

Pacific University

CommonKnowledge

College of Optometry

Theses, Dissertations and Capstone Projects

1-1990

Change in focus due to motion

Richard B. McMaster
Pacific University

Recommended Citation

McMaster, Richard B., "Change in focus due to motion" (1990). *College of Optometry*. 924.
<https://commons.pacificu.edu/opt/924>

This Thesis is brought to you for free and open access by the Theses, Dissertations and Capstone Projects at CommonKnowledge. It has been accepted for inclusion in College of Optometry by an authorized administrator of CommonKnowledge. For more information, please contact CommonKnowledge@pacificu.edu.

Change in focus due to motion

Abstract

The physical properties of a system at rest are not the same when the system is in motion. These relativistic effects, such as changes of length and mass, are usually minute when the velocity of the system is small compared to the speed of light and therefore of little practical interest. However, this paper reports changes of the velocity of light in a moving medium which lead to changes in refractive index and therefore focal length that are not small. The change in focal length can be as much as a fraction of a millimeter when the velocity of the system is as low 1/10,000th the speed of light. Conceivably these effects could cause ' defocusing of optical instruments carried on space flights.

Degree Type

Thesis

Degree Name

Master of Science in Vision Science

Committee Chair

Jurgen R. Meyer-Arendt

Keywords

Angle of incidence, angle of refraction, refraction, relativity, refractive index

Subject Categories

Optometry

Copyright and terms of use

If you have downloaded this document directly from the web or from CommonKnowledge, see the "Rights" section on the previous page for the terms of use.

If you have received this document through an interlibrary loan/document delivery service, the following terms of use apply:

Copyright in this work is held by the author(s). You may download or print any portion of this document for personal use only, or for any use that is allowed by fair use (Title 17, §107 U.S.C.). Except for personal or fair use, you or your borrowing library may not reproduce, remix, republish, post, transmit, or distribute this document, or any portion thereof, without the permission of the copyright owner. [Note: If this document is licensed under a Creative Commons license (see "Rights" on the previous page) which allows broader usage rights, your use is governed by the terms of that license.]

Inquiries regarding further use of these materials should be addressed to: CommonKnowledge Rights, Pacific University Library, 2043 College Way, Forest Grove, OR 97116, (503) 352-7209. Email inquiries may be directed to: copyright@pacificu.edu

CHANGE IN FOCUS DUE TO MOTION

Written By : Richard B. McMaster

Advisor : Jurgen R. Meyer-Arendt

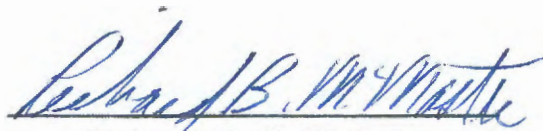
**A thesis submitted to the faculty of the
College of Optometry
Pacific University
Forest Grove, Oregon
for the degree of
Doctor of Optometry**

January, 1990

SIGNATURE PAGE

CHANGE IN FOCUS DUE TO MOTION

Submitted by :



Richard B. McMaster

Advisor :

Jurgen R. Meyer-Arendt

ACKNOWLEDGEMENTS

I would like to thank Dr. Jurgen Meyer-Arnedt for his ideas, inspiration, and guidance. Working with him was an honor and a pleasure.

I would also like to thank my family, especially my wife Ann, for their understanding and support.

ABSTRACT

The physical properties of a system at rest are not the same when the system is in motion. These relativistic effects, such as changes of length and mass, are usually minute when the velocity of the system is small compared to the speed of light and therefore of little practical interest. However, this paper reports changes of the velocity of light in a moving medium which lead to changes in refractive index and therefore focal length that are not small. The change in focal length can be as much as a fraction of a millimeter when the velocity of the system is as low $\frac{1}{10,000}$ th the speed of light. Conceivably these effects could cause defocusing of optical instruments carried on space flights.

KEY WORDS

Angle of incidence, angle of refraction, refraction, relativity, refractive index.

INTRODUCTION

Einstein postulated that physical properties of a system at rest will not be the same when the system is in motion. These relativistic effects, such as changes in mass and length, are generally minute when the velocity of the system is small compared to the speed of light. This paper reports of relativistic changes in refractive index and focal length of a lens traveling at fairly low speeds, that are not minute.

