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VEHICLES WITH COOPERATIVE REDUNDANT MULTIPLE STEERING SYSTEMS:

ALTERNATIVE DRIVER INTERFACES

I.J. SPARK (Monash University, Australia) and A PERCY (Monash University, Australia.

Corresponding author: Andrew.percy@monash.edu)

KEYWORDS: Alternative vehicle designs, driving modelling/ simulation, environmental impact of vehicles, four-wheel drive vehicles, Steering system, Agricultural vehicles, automotive systems, Fuel efficiency/economy, off-road vehicles.

ABSTRACT:

This paper presents the results of calculations of wheel angles and drive wheel speeds to ensure that the steering effect of the wheel angles and the steering effect of the speeds of the drive wheels are identical. These calculations are general insofar as the centre of curvature of the path of the centre of the vehicle can lie anywhere in the "horizontal" plane, including within the plan view of the vehicle. These minimal turning circles at times require large wheel angles and large differences in drive wheel speeds. When the driver selects a centre of curvature inside the rectangle defined by the wheel base and track, problems arise due to the multiple solutions of the arctan function. This problem is solved so that flipping of the wheels through 180 degrees is avoided. Similar problems can arise in the calculation of correct wheel speed due to the ambiguity of the square root function, having both positive and negative roots. This problem is also solved.

Alternative driver interfaces are described in detail.

Vehicles with cooperative redundant multiple steering systems promise safety benefits relative to vehicles with a single non-redundant steering system and environmental benefits relative to vehicles with conflicting redundant multiple steering systems. The safety benefits result from increased traction, stability and manoeuvrability (especially on hills). The environmental benefits include reduced ground damage, tyre wear and fuel wastage on turning. These vehicles would be used to best advantage as extreme off-road vehicles. The general case of vehicles described is capable of both pure rotation and pure translation in any direction, and all motion in between. This maximised manoeuvrability also makes the system ideal for vehicles operating in confined spaces, such as fork lift trucks.

NOTATION:

b	Wheel base
COC	Centre of curvature
COV	Centre of vehicle
CRMSS	Cooperative Redundant Multiple Steering System
d	Total displacement of rotatable joystick from the null position or forward/backward displacement of normal joystick or speed control lever or pedal
R	Distance between COV and COC (positive if COC is to right of COV)
Rn	Distance between kingpin of nth wheel and COC
Rx	Distance of COC to right of COV
Ry	Distance of COC forward of COV
RMSR	Root mean square radius
RMSWS	Root mean square wheel speed
t	Track

2WS/2WD	2 wheel steering/2 wheel drive
2WS/4WD	2 wheel steering/4 wheel drive
4WD/4WD	4 wheel steering/4 wheel drive
ZTR	Zero Turn Radius
Ψ = arctanRy/Rx	Direction of displacement of joystick
θ	Rotation of rotatable joystick or steering wheel or sideways displacement of joystick
θmax	Maximum rotation of rotatable joystick or steering wheel (full lock)
$\theta'=\theta/\theta$max	Relative rotation of rotatable joystick or steering wheel
θ''	Relative rotation of steering wheel or sideways displacement of joystick when Ry=+/-b/2

1 INTRODUCTION:

This work is the result of the first author posing two fundamental questions. They are:

1. What are the ideal performance characteristics of a Limited Slip Differential?
2. How can these ideal characteristics be achieved?

The answer to the first question is that the steering effect of the speeds of the driven wheels must be identical to the steering effect of the wheel angles. The answer to the second question is that the speed of the driven wheels and the angle of the wheels must satisfy the equations derived in this article.

There are two ways of steering a wheeled vehicle. One method is to turn one or more steerable wheels. The other method is to drive one or more left hand wheels independently of one or more right hand wheels. Each system if acting alone would cause the vehicle to follow a path with a specific centre of curvature (COC). If both steering systems are enabled they will in general have different centres of curvatures. This conflict between the two steering systems will cause ground damage, tyre wear and fuel wastage. The fuel wastage occurs because the energy required to inflict ground damage and tyre wear comes from the consumption of extra fuel. Conflict between two (or more) steering systems causes a braking effect that can only be overcome by increasing the power delivered by the engine.

The traditional means of avoiding conflict between the two steering systems is to disable one steering system. In road cars, the steering effect of the driven wheels is eliminated by incorporating a differential into the drive train to the driven wheels. See cells A1, A2 and B1 in Table 1. Alternatively in Zero Turn Radius (ZTR) mowers, the steering effect of the non-driven wheels is eliminated by making them castors. See cell E1 in Table 1. In both cases a single non-redundant steering system results.

In general a single non-redundant steering system is adequate on level ground. However on difficult terrain, such as steep and slippery slopes, the single steering system is prone to failure – resulting in traction, stability and safety problems.

The above problems can be overcome to some extent by allowing both the wheel angle steering system and the drive wheel steering system to operate, but allowing one system to over-power the other. In vehicles with locked differentials, the steering effect of the wheel angles over-powers the steering effect of always driving both driven wheels at the same speed. See cells C1, C2 and C3 in Table 1. Alternatively in Skid Steer Loaders, the steering effect of the speeds of the driven wheels over-powers the steering effect of the constant (i.e. zero) wheel angles. . See cell E3 in Table 1. In both cases a conflicting redundant multiple steering system results. These systems result in increased ground damage, tyre wear and fuel consumption, the latter due to un-desirable (and un-necessary) work done at the tyre/terrain interface.

However if the two steering systems are integrated so that each (if acting alone) produced the same centre of curvature, then the two steering systems would reinforce each other. In this case a cooperative redundant multiple steering system results. The purpose of this paper is to explore the implications of such a system. . Hereafter the “Cooperative Redundant Multiple Steering System” will be abbreviated to CRMSS.

This work bears some relationship to work on “over-actuated” systems, where the number of control inputs exceeds the number of variables to be controlled. Over-actuation is used in aircraft [1], [2], ships [3], [4] and automobiles [5]. It is also used in robots and machine tools [6]. Vehicles where both the wheel angle steering system and the wheel speed steering system are both enabled are over-actuated. In the CRMSS advocated here both the wheel angle “actuator” and the drive wheel speed “actuator” are integrated so that they are both attempting to impose the same COC on the path of the vehicle. In this case the two “actuators” reinforce each other so that when one actuator starts to fail, it is backed up by the other. This should defer the failure of the cooperative redundant steering systems (CRSMSS) to more onerous operating conditions.

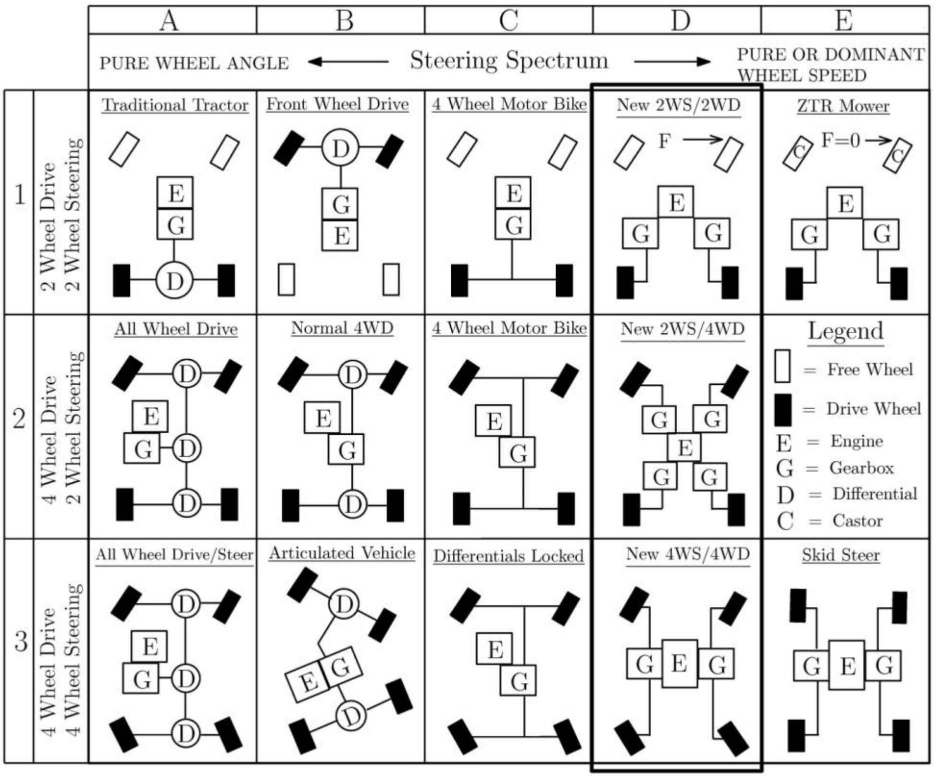
2 COMPARISON OF COOPERATIVE REDUNDANT MULTIPLE STEERING SYSTEM WITH EXISTING SYSTEMS:

Table 1 depicts existing steering/drive systems for four wheeled vehicles. Steering/drive systems vary from pure wheel direction steering on the left to pure or dominant wheel speed steering on the right. The first row depicts vehicles with two driven wheels and two steerable wheels (with the exception of the zero turn radius (ZTR) vehicle where the non-driven wheels are castors). The second row depicts vehicles with four driven wheels and two steerable wheels. The third row depicts vehicles with four driven wheels and four steerable wheels (with the exception of the skid steer vehicles where no wheels are steerable).

The three novel vehicles with cooperative redundant multiple steering systems (CRMSS) are depicted in the fourth column (i.e. D1, D2 and D3). Note that there are no differentials or castors in these three variants of the CRMSS depicted in column D. The two wheel drive/two wheel steer vehicle (in cell D1) can be regarded as a modified ZTR vehicle where the castors are replaced with steerable wheels, the angles of which are integrated with the speeds of the driven wheels. In this case two independent systems are required to control the speed of the two driven wheels and two independent systems are required to control the angles of the two steerable wheels (which replace the ZTR castors).

The four wheel drive/four wheel steer vehicle (in cell D3) can be regarded as a modified skid steer vehicle where the four driven wheels are steerable where their angles are integrated with the speeds of the driven wheels. In this case two independent systems are required to control the speeds of the four driven wheels, and two independent systems are required to control the angles of the four wheels.

Table 1: The relationship between the novel CRMSS vehicles and existing vehicles



Alternatively the new vehicle depicted in cell D3 can be regarded as an articulated loader where the front and rear differentials are removed and the speed of the left hand and right hand wheels are integrated with the angles of these wheels. In this case two independent systems are required to control the speeds of the four driven wheels and one system is required to control the angles of the four steerable wheels.

The four wheel drive/two wheel steer vehicle (in cell D2) can be regarded as a four wheel drive/two wheel steer tractor where the speeds of the four driven wheels are integrated with the angles of the two steerable wheels. In this case four independent systems are required to control the speeds of the four driven wheels and two independent systems are required to control the angles of the two steerable wheels.

Vehicles with a single non-redundant steering system are depicted in cells A1, A2, A3, B1, B3 and E1. Vehicles with (two) conflicting redundant steering/drive systems are depicted in cells B2,C1, C3, C2 and E3 in order of increasing conflict. The mismatch between the effects of the two steering systems can be regarded as progressive errors. Examples of existing vehicles are listed in the appropriate cells. These examples are not exhaustive.

Wong [7] has shown theoretically that, for wheel loadings that are proportional to wheel diameter, the maximum slip efficiency (which is the major component of traction efficiency) occurs when the slip of the front and rear drive wheels is equal. Wong's theoretical work is supported by experimental work [8, 9] which showed that the maximum slip efficiency occurred when the slip of the front and rear drive wheels is equal or almost equal. Besselink [10] showed theoretically that for a four wheel drive vehicle operating under non-uniform traction conditions, the maximum slip efficiency can occur when the slips of the front and rear wheels are not equal, so that unequal drive wheel speeds may be required to maximise efficiency. Vantsevich [11] deduced the transport efficiency of an eight wheel drive vehicle is, in general, a maximum when the slip of all wheels is the same. In other words the actual speeds of all wheels should correspond to the "theoretical" speeds. Exceptions to the "equal slip" rule occur in unusual circumstances, and the deviations are small [11].

