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LIX. On the measurement of the power supplied to the primary coil of a transformer

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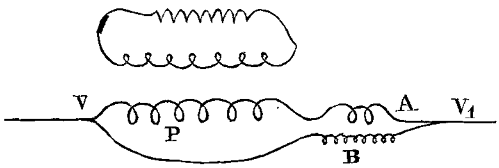
drawn from them is that although the watts per candle for green light are not quite the same for direct and for alternating currents, and although the volts per candle for red light are also not exactly the same for direct and alternating, still the difference between the results for the same colour is so small that it may be put down to experimental errors; and this, combined with the fact that the mean of all the 75 experiments gives practically the same number of watts per candle for both direct and alternating currents, leads to the practical certainty that *the efficiency of an incandescent lamp is the same for both direct and alternating currents.*

LIX. *On the Measurement of the Power supplied to the Primary Coil of a Transformer.* *By* E. C. Rimington*.

In the discussion on Mr. Kapp's paper on Transformers, at the Society of Telegraph Engineers, Professor Ayrton gave a formula for calculating the true power supplied to the primary from the reading of a Siemens wattmeter. The thick wire coil of the wattmeter is in series with the primary



coil, and the fine wire coil connected as a shunt on the two, as in the diagram, where P is the primary coil, A the thick and B the fine wire coil of the wattmeter. Let r_1 be the resistance of the primary P including A, and L_1 its coefficient of self-induction also including A; let r_2 and L_2 be the resistance and coefficient of self-induction of B; i_1 and i_2 the currents in P and B respectively. Let e be the potential-difference between the points V and V_1. Now

$$e = E \sin at, \text{ where } a = \frac{2\pi}{T},$$

T being the periodic time, and E = the maximum value of e. Also

$$i_1 = A_1 \sin(at - \psi_1), \text{ where } \tan\psi_1 = \frac{aL_1}{r_1},$$

and

$$i_2 = A_2 \sin(at - \psi_2), \text{ where } \tan\psi_2 = \frac{aL_2}{r_2}.$$

* Communicated by the Physical Society: read March 10, 1888.

Let δ be the reading on the torsion-head ; then

$$\delta = \frac{k}{T}\int_0^T i_1 i_2 \, dt \text{ ; } k \text{ being some constant.}$$

$$\delta = \frac{k}{T} A_1 A_2 \int_0^T \sin(at - \psi_1)\sin(at - \psi_2)\, dt,$$

$$= \frac{kA_1A_2}{2T}\int_0^T \{\cos(\psi_2 - \psi_1) - \cos(2at - \psi_1 - \psi_2)\}\, dt,$$

$$= \frac{kA_1A_2}{2}\cos(\psi_1 - \psi_2).$$

Now $$A_2 = \frac{E}{S}, \text{ where } S = \sqrt{r_2^2 + a^2 L_2^2}.$$

Hence

$$\delta = \frac{k}{S} \cdot \frac{EA_1}{2}\cos(\psi_1 - \psi_2).$$

Now mean power, or

$$p_m = \frac{1}{T}\int_0^T e i_1 dt,$$

$$= \frac{1}{T} EA_1 \int_0^T \sin at \,.\, \sin(at - \psi_1)\, dt$$

$$= \frac{EA_1}{2}\cos\psi_1.$$

Therefore

$$p_m = \frac{\delta}{k} \,.\, S \,.\, \frac{\cos\psi_1}{\cos(\psi_1 - \psi_2)}.$$

But

$$\frac{\cos\psi_1}{\cos(\psi_1 - \psi_2)} = \frac{\cos\psi_1}{\cos\psi_1\cos\psi_2 + \sin\psi_1\sin\psi_2}$$

$$= \frac{\frac{1}{\cos\psi_2}}{1 + \tan\psi_1\tan\psi_2},$$

$$\frac{\cos\psi_1}{\cos(\psi_1 - \psi_2)} = \frac{\sqrt{1 + \tan^2\psi_2}}{1 + \tan\psi_1\tan\psi_2} = \frac{\sqrt{1 + \frac{a^2 L_2^2}{r_2^2}}}{1 + \frac{a^2 L_1 L_2}{r_1 r_2}}$$

$$= \frac{r_1 S}{r_1 r_2 + a^2 L_1 L_2}.$$

Hence

$$_m = \frac{\delta}{k}\ \frac{r_1 S^2}{r_1 r_2 + a^2 L_1 L_2}.$$

Now for permanent currents,

$$\delta = k\frac{e}{r_2}C, \text{ and } eC = K\delta,$$

where K is the constant of the instrument for watts.

