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Laser and conventional near point cross cylinder and astigmatism measurements compared

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Laser and conventional near point cross cylinder and astigmatism measurements compared

Abstract

Laser and conventional near point cross cylinder and astigmatism measurements compared

Degree Type Thesis

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Committee Chair Pratt

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Laser and Conventional Near Point Cross Cylinder and Astigmatism Measurements Compared

A Thesis Presented to the Faculty

of

Pacific University College of Optometry

In Partial Fulfillment of Recuirements for the Degree Doctor of Optometry

May 10, 1973

Richard G. Brown David Korver William Mortvedt James R. Ogden We would like to thank Dr. C. B. Pratt and Dr. W. T. Griffith for their valuable assistance with this thesis project.

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INTRODUCTION

Statement of the Problem:

The purpose of this study was to determine the relationship that the near point refraction by a laser light target has to the conventional OEP finding $#14A$ (monocular crosscylinder equality) and to the Pratt near cylinder finding (Pratt).

Im�ortance of the Study:

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Only within the past eight years has the laser been used for determining the refractive state of the eye. There have been studies done on distance refraction and the determination of the accommodative state of the eye, but nothing has been done in determining a near cylinder finding by the use of a laser target. Also, there have been no studies investigating the relationship between conventional cross cylinder (14A) results and similar findings obtained by laser refraction techniques.

REVIEW OF THE LITERATURE AND HISTORY

The process of stimulated emission of radiation was first described theoretically by Albert Einstein in 1917.^{(Silfvast n.} 91) This is a process whereby a light beam of a specific wavelength can interact with an atom in such a way that the atom will admit additional light that moves in the same direction and has the same wavelength as the original beam. The word "laser" stands for "Light amplification by stimulated emission of radiation."

If a beam of light were fed into a suitable laser amplifier, the ampli fier would maintain the original direction and wavelength of the input beam and the intensity of the light would increase. An oscillation can be produced by placing mirrors at the ends of the amplifier, forcing the light to bounce back and forth between the mirrors. In such an oscillator, the intensity of the light can build up to many orders of magnitude greater than the intensity of the original light signal, producing a highly directional laser beam of an extremely pure color. The light is extracted from the laser through one of the mirrors, which is designed to transmit a small percentage of the light. (Si^l fva^st p. 91)

Since 1960, when a laser was first successfully demonstrated, laser action has been obtained in many different physical

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systems. The original ruby laser was succeded by other solidstate lasers, including the neocymium laser and the large class of semiconductor lasers. Gas lasers appeared soon after the first ruby laser. (Maiman p. 494) Such an example of a gas laser is the helium-neon laser, which was used in this study. Eelium has the large st ionization energy of any atom (?4. 6 Electron Volts).

The use of the laser for refraction has been of interest to those in the visual science field in recent years. In 1962, Rigden and Gordon described several phenomena associated with laser light and gave the first published explanation of this speckled pattern phenomenon. They stated that the sig nificant feature of the laser light in producing the phenomenon is not the spectral purity, but rather the fact that the angular spread is diffraction limited. The motion of tbe granularity when the observer moves can be explained as a result of parallax. The relative direction of motion reverses as tbe plane of focus is moved from behind the plane to in front of the scattering plane. (Rigden and Gordon p. 2367) Knoll (1966) noted that the motion is more apparent if the subject's head is held stationary and the surface on which the laser is projected is moved. Rigden and Gordon (1962) observed that it was necessary for the surface to be moved extremely slow in order for the pattern to be seen. Wittenberg (1972) found that the size of the granules in the pattern will vary directly with the distance between the plane of regard and the plane conjugate with the retina;

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directly with the absolute magnitude of the radius; directly with the wavelength of the light used; and will vary inversely with the diameter of the pupil. The apnarent spread of motion of the pattern (for a constant rate of drum movement) is directly proportional to the distance of the conjugate plane from the reference plane to the radius of curvature of the reflecting surface and to the reciprocal of the wavelength. ^(Wittenberg p. 30)

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From: Mohon and Rodemann, "Laser Speckle for Determining Ametropia and Accommodation Resconse of the Eve," Applied Ontics. 12: 784, April, 1973.

 $5.$

METHOD

Subjects:

Thirty-nine subjects were used in this experiment. These subjects included thirty-four male optometry students, age 22 to 30 and five clinical advisors whose ages were 37, 53, 34, 32, and 55. All subjects were able to attain visual acuity of 20/20 at sixteen inches with a reduced Snellen acuity chart. All subjects appeared to be in good physical health with no uncorrectable refractive errors. All subjects were, to a degree, trained observers.

