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# XXXIV. The most economical potential-difference to employ with incandescent lamps

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# XXXIV. *The most Economical Potential-difference to employ with Incandescent Lamps. By* Professors W. E. Ayrton, *F.R.S., and* John Perry, *M.E.**

THE subject in connection with which the accompanying paper is a small contribution is one of considerable commercial importance. It has long been known that the luminous power of an electric lamp increased much more rapidly than the power electrically expended on it; or, that the number of candles per horse-power increased as the lamp was made to become brighter and brighter. Perhaps the earliest experiments on the subject were those published by Sir William Thomson in 1881, and those made by our students in 1880. Since that time (that is, during the last five years) tests of the efficiency of various types of incandescent lamps have been made over and over again by various persons, without perhaps its being clearly realized that such efficiency-experiments by themselves gave us no idea of the commercial value of any particular lamp.

It is not sufficient to know that when a lamp is giving out a certain number of candles it absorbs so much power per candle, and when giving out a much larger number of candles it absorbs so much less power per candle; but what must be known in addition is the life of the lamp at each of these two candle-powers, before we can decide whether it is more economical to use the lamp with the filament at not much more than a dull red or when brilliantly luminous with a bluish tint. For with the filament at a comparatively low temperature, although the efficiency of the lamp is low, its life will be great; whereas if the temperature of the filament be high, the large efficiency will be to some extent balanced by its short life and the consequent large cost due to lamp renewals.

In the 'Electrician' for January 31st of this year M. Foussat gives a table of values of the life of a 100-volt Edison lamp for different potential-differences; and M. Foussat has assured us that these numbers were obtained experimentally, and not from calculation according to any theory. That the results should lie so nearly in a curve is undoubtedly at first sight a little striking; but if it be remembered that they are stated to represent the results obtained from the *averages* of a very extended series of tests, their great regularity is not more striking than is a curve showing graphically the result of mortality tables. We asked M. Foussat whether he had also the results of experiments on the efficiency of the same type of 100-volt lamps, as our own experiments on the efficiency

* Communicated by the Physical Society, having been read at the Meeting on February 28th, 1885.

of Edison lamps had been made with 55-, 108-, and 110-volt lamps. As, however, he had no such results, and as his tests of lives are the only ones that we now have, and possibly may have, until the completion of the excellent work at present being carried out by the Electric-Lighting Jury of the Health Exhibition, we commenced this investigation by endeavouring to combine the lives given by M. Foussat with the results previously published for efficiency. To enable us to do this, we assumed that the life of 1000 hours given by M. Foussat for his 100-volt lamps when used with a potential-difference of 100 volts would be the same as that for our 108-volt lamp when employed with a potential-difference of 108 volts; or, in other words, we regarded lives given by him as applicable to our 108-volt Edison lamps when each of his potential-differences was multiplied by $\frac{108}{100}$.

To ascertain the most economic potential-differences, we must proceed as follows:—Let $f(v)$ be the life in hours as a function of the number of volts constantly kept on the lamps, $\theta(v)$ the number of candles emitted by one lamp as a function of the potential-difference in volts employed, let $p$ be the price in pounds paid for one lamp, and $n$ the number of hours per year that a lamp is kept burning; then

$$\frac{p \times n}{f(v) \times \theta(v)}$$

stands for the cost per year per candle, as far as the renewal of lamps is concerned.

Next, let H stand for the cost in pounds of an electric horse-power per year for the number of hours an electric horse-power is employed. It is often assumed that H is proportional to the number of hours per day that the power is used, or that the yearly bill for power should be based on the horse-power hours, or total energy consumed; but this idea, which runs even through the Electric Lighting Act, is quite an erroneous one. H will be of the form $h + \mathrm{F}(n)$, where $h$ is a constant independent of the number of hours and depending on the rent of the site, capital expended upon engines, dynamos, leads, &c.; and $\mathrm{F}(n)$ is some function of the number of hours during which electric power is required, and depends on the cost of coal, superintendence, &c. If the light were only required for one or two hours in a district where rent was very high, $h$ would be the all-important term and $\mathrm{F}(n)$ would be unimportant; whereas if the electric power were required for various purposes, for, say, 15 or 20 hours out of the 24, in a place where rent was low but coal dear, $\mathrm{F}(n)$ would be the important item in the yearly bill for electric power supplied.

