A New Algorithm for the Placement of WLAN Access Points Based on Nonsmooth Optimization Technique

S. Kouhbor, Julien Ugon, Alex Kruger, Alex Rubinov

School of Information Technology and
Mathematical Sciences
Centre for Informatics and Applied Optimization
University of Ballarat
Ballarat, Vic 3353
skouhbor@students.ballarat.edu.au

Philip Branch

Centre for Advanced Internet Architectures
Swinburne University of Technology
Hawthorn, Vic 3122
pbranch@swin.edu.au

***Abstract* — In wireless Local Area Network (WLAN), signal coverage is obtained by proper placement of access points (APs). The impact of incorrect placement of APs is significant. If they are placed too far apart, they will generate a coverage gap but if they are too close to each other, this will lead to excessive co-channel interferences. In this paper, we describe a mathematical model we have developed to find the optimal number and location of APs. To solve the problem, we use an optimization algorithm developed at the University of Ballarat called Discrete Gradient algorithm. Results indicate that our model is able to solve optimal coverage problems for different numbers of users.**

***Keywords* — Wireless Local Area Network (WLAN), access point (AP), design area, optimal placement, optimization, users coverage**

I. Introduction

Since the IEEE 802.11 WLAN was approved in the late 1990s, growth in the deployment of WLAN has surpassed all projections. The sales of WLAN equipment in the first two years were estimated to be $1.1 billion. By this year the value of WLAN installation is expected to reach $34 billion [1].

The key issue in the deployment of WLAN infrastructure is to provide optimal coverage. WLAN coverage is determined by the number of Access Points (APs) and their location in the design area.

Often the only method used to provide coverage is "trial-and error". However, experience has shown that this method is not accurate and reliable especially for large buildings [2]. Consequently, the use of optimization techniques has begun to attract attention. In general, the results obtained from the optimization approach are more reliable and enable trade-offs between AP parameters to be evaluated more easily than trial-and-error approaches. Several researchers [2-12] have used optimization techniques to obtain the optimal placement of APs. Most of the authors [2 - 9] use discrete mathematical models to find the position of APs. In this case, the design area is divided into rectangles (grids). APs are only allowed to be placed in the centres of the rectangles. To obtain satisfactory results, the size of the grid must be sufficiently small. However, in this case, the dimension of the problem can be very high. For this reason, some authors [12] prefer continuous mathematical models. Others [10-11] have tried to solve their optimization problem using both methods.

The model investigated in the current paper is also based on applying continuous optimization techniques with no restrictions on the position of APs.

It should be noted that there are different approaches to solve the optimization problem in hand. In most of them there are two types of variables in the model: integer variables and continuous ones. Continuous variables describe the location of APs whereas integer variables describe the membership degree of receivers to clusters. Since each receiver can belong only to one cluster, integer variables can attain values of 0 and 1 only. As a result one gets a mixed integer nonlinear programming problem. It is well known that these problems are difficult to solve in many situations. A nonsmooth optimization approach described in this paper allows one to exclude integer variables, to reduce significantly the number of variables in the optimization problem and to replace the mixed integer nonlinear programming problem with a continuous nonlinear programming problem.

The paper is organised as follows. Section II shows the model notation. Section III presents the mathematical model. The solution to the optimization problem is explained in section IV. The testing method is described in section V. Results and the effect of AP parameters on coverage are discussed in section VI. Finally section VII summarises the paper and discusses future research in this area.

II. General Notations

Throughout this paper the following notations are used:

a_j $\quad j = 1,\ldots,N$ $\quad$ Access point (AP)

$r_i \quad i = 1,\ldots,M$	Receiver/user
$d(a_j, r_i)$	Distance between AP and receiver
$g(a_j, r_i)$	Path loss from i^{th} user to access point j
g_{max}	Maximum tolerable path loss
μ	Penalty parameter for violating the maximum tolerable path loss
P_t	Transmit power
R_{th}	Receive threshold
M	Number of users
a_p	Position of AP
N	Number of AP

All points in the design area are represented by their coordinates (either two–on the plane or three – in three dimensional space). We assume the distance function to be Euclidean, hence the distance (d) between an AP a_j and a receiver r_i is given by:

$$d(a_j, r_i) = \sqrt{(r_i^1 - a_j^1)^2 + (r_i^2 - a_j^2)^2},$$

where $a_j = a_j\,(a_j^1, a_j^2)$, and $r_i = r_i\,(r_i^1, r_i^2)$.