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Laser target apparatus :

In 1966, the Bausch and Lomb company experimented with, and eventually placed on the market, a laser target for use with far point refractive techniques. Knowing that a coherent light beam of a visible spectral wavelength, projected on a randomly diffusing surface would create a random interference pattern, they constructed such a surface on a drum 7 inches high and 6 inches in diameter. Also knowing that the interference pattern thus created would be percieved as flowing in a particular direction, or seem to "swirl" in place, depending on the conjugate point of the refractive system of a given observer viewing the drum as it was slowly rotated, they incorporated a drive system that would rotate the drum about its cylindrical axis at the rate of $\frac{1}{2}$ revloutions per minute. This. now rotating drumwas given the added capability of being rotated about an axis perpendicular to and bisecting its cylindrical axis. Using the above target with a laser beam projected on it from the

viewer's side, and by manipulating lenses before the viewer's eyes, refractive errors could be neutralized in the various major meridians of concern. (Knoll p. 415-418)

The apparatus design used in this project makes use of some of the features used in this Bausch and Lomb instrument, particular-Iv its rotational capabilities. It also differs greatly from the Bausch and Lomb instrument in that it is of a "unit" design. That is, the coherent light source and the rotational target are housed in one portable unit, and the unit is used at the near point $(16")$ instead of at far $(20")$. Instead of a drum target, the near point laser unit has a small rotating rod on which the laser light is projected.

A box, measuring 13 inches long, 8 inches wide, and 13 inches high was constructed from $1/2$ inch plywood. The box sits upright on its 8 by 13 inch bottom, and one side was left open and fitted with a door. A full shelf was fitted parallel to and about halfwar between the bottom and top. The rotational rod target was mounted in the lower compartment and a 3 inch diameter hole was cut in the target end of the compartment to afford an aberture in which the rod target could be seen from outside of the box. The rod consisted of a 1 and $1/8$ inches long, one inch diameter dowel of birch wood. The random cellular structure of this type of material provides an efficient diffusing surface, dowel was drilled through its cylindrical axis, and mounted on a $1/8$ inch steel shaft. This shaft in turn was mounted in a bearing yoke which allowed the dowel to rotate about its cylindrical axis as well as perpendicular to and bisecting its cylindrical axis.

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Rotational drive about the cylindrical axis was provided by a. small 1.5 Volt electric motor through a system of gears and pullies giving the dowel a rotational rate of one revolution every two minutes and forty seconds. Power for the drive system was supplied by two 1.5 Volt "D" cell dry charge batteries wired in parallel. Rotation of the dowel about the axis perpendicular to and bisecting its cylindrical axis was accomplished by a hand adjustment on the back side of the target box. This adjustment rotated a shaft upon which the drive motor' and dowel yoke mount were fitted. The dowel could be rotated 180 degrees about this axis, and could be locked in place at 45, 90, 135, and 180 degrees I with a spring loaded drop pin lock. A pointer was also fixed to this axis adjustment, and pointed to degree readings on a protractor scale that was fastened to the back of the target box and set so as to indicate the true meridians of dowel rotation.

A flat-black construction paper disc with a 9/16 by 13/16 inch rectangular aperture was fastened in place directly in front of the rotating dowel and on the dowel yoke so as to turn with the dowel when it was turned about the axis perpendicular to its axis of constant rotation. This aperture was bisected from side to side and likewise end to end by two waxed threads so positioned as to leave a space between them that subtended about 1. 5 minutes of visual angle at the viewing distance of sixteen inches. These "spaced threads" provided an acuity demand for control of the subject's accommodation.

The upper compartment of the box housed a Meteorlogical Helium-Neon Laser that emitted light at 632 nanometers with a

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power of 0.5 Milliwatt. The laser was mounted with its axis parallel to that of the drive unit and dowel bearing yoke of the lower compartment. The laser was powered by a standard 60 cycle, 110 Volt source and was cooled by internal heat sinks. When turned on, the beam of the laser passed through a $15%$ transmittance neutral grey filter, a 1/2 inch diameter aperture hole in the target end of the box, and was diffused by a minus 20. Diopter biconcave, clear lens. The laser light was then reflected from a front surface mirror mounted 8 inches from the front of the box and was projected onto the rotational dowel target. The beam traversed an optical path of 20 inches.

