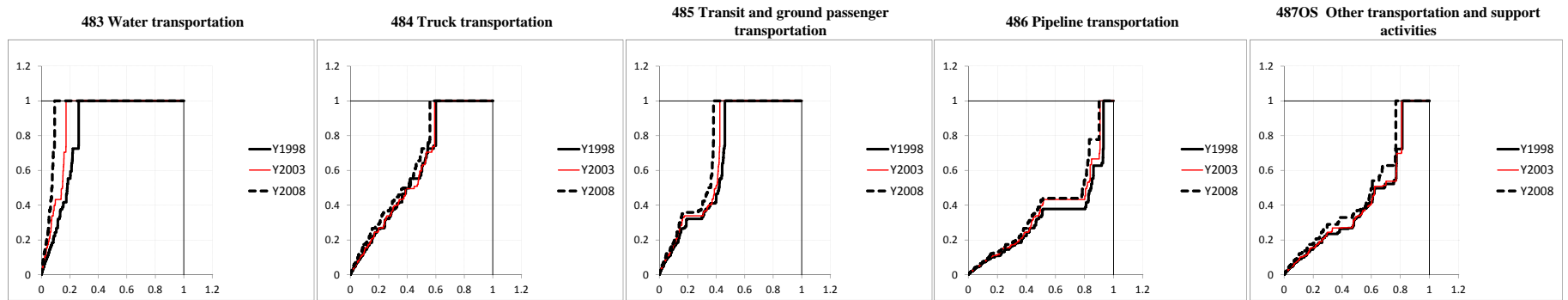


Vertical Structure of U.S. Industries (commodity base) ---continued

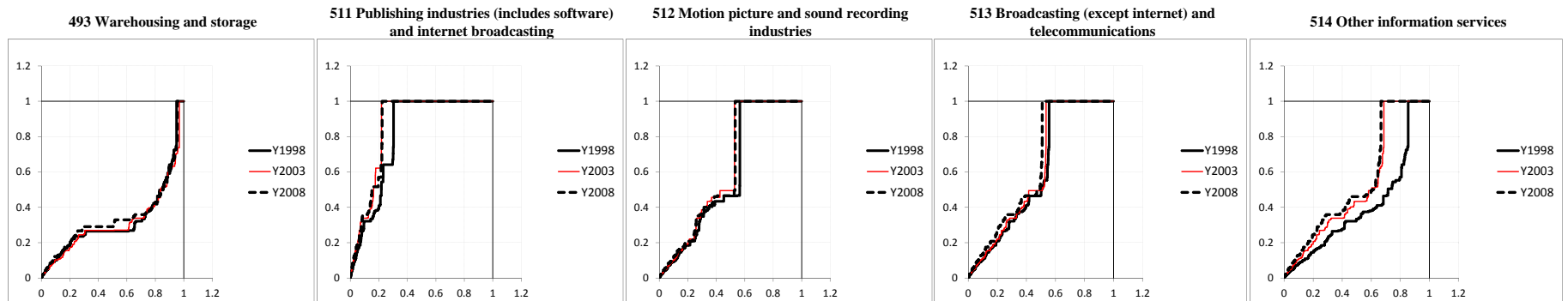


Vertical Structure of U.S. Industries (commodity base) ---continued

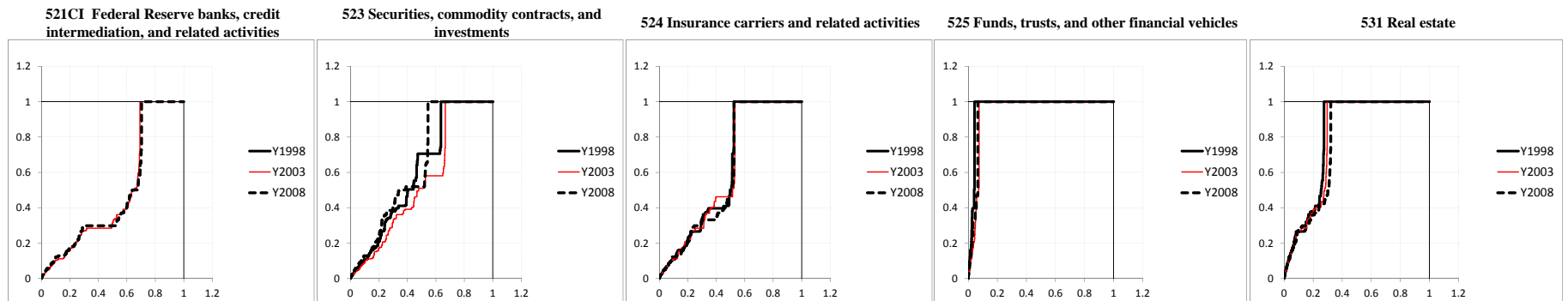
35



40

