

COPYRIGHT NOTICE



FedUni ResearchOnline

<http://researchonline.federation.edu.au>

This is the author's accepted version of the following publication:

Khandelwal, M., Armaghani, D. (2015). Prediction of drillability of rocks with Strength Properties Using a Hybrid GA-ANN Technique. Geotechnical and Geological Engineering Vol. p.1-16.

The version displayed here may differ from the final published version.

The final publication is available at:

<http://doi.org/10.1007/s10706-015-9970-9>

Copyright © 2015, Springer-Verlag Berlin Heidelberg

1 **PREDICTION OF DRILLABILITY OF ROCKS WITH STRENGTH PROPERTIES USING A**
2 **HYBRID GA-ANN TECHNIQUE**

3 **Manoj Khandelwal^{1*} and Danial Jahed Armaghani²**

4 ^{1*}Faculty of Science and Technology, Federation University Australia, PO Box 663, Ballarat, Victoria
5 3353, Australia Phone: +61 3 5327 9821 Email: m.khandelwal@federation.edu.au.

6 ²Department of Geotechnics and Transportation, Faculty of Civil Engineering, Universiti Teknologi
7 Malaysia, 81310, UTM, Skudai, Johor, Malaysia. Email: danialarmaghani@gmail.com.

8 **Abstract**

9 The purpose of this paper is to provide a proper, practical and convenient drilling rate index (DRI)
10 prediction model based on rock material properties. In order to obtain this purpose, 47 DRI tests were
11 conducted in the laboratory. In addition, the relevant strength properties i.e. uniaxial compressive strength
12 (UCS) and Brazilian tensile strength (BTS) were determined and selected as input parameters to predict
13 DRI. Examined simple regression analysis showed that the relationships between the DRI and predictors
14 are statistically meaningful but not good enough for DRI estimation in practice. Moreover, multiple
15 regression, artificial neural network (ANN) and hybrid genetic algorithm (GA)-ANN models were
16 constructed to estimate DRI. Several performance indices i.e. coefficient of determination (R^2), root
17 mean square error (RMSE) and variance account for (VAF) **were used for evaluation of performance**
18 **prediction the proposed methods. Based on these results and the use of simple ranking procedure,** the best
19 models were chosen. It was found that the hybrid GA-ANN technique can performed better in predicting
20 DRI compared to other developed models. **This is because of the fact that the proposed hybrid model can**
21 **update the biases and weights of the network connection to train by ANN.**

22 **Keywords:** Drilling rate index, Rock material properties, Artificial neural network, Hybrid model.

24 **1. Introduction**

25 Drillability is defined as the resistance of rock to penetrate the rock mass by a drilling system. Drilling
26 rate index (DRI) is one of the tool to evaluate drillability of rocks. The drilling rate index (DRI) was
27 proposed by Selmer-Olsen and Lien (1960) in order to evaluate the drillability of rocks by percussive
28 drilling. The influential factors on DRI can be categorized into two parts, i.e. controllable and
29 uncontrollable parameters. Bit type and diameter, thrust, blow frequency, rotational speed and flushing
30 are considered as controllable factors on DRI, while some other parameters, like geological conditions
31 and rock properties are defined as uncontrollable parameters of DRI (Yarali and Kahraman 2011).
32 Drilling has a direct and may be close relationship with the rock mass and material properties (Hoseinie et
33 al. 2008). Strength of rock has a considerable impact on drilling thrust. Strength properties of rocks play
34 an important role in design, safety and stability of any rock structures (Khandelwal and Ranjith, 2010;
35 Khandelwal, 2013). Therefore, recognition the most effective parameters on DRI and subsequently proper
36 DRI prediction would help designers to select the appropriate type of drilling system.

37 Many studies have been conducted in order to demonstrate the effects of rock (mass and material)
38 properties on DRI (e.g. Wijk 1989; Karpuz et al. 1990; Kahraman 1999; Kahraman et al. 2000; Kahraman
39 et al. 2003; Hoseinie et al. 2009; Dahl et al. 2012; Yarali and Soyer 2013; Macias et al. 2014; Tripathy et
40 al. 2015; Ataei et al. 2015). A penetration rate model was proposed using stepwise linear regression
41 analysis in the study conducted by Selim and Bruce (1970). Schmidt (1972) related the penetration rate
42 with tensile strength, density, Young's modulus, Shore hardness, shear modulus, longitudinal wave
43 velocity, compressive strength, Poisson's ratio and shear wave velocity. A rating classification for DRI
44 prediction was established in the study carried out by Hoseinie et al. (2008). They used six rock mass
45 properties namely Mohs hardness, grain size, uniaxial compressive strength (UCS), joint filling, joint
46 spacing and joint dipping to predict DRI. Yarali and Kahraman (2011) proposed new relations for
47 predicting DRI by using brittleness values of 32 different rocks. Cheniany et al. (2012) developed linear
48 and non-linear multiple regression to estimate specific rock mass drillability (SRMD) index. In their

49 models, UCS, quartz content, Schmidt hammer hardness value, joint dip, alteration and fragment size
50 (d80) were considered as predictors. Single regression models were introduced by Yarali and Soyur
51 (2013) in order to relate DRI with several properties of rocks including UCS, Brazilian tensile strength
52 (BTS), point load strength, Schmidt rebound hardness and Shore scleroscope hardness. They showed that
53 rock strength is the most effective parameter on DRI. Moein et al. (2014) measured DRI values of
54 carbonate rock in the laboratory and indicated good relationships for predicting DRI using the alteration
55 index and specific energy.

56 In the field of artificial intelligent systems, there were also several attempts by previous researchers in
57 order to predict penetration rate. An artificial neural network (ANN) approach was selected and proposed
58 for predicting penetration rate by Akin and Karpuz (2008). They concluded that their developed approach
59 can provide satisfactory results in estimating penetration rate. Arabjamaloei and Karimi Dehkordi (2012)
60 utilized adaptive neuro-fuzzy inference system (ANFIS), ANN and statistical techniques in estimating
61 penetration rate and concluded that ANN is the best model among all developed models. A penetration
62 rate model based on ANFIS was proposed in the study conducted by Basarir et al. (2014). They
63 successfully showed that ANFIS results are better than the results of statistical model. In the present
64 study, several linear and non-linear models i.e. multiple regression analysis, ANN and hybrid genetic
65 algorithm (GA)-ANN were applied and developed to predict the DRI values using strength properties of
66 rocks.

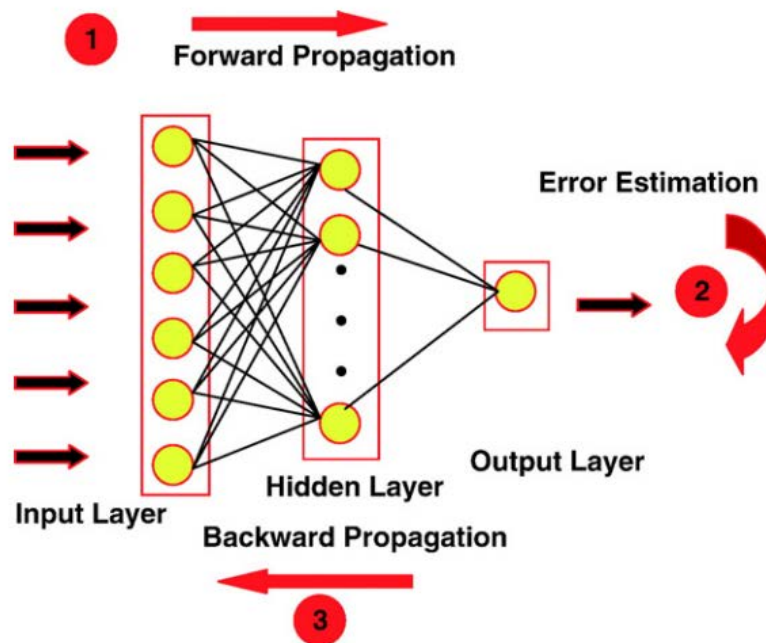
67

68 **2. Method**

69 **2.1 Artificial Neural Network**

70 In ANNs, which are function approximation tools, the process of information-transfer in the human brain
71 is imitated. Generally, ANNs are applicable to cases in which there is very complex and nonlinear contact
72 nature between input variable(s) or predictor(s) and output of the system (Garrett 1994; Jahed Armaghani

73 [et al. 2015a](#)). ANNs have been designed in many types, and the most commonly-used one is the
74 multilayer feed-forward ANN that comprises multiple layers that are connected together by a number of
75 hidden nodes (neurons) with different connection weights ([Simpson 1990](#)). For the achievement of a
76 desirable outcome, ANNs should be trained by means of some learning algorithms. For training ANNs,
77 the back-propagation (BP) algorithm is the most widely-used among other learning algorithms ([Dreyfus](#)
78 [2005](#); [Hajihassani et al. 2014](#); [Jahed Armaghani et al. 2015b](#)). By using BP algorithm, system error
79 between desired and predicted values can be minimized. The output of each hidden node is determined
80 subsequent to the application of a transfer function, which is mostly sigmoidal function, to the net input of
81 the hidden node. A comparison is made between the desired output (targets) and the predicted one, and
82 then the error is computed. If this error is bigger than mean square error (MSE) or root mean square error
83 (RMSE) values, the network should be propagated back for adjusting the connection weights. Figure 1
84 shows structure of BP ANN algorithm with one hidden layer.