Let $\phi(v)$ be the watts per candle, expressed as a function of the number of volts employed at the terminals of a lamp; then

$$\frac{\mathrm{H}}{746} \times \phi(v)$$

represents the cost per year per candle as far as the production of electric power is concerned.

The total cost, therefore, per year per candle is

$$\frac{p \times n}{f(v) \times \theta(v)} + \frac{\mathrm{H}}{746} \phi(v) \text{ pounds,} \quad . \quad . \quad . \quad (\mathrm{A})$$

and we must find the value of $v$ that makes this a minimum.

There are two ways in which such a problem can be solved: the one a graphical method, the other an analytical method. The former may be used by even elementary students, and will be given first. It consists in drawing curves to represent, 1st, $f(v)$ in terms of $v$, 2nd, $\theta(v)$ in terms of $v$, and 3rd, $\phi(v)$ in terms of $v$; and from these the values of $f(v)$, $\theta(v)$, and $\phi(v)$ are each determined graphically for many values of $v$, and the value of A calculated for each of these values. A fourth curve is then drawn, connecting the values of A with those of $v$, when it is easy to see by inspection for what value of $v$ the expression A has a minimum value.

The following is the result so obtained for the 108-volt Edison lamps used for lighting the Finsbury Technical College :—

$p$ is taken at 5*s*., or £0·25 ;

$n$ as 560 hours, the time per year, approximately, during which the lamps are lighted ;

H as £5 : this is perhaps a rather high estimate for the cost of power, considering that the interest on the steam-engine, dynamos, &c., price of coal burnt, wages of the engine-driver and stoker have to be mainly debited to the driving of the College workshops, supplying power for the dynamos worked for experimental purposes; but it will be accurate enough to take the sum of £5 per year as representing yearly interest on extra plant and the yearly interest on the extra cost of supplying one electric horse-power during the 560 hours.

Then the value of $v$ which makes A a minimum turns out to be about **106** *volts*; and on account of the flatness of the curve connecting the expression A with $v$, we see that in this particular case the annual cost of supplying light is only increased by 3·5 per cent., if the potential-difference between the mains be kept diminished to about 104·8 or kept up to about 108 volts. Also, that keeping the potential-difference diminished to about 104·5, or increased to about 108·5 volts, increases our total annual cost of lighting by 5 per cent.

More recently one of our students, Mr. Robertson, has been making efficiency-experiments on some Edison lamps obtained from France; and by trial he has found some that give 16 candles at about 100 volts, and are therefore presumably of the same type as those employed in the life-experiments given by M. Foussat in the 'Electrician' for January 31st.

Mr. Robertson's results from one such lamp are as follow:—

TABLE I.

