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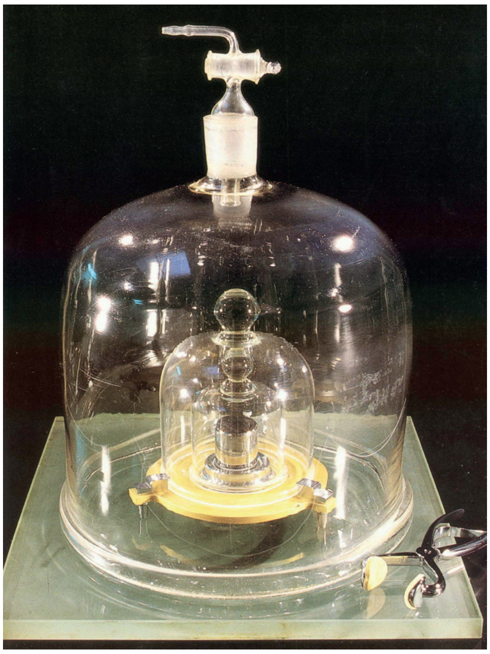
The General Conference on Weights and Measures is expected to adopt a fundamental revision of the International System of Units (SI) on the 16th of November 2018. Newly, a set of seven defining constants with fixed values completely sets the system and forms the basis for defining the units. After more than one hundred years of use, the revision will release the last artifact in the SI, the prototype kilogram, from service and replace it with a mass unit based on natural constants.

In almost all areas of modern society, from science and technology to industrial manufacturing and commerce to daily life, the International System of Units (SI) is used to express the results of measurements in a clear and comparable manner. With the advancement in science and technology, the SI has to evolve and adapt to the needs of users. The revision, to be approved in November 2018, marks a milestone in the further development of the SI. The changes will make measurement results even more consistent, reliable and accurate, enabling new scientific discoveries and innovations.

What distinguishes a system of units?

The choice of a system of units is not a strictly scientific process. It is characterized by historical compatibility conditions, practical considerations, and the knowledge of physical contexts but also some arbitrariness. Thus, the SI introduced worldwide today is the result of a long historical development. Above all, increasing demands on the accuracy of measurements repeatedly led to improvements in the definitions of the units [1]. An important constraint for changes is backward compatibility. Measurement results, such as the climate data should be comparable over a long period. This is only possible if the units used are stable and comparable in time within their uncertainties.

The current SI distinguishes base and derived units. The values of the base units, seven for the moment, are chosen by convention. The derived units are constructed as products of powers of the base units.



1: International prototype of the kilogram. Cylinder made of a platinum-iridium alloy, kept at the International Bureau for Weights and Measures (BIPM) in Sèvres, Paris. (Source: BIPM)

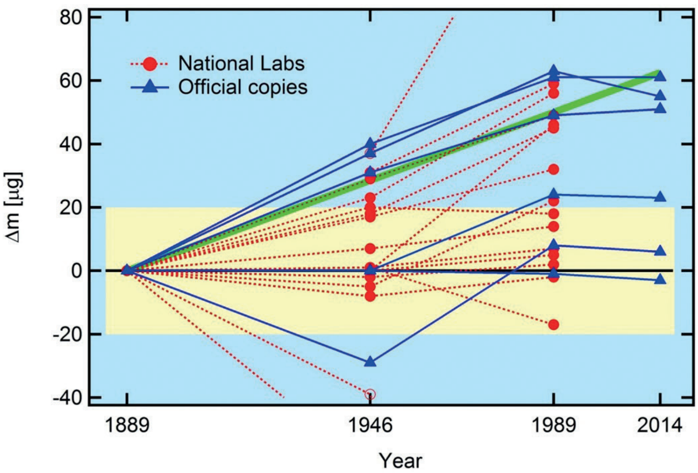
The definitions of the base units, as they have been used over time, can be divided in a simplified manner into different classes:

1.) An appropriate artifact is selected as the unit realization for the desired quantity. Until the upcoming revision, the kilogram is the last SI unit defined in this way: The kilogram is the mass of the International Kilogram Prototype, a cylinder made of a platinum-iridium alloy, which is stored at the International Bureau for Weights and Measures (BIPM) in Paris (Figure 1). This definition obviously has a local character. The unit is only available in one place, the BIPM. The transfer of the unit is done by comparison with the original standard and the accuracy is thus limited by the accuracy of the comparison method. Since the prototype kilogram is a macroscopic body with an unstable surface, the temporal evolution of the unit is not known exactly. This is the biggest disadvantage of the definition.

2.) A unit may also be realized on the basis of a suitable physical state. In this way, the second is defined by the period of the radiation of an atomic transition in the cesium atom. Prior to the revision, the realization of the temperature unit kelvin, makes use of the fact that the thermodynamic temperature of the water at the triple point keeps a stable value independent of environmental influences. The triple point is the state in which all three phases of the water, solid, liquid and gaseous, are in equilibrium. The unit realizations supported in this way have a universal character. This means that the units can be realized everywhere and at any time. All Cs atoms have the same properties that do not change over time. However, the states cannot be described with sufficient accuracy by an analytic model equation. In addition, the accuracy of the unit realization is limited by the characteristics of the chosen physical process itself.

3.) Finally, units can be based on natural constants. These also appear as proportionality constants and quantitative connecting points in the physical theories. Their value cannot be influenced and does not change spatially or temporally. Natural constants are thus the "natural" units and, in an ideal manner serve as the basis for the determination of SI units. Until the revision, the meter and the ampere are examples of this unit class. The meter definition assigns a fixed value to the speed of light in vacuum. In the case of the ampere definition, a fixed value is attributed to the magnetic permeability of the vacuum. Basic units of this type are universal in nature, such as those of type 2. However, they are not bound to specific physical states, which allows for a progressive improvement of the realization with the advancement of physics and technology.

Why a revision of the SI is needed?
Up to the SI revision, the kilogram is the last base unit still represented by an artifact. The kg is defined as the mass of the kilogram prototype. Copies of this standard are preserved by many National Metrology Institutes (NMIs) around the world. Since 1889, these copies have been compared three times to the international prototype. A number of copies were produced later and a comparative measurement with the prototype took place only twice. For both groups, it has been found that the mass of national copies has increased on average compared to the international prototype [2] (see also Figure 2). The mean relative change of about 50 µg in 100 years is very small. However, because the electric units refer to the force and thus to the kilogram through the ampere definition, a drift of the kilogram induces a similar drift in the electrical units.

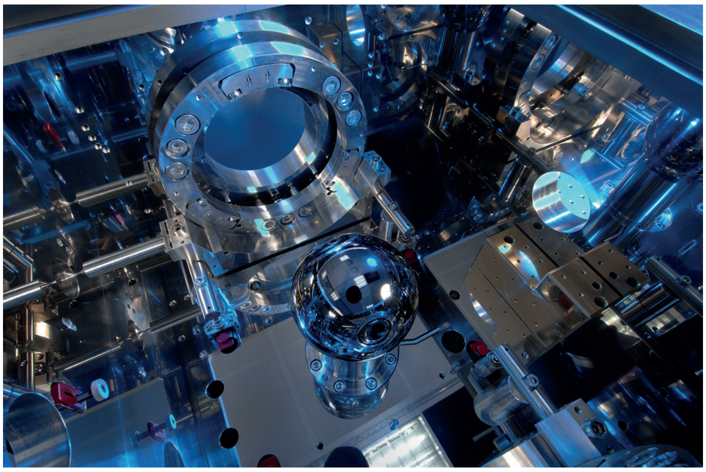


2: Periodic Verification: Comparison of National Copies and Official Copies with the kilogram prototype. The comparisons took place at the time of introduction in 1889, then in 1946 and finally in1989. The comparison of 2014 is not an official "periodic verification" because only one subset of the standards was involved.

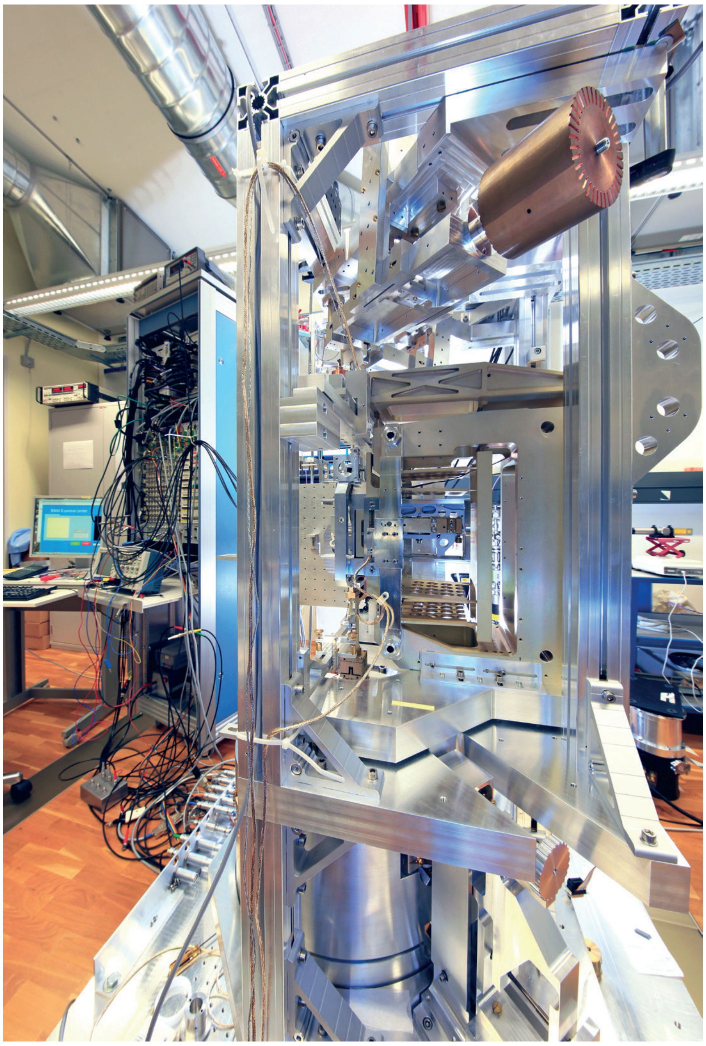
The ampere definition links electrical to mechanical units. For the realization of the electrical units, complicated electro-mechanical experiments are necessary (Kibble balance, calculable capacitor, ...). In modern electrical measurement technology, however, very reproducible voltage and resistance values are realized with the Josephson and quantum Hall effects, which according to the state of knowledge depend only on natural constants [3], [4]. The voltage of a Josephson standard is inversely proportional to the Josephson constant $K_J = 2e/h$. The quantized Hall resistance is proportional to the von Klitzing constant $R_K = h/e^2$. Josephson and quantum Hall standards are, thus, directly traceable to the elementary charge e and the Planck constant h. K_J and R_K could be determined before the SI revision with a relative uncertainty of 10^{-7}. This is about 100 times worse than the reproducibility of quantum effects in the laboratory. As a result, the International Committee of Weight and Measures introduced K_{J-90} and R_{K-90} on 1.1.1990 by convention: K_{J-90} = 483 597.9 GHz V^{-1}, R_{K-90} = 25 812.807 Ω.

This step has dramatically improved the global consistency of electrical measurements. On the other hand, however, it led to a practical subsystem in the SI, which is unsatisfactory from a conceptual point of view.

Also in temperature measurement, the previous definition of the base unit kelvin via the water triple point cell (type 2 according to the classification above) reaches its limits. The realization is sensitive to impurities in the triple point cell and the isotopic composition of the water used. In addition, the realization of the scale from the zero point and the triple point is very time consuming.



3: The sphere interferometer of the German Physikalisch Technische Bundesanstalt (PTB), with which the diameter of the silicon spheres in the XRCD experiment can be measured accurately down to a few nanometers. (Source: PTB)



4: Kibble balance at METAS. The Kibble balance compares mechanical with electrical power. If the electrical quantities are measured in terms of the Josephson and the quantum Hall effect resp., a relation between the test mass and the Planck constant is established.