85

86 **Figure 1.** Structure of BP ANN algorithm with one hidden layer ([Saemi et al. 2007](#))

87

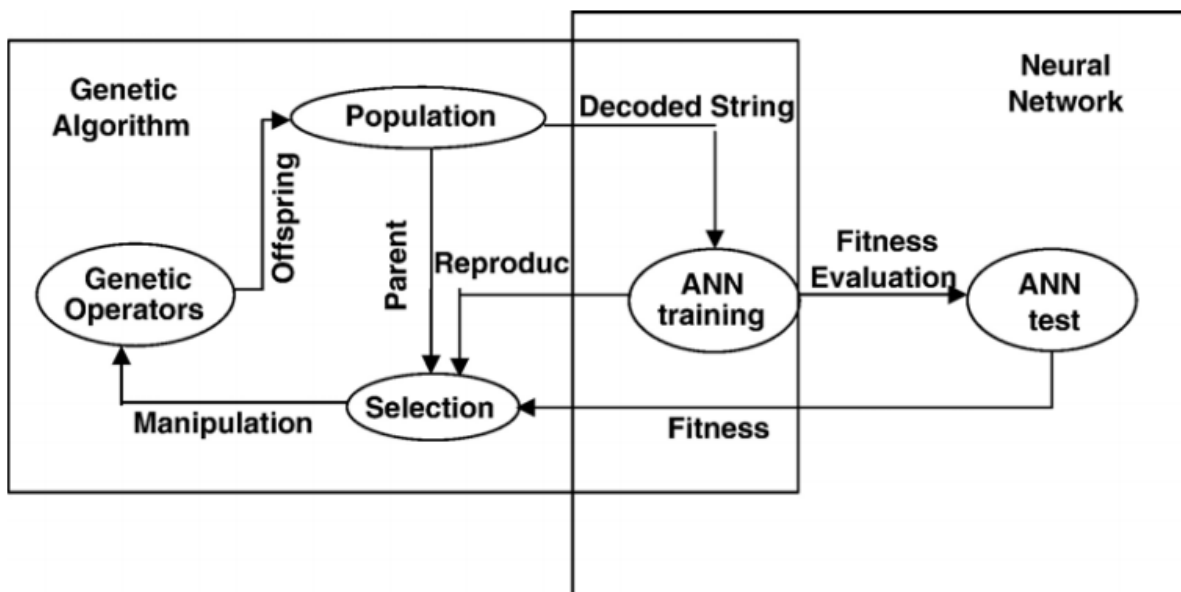
88 2.2 Genetic Algorithm

89 Genetic algorithm (GA) which was developed by Holland (1975) is considered as an optimization
90 technique. This algorithm mimics the natural selection mechanism and the biological species evolution.
91 To advance, in each decision variable, objective function evaluation is needed by GA. This is because the
92 stochastic-based technique of GA does not need any specific information for guiding the search
93 (Chipperfield et al. 2006).

94 Generally, in GA, there are populations of individuals that are known as candidate solutions; each
95 individual gradually converges over time to an optimal solution. Each candidate solution is denoted by a
96 linear string that consists of chromosomes represented by 0s and 1s. Total solutions form the population
97 size together with the optimization process of each iteration is known as a generation. In GA, for the
98 creation of the next generation, three basic genetic operators i.e. reproduction, cross-over, and mutation
99 should be applied. The first operator or reproduction is defined as a process through which the best
100 chromosomes are selected according to their scaled values with considering the given criteria of fitness,
101 and then the selected chromosomes are directly transferred to the next generation. Through the cross over
102 operator, offspring or new individuals are produced through combining particular parts of individuals
103 (parent). Recombination is done through several ways, including single-point cross over and two-point
104 cross over. Nevertheless, during the process of cross over, a random cross over point and two parents are
105 chosen. The creation of the first offspring is through the combination of the left side genes of the first
106 parent with the right side genes of the second parent. To form the second offspring, an inverse procedure
107 is repeated (Momeni et al. 2014). Mutation is defined as a process during which a random change occurs
108 in elements of a chromosome.

109 Several studies have been conducted to enhance the performance quality and generalisation capabilities of
110 ANNs through the use of GA algorithm (e.g. Monjezi et al. 2012; Aghajanloo et al. 2013; Momeni et al.
111 2014). GA is known as stochastic search algorithm; as a result, it can be performed for adjusting the

112 biases and weights of the ANNs to increase the performance prediction of ANNs (Momeni et al. 2014).
 113 At a local minimum, by ANNs, there is normally more probability of convergence, while GA can find a
 114 global minimum. So, a combination of GA and ANN model (GA-ANN model) utilizes the search
 115 properties of both algorithms to enhance the network power. In this model, first, GA finds global
 116 minimum in search space, and then ANN employs it to discover the best results. A hybrid GA-ANN
 117 algorithm is displayed in Figure 2.



118

119 **Figure 2.** Combination of GA-ANN (Saemi et al. 2007)

120

121 3. Laboratory Testing

122 Rock mass samples were collected from different published literatures (Yarali and Kahraman, 2011;
 123 Adebayo et al, 2010; Ekincioglu et al, 2013) to fulfill the aim of this research.

124

125

126

127

128 **3.1 Uniaxial compressive strength**

129 Determination of UCS involves the use of a NX size (54 mm diameter) cylindrical specimen with length
130 to diameter ratio of 2.5 which is loaded axially as suggested by ISRM (1979). UCS can be calculated with
131 the help of following formula:

132
$$UCS = P / A \tag{1}$$

133 where,

134 P – Failure load, and

135 A – Cross-sectional area of the cylindrical specimen

136

137 **3.2 Tensile strength**

138 Brazilian test is used in order to determine tensile strength in the laboratory. This test is conducted based
139 on the fact that mostly rocks in biaxial stress fail in tension at their uniaxial tensile strength (Jaeger 1967).

140 The test should be conducted in accordance with ISRM (1978) standard. Tensile strength can be
141 calculated with the help of following formula:

142
$$TS = 2.P / \pi.d.t \tag{2}$$

143 Where,

144 P = Failure load, and

145 d = Diameter of the disc

146 t = Thickness of the disc

147

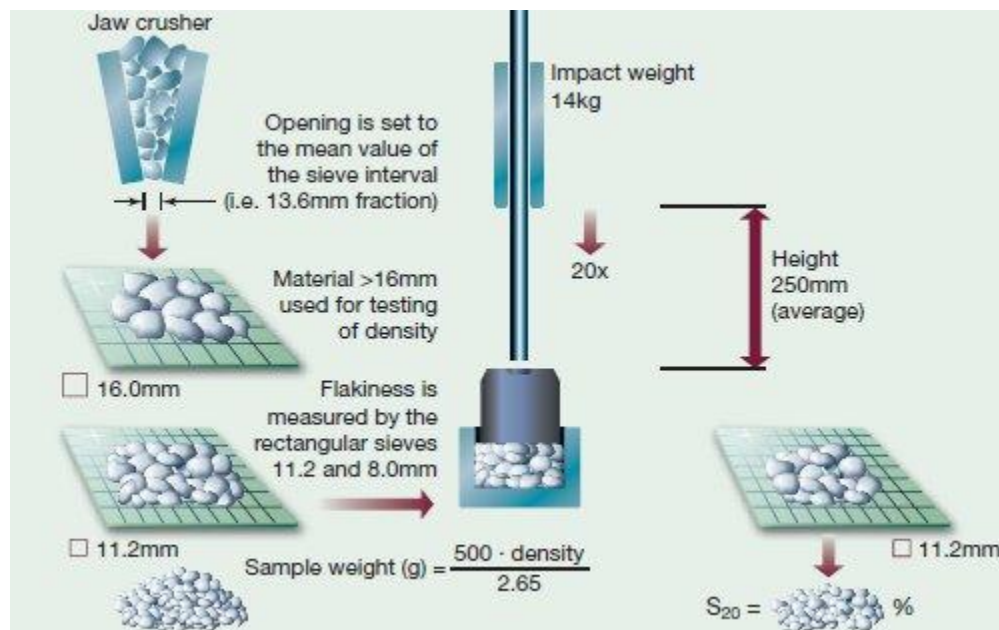
148 **3.3 Drilling Rate Index (DRI)**

149 Drillability of rocks are examined on the basis of the DRI. The DRI is defined as a combination of the
150 intact rock specimen brittleness value (S20) test which was proposed by Matern and Hjelmer (1943) and
151 Sievers' J-Value (SJ) miniature drill-test which was proposed by Sievers (1950). The SJ test is considered
152 as an indirect measure of rock resistance to tool indentation (surface hardness); the brittleness value, S20,
153 is an indirect measure of rock resistance to crack growth and crushing.

154 **3.3.1 The Brittleness Test**

155 In this study, S20 values were measured by using the Swedish Stamp Test (see Figure. 3). **The test is**
156 **started by putting the rock aggregate in a mortar and then by using a 14 kg hammer, struck 20 times.** The
157 mortar aggregate volume corresponds to that of a 0.5 kg aggregate with a density of 2.65 tons/m³ in the
158 fraction 11.2 - 16.0 mm. S20 equals the percentage of undersized material that passes through a 11.2 mm
159 mesh after the drop-test. S20 should be taken as a mean value of three or four parallel tests.