With regard to turning vehicles the theoretical development of cooperative redundancy is well ahead of practice. Despite the amount of work that has been done on the tractive performance of vehicles travelling in a straight line, little has been done on turning vehicles. Dequesne et al [12] have found that the use of a "controlled" differential increases the tractive force of a 2WD tractor when the turning radius is less than 10m. They also found that manoeuvrability and ground damage were increased and decreased respectively.

Forbes and Lewis [13] have used the ADAMS simulation software to compare the expected performance of the following vehicles on both level and sloping ground:

1. Traditional 2WD agricultural tractor (single **non-redundant** steering system. Cell A1 in Table 1)
2. 2WS/4WD agricultural tractor with locked differentials (**conflicting redundant** multiple steering system. Cell C2 in Table 1)
3. 4WD skid steer loader (**conflicting redundant** multiple steering system. (Cell E3 in Table 1)
4. 4WS/4WD vehicle with CRMSS. (**Cooperative Redundant** Multiple Steering System. Cell D3 in Table 1).

For the vehicles with steerable wheels, pure Ackermann steering was assumed where the axes of rotation of all wheels intersect at a single point (or vertical line). As this cannot be achieved by the traditional four-bar-linkage, especially for large wheel angles, a dedicated actuator was assumed for each steerable wheel. Slip angles and longitudinal slip were central to the ADAMS modelling work. Vehicles with conflicting redundant steering systems were characterised by large slip angles and longitudinal slips. Vehicles with a single non redundant steering system or CRMSS were characterised by much smaller slip angles and longitudinal slips.

These model vehicles were identical except for their steering and drive systems. The vehicles were required to make a circular uphill turn starting from the transverse position. The speed of the vehicle, the radius of the turn and the steepness of the slope were all varied. The slope at which the vehicles became unstable was noted. The simulation indicated that the CRMSS vehicles equalled or exceeded the performance of the other vehicle configurations in all respects. Vehicles with CRMSS demonstrated safety benefits relative to vehicles with a single non-redundant steering system such as increased traction, stability and manoeuvrability (especially on hills). CRMSS was also shown to provide environmental benefits relative to vehicles with a conflicting redundant steering drive system.

Restricting the simulations to the relatively simple "steady-state" situation of curves of continuous radius made it possible to calculate the wheel angles for the CRMSS vehicle on a slope, however equations had not been developed to allow movement of the COC from a fixed point. The current article provides calculation of wheel angles and speeds for general paths which are not necessarily circular. However, at this stage, the calculations are limited to paths in a horizontal plane where slip angles and longitudinal slip are assumed to be negligible. However means of accounting for slip are given in [21-24].

3 PROGRESS TO DATE:

Spark and Besselink [14] commenced work on the cooperative redundant steering/drive vehicle depicted in cell E1. They modified a Dixon ZTR mower by replacing the front castors with "intelligent castors". The primary steering system is the independent friction drives to the non-steerable rear wheels. Proximity sensors measured the speed and direction of rotation of each drive wheel by counting the passing sprocket teeth. An on-board computer calculated the appropriate angles and implements these angles. Besselink [15] has subsequently designed and build an instrumented ZTR test bed where the angle of a single front wheel (intelligent castor) is integrated with the speeds of two independently driven non-steerable rear wheels.

Blair and Spark [16] extended the principle of cooperative redundancy to four wheel drive/four wheel steering vehicles. They proposed modifying a skid steer vehicle (see E3) by turning the four driven wheels so that the steering effect of the wheel angles was identical to the steering effect of the wheel speeds (see D3). In this case the primary steering effect is the independent hydrostatic drives to the left hand and right hand wheels. Wheel speed sensors would measure the speed and direction of rotation of the left hand and right hand wheels. An on-board computer calculates and then implements these appropriate wheel angles. It was found that there was a limited range of wheel speed ratios where scuffing could be avoided by turning the wheels. Outside this range of wheel speed ratios there are no wheel angles that avoided scuffing. It was also found that for each allowable driver-selected wheel speed ratio there were two sets of wheel angles that avoided scuffing, so that the driver has to nominate which solution he desired.

Lu et al [17] overcame the above two problems by allowing the driver to select the desired centre of curvature with a steering wheel or joystick. The on-board computer then calculates and implements both the appropriate wheel speeds and wheel angles.

Spark and Ibrahim [18-20] extended the above theory to the general case where the centre of curvature (COC) selected by the driver can lie anywhere in the horizontal plane. In this case the on-board computer must calculate and implement four wheel speeds and four wheel angles. Unfortunately, ambiguity occurred when the COC was located inside the plan of the vehicle.

Spark and Ibrahim [21-24] have recognised that in general the effective wheel angle will differ from the apparent wheel angle by the slip angle. Similarly, in general the effective wheel speed will differ from the apparent wheel speed by the longitudinal slip. Means of deducing the linear elastic portion of slip angle and longitudinal slip are described in [21]. Alternatively means of measuring the total slip angle and total longitudinal slip are described in [22].

The extension of the principle of cooperative redundancy between two or more steering systems to multi axle and/or articulated vehicles has been described in [25-30].

4 THE PREFERRED DRIVER INTERFACE

Note that the novel vehicles with CRMSS depicted in Table 1 are special cases. In the new vehicles in the first two rows (D1 and D2) either the front or rear wheels are not steerable so that the centre of curvature always lies on the axes of these non-steerable wheels. In the new vehicle in the third row (D3) the front and rear wheels on one side of the vehicle turn by the same amount and the front and rear wheels on one side rotate at the same speed. In this case the centre of curvature will always lie on the transverse axis of the vehicle. These restrictions do not apply to the vehicle shown in Fig. 1

The vehicle depicted in Fig. 1, below, is a design concept in as much as a test bed is yet to be constructed and tested in the field. It is assumed for this article that the vehicle is on a horizontal surface and travelling at low speeds. The steering system, as presented here, is therefore suitable for vehicles, such as fork-lift trucks, operating in the confined environment of a warehouse. Note that the location of the centre of curvature is strictly only true if the slip angles and longitudinal slips of all the wheels are zero [21-24].

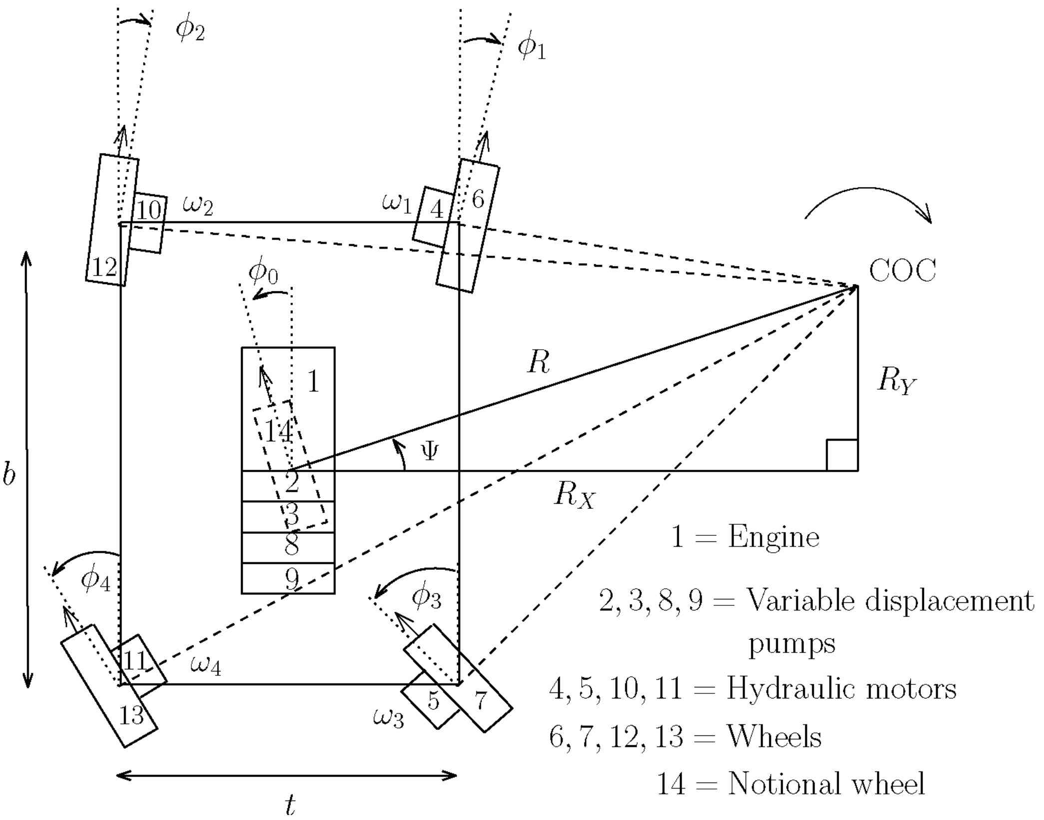


Figure 1: Four wheel steering/four wheel drive vehicle

This paper deals with the general case where the centre of curvature of the path of the vehicle can lie anywhere in the horizontal plane. In this case four independent systems are required to control the speeds of the four driven wheels and four independent systems are required to control the angles of the four steerable wheels.

Note that the origin (or reference point) is the Centre of the Vehicle (COV) as defined by the intersection of the longitudinal and transverse axes. Ideally the Centre of Gravity (COG) would be located close to the COV and as low as possible.

The most general case of the four wheel steering/four wheel drive variant of the vehicle, in which the COC may be located anywhere in the plane of motion, is depicted in Fig. 1. Here, an internal combustion engine 1 drives two right hand variable displacement hydraulic pumps 2 and 3 and two left hand variable displacement pumps 8 and 9. These displacement pumps in turn drive hydraulic motors 4 and 5 mounted in the steerable front and rear right hand wheels respectively and two hydraulic motors 10 and 11 which are mounted in the steerable front and rear left hand wheels 12 and 13 respectively. .

Although hydrostatic drives are extensively used in industrial mowers, skid-steer loaders and bulldozers, other drive systems could be used in CRMSS vehicles. Electric wheel motors with robust speed control could also be used. The first author has devised a hybrid system where most of the power is delivered to the drive wheels via high efficiency shaft drives. In this case, hydrostatic drives are only used to “correct” the drive wheel speeds [24, 31]. Inclusion of notional hydraulic wheel motors in the Figures clearly identifies the inside of the wheel. Details of alternative drive systems which allow a 180 degree range in drive wheel angles will be the subject matter of a future publication.

The angles of the wheels 6, 12, 7 and 13 are shown as ϕ_1, ϕ_2, ϕ_3 and ϕ_4 respectively. The rotational speed of the wheels 6, 12, 7 and 13 are ω_1, ω_2, ω_3 and ω_4 respectively. If a notional wheel 14 with the same centre of curvature (COC) is located at the centre of the vehicle, its angle θ_0 will be identical to ψ.

