# AN ABSOLUTE SCALE OF X-RAY WAVE-LENGTHS. 

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## Synopsis.

Absolute Scale of X-Ray Wave-Lengths Based on Discontinuities Due to Analyzing Crystal.-In a previous study of the continuous spectrum from a tungsten target, an absorption line was observed at $0.7928 \times 10^{-8} \mathrm{~cm}$. This result has been confirmed with a molybdenum target, light lines being obtained at 0.7926 and $0.4288 \times \mathrm{ro}^{-8}$ cm . To get these lines with a rock salt crystal a thickness of less than $10^{-2} \mathrm{~cm}$. was necessary. In explanation of the discontinuities it was suggested by A. W. Hull that when any wave-length is reflected by two sets of crystal planes, the energy is divided so that the intensity of the part which reaches the photographic plate is less than for wave-lengths on either side. For each pair of sets of planes the critical wavelength is fixed by the geometry of the crystal alone. Taking the grating constant for rock salt as $2.814 \times 10^{-8} \mathrm{~cm}$., the wave-lengths computed for the planes $100 / 210$ and $100 / 310$ are 0.7959 and $0.4316 \times 10^{-8} \mathrm{~cm}$. respectively. Using these as reference points, the center of the scale of wave-lengths may be accurately determined and a scale of wave-lengths may be established independent of everything except the grating constant.

Bromine and Silver X-Ray Absorption Limits come near the above reference points and were thus accurately located at 0.9186 and $0.484^{2} \times 10^{-8} \mathrm{~cm}$., in close agreement with the values given by Blake and Duane.

K Series of Molybdenum.-The wave-lengths of five lines were determined by reference to $0.7959 \times 10^{-8} \mathrm{~cm}$. more accurately than in previous measurements.
TN a former paper ${ }^{1}$ the author has described some work done at the University of Iowa on the tungsten spectrum and has shown a photograph of a light line which appeared very clearly in the region of the bromine absorption band at a wave-length of $.7928 \times 10^{-8} \mathrm{~cm}$.

Later de Broglie ${ }^{2}$ has published a paper in which he states that he has verified all the other results of the author on the tungsten spectrum but that he has not found this line. Since de Broglie's work has evidently been done with great care, this line might appear to have been a false effect.

The experiment has there-


Fig. 1.
${ }^{1}$ Phys. Rev., N. S., Vol. XVI., Dec., 1920, p. 526.
${ }^{2}$ Phil. Mag., Nov., 1919.
molybdenum tube at the University of Chicago. A light line has again been found to appear unmistakably at a wave-length of $.7926 \times 10^{-8}$ cm . as compared with the wave-length $.7928 \times \mathrm{IO}^{-8} \mathrm{~cm}$. previously found.

## Apparatus and Method.

The source of the rays was a Coolidge tube with a molybdenum target excited by means of a transformer, the tube rectifying its own current. The continuous spectrum was produced by a thin crystal of rock salt rotated slowly by clockwork. The exposure was twenty hours per degree of rotation with a constant input of 150 watts. The distance from plate to crystal axis was kept in the neighborhood of 30 cm . and the slit was .030 cm . in width. The wave-lengths were measured by Dershem's method ${ }^{1}$ thus eliminating the necessity of correcting for depth of penetration into the crystal. The spectrometer although differing widely in detail, was constructed on the same principle as that used by Dershem and will therefore require no description here.

Results.
It was found that these lines could not be produced clearly using a thick crystal. Indeed, one crystal which failed to produce an effect when it had a thickness of . 013 cm . gave a clear photograph of the line $.7926 \times 10^{-8} \mathrm{~cm}$. when ground down to a thickness of .006 cm . It would seem that the crystal must have a thickness less than about .or cm . in order to produce a clear image of this line. The crystal which produced the best photograph had an average thickness of .0059 cm . Under conditions as outlined above, this line will appear very readily.

The line of wave-length $.4288 \times 10^{-8}$ appears very feebly as would be expected from the fact that there are fewer atoms in the planes reflecting the radiation away from the plate at this point and it is therefore very difficult to measure it accurately. It has been found on only one plate since lack of time prevented my finding it a second time. For this reason the measurement is probably somewhat uncertain.

