Calculus Made Easy. By F. R. S. 2nd Edition, enlarged. (Macmillan & Co.)

This book is apparently written for science- and engineer-students who find it necessary to be able to understand such applications of the Calculus as they would meet with in their studies, but have not sufficient time to read a more systematic treatise on the subject.

Considerable ground is covered by the book, viz. the differentiation of all the standard functions, with applications to maxima and minima, and to speed and acceleration, and to various practical problems in physics and engineering, followed by a fairly full but rather more hasty treatment of integration, finishing with a short excursus into the solution of a few important types of differential equations. Just before the answers at the end there is a useful table of standard forms of integrals and differential coefficients for reference.

The explanations are of the colloquial type, mixed with jokes on the folly of mathematicians and others, which it is to be presumed the author has found useful in stimulating the ardour and courage of his students. Professor Perry in his books makes use of the same odd stimulus. It is a curious psychological fact if such banter does overcome intellectual diffidence and encourage effort.

It is a pity that the explanations are in some cases more colloquial than accurate: for instance, at the very beginning, d is said to mean "a little bit of," whereas in the subsequent pages it is quietly treated as "a little bit at the end of," or "a little addition to," which may be either positive or negative according to circumstances. The temptation to call it "a little bit of" occurs in integration when considered as a summation, but the temptation can be resisted, for, to borrow and modify the author's illustration, $\int dx$ means the sum of all the bits by which x *grows* (positively or negatively) from some initial value to such stature as at any point it has attained. It is quite true that it is also, as the author says, "the sum of all the little bits of x," but only because each 'dx' in turn has been added to the previous collocation of orderly 'bits.' When we turn to p. 6 we find 'dx' used in its proper sense, and so, of course, in all the work on differentiation.

Other small blemishes are:

On p. 12. The top of the ladder being y feet up when its foot is x feet from the wall, the author says it will be $y-dy$ feet up when the foot is $x+dx$ feet from the wall, whereas it is necessarily $y+dy$ feet up, and the peculiarity is that dy is negative.

,, p. 6, $(dx)^2$ is described as being a bit of a bit of x^2; but

,, p. 19, $(dx)^2$ is said to mean a little bit of a little bit of x.

,, p. 31, Ex. (4). In the second part it is not clear whether h is to increase with r in accordance with the condition $r=h$, or whether h is to be constant, *i.e.* whether the function to be differentiated is πr^3 or $\pi r^2 h$. Moreover, in either case, when r is only a few inches, it is rather a stretch to take $dr=1$ inch as a *differential*; it gives a result somewhat in excess of the true value.

,, p. 104, line 7. It is impossible if a, b have the same sign, but not otherwise.

,, p. 109, lines 10, 11. The quantities should be written $\frac{dS}{dt}, \frac{dr}{dt}, \frac{dV}{dt}$.

,, p. 166. No emphasis is laid on the fact that θ must be in radians, nor any explanation as to why we may take $\sin \frac{1}{2}d\theta$ as being the same as $\frac{1}{2}d\theta$.

,, p. 195, foot-note to top line. This is very confused and confusing, and needlessly so. x^2dx is $d(\frac{1}{3}x^3)$, or, more generally, $d(\frac{1}{3}x^3+c)$; $\therefore \int x^2dx = \int d(\frac{1}{3}x^3+c)$, which $=\frac{1}{3}x^3+c$ in exactly the same way as $\int dy = y$.

In spite of such blemishes, the work is very sound as a whole, and many of the examples are of considerable interest, and it is not surprising to see that the book has met with a good deal of success, as it carries the practical student to a very useful point.

There is a slip at foot of p. 170. We should read $\frac{d^2(\sin\theta)}{d\theta^2} = -\sin\theta$.

A. LODGE.

Geometry of Four Dimensions. By H. P. MANNING. Pp. ix + 348. 8s. 6d. net. 1914. (New York: The Macmillan Company.)

Although the study of Geometry of four and more dimensions was really inaugurated by British mathematicians—Cayley, Sylvester and Clifford—it is now almost neglected in this country, and is treated only in a subsidiary fashion as a mode of representation of analytical relations, and not as a field of geometrical enquiry. Thus, the latest edition of the *Encyclopedia Britannica*, which gives a detailed account of the non-euclidean geometries, dismisses the "fourth dimension" in four lines. For this reason—or is it the other way round?—the subject has never been presented systematically to the English reader. We therefore owe a debt of gratitude to Professor Manning for the preparation of this logical and comprehensive treatise, and for the care and thought which he has devoted to removing the difficulties which are inherent in the subject.

There are two classes of people whose interests have led them to a study of hyperspace: the mathematician and the philosophically (or spiritualistically) inclined layman. To the former the subject suggests mostly equations in n variables and a straining of geometrical language; the latter loves to imagine objects disappearing from closed rooms and knots tied on endless strings. In the book before us both of these aspects are ignored, and the subject is treated in the spirit of pure synthetic geometry. As the author explains in his introduction, there is much to be said for such a treatment; it certainly exercises the reader's powers of visualisation and deductive reasoning in a way which no manipulation of symbols will.

The scope of the book is limited to the elementary figures and the "round bodies," and deals with their descriptive and metrical relations. A characteristic feature is that the treatment is carried as far as possible without any assumption relating to parallel lines, so that the first five chapters, or 200 pages, apply equally well to non-euclidean space. To secure this result, however, many theorems (*e.g.* § 27, Th. 3, and § 53) have been expressed in a form which the reader, without a knowledge of non-euclidean geometry, will have difficulty in understanding. From the logical point of view there is an advantage in point of generality in dispensing with auxiliary aid which is not essential, but such a course tends to perplex the beginner. The exposition would be much easier to follow if at each stage the attention were confined at first to the euclidean case in which two coplanar lines either intersect or are parallel. But, after all, it the student wishes fully to understand the subject, especially the geometry of planes through a point, he must make up his mind first to acquire a knowledge of non-euclidean geometry.

The following is a brief summary of the contents. First comes a short introduction of twenty pages, which contains some valuable historical references. Chap. I. deals with the fundamental relations of lines, planes and hyperplanes, convex polygons, pyramids, hyperpyramids, hypercones and "plano-conical hypersurfaces" (or hypercones—"hyperconical hypersurfaces"—of the second species). The limits of the book have precluded any mention of hyperquadrics in general, or indeed of hypercones of either species with directing surface or curve other than a sphere or a circle. The interesting properties of ruled hypersurfaces are therefore excluded. Chap. II. deals with perpendicularity, and Chap. III. with the angles between two planes with only one point in common. This part of the subject is apt to present special difficulty to the reader who is accustomed to picture two planes as always cutting in a line and being fixed relatively to one another by a single dihedral angle. The configuration of two planes with only one point in common is analogous to that of two skew lines in elliptic space of three dimensions; these have two common perpendicular lines on which are measured the shortest distances between the two lines. The author establishes the existence of a common perpendicular to two skew lines by obtaining a sequence of points