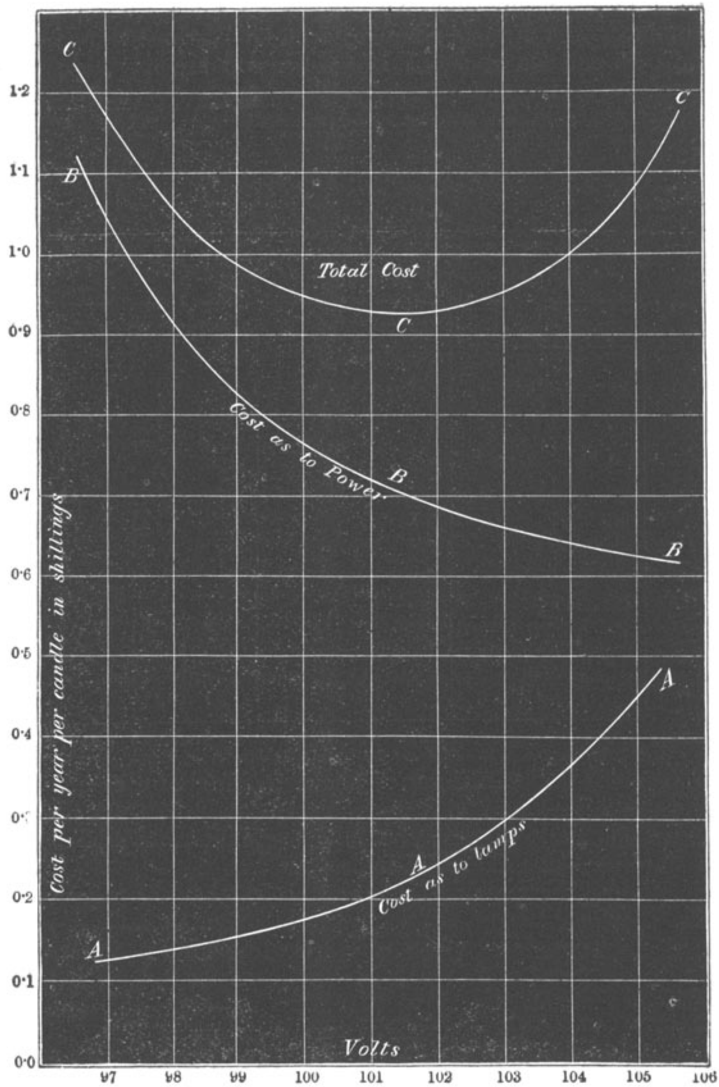
| Candles, or $\theta(v)$. | Volts, or $v$. | Watts. |
|---|---|---|
| 3 | 84 | 56·62 |
| 4 | 87 | 63·51 |
| 5 | 87·8 | 66·03 |
| 6 | 90·5 | 69·05 |
| 7 | 93·5 | 75·74 |
| 10 | 96·75 | 79·34 |
| 11 | 97·61 | 81·01 |
| 12 | 98·04 | 82·35 |
| 14 | 98·9 | 85·55 |
| 16 | 100·6 | 90·54 |
| 18 | 101·5 | 92·37 |
| 20 | 103·6 | 96·66 |
| 22 | 105·4 | 100·10 |
| 24 | 107·5 | 104·28 |
| 26 | 109·2 | 108·11 |
| 28 | 110 | 111·10 |
| 30 | 111·4 | 113·63 |
| 35 | 114 | 118·56 |
| 40 | 115·2 | 122·11 |

We are therefore now in a position to take up the problem in a more direct manner, without making any assumption beyond this—that Mr. Robertson's 16-candle 100-volt French Edison lamp is of the same kind as M. Foussat's 16-candle 100-volt French Edison lamp.

Solving the problem graphically, in the way previously described, and using the same values of $p$, $n$, and H—viz. cost of a lamp 5*s*., number of hours of burning per year 560, and annual cost of an electric horse-power for those 560 hours £5,—we obtain the curve A A A to represent the cost per candle per year as regards renewal of lamps, B B B the cost per candle per year as regards power, and the resultant curve C C C as the total cost per candle per year.

The minimum value of this cost appears from the curve C C C to be at about **101·4** *volts, and to equal about* **11d.** *per candle per year.* If the potential-difference be maintained constantly down at 98·7 volts, or up at 104 volts, then the cost becomes 1*s*. per candle per year. It will also be seen from the curves that the yearly cost for renewals of lamps is

Diagram for a 16-candle 100-volt Edison Lamp.




but a small fraction of the total yearly cost, as long as we are using potential-differences of not more than about 100 volts; so that when using, with these lamps, this or a lower potential-difference, the actual sum paid for the lamps is not so very serious an item in the yearly lighting bill. But if, on the other hand, we maintain a potential-difference of, say, 104 volts constantly at the terminals, then the lamp renewals represent, in our particular case, nearly one third of the yearly expenditure, and consequently any change in the price of lamps becomes extremely important.