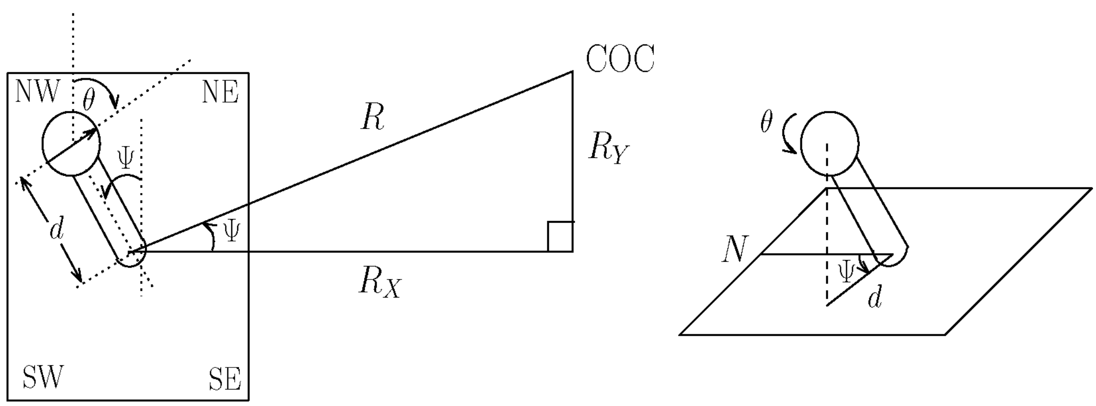


Figure 2: Preferred driver interface (plan view and 3 dimensional)

The preferred driver interface is a rotatable joystick as shown in Fig. 2. The advantage of this interface is that only one element is required to control the trajectory and orientation of the vehicle. The vehicle is controlled by the driver selecting the radius of curvature of the vehicle's path, R, and the sense of rotation by rotating (or twisting) the joystick, that is, selecting a positive or negative angle of *Θ*. When the joystick is not twisted, *Θ=0,* the R will be infinite and the vehicle will move in a straight line parallel to the direction of displacement of the joystick. This pure translation in any direction is often referred to as “Crab Steering”. When the joystick is twisted as far as it will go in a clockwise direction, the radius of R will be zero and the vehicle will rotate clockwise about its own centre (i.e. execute pure rotation). Between these two extremes the radius of curvature of the path of the vehicle is defined as:

$$\frac{R}{t} = \cot(90^0 \theta / \theta_{\max}) \;=\; \cot(90^0 \theta')$$

Where *t* is the track of the vehicle, θ is the rotation of the joystick, $\theta_{\max}$ is the maximum rotation of the joystick, and θ' (= θ/θ_{max}) is the relative rotation of the joystick. Note that when θ' is negative R will be "negative" and the vehicle will generally execute an anti clockwise (ACW) turn.

If the driver displaces the rotatable joystick so that it's projection onto the horizontal is at an angle Ψ to the straight ahead position (where ACW angles are deemed to be positive), the direction of the centre of curvature (COC) of the path of the vehicle will be at right angles to the direction of joystick displacement and R_X and R_Y will be defined by the following equations:

$$R_X \;=\; R\cos\Psi \;=\; t\cot(90^0\theta')\cos\Psi$$

and $$R_Y \;=\; R\sin\Psi \;=\; t\cot(90^0\theta')\sin\Psi$$

To select the direction of the centre of curvature the driver displaces the joystick at right angles to this direction. The centre of curvature of the path of the vehicle is now specified by the two components R_X and R_Y. Additionally, the driver selects the root mean square of the four wheel speeds by the amount of displacement d of the joystick's projection onto the horizontal.

The control system then rotates the four drive wheels to the following angles:

$$\tan\phi_1 \;=\; (b/2 - R_Y)/(R_X - t/2) \;=\; (b/2 - t\cot(90^0\theta')\sin\Psi)/(t\cot(90^0\theta')\cos\Psi - t/2)$$

$\tan\phi_2 = (b/2 - R_Y)/(R_X + t/2)$ = (b/2 - t cot (90^0 θ’) sinΨ) / (t cot (90^0 θ’) cosΨ + t/2)

$\tan\phi_3 = (b/2 + R_Y)/(R_X - t/2)$ = (b/2 + t cot (90^0 θ’) sinΨ) / (t cot (90^0 θ’) cos Ψ – t/2)

$\tan\phi_4 = (b/2 + R_Y)/(R_X + t/2)$ = (b/2 + t cot (90^0 θ’) sinΨ) / (t cot (90^0 θ’) cos Ψ + t/2)

and $\phi_0 = \Psi$

where b is the wheel base of the vehicle, R_Y is the displacement of the centre of curvature forward of the centre of the vehicle and R_X is the displacement of the centre of curvature to the right of the centre of the vehicle.

The amount of displacement d of the joystick from its null position determines the root mean square of the four wheel speeds (RMSWS) according to the equation:

$$RMSWS = Kd = (\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)^{1/2}/2$$

where K is an appropriate constant.

The individual wheel speeds are given by the equations:

$\omega_1 = RMSWS(R_1/RMSR)$ where $R_1^2 = (b/2 - R_Y)^2 + (R_X - t/2)^2$

= (b/2 - t cot (90^0 θ’) sinΨ)^2 + (t cot (90^0 θ’) cosΨ – t/2)^2

$\omega_2 = RMSWS(R_2/RMSR)$ where $R_2^2 = (b/2 - R_Y)^2 + (R_X + t/2)^2$

= (b/2 - t cot (90^0 θ') sinΨ) ^{2} + (t cot (90^0 θ') cosΨ + t/2) ^{2}

$\omega_3 = RMSWS(R_3 / RMSR)$ where $R_3^2 = (b/2 + R_Y)^2 + (R_X - t/2)^2$

= (b/2 + t cot (90^0 θ') sinΨ) ^{2} + (t cot (90^0 θ') cosΨ − t/2) ^{2}

$\omega_4 = RMSWS(R_4 / RMSR)$ where $R_4^2 = (b/2 + R_Y)^2 + (R_X + t/2)^2$

= (b/2 + t cot (90^0 θ') sinΨ) ^{2} + (t cot (90^0 θ') cosΨ + t/2) ^{2}

and the RMSR is the root mean square radius, which is given by:

$RMSR = (R_1^2 + R_2^2 + R_3^2 + R_4^2)^{1/2} / 2$ = $(R_X^2 + R_Y^2 + t^2/4 + b^2/4)^{1/2}$

= ((t cot (90^0 θ'))^2 + t^2/4 + b^2/4)$^{1/2}$

The notional speed ω_0 is given by:

ω_0 = RMSWS(R/RMSR) where R = t cot(90^0 θ')

Note that the use of the RMSWS as the driver's speed input greatly simplifies the individual wheel speed equations. More importantly the equations will always yield values for the individual wheel speeds regardless of the COC selected by the driver. Note that if the average wheel speed were used as the driver's speed input, the individual wheel speeds would become indeterminate when the average speed is zero. Similarly if the speed of a particular point on the vehicle was used as the driver's speed input, then the speed of individual wheels would become indeterminate if the driver selects this point as the COC of the vehicle's path.

For this general case the driver can select any centre of curvature (COC) by selecting θ and Ψ. See Figs 1 and 2. The only restriction is that if Ψ is held constant the centre of curvature will lie on a straight line which passes through the centre of the vehicle and is inclined at angle Ψ to the transverse axis of the vehicle. See Figs 1 and 2.

If the vehicle is to be capable of rotating about any centre of curvature (COC) in the horizontal plane all wheels must be capable of turning through an angle range of 360^0 if the driven wheels can only rotate in the forward direction. However if the driven wheels can also be driven in reverse then the required angle range is reduced to 180^0

Two problems arise when the above equations are used to define the appropriate angle and speed for each wheel to avoid conflict. These problems only occur when the COC selected by the driver lies inside the rectangle formed by the track t and the wheel base b (i.e. the COC lies within the vehicle).

The first problem is associated with the arctan function, which gives only one of an infinite number of solutions when used as an inverse function of tan. As the function repeats itself every 180^0, there are two solutions within a range of 360^0. The computer however only returns angle values between -90^0 and $+90^0$. If θ' is increased ϕ_1 will increase. However when ϕ_1 reaches $+90^{0,}$ it will jump to -90^0 and continue to increase. If ϕ_1 relates to a driven wheel then its direction of rotation would have to be reversed when the wheel is flipped through 180. This situation is clearly undesirable since this wheel flipping would cause an instantaneous braking effect, which could send the vehicle into a spin. Furthermore the instantaneous reversal of wheel rotation would be difficult for the control system to achieve, and subject the drive train to shock loading. If flipping of the wheels is to occur, it is best that it occurs when the vehicle is travelling at minimum speed. When the vehicle is turning about a centre of curvature which corresponds to the centre of the vehicle (COV) (i.e. $R_X = R_Y = 0$), a low RMSWS is likely to be selected by the driver. The driver can reverse the direction of rotation by twisting the joystick to opposite "lock". This would cause the wheels to turn through 180^0.

Alternatively the driver could displace the joystick backwards, which would cause the wheels to rotate in the opposite direction, but not flip.

Undesirable flipping of the wheels occurs because the computer has returned the "inappropriate" angle in the 360 ^{0} range. It can be avoided by applying the following logic. Firstly the limiting wheel angles (where $\theta' = \theta / \theta_{\max}$ =+/-1) must be calculated. These are the wheel angles that would cause the vehicle to rotate about its centre. For clockwise rotation these limiting wheel angles are:

ϕ_{1L} = ϕ_{3L} = 180 - arctan (b/t)

and ϕ_{2L} = ϕ_{4L} = arctan (b/t);

For anti-clockwise rotation the limiting angles are;

ϕ_{1L} = ϕ_{3L} = - arctan (b/t)

and ϕ_{2L} = ϕ_{4L} = -180 + arctan (b/t);

Subtracting the angles for anti-clockwise rotation from the angles for clockwise rotation shows that all wheels must be capable of turning through 180 ^{0} if pure rotation of the vehicle in both directions is to be possible.

To reiterate: -arctan (b/t) < ϕ_1 and ϕ_3 < 180 – arctan (b/t)

and arctan (b/t) – 180 < ϕ_2 and ϕ_4 < arctan (b/t)

For the right hand wheels the appropriate range of wheel angles is from –arctan (b/t) to 180 – arctan(b/t). Therefore if the computer returns an angle which is > -90 and < -arctan (b/t) then 180 must be added to this angle. Similarly for the left hand wheels, if the computer returns an angle <90 and > arctan (b/t) then 180 must be subtracted from this angle.

The second problem arises when the line on which the COC lies passes through the vertical axis (kingpin) of a drive wheel. This is the case when Ψ = arctan (+/- b/t). When Ψ = arctan(b/t) the COC will pass through the vertical axis of the front left hand and rear right hand wheels. When

Ψ = arctan (- b/t) the COC will pass through the vertical axis of the front right hand and rear left hand wheels. Since these wheels will always be at right angles to the line on which the COC lies, the angle of these wheels will be constant and independent of where the COC lies on the line of the COC. This result is confirmed by the above equations for wheel angles when R_Y/R_X = +/- (b/t), (bearing in mind that there is an equally valid angle where 180 is added or subtracted to or from the angle returned by the computer).

The general equation for relative wheel speed is:

ω_n /RMSWS = R_n /RMSR where R_n is calculated using Pythagoras's equation, and is the distance between the vertical axis of wheel n and the COC. As the computer only returns positive square root values, the calculated ω_n will always be positive. However, when the COC passes from outside to inside the vehicle, the wheel through which it has passed should rotate in the opposite direction. Although this could be achieved by turning this wheel through 180 as the COC passes through the wheel axis, an alternative solution to the problem will be outlined below. Note that if the COC passes very close to the wheel axis, and the arctan function is modified as described above, equations predict that the wheel will quickly (but monotonically) turn through 180, and reversal of rotation is not required. If the arctan function is not modified the wheel angle will flip from $+90^0$ to -90^0 or vice versa. If the arctan function is corrected a reversal of wheel rotation is not required.