Therefore

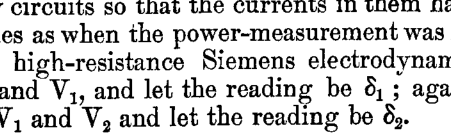
$$k = \frac{r_2}{K},$$

or

$$p_m = K\delta \cdot \frac{r_1(r_2{}^2 + a^2L_2{}^2)}{r_2(r_1r_2 + a^2L_1L_2)}.$$

In order to be able to make use of this result it is necessary to know the coefficients of self-induction of P + A and B; the latter may be found once for all by any of the well-known methods and its value marked on the instrument; but the former will require to be determined for the same values of the currents in the primary and secondary coils of the transformer as are flowing when the power is measured, since the apparent coefficient of self-induction of the primary coil depends on the saturation of the iron of the transformer and also on the current in the secondary coil. The best method of measuring L_1 under these conditions is that due to Joubert.

Connect an inductionless resistance R in series with the



primary, and pass an alternating current of known period through the two; arrange the resistances of the primary and secondary circuits so that the currents in them have about the same values as when the power-measurement was made. Now connect a high-resistance Siemens electrodynamometer* between V and V_1, and let the reading be δ_1; again connect it between V_1 and V_2 and let the reading be δ_2.

Then

$$\frac{\sqrt{r_1{}^2 + a^2L_1{}^2}}{R} = \sqrt{\frac{\delta_1}{\delta_2}},$$

$$L_1 = \frac{1}{a}\sqrt{R^2\frac{\delta_1}{\delta_2} - r_1{}^2}.$$

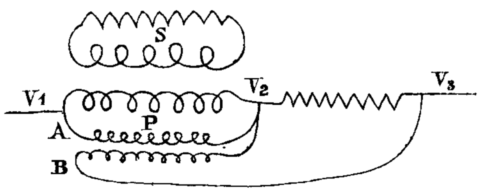
* A Cardew voltmeter may be employed, in which case

$$L_1 = \frac{1}{a}\sqrt{R^2\frac{\delta_1{}^2}{\delta_2{}^2} - r_1{}^2}.$$

This method of calculating the mean power supplied to the primary of a transformer is, however, inconvenient, as it entails the use of a high-resistance dynamometer, with which two observations must be taken to obtain L_1, in addition to the trouble of adjusting the resistances of the two circuits to obtain the same conditions as when the wattmeter-reading was taken.

The following method, in which a high-resistance electro-dynamometer is employed, enables the power given to the primary coil to be measured from one reading without the knowledge of either the resistance or the coefficient of self-induction of the primary.

Let the two coils of the electrodynamometer be A and B, and let their resistances and coefficients of self-induction be $r_1 r_2$ and $l_1 l_2$ respectively; moreover, let $\frac{l_1}{r_1} = \frac{l_2}{r_2}$, that is to say, let the time-constants of the two coils be the same; this can be easily effected by putting an inductionless resistance in series with one or other of the coils. Connect as in the diagram.



Let the primary coil be put in series with an inductionless resistance R. Let the potential-difference between V_1 and V_2 $= E_1 \sin at$; then i, the current through the primary coil and through $R = \frac{E_2}{R}\sin(at-\psi)$, where E_2 is the maximum potential-difference between V_2 and V_3.

Let ψ_1 and ψ_2 be the angles of lag of the coils A and B respectively, and let i_1 and i_2 be the currents through them at some instant t.