## Theory.

I am indebted to Dr. A. W. Hull for suggesting the following very interesting interpretation of the meaning of these light lines.

The depth of penetration of the reflected rays into the crystal and hence the intensity of the photograph is limited by absorption. For the small angles under consideration a large part of this absorption is scattering. But the scattering due to any set of planes is small except when it is at the angle of perfect reflection.
${ }^{1}$ Phys. Rev., N. S., Vol. XI., June, 1918, p. 46 I.

Let us suppose, for example, that the homogeneous spectrum is being reflected to the photographic plate by a set of planes $A$. As the crystal is rotated it may come to the angle of perfect reflection for another set of planes $B$ with the result that the crystal suddenly becomes more opaque to the rays which should be reflected from $A$; i.e., some of the radiation which we should expect to be reflected by the set $A$ is reflected in another direction by the set $B$ and does not reach the photographic plate. The intensity of the photograph will therefore be diminished at the wave-length where this occurs and a light line will result.

The angle at which this will occur depends upon the relative positions of the two sets of planes. To illustrate, let us suppose that the continuous spectrum is being produced by the 100 planes and that the crystal is rotated until it reaches a position where the same wave-length, $\lambda$, is reflected simultaneously by the 100 and 210 planes.

Then

$$
\lambda=2 d_{100} \sin \theta_{100}=2 d_{210} \sin \theta_{210},
$$

or

$$
\sin \theta_{210}=\frac{d_{100}}{d_{210}} \sin \theta_{100}
$$

But since

$$
\frac{d_{100}}{d_{210}}=\sqrt{5} .
$$

We have

$$
\begin{equation*}
\sin \theta_{210}=\sqrt{5} \sin \theta_{100} \tag{I}
\end{equation*}
$$

Another relation is evident from Fig. I, namely,

$$
\begin{equation*}
\theta_{100}+\theta_{210}=\tan ^{-1} \frac{1}{2} \tag{2}
\end{equation*}
$$

Equations (1) and (2) fully determine $\theta_{100}$ and $\theta_{210}$. Solving, we get,

$$
\begin{aligned}
& \theta_{100}=8^{\circ} 7^{\prime} 48^{\prime \prime} \\
& \theta_{210}=18^{\circ} 26^{\prime} 06^{\prime \prime} .
\end{aligned}
$$

And

$$
\lambda=2 d \sin \theta=.7959 \times 10^{-8} \mathrm{~cm}
$$

Other combinations of atomic planes should produce other lines as indicated in Table I.

Table I.
Grating constant for rock salt $=d=2.814 \times 10^{-8} \mathrm{~cm}$.

| Planes. | $\begin{gathered} \text { Wave-Lengths } \\ \text { in } \\ 10^{-8} \mathrm{~cm} . \end{gathered}$ | Observed Values. | Correction. |
| :---: | :---: | :---: | :---: |
| 100 and 210 . | . 7959 | . 7926 and .7928 | +.0033 and +.0031 |
| 100 and 310 . | . 4316 | . 4288 | +. 0028 |
| 100 and 320. | 1.103 |  |  |
| 100 and 110. | 1.779 |  |  |

The intensity of these lines would vary in accordance with the relative number of atoms in the planes producing them. The absorption coefficients of the crystals used would also have some effect in changing the relative intensity. However, it seems reasonable to suppose that the first three lines mentioned in the table should be visible with a rock salt crystal.
Several attempts were made to find the first three lines. As shown in the third column, these attempts have met with success in the first two cases only. The line of wave-length $1.103 \times 10^{-8}$ was obscured on the photographs (if it did exist) by the fact that it is very close to the $\gamma_{1}$ line of tungsten which appeared on all the molybdenum photographs of this region.

## An Absolute Scale of X-Ray Wave-Lengths.

The wave-lengths .7959 and .4316 , as determined above, depend only upon constants of the crystal and are totally independent of the apparatus. They therefore furnish a method of finding the center correction more accurate than any other yet devised.

Furthermore, any wave-lengths of lines appearing on the same plat in the immediate region of one of these light lines may be corrected bye adding the difference between the calculated and observed values of these light lines as shown in the fourth column. Wave-lengths so found should be more accurate than those found by other methods provided the measurements are accurate. They would be subject to twice the error in the dividing engine used in measuring the plates and to the error in the grating constant but to no other error. An absolute scale of wavelengths is thus established.

