Teaching with CAS in a Time of Transition

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Abstract

Integrating a powerful instrument such as CAS into teaching and learning mathematics requires changes to many aspects of the classroom, which teachers will make from the base of their prior teaching styles and their beliefs about mathematics and how it should be taught. The paper describes the different ways in which two pioneering Australian teachers adapted their teaching to use CAS. One teacher used CAS with the primary goal of increasing understanding but restricted students' use. The other teacher adopted CAS as an extra technique for solving standard problems, emphasising timesaving routines by hand and with CAS. Through these case studies we comment on the following issues related to teaching with CAS: different ways of organising the classroom, variety in approaches to teaching the use of CAS, the increased range of methods for solving problems and for teaching, the contrast between using of graphics calculators and CAS, the challenge of finding the place of by-hand skills and CAS use, and the curriculum and assessment changes required in schools.

Introduction

The technological tool, CAS, with its powerful symbolic, graphical and numerical capabilities is becoming increasingly available to students of mathematics. This paper describes the experiences of some of the teachers who are pioneering the use of CAS in Australia. Some of the findings and constraints are universal while others reflect their individual context. As is the case in many countries, teaching mathematics in the senior secondary years in Australia is largely determined by the external examinations that the students undertake at the end of school. A research project (Stacey, McCrae, Chick, Asp & Leigh-Lancaster 2000; http://www.edfac.unimelb.edu.au/DSME/CAS-CAT) is investigating the changes that regular access to CAS may have on the formal curriculum of the senior mathematics subjects and the questions asked on the examinations. However, the teachers whose work is described in this paper were operating earlier in an unchanged external environment.

Looking through the windows of two classrooms

This paper begins with a look through the windows of their classrooms, where the teaching and learning have been carefully documented. The paper highlights the ways in which the teachers have changed to accommodate CAS in their classrooms and the benefits and challenges that it has brought. Benoit and Andre were volunteers participating in a research project of the University of Melbourne in 1998 and 1999. During the CAS Calculus project, they taught their Year 11 students introductory calculus for eight weeks using CAS calculators (TI-92's). Neither had had experience with teaching with CAS before, although both teachers and students used graphics calculators routinely in all their other work. Together, the research team and the teachers planned the lessons, aiming to primarily to develop students' conceptual foundation for differentiation, especially by use of multiple representations linking graphs, symbols and tables of function data. Designing a course where understanding could precede procedures, and hence receive the major emphasis, was the prime intention. Studies by Heid (1988), Palmiter (1991), and Repo (1994) showed that this was feasible. In 1999, the study was repeated and Andre and Benoit participated again, with their new classes in the same school. Results of the 1998 study are fully reported including a description of each teacher's pedagogy and how each teacher's privileging impacted on student learning outcomes (Kendal & Stacey, 1999; McCrae, Asp, & Kendal, 1999). The changes that occurred in the teachers' pedagogy in the second year, showing gradual evolution, are reported in Kendal and Stacey (2000; in press). The teachers' experiences serve to pinpoint a wide range of issues that emerge when teachers begin teaching with CAS technology.

Benoit: teaching for understanding

Benoit was a very experienced teacher and Head of the Mathematics Department in his school. He was very interested in teaching with the graphics calculator (officially endorsed for all forms of assessment including state-wide examinations) and he actively encouraged other teachers to use it in their classes. One indicator of his interest and expertise is that he collected a wide range of programs and then downloaded them onto the students' calculators for their use in class and in examinations.

Benoit was especially keen to use CAS calculators to give students a better conceptual understanding of calculus. With and without technology, he emphasized understanding the concepts being taught. He frequently used enactive representations (e.g., making purposeful hand and arm movements in the air) and visualization techniques to explain symbolic ideas. He constantly linked the symbolic derivative to gradient of the tangent to the curve (represented by his outstretched arms). He also used real world phenomena to explain mathematical ideas. For example, after discussion with the research team, he explained the rule for finding the derivative of a sum of two functions by considering the speed of a person running on a moving platform.