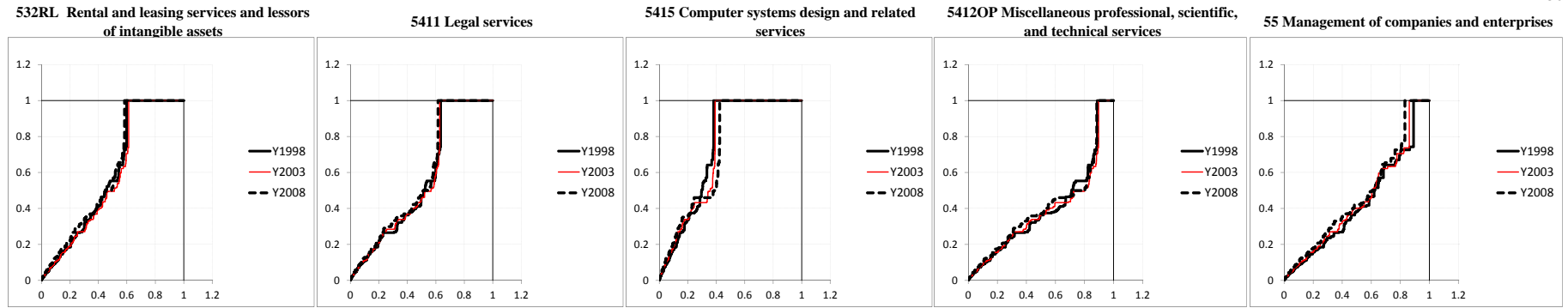


45

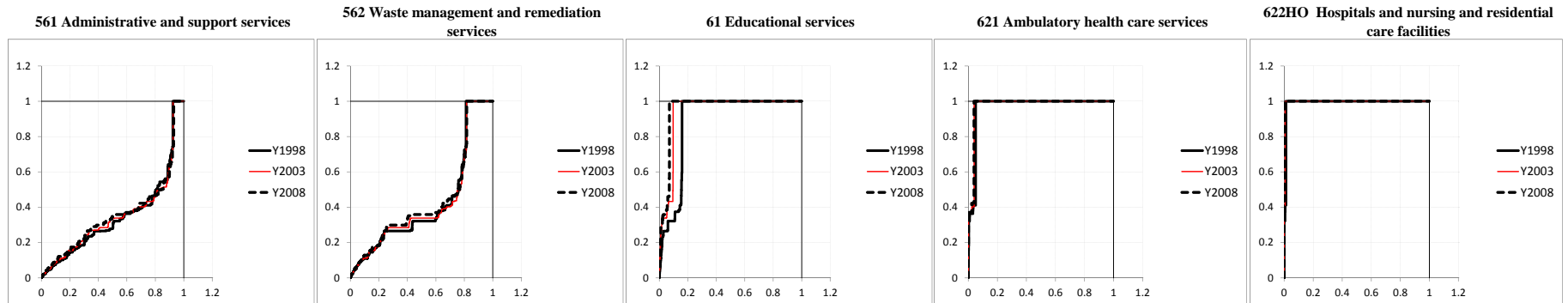


Vertical Structure of U.S. Industries (commodity base) ---continued

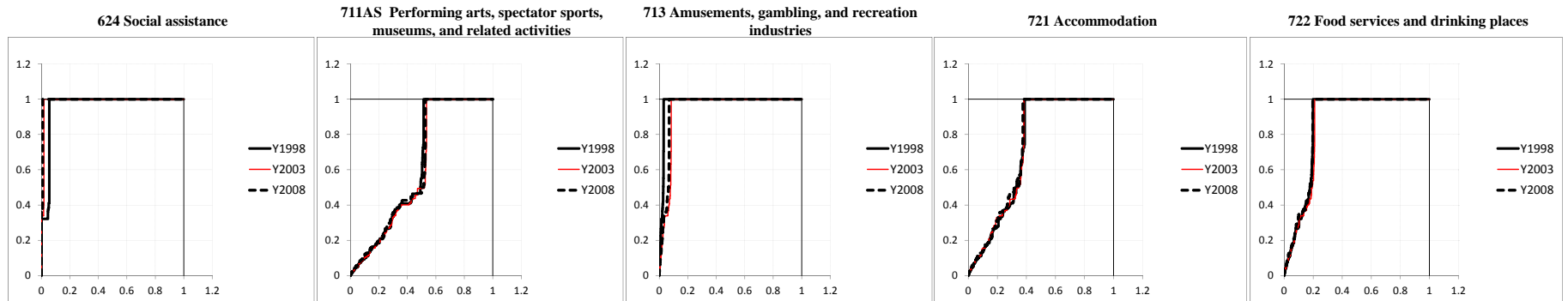
50



55



60



Vertical Structure of U.S. Industries (commodity base) ---continued

