A numerical control algorithm for a B-double truck-trailer with steerable trailer wheels and active hitch angles, Part II: Reversing.

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Abstract

The authors have previously proposed a solution to the twin problems of wheel scuffing and off tracking of B-Double truck-trailer vehicles thereby reducing tyre wear and environmental damage as well as improving maneuverability. The solution to the scuffing problem requires that trailer axles in excess of one per trailer must have steerable wheels. However if all trailer wheels are steerable then the off tracking problem can also be solved. The previous work devised an algorithm for a B-Double in forward motion, whereby an on board computer would be used to calculate the correct wheel and hitch angles and a control system would implement these angles.

The purpose of the present technical note is to complete the study of a numerical algorithm for navigating a B-double truck-trailer vehicle by considering travel in the reverse direction. In this case the angle of the front wheels of the truck must also be controlled by the on board computer. The algorithm for determining the effective angle of the truck's steerable wheels is derived using an innovative combination of vector geometry and calculus and completes the total control system for these B-double vehicles.

The paper concludes with a simulation study of the control algorithm demonstrating its versatility for reversing along twisting paths and effectiveness in reducing off-tracking.

KEYWORDS: B-double truck-trailer, reversing, control system, off-tracking, cooperative redundancy.

1 Introduction

The problem of accurately reversing a truck-trailer system has been studied by many researchers. However, the authors have found no reference in the literature to control systems for reversing B-double truck-trailer systems. Most systems involve a single trailer with a single axle. Some systems involve multiple trailers [1, 2] but again, each trailer has only one axle and one pair of wheels. Control is often for predetermined paths and uses sensors to determine hitch angles as the mechanism by which steering is effected. The modern control systems used are based on neural networks, fuzzy logic, genetic or learning algorithms (see [3, 4] for surveys of the literature). Other methods and systems have been experimented with to reduce off-tracking such as sliding kingpins [5] or the "trackaxle self-steering system" [6] but these have not been tested in reverse.

In [7] a different point of view is taken and the trailer wheels of a B-double truck-trailer system are allowed to be steerable. The truck is a standard type with front wheels using Ackerman steering and rear wheels on fixed axles. A numerical algorithm was given, geometrical in construction, to determine the effective wheel angles for trailers that minimize off-tracking by ensuring that a sequence of reference points all traverse the same path. The term "effective wheel angles" acknowledges the slip angle correction that would need to be accounted for at all but slow speeds as is discussed in [7]. The reference points chosen to minimize off-tracking were the truck hitch point, first trailer hitch point and the middle point of the middle axle of the second trailer. Scuffing is minimized by ensuring all wheels are directed tangentially to their path. The algorithm was given for the forward direction only.

B-double truck-trailers are not symmetrical in terms of forwarding and reversing navigation. In forward motion, the truck prescribes the path and pulls the trailers. In reverse motion, the truck pushes the trailers and this is an unstable inverted pendulum system for standard fixed axle trailers. Allowing the trailers to have steerable wheels effectively removes the instability of control when reversing with multiple trailers. This stabilization provides an added benefit to the minimal off-tracking and reduced scuffing because any errors that may occur will not be unstable.

For reverse motion the master reference is the rear reference point and the slave reference points are the second hitch point and the truck hitch point. The path and speed of the master reference point is not predetermined; it is controlled by the operator who selects the curvature of the path and the speed of translation of the rear reference point along that path. This may be done with a joystick or small steering wheel from outside the vehicle or with the aid of a reversing camera fitted to the rear of the last trailer. External "control by wire" of articulated vehicles has been successfully tested for operational performance [8] as well as systems of control by camera vision [9]. The trailer wheel angles are controlled by electric or hydraulic actuators. The truck front wheel angles could be controlled in a similar manner to GPS controlled tractors. Here a rubber wheel is pressed against the steering wheel rim and turned with a computer controlled electric motor. Alternatively the correct angles could be implemented through the power steering system especially if the power steering is electric.