A standard Keratometer mounting plate was fastened to the bottom of the target box. This facilitated the convenient use of the.unit on the Keratometer mount arm of any standard refracting .. unit stand. Photographs of the laser apparatus are included below.

Front View

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EXPERIMENTAL TESTING PROCEDURES

Each subject was seated in a standard examination chair, with the headrest in proper adjustment. A bausch and Lomb Greens refractor was used in all cases. The subject's near interpupillary distance was measured and set in the refractor. Through the refractor, the subject viewed a near Snellen card which was pleced on the reading rod at a. distance of sixteen inches. Each sub ject's most recent spectacle correction was recorded.

Non-laser testing

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A #21 monocular finding (CEP) (Maximum plus for 20/20 at 16") was taken without the subject's habitual cylinder. If the subject wore a high cylindrical correction, one-half to threefourths of this cylinder was used and this value recorded.

A near cylinder testing technique (as used by Pratt) was used to determine the power and axis of astigmatism.

A #21 moncoular was done again if the near cylinder power exceeded 0.50 Diovters. With this cylinder in the refractor, a standard $#14A$ (OEP) (monocular cross cylinder at $16"$) was administered to each subject. All findings were recorded on prepared recording forms.

Laser testing Procedure #1 (Laser #14A)

The laser unit, which was mounted on the Keratometer arm, was moved into position $16"$ from the subject's eye at eye level. The $#21$ monocular with the Fratt cylinder was placed in the refractor. This gave an excessive plus preset for the laser

 $#14A$ procedure. The left eye was occluded and the subject was directed to relax and to look into the upper right quadrant of the illuminated screen and report what he observed. With excess plus (as in #21) the pattern will appear to move in a direction with the rotation of the turning dowel.

In this instance, the motion was in an upward direction, since the dowel was rotating upward for this test. Plus was then reduced in 0.25 Diopter steps until a stationary position or swirling motion was noticed. This value was then recorded directly. In cases where a change of 0.25 Diopter reversed the direction of motion, the last upward response value was recorded

An identical procedure was instituted for the left eye, except that the subject was instructed to look in the upper left quadrant of the aperture or screen.

The near cylinder value was removed from the refractor and the #21 without the Fratt cylinder correction was used as ^a starting point.

The subject was instructed to look at the cross hairs on the target while observing the background motion on the rotating dowel. He was then asked to determine if the pattern flow was parallel to the sides of the aperture at meridians $45, 90, 135,$ and 150 degrees. If the motion was not parallel, the aperture was rotated until the motion was parallel. Plus was then reduced in 0.25 Diopter steps until the first reversal was reached. This power and meridian were recorded. The dowel target was then rotated to a position 90 degrees from the first position and plus was added until a strong "with" motion was observed. Plus was

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afain reduced until re versal, and power as well as meridian were recorded. This same procedure was carried out on the other eye.

After attempting this procedure on a number of subjects, it was found that few responded to this type of testing. Therefore, we did not analyse this data.

Laser Procedure #2

This method was used to determine the refractive error in four major meridians. From this data, the cylinder power and axis were calculated. The dowel target was successively positioned at the 45, 90, 135, and 180 degree meridians. Plus was added until there was a pronounced "with" motion and was then reduced in 0.25 Diopter steps until first reversal of the motion. The subject was constantly reminded to keep the cross hairs clear for purposes of accommodative control. The refractive findings for each measured meridian were recorded. An identical procedure was carried out for the other eye.

The room illumination for the $#21$ monocular and $#14A$ were according to OEP specifications. The Pratt near cylinder was administered with standard near point illumination, using a 40 Watt incandescent bulb. The laser data was obtained using reduced illumination with a range of 1.5 to 2.5 foot candles.

Data Worku^p

The data was worked up according to a method suggested by Dr. Pratt. The differences between meridians which were measured by the laser technique were converted into a pair of cylinders

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.crossed at 45 degrees. Using these two cylinders and an eauation supplied by Dr. Pratt, it is possible to calculate a spherocylinder prescription. Likewise, a sphero-cylinder lens as obtained by the Fratt near cylinder test can be resolved into a pair of crossed cylinders and an average sphere. For convenience, Dr. Pratt's equation orients one of these cylinders at axis 90 or 180 (rectilinear), while the other cylinder is oriented at either axis 45 or 135 (oblique).

Direct axis comparisons were not made between the calculated laser cylinder and the measured near cylinder findings. Instead, comparisons were made on the basis of the rectilinear and oblique cylindrical components.

