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#### DISCUSSION AND CORRESPONDENCE

##### MASS AS QUANTITY OF MATTER

THAT the words "quantity of matter" are of service in explaining the significance of "mass" in dynamics has been assumed either explicitly or tacitly by many authorities, including Newton, Maxwell, Kelvin, Tait and Clifford, and this view is obviously held by several of those who have contributed to the recent discussion in SCIENCE. There are, however, those who dissent from this view,<sup>1</sup> maintaining that the word mass as used in dynamics has no meaning except that given to it by the "law of acceleration" (Newton's second law), and that the statement that "the mass of a body is a measure of its quantity of matter" contributes nothing to our understanding of the definition. My present object is to call attention to a consideration which appears to be lost sight of by those who take this latter position. This consideration, stated briefly, is that *the mass of a body is distributed in a perfectly definite way among the individ-*

<sup>1</sup> The dissenting view is vigorously advocated by Professor Huntington in his latest letter (SCIENCE, July 30, 1915). It should be noted that this question is aside from the question whether mass should appear in the fundamental equations. Whatever definition of mass may be adopted, the fact remains that the quantity ordinarily called mass is a part of the fundamental data of dynamics. That Professor Huntington's formulation of principles obscures this fact is my chief reason for dissenting from it. Further discussion of this point by me would, however, be a reiteration of what has been said in a former communication (SCIENCE, April 23, 1915). Any reader who is interested in Professor Huntington's reference to my text-book on theoretical mechanics may find by consulting the book that the explanation of the laws of motion contained in it is substantially that which I have recently favored in the pages of SCIENCE; but it is my present belief that the notion of quantity of matter might have been used more effectively in this book, as well as in most other text-books that are known to me.

*ual portions of matter of which the body is composed.*

Dynamics deals with the motions of bodies. By a body we mean any connected aggregate of matter. Without attempting to define matter, we recognize the applicability to it of the notions that *the whole is greater than any part* and *the whole is equal to the sum of its parts*. These are quantitative notions; and it will be seen that they are an essential part of the notion of *mass* which we habitually use in interpreting the second law of motion.

Consider the following proposition:

I. *If two bodies be acted upon by equal forces, the body having the greater mass will have the lesser acceleration.*

According to one view this is merely an arbitrary definition of the meaning of greater and less as applied to mass; *i. e.*, the statement that "the mass of a body *A* is greater than that of a body *B*" is held to mean nothing more than that "if *A* and *B* be acted upon by equal forces the acceleration of *A* will be less than that of *B*." If, however, we are to regard proposition I. as having any application to actual physical bodies, it is easy to show that it is not a mere definition, but a partial expression of a physical law, enabling us in certain cases to make predictions. Thus, suppose material to be removed from a body *A*, leaving a body *B*; we know that, if a certain force be applied to *A* and an equal force afterward to *B*, the acceleration of *B* will be greater than that of *A*; and the truth of this is recognized because we know that *B* contains less material than *A*. That is, in applying I. to this case we associate with *mass* the notion of *quantity of matter*.

Consider now the following more definitely quantitative proposition:

II. *If different bodies are acted upon by equal forces, the resulting accelerations are in the inverse ratios of the masses of the bodies.*

The interpretation we put upon this proposition becomes evident from a consideration of particular cases.

As a first illustration let *A* be a body which, when acted upon by a force *F*, has the acceleration *a*; and suppose *A* to be divided into two

bodies  $B$  and  $C$  and that forces equal to  $F$ , applied to  $B$  and  $C$ , cause accelerations  $a'$ ,  $a''$ . We recognize the truth of the following statements about the values of  $a'$  and  $a''$ :

Both  $a'$  and  $a''$  are greater than  $a$ .

If one of the accelerations  $a'$ ,  $a''$  is less than  $2a$  the other is greater than  $2a$ .

If  $a'$  and  $a''$  are equal, each is equal to  $2a$ .

The accelerations satisfy the equation  $1/a' + 1/a'' = 1/a$ . (This of course includes the three preceding statements.)

These statements are consequences of II.; but the reason we recognize this is because we recognize that  $A$  contains more matter than either  $B$  or  $C$ , and that the sum of the quantities of matter of  $B$  and  $C$  is equal to that of  $A$ . That is, in interpreting II. we regard *mass* as a measure of *quantity of matter*.

As another illustration, let  $A$  and  $B$  be any two distinct bodies such that when equal forces are applied to them the acceleration of  $A$  is less than that of  $B$ . Proposition II. tells us that the mass of  $A$  is greater than that of  $B$ ; but is there any reason for saying that  $A$  contains more matter than  $B$ ? There is this reason: We know that, by removing from  $A$  some quantity of matter, there will remain a body  $A'$  such that, if equal forces be applied to  $A'$  and  $B$ , their accelerations will be equal; or by adding to  $B$  some quantity of matter there will be produced a body  $B'$  such that, if equal forces be applied to  $A$  and  $B'$ , their accelerations will be equal. Moreover, we know that the matter which must be taken from  $A$  to produce  $A'$ , and that which must be added to  $B$  to produce  $B'$ , have equal masses  $m$  as tested by II.; and that if the accelerations of  $A$  and  $B$  due to equal forces  $F$  are  $a'$  and  $a''$ , a body of mass  $m$  acted upon by a force  $F$  would have an acceleration  $a$  such that  $1/a = 1/a' - 1/a''$ . These facts are all recognized as consequences of II. because we regard *mass* as therein used to be a measure of *quantity of matter*; they would not follow if our notion of mass were derived wholly from proposition II. itself.

The significance of mass in the second law of motion is sometimes stated in the following form:

III. *The forces required to give equal accelerations to different bodies are proportional to their masses.*

It is easy to cite illustrative cases showing that in applying this proposition also we interpret mass as a measure of the matter of which bodies are composed. Thus the statement that "body  $A$  has three times the mass of body  $B$ " means more than that "body  $A$  requires three times as much force as body  $B$  to give it a specified acceleration"; it means that the material contained in body  $A$  might be made into three bodies, each of which would require the same force as body  $B$  to give it a specified acceleration.

It is of course true that an important part of the import of propositions II. and III. consists in giving precision to the definition of mass. But the illustrations which have been given show that the preliminary definition of mass as quantity of matter is not without important meaning, and serves a useful purpose in explaining the significance of mass in the laws of motion.

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IS SIVAPITHECUS PILGRIM AN ANCESTOR OF MAN?

IN the "Records of the Geological Survey of India" for February, 1915, Dr. Guy E. Pilgrim has described the fossil anthropoids of India, including several new forms of great interest from the Lower, Middle and Upper Siwaliks. Through the kindness of Dr. Pilgrim the American Museum of Natural History has received casts of his types and principal specimens of Siwalik anthropoids, consisting of fragments of jaws and isolated molars. These casts, together with Dr. Pilgrim's excellent illustrations, have enabled the writer to make a critical comparison of the extinct Indian anthropoids with the existing anthropoids and with recent and extinct races of man.

Pilgrim describes several new species of *Dryopithecus*, a genus characteristic of the Upper Miocene of Europe. Its known range is thus extended to the Upper Miocene of