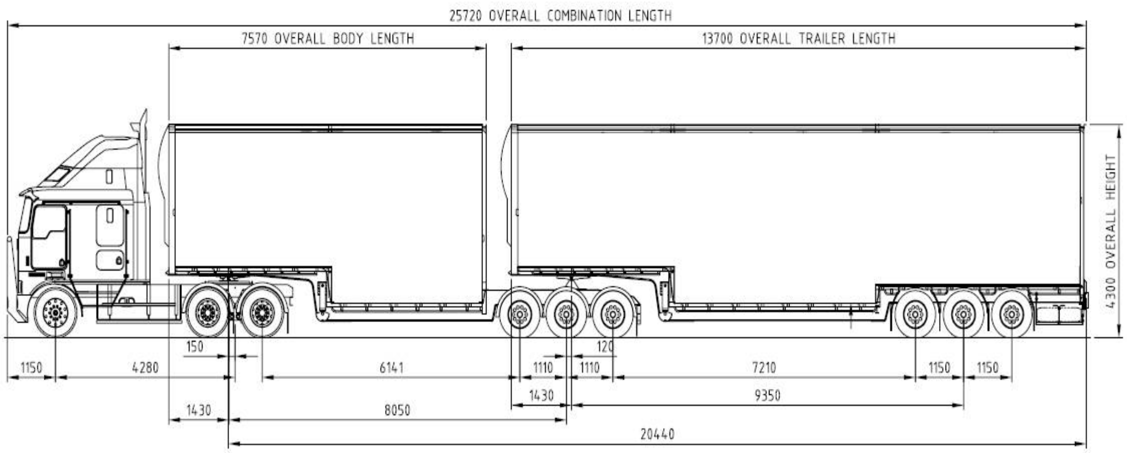


Figure 1: Typical dimensions, of a B-double truck-trailer vehicle. Lengths are in mm.

Moreover, a further level of redundant control can be achieved by control of active hitch angles between trailers and truck and first trailer. This control by hydraulic actuator implements the calculated correct hitch angle, reinforcing the steering control by wheel angle and providing a mechanism to prevent jack-knifing. The algorithm for reversing may be incorporated with the program for forward navigation with a simple if/else command dependent on speed being negative or positive.

Although trailer wheel algorithms are similar to that of forward case the calculation of truck wheel angles is completely different and does not rely on the centre of curvature

(COC) of the truck. An interplay of vector calculus and analytical geometry shows the angle between tangents to the hitch path and the nominal central front wheel path determines that nominal wheel angle. This angle is given by $\alpha = \arctan((b-d)\kappa_1)$ using the notation of [7] shown in Figure 2 and with κ_1 the path curvature at the truck hitch point and $b-d$ the distance from truck hitch to front axle.

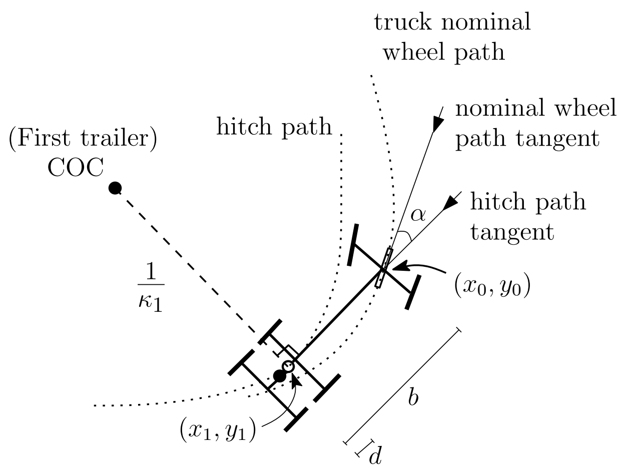


Figure 2: Trucks steering angle and radius of curvature

Simulation results are provided in section 3 verifying the effectiveness of the algorithm in reducing off-tracking for a complicated reversing procedure.