Refraction is defined as the bending of light at the interface of two different media. It is caused because the speed of light in a medium is less than in free space and also different in different media. Advanced theory shows how to correlate electromagnetic wave velocity with the electrical and magnetic properties of a material. It is probably sufficient to say that in free space light encounters no atoms and therefore its velocity is at a maximum. In matter, the velocity is the same between the atoms, but near the atoms the light interacts with the orbiting electrons and its resulting velocity is less. The velocity of light through a medium, therefore, is inversely proportional to the density of the atoms along the path of the light.

The absolute refractive index of a material (n) refers to the ratio between the speed of light in free space (c) and the speed of light in the material (v).

$$n = \frac{c}{v} \quad (\text{Equation 1})$$

The refractive index therefore is proportional to the density of the material.

If a refractive medium is moving toward a light source, the light in the medium encounters more atoms per unit time so the resulting velocity of the light within the medium should decrease. Conversely, as the medium moves away from the light source, the velocity of the light in the medium increases. If we merely add the velocity of the medium (w) and the light in the medium (noting from Equation 1 that $v = \frac{c}{n}$), the resulting velocity of the light in the moving medium (v') is numerically too high.

$$v' = \frac{c}{n} + w \quad (\text{Equation 2})$$

In 1818, J.A. Fresnel predicted that light is partially "dragged along" by the moving medium. The velocity of light in the moving medium is, by his prediction:

$$v' = \frac{c}{n} + w \left(1 - \frac{1}{n^2} \right) \quad (\text{Equation 3})$$

The factor $(1 - \frac{1}{n^2})$ is called the Fresnel drag coefficient. The speed of light is changed from c/n by the motion of the medium. Because the drag coefficient is always less than one, the change is less than the speed of the medium. This effect was confirmed experimentally by H.Fizeau in 1851².

The currently accepted method of calculating the velocity of light in a moving medium is by using Einstein's relativistic velocity addition theorem.

$$v' = \frac{v + w}{1 + \frac{v \cdot w}{c^2}} \quad (\text{Equation 4})$$

Expansion of this equation by the binomial theorem yields :

$$v' = (v + w) \left(1 - \frac{v}{c} \frac{w}{c} - \dots \right) = v + w - \frac{v^2 w}{c^2} - \frac{w^2 v}{c^2} - \dots$$

(Equation 5)

When the velocity of the medium is much smaller than c , the last term (a second-order quantity) can be dropped. The velocity of light relative to the system ($\frac{c}{n}$) can replace v . The term $\frac{1}{n}$ can replace $\frac{v}{c}$ (because $n = \frac{c}{v}$):

$$v' = \frac{c}{n} + w \left(1 - \frac{1}{n^2} \right)$$

Therefore, the resulting first-order equation for the velocity of light in the medium is exactly what Fresnel had predicted.

This formula has been derived in different ³⁻⁶ ways and all these derivations agree.

Snell's law of refraction

$$n \sin i = n' \sin i' \quad (\text{Equation 6})$$

can be used to calculate the angle of refraction (i') if the refractive indices (n and n') and the angle of incidence (i) are known. Since the velocity of light in a moving system and the new refractive index resulting from the motion can be calculated, so can the change of the angle of refraction. When a lens is in motion, the angle of refraction and therefore the focal length will change. It is the purpose of this paper to explore the size of this change with variations in the angles of incidence, velocity of the system, and the refractive index of a medium limited by a simple, curved surface.

APPLICATION

To demonstrate how to quantify the change in focal length between a stationary and a moving refractive surface the following construct is used. A single curved refractive surface with a radius of curvature (R) of +20 cm separates free space on the left from a medium on the right. The refractive index (n') of the medium is 1.496. A ray of light coming from the left, parallel to the optic axis, is incident on the surface at a height (h) of 2 cm.

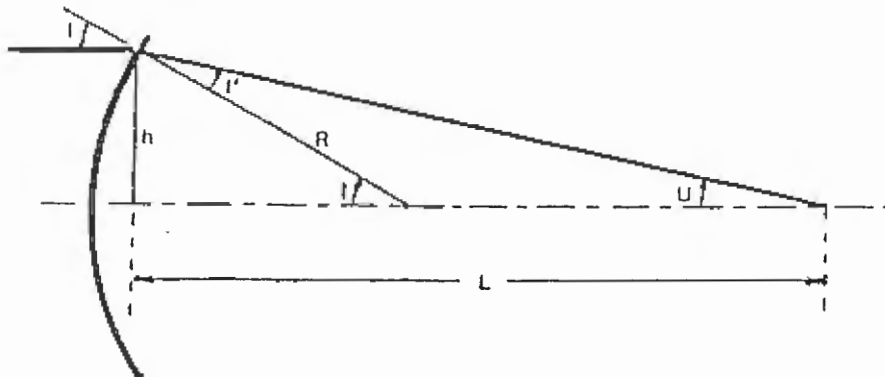


Figure 1 : Refraction of light at a single surface.