Then, if δ be the reading of the torsion-head,

$$\begin{aligned}\delta &= \frac{k}{T}\int_0^T i_1 i_2 \, dt, \\ &= \frac{k}{T}\cdot\frac{E_1 E_2}{S_1 S_2}\int_0^T \sin(at-\psi_1)\sin(at-\psi-\psi_2)\,dt, \\ &= \frac{k}{2}\cdot\frac{E_1 E_2}{S_1 S_2}\cos(\psi_1-\psi_2-\psi);\end{aligned}$$

where

$$S_1=\sqrt{r_1^2+a^2l_1^2} \quad \text{and} \quad S_2=\sqrt{r_2^2+a^2l_2^2}.$$

Now

$$\tan\psi_1=\frac{al_1}{r_1}, \quad \text{and} \quad \tan\psi_2=\frac{al_2}{r_2}.$$

Hence

$$\tan\psi_1=\tan\psi_2, \text{ since } \frac{l_1}{r_1}=\frac{l_2}{r_2},$$

or

$$\psi_1-\psi_2=0.$$

Therefore

$$\delta=\frac{k}{2}\cdot\frac{E_1E_2}{S_1S_2}\cos\psi.$$

Now mean power given to the primary coil, or

$$p_m=\frac{E_1E_2}{2R}\cos\psi,$$

$$=\frac{\delta}{k}\cdot\frac{S_1S_2}{R}.$$

For permanent currents,

$$\delta=k\frac{E_1}{r_1}\cdot\frac{E_2}{r_2}=kE_1C\,.\frac{R}{r_1r_2}.$$

Also

$$E_1C=\frac{K}{R}\delta,$$

where $\frac{K}{R}$ is the constant for watts.

Hence

$$k=\frac{r_1r_2}{K}.$$

Therefore

$$p_m=\frac{K}{R}\cdot\delta\cdot\frac{S_1S_2}{r_1r_2}.$$

$S_1\,S_2$ may be written

$$r_1r_2\sqrt{\left(1+\frac{a^2l_1^2}{r_1^2}\right)\left(1+\frac{a^2l_2^2}{r_2^2}\right)}=r_1r_2(1+\tan^2\psi_1).$$

Hence

$$p_m=\frac{K}{R}\cdot\delta\,.\,(1+\tan^2\psi_1).$$

In order that the non-inductive resistance R shall not absorb too much power, B should be the movable coil of the electro-dynamometer, as this is generally the one of lower resistance; $\frac{E_1}{S_1}$ should have about the same value as $\frac{E_2}{S_2}$, or

$$\frac{E_2}{E_1}=\frac{S_2}{S_1} \text{ approximately.}$$

Now

$$\frac{S_2}{S_1} = \frac{r_2\sqrt{1+\tan^2\psi_2}}{r_1\sqrt{1+\tan^2\psi_1}} = \frac{r_2}{r_1};$$

that is, if r_2 is considerably lower than r_1, E_2 will be in the same proportion lower than E_1, and hence R may be a good deal smaller than the impedance of the primary.

LX. *On the Polarization of Platinum Plates.* *By* C. H. Draper, *B.A., D.Sc.**

THE fact of the decomposition of dilute sulphuric acid by the passage between platinum electrodes of electricity through a voltameter containing it, involves an expenditure of energy. This energy is made available by a sudden fall of the current through a certain difference of potential numerically equal to the energy absorbed by the quantity of water which one unit of electricity decomposes. The cause of this sudden fall of potential within the voltameter over and above that due to the resistance of the liquid as a conductor is the modified condition assumed by the platinum plates, which leads to the phenomenon called polarization. Those portions of the gaseous products which come first into contact with the platinum, especially in the case of hydrogen, either form with the platinum a chemical combination, or undergo such physical or chemical modification by occlusion or condensation as results in a loss of energy and resulting fall of potential. This modification of condition progressively diminishes in the successive layers as we proceed outwards from the platinum plate, until a layer is reached which is beyond the reach of the influence of the platinum, and where the gas escapes freely as fast as it is formed. The result of this polarized condition of the electrodes is manifested as an electromotive force opposed to that which produces the current. The electromotive force of polarization has for each electrolyte a theoretically fixed maximum value, and experimentally there appears to be an approximately constant maximum value which is always above the theoretical value when any considerable current is passing, and which varies with the conditions of the experiment. Thus Professor Tait, in some experiments described in the 'Philosophical Magazine' for September 1869, found (taking the electromotive

* Communicated by the Author.