III. Optimization Model

In this model we use path loss to find the optimal placement of APs. The path loss model is the core of the signal coverage calculation for any environment [13]. The path loss model describes the loss of signal strength due to distance between the transmitter and the receiver. Path loss models can be used to calculate the coverage area of AP and maximum distance between the two terminals.

A. Model Description

The objective function based on path losses can be developed in different formats such as the average path loss, the maximal path loss, the sum of squared path losses, and a convex combination (an approach is used to combine two extreme cases) of the average and the maximal path losses with different coefficients. In this paper, we consider minimising the average path loss:

$$F_1\,(a) = \frac{1}{M}\sum_{i=1}^{M} \min_j g\left(a_j\,,\, r_i\right). \qquad (1)$$

This objective function (1) provides coverage for the users in the design area. The drawback with this objective function is that it might ignore a few remotely located users. F_1 is minimized subject to the following constraint:

$$\min_j\, g(a_j\,,\, r_i) \le g_{\max} \quad \forall i = 1,\ldots,M\,. \qquad (2)$$

This constraint states that path loss is evaluated against the maximum tolerable path loss g_{max}. This ensures that the quality of coverage at each receiver location is above the given threshold. This given value, g_{max} can be calculated by subtracting the receiver threshold from the transmitter power

$$g_{\max} = P_t - R_{th}\,.$$

The above inequality (2) can be expressed in the equality form as

$$\left(\min_j g\left(a_j\,,\, r_i\right) - g_{\max}\right)^+ = 0,$$

where $(\alpha)^+ = \max(\alpha\,, 0)$.

The above problem of minimizing (1) subject to (2) can be converted into the unconstrained one by using a penalty approach. If the path loss threshold (g_{max}) at a receiver location is violated, a penalty term depending on a parameter μ will be added:

$$F_2\,(a) = \mu \sum_{i=1}^{M}\left(\min_j\, g\left(a_j\,,\, r_i\right) - g_{\max}\right)^+ .$$

The total objective function is the sum of F_1 and F_2:

$$F_3\,(a) = \frac{1}{M}\sum_{i=1}^{M}\, \min_j g\left(a_j\,,\, r_i\right) +$$
$$\mu \sum_{i=1}^{M}\left(\min_j g\left(a_j\,,\, r_i\right) - g_{\max}\right)^+$$

The path loss function at the *i-th* receiver as described in [14-15] can be written as:

$$g\left(a_j\,,\, r_i\right)[dB] = g\,(d_0)[dB] + 10\,\log\left(\frac{d\left(a_j\,,\, r_i\right)}{d_0}\right)^n . \quad (3)$$

n is the path loss exponent which indicates the rate at which the path loss increases with the increase of the distance. Its value depends on the specific propagation environment. For free space environment [14 -15], where line of sight exists, $n = 2$. $g\,(d_0)$ is the path loss at the reference distance d_0 and d is the distance between transmitter and receiver. The objective function (3) is valid for $d \ge d_0$. For $d < d_0$ we take

$$g\left(a_j\,,\, r_i\right)[dB] = g\,(d_0)\,[dB]..$$

As described in [15], the path loss at the reference distance can be calculated from

$$g(d_0)[dB] = 20\log\frac{4\pi d_0 f}{c},$$

where f is the carrier frequency and c is the speed of light.

IV. Solution of Problem (1)–(2)

The objective function (1) is nonsmooth and nonconvex and it has many local minima. When the number of AP and receivers is large, we get a large scale global optimization problem. However, traditional global optimization methods cannot be directly applied to it.

Computation of subgradients of the function F_3 is a very difficult task. Therefore, methods requiring subgradient evaluation at each iteration cannot be effective.

Direct search methods seem to be the best option for solving problem (1)–(2). However, for many such methods the number of variables and/or constraints, which can be efficiently handled, is restricted. For example, one of the most efficient direct search methods – the Powell method [16] performs well when the number of variables is less than 20. However, in the problem under consideration the number of AP can be large, therefore, the number of variables can be much more than 20. (The number of variables is the number of coordinates of AP multiplied by their number. For example, in case of two APs in three dimensional space, the number of variables is 6.)