We find that the simplest expression for the candle-power of this incandescent lamp in terms of volts is

$$\theta(v)=a(v-b)^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Thus, for example, it will be found that if the cube roots of the values of $\theta(v)$, given in the preceding table, are plotted with the corresponding values of $v$ as coordinates of points on squared paper, the points lie in no regular curve so nearly as in a straight line, discrepancies being apparently due to errors of observation, and these errors of observation follow a periodic law which may be of interest physiologically. Or it may be that there really is a point of inflexion in the curve.

For the particular lamp in question we find that

$$a=0{\cdot}0002621,$$
$$b=62{\cdot}12\,;$$

or, if the law remains true for a less number of volts than 84, the lowest used in this experiment, then when $v=62{\cdot}12$, the candle-power is zero.

We do not, however, find that the cube roots of the candle-powers of all sorts of incandescent lamps follow a line-function of the difference of potential at their terminals; for on examining the results of experiments that have been made with various types of lamps, we find that with the first five of the following set of lamps the law

$$\sqrt[3]{\theta(v)} \propto v-b$$

is very nearly true.

| | | |
|---|---|---|
| 1. A Lane-Fox lamp . . . . . . | from | 50 to 80 volts. |
| 2. A British Electric Light Co.'s lamp | ,, | 50 to 68 ,, |
| 3. An Edison "7 B" 8-candle lamp . | ,, | 45 to 63 ,, |
| 4. An old Swan lamp . . . . . . . | ,, | 40 to 64 ,, |
| 5. A low-resistance 8-candle Swan lamp | ,, | 16 to 40 ,, |

Whereas with the next five lamps it is not so true, although for half the range the law might be regarded as true in all the cases.

| | | |
|---|---|---|
| 6. A Maxim lamp . . . . . . . . | from | 42 to 57 volts. |
| 7. A Maxim lamp . . . . . . . . . | ,, | 35 to 110 ,, |
| 8. An old form Swan lamp . . . . . | ,, | 25 to 54 ,, |
| 9. A high-resistance Swan lamp . . | ,, | 45 to 90 ,, |
| 10. An Edison B 8-candle lamp . . | ,, | 36 to 85 ,, |

We may here mention that we have found the following plan very useful in helping us to draw correctly a curve at a place where there is a sudden bend and an absence of points, determined from experiments to guide us in drawing the curve. Instead of plotting $x$ and $y$, and obtaining a curve which it would be very difficult to draw correctly, we may by using some simple function of $y$ obtain points which obviously lie in a curve which it is easy to draw. Thus we may plot $\sqrt{y}$ and $x$, or $\sqrt[3]{y}$ and $x$, or $\log y$ and $x$. When the curve connecting $y$ and $x$ is not roughly asymptotic to the axis of $x$, but to some line parallel to this axis, it is obvious that there is a greater likelihood of obtaining simple curves by plotting $\sqrt{y \pm \alpha}$ and $x$, or $\sqrt[3]{y \pm \alpha}$ and $x$, or $\log(y \pm \alpha)$ and $x$, where $\alpha$ is some constant obtainable by inspection, than by simply plotting the function of $y$ alone.

The following table gives M. Foussat's lives in terms of $v$ and the corresponding values which we have calculated of $\log f(v)$, $\log \theta(v)$, and of $\log f(v)\theta(v)$ from the values of $\theta(v)$ given in the previous table.

TABLE II.

| $v$. | $f(v)$. | $\log f(v)$. | $\log \theta(v)$. | $\log f(v)\theta(v)$. |
|---|---|---|---|---|
| 95 | 3595 | 3·5557 | 0·9729 | 4·5286 |
| 96 | 2751 | 3·4395 | 1·0119 | 4·4514 |
| 97 | 2135 | 3·3294 | 1·0485 | 4·3779 |
| 98 | 1645 | 3·2161 | 1·0851 | 4·3012 |
| 99 | 1277 | 3·1062 | 1·1199 | 4·2261 |
| 100 | 1000 | 3·0000 | 1·1556 | 4·1556 |
| 101 | 785 | 2·8949 | 1·1898 | 4·0847 |
| 102 | 601 | 2·7789 | 1·2216 | 4·0005 |
| 103 | 477 | 2·6785 | 1·2531 | 3·9316 |
| 104 | 375 | 2·5740 | 1·2852 | 3·8592 |
| 105 | 284 | 2·4533 | 1·3152 | 3·7685 |

When $\log f(v)\,\theta(v)$, and $v$ are plotted as coordinates of points, these points are found to lie so nearly on a straight line that the formula

$$\frac{1}{f(v)\theta(v)} = 10^{0\cdot07545\,v - 11\cdot697} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

is found to be true with considerable accuracy, and such a formula lends itself with great ease to calculation.