2 Calculating wheel angles and hitch angles

Many aspects of the calculation of wheel angles for the reversing B-double require minor modification to the forwarding case the details of which are given in [7]. For this reason, a brief description of how they are calculated is given but the equations left to be derived by the reader with reference to [7]. The significantly different calculation of the truck steering angle is derived in full detail as well as formulae to relate the "nominal wheel" angle α to the actual truck front wheel angles (which was not given in [7]).

Reversing speeds are given as negative. Wheel and hitch angles are given as positive if anticlockwise and negative if clockwise relative to the forward orientation as in [7].

Similarly to the forwarding algorithm [7], the operator prescribes the velocity $v_3(t)$ and curvature $\kappa_3(t)$ of the path of the rear reference point at time t. Recall that the radius of curvature of the path is $\frac{1}{\kappa_3}$.

The truck-trailer system is assumed to be moving in a flat plane overlaid by the plane $\mathbb{R}^2$ with standard orthonormal basis $\mathbf{i}$ in the x-direction and $\mathbf{j}$ in the y-direction. From the operator parameters, calculation of the path in time $\mathbf{x}(t) = x_3(t)\mathbf{i} + y_3(t)\mathbf{j}$ can be achieved by differential geometry. The parameter θ_3 is introduced as the angle made by the tangent to the path and $\mathbf{i}$, so that the tangent is $\mathbf{t}_3(t) = \cos\theta_3\mathbf{i} + \sin\theta_3\mathbf{j}$. The Serret-Frenet equations [10] then solve to give

$$\theta_3(t) = \int_0^t \kappa_3(t)\ v_3(t)\ dt \tag{1}$$

$$x_3(t) = \int_0^t v_3(t)\cos\left(\theta_3(t)\right)\ dt \tag{2}$$

$$y_3(t) = \int_0^t v_3(t)\sin\left(\theta_3(t)\right)\ dt \tag{3}$$

To allow for calculation time, these analytical equations are replaced by numerical approximations over a discrete time interval Δt using trapezoidal approximation and iteration. Similar numerical estimation is used in controller design of mobile robots [11]. This leads to a calculation of a discrete set of path points $(x_3(t_i), y_3(t_i))$ which are stored together with $\kappa_3(t_i)$ and the unit normal to the path at that point $\mathbf{\hat{n}_3}(t_i) = -\sin(t_i)\mathbf{i}+\cos(t_i)\mathbf{j}$. This unit normal is calculated as the negative of the unit normal in the forwarding situation [7] because negative velocity is used to indicate reversing.

The sequence of discrete path points are initialized under the assumption that the second trailer moves with $\kappa_3 = 0$ for the length of the longest trailer. The positions of the first trailer and truck hitch points are calculated by determining the discrete path point that is closest to the correct length from the previous hitch as in [7]. In this way, a stored path point for the rear reference point $(x_3(t_j), y_3(t_j))$ at a previous time $j \leq i$ is relabeled $(x_2(t_1), y_2(t_i))$ and $(x_1(t_i), y_1(t_i))$ respectively as the location of the discrete path of the first trailer and truck hitch points at time t_i. The recorded value of $\kappa_3(t_j)$ at the truck hitch is relabled $\kappa_1(t_i)$. The recorded unit normals along with these hitch points are relabelled $\mathbf{\hat{n}_2}(t_1)$ and $\mathbf{\hat{n}_1}(t_i)$, allowing for calculation of the COC for each of the trailers, as the intersection of lines from the hitch points in the direction of the unit normals. From these the effective wheel angles for the trailer wheels are calculated as in [7].

The hitch point velocities can be calculated since the ratio of these velocities is equal to the ratio of their distances from the COC. This allows calculation of the velocity of the truck hitch point $v_1(t_i)$ which is equal to the translational speed of the truck drive wheels. So, although the operator is prescribing the velocity of the rear reference point, the drive comes from the truck and the velocity of the drive wheels can be determined. They will have the same velocity as the hitch point because one of the conditions of the reversing system is that the truck axis from rear to front, must remain tangential to the hitch path. This ensures the force of movement is always in the direction of movement.