160



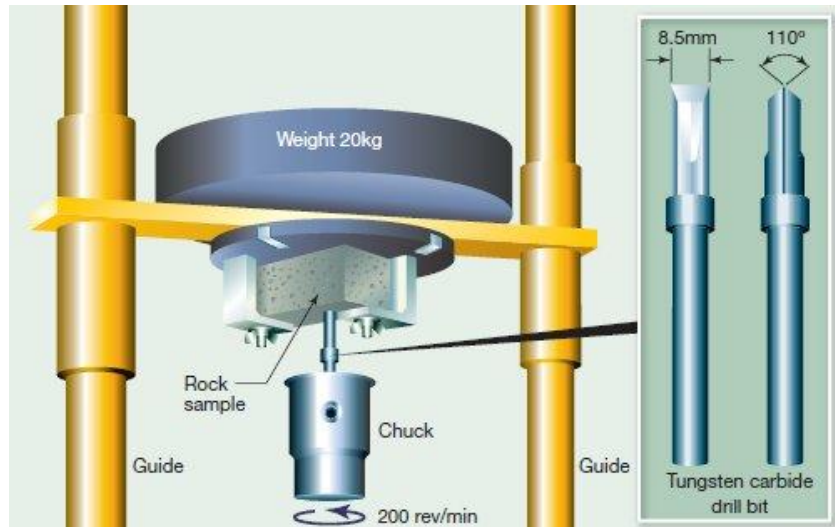
161

162 **Figure 3.** Outline of the brittleness test (Dahl 2003)

163

164 **3.3.2 The Sievers' J (SJ) miniature drill test**

165 The second DRI parameter which is the SJ value, can be obtained from a miniature drill test. After 200
166 revolutions, in the rock sample, the hole depth is measured in 1/10 mm. A mean value of four - eight test
167 holes should be used. Parallel to rock foliation, the SJ values are always measured for created holes.
168 Outline of the Sievers' J miniature drill test is shown in Figure 4.



169

170

Figure. 4. Outline of the Sievers' J miniature drill test (Dahl 2003)

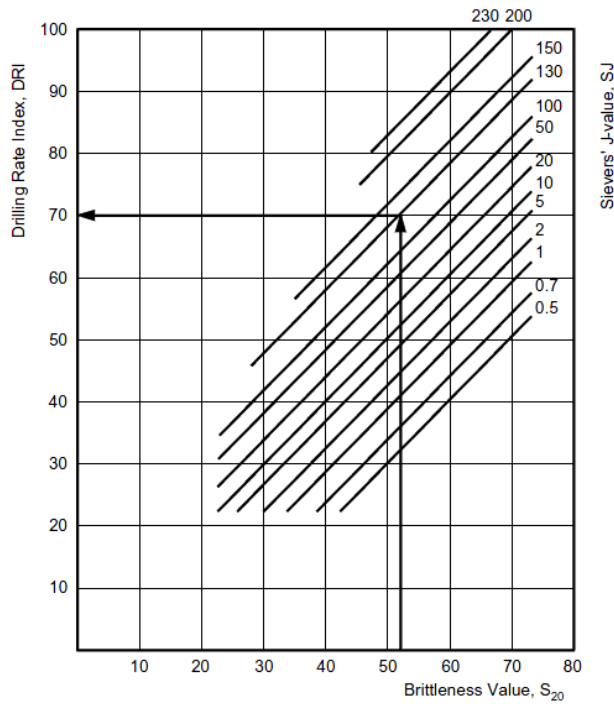
171

172 3.3.4 Assessment of DRI

173 After measuring S₂₀ and SJ values, DRI can be determined by using the Figure 5. Based on this figure,

174 DRI values can be determined using both brittleness and SJ values. Table 1 shows DRI classification

175 rating for various categories.



176

177

Figure. 5 The graph for determination of DRI using S₂₀ and SJ (Bruland 1998)

178

Table 1 Classification of rocks considering DRI rating

S. No.	Category	DRI
1	Extremely low	21
2	Very low	28
3	Low	37
4	Medium	49
5	High	65
6	Very high	86
7	Extremely high	114

179

180 **4. DRI Prediction**

181 In order to solve the engineering problem, simple regression equations are conducted in the first step. The
 182 used parameters and their categories and ranges are shown in Table 2. Based on this table, UCS and BTS
 183 were considered as model inputs in this study to predict DRI.

184 **Table 2** Basic statistical description of input and output parameters

Parameter	Unit	Category	Symbol	Min	Max	Mean
Uniaxial compressive strength	MPa	Input	UCS	28.6	182.1	95.5
Brazilian tensile strength	MPa	Input	BTS	2.57	17.07	8.68
Drilling rate index	-	Output	DRI	22	86	55.26

185

186 The simple regression analyses were performed between the DRI and predictor parameters i.e. UCS and
 187 BTS. The obtained results from simple regression analysis are not good enough to be utilized to solve the
 188 problem. Due to this reason, to obtain the better model for prediction of DRI, multiple regression
 189 analysis, ANN and hybrid GA-ANN techniques were also conducted using established dataset. The
 190 procedure of each modelling technique was described in the following sections.

191

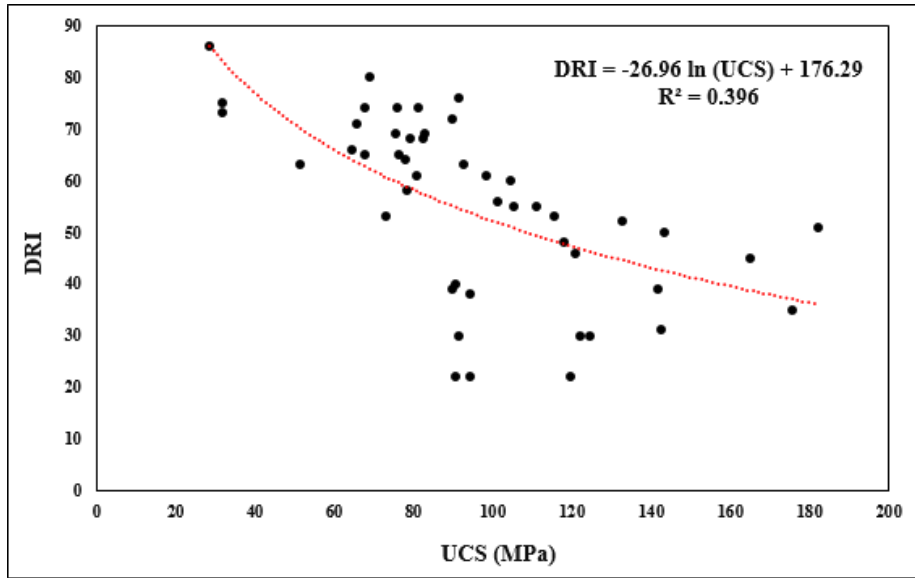
192 **4.1 Simple Regression**

193 In order to examine the effect of input parameters, the simple regression analyses were constructed
194 between the DRI and other mentioned parameters including UCS and BTS. Subsequently, new equations
195 introduced for estimation of DRI. In order to obtain equations with higher performance capacity, various
196 simple regression analyses such as; linear, exponential, power and logarithmic were performed. The
197 selected equations to predict DRI using UCS and BTS are presented in Equations 3 and 4, respectively.

198
$$\text{DRI} = - 26.96 \times \ln (\text{UCS}) + 176.29 \quad (\text{R}^2 = 0.396) \quad (3)$$

199
$$\text{DRI} = - 31.15 \times \ln (\text{UCS}) + 120.86 \quad (\text{R}^2 = 0.411) \quad (4)$$

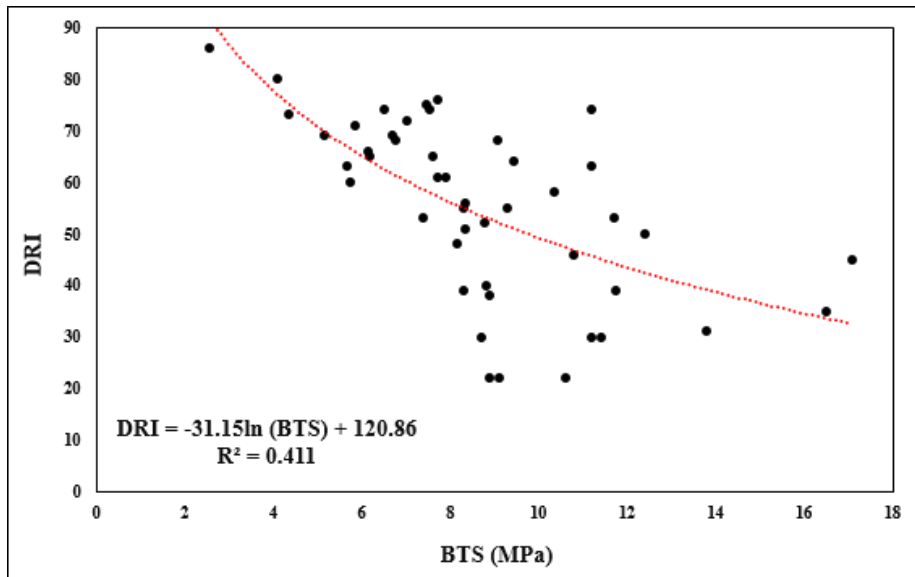
200 The reliability of the developed relationships was evaluated by comparing the obtained coefficient of
201 determination (R^2) values for each analysis. As it can be seen in Equations 3 and 4, the logarithmic
202 relationships give the best relatively results in estimating DRI among all utilized-equation types. R^2
203 values of 0.396 and 0.411 were obtained for predicting DRI considering UCS and BTS data, respectively.
204 The purposed relationships between the DRI and input parameters i.e. UCS and BTS are given in Figures
205 6 and 7. The results indicated that the relations between the input parameters and DRI are meaningful but
206 not good enough for estimation of the DRI in practice. These relationships indicated that maybe multi-
207 inputs are required to predict the DRI, so, various modelling techniques namely multiple regression
208 analysis, ANN and GA-ANN were also constructed.