The formula, which we used for crossed cylinder calculations is derived from Thompson's Formulae, which deal with quantifying obliquely crossed cylinders. For further information on this calculation.method, an explanation has been included at the end of this paper.

Comparisons were made between the conventional $#14A$ finding and the laser "14A"; between the Fratt near cylinder power and the calculated laser cylinder power and ; between the rectilinear and oblique cylindrical components of the calculated laser and the measured Pratt near cylinder findings. In the oblique and rec tilinear cylinder comparisons, all cylinders are expressed as astigmatic intervals such that all minus cylinders axis 180 or plus c^y linders axis 90 are expressed as positive astigmatic intervals. All minus cylinders axis 135 or positive cylinders axis 45 represent a positive oblique astigmatic interval. The

oppositely directed astigmatic intervals are expressed as minus.

In all instances, the non-laser findings were subtracted from the laser findings. Therefore, a minus difference indicates a laser finding of more minus than in the comparable non-laser finding.

Means, standard deviations, and frequency distribution histograms were calculated for all of the compared findings.

RESULTS

The results of this experiment are summarized by the following table:

"0" indicates that laser finding is equal to the non-laser plus finding. "Plus" indicates that the laser finding is greater/than the non-laser finding. "Minus" means that the laser finding is more minus than the non-laser finding.

The total number of findings compared in each case was 77.

The greatest positive correlation exists between the cylinder power obtained by Pratt's near method and the cylinder power

obtained by the laser method. 72% of the eyes measured were within 0.25 Diopters of equality.

The poorest correlation was between the obliaue cylinder components, of which only 45% of the findings were within 0.25 Dioters of equality.

From the graphs at the end of this paper, the range of findings can be observed for each set of data which was compared_. The largest range was 5.00 Diopters and was observed in the oblique cylinder component comparisons. The smallest range was seen in I the power comparisons. This range was 2.50 Diopters.

DISCUSSION AND COWCLUSIONS

In all but one comparison, more of the laser findings were more minus (or less plus) than were the non-laser findings. This seems to be a contradictory statement when compared to available literature which states that findings taken with a red laser are usually more plus, due to chromatic aberration of the eye. (Sinclair pp. 575-576}

This observation may possibly be explained by a type of empty field myopia which has been reported. (Borish p. 812) Under conditions of a field with little detail, the accommodative system sometimes reacts more dynamically in the positive direction, causing measures of accommodative response (such as $#14A$) to be more minus than normal. Since the laser speckle pattern has little acuity demand, such an empth field effect may have resulted in some of our subjects, despite our attempt to control

accommodation by the use of a cross hair target. This question can be answered only by further experimentation in which the actual accommodative nosture is known.

The laser technique is a very sub jective one and reouires a fair amount of patient attention. A few of our subjects gave seemingly unreasonable responses which may be explained by poor observation on their part.

As can be seen from the table on page 15, the poorest correlation between laser and non-laser technicues was found for the oblique cylinder component. This may be due to the fact that most subjects found the laser pattern motion more difficult to observe when the dowel target was oriented in oblique meridians. Recilinear orientation produced better results in most cases.

Although all subjects were able to see the laser pattern movement, some reported that the motion direction did not reverse as conjugacy was approached. Instead, the pattern appeared to rotate in a clockwise manner until it was flowing in the opposite direction. This observation was made more often with subjects having moderate to high astigmatism.

Some subjects felt that the laser 14A determination was ^easier for them to respond to than was the standare #14A test. This laser technique may have possibilities as a screening device, provided that the patient is capable of observing and reporting the mo tion direction .

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SUGGESTIONS FOR FURTHER STUDY

A similar experiment done at the far point may be more use ful for determining cylinder axis and power than was this study. Although we made an attempt to control accommodation by an acuity target, we have no way of knowing that accommodation. was really held in a relatively fixed position. It seems that controlling accommodation at far would be less difficult and more certain than trying to control it at the near point.

If a safe and reasonably priced laser which emits blue or green light could be obtained, the effects of chromatic aberration could be investigated by either the method which we used or by a similar method at far.

SUMMARY

Near cylinder and #14A findings were compared to similar findings taken using a laser target apparatus. Measurements were taken on 39 male subjects, age 22 to 55. Although direct cylinder axis determination was relatively poor by the laser technique, fairly high correlations were found between the $14A$ findings and between the cylinder powers measured by each method.