The Bromine Absorption Band.
The bromine absorption band always appears on photographs with a sharp critical absorption edge and so furnishes a convenient reference line. It lies very close to the line .7959 and hence the above-mentioned correction can be applied to it.

Table II.
Bromine Critical Absorption Wave-Length $\times 10^{8}$.

|  | Obs. Value. | Correction. | Corrected Value. | Blake and Duane. ${ }^{2}$ |
| :--- | :---: | :---: | :---: | :---: |
| Plate A. . . . . . | .9151 | +.0033 | .9184 | .9179 |
| Plates $210,213 \ldots \ldots$ | .9157 | +.0031 | .9188 | .917 |

The average value of the critical absorption wave-length found by this method is seen from Table II. to be $.9186 \times 10^{-8} \mathrm{~cm}$. which agrees within one tenth of one per cent. with the value found by Blake and Duane. ${ }^{1}$

The Silver Absorption Band.
The silver critical absorption wave-length appears on the same plate as the line $.4288 \times 10^{-8}$ and lies near it in the spectrum. Hence it can be determined as follows:

$$
\begin{aligned}
& \text { Observed Ag. abs. wave-length . ............... . . } 4814 \times 10^{-8} \mathrm{~cm} \text {. } \\
& \text { Correction......................................... . . }+.0028 \\
& \text { Corrected value. . . . . . . . . . . . . . . . . . . . . . . . . . . } \overline{.4842} \times 10^{-8} \mathrm{~cm} \text {. }
\end{aligned}
$$

Here the corrected value is within two tenths of one per cent. of the value given by Blake and Duane, namely, $.4850 \times 10^{-8} \mathrm{~cm}$.

## The K Series of Molybdenum.

The K series lines of molybdenum all appear on the same plates with the line $.7959 \times 10^{-8}$ and can be corrected as follows:

| Plate A. |  |  | Plates 210 and 213. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Designation. | Observed Value. | Corrected Value. | Observed Value. | Corrected Value. | Average Corrected. |
| Light 210/100.. | . 7926 | . 7959 | . 7928 | . 7959 | . 7959 |
| Mo $\alpha_{2}$. | . 7099 | . 7132 | . 7099 | . 7130 | . 7131 |
| Mo $\alpha_{1}$. | . 7055 | . 7088 | . 7056 | . 7087 | . 7087 |
| Mo $\beta$. | . 6294 | . 6327 | . 6291 | . 6322 | . 6324 |
| Mo. | . 6235 | . 6268 | - | - | . 6268 |
| Mo $\gamma . . . . . .$. | . 6181 | . 6214 | - | - | . 6214 |

These corrected wave-lengths should be more reliable than any other values heretofore found.

## The K Series of Tungsten.

For comparison, one photograph has been made of the second order K series lines of tungsten. Only $\alpha_{2}$ and $\alpha_{1}$ appeared on the plate. These lines are quite close to the silver absorption edge and so can be calibrated from its wave-length after this has been corrected by means of these light lines. The distance between crystal and plate for this photograph was 48.136 cm . and the center of the scale was found by calling the wave-length of the silver absorption edge $.4842 \times \mathrm{IO}^{-8}$ as determined above. This gives the correct values at once. The measurements are as follows:
${ }^{1}$ Phys. Rev., N. S., Vol. X., 1917, p. 700.


The results of Siegbahn ${ }^{2}$ and Duane and Patterson ${ }^{1}$ are listed for comparison, those of Duane and Patterson being changed slightly to make them conform to the grating constant here used.
The extreme faintness of the line $.4316 \times 10^{-8}$ by which the silver absorption edge is calibrated introduces a certain degree of uncertainty into these results. However, the agreement is as good as could be expected under the circumstances.

More accurate determinations of the wave-lengths of these light lines would seem to be of great importance to the study of x-ray spectroscopy. Further work along these lines is now being done at the Ryerson Physical Laboratory.

In conclusion, the author wishes to express his thanks to Dr. A. W. Hull for his suggestions, to Professor R. A. Millikan for his kindly interest, and to Mr. Erik Andersen for valuable assistance rendered in the experimental work.