It is quite true that we might have obtained a still more accurate formula than (2) since, as published by Mr. Wright, one of the members of our class, in the 'Electrician' for February 21st, the logarithm of the life of a lamp is shown to be a line-function of the difference of potentials at which it is worked.

We may in fact with this particular type of lamp put

$$f(v) = 10^{14 - 0\cdot11\,v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

with very great accuracy indeed.

Combining this with (1), we obtain as a more accurate formula,

$$\frac{1}{f(v)\theta(v)}=\frac{1}{a}10^{0\cdot11v-14}(v-b)^{-3}. \quad . \quad . \quad . \quad (4)$$

But unfortunately this more accurate formula does not lend itself to mathematical calculation, whereas that given in (2) is very suitable for this, and has a sufficient degree of accuracy for our purpose. In using (2) we are really using for the candle-power

$$\theta(v)=10^{0\cdot03455v-2\cdot303} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

instead of (1).

It will be also found that from 95 to 105 volts, the range of volts given in Table II., the values of $\phi(v)$, or watts per candle-power, when corrected for errors of observation, satisfy with considerable accuracy the equation

$$\phi(v)=3\cdot7+10^{8\cdot007-0\cdot07667v}. \quad . \quad . \quad . \quad . \quad (6)$$

If, however, we take the whole range of values given in Table I., then it will be found that the equation

$$\phi(v)=2+10^{4\cdot424-0\cdot03793v} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

is better satisfied than (6).

The following Table III. gives the numbers we have actually employed in making these calculations.

Table III.

| $v$. | $\sqrt[3]{\theta(v)}$. corrected. | $\theta(v)$ corrected. | Watts corrected. | Watts per candle-power $\phi(v)$. | $\log\{\phi(v)-2\}$. |
|---|---|---|---|---|---|
| 84 | 1·4 | 2·744 | 56 | 20·408 | 1·2650 |
| 86 | 1·53 | 3·582 | 60·15 | 16·79 | 1·1700 |
| 88 | 1·66 | 4·574 | 64·25 | 14·27 | 1·0888 |
| 90 | 1·786 | 5·7 | 68·4 | 12·0 | 1·0000 |
| 92 | 1·914 | 7·0 | 72·6 | 10·37 | ·9227 |
| 94 | 2·044 | 8·55 | 76·85 | 9·0 | ·8451 |
| 96 | 2·175 | 10·28 | 81 | 7·882 | ·7695 |
| 98 | 2·3 | 12·167 | 85·15 | 7·0 | ·6990 |
| 100 | 2·428 | 14·24 | 89·3 | 6·271 | ·6305 |
| 102 | 2·556 | 16·69 | 93·5 | 5·6 | ·5563 |
| 104 | 2·682 | 19·29 | 97·8 | 5·07 | ·4871 |
| 106 | 2·81 | 22·188 | 101·9 | 4·593 | ·4138 |
| 108 | 2·94 | 25·412 | 106 | 4·172 | ·3369 |
| 110 | 3·065 | 28·79 | 110·1 | 3·824 | ·2610 |
| 112 | 3·195 | 32·61 | 114·25 | 3·504 | ·1772 |
| 114 | 3·32 | 36·594 | 118·3 | 3·233 | ·0910 |

Using (6) and (2), the total cost of one candle per year is

$$p\,n\,10^{0\cdot07545v-11\cdot697}+\frac{\mathrm{H}}{746}(3\cdot7+10^{8\cdot007-0\cdot07667v}).$$

This is a minimum when

$$v = 110{\cdot}66 + 6{\cdot}574 \log \frac{\mathrm{H}}{p\,n}.$$

Hence taking $p=0{\cdot}25$, $n=560$, $\mathrm{H}=£5$, we find

$$\mathbf{v} = \mathbf{101{\cdot}15} \text{ volts.}$$

Using (7) and (2), we should find in the same way that the minimum is obtained when

$$\mathbf{v} = \mathbf{101{\cdot}46} \text{ volts ;}$$

and these minimum values of $v$ agree very closely with that previously determined graphically.