By definition, the angle of incidence is subtended by the surface normal and the incident ray. The surface normal is a line drawn perpendicular to the surface from the point of incidence of the ray. A line representing the radius of curvature intersects the surface at a 90 degree angle, thus the angle subtended by the radius (R) and the incident ray is the angle of incidence (i). Since the incident ray is parallel to the optic axis, the angle subtended by the radius and the optic axis is equal to the angle of incidence.

The angle of incidence at the surface, from Figure 1 is found from

$$\sin(i) = \frac{h}{R}$$

so that

$$i = \arcsin\left(\frac{2}{20}\right) = 5.73917 \text{ degrees.}$$

At the surface the ray is refracted according to Snell's law of refraction:

$$n \sin(i) = n' \sin(i')$$

Solving for the angle of refraction gives

$$i' = \arcsin\left(n * \frac{\sin(i)}{n'}\right)$$

and so, with $n = 1.00$,

$$i' = \arcsin\left(\frac{\sin(5.73917)}{1.496}\right)$$

$$i' = \arcsin\left(\frac{0.1}{1.496}\right) = 3.83279 \text{ degrees.}$$

Next the slope angle, U , at which the refracted ray intersects the axis, is determined.

I' and U are interior angles in a triangle whose opposite exterior angle is I . The sum of two interior angles of a triangle is equal to the opposite exterior angle. Therefore,

$$I = I' + U$$

and, solving for U ,

$$U = I - I'$$

$$U = 5.73917 - 3.83279 = 1.90638 \text{ degrees.}$$

Since the height (h) and the angle U are known, the distance (L) from the axial intersect to h can be found by using

$$\tan(U) = \frac{h}{L}$$

or

$$L = \frac{20}{\tan(1.90638)} = 600.873 \text{ mm.}$$

With the medium in motion, the velocity of the light within the medium will change. This new velocity is calculated using Fresnel's formula (Equation 3). Since velocity is a vector quantity, the cosine of the

angle that the refracted ray subtends with the direction of motion must be included :

$$v' = \frac{c}{n} + w (\cos \theta) \left(1 - \frac{1}{n^2} \right) \text{ (Equation 7)}$$

The cosine factor is necessary in order to find the velocity of the light in the moving medium. With the medium traveling toward the light source this angle will be a few degrees less than 180° and its cosine is negative. Consequently the last three terms in Equation 7 need to be subtracted from $\frac{c}{n}$. Substituting the actual figures shows that the velocity of the light in the medium is

$$v' = \frac{299792458}{1.496} + (30000)(\cos 176.167) \left(1 - \frac{1}{1.496^2} \right)$$

$$v' = 200379469 \text{ m/s.}$$

The new refractive index of the system in motion is

$$n' = \frac{c}{v'} = \frac{299792458}{200379469} = 1.49612$$

and the angle of refraction within the moving medium is

$$i'' = \arcsin\left(\frac{0.1}{1.49612}\right) = 3.83248 \text{ degrees.}$$

The slope angle is

$$U = i - i'' = 1.9067 \text{ degrees}$$

and the axial intercept is

$$L = \frac{20}{\tan(1.9067^\circ)} = 600.772 \text{ mm.}$$

We now have two axial intercept distances (L), one for the medium at rest and one with the medium in motion. These distances are measured from h not from the surface, but the essential point is that their difference (ΔL), is the change of focal length due to motion.

$$\Delta L = 0.1 \text{ mm}$$

DISCUSSION

There are two areas of potential error in this method of calculating focal length change caused by motion. If we use both Fresnel's and the full relativistic equations (Equations 3 & 4) and calculate the velocity of light in a medium of refractive index = 1.5, moving toward a distant light source at a velocity of 1,000,000 meters per second, the relativistic formula yields a 0.000615% smaller result. The velocities calculated with the relativistic formula, although virtually identical to that found with Fresnel's, result in the focal length change induced by motion being slightly greater.