Benoit's teaching style was based around discussion with students. He involved every student in the class by challenging them to explain their ideas and encouraged them to construct meaning for mathematical ideas through conjecture, analysis, and discussion with others. Benoit moved around the classroom and checked individual students' work as they solved problems from worksheets (or textbook) to work on in class and complete at home. He responded to common problems by initiating further class discussion. His blackboard notes incorporated key aspects of the class discussion.

Benoit also used orchestrated discussion to teach his students to use the calculators. He did not use an overhead projector or any other special classroom arrangement to demonstrate CAS procedures. Instead, he would say what he was doing on his calculator, slowly enough so that all the students could follow on their own machines. Benoit would wait until the whole group reached each stage and walk around the classroom helping students who were in trouble. Because of his expertise in group management, all the students participated. He only used an overhead projector for special demonstrations such as a dynamic experiment involving the collection of real data by a data-logging device attached to the calculator.

Benoit's concern for understanding the concept of differentiation was displayed in his strong emphasis on the links between functions and their graphs and between differentiation and the slope of tangents to curves. To Benoit, knowing these links was the essential aspect of understanding differentiation. Benoit had embraced graphics calculators wholeheartedly because they had provided excellent technological support for this. Our testing showed that this aspect of his teaching was reflected in strong class results: most of his students were able to interpret a derivative in terms of the slope of a tangent or as a rate of change. He therefore used the graphical facility of the CAS calculators constantly.

On the other hand, Benoit's concern for understanding led to him restricting the use of the symbolic facility of the CAS calculator. He strongly believed that doing algebra by hand was extremely important for understanding and that if he allowed his students to do algebra with CAS he would be depriving them of an opportunity to understand. He stated in an interview that there are ?*certain [algebraic] skills that kids have to have, even if you can use the technology. . . .They've still got to have hands-on; they've still got to get pen and paper skills*?.

The main use he made of the symbolic facility was to perform repetitive routine tasks quickly as a preliminary step to developing understanding concepts through exploration, investigation and induction. ?*Potentially, it enables you to do a bit more investigation, in terms of looking at more complex functions . . . It's good for discovery, and it's a lot easier in terms of discovering [mathematical properties] because it takes a lot of hack work out of [it]*?. Benoit reported, for example, that the class had constructed tables of derivative values of polynomial functions and deduced the rules for the differentiation of x^n, ax^n and sums of these. He commented: ?*I think we've done very nicely with the calculator. One thing I like is the routine procedures. You haven't got all that time wasting. You can do very nicely a lot of the algebra. You can do it so simply on the calculator and you're avoiding in some ways the time that goes by when you're doing a lot of repetitive calculations.?* Beyond uses such as this, however, Benoit carefully controlled how students used their calculators.

In the second year, Benoit assessed his class as being less mathematically able. He further reduced student use of CAS for symbolic differentiation and increased his emphasis on by-hand algebra and differentiation calculations. He also omitted all work with the numerical, tabular representation of differentiation, since he believed that this particular class would be confused by the third representation. However, he continued with his strong use of the graphical representation.

Benoit's concern about allowing students to use the symbolic facility arose from a mix of concerns. Firstly he believed that performing algebra step by step and by hand contributes to ?understanding?. Secondly he was acutely aware that his students would not be able to use CAS in their future school examinations. Amongst the graphics calculator programs that he provided for the examinations was one that factorised quadratic expressions. This indicates that it is perhaps not the by-hand algebra skills themselves that Benoit valued, but students' performance in the examination.

Andre: teaching for performance

Andre, Benoit's colleague and also an experienced teacher, taught the second class. Unlike Benoit, Andre did not enjoy teaching with the graphics calculator and recalled his previous experience in an interview ?*Actually, I tried to bring in the [graphics calculator] but I had real trouble with it. I thought 'I just can't be bothered' and I haven't [used it in class] since. I didn't feel comfortable with the [graphics calculator] because I had so many problems.*? In contrast, Andre enjoyed using the symbolic capabilities of the CAS calculator himself and also in class. As the project progressed he began to use it constantly for demonstrating procedures while teaching. He stated: ?*I hooked it up at the beginning of the lesson and I used it much more than I would use a graphics calculator in the classroom. . . I was in the mode of always having it there and having it set up so that the overhead projector was there and I just slipped on the screen, hooked it up and it was there. . . It was on all the time and I felt comfortable*.?