One very important problem in connection with incandescent lamps, and one that cannot yet be regarded as solved, is the determination of the life of a lamp for any given number of volts, from experiments made either on the efficiencies at several different potentials, or from experiments on the life made at so high a potential-difference that the life will be short, and the experiment made therefore in a comparatively short time.

If, however, an expression of the form

$$f(v) = 10^{a-bv}$$

can be regarded as representing with sufficient accuracy the law of life for all types of incandescent lamps, then if sufficient experiments be made with a number of lamps at each of two different potential-differences to enable us to determine the average life at each of these potential-differences, the constants $a$ and $b$ can be calculated from the equation, and hence the value of $f(v)$, the life, calculated for any other potential-difference.

In connection with this investigation we have endeavoured, with the aid of one of our Assistants, Mr. Walmsley, to ascertain whether some form of ordinary cheap candle could be used, at any rate for rough photometric measurements, in place of the much dearer standard-candles, and, as far as the following results obtained with seven candles selected at random from a packet of No. 8 sperm-candles go, it would seem that these candles do not differ so very much more in intensity from one another than standard candles are said to do. Of course many more experiments on this subject must be made before the possibility of using cheap candles as a rough standard can be decided on, but in the meantime the following experiments may be interesting.

This particular type of candle, No. 8 sperm, and costing 11*d.* per pound, was selected, because such candles were found to resemble in thickness the standard candles that we have been accustomed to use, and which cost 2*s.* 9*d.* per pound.

TABLE IV.

| Name of candle. | Grammes of wax burnt per hour. | Light in terms of that emitted by the Standard candle. |
|---|---|---|
| Standard candle ... | 7·82 | 1·00 |
| 1st Sperm " ... | Doubtful | 1·14 (?) |
| 2nd " " ... | 6·102 | 1·00 |
| 3rd " " ... | 7·188 | 1·00 |
| 4th " " ... | 7·29 | 1·02 |
| 5th " " ... | 7·1 | 1·02 |
| 6th " " ... | 6·84 | 1·05 |
| 7th " " ... | 6·66 | 0·99 |

These tests of candle-power were not made with any very high degree of accuracy; but the comparison of these No. 8 sperm-candles with the standard was carried out with probably quite as much accuracy as is employed in making ordinary commercial experiments on the luminosity of incandescent lamps.



## XXXV. *Intelligence and Miscellaneous Articles.*

### ON THE LIMIT OF THE DENSITY AND ON THE ATOMIC VOLUME OF GASES, AND PARTICULARLY OXYGEN AND HYDROGEN. BY E. H. AMAGAT.

THE recent researches on the density of liquid oxygen by MM. Cailletet and Hautefeuille, Pictet, and by Wroblewski, have led to values all of which are less than unity, in the different conditions in which this density has been determined. It has been concluded from this that, in agreement with the previsions of Dumas, it will be equal to unity under a sufficiently powerful pressure, or a sufficiently low temperature, and that accordingly the quotient of the atomic weight by the density, or the atomic volume, would be virtually the same for oxygen, sulphur, selenium, and tellurium.

In my second memoir on the compressibility of gases under strong pressures (*Annales de Chimie et de Physique*, 5th series, vol. xxii. 1881), I showed that at sufficiently high temperatures the law of the compressibility of gases is ultimately represented by straight lines which correspond to the ratio $p=(v-\alpha)=$constant, which at once gives the limiting volume $\alpha$ for $p$ equal to infinity, and therefore the limiting density; and that for lