Philosophical Magazine Series 6

ISSN: 1941-5982 (Print) 1941-5990 (Online) Journal homepage: http://www.tandfonline.com/loi/tphm17

XLVI. A reciprocal relation in diffraction

A.A. Michelson

To cite this article: A.A. Michelson (1905) XLVI. A reciprocal relation in diffraction , Philosophical Magazine Series 6, 9:52, 506-507, DOI: 10.1080/14786440509463300

To link to this article: http://dx.doi.org/10.1080/14786440509463300

Published online: 08 Jun 2010.

XLVI. *A Reciprocal Relation in Diffraction.* *By* A. A. Michelson *.

SUPPOSE the vibration at the surface A of a sphere whose centre is at O to be a known function of x and y, the origin being a point P on the sphere,

$$V = \phi \cos nt + \psi \sin nt. \quad \quad (1)$$

Then the vibration † on a sphere B passing through O whose centre is at P will be

$$W = -\frac{1}{\lambda f}\iint \phi \, dx \, dy \sin n(t-\tau) - \frac{1}{\lambda f}\iint \psi \, dx \, dy \cos n(t-\tau) \quad (2)$$

Putting

$$\tau = \frac{f}{a}\left(1 - \frac{x\xi + y\eta}{f^2}\right), \quad t - \frac{f}{a} = t_1, \quad \frac{n\xi}{fa} = u, \quad \frac{n\eta}{fa} = v,$$

$$\left.\begin{aligned} W = &-\frac{1}{\lambda f}\left[\iint \phi \, dx \, dy \cos(ux+vy) - \iint \psi \, dx \, dy \sin(ux+vy)\right]\sin nt \\ &-\frac{1}{\lambda f}\left[\iint \phi \, dx \, dy \sin(ux+vy) + \iint \psi \, dx \, dy \cos(ux+vy)\right]\cos nt \end{aligned}\right\} \quad (3)$$

or

$$W = P \sin nt_1 + Q \cos nt_1 . \quad \quad (4)$$

If now a spherical mirror be made to coincide with the sphere B an image of the source will be formed at A.

This image may also be considered as the resultant of the vibrations at B. Hence, if we designate by DW the

* Communicated by the Author.

† Scientific Papers of Lord Rayleigh, vol. iii. p. 80. The results given by Lord Rayleigh for the *intensity* of the diffraction figure in the focal plane do not apply to the phase of the vibration. This restriction is removed if the surface considered be the sphere B; for the distance between two points on the spheres is

$$\rho^2 = (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2,$$

or if $f^2 = x^2+y^2+z^2 = \xi^2+\eta^2+(\zeta-f)^2$

$$\rho^2 = f^2 - 2x\xi - 2y\eta - 2z\zeta + 2f\zeta.$$

If ξ and η are small, ζ will be of the second order, and so is $f-z$, so that $(z-f)\zeta$ is of the fourth order and may be neglected. Hence

$$\rho^2 = f^2 - 2x\xi - 2y\eta$$

or approximately

$$\rho = f - \frac{x\xi + y\eta}{f}.$$

operation by which W is obtained from V,

$$\left.\begin{aligned} W &= DV \\ V &= DW \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or

$$V = DDV. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

These equations hold for light of any colour, and—with corresponding extension of the definition of D—to any combination of colours.

The analogy with Fourier's formula is apparent. In fact, the second of equations (5) is, putting $n/fa = p$,

$$V = -\frac{1}{\lambda f}\left[\iint Q\,d\xi\,d\eta\cos p(x_1\xi+y_1\eta) - \iint P\,d\xi\,d\eta\sin p(x_1\xi+y_1\eta)\right]\cos nt_2$$
$$-\frac{1}{\lambda f}\left[\iint Q\,d\xi\,d\eta\sin p(x_1\xi+y_1\eta) + \iint P\,d\xi\,d\eta\cos p(x_1\xi+y_1\eta)\right]\sin nt_2$$

Substituting the values of P and Q from (4),

$$V = \left.\begin{aligned} &\frac{1}{\lambda^2 f^2}\iiiint dx\,dy\,d\xi\,d\eta\,\psi(x,y)\sin\left[(x_1-x)p\xi+(y_1-y)p\eta\right] \\ &+\frac{1}{\lambda^2 f^2}\iiiint dx\,dy\,d\xi\,d\eta\,\phi(x,y)\cos\left[(x_1-x)p\xi+(y_1-y)p\eta\right] \end{aligned}\right\}\cos nt_2$$

$$\left.\begin{aligned} &+\frac{1}{\lambda^2 f^2}\iiiint dx\,dy\,d\xi\,d\eta\,\phi(x,y)\sin\left[(x_1-x)p\xi+(y_1-y)p\eta\right] \\ &+\frac{1}{\lambda^2 f^2}\iiiint dx\,dy\,d\xi\,d\eta\,\psi(x,y)\cos\left[(x_1-x)p\xi+(y_1-y)p\eta\right] \end{aligned}\right\}\sin nt_2$$

The first and third integrals are identically zero, so that putting

$$u = \frac{n\xi}{fa} \quad \text{and} \quad v = \frac{n\eta}{fa}$$

and disregarding the phase difference between t and t_2, we have by (1)

$$\frac{4\pi^2}{f^2}\phi(x_1,y_1) = \iiiint du\,dv\,dx\,dy\,\phi(x,y)\cos\left[(x_1-x)u+(y_1-y)v\right]$$

This, disregarding the intensity factor f^2, is the Fourier formula extended to two dimensions.

Formulæ (5) express the fact that if

W is the diffraction image of V, then
V is the diffraction image of W.

In applying the formulæ it must be remembered that V and W represent the vibration—not merely the intensity.