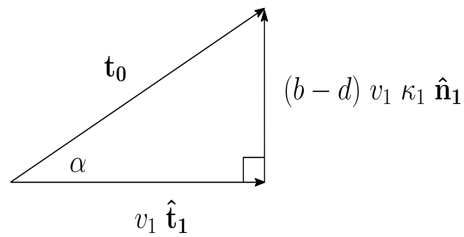
The angle α of the nominal steered wheel of the truck front wheels can be calculated analytically using the path tangent and truck hitch position. The analytical solution can then be transcribed to the discrete version allowing a time increment necessary for calculation of the algorithm.

First, the position vector of the nominal front wheel in real time is given by (see also Figure 2) $\mathbf{x_0}(t) = \mathbf{x_1}(t) + (b - d)\ \hat{\mathbf{t}}_\mathbf{1}(t)$ where $\mathbf{x_1}(t)$ is the position vector of the truck hitch and $\hat{\mathbf{t}}_\mathbf{1}(t)$ the unit tangent to the path at position $\mathbf{x_1}(t)$ at time t.

Arc length s is the natural parameter but differentiation with respect to time t can be accomplished using the chain rule (see [12] or [10] for derivations using the natural parameter).

$$\begin{aligned}
\mathbf{t_0}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x_0}(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\mathbf{x_1}(t) + (b-d)\ \hat{\mathbf{t}}_\mathbf{1}(t)\right) \\
&= \frac{\mathrm{d}\mathbf{x_1}}{\mathrm{d}s}\ \frac{\mathrm{d}s}{\mathrm{d}t} + (b-d)\frac{\mathrm{d}\hat{\mathbf{t}}_\mathbf{1}}{\mathrm{d}\theta_1}\ \frac{\mathrm{d}\theta_1}{\mathrm{d}t} \\
&= v_1(t)\ \hat{\mathbf{t}}_\mathbf{1}(t) + (b-d)\ v_1(t)\ \kappa_1(t)\ \hat{\mathbf{n}}_\mathbf{1}(t) \qquad (4)
\end{aligned}$$

The derivative $\frac{\mathrm{d}\hat{\mathbf{t}}_\mathbf{1}}{\mathrm{d}\theta_1} = -\sin\theta_1\mathbf{i} + \cos\theta_1\mathbf{j}$ is the unit normal $\hat{\mathbf{n}}_\mathbf{1}(t)$ and $\frac{\mathrm{d}\theta_1}{\mathrm{d}t} = v_1(t)\ \kappa_1(t)$ by the fundamental theorem of calculus and equation (1). By a subtle interpretation, equation (4) describes a triangle of vectors



and since $\hat{\mathbf{t}}_{\mathbf{1}}$ and $\hat{\mathbf{n}}_{\mathbf{1}}$ are unit length, by trigonometry $\tan\alpha = \dfrac{(b-d)_1\kappa_1}{v}$ so, the discrete version for calculating truck steering angle is

$$\alpha(t_i) = \arctan\left((b-d)\kappa_1(t_i)\right) \tag{5}$$

In [7] a functional relation was assumed between steering wheel rotation and the angle α of a "nominal wheel" in the centre of the truck's front axle. Although this function will vary between trucks, assuming the geometry of the Ackerman steering system is precise, the actual left and right front truck wheel angles can be calculated from α and this should be consistent across all types of truck. The actual wheels can then be monitored in the reversing situation to ensure they are maintained at the algorithm's prescribed value. By trigonometry, the steered wheels of the truck have angles given by

$$\phi_{\text{left}}(t) = \arctan\left(\frac{b\tan\alpha}{b-e\tan\alpha}\right) \qquad \phi_{\text{right}}(t) = \arctan\left(\frac{b\tan\alpha}{b+e\tan\alpha}\right) \tag{6}$$

where $2e$ is the axle width. These angles can be monitored and adjusted by the control mechanism.