209

210

Figure 6. Relationship between measured DRI and UCS values



211

212

Figure 7. Relationship between measured DRI and BTS values

213

214 **4.2 Multiple Regression**

215 The multiple regression (MR) technique aims at determining the values of parameters for a function that

216 causes the function to best fit a provided set of data observations. The function is a linear (straight-line)

217 equation in this technique. In cases where more than one independent variable exists, MR is employed in
218 order to achieve the best-fit equation. MR can solve the engineering problems through performing a least
219 squares fit. By employing this techniques, some coefficients are suggested by means of the backslash
220 operator (Khandelwal and Monjezi 2013). The MR equation type is presented as follows (Jahed
221 Armaghani et al. 2015c):

$$222 \quad y = a + b_1x_1 + b_2x_2 + b_3x_3 + \dots + b_nx_n \quad (5)$$

223 where,

224 $x_1, x_2, x_3, \dots, x_n$ are independent variables,

225 $b_1, b_2, b_3, \dots, b_n$ are coefficients of independent variables, and

226 y is output of the system.

227 To predict DRI using MR technique, actual DRI values are considered to be the product of the 2 input
228 parameters namely UCS and BTS. As a first step of MR modelling, all data should be normalized
229 considering the below equation:

$$230 \quad X_{norm} = (X - X_{min}) / (X_{max} - X_{min}) \quad (6)$$

231 Where,

232 X_{norm} is the normalized value of the measured parameters,

233 X , X_{min} and X_{max} are the measured, minimum and maximum values of the measured
234 parameters, respectively.

235 Afterwards, 5 datasets were chosen randomly to train and test for proposing MR models to evaluate the
236 capability of the purposed model for estimation of the DRI as suggested by Zorlu et al. (2008), Yagiz et
237 al. (2009). In the literature, 20% (Swingler 1996) of whole datasets and also a range of 20%-30%
238 (Nelson and Illingworth 1990) of whole datasets were recommended for testing of the system. Based on
239 above discussion, 80% (38 datasets) of whole datasets (47 datasets) was chosen randomly for developing

240 the models, whereas the remained 20% (9 datasets) of data was assigned for testing. It should be noted
 241 that, an ANN code was used for the selection of the random data. Using the built datasets, five multiple
 242 input equations were developed as shown in Table 3.

243 It is concluded that, the R^2 values range from 0.391 to 0.451 for training and 0.325 to 0.760 for testing of
 244 the MR models. In these models, UCS and BTS were considered as inputs and then, the DRI was
 245 estimated as function of them. As a result, it is found that there is no salient difference among the
 246 developed models. More details regarding evaluation of the developed MR equations are given later. Note
 247 that, simple and MR regression analysis were performed using statistical software package of SPSS
 248 version 16 (SPSS 2007).

249 **Table 3** MR equations together with the coefficient of correlation for testing and training

Dataset No.	Developed Relationship	Training	Testing
		R^2	R^2
1	$DRI = -0.342 \times UCS - 0.451 \times BTS + 0.845$	0.403	0.714
2	$DRI = -0.638 \times UCS - 0.315 \times BTS + 0.927$	0.451	0.325
3	$DRI = -0.370 \times UCS - 0.447 \times BTS + 0.892$	0.391	0.501
4	$DRI = -0.391 \times UCS - 0.493 \times BTS + 0.898$	0.430	0.344
5	$DRI = -0.406 \times UCS - 0.447 \times BTS + 0.879$	0.398	0.760

250
 251

252 4.3 ANN Modelling

253 In the ANN modeling, the same datasets utilized in the multiple regression part were performed. As
 254 mentioned by Kanellopoulas and Wilkinson (1997) and Hush (1989), ANN ability is directly related to its
 255 architecture. So, to design a desirable ANN model, determining the optimal architecture is needed. As a
 256 well-known fact, architecture of an ANN model is defined as the number of hidden layer or layers and the
 257 number of neuron or neurons in each hidden layer. Based on several scholars (e.g. Hecht-Nielsen 1987;

258 Hornik et al. 1989), hidden layer equal to 1 can approximate any complicated function. Then, hidden
 259 layer = 1 was chosen to construct the ANN networks. Additionally, Table 4 presents some of the available
 260 proposed equations for determining the number of neuron(s) together with their references. According to
 261 this table and considering $N_i = 2$ and $N_o = 1$, a range of 1-5 should be utilized in the hidden layer.

262 **Table 4** The proposed number of neuron for hidden layer (Sonmez et al. 2006)

Heuristic	Reference
$\leq 2 \times N_i + 1$	Hecht-Nielsen (1987), Caudill (1988)
$(N_i + N_o)/2$	Ripley (1993)
$\frac{2 + N_o \times N_i + 0.5 N_o \times (N_o^2 + N_i) - 3}{N_i + N_o}$	Paola (1994)
$2N_i / 3$	Wang (1994)
$\sqrt{N_i \times N_o}$	Masters (1994)
$2N_i$	Kaastra and Boyd (1996)
	Kannellopoulos and Wilkinson (1997)

N_i : number of input neuron, N_o : number of output neuron.

264 To determine the optimum number of neurons in the hidden layer, various ANN networks were modelled
 265 using one hidden layer and number of hidden neurons in the range of 1 to 5. The relevant results in terms
 266 of R^2 and RMSE can be seen in Tables 5 and 6, respectively. According to these tables, considering
 267 average R^2 and RMSE values of both training and testing datasets, model No. 3 with hidden neurons of 3
 268 outperforms the other ANN models. Therefore, 3 was selected as number of hidden neuron in
 269 constructing ANN models in this study. Levenberg–Marquardt (LM) learning algorithm was used in
 270 constructing ANN models. The efficiency of the LM algorithm in comparison with the other conventional
 271 gradient descent techniques has been highlighted in the study conducted by Hagan and Menhaj (1994).
 272 ANN results of model No. 3 (all five iterations) were considered as the best ANN results for predicting
 273 DRI. More explanations regarding the selecting the best ANN network are given later.

274

275

276

277 **Table 5** R² values of the constructed ANN models to predict DRI for selecting the optimum number of
 278 hidden node

Model No.	Nodes in hidden layers	Obtained Results of Network											
		Run 1		Run 2		Run 3		Run 4		Run 5		Average	
		R ²		R ²		R ²		R ²		R ²		R ²	
		Train ing	Testi ng	Train ing	Testin g	Train ing	Testin g	Train ing	Testi ng	Train ing	Testi ng	Train ing	Testin g
1	1	0.773	0.298	0.812	0.183	0.769	0.429	0.814	0.337	0.787	0.404	0.791	0.330
2	2	0.801	0.348	0.823	0.581	0.826	0.654	0.830	0.638	0.839	0.674	0.824	0.579
3	3	0.855	0.824	0.827	0.839	0.835	0.838	0.819	0.807	0.859	0.821	0.839	0.826
4	4	0.841	0.811	0.837	0.821	0.801	0.792	0.822	0.81	0.833	0.832	0.827	0.813
5	5	0.845	0.834	0.811	0.838	0.817	0.820	0.829	0.815	0.842	0.809	0.829	0.823

279

280 **Table 6** RMSE values of the constructed ANN models to predict DRI for selecting the optimum number
 281 of hidden node

Model No.	Nodes in hidden layers	Obtained Results of Network											
		Run 1		Run 2		Run 3		Run 4		Run 5		Average	
		RMSE		RMSE		RMSE		RMSE		RMSE		RMSE	
		Train ing	Testi ng	Train ing	Testin g	Train ing	Testin g	Train ing	Testi ng	Train ing	Testi ng	Train ing	Test
1	1	0.135	0.319	0.137	0.274	0.155	0.251	0.149	0.231	0.132	0.244	0.142	0.264
2	2	0.134	0.460	0.148	0.252	0.146	0.226	0.140	0.211	0.144	0.144	0.142	0.259
3	3	0.157	0.106	0.157	0.090	0.130	0.122	0.144	0.108	0.106	0.088	0.139	0.103
4	4	0.162	0.111	0.172	0.096	0.122	0.134	0.139	0.129	0.145	0.110	0.148	0.116
5	5	0.160	0.092	0.170	0.103	0.133	0.125	0.141	0.131	0.128	0.134	0.146	0.117

282

283 4.4 GA-ANN Modelling

284 As mentioned before, GA can efficiently improve the ANN performance and remove its limitations (e.g.
 285 [Lee et al. 1991](#); [Majdi and Beiki 2010](#); [Rashidian and Hassanlourad 2013](#)). The most frequently-cited
 286 advantage of GAs is the capability of these algorithms in escaping from being trapped in a local optimum
 287 ([Chambers 2010](#)). [Chambers \(2010\)](#) showed that with the use of a GA or at least a hybrid GA, an
 288 appropriate objective function can be freely selected. It can be concluded that the network connection

289 weights and biases are optimized with GA instead of random generation. The hybrid GA-based ANN
290 model can be referred to (Hagan and Menhaj 1994) for more details.