The second area of potential error is that the angle of refraction in the system at rest is used to calculate the new velocity of the light when the system is in motion. To be completely accurate this formula should be optimized in order to account for the fact that as the system speed increases, the refractive index and therefore the angle of refraction increase also. The error caused by using this method is minute because the difference between the angles of refraction in the system at rest and in motion is very small.

Both possible errors are small and if taken into account would make the focal length change even greater.

Although the velocity used as an example is only $\frac{1}{10,000}$ th the speed of light, it is still faster than any manmade interplanetary craft to date.

However, as higher speeds become possible, the focal planes of optical instruments carried on flights will shift toward the lens and at some speed defocusing could occur. The magnitude of this shift is dependent on three parameters:

(1.) The lower the refractive index the larger the focal point shift will be.

(2.) As the distance between the incident ray and the optic axis decreases, the focal point change increases.

(3.) As the radius of curvature of the surface increases, the focal length change increases.

These parameter changes all have the effect of moving the axial intercepts away from the lens. As the distance from the lens to the pair of intercepts increases, so will the distance between them.

If all the parameters mentioned above remain constant, an increase in the velocity of the system toward the light source will move the axial intercept closer to the lens. If the system moves away from the light source at the same velocity the axial intercept will move farther from the lens. If the system moves away from the light source at the same

velocity the axial intercept will move farther from the lens.

The angles between the intercepts of the system at rest and in motion will be the same in both directions but the distance between them will be greater when the system is moving away. This again is because the distance to these intercepts is greater and therefore, the distance between them will be greater also.

Included in this thesis is a Pascal program that I've written which can be used to explore the effects that changes in these parameters have on a single refracting surface. This program accepts inputs for the height of the incident ray, radius of curvature, refractive index, and velocity of the system. It incorporates the ray tracing algorithm outlined in the Application section and will output the velocity of light in the moving system, the new refractive index induced by motion, the angles of incidence and refraction, the slope angles, the distance to the axial intercepts, and the distance between them.

```

(*****
(*****
PROGRAM FOCAL_MOTION ;
(* Compiled version = FOCAL.COM *)

(*****
(*****

{$I Constants.Box }
{$I Utility.Box }
{$I Math.Box }

(*****
(*****

function snell_1(n1,n2,incidence :real) : real;
{ Receives angle of incidence in degrees, converts it to radians
and returns the angle of refraction in degrees. }
var
ang_incid : real ;
begin
ang_incid := incidence * degree_to_rad ;
snell_1 := arcsin((n1 * sin(ang_incid))/n2)
end ;

function beta_angle(veloc_1,veloc_2,ang_alph : real ) : real ;
{ Receives two velocities and the incident angle and returns
the refracted angle of the moving system. } var
angle_alpha : real ;
begin
angle_alpha := ang_alph * degree_to_rad ;
beta_angle := arcsin((sin(angle_alpha) * veloc_2)/veloc_1) ;
end ;

function drag2(n1,n2,incid,medium_speed :real) :real ;
{ Receives two refractive indices and the velocity of the medium
and returns the velocity of the light in the system. This
function uses the drag coefficient formula. }
begin
drag2 := (light_speed / n2) - cos(snell_1(n1,n2,incid) * degree_to_rad)
* (medium_speed * (1 - 1/sqr(n2))) ;
end ;

```

```

function total(incid,ang_refract :real ) :real ;
begin
  total := ang_refract + ( 90 - incid) ;
end ;

procedure focal ;
begin
  ClrScr ;
  ans := 'y' ;
  while ans in ['Y','y'] do
  begin
    Zero ;
    ClrScr ;
    write('Enter the refractive index of the system : ') ; readln(ndx2) ;
    writeln ;
    write('Enter the height of the incident ray (in cm.) : ') ;
    readln(height) ;
    writeln ;
    write('Enter the radius of curavture (in cm.) : ') ; readln(radius) ;
    writeln ;
    write('Enter the velocity of the system (in meters / second) : ') ;
    readln(velocity) ;
    anglof_incid := arcsin(height / radius) ;
    value := snell_1(ndx1,ndx2,anglof_incid) ;
    writeln ;
    writeln(Conststr('*',79)) ;
    writeln ;
    writeln('Velocity of light through stationary system : ',
            light_speed / ndx2 :10:5, ' meters /second.') ;
    writeln('Velocity of light through the moving system : ',
            drag2(ndx1,ndx2,anglof_incid,velocity):10:5, ' meters /second.') ;
    writeln ;
    writeln('Total angle ( T ) with no motion : ',
            total(anglof_incid,value):3:10) ;
    writeln('Total angle with the system in motion : ',
            total(anglof_incid,beta_angle(light_speed,
            drag2(ndx1,ndx2,anglof_incid,velocity),anglof_incid)):3:10) ;
    difference2 := total(anglof_incid,value) -
    total(anglof_incid,beta_angle(light_speed,drag2(ndx1,ndx2,anglof_incid,
            velocity),anglof_incid));
    writeln('Difference between the angles : ',difference2:3:10,
            ' degrees or ', difference2 * 3600:3:5, ' arcsec.' ) ;
    writeln ;
    focal1 := height / cotan(total(anglof_incid,value)) ;
    focal2 := height / cotan(total(anglof_incid,beta_angle(light_speed,
            drag2(ndx1,ndx2,anglof_incid,velocity),anglof_incid)));
    writeln('The focal length of stationary system : ',focal1 :5:8,'cm.') ;
  end ;
end ;