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If two cylinders exist together with their axes separated 45° , the resultant single cylindrical effect will have an axis between the two originals, and the magnitude of the resultant cylinder will be greater than either of the originals (presuming the original cylinders were of the same sign + or \sim). If the original cylinders are equal in amount, the resultant axis will be 22 1/2° from either primary axis and the cylinder amount will be 1.43 times the primary amount. If the originals are unequal in amount, the resultant axis will be less than 22 1/2° different from the axis of the larger cylinder and resultant amount will be greater than the larger cylinder but less than 1.43 times the larger.

If we call S the smaller of the two original cylinders and L the larger, we can use the ratio S/L , ranging from 0 to 1.0, to determine both axis, X, in degrees from the larger axis in the direction of the smaller, and the resultant cylinder amount, C, as a proportion of the larger cylinder or of proportion of both original cylinders. The enclosed graph indicates the exact amounts of X and C when related to the ratio S/L. The dotted lines represent approximations which can be used if the cylinders are less than 30 and the axes needed are within 2°.

$$
X^0 = 25 S/L
$$
 $C = .85 L + .57 S = .9 L + .5 S = 12.5 L$

In a number of instances it is useful to consider any lens having spherical and cylindrical components as a combination of average dioptric powers in association with a pair of crossed-cylinders, axes 45° apart, whose net dioptric powers are zero. For instance a lens $+1.00$ cyl \times 90 may be considered +.50 sph ϵ (+.50 sph \equiv -1.00 cyl x 180) where the latter two quantities represent a cross-cylinder. The same lens in a different orientation is +1.00 cyl x 75 which may be represented as +.50 sph \in $(*.43 - .86 \times 180) \oplus (+.25 - .50 \times 135)$ in which we have two crosscylinders along with the sphere power.

Conversely, such a combination of cylinders and spheres may be recombined into a single usual ophthalmic lens. In the instance above such powers may be measured: in direction 180 (\degree) +.93 D, direction 45 \degree +.25 D direction 90° +.07 D, direction 135° +.75 D

The vertical/horizontal difference in power (astigmatic power) is $-93 - (+.07) = .86$ D The oblique difference in power (astigmatic power) is $-75 - (+.25) = .50$ D

The average spherical power is $1/9$ (+.93 + .25 + .07 + .75) = + 0.50 0

The \pm .50 sphere may be combined with two cross-cylinders (no net power) with their minus axes in the directions of the larger plus amounts, i.e. x 180° and x 135°.

.. +.50 sph $\frac{2}{5}$ +.43 sph. $\frac{3}{5}$ -.86 x 180° $\frac{2}{5}$ +.25 sph $\frac{2}{5}$ -.50 x 135° The ratio of the smaller cylinder to the larger is $S/L = 50/86 = .58$. The resultant single ophthalmic cylinder axis is: 180 ° - (.25 x .58) = 165°. The resultant single ophthalmic cylinder power is: $C = .86 (-0.86) + .57 (-50)$ $\frac{1}{25} = -.74 - .28 = -1.02 \text{ D}.$

The total spherical average power of a "lens" complex is not changed by any number of additional cross-cylinders. The total spherical power of +.50 D combined with a cross-cylinder of the cylindrical amount and axis indicated above, namely, +0.51 sph $\frac{2}{5}$ -1.02 cyl x 165°, gives the following: $+0.50 \n\hat{\boldsymbol{\epsilon}} +0.51 \n\hat{\boldsymbol{\epsilon}} -1.02 \times 165^{\circ} = +1.01 \n\text{ sph} \n\hat{\boldsymbol{\epsilon}} -1.02 \n\text{ cyl} \times 165^{\circ}.$ $= +1.01$ cyl $\times 75^{\circ}$

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 \mathbb{C} $\left(\times L\right)$ $74 + 55$ $\begin{picture}(20,20) \put(0,0){\line(1,0){15}} \put(15,0){\line(1,0){15}} \put(15,0){\line(1$ $C = [1 + (\frac{5}{2} - 2) \frac{1}{2}]$ $.5209$ C_7 1.2- 5.8644575 $-1,0$ 20° $X^{\circ} = \frac{1}{\frac{1}{2}} \sum_{L} 100$ x° 18 - indirection of smaller frame $\overrightarrow{\sigma}_{\overrightarrow{C}}$ -2 -4 -6 -8 -1.0
 $-5/6$ 5 = smaller magnitude of cylis 45° sport 1. abotted tata from the wall $1.14.$ $10 - 1$ $\begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

O.D. Thosics B. Broth, Korver, Mortvedt, Opden