Reversal of rotation of the wheels when appropriate can be achieved by applying the following logic: Once the driver selects a value of Ψ a check must be made to ascertain whether the line of the COC passes through the vertical axis of any driven wheels. If tan Ψ = +/- b/t some wheels will have constant wheel angles and a reversal of rotation will be required. This can be achieved by multiplying the calculated wheel speed by -1. If the modulus of R, $|R| = |\, t \cot (90^0\theta')| > (b^2 + t^2)^{1/2}/2$ no action is required. However if the modulus $|R| < (b^2 + t^2)^{1/2}/2$, where $R = t \cot (90^0\theta')$, then the wheel speed as calculated by the above equations must be reversed for the wheel through which the COC has passed. Alternatively if $R_Y = +/- b/2$ the line of the COC will pass through the vertical axis of either the front or rear wheels. In this case, if the modulus $|R| < (b^2 + t^2)^{1/2}/2$, where $R = ((b/2)^2 + (R_x - t/2)^2)^{1/2}$ and $R_X = t \cot (90^0\theta')$, then the wheel speed calculated by the above equations must be reversed for the wheel though which the COC has passed.

If $\tan \Psi > b/t$ another problem occurs. In this case modification of the raw wheel angles to bring them within the correct range will cause the slope of the wheel angle versus relative joystick rotation for some wheels to become negative. The rotation of these wheels must be reversed by multiplying the raw wheel speed by -1.0. Note that when the COC selected is coincident with the kingpin of a wheel, the correct angle for this wheel becomes indeterminate. In this case the control system implements the last determined angle (where angles are given at discrete time intervals allowing for machine calculation). This value could also be described as the limiting value as the COC approaches the kingpin.

Note that even if angle range for the steerable wheels is less than 180 degrees, a pure rotation about the centre of the vehicle is still possible if both the front and rear wheels can be toed in so that they are tangential to a circle passing through all four kingpins, and either the left hand or right hand wheels are rotated in a reverse direction (for CW and ACW turns respectively) [23,24]. In this case the driver can increase the curvature of the path monotonically until "full lock" is reached. If he selects "Pure Rotation" CW or ACW, all wheels will go to the tangential angle and the wheels on one side will rotate in reverse to yield a path curvature of +/- infinity.

In the above derivations equal front and rear tracks have been designated. If unequal tracks are designated, the trigonometry becomes only slightly more complicated [15].

4.1 THE FIRST SPECIAL CASE

If $R_Y = 0$ then $\psi = 0$ and a plane of symmetry will contain the transverse axis of the vehicle. In this case $\phi_1 = \phi_3$, $\phi_2 = \phi_4$, $\omega_1 = \omega_3$ and $\omega_2 = \omega_4$. The advantages of this configuration are as follows;

- As the vehicle can rotate about its own centre the turning circle will be minimised.
- Only two independent wheel angle control systems are required.
- Only two independent wheel speed control systems are required
- A rotatable joystick is not required. A normal joystick will suffice.

Note that the above advantages only strictly apply if slip angles and longitudinal slips are zero.

The main disadvantage of this configuration is that all wheels must be steerable.

If $R_Y = 0$ the eight general control equations become:

$$\tan\phi_1 = (b/2)/(R_X - t/2) = \tan\phi_3 \quad , \qquad \tan\phi_2 = (b/2)/(R_X + t/2) = \tan\phi_4$$

and $\omega_1 = KdR_1 / RMSR = \omega_3$ where $R_1^2 = b^2/4 + (R_X - t/2)^2$

$$\omega_2 = KdR_2 / RMSR = \omega_4 \quad \text{where } R_2^2 = b^2/4 + (R_X + t/2)^2$$

$$\omega_0 = KdR / RMSR \quad \text{where} \quad RMSR = (R_X^2 + b^2/4 + t^2/4)^{1/2}$$

4.2 THE SECOND SPECIAL CASE

In this case the line on which the COC lies coincides with the front or rear “axle”. If R_Y = b/2 the COC will lie on the line of the front axle, whereas if R_Y = -b/2 the COC will lie on the line of the rear axle. Wheels that lie on the line of the COC need not be steerable. This makes it easier to transmit power to these wheels as shaft drives without universal joints can be used. In this case the relative rotation of the joystick is denoted $\theta'' = \theta / \theta_{\max}$, and consequently, R_X is defined as

$$\frac{R_X}{t} = \cot(90^0 \theta'')$$

The main advantages of this configuration are as follows:

- Only two wheels need to be steerable.
- A rotatable joystick is not required. A normal joystick will suffice where the X displacement corresponds to θ''
- Only two wheel angle control systems are required.

The main disadvantages of this configuration are as follows:

- The turning circle of the vehicle will be slightly larger than that of vehicles which can rotate about their own centre. In this case the minimum turning circle occurs when the COC coincides with midpoint between the non steered wheels.
- Four wheel speed control systems will be required.

Since the unsteered wheels lie on the line of the COC, their speeds as given by the above equations may need to be corrected. The general equation for relative wheel speed is:

ω_n /RMSWS = R_n /RMSR, where R_n is the distance between the vertical axis of wheel n and the COC, and is calculated using Pythagoras’s equation. As the computer only returns positive square root values, the calculated ω_n will always be positive. However when the COC passes from outside to inside the vehicle, the wheel through which it has passed should turn in the opposite direction. Although this could be achieved by turning this wheel through 180 as the COC passes through the wheel axis, an alternative solution to the problem will be outlined below. Reversal of rotation of the wheels when appropriate can be achieved by applying the following logic:

If R_Y = +/- 0.5 b some wheels will have constant wheel angles (of zero) and a reversal of rotation will be required. This can be achieved by multiplying the calculated wheel speed by -1.If $R_X^2 > t^2/4$ no action is required. However if $R_X^2 < t^2/4$ then the wheel speed as calculated by the above equations must be reversed for the wheel though which the COC has passed.

The same effect could be achieved by flipping the wheels in question through 180^0. However this course of action would negate the advantage of having non steered wheels.

If $R_Y = +b/2$ the front wheels are unsteered and the eight control equations become:

$$\tan\phi_1 = \tan\phi_2 = 0\ , \quad \tan\phi_3 = b/(R_X - t/2)\ , \quad \tan\phi_4 = b/(R_X + t/2)\ , \quad \tan\phi_0 = b/(2R_X)$$

$$\omega_1 = KdR_1 / RMSR \qquad \text{where } R_1^2 = (R_X - t/2)^2$$

$$\omega_2 = KdR_2 / RMSR \qquad \text{where } R_2^2 = (R_X + t/2)^2$$

$$\omega_3 = KdR_3 / RMSR \qquad \text{where } R_3^2 = b^2 + (R_X - t/2)^2$$

$$\omega_4 = KdR_4 / RMSR \qquad \text{where } R_4^2 = b^2 + (R_X + t/2)^2$$

$$\omega_0 = KdR_0 / RMSR \qquad \text{where } R_0^2 = R_X^2 + b^2/4$$

where $RMSR = (R_X^2 + b^2/2 + t^2/4)^{1/2}$

5 ALTERNATIVE DRIVER INTERFACES

5.1 The General Case:

Although the rotatable joystick depicted in Fig.2 is the preferred driver interface, other driver interfaces are possible. Two alternatives are outlined below.

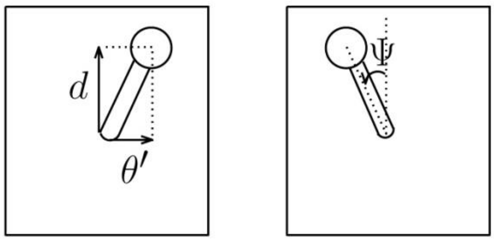


Figure 3: Two joystick interface

Fig. 3 depicts an interface consisting of two joysticks. With the first joystick the X and Y directions control radius of curvature (ROC) and RMSWS respectively according to the equations:

R = t cot ($90^0\,\theta'$) and RMSWS = Kd

With the second joystick the direction of displacement determines ψ. The line on which the COC lies will be at right angles to the joystick displacement.

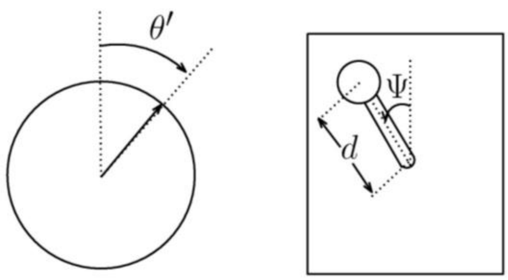


Figure 4: Steering wheel and joystick

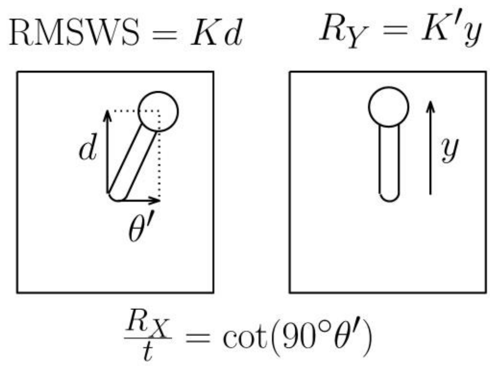


Figure 5: Joystick and detented lever

Fig. 4 depicts an interface consisting of one steering wheel, knob or lever and one joystick. The steering wheel, knob or lever controls the ROC according to the equation:

$R = t \cot (90^0 \theta')$

The direction and amount of displacement of the joystick control ψ and RMSWS respectively.

Fig. 5 depicts an interface consisting of a joystick and a detented lever. With the joystick the X and Y directions control the R_X and RMSWS respectively according to the equations:

$R_X = t \cot (90^0 \theta'')$ and RMSWS = Kd

The lever controls R_Y according to the equation $R_Y = K' y$ where y' is the displacement of the lever and K' is a constant.

5.2 The First Special Case (ψ = 0):

In this case a single joystick will suffice. The X and Y displacements control the ROC and RMSWS respectively (See Fig. 6(a)). Alternatively a steering wheel, knob or lever and one speed control lever or pedal could be used (See Fig. 6(b)).

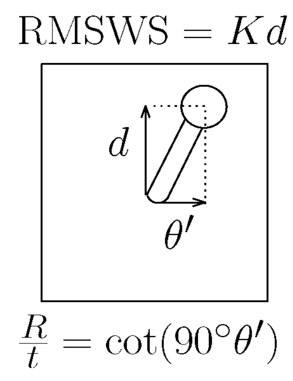


Figure 6(a): Alternative driver interface when ψ = 0

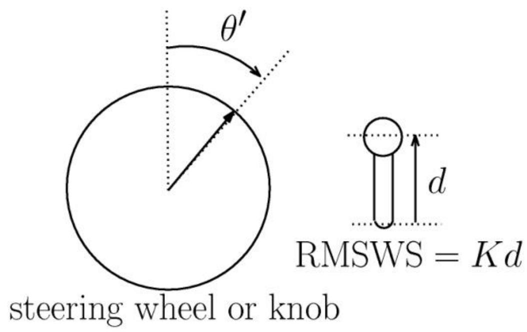


Figure 6(b): Alternative driver interface when ψ = 0

5.3 The Second Special Case (R_Y = +/-b/2):

Once again a single joystick will suffice. The X and Y displacements control the ROC and RMSWS respectively. Alternatively a steering wheel, knob or lever and one speed control lever or pedal could be used. Figures for these driver interfaces would be as shown in Fig 6(a) and (b) with R and θ' replaced by R_X and θ''.

6 RESULTS:

6.1 Simulations of required wheel angles and wheel speeds according to the protocols described in section 7

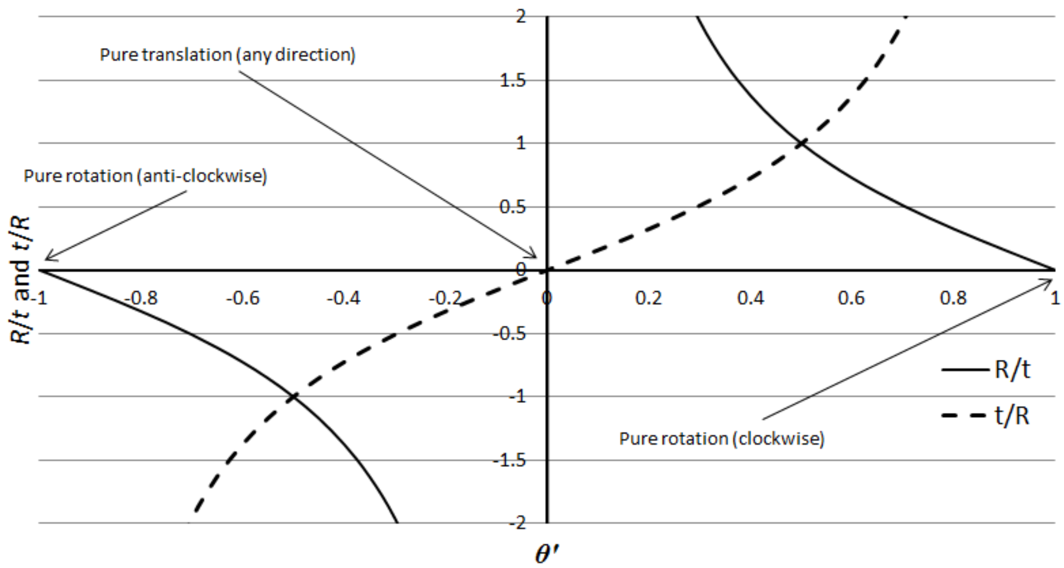


Figure 7(a): Values of R/t and t/R as functions of relative joystick rotation, θ', with b = t and ψ = 0°

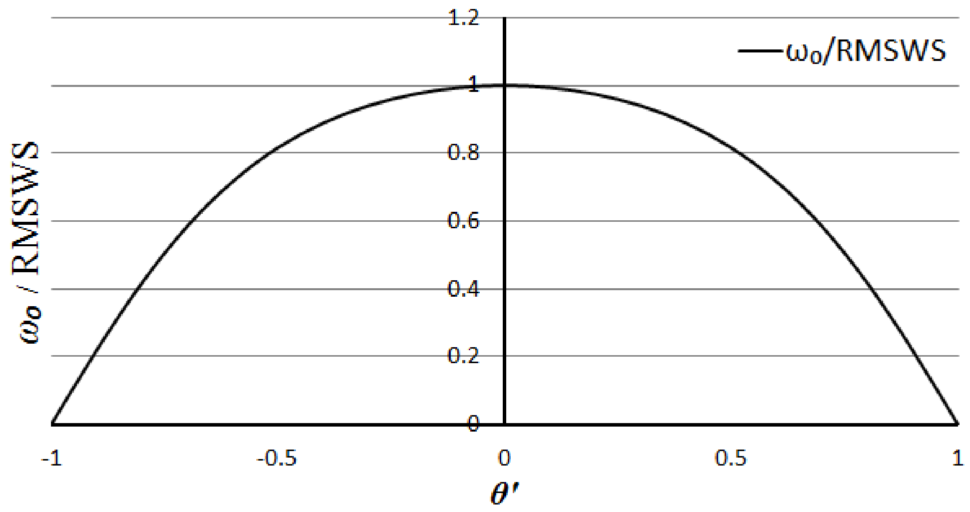


Figure 7(b): Variation of ω_0 as a function of relative joystick rotation, θ', with b = t and ψ = 0°

Fig 7(a) shows the variation of R/t and t/R as a function of the driver input θ' when t = b and the line on which the centre of curvature (COC).lies passes through the centre of the vehicle (COV). Note that t/R is the relative curvature of the path. Its reciprocal R/t is only plotted because t/R approaches +/- infinity (and is therefore hard to plot) when θ' approaches +/-1.0 (i.e. pure rotation CW/ACW).

Fig 7(b) shows the variation of the speed ω_0 of the centre of the vehicle (COV) divided by the RMSWS as a function of the driver input θ' when t = b. The speed of the centre of the vehicle is the speed of a notional castor located at the vehicle centre. Note that ω_0 = 0 when θ' = +/-1.0.

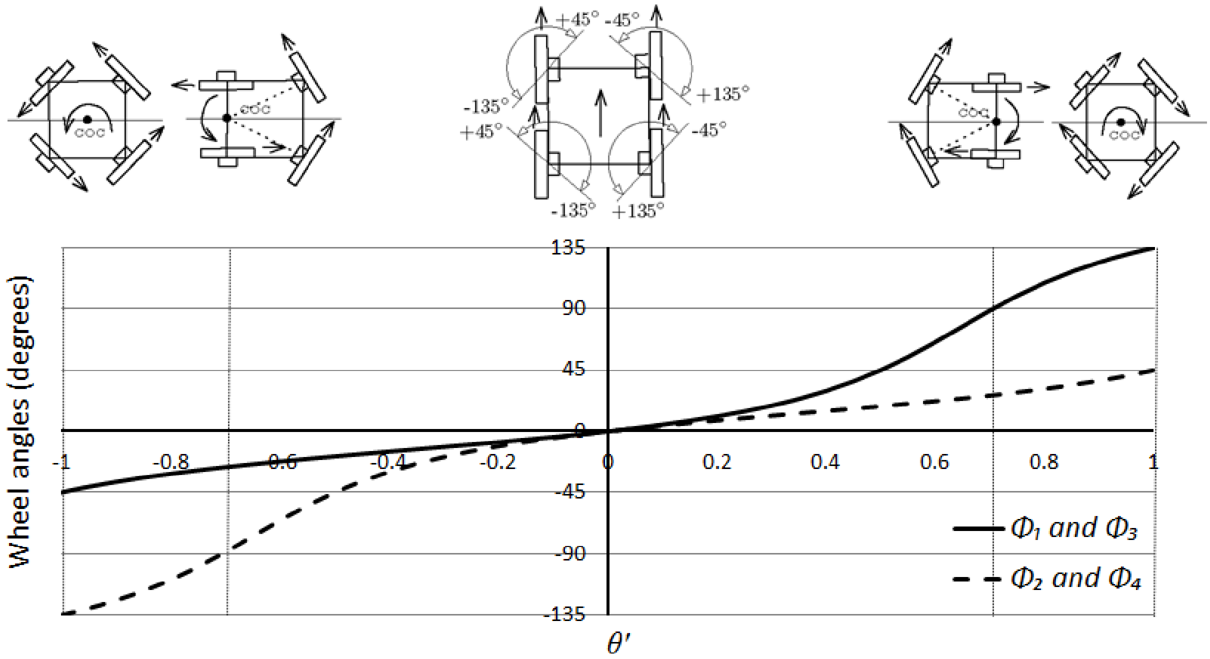


Figure 8(a): Required wheel angles as functions of relative joystick rotation, θ', with b = t and ψ = 0º

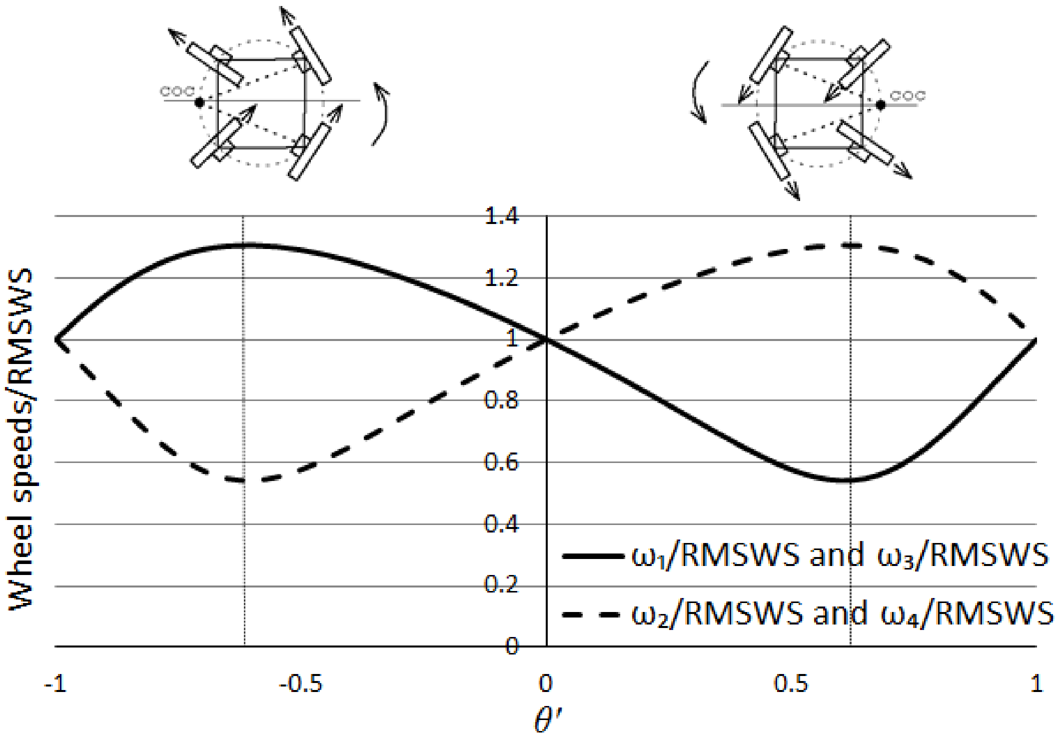


Figure 8(b): Required drive wheel speeds as functions of relative joystick rotation, θ', with t = b and ψ = 0º

Fig. 8(a) shows the four wheel angles required as a function of the driver input θ' when driver input ψ = 0 and t = b. In this case $\Phi_1 = \Phi_3$ and $\Phi_2 = \Phi_4$. The central diagram of Fig 8(a) shows the required range of wheel angles when b = t as being +/- 45^0 and +/- 135^0. Note this range does not change with changes in ψ.

Fig. 8(b) shows the four wheel speeds required divided by the RMSWS as a function of the driver input θ' when driver input ψ= 0 and t = b. In this case $\omega_1 = \omega_3$ and $\omega_2 = \omega_4$. Note that the maximum drive wheel speeds occur when the COC lies on a circle passing through the four kingpins.