We use the Discrete Gradient method to solve Problem (1)–(2). The description of this method can be found in [17, 18]. This is a derivative-free method. The Discrete Gradient method consists of two main steps: the computation of a descent direction which is reduced to a certain quadratic programming problem and a line search. We use so-called Armijo-type line search in this method. As it is shown in [17], computation of the descent direction is a terminating process: after a finite number of steps the algorithm either computes the descent direction or finds out that the current iteration is a stationary point. The Discrete Gradient method is efficient for solving large scale nonsmooth optimization problems. It is suitable for finding local minimizers in Problem (1)–(2). This algorithm allows a continuous search to find the optimal placement of APs meaning no restriction is placed on their positions.

A. *Finding the Number of APs*

Initially we set the number of AP to 1: $N = 1$; then the necessary number of APs is found through the following steps:

1. Try to solve Problem (1)-(2);
2. If the solution exists, then N is the desired number;
3. Otherwise, N is increased by 1: $N = N+1$;
4. Go to step 1.

B. *Calculating Path Loss at Reference Distance*

We have chosen the reference distance $d_0 = 1$ m. According to the 802.11b standard (standard written by IEEE for WLAN) [19], the carrier frequency is $f = 2.4 \times 10^9$ Hz. The speed of light is $c = 3 \times 10^8$ m/s. Based on these values, the path loss at the reference distance, $g(d_0)[dB]$ is 40.04 dB (decibels).

V. Method of Testing

Two simple cases were considered for conducting the test in order to examine the model. In the first case, the size of the design area was 100 m^2. In this area we located 4, 9, and 16 users. It is important for us to examine the model with different numbers of users, as the model is developed to cover the largest number of users rather than maximising the coverage of the design area. Fig 1 shows how users are distributed in the design area. In the second case, we extended the design area to 1000 m x 1500 m, increased the number of users to 75, and distributed them in different parts of the area. Fig 2 shows the location of users and the shape of the area.

The specifications of two models of APs developed by Cisco (Aironet 1200 series and Aironet 340 series) [20 - 21] and IEEE 802.11b standard [19] are used to test the model. This gives us the opportunity to examine the model with different values of P_t and R_{th} and also to find the effect of AP parameters on the capacity that is going to be considered for further development of the research.

VI. Results

A. *Case One: Design Area 100 m^2*

In this case we considered the design area to be 100 m^2 and users to be distributed uniformly. We tested the model with different values of P_t and R_{th}. Table I shows the results for different values of P_t, R_{th} and numbers of users.

The results indicate that when P_t changes from 15 dBm to 20 dBm and R_{th} changes from -94 dBm to -70 dBm, one AP is required to cover the users. In this case AP is placed on the top corner of the area. The result is the same for 4, 9, and 16 users in the area. Fig 3 shows coordinates of the AP for 4 users in the area. The reason that AP is placed in corner of the area in this case is due to the properties of the function (3). That is the path loss is a concave function of the distance. For an AP serving a number of users, the minimal average path loss is obtained by placing the AP near one of the users.

When P_t is 20 dBm and R_{th} is increased to −60 dBm, again one AP is able to cover all users. However, the position of AP is changed. Coordinates of AP are shown in Fig 4. As R_{th} is further increased to −55 dBm, the coverage range is decreasing and consequently two APs are found to be required to cover all users. The same number of APs is required when P_t is reduced to 17 and 15 dBm, and R_{th} is kept at -60 dBm. Fig 5 shows the coordinates of APs for 9 users in the area.

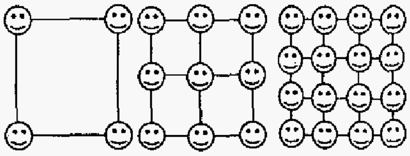

Figure 1 Position of 4, 9 and 16 users in the design area

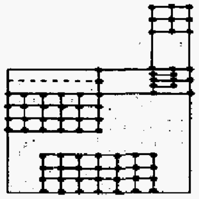

Figure 2 Extended design area with 75 users

With P_t at 17 dBm and R_{th} at –55 dBm, all users are covered with four APs. Fig 6 shows the coordinates of APs when there are 16 users in the area. When P_t is further reduced to 15 dBm, and R_{th} is kept at -55 dB, four, six and five APs are needed to cover 4, 9 and 16 users respectively. Fig 7 shows the coordinates of APs when there are 9 users in the area.

B. *Case Two: Extending the Area and Increasing the Number of Users to 75*

Table II shows the optimal placement of AP/APs for different values of R_{th} and P_t.

As the design area extends and the number of users increases, the required number of APs changes dramatically in comparison to case A. In the case, where both P_t and R_{th} are high, one AP is covering all users. This position changes as the parameters of AP change. When P_t = 17 dBm and R_{th} = -76 dBm, three APs are covering all users. Fig 8 shows the position of APs and the coverage of users. The number of APs doubled when R_{th} increased to -73 dBm. Fig 9 shows the position of APs and coverage of users. Decreasing P_t to 15 dBm is causing the change in the position of APs. Fig 10 shows the position of APs and the coverage of users. The required number of APs to cover all users increases to 49, when Pt is kept at 15 dBm and R_{th} is further increased to -60 dBm. This is due to the decreasing of the coverage range.

VII. Conclusion

This paper describes a model based on path losses that can be used to find the optimal position of APs while covering as many users as possible. We used a new algorithm called Discrete Gradient that uses continuous search to solve the optimization problem. In continuous search the positions of APs are not fixed. We are also providing good quality of coverage for the majority of users by minimising the average path loss over the design area. The results obtained show that all users are covered in all cases. It was also observed that both parameters of AP (P_t and R_{th}) have an effect on the position and number of AP. This effect is more noticeable when the area and the number of users are large. These parameters can be used to limit the number of users per an AP in order to increase the throughput of the network.

Further work will involve extending the objective function to include the convex combination of the average and the maximal path losses with different coefficients and also to develop models based on considering distances and powers. To examine the model with real type environment, we will include areas with obstacles. Finally, we will limit the number of users per an AP in order to increase capacity for all users if required.

Table I Optimal placements of APs for design area = 100 m^2

P_t (dBm)	R_{th} (dBm)	M	a_p (m)
20	-94 to -70	4	[99.29, 99.29]
	-60		[70.39, 70.39]
	-55		[55.97, 0],[44, 100]
17	-94 to -70		[99.29, 99.29]
	-60		[29.5, 0],[29.5, 100]
	-55		[0.3, 0.28],[100,0.9], [0.33,99.7],[100, 100]
15	-94 to -70		[99.29, 99.29]
	-60		[55.97, 0],[44, 99.99]
	-55		[0,100],[0.88,0.32], [100,100],[100,0]
20	-94 to -70	9	[99.29, 99.29]
	-60		[50.11, 50.99]
	-55		[50, 25.1],[51, 99.99]
17	-94 to -70		[99.29, 99.29]
	-60		[50, 0],[50, 51]
	-55		[87.65, 12.34],[39.6, 100], [12.34, 37.6],[100, 100]
15	-94 to -70		[99.29, 99.29]
	-60		[50, 25.1],[51, 99.99]
	-55		[100, 31.47],[0, 68.5], [31.4, 0], [100, 100],[50, 100], [50, 50]
20	-94 to -70	16	[99.29, 99.29]
	-60		[64.2, 64.2]
	-55		[47.9, 20.5], [47.9, 79.4]
17	-94 to -70		[99.29, 99.29]
	-60		[66.9, 37.7],[33, 37.7]
	-55		[35, 0.98],[22.6, 67.5], [72.9, 71],[100, 0.99]
15	-94 to -70		[99.29, 99.29]
	-60		[47.9, 20.5],[47.9, 79.4]

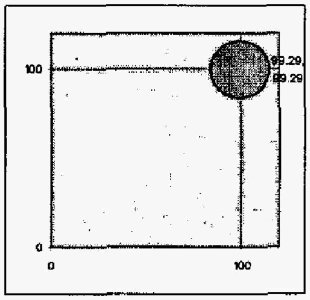

Figure 3 Coordinates of AP for 4 users when P_t=15 to 20 dBm and R_{th} = -70 to -94 dBm

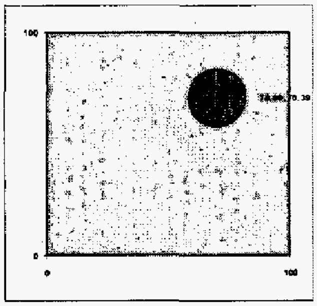

Figure 4 Coordinates of AP for 4 users when P_t = 20 dBm And R_{th} = -60 dBm

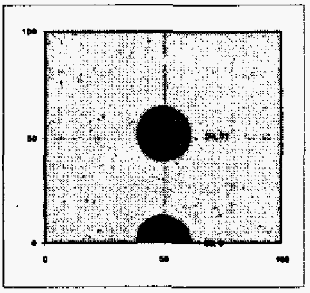

Figure 5 Coordinates of APs for 9 users when Pt = 17 dBm and Rth = - 60 dBm

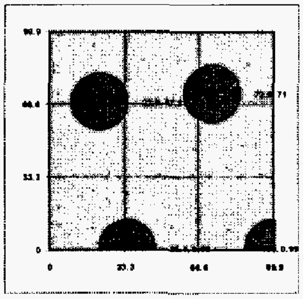

Figure 6 Coordinates of APs for 16 users when Pt =17 dBm and Rth = - 55 dBm

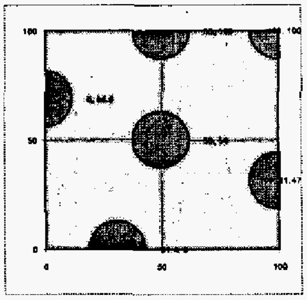

Figure 7 Coordinates of APs for 9 users when Pt = 15 dBm and Rth = - 55 dBm

Table II Optimal placement of APs for extend area

P_t (dBm)	R_{th} (dBm)	a_p (m)
20	≤ -85	[999.45, 1499.1]
	-83	[799.94, 999]
	-76	[499, 499.8], [999.4, 1499.2]
	-73	[500,299],[897.3,1067.38],[299.78, 500.1]
	-60	25 APs were found
17	≤ -89	[999.45, 1499.1]
	-85	[799.96, 999]
	-83	[499.23, 699.36]
	-76	[500, 299], [879.5, 1072], [299.8, 501]
	-73	[600.9,100.3],[900,1301],[236.8,672.6] [999.3,800.7],[399,199.7],[500,999]
	-60	47 APs were found
15	≤ -90	[999.45, 1499.1]
	-89	[899.4, 1299.2]
	-88,-87	[799.9, 999]
	-85	[499.23, 699.36]
	-83	[399.5, 599.2], [999.4,1499.2]
	-76	[500, 186.4], [750.4, 1250.2], [203.5, 771.25], [999.4, 800.8]
	-73	[608.9,150.6],[899.8,1299.34] [198.2,647.6],[497.8,750],[396.3,150.3],[971,830.6]
	-60	49 APs were found

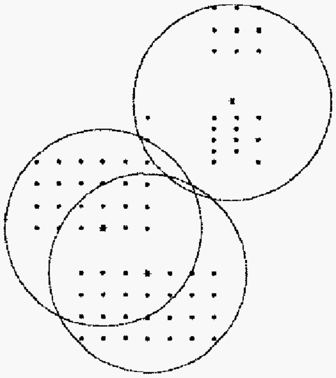

Figure 8 Position of APs for 75 users when Pt = 17 dBm and Rth = -76 dBm

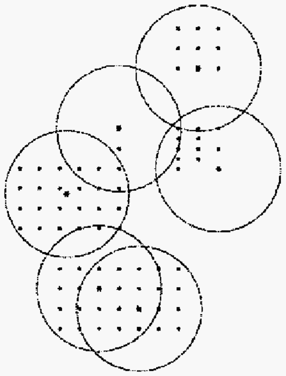

Figure 9 Position of APs for 75 users when Pt = 17 dBm and Rth = -73 dBm

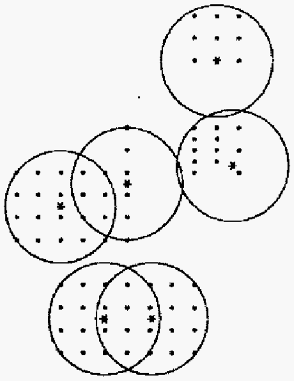

Figure 10 Position of APs for 75 users when Pt =15 dBm and Rth = -73 dBm

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