291 To propose hybrid GA-ANN model for DRI prediction, the most influential GA parameters should be
292 designed. To do this, several parametric investigations were carried out to find optimum GA parameters.
293 In the hybrid GA-ANN model, the mutation probability was set to 25% of the population size; whereas
294 the percentage of recombination was fixed at 9% and value of 1% was applied as utilized by Momeni et
295 al. (2014). The single point cross-over was used with 70% possibility. Numerous selection methods have
296 been proposed in the literatures regarding cross-over operation; however, the tournament selection
297 method was employed to generate two offspring from two parents (Momeni et al. 2014). It should be
298 mentioned that the mutation probability and cross-over possibility were determined using trial-and-error
299 method.

300 Finding the best population size is the next step of the hybrid GA-ANN. In this regard, several GA-ANN
301 models were built with population sizes in range of 25 to 600 as shown in Table 7. In these models, the
302 suggested ANN architecture and maximum generation of 100 were utilized. In the Table 7, the R^2 and
303 RMSE values were tabulated for training and testing datasets of each model. Generally, increment in
304 population size causes the increase in R^2 values and decrease in RMSE values. Since selection of the best
305 model is too difficult, a simple ranking method proposed by Zorlu et al. (2008) was performed to obtain
306 the optimum population size. Based on this method, each performance index was ordered in its class and
307 the best performance index was assigned the highest rating. As an example, R^2 values of 0.426, 0.437,
308 0.425, 0.386, 0.420, 0.372, 0.443, 0.480, 0.661, 0.753, 0.842, 0.921, 0.927 and 0.931 were obtained for
309 training datasets of models 1 to 14, respectively. Hence, their ratings were assigned as 5, 6, 4, 1, 3, 2, 7, 8,
310 9, 10, 11, 12, 13 and 14, respectively. This procedure was repeated for results of RMSE as well. After
311 this process, the obtained ratings of performance indices for training and testing datasets were summed up
312 in each model as shown in the last column of Table 7 (total rank). Based on obtained total rank values,

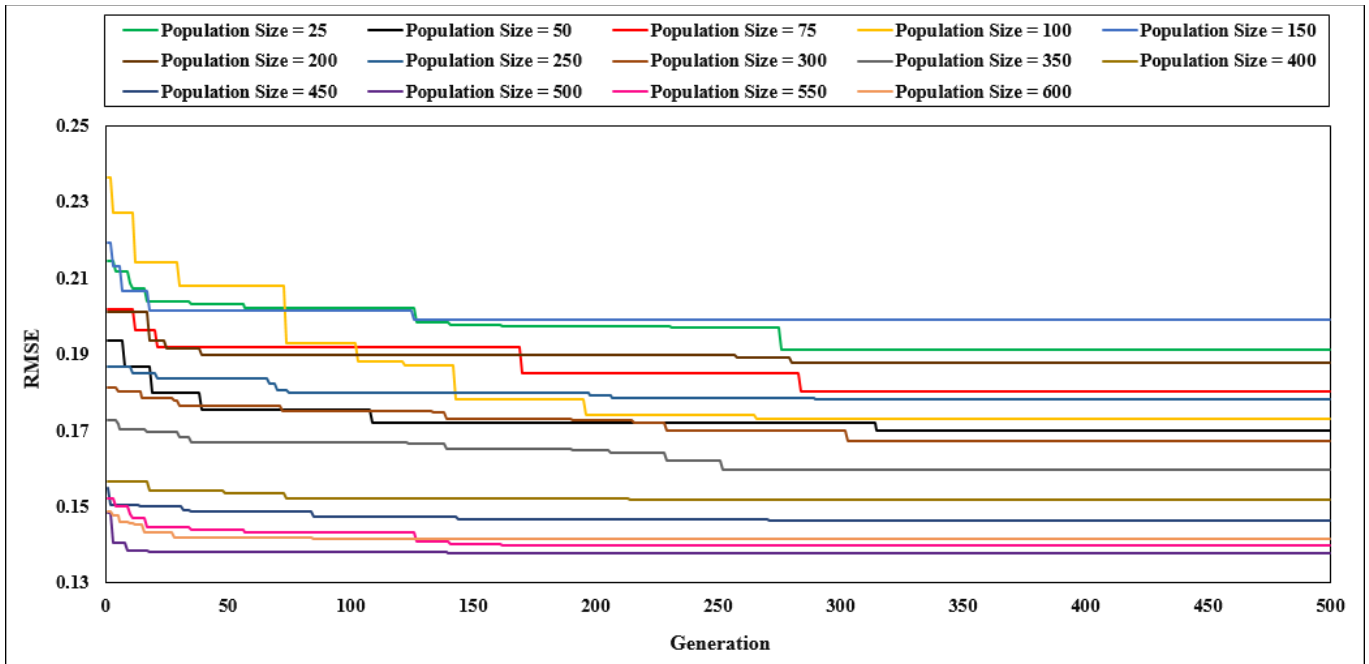
313 model No. 12 can provide higher performance capacity compared to other models. Therefore, population
 314 size of 500 was chosen in modeling of GA-ANN technique.

315 **Table 7** Effects of population size on network performance

Model No.	Population Size	Network Result				Ranking				Total Rank
		Train		Test		Train		Test		
		R ²	RMSE	R ²	RMSE	R ²	RMSE	R ²	RMSE	
1	25	0.426	0.200	0.223	0.217	5	5	1	2	13
2	50	0.437	0.194	0.561	0.181	6	7	6	4	23
3	75	0.425	0.182	0.341	0.253	4	8	2	1	15
4	100	0.386	0.195	0.486	0.213	1	6	4	3	14
5	150	0.420	0.201	0.506	0.175	3	4	5	5	17
6	200	0.372	0.207	0.658	0.153	2	2	9	7	20
7	250	0.443	0.203	0.456	0.158	7	3	3	6	19
8	300	0.480	0.182	0.632	0.147	8	8	8	9	33
9	350	0.661	0.171	0.592	0.152	9	9	7	8	33
10	400	0.753	0.154	0.778	0.137	10	10	10	11	41
11	450	0.842	0.148	0.851	0.131	11	11	11	13	46
12	500	0.921	0.139	0.932	0.122	12	13	14	14	53
13	550	0.927	0.140	0.921	0.135	13	12	13	12	50
14	600	0.931	0.138	0.913	0.144	14	14	12	10	50

316

317 Determination of maximum number of generation (G_{max}) is the next step of GA-ANN modelling
 318 procedure. To recognize the effect of G_{max} on the network's performance, one more parametric study was
 319 conducted. The number of generation was set to be 500 in order to determine the optimum number of
 320 generation. To do this, 14 models presented in Table 7 were constructed again using the mentioned
 321 maximum generation number (500). Figure 8 shows the importance of the number of generation to the
 322 network performance for predicting DRI. As displayed in this figure, there is no changes in the network
 323 performance (RMSE) after generation number = 300. Hence, the optimum number of generation was set
 324 to be 300 in design of GA-ANN models. It is worth mentioning that in determining number of generation,
 325 the other mentioned network parameters were kept constant.



326

Figure 8. The effect of the number of generation on the network performance

327

328 In the final step of hybrid GA-ANN modelling, by using three different factors namely the suggested
 329 ANN structure ($2 \times 3 \times 1$), 5 randomly selected datasets, and determined GA parameters, five hybrid
 330 models were constructed. Evaluation of the obtained results of the hybrid models together with its
 331 discussion will be given later.