Experimental precondition for the revision

In order to remedy the identified weaknesses of the system, extensive work was required on two fronts on the experimental side: the establishment of a link between the kg and the Planck constant with a relative uncertainty of $\leq 2 \times 10^{-8}$ as required by the specialists, and the determination of the Boltzmann constant *k* with a relative uncertainty $\leq 10^{-6}$. Especially the first problem turned out to be very persistent. Two fundamentally different approaches were pursued:
In the X-Ray Crystal Density (XRCD) experiment, the mass of a silicon atom is measured with high accuracy by "counting" atoms in a nearly perfect Si crystal [5] (Figure 3). The atomic mass in turn can be linked to the Planck constant *h* with very high accuracy. Therefore, the XRCD experiment offers the possibility to refer the kilogram either to an atomic mass or to the Planck constant. Another experimental approach is the so-called "watt balance" (or after its inventor, the "Kibble balance") [6] (Figure 4). The balance compares mechanical and electrical power. If the electrical power is measured with quantum standards, the mass can be related to Planck's constant [5]. Of course, the results of the two different approaches must agree.

There are a number of methods for determining the Boltzmann constant *k* [7]. The most accurate is the acoustic gas thermometer, where *k* is determined from the velocity of sound in a gas as a function of temperature.

The demands on consistency and accuracy in determining the Planck resp. Boltzmann constants and thus the conditions for a revision of the SI were achieved by spring 2017.

A set of constants defines the system

We have seen in the previous sections how the definition of the SI units ranges from a one to one relationship to an artifact (kilogram), through reference to a physical system or state (triple point of water for the kelvin), to the direct use of a natural constant as reference (speed of light for the meter). In the final step, the realization of the unit is conceptually detached from the definition. A unit defined by the fixed value of natural constants can be realized in accordance with the physical laws of science and technology. Improvements in the realization are possible without having to redefine the unit.

With the advances in the experiments, it becomes possible for the first time to base the entire SI on a set of constants with fixed values. In the SI, we have made the choice to select seven base units by convention. For this reason, we also have seven constants to set. The selected set of constants is as follows (see also info box and figure 5):

5: The revised SI: The inner circle shows the 7 defining constants. They form the building blocks for the realization of units shown on the external circle. The seven base units of the SI are shown. All other units can be derived from combinations of the defining constants. Base and derived units are equivalent.

- $\Delta\nu$: *Frequency of the unperturbed ground-state hyperfine transition of the* ^{133}*Cs atom*.
  This constant defines the second. The SI revision does not change the practical realization of the unit.
- *c*: *Speed of light in vacuum*.
  With *c* and the second based on $\Delta\nu$, the meter can be realised. Again, the revision does not change anything in the immediate practice.
- *h*: *The Planck-constant,*
  together with *c* and $\Delta\nu$ and the appropriate experiments, linking the microscopic to the macroscopic mass scale, enables the realization of the kg. This is the most important result of the revision.
- *e*: *The elementary charge,*
  together with the second based on $\Delta\nu$ redefines the ampere. The ampere can be realized directly via single-electron circuits. The advantage of the definition of A, however, lies mainly in the fact that when fixing the Planck constant and the elementary charge, the Josephson and the Klitzing constant are also fixed and thus the Josephson and the quantum Hall effect realize the volt and the ohm resp. directly in the revised SI. This eliminates the need for the conventional constants K_{J-90} and R_{K-90}, as well as the practical subsystem.
- *k*: *The Boltzmann constant*,
  together with $\Delta\nu$, *c*, *h* and a suitable primary experiment (e.g., acoustic gas thermometer), realizes the kelvin. Therefore, the temperature of the triple point of water is no longer fixed and is now subject to uncertainty.
- N_A: *Avogadro constant*.
  By fixing the value of the Avogadro constant, the mole is defined as the amount of the substance containing $6.022\,140\,76 \times 10^{23}$ specified elementary particles. The link to the kg, as it was previously made in the mole definition, is eliminated. Thus, the mass of ^{12}C no longer has a fixed value, but carries an uncertainty.
- K_{cd}: *Luminous efficacy of monochromatic radiation of frequency* 540×10^{12} *Hz.*
  With this definition, the realization of the candela remains unchanged.

The SI is a practical system and in this sense, it is not surprising that the constants listed above do not all have the same significance. The speed of light *c* and the Planck constant *h* are considered as truly fundamental constants in modern physics. They refer to general properties of space, time and physical processes, which apply equally to every kind of particle and interaction. The Boltzmann constant *k* can be considered as a conversion factor between energy and temperature. The ground state hyperfine splitting frequency of the cesium 133 atom $\Delta\nu$ is the property of a particular atom. It cannot be easily expressed in more fundamental terms. The accuracy of the realization of the unit second, associated with this constant, is limited by the natural linewidth of the atomic transition. Considerable efforts are being made to define the unit of time in the foreseeable future through a more fundamental constant. The Avogadro constant N_A and the luminous efficacy K_{cd} are chosen for practical reasons; physicists do usually not consider them "basic".

Definition of the SI:

The International System of Units, SI is defined by fixing the values of 7 constants. The numerical values are taken from CODATA's least square adjustment carried out in the summer of 2017. The SI is the system of units in which:

- the unperturbed ground state hyperfine splitting frequency of the ^{133}Cs atom $\Delta\nu = 9\ 192\ 631\ 770\ s^{-1}$
- the speed of light in vacuum $c = 299\ 792\ 458\ m\ s^{-1}$
- the Planck constant $h = 6.626\ 070\ 15 \times 10^{-34}$ J s (J s = kg m^2 s^{-1})
- the elementary charge e = $1.602\ 176\ 634 \times 10^{-19}$ C (C = A s)
- the Boltzmann constant $k = 1.380\ 649 \times 10^{-23}$ J K^{-1} (J K^{-1} = kg m^2 s^{-2} K^{-1})
- the Avogadro constant $N_A = 6.022\ 140\ 76 \times 10^{23}\ mol^{-1}$
- the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz $K_{cd} = 683\ lm\ W^{-1}$

Using the fixed constants and with the help of the laws of physics all units can be realized in the SI. The constants are the building blocks and set the standard for the entire system. Consequently, it is no longer necessary to distinguish between base and derived units. All units of the SI are derived from the chosen set of seven constants and are, thus, equivalent.

The Committee on Data for Science and Technology Committee (CODATA) periodically provides the scientific and technological community with a self-consistent set of internationally recommended values of natural constants and conversion factors for physics and chemistry through its Task Group on Fundamental Constants. Because of this role, the General Conference for Weights and Measures invited the CODATA Task Group to undertake a special adjustment to determine the values of the defining constants for the revised SI. All relevant data published or accepted for publication before the end of June 2017 have been taken into account in the analysis. The results of this calculation are listed in Box 1 [8], namely the numerical values of *h*, *e*, *k* and N_A, each with a sufficient number of digits to ensure consistency between the previous and the revised SI. The next regular CODATA periodic adjustment of the fundamental constants takes place in 2018. It will be unique as it will be the first one based on the exact fundamental constants of the revised SI.

What are the changes for the user?

The revised SI will enter into force on 20th May 2019, the World Metrology Day 2019. Although, on this day the most fundamental change since the introduction of SI will be implemented, it will not have any immediate impact on daily life. Despite the new definition, the values of the units kilogram, kelvin and mole remain initially unchanged. Only with the electrical units, small corrections will be necessary. The redefinition of the ampere makes obsolete the practical units defined by the conventional values of the Josephson and Klitzing constants. The "return" to SI means a relative change of 1.07×10^{-7} for voltage measurements and 1.78×10^{-8} for resistance measurements. These corrections are so small that they are only relevant to a few users outside the NMIs.

Conclusion

Thanks to the revision, the International System of Units is fit for the future. It is designed to allow better realizations of the units over time without being explicitly dictated by the system. Thus, the SI is on a solid basis in the long term and remains the foundation worldwide for measurements with the accuracy required by society, industry and science.

Beat Jeckelmann graduated in 1986 from the University of Fribourg with a doctoral thesis in the field of experimental physics. He then worked as a post-doc in the Department of Physics at ETH Zurich and at the Massachusetts Institute of Technology. At the end of 1988 he joined the electricity sector of METAS. There his research work was mainly focused on the application of the quantum Hall effect as resistance standard and the associated measurement techniques. From 1997 to August 2015, he headed the electricity sector. Since 2011 he holds the position of Chief Science Officer at METAS.
One of the focal points of his work is the international cooperation in the metrology community, mainly in the framework of the European Association of National Metrology Institutes (EURAMET). He has been the Chair of the Technical Committee for Electricity and Magnetism; he is member of EURAMET's Board of Directors since 2010 and Chair of the association between 2015 and 2018. He is the Swiss representative in the Consultative Committee for Units of the Metre Convention.

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