```

```
writeln('The focal length of system in motion : ',focal2:5:8,'cm. ');
writeln('Difference in the focal lengths (in mm.) : ',
        ((focal1 - focal2) * 10):3:8);
writeln;
Delay(2000);
ChkRes(ans,'Continue ? (y/n)',['y','Y','n','N']);
end;
X_it;
end;

begin
    focal;
end.

(*****)
```

```
(*****)
(***** Include File of Utility Procedures *****)
(***** Utility . Box *****)
(*****)
```

```
procedure zero;
begin
  FillChar( zero1, ofs( zero2 ) - ofs( zero1 ) + sizeof( zero2 ), 0 );
end; { Used to initialize the whole data segment }
```

```
procedure ChkRes( var ch: char; Msg: Prompt; RightSet: CharSet );
begin { To prompt user and check response }
  repeat
    GOTOXY( 26, 23 ); writeLn( Msg ); GOTOXY( 43, 23 ); readLn( ch );
    ch := ( UpCase( Ch ) );
  until Ch in RightSet { Used in Focal.pas & focal2.pas }
end;
```

```
procedure ChkResLow( var ch: char; Msg: Prompt; RightSet: CharSet );
begin { To prompt user and check response and write in Lowvideo }
  repeat
    GOTOXY( 26, 23 );
    Lowvideo ;
    write( Msg );
    GOTOXY( 43, 23 );
    Highvideo ;
    readLn( ch );
    ch := ( UpCase( Ch ) );
  until Ch in RightSet { Used in Focal2.pas }
end;
```

```
function ConstStr( C : Char; N : Integer ) : Str80;
var
  S : string[ 80 ];
begin
  if N < 0 then
    N := 0;
  S[ 0 ] := Chr( N );
  FillChar( S[ 1 ], N, C );
  ConstStr := S;
end;
```

```

procedure wrt(x,y:integer;msg:prompt);
begin
  GotoXY(X ,Y); Write(msg);
end ;

procedure wrt2(x,y :integer; msg : prompt; number : real; a,b:integer);
begin
  GotoXY(X ,Y);
  Lowvideo ;
  Write(msg);
  Highvideo ;
  Write(number:a:b) ;
  Normvideo ;
end ;

procedure wrt3(x,y :integer; msg : prompt);
begin
  GotoXY(X ,Y); WriteLn(msg);
end ;

procedure Frame(UpperLeftX, UpperLeftY, LowerRightX, LowerRightY: Integer);
var l : Integer;

begin {Frame}
  GotoXY(UpperLeftX, UpperLeftY);
  Write(chr(218));
  for l := (UpperLeftX + 1) to (LowerRightX - 1) do
    begin
      Write(chr(196));
    end;
  Write(chr(191));
  for l := (UpperLeftY + 1) to (LowerRightY - 1) do
    begin
      GotoXY(UpperLeftX , l); Write(chr(179));
      GotoXY(LowerRightX, l); Write(chr(179));
    end;
  GotoXY(UpperLeftX, LowerRightY);
  Write(chr(192));
  for l := (UpperLeftX + 1) to (LowerRightX - 1) do
    begin
      Write(chr(196));
    end;
  Write(chr(217));
end; {Frame}

```



```

procedure FrameLow(UpperLeftX, UpperLeftY, LowerRightX, LowerRightY: Integer);
var I: Integer;
begin {FrameLow}
  Lowvideo;
  GotoXY(UpperLeftX, UpperLeftY);
  Write(chr(218));
  for I := (UpperLeftX + 1) to (LowerRightX - 1) do
  begin
    Write(chr(196));
  end;
  Write(chr(191));
  for I := (UpperLeftY + 1) to (LowerRightY - 1) do
  begin
    GotoXY(UpperLeftX, I); Write(chr(179));
    GotoXY(LowerRightX, I); Write(chr(179));
  end;
  GotoXY(UpperLeftX, LowerRightY);
  Write(chr(192));
  for I := (UpperLeftX + 1) to (LowerRightX - 1) do begin
    Write(chr(196));
  end;
  Write(chr(217));
  Normvideo;
end; {framelow}

```

```

procedure Beep;
begin
  Write('^G');
end;

```

```

procedure X_it;
begin
  wrt(26,23,'Program terminated. ');
  Beep;
  exit;
end;

```

(*****)

```
(*****  
(*****          Constants.Box          *****  
(***** Constants and Variables Used in Focal.pas *****  
(*****  
(*****
```

```
type
```

```
prompt = string[80];  
charset = set of char;  
Str80 = string[80];
```

```
const
```

```
pi = 3.141592654;  
rad_to_degree = 57.295779506;  
degree_to_rad = 0.01745329252;  
light_speed = 299792458.0;  
ndx1 = 1.000;
```

```
var
```

```
zero1 : byte;  
anglof_refract,angle_of_refract,anglof_incid,ndx2 : real;  
value, medium_velocity,difference,difference2 : real;  
height, radius,focal1,focal2,slope_angle,slope_angle2 : real;  
slope_difference : real;  
new_index, ax_intercept,ax_intercept2 : real;  
ans : char;  
zero2 : byte;
```

```
(*****
```

```
(*****  
(*****          MATH.BOX          *****  
(***** Math Procedures Not Included *****  
(*****          In Turbo Pascal          *****  
(*****
```

```
function arcsin(X:real) : real;  
{ Takes radians and returns degrees. }  
begin  
  arcsin := arctan(X/sqrt(1-sqr(X))) * rad_to_degree ;  
end;
```

```
function tan(angle :real) : real;  
{ Takes argument in degrees and returns tangent value. }  
var  
  ang : real ;  
begin  
  ang := angle * degree_to_rad ;  
  tan := sin(ang) / cos(ang) ;  
end ;
```

```
function cotan(angle : real) : real ;  
{ Takes argument in degrees and returns the cotangent value. }  
var  
  ang : real ;  
begin  
  ang := angle * degree_to_rad ;  
  cotan := cos(ang) / sin(ang) ;  
end ;
```

CONCLUSION

A refractive element in motion has a different refractive index than when it is at rest. This change in refractive index leads to a change in focal length that can be as large as a fraction of a millimeter at speeds as low as $\frac{1}{10,000}$ th the speed of light. It is easy to speculate that these changes in refractive index will need to be considered when designing optical instruments to be carried on space flights of the future. The important point though is that this motion induced change in focal length is considerably larger than what is known to date from "relativistic" effects.

REFERENCES

1. A. Fresnel, "Sur l'influence du mouvement terrestre dans quelques phenomenes d'optique," Ann.Chim.Phys. (2) 9 (1818), 57-66.
2. H. Fizeau, "Sur les hypothèses relatives à l'éther lumineux, et sur une expérience qui paraît démontrer que le mouvement des corps change la vitesse avec laquelle la lumière se propage dans leur intérieur," Compt. rendu 33 (1851), 349-355.
3. A. Sommerfeld, Optics, Academic Press, Inc., New York 1954. Pp. 69 - 72.
4. R. Resnick, Introduction to Special Relativity, John Wiley & Sons, Inc., New York 1968. Pp. 30 - 33, 81.
5. W. Rindler, Introduction to Special Relativity, Oxford University Press, New York 1982. Pp 44-45.
6. V.Santarelli, Fresnel's Coefficient from the Photon Theory, Am. J. Physics 52 , 659 (1984).

7. Meyer-Arendt, Jurgen R., Introduction to Classical and Modern Optics.
Third Edition. Englewood Cliffs; Prentice Hall., 1989

8. Carl Sagan, Cosmos., Random House, New York 1980, Pp 199-206.