3 Results

The following are the results of a simulation of 25 seconds of reversing with a time increment $\Delta t = 0.005$ seconds. The trailer makes a standing start and travels in a straight line ($\kappa_3 = 0$) for the length of the second trailer to initialize the discrete path of points. The velocity of the rear reference points decreases linearly to -10 m/s which it holds for seconds 10 to 22 at which time it accelerates linearly back to a stop. In this time the system traverses a left circle and then a right circle, each of radius 10m. Between seconds 5 and 7 the curvature decreases linearly from 0 to the left circular arc with $\kappa_3 = -0.1$, in seconds 11 to 15 the curvature increases linearly to the right circular arc with $\kappa_3 = 0.1$ and finally in seconds 19 to 21 the curvature decreases linearly back to 0.

It should again be noted that this is not a pre-determined path, rather, it is a simulation of the operators instructions. These instructions are contained in a separate subprogram and are sampled in the same way that the operator's given parameters would be sampled at each time increment.

The accuracy of the calculations was extensively tested in [7] and it is assumed in this paper that the errors are acceptably small given the smaller magnitude of speeds when reversing.

Since each subsequent hitch point is prescribed as a previous point of the hitch in front (in the sense of the reversing direction) all hitch paths are the same except for the start and finish points. The initial points are clearly shown in Figure 3 to be the correct trailer

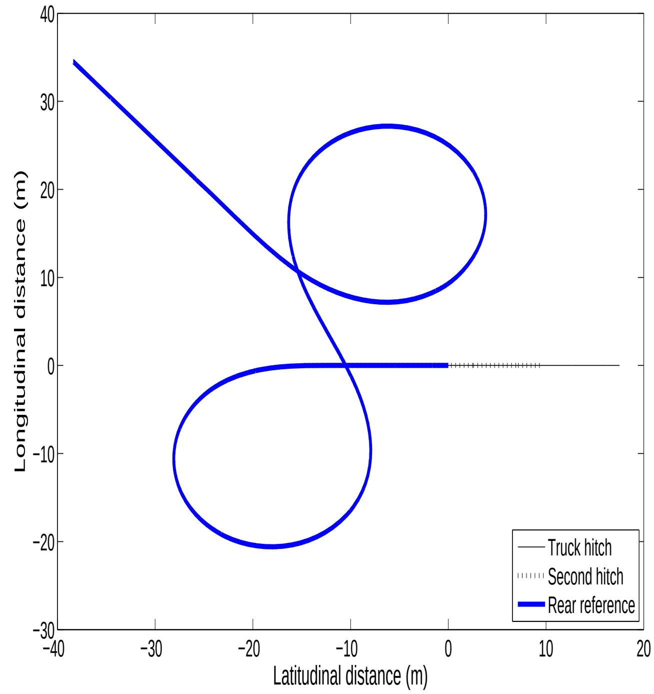


Figure 3: Hitch paths, with rear reference point starting at (0,0) and moving to the left

length to the right (a positive latitudinal direction) of each other.

The wheel angles were calculated and are shown in Figure 4. The ordering of wheel number can be described by referring to Figure 2. In Figure 2 the left hand side wheels are shown and for the second trailer from rear to front we see wheels 12, 10 and 8. For the first trailer from rear to front we see wheels 6, 4 and 2. The wheels on the right side of the vehicle are odd numbers in similar sequence. This is consistent with [7].

Figure 4 demonstrates the expected symmetry of inner and outer wheels whilst performing left followed by right turns of equal but opposite curvature. Also expected is the

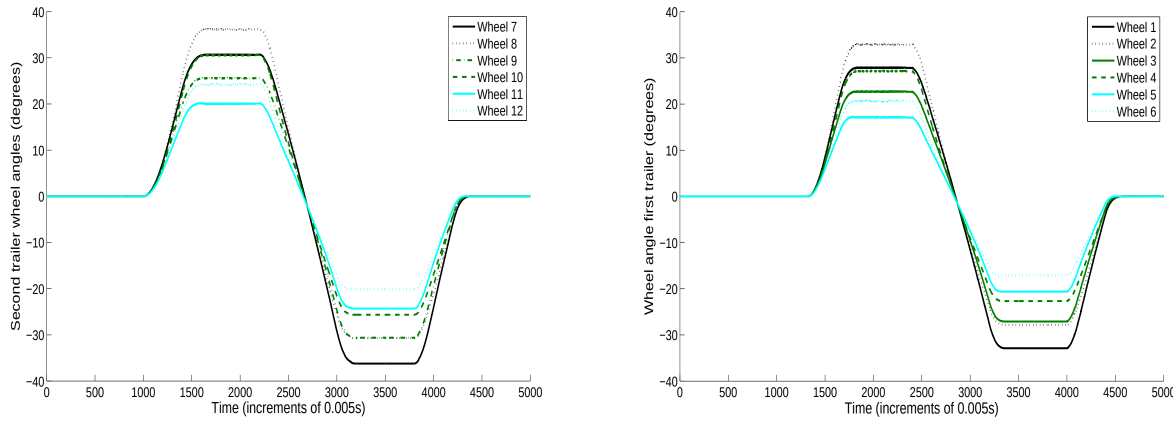


Figure 4: Wheel angles of second and first trailers as functions of time

lesser angle for wheels closer to the rear of the trailer due to the geometry with these wheels being further behind the COC of that trailer (in the direction of the line between hitch points). The movement of wheel angles of the first trailer is delayed behind those of the second trailer as they perform the manoeuvres later in time. At a smaller scale the delay of trailing wheels of each trailer would be clearly seen though, even in the given scale, there is a small but discernible difference delay between wheel 11 and wheel 7 reaching their first steady state of constant angle.

The nominal truck wheel angle α is calculated as well as the translational speed of the rear reference point and truck hitch point (which provides the necessary truck wheel speed). These are plotted in Figure 5.

As expected, the truck wheel manoeuvres are delayed relative to the first trailer since it follows it, and are in the opposite direction since this wheel is at the opposite end of

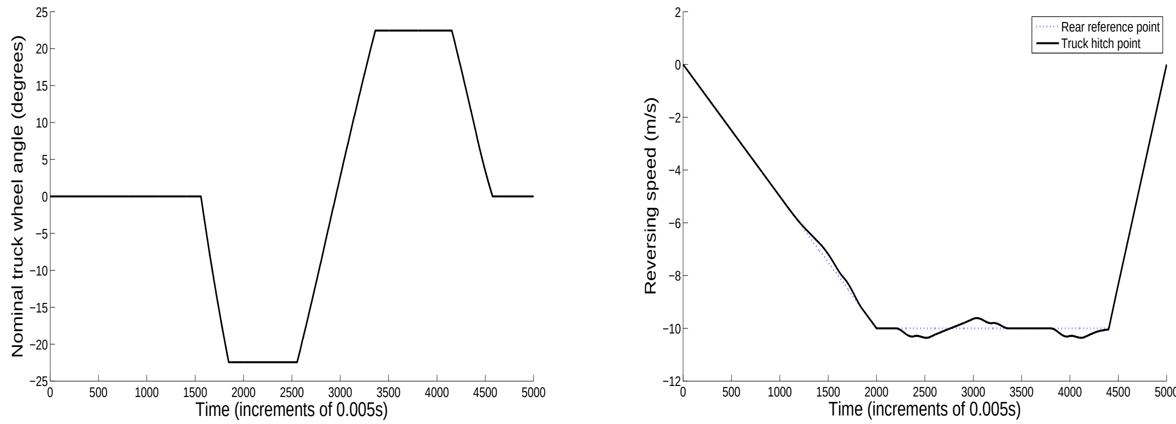


Figure 5: Truck steering angles and drive wheel velocity

a section of the vehicle. The change of angles are sharper for the truck than the trailer wheels. This is because the truck steering angle $\alpha(t)$ is directly related to the path curvature which changes sharply from constant to linearly increasing or decreasing whereas the trailer wheels are related to movements in the COC which are more continuous and smooth.

The speed of truck and rear reference points are the same for approximate time intervals 0 to 1200, 1700 to 2300, 3300 to 3800 and 4500 to 5000 time increments. These are time intervals when the truck and two trailers are in steady state, either all going straight or all on the same circular arc. A comparison of the intervals of constant wheel angles in Figures 4 and 5 show that this should be so.

As a second cooperatively redundant steering system the trailer and truck hitch angles were calculated. These are shown in Figure 6. The truck hitch angle is approximately half that of the trailer hitch angle in steady state because the trailer angle is between two

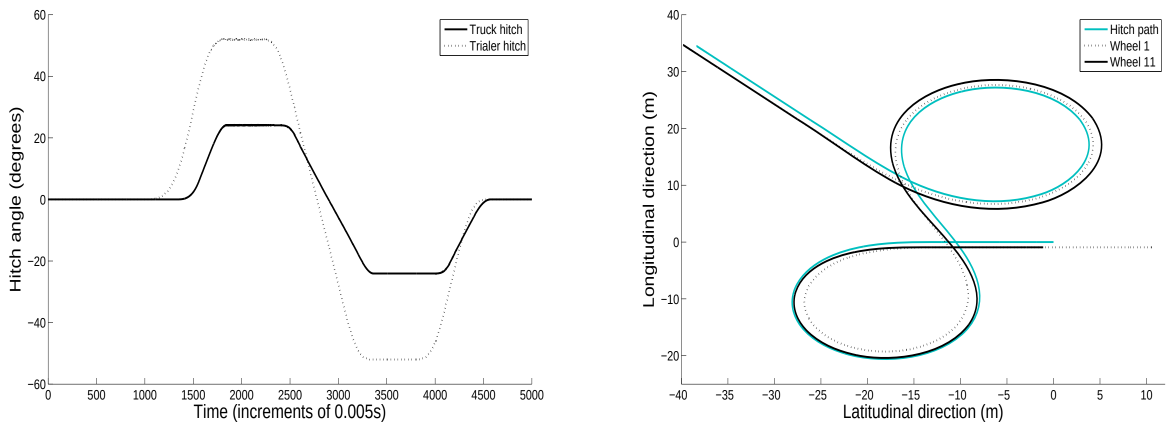


Figure 6: Hitch angles and off-tracking simulations

chords of the circle and the truck angle between a chord and a tangent.

Off-tracking for the trailer wheels 1 and 11 is also plotted in Figure 6. Unlike the forward direction [7] in which fixed axle truck-trailer system were modelled for off-tracking to compare to the steerable trailer wheel system, the reversing of a fixed-axle B-double is extremely difficult to model without assumptions about control of hitch angles. However, it is certainly clear from Figure 6 that off-tracking is minimal for such tight cornering of a B-double system.

4 Conclusion

It has been shown that an operator can easily reverse a B Double truck-trailer vehicle by controlling the path of the rearmost reference point (the midpoint of the second last axle). The operator can use a reversing camera or vantage point external to the truck and select the instantaneous curvature of the path of the rearmost reference point with a joystick, joy pad, auxiliary steering wheel or knob. The middle reference (the hitch point connecting the first and rearmost trailer) and foremost reference point (the hitch point connecting the first trailer to the truck) can be made to follow the same path by control of the steerable trailer wheels resulting in minimal off-tracking. The effective wheel angles are calculated numerically and eliminate scuffing at slow speeds. The correct hitch angles are also calculated and can be actuated to provide a cooperative steering effect on the vehicle.

The angle of the truck wheels required to ensure the foremost reference point also follows the master path whilst keeping the drive force tangential to the master path are calculated by analytical means and transposed for compatibility with the numerical control algorithm. These truck wheel angles are implemented by means of an automatic steering system.

The effectiveness of the control algorithm was tested successfully by simulation.

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