332

333 5. Evaluation of the Results

334 In this study, several techniques i.e. multiple regression, ANN and GA-ANN were applied and proposed
 335 for DRI prediction. Here, all 47 datasets were randomly selected to 5 datasets (to training and testing
 336 purposes) for developing the linear and non-linear models. For evaluation of the prediction performance,
 337 several performance indices including R^2 , amount of variance account for (VAF) and RMSE were
 338 considered and computed:

339
$$R^2 = 1 - \frac{\sum_{i=1}^N (y-y')^2}{\sum_{i=1}^N (y-\bar{y})^2} \quad (7)$$

340
$$VAF = [1 - \frac{\text{var}(y-y')}{\text{var}(y)}] \times 100 \quad (8)$$

341
$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y - y')^2} \quad (9)$$

342 Where,

343 y, y' and \bar{y} are the measured, predicted and mean of the y values respectively,

344 N is the total number of data and

345 P is the number of predictors.

346 Results of the mentioned indices for training and testing datasets are tabulated in Table 8. As shown in
 347 this table, selecting the best model for DRI estimation is not easy. To overcome this difficulty, as
 348 mentioned before, a simple ranking procedure developed by Zorlu et al. (2008) was used. A ranking value
 349 was computed and assigned for each training and testing dataset separately (see Table 8). **The obtained**
 350 **total rank results for the developed models are shown in Table 9.** Based on Table 9, model No. 4
 351 exhibited the best performance of DRI prediction for MR technique, while models No. 5 and 1 yielded the
 352 best results of ANN and GA-ANN techniques, respectively. **Therefore, the hybrid GA-ANN models** can
 353 provide higher prediction performances in predicting DRI compared to other developed models (ANN
 354 and MR). The selected MR equation (model No. 4) is shown as follows:

355
$$DRI = -0.391 \times UCS - 0.493 \times BTS + 0.898 \quad (10)$$

356

357

358

359

360 **Table 8** The obtained performance indices for training antesting and their ranges for proposed models

Method	Model	R ²	RMSE	VAF	Rating for R ²	Rating for RMSE	Rating for VAF	Rank value
MR	Training 1	0.403	0.201	40.304	3	4	3	10
	Training 2	0.451	0.205	45.051	5	2	5	12
	Training 3	0.391	0.202	39.065	1	3	1	5
	Training 4	0.430	0.199	43.028	4	5	4	13
	Training 5	0.398	0.221	39.750	2	1	2	5
	Testing 1	0.714	0.204	45.537	4	1	3	8
	Testing 2	0.325	0.185	0.539	1	3	1	5
	Testing 3	0.501	0.195	48.802	3	2	4	9
	Testing 4	0.344	0.150	34.134	2	4	2	8
	Testing 5	0.760	0.093	71.843	5	5	5	15
ANN	Training 1	0.855	0.157	85.466	4	2	4	10
	Training 2	0.827	0.157	82.576	2	2	2	6
	Training 3	0.835	0.130	83.419	3	4	3	10
	Training 4	0.819	0.144	81.438	1	3	1	5
	Training 5	0.859	0.106	85.934	5	5	5	15
	Testing 1	0.824	0.106	82.316	3	3	3	9
	Testing 2	0.839	0.090	83.882	5	4	5	14
	Testing 3	0.838	0.122	82.773	4	1	4	9
	Testing 4	0.807	0.108	77.351	1	2	1	4
	Testing 5	0.821	0.088	82.061	2	5	2	9
GA-ANN	Training 1	0.933	0.071	93.066	3	4	4	11
	Training 2	0.926	0.111	92.090	2	2	2	6
	Training 3	0.948	0.066	94.772	5	5	5	15
	Training 4	0.937	0.076	93.046	4	3	3	10
	Training 5	0.921	0.119	91.909	1	1	1	3
	Testing 1	0.940	0.077	94.037	3	3	4	10
	Testing 2	0.945	0.058	94.457	4	4	5	13
	Testing 3	0.935	0.090	92.341	2	2	1	5
	Testing 4	0.929	0.053	92.845	1	5	2	8
	Testing 5	0.946	0.098	93.636	5	1	3	9

361

362

363

364

365

366

367

368

369

370

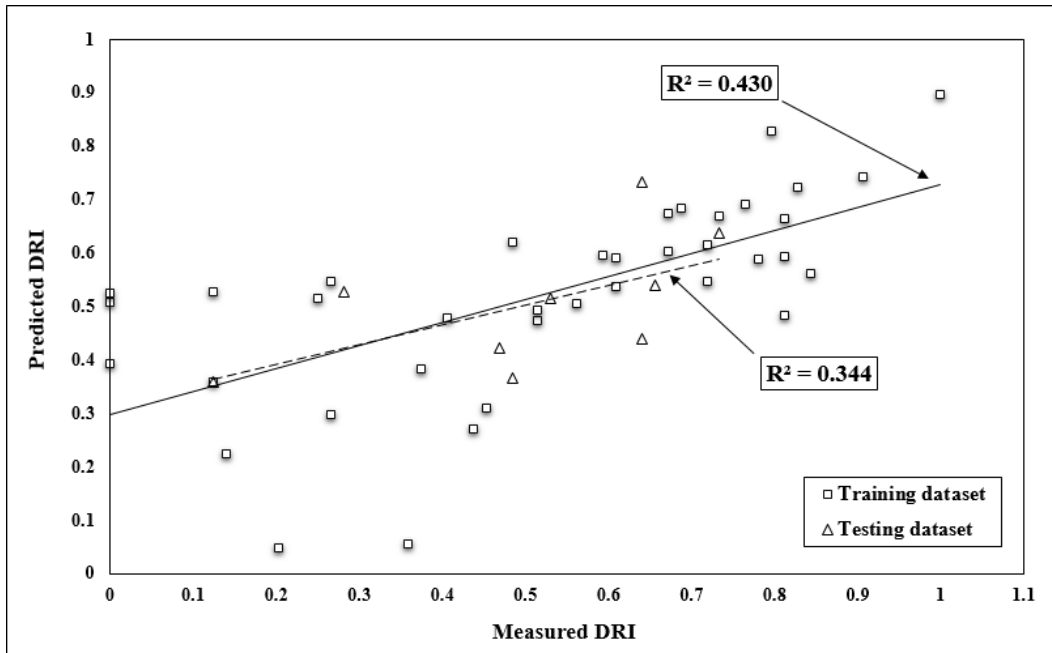
371

Table 9 Obtained total rank results for the developed models

Method	Model	Total rank
MR	1	18
	2	17
	3	14
	4	21
	5	20
ANN	1	19
	2	20
	3	19
	4	9
	5	24
GA-ANN	1	21
	2	19
	3	20
	4	18
	5	12

372

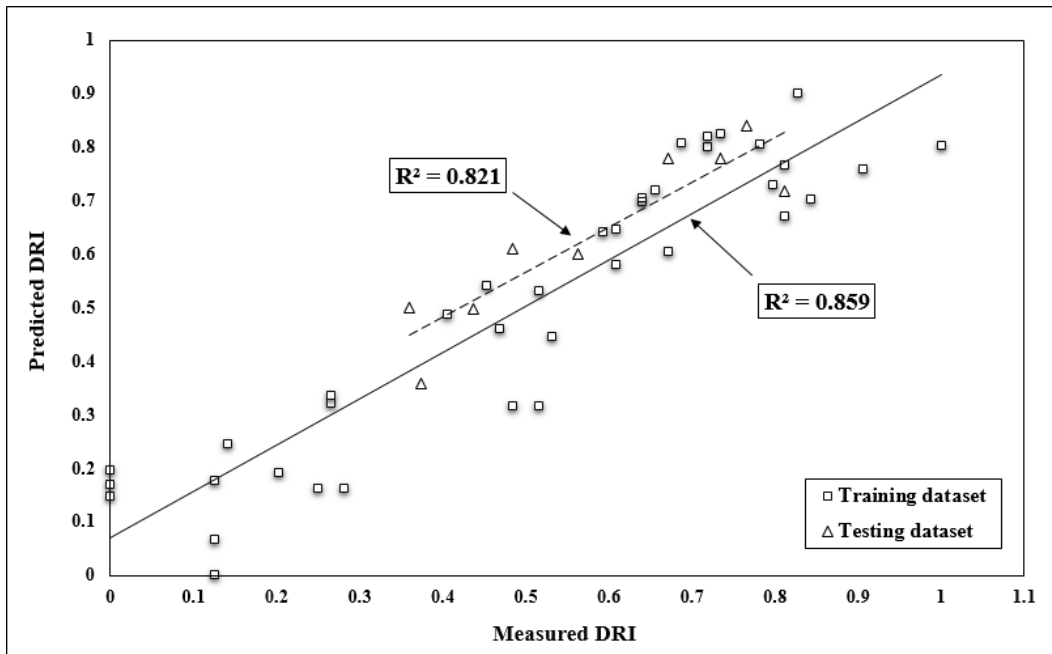
373 The graphs of predicted DRI using the MR, ANN and GA-ANN techniques against the measured DRI for
374 training and testing datasets are shown in Figures 9 to 11, respectively. Based on the presented figures,
375 the GA-ANN model can perform better in estimating DRI compared to other proposed models. Based on
376 these figures, the R^2 equal to 0.940 for testing dataset suggests the superiority of the hybrid GA-ANN
377 model, while these values are 0.821 and 0.344 for ANN and MR models, respectively. This shows the
378 capability of the hybrid GA-ANN technique to predict DRI.