Andre's normal teaching style was to lecture, emphasizing rules and procedures, with few teacher-student or student-student interactions. The students mostly worked alone but occasionally consulted with their neighbours, using the same worksheets (or textbook problems) as Benoit's class. Andre used an overhead projector to demonstrate how to use the CAS calculator to achieve given results. His students copied down two sets of notes as he wrote them up on the blackboard: first, the by-hand procedures, and second, the corresponding set of step-by step CAS calculator (TI-92) procedures. An example of Andre's blackboard notes is given in Figure 1. Andre also liked compressed use of calculator to carry out routine procedures in a minimum of steps such as the one line command *solve* ($d(x(32 – 2x), x) = 0, x$) which differentiates the expression $x(32 – 2x)$ and finds the value of x where the derivative is zero. In contrast, Benoit gave much less emphasis to the procedures for using the calculator itself, and he generally managed to teach students how to carry out the procedures by demonstrating them to the whole class and then assisting individuals.

The number of bacteria (N millions) present in a culture at time t (in seconds) is

$$N = 10t^3 – 10t + 40$$

Find the rate of growth of the number of bacteria after 5 seconds.

By hand:	With TI-92
$N = 10t^3 – 10t + 40$	[2nd 8]
$\frac{dN}{dt} = 30t^2 – 10$	$d(10x^3 – 10x + 40, x)$
$\frac{dN}{dt} = 30(5)^2 – 10$	[2nd K]
$= 750 – 10 = 740$ million	$d(10x^3 – 10x + 40, x)/x = 5$
	[E]
	$= 740$

Figure 1. Andre's blackboard notes showing both by-hand and TI-92 (CAS) procedures to solve a rate problem.

The two teachers had very different attitudes to student use of the CAS. Benoit controlled the use that students made of the CAS calculator, especially its symbolic facility, suggesting when students should and should not use it. In contrast, Andre gave students complete freedom about when they could use their calculators.

Whereas Benoit privileged the symbolic and graphical and representations of derivative and the link between them, Andre had a strong preference for the symbolic representation. This seemed to be because it led to exact answers in two senses: it is able to give answers as surds and rational numbers etc rather than only as a decimal but also there is no error of measurement from symbolic methods such as that which could come from reading a graph or interpolating in a table. In an interview, when asked to discuss alternative methods of finding the gradient of the curve at a point given the function and its graph, he indicated that his preference was the accurate method to differentiate and substitute: ?*Oh well, [differentiation and substitution] is accurate. An approximate method . . . would be actually drawing a tangent at x = 1 and then working out the gradient. But I don't think the girls would ever attempt that because they hate anything where they have to guess or where the answer might be really different. But that doesn't necessarily mean that they would use the exact method like mine*.?

During the trial, Andre's skills in teaching with technology grew. He increased his use of the CAS calculator including ?new? graphical and numerical differentiation procedures with CAS. Since he really liked using the CAS calculator, he also experimented with the data logger and several times enthusiastically demonstrated (using projection) a dynamic program that linked the numerical and graphical representations of derivative. Both of these were new initiatives for Andre, reflecting new confidence.

Teaching in a time of transition

Our teachers are pioneers, working in a time of transition from old to new ways of doing and teaching mathematics. Teachers need to support students' learning of both the technology and the mathematics, whilst common mathematical practice is itself is changing. In this section we draw together insights from what the teachers have done and the issues they have faced.

More options for solving problems: more options for teaching

The advent of new technological tools such as CAS is accompanied by an increased number of ways of solving mathematical problems. Methods that in the past were extremely tedious and so were only available *in principle* are now available *in practice*. So, for example, it has always been possible *in principle* to solve equations by graphing (with standard cautions about using mathematical analysis to know the number of solutions expected and in what general regions they might be located). However, solving equations graphically used to be a method of last resort, when other methods failed, not a method of choice. With advanced graphing capabilities on calculators and computers, this is no longer the case: solving equations graphically can be quick and easy. A second example of the different status of mathematical methods with improved calculation is from differentiation. Before scientific calculators, differentiation of the square root function might have been used to estimate $\sqrt{100.2}$ rather than calculate it numerically. This could be done by using the approximation of $f(x) + f'(x).h$ to $f'(x+h)$ with $x = 100$ and $h = 0.2$ to quickly see that $\sqrt{100.2}$ is approximately 10 + 0.2/(20) i.e. 10.01. Now we have the reverse situation: the calculation $\frac{\sqrt{100.2} ? \sqrt{100}}{0.2} ? 0.049975$ can be used to quickly find the derivative (0.05) of the square root function at 100. Alternatively, the function can be graphed and zooming in or automatically drawing the tangent gives the derivative. The United Kingdom's National Council for Educational Technology acknowledged this explosion of feasible methods.

> For any one problem there may now be a range of methods of solution. Typically, there may be numerical and graphical approaches as well as algebraic and analytic approaches. Indeed there may be a variety of algebraic approaches. Hence it is more likely that a problem will be tackled with a view to comparing and contrasting different methods, with each solution possibly giving rise to some new mathematics. (NCET Report, 1994)

The growth in options for solving problems with new technologies is accompanied by a growth in options for teaching. A consequence of having a greater choice of methods was that Andre and Benoit taught different ways of solving differentiation problems, even though the lessons were planned together. Benoit taught his students to work primarily from the symbolic derivative, calculated by hand, and interpreted as the gradient of the tangent to a curve. Andre's students had a wider range of methods for calculating derivatives (at a point) since he taught them to differentiate symbolically or calculate a difference quotient from a table of values or get the calculator to draw a tangent to the graph and then write down its gradient. We expect that this explosion in methods will be the norm – we do not think that differentiation is special in this regard.

Andre and Benoit developed teaching practices that fitted with their beliefs about mathematics and their previous styles of teaching mathematics. We use the term ?privileging? to describe the emphases that they made. The two teachers had very different conceptions of mathematics and their teaching styles, use of representations (numerical, graphical or symbolic), and use of technology were distinctly different. In consequence, although the two classes had similar overall achievement, the students learnt rather different mathematics. See Kendal and Stacey (in press) for a more detailed analysis.

CAS enabled Andre to extend his teaching and his students' skills with a new set of routine procedures of using CAS, and the use matched his usual lecture/demonstration style of teaching, for teaching rules. He really appreciated the symbolic capability of CAS and enjoyed using it. Benoit privileged pedagogical use of CAS. He saw this pedagogical use residing in two possibilities to increase understanding. Firstly, he believed that linking the symbolic and graphical representations of a function (or a derivative function) was a key to understanding. Secondly, the CAS enabled students readily to collect data for class discussions during which students would induce the rules for differentiation etc. Beyond use of this nature, the symbolic algebra facility was of little interest to Benoit.

Preferences for a graphics or CAS calculator

Interesting differences were observed between the ways the teachers taught with a graphics calculator and with a calculator with a symbolic algebra facility. Andre preferred CAS to the graphics calculator and Benoit preferred the graphics calculator to CAS. These preferences are consistent with observations by other researchers. Jost (1992) noted that teachers who viewed the graphics calculator as a tool for computation tended to stress content-orientated goals and viewed learning as listening. Tharp, Fitzsimmons, and Brown Ayres (1997) noticed that rule based teachers (like Andre) subscribed to the view that graphing calculators may hinder instruction and restricted student use of the graphics calculator for investigations. We believe that Andre did not like using the graphics calculator since he was expected to use it in ways (such as experimentation and discovery, getting approximate results) that conflicted with his rule-based conception of mathematics and his preferred lecture/demonstration style of teaching. In contrast, when using the CAS calculator, Andre was very enthusiastic.

> I loved it [CAS calculator]. I thought it was great. I really liked the exact and approximate [i.e. the modes which specify whether answers are given as decimals or as rational numbers or roots], the spreadsheets [tables facility] and graphing from the spreadsheets. Yes, I thought they were fantastic. And the girls did too. I pined for it when I went back to the graphics

calculator, which I found very limited and inaccurate]. [*Some comments on the usefulness of the larger screen and better menu structure of the CAS calculator followed*.]

Benoit’s reactions to the two types of calculators were the opposite of Andre’s. As indicated earlier, he was highly skilled in using graphics calculators and fully embraced its use personally and he enjoyed teaching with it. He used it to help students understand concepts, particularly in explaining symbolic ideas graphically, and for explorations. His behaviour was consistent with other research. Jost (1992) found that teachers (like Benoit) who tended to employ interactive or inquiry-orientated methodologies used graphics calculators more than teachers who used other approaches and that teachers who saw the graphics calculator as a tool for learning had student-centred goals, interactive inquiry driven teaching styles and student-centred views in learning. Tharp et al (1997) noticed that the teachers who were not rule-based teachers did not restrict student use of the graphics calculator for investigations and were more likely to be concerned with student conceptual understanding and thinking. However, Benoit taught differently with a CAS calculator. In both studies, he restricted student use of CAS for symbolic procedures.

Teaching with technology

Learning to use such a complicated machine as a CAS calculator cannot be left to the student alone. The teachers developed their own styles of doing this and of managing the class using technology. During the project, Andre showed considerable growth in his skills of teaching with technology and in his confidence to use other technologies in the classroom. This seemed to be because of his positive experience of the convenience of using the overhead projector and his admiration of the features of the CAS calculator that was used, including its exact answers, large screen and clear menu structure. His systematic and somewhat ?procedural? approach to mathematics was evident again in his use of flowcharts and notes about calculator procedures. He taught his students mathematical procedures and CAS calculator procedures simultaneously, emphasising both.

Benoit, in contrast, began the project already accomplished in teaching with graphics calculators. He also taught his students mathematical procedures and CAS calculator procedures simultaneously, but emphasised the latter much less. It was not as important to Benoit that the students could use their CAS calculators efficiently. In our testing, we saw that his students under-utilised the machines and often made errors that calculator use would have avoided (Kendal & Stacey, 1999). We suspected that Benoit’s method of teaching technology use orally and without visual aids worked only because of his exceptional classroom management skills.

In summary, the different strategies that the teachers employed suggest that there will be a variety of successful solutions to the problem of teaching both mathematics and technology use. Guin and Trouche (1999) describe an alternative classroom structure, which has been trialled in French classrooms.

Using time for mathematics or technology

Many authors believe that using CAS in school mathematics will save time that can be reallocated to improve students’ understanding. Is this the experience of our two teachers?

Both teachers found that teaching CAS procedures at the same time as teaching the new mathematics content was possible. He commented that, on occasions, using CAS saved time such as when data gathering for lessons based on an investigation of patterns. This additional time was absorbed into classroom discussion and there was no real reduction in the expectations of by-hand skills. Since Andre freely permitted CAS techniques to be used by students, he was in a position to save time by reducing practice of by-hand skills. However, he reallocated this time to teaching efficient calculator procedures.

It is significant that Andre taught CAS procedures in a way that did not integrate the technology and mathematics skills. Andre emphasised in speech and notes, the button sequences on the machine: press F4, then F6, and so forth. In contrast Benoit spoke about the mathematical procedures using standard mathematical vocabulary: differentiate, then solve, and so forth, indicating the buttons incidentally. Our research team believes that this is an important difference. Standard advice to teachers embarking on teaching with technology should be to use primarily the standard mathematical terms, with button sequences indicated secondarily.

It is apparent that beyond this time of transition, course time should become available for teachers to reallocate: but whether this is to additional topics, increased understanding, to develop better capabilities for formulating real problems in mathematical terms, or some other goal is an important future choice. Stacey, McCrae and Asp (2000) present options.

Making full use of the symbolic facility

As we noted above, Benoit used the symbolic facility of CAS in a constrained way, embracing its use only for gathering data for students to guess patterns and rules. We explained above that he was cautious about CAS use, but, on reflection, we see that the lessons we developed did not really need symbolic algebra for solving problems and this may have contributed to Benoit's decision to reduce its emphasis. Nearly all of the algebra and calculus procedures required were within the normal by-hand expectations. There was only re-ordering of material, so that students could observe the properties of differentiation etc before the more formal aspects were taught.

This is in direct contrast to the many problems that easily benefited from machine graphing. Graphing is a conceptually relatively simple procedure that is very tedious to carry out in practice without technology. Moreover, a computer graph has more functionality than a paper graph: one can zoom in and out to change the picture, read off points etc. Incorporating graphics calculators into teaching therefore has obvious benefits and can make problems easier for students. In our schools, we are now seeing a rise in the number of students who solve problems graphically. For example, Charles (from a different school), the third teacher in the first CAS Calculus teaching trial, stressed a graphical approach to a wide range of problems. In our testing, his students used a high proportion of graphical methods, and were relatively better at solving problems than students who mainly used an algebraic approach (Kendal & Stacey, 1999). It seems likely that problems that really require the symbolic algebra facility of CAS will be more complicated or sophisticated than those in our standard curriculum. This issue is receiving ongoing attention as part of our work on creating a new formal subject that permits CAS in its external examinations (Stacey et al, 2000).

Finding the place of by-hand skills in a CAS curriculum

The teachers faced decisions about which skills were essential for students to master by hand. This issue was the least problematic for Andre, who could accept the ability to carry out a routine procedure (such as differentiating) on the CAS calculator as equivalent to the ability to carry it out by-hand. Andre's procedural view of mathematics led him to accept that there are alternative procedures. In contrast, Benoit wanted ?understanding? and felt that implementing rules by hand (e.g. differentiating $x^5 + 2x^3$ and $(x^3 +5)(3x^2 - x)$ by hand) contributed to this to such an extent that it was irreplaceable. As we noted above, this was also fuelled by concern that students needed to learn by-hand procedures for the examinations they would take 18 months hence. Monaghan (1997) suggests that the more important algorithms to carry out by hand are those that play an important part in students' cognitive development or those that contain a principle that is important for later development. But which are these? Until the external curriculum environment changes, teachers and students will live in an ambiguous situation about by-hand skills. It seems to us not to be within the capacity of individual teachers to make changes here.

Conclusion

With CAS, students have the opportunity to fulfil their mathematical potential with less computational skill. Using suitable teaching materials, competent teachers will focus student attention on mathematical activities that require them to explore the meaning of the mathematics under consideration. Students will have the opportunity to actively construct knowledge, acquire insightful problem solving skills, develop deep conceptual understanding, develop higher levels of thinking, and gain an understanding of how to validate and interpret solutions. CAS technology should prove to be a powerful mathematical partner.

The experiences of our pioneering teachers show some of the first steps along the road to this ideal situation. The classrooms of Andre and Benoit illustrate how current differences between teachers will not disappear and may even be exaggerated by intelligent tools. The technology can be used to support learning and teaching of many different styles, including both teaching emphasising routine procedures and teaching emphasising understanding. Whereas graphics calculators, for many teachers, slotted easily into the curriculum and enhanced teaching with little threat, CAS demands a more thorough response. Neither Andre nor Benoit, working in a somewhat artificial environment, got far along this track. Their first reactions were interestingly different, in the ways they allowed it to change the curriculum (especially how they came to regard by-hand skills), what they valued when they taught with it, and how they managed their classrooms. The task for educators is now to move ahead simultaneously on curriculum and assessment, teaching styles and classroom organization to develop a viable and coherent response to teaching mathematics in the information age.

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