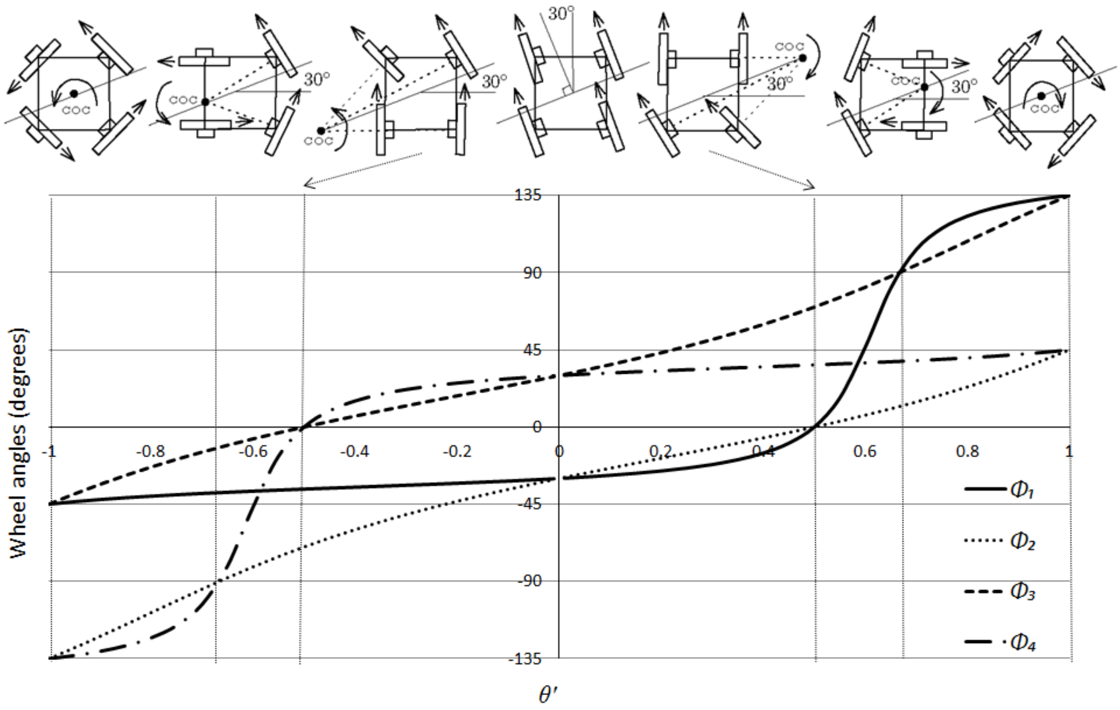


Figure 9(a): Required wheel angles, Φ, as functions of relative joystick rotation, θ', when b = t and ψ = 30°

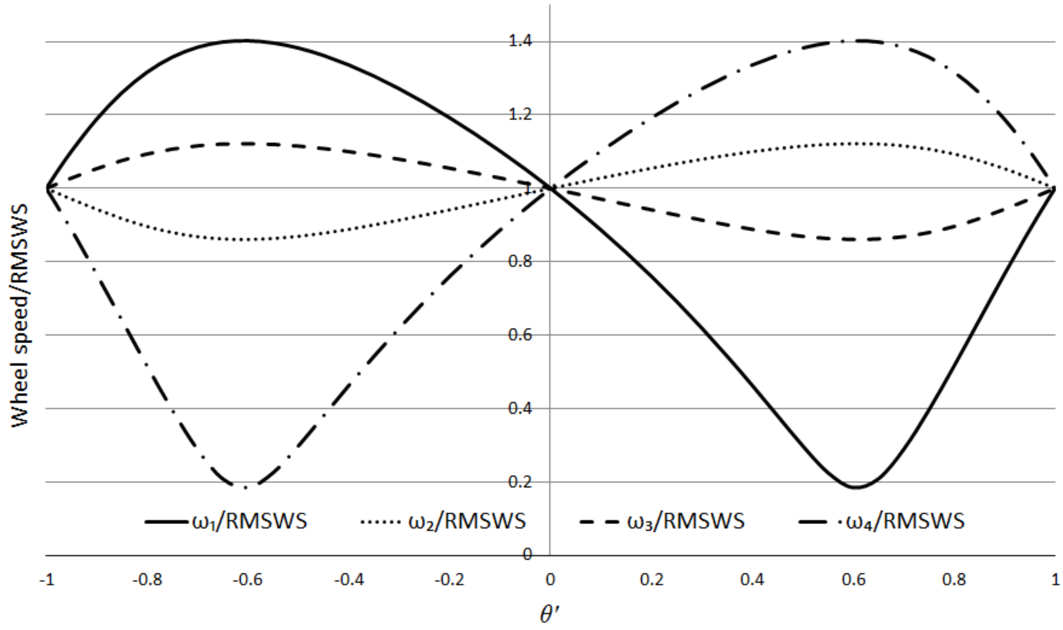


Figure 9(b): Required drive wheel speeds, ω/RMSWS, as functions of relative joystick rotation, θ', when t = b and ψ = 30º

Fig. 9(a) shows the four wheel angles required as a function of the driver input θ' when driver input ψ= 30 and t = b. The limiting wheel angles are unchanged at +/- 45^0 and +/- 135^0.

Fig. 9(b) shows the four wheel speeds required divided by the RMSWS as a function of the driver input θ' when driver input ψ= 30 and t = b. Note that the speed variation of the two wheels closer to the line on which centre of curvature lies is much greater than for the other two wheels.

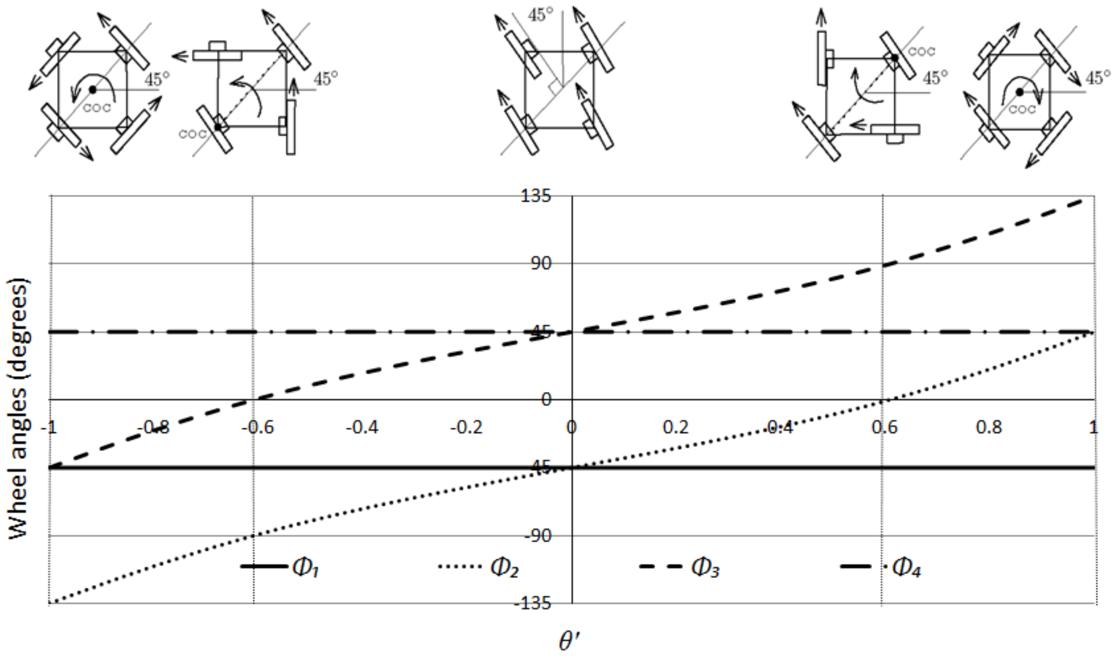


Figure 10(a): Required wheel angles, Φ, as functions of relative joystick rotation, θ', when t = b and ψ = 45º

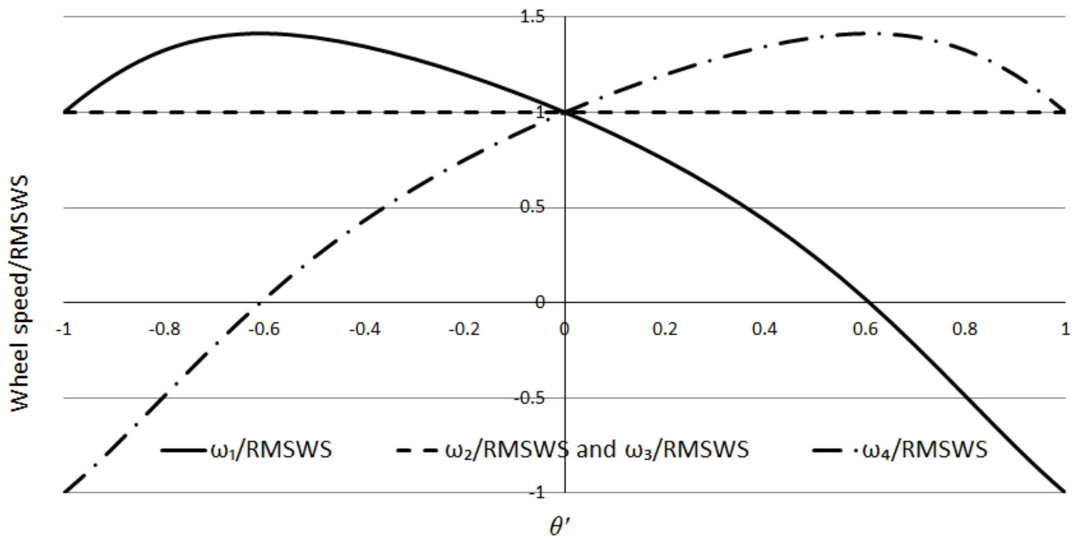


Figure 10(b): Required drive wheel speeds, ω/RMSWS, as functions of relative joystick rotation, θ', when t = b and ψ = 45º

Fig. 10(a) shows the four wheel angles required as a function of the driver input θ′ when driver input ψ= 45 and t = b. In this case TAN ψ = +t/b so that the line on which the centre of curvature lies passes through the vertical axes of the front left wheel and the rear right wheel. The angles of these wheels remain unchanged at +45^0 and -45^0 respectively.

Fig. 10(b) shows the four wheel speeds required divided by the RMSWS as a function of the driver input θ' when driver input ψ= 45 and t = b. Note that the speed variation of the two wheels which lie on the line on which centre of curvature lies is much greater than for the other two wheels (where the speed remains constant). Also note that the speed of the former wheels becomes negative when the centre of curvature passes from inside to outside the vehicle.

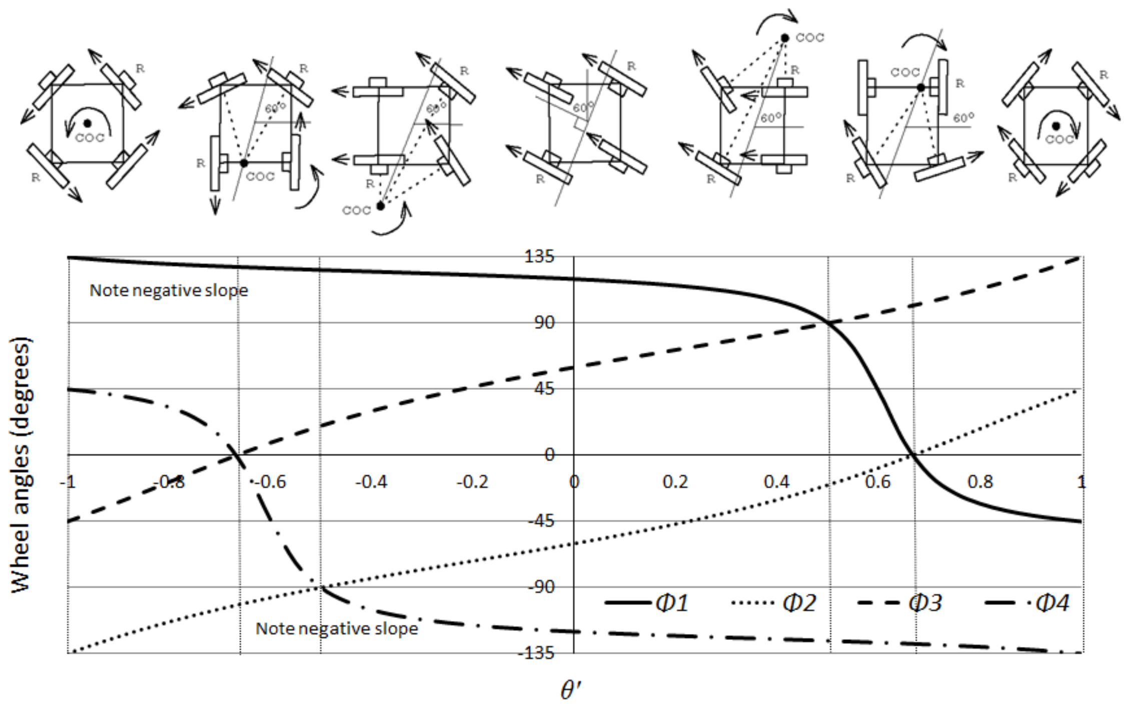


Figure 11(a): Required wheel angles, Φ, as functions of relative joystick rotation, θ', when t = b and ψ = 60°

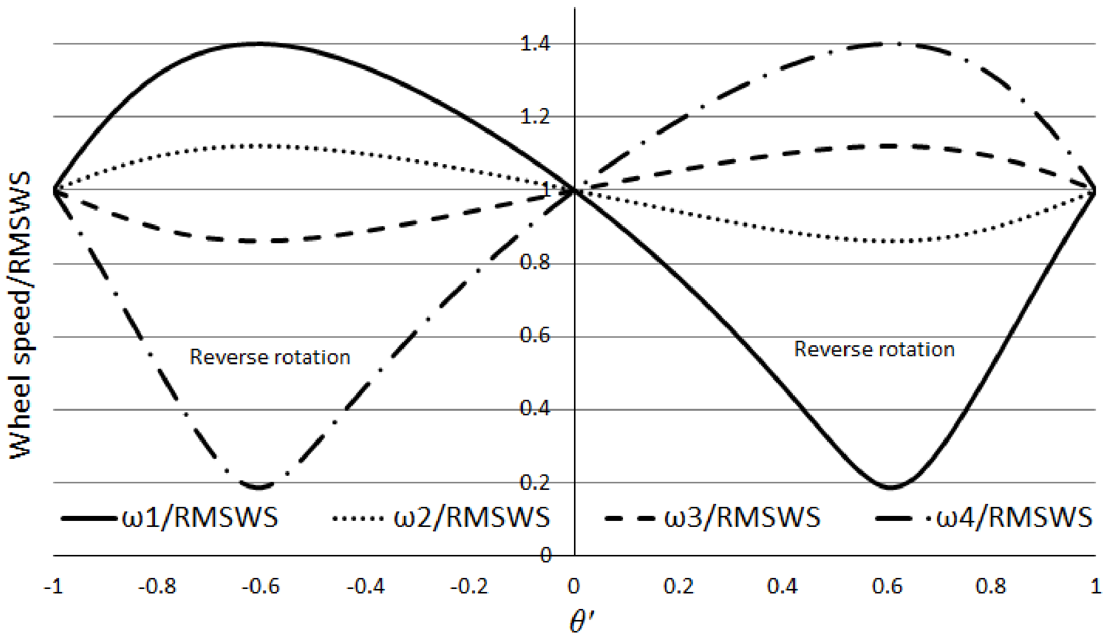


Figure 11(b): Required drive wheel speeds, ω/RMSWS, as functions of relative joystick rotation, θ', when t = b and ψ = 60°

Fig.11(a) shows the four wheel angles required as a function of the driver input θ′ when driver input ψ= 60 and t = b. Since tan 60^0 > b/t the rotation of two drive wheels must be reversed. Since $\phi_{\theta=0.99}-\phi_{\theta=-0.99}<0$ for wheels 1 and 4 these wheels must be rotated in the opposite direction.

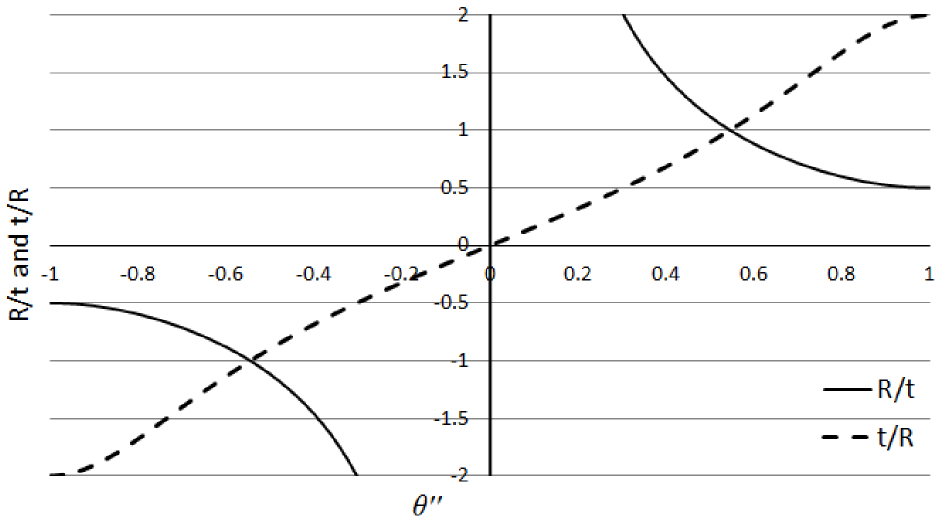


Figure 12(a): Values of R/t and t/R as functions of relative joystick rotation, θ'', with t = b and R_Y = b/2

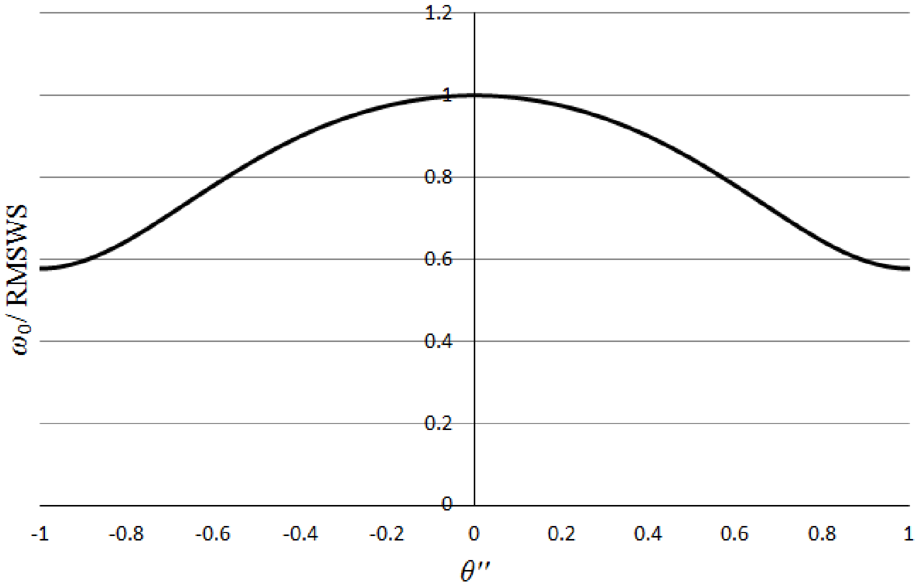


Figure 12(b): Variation in ω_0 as a function of relative joystick rotation, θ'', with t = b and R_Y = b/2

Figures 12 and 13 relate to the special case where the line on which the centre of curvature lies passes through the vertical axes of the front wheels. In other words the line of the COC coincides with the front "axle" of the vehicle.

Fig 12(a) shows the variation of R/t and t/R as a functions of the driver input, θ'', where

R_X /t = cot(90 θ''), R_Y = b/2 and b = t. Note that R/t does not become zero when θ'' = +/-1.0.

Fig 12(b) shows the variation of the speed ω_0 of the centre of the vehicle (COV) divided by the RMSWS as a function of the driver input θ'' when R_Y = b/2 and b = t . The speed of the centre of the vehicle is the speed of a notional castor located at the vehicle centre. Note that ω_0 does not decrease to zero when θ' = +/-1.0.

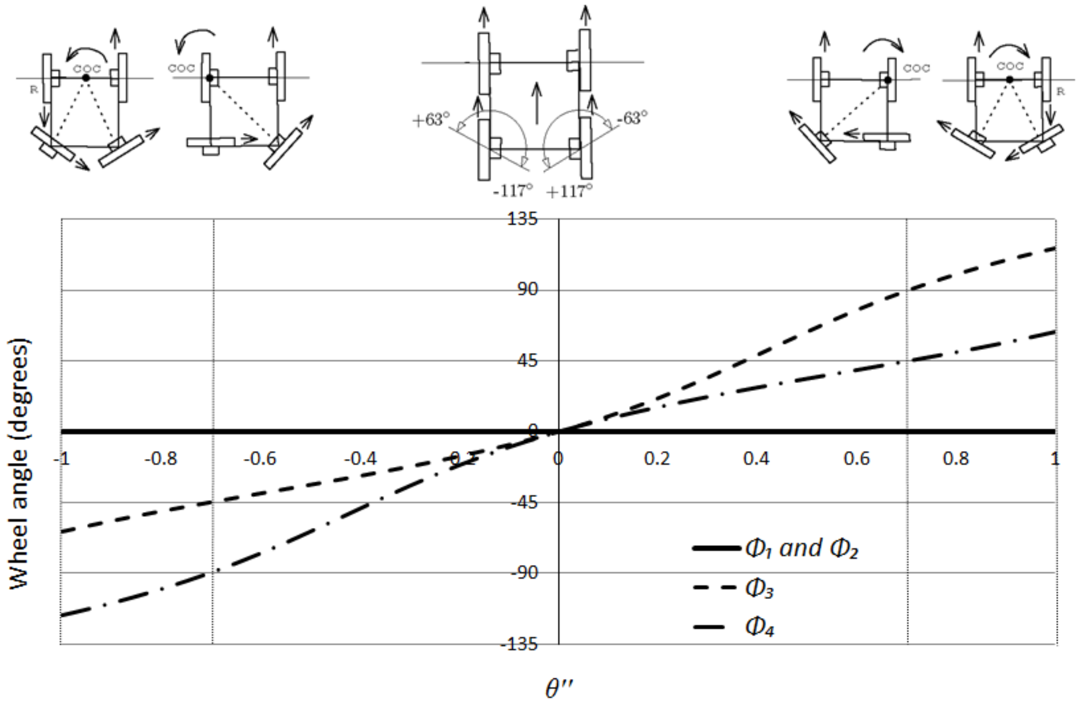


Figure 13(a): Required wheel angles as functions of relative joystick rotation, θ'', with t = b and R_Y = b/2

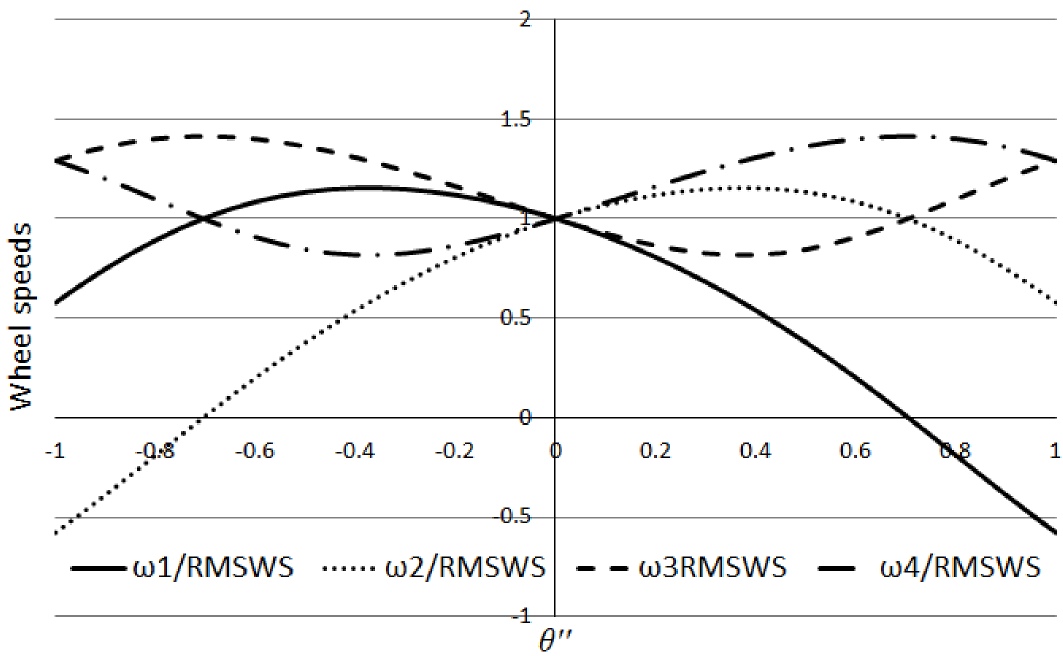


Figure 13(b): Required drive wheel speeds, ω/RMSWS, as functions of relative joystick rotation, θ'', with t = b and Ry = b/2

Fig.13(a) shows the four wheel angles required as a function of the driver input θ'' when R_Y = b/2 and b = t. In this case the angle of the front wheels are always zero. The central diagram of Fig 13(a) shows the required range of rear wheel angles, when b=t, as being +/- 63^0 and +/- 117^0.

Fig 13(b) shows the four wheel speeds required divided by the RMSWS as a function of the driver input θ'' when R_Y = b/2 and b = t . Note that the speed variation of the front wheels is much greater than for the rear wheels. Also note that the wheel speed of the left and right wheels becomes negative as θ' approaches -1 and +1 respectively.

7 PROTOCOL FOR DETERMINING THE SET POINTS FOR THE CONTROL SYSTEMS

7.1 General Case

The following preliminary calculations are executed:

1. From the values of wheel base b and track t for the vehicle b/t is calculated.
2. From b/t, the 180^0 angle range for each wheel is calculated.

In operation the driver selects ψ, θ and RMSWS via the driver interface.

1. From θ, R/t is calculated. Note that if θ' is negative then R will be negative.
2. From ψ, R_x/t and R_y/t are calculated
3. Calculate raw wheel angles
4. Correct angles outside allowable range by adding or subtracting 180^0
5. Calculate raw wheel speeds from RMSWS. Note that if RMSWS is negative all wheel speeds must be multiplied by -1.
6. If tan ψ =b/t the angle of two wheels will remain constant at +/- ψ. These wheels must be rotated in the reverse direction if $0 < 4R < (b^2 + t^2)^{0.5}$ or $0 > 4R > -(b^2 + t^2)^{0.5}$

7. If $\Phi_{\theta' = 0.99} - \Phi_{\theta' = -0.99} < 0$ the related wheels must be rotated in the opposite direction.
8. The control system implements the required wheel angles and wheel speeds.

7.2 Second Special Case when front wheels are unsteered (i.e. $R_Y = b/2$)

The following preliminary calculations are executed:

1. From the values of wheel base b and track t for the vehicle b/t is calculated.
2. From b/t, the 180^0 angle range for each steerable (rear) wheel is calculated.

In operation the driver selects θ'' and RMSWS via the driver interface.

1. From θ'', R_X/t is calculated
2. Calculate raw wheel angles for the steerable (rear) wheels.
3. Correct angles outside allowable range by adding or subtracting 180^0
4. Calculate raw wheel speeds from RMSWS. Note that if RMSWS is negative all wheel speeds must be multiplied by -1.
5. If $0 < R_X < t/2$ then the right front wheel must be rotated in the opposite direction. If $0 > R_X > -t/2$ then the left front wheel must be rotated in the opposite direction.
6. The control system implements the required wheel angles and wheel speeds.

8 APPLICATION OF THE COOPERATIVE REDUNDANT MULTIPLE STEERING SYSTEM

The vehicles described above could be used to best advantage to replace current vehicles operating near the limits of traction. In general it is not feasible to retrofit CRMSS to existing vehicles. However CRMSS vehicles could be constructed by fitting off the shelf components to a purpose designed chassis. The four most promising applications are vehicles to replace industrial mowers, skid steer loaders, all terrain forklifts and articulated loaders. However they could also be used to replace existing forestry, agricultural, military, mining and recreational all-terrain vehicles. Application to road vehicles would offer minimal benefits since these vehicles rarely operate near the limit of traction and very tight turns, or crab steering are rarely required.

Note that the CRMSS represents the pinnacle of "X by Wire" systems. Consequently the computer architecture employed must be at least "fail safe" and preferably "fail operating".

To reiterate, vehicles with cooperative redundant multiple steering systems promise safety benefits relative to vehicles with a single non-redundant steering system and environmental benefits relative to vehicles with a conflicting redundant multiple steering system. The safety benefits are increased traction, stability and manoeuvrability (especially on hills) and the environmental benefits are reduced ground damage, tyre wear and fuel wastage on turning.

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9 CONCLUSIONS

This paper presents the results of calculations of wheel angles and drive wheel speeds that ensure that the steering effect of the wheel angles is identical to the steering effect caused by the speeds of the drive wheels.

These calculations include the general case where the centre of curvature of the path of the centre of the vehicle can lie anywhere in the operating plane, including within the plan view of the vehicle. These minimal turning circles require wheel angle ranges of at least 180^0. Undesirable flipping of the wheels when a wheel angle reaches +/- 90^0 (due to the ambiguity of the arctan function) is prevented.

Large differences in drive wheel speeds are also required including cases where some wheel speeds must be negative. These negative wheel speeds are not obvious due to the ambiguity of the sign of the square root function, and must be logically deduced.

The general case of the novel vehicle described above is capable of both crab steering in any direction and pure rotation about the centre of the vehicle and any motion in between these extremes. That is, the driver can select a COC anywhere in the operating plane.

REFERENCES

[1] Boskovic, J.D.and Mehra, R.K. “A multiple model-based decentralised system for accommodation of failures in second-order flight control actuators”, Proceedings of the American Control Conference, pp 4436-4441,Minneapolis, USA, 2006

[2] Boskovic,J.D, Ling,B, Prasanth,R..and Mehra, R.K. “Design of control allocation algorithms for overactuated aircraft under constraints using lmis”, Proc 41st IEEE Conference on Decision and Control, pp 1711-17816. IEEE, 2002

[3] Fossen, T. and Johansen. T.A., “a survey of control allocation methods for ships and underwater vehicles”, Proc. 14th Mediterranean Conference on Control and Automation, Ancona, June 2006

[4] Johansen. T.A., Fossen, T. and Berge, S.P., “Constrained nonlinear control allocation with singularity avoidance using sequentialquadratic programming”, IEEE Trans on Control Systems Technology, vol12, pp 211-216, 2004

[5] Vermillion, C.Sun, J. and Butts, K., “Model predictive control allocation for overactuated systems- stability and performance”, Proc 46th Conference on Decision and Control, pp 1251-1256, New Orleans, USA, December 2007

[6] Muller, A., “Internal preload control of redundantly actuated parallel manipulators –its application to backlash avoiding control”, IEEE Trans on Robotics, vol 21, pp 668-677, August 2005

[7] Wong, J.Y. “Optimization of the tractive performance of four-wheel-drive off-road vehicles.” SAE Trans 1970; **79** 2238

[8] Wong, J.Y., McLaughlin, N.B., Knezevic, Z. and Burtt, S. “Optimization of the tractive performance of four-wheel-drive tractors – Theoretical analysis and experimental substantiation.” Proc of the Institution of Mechanical Engineers, Part D 1998, Journal of Automobile Engineering **212**: 285

[9] Wong, J.Y., Zhao, Z, Li,J., McLaughlin, N.B. and Burtt, S. “Optimization of the tractive performance of four-wheel-drive tractors – correlation between analytical predictions and experimental data.” SAE Technical Paper series, 2000-01, 2596

[10] Besselink, B.C. “Tractive efficiency of four-wheel-drive vehicles: An analysis for non-uniform traction conditions.” Proc of the Institution of Mechanical Engineers Part D: Journal of Automobile Engineering **217** (5): 363

[11] Vantsevich, V.V. “Multi-wheel drive vehicle energy/fuel efficiency and tractive performance: Objective function analysis.” Journal of Terramechanics **44:** 239

[12] Dequesne, F.,Kermis, L. and Verchoore, R. "Influence of differential locking on tractor work rate: Part 2, Simulationof a two-wheel-drive tractor on turning." Journal of Agricultural Engineering Research, **64**, 79 (1996)

[13] Forbes, C. and Lewis, R. "Computer-integrated steering/drive system." Technical Report, Dept of Mechanical and Manufacturing Engineering, University of Melbourne (2004)

[14] Spark, I.J. and Besselink, B.C. "Zero turn radius vehicle incorporating computer controlled intelligent castors." ASME Computers in Engineering Conference, Minneapolis , USA (1994)

[15] Besselink, B.C., "Development of a vehicle to study the tractive performance of integrated steering-drive systems." Journal of Terramechanics, **41**, 187, (2004)

[16] Blair, D.B. and Spark, I.J., "Computer integrated steering/drive systems for vehicles." ASME Computers in Engineering Conference, Irvine, California, (1996)

[17] Lu, J., Spark, I.J., Vains, G.G. and Spriggs, K.R. "Computer integrated steering/drive systems with steering wheel control." IE Australia Conference, Sydney, Australia (1997)

[18] Spark, I.J. and Ibrahim, M.Y. "Integrated mechatronics solution to maximise tractability and efficiency of wheeled vehicles." IEEE/ASME International Conference on Advanced Intelligent Mechatronics, Como, Italy, 320, (2001)

[19] Spark I, US Patent 7,191,865, issued 20 Mar 2007

[20] Spark I, Australian Patent 2001293507, granted 17-6-2004

[21] Spark, I.J. and Ibrahim, M.Y. “Cooperative redundant steering/drive system: Mechatronic correction for slip angles and longitudinal slip.” Proceedings of IEEE International Conference on Mechatronics, vol 1, pp. 451-456, Budapest, Hungary, July 2006, ISBN: 1-4244-9713-4.

[22] Spark, I.J. and Ibrahim, M.Y. “Slip angles and longitudinal slip measurement of the cooperative redundant steering/drive system.” Proceedings of IEEE International Symposium on Industrial Electronics, Montreal, Canada, July, 2006, IEEE/USA, ISBN 1-1422-0497-5.

[23] Spark I J, US Patent 7,464,785, issued 16 Dec 2008

[24] Spark I J, Australian Patent 2003201206, granted 23-3-2006

[25] Spark, I.J. and Ibrahim, M.Y. “Manoeuvrable gantry tractor comprising a “chorus line” of synchronised modules”, Proceedings of IEEE Symposium of Industrial Electronics, pp 2208-2213, Vigo, Spain, June 2007. IEEE, ISBN 1-4244-0755-9.

[26] Percy, A., Spark, I.J. and Ibrahim, M.Y. “On-line determination of wheel angles and speeds for manoeuvrable gantry tractor comprising a “chorus line” of synchronised modules.” IEEE-ICIT 09 International Conference on Industrial Technology, pp 320-325, Churchill, Australia, Feb 2009.

[27] Ibrahim, M.Y., Spark, I.J. and Percy, A. “New control concept for a gantry tractor comprising a “chorus line” of synchronized modules.” IEEE Transactions on Industrial Electronics, vol 57, number 2, pp 762-768, Feb 2010.

[28] Percy A, Spark I, Ibrahim Y and Hardy L, A numerical control algorithm for navigation of an operator driven snake-like robot with 4WD-4WS segments', *Robotica,2011, 29, 471-482*

[29] Percy A and Spark I, A numerical control algorithm for a B Double truck trailer system with steerable wheels and active hitch angles. *Proc I Mech E. Part D: J Automobile Engineering 2012; 289-300*

[30] Percy A and Spark I, A numerical control algorithm for a B Double truck trailer system with steerable wheels and active hitch angles. Part 2: Reversing. *Proc I Mech E. Part D: J Automobile Engineering 2013; 899-904*

[31] Spark I, US Patent 7,857,085, issued 28 December 2010.