379

380

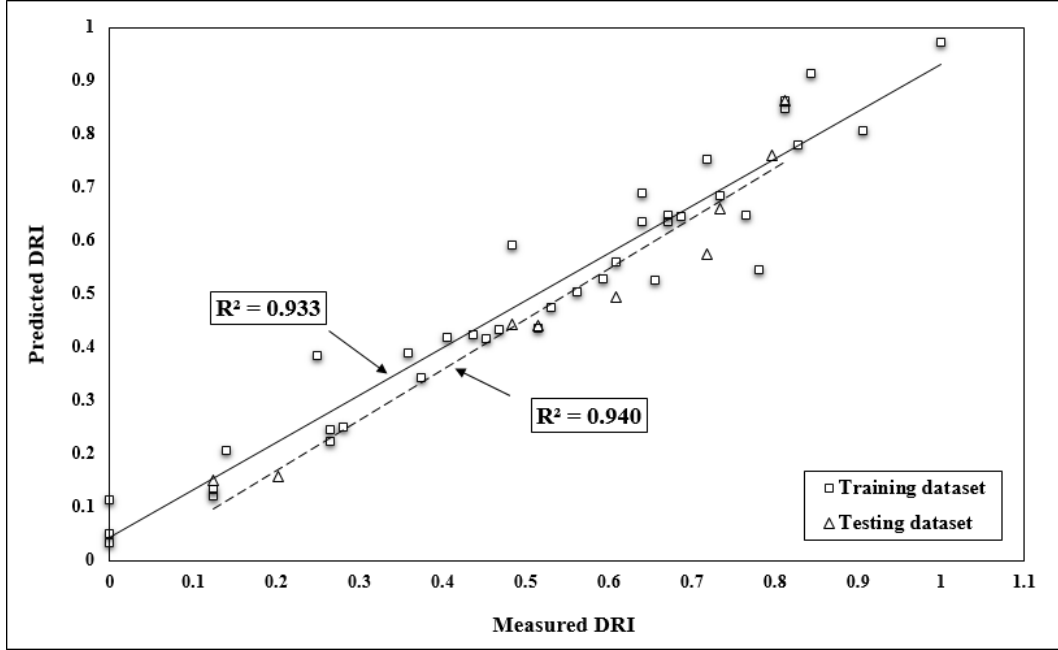
Figure 9. Predicted DRI values by MR model against the Measured DRI



381

382

Figure 10. Predicted DRI values by ANN model against the Measured DRI



383

384

Figure 11. Predicted DRI values by GA-ANN model against the Measured DRI

385 **6. Sensitivity Analysis**

386 In this study, sensitivity analysis was performed to investigate the impacts of each input parameter on the
 387 output(s) using the cosine amplitude method (Yang and Zang 1997). All data pairs were utilized to
 388 construct a data array X as follows:

389
$$X = \{x_1, x_2, x_3, \dots, x_i, \dots, x_n\} \quad (11)$$

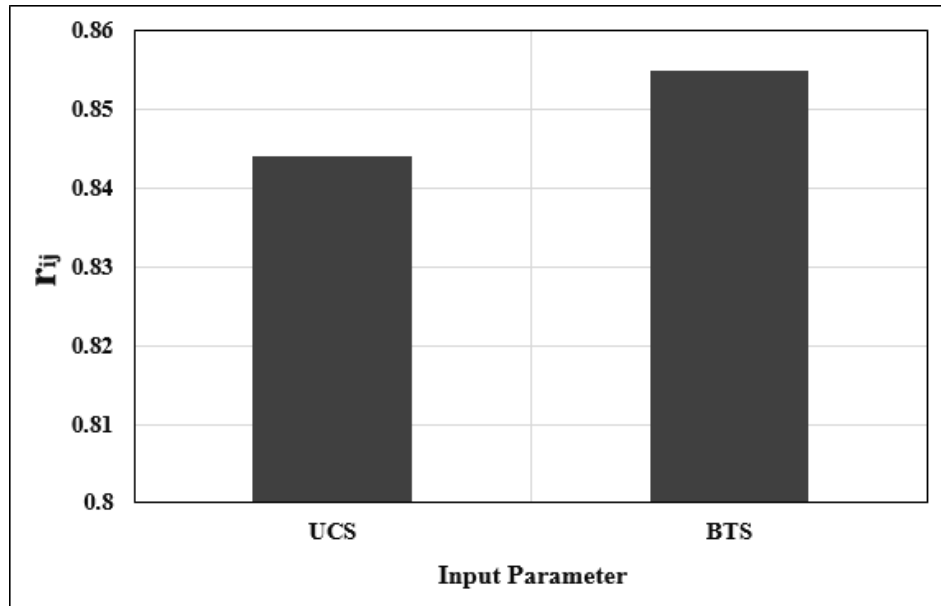
390 Variable x_i in array X is a length vector of m as:

391
$$x_i = \{x_{i1}, x_{i2}, x_{i3}, \dots, x_{im}\} \quad (12)$$

392 The strength of the relationship (r_{ij}) between datasets X_i and X_j can be expressed as follows:

393
$$r_{ij} = \frac{\sum_{k=1}^m x_{ik}x_{jk}}{\sqrt{\sum_{k=1}^m x_{ik}^2 \sum_{k=1}^m x_{jk}^2}} \quad (12)$$

394 Figure 12 displays the strengths of the relationships between the input variables and output (DRI). The
395 results show that among UCS and BTS, BTS is the most effective factor on the DRI.



396

397 **Figure 12.** The effect of input parameters on the DRI

398 7. Conclusions

399 In this study, an attempt has been made to predict DRI by using strength properties of rock. To achieve
400 this aim, DRI tests were conducted in the laboratory. In order to estimate DRI, two strength properties of
401 rock namely UCS and BTS were chosen as model inputs. Based on simple regression models, the
402 relationship between the DRI and input variables are acceptable and meaningful. Since each mentioned
403 parameter has good relationship with the DRI, multiple regression, ANN and GA-ANN models were also
404 generated to achieve the best accurate result.

405 In order to develop multiple-input models, the established datasets were divided into training and testing
406 parts as suggested in the literature. Further, five different dataset for training and testing were established
407 randomly to obtain the best models for each modeling technique. Developed models are compared to each
408 other for choosing the best model among them. For selecting the best model, obtained R^2 and total rank

409 for each model were computed and compared. As considering the testing datasets, the prediction
410 performance of the GA-ANN model ($R^2 = 0.940$) is higher than that of the ANN model ($R^2 = 0.821$) and
411 MR ($R^2 = 0.344$). Also, on taking into considering the training datasets, similar results were also obtained
412 ($R^2 = 0.430$; 0.859 ; 0.933 , respectively). **It was found that the hybrid GA-ANN technique shows the best**
413 **result compared to other models. Additionally, results of sensitivity analysis showed that the effect of**
414 **BTS on DRI is slightly higher than the effect of UCS.**

415 **References**

- 416 **Adebayo B, Opafunso Z O, Akande J M (2010), Drillability and strength characteristics of selected rocks**
417 **in Nigeria. AU Journal of Technology 14(1):56-60**
- 418 Aghajanloo MB, Sabziparvar AA, Talaei PH (2013) Artificial neural network–genetic algorithm for
419 estimation of crop evapotranspiration in a semi-arid region of Iran. *Neural Comput Appl* 23(5):
420 1387-1393
- 421 **Akin S, Karpuz C (2008) Estimating drilling parameters for diamond bit drilling operations using**
422 **artificial neural networks. Int J Geomech 8(1):68-73**
- 423 **Arabjamaloei R, Karimi Dehkordi B (2012) Investigation of the Most Efficient Approach of the**
424 **Prediction of the Rate of Penetration. Energy Sources, Part A: Recovery, Utilization, and**
425 **Environmental Effects 34(7):581-590**
- 426 Ataei M, KaKaie R, Ghavidel M, Saeidi O (2015) Drilling rate prediction of an open pit mine using the
427 rock mass drillability index. *Int J Rock Mech Min Sci* 73:130-138
- 428 **Basarir H, Tutluoglu L, Karpuz C (2014) Penetration rate prediction for diamond bit drilling by adaptive**
429 **neuro-fuzzy inference system and multiple regressions. Eng Geol 173:1-9**
- 430 Bruland A (1998) *Drillability Test Methods*. Trondheim: NTNU
- 431 Caudill M (1988) Neural networks primer part III. *AI Expert* 3:53–9
- 432 Chambers LD (2010) *Practical Handbook of Genetic Algorithms: Complex Coding Systems*, CRC Press

- 433 Cheniany A, Hasan KS, Shahriar K, Hamidi JK (2012) An estimation of the penetration rate of rotary
434 drills using the Specific Rock Mass Drillability index. *Int J Rock Mech Min Sci* 22:187-193
- 435 Chipperfield A, Fleming P, Pohlheim H (2006) Genetic algorithm toolbox for use with MATLAB User's
436 guide. version 1.2. University of Sheffield
- 437 Dahl F (2003) DRI, BWI, CLI Standards. NTNU, Angleggsdrift, Trondheim
- 438 Dahl F, Bruland A, Jakobsen PD, Nilsen B, Grøv E (2012) Classifications of properties influencing the
439 drillability of rocks, based on the NTNU/SINTEF test method. *Tunnel Undergr Sp Technol*
440 28:150-158
- 441 Dreyfus G (2005) *Neural Networks: Methodology and Application*, Springer Berlin Heidelberg, Germany
- 442 **Ekincioglu G, Altindag R, Sengun N, Demirdag S, Guney A (2013) The relationships between drilling**
443 **rate index (DRI), physico-mechanical properties and specific cutting energy for some carbonate**
444 **rocks, *Rock Mechanics for Resources, Energy and Environment*, Taylor & Francis group,**
445 **London, 867-873**
- 446 Garrett J (1994) Where and why artificial neural networks are applicable in civil engineering. *J Comput*
447 *Civil Eng* 8:129–130
- 448 Hagan MT, Menhaj MB (1994) Training feed forward networks with the Marquardt algorithm, *IEEE*
449 *Trans. Neural Networks* 5:861– 867
- 450 Hajihassani M., Jahed Armaghani D., Marto A., Tonnizam Mohamad E (2014) Ground vibration
451 prediction in quarry blasting through an artificial neural network optimized by imperialist
452 competitive algorithm. *Bull Eng Geol Environ* doi:10.1007/s10064-014-0657-x
- 453 Hecht-Nielsen R (1987) Kolmogorov's mapping neural network existence theorem. In: *Proceedings of the*
454 *First IEEE International Conference on Neural Networks*, San Diego, CA, USA, pp 11–14
- 455 Holland J (1975) *Adaptation in Natural and Artificial Systems*, The University of Michigan Press, Ann
456 Arbor
- 457 Hornik K, Stinchcombe M, White H (1989) Multilayer feedforward networks are universal
458 Approximators. *Neural Networks* 2:359-366

- 459 Hoseinie SH, Aghababaei H, Pourrahimian Y (2008) Development of a new classification system for
460 assessing of rock mass drillability index (RDi). *Int J Rock Mech Min Sci* 45:1-10
- 461 Hoseinie SH, Ataei M, Osanloo M (2009) A new classification system for evaluating rock penetrability.
462 *Int J Rock Mech Min Sci* 46:1329-1340
- 463 Hush DR (1989) Classification with neural networks: a performance analysis. In: Proceedings of the
464 IEEE International Conference on Systems Engineering. Dayton, OH, USA, pp 277–280
- 465 ISRM (1978) Suggested methods for determining tensile strength of rock materials. *Int. J. Rock Mech.*
466 *Min. Sci. Geomech. Abstr* 15:101-103
- 467 ISRM (1979) Suggested methods for determining the uniaxial compressive strength and deformability of
468 rock materials. *Int J Rock Mech Min Sci Geomech Abstr* 16:135-140
- 469 Jaeger JC (1967) Failure of rocks under tensile strength. *Int J Rock Mech Min Sci* 4: 219-227
- 470 Jahed Armaghan D, Tonnizam Mohamad E, Hajihassani M, Alavi Nezhad Khalil Abad SV, Marto A,
471 Moghaddam MR (2015b) Evaluation and prediction of flyrock resulting from blasting operations
472 using empirical and computational methods. *Eng Comput* doi:10.1007/s00366-015-0402-5
- 473 Jahed Armaghani D, Hajihassani M, Sohaei H, Mohamad ET, Marto A, Motaghedi H, Moghaddam MR
474 (2015c) Neuro-fuzzy technique to predict air-overpressure induced by blasting. *Arab J Geosci* 1-
475 14
- 476 Jahed Armaghani D, Momeni E, Alavi Nezhad Khalil Abad SV, Khandelwal M (2015a) Feasibility of
477 ANFIS model for prediction of ground vibrations resulting from quarry blasting. *Environ Earth*
478 *Sci.* DOI 10.1007/s12665-015-4305-y
- 479 Kaastra I, Boyd M (1996) Designing a neural network for forecasting financial and economic time series.
480 *Neurocomputing* 10:215-36
- 481 Kahraman S (1999) Rotary and percussive drilling prediction using regression analysis. *Int J Rock Mech*
482 *Min Sci* 36: 981–989

483 Kahraman S, Balcı C, Yazıcı S, Bilgin N (2000) Prediction of the penetration rate of rotary blast hole
484 drills using a new drillability index. *Int J Rock Mech Min Sci* 37:729-743

485 Kahraman S, Bilgin N, Feridunoglu C (2003) Dominant rock properties affecting the penetration rate of
486 percussive drills. *Int J Rock Mech Min Sci* 40:711–723

487 Kanellopoulos I, Wilkinson GG (1997) Strategies and best practice for neural network image
488 classification. *Int J Remote Sens* 18:711–725

489 Karpuz C, Pasamehmetoglu AG, Dincer T, Muftuoglu Y (1990) Drillability studies on the rotary blast
490 hole drilling of lignite overburden series. *Int. J Surf Min Recl* 4:89–93

491 **Khandelwal M (2013) Correlating P-wave velocity with the physico-mechanical properties of different
492 rocks. *Pure and Applied Geophysics* 170(4):507-514**

493 Khandelwal M, Monjezi M (2013) Prediction of backbreak in open-pit blasting operations using the
494 machine learning method. *Rock Mech Rock Eng* 46(2):389-396

495 **Khandelwal M, Ranjith P G (2010) Correlating index properties of rocks with P-wave measurements.
496 *Journal of Applied Geophysics* 71(1):1-5**

497 Lee Y, Oh SH, Kim MW (1991) The effect of initial weights on premature saturation in back-propagation
498 learning, In: *Proceedings of the International Joint Conference on Neural Networks* 765–770

499 Macias FJ, Jakobsen PD, Seo Y, Bruland A (2014) Influence of rock mass fracturing on the net
500 penetration rates of hard rock TBMs. *Tunnel Undergr Sp Technol* 44:108-120

501 Majdi A, Beiki M (2010) Evolving neural network using a genetic algorithm for predicting the
502 deformation modulus of rock masses, *Int J Rock. Mech. Min Sci* 47:246–253

503 Masters T (1994) *Practical neural network recipes in C++*. Boston MA: Academic Press

504 Matern N von, Hjelmer A (1943) Försök med pågrus (“Tests with Chippings”), *Medelande nr. 65*,
505 *Statens väginstitut, Stockholm*, 65 pp. (English summary, pp. 56–60).

- 506 Moein MJA, Shaabani E, Rezaeian M (2014) Experimental evaluation of hardness models by drillability
507 tests for carbonate rocks. *J Petroleum Sci Eng* 113:104-108
- 508 Momeni E, Nazir R, Jahed Armaghani D, Maizir H (2014) Prediction of pile bearing capacity using a
509 hybrid genetic algorithm-based ANN. *Measurement* 57:122-131
- 510 Monjezi M, Khoshalan HA, Varjani AY (2012). Prediction of flyrock and backbreak in open pit blasting
511 operation: a neuro-genetic approach. *Arab J Geosci* 5(3):441-448
- 512 Nelson M, Illingworth WT (1990) *A Practical Guide to Neural Nets*. Addison- Wesley, Reading MA
- 513 Paola JD (1994) *Neural network classification of multispectral imagery*. MSc thesis, The University of
514 Arizona, USA
- 515 Rashidian V, Hassanlourad M (2013) Predicting the shear behavior of cemented and uncemented
516 carbonate sands using a genetic algorithm-based artificial neural network. *Geotech. Geol Eng*
517 2:1–18
- 518 Ripley BD (1993) Statistical aspects of neural networks. In: Barndoff- Neilsen OE, Jensen JL, Kendall
519 WS, editors. *Networks and chaos-statistical and probabilistic aspects*. London: Chapman & Hall,
520 pp 40-123
- 521 Saemi M, Ahmadi M, Varjani AY (2007) Design of neural networks using genetic algorithm for the
522 permeability estimation of the reservoir. *J Petroleum Sci Eng* 59:97-105
- 523 Schmidt RL (1972) *Drillability Studies – Percussive Drilling in the Field*, US Bureau of Mines RI 7684
- 524 Selim AA, Bruce WE (1970) Prediction of penetration rate for percussive drilling. USBM. RI; p 7396
- 525 Selmer-Olsen R, Lien R (1960) Bergartens borbarhet og sprengbarhet, *Teknisk Ukeblad*, 34, Oslo, pp 3–
526 11
- 527 Sievers H (1950) Die Bestimmung des Bohrwiderstandes von Gesteinen, *Glückauf* 86: 37/38, pp 776–
528 784. Glückauf G.M.B.H., Essen
- 529 Simpson P (1990) *Artificial Neural System: Foundation, Paradigms, Applications and Implementations*,
530 Pergamon, New York

531 Sonmez H, Gokceoglu C, Nefeslioglu HA, Kayabasi A (2006) Estimation of rock modulus: for intact
532 rocks with an artificial neural network and for rock masses with a new empirical equation. Int. J.
533 Rock Mech Min Sci 43:224-235

534 SPSS Inc. (2007). SPSS for Windows (Version 16.0). Chicago: SPSS Inc

535 Swingler K (1996) Applying Neural Networks: A Practical Guide. Academic Press, New York

536 Tripathy A, Singh TN, Kundu J (2015) Prediction of abrasiveness index of some Indian rocks using soft
537 computing methods. Measurement 68: 302-309

538 Wang C (1994) A theory of generalization in learning machines with neural application. PhD thesis, The
539 University of Pennsylvania, USA

540 Wijk G (1989) The stamp test for rock drillability classification. Int J Rock Mech Min Sci Geomech
541 Abstracts 26:37-44

542 Yagiz S, Gokceoglu C, Sezer E, Iplikci S (2009) Application of two non-linear prediction tools to the
543 estimation of tunnel boring machine performance. Eng Appl Artif Intel 22(4): 808-814

544 **Yang Y, Zang O (1997) A hierarchical analysis for rock engineering using artificial neural networks.**
545 **Rock Mech Rock Eng 30:207–222**

546 Yarali O, Kahraman S (2011) The drillability assessment of rocks using the different brittleness
547 values. Tunnel Undergr Sp Technol 26:406-414

548 Yarali O, Soyer E (2013) Assessment of relationships between drilling rate index and mechanical
549 properties of rocks. Tunnel Undergr Sp Technol 33:46-53

550 Zorlu K, Gokceoglu C, Ocakoglu F, Nefeslioglu HA, Acikalin S (2008) Prediction of uniaxial
551 compressive strength of sandstones using petrography-based models. Eng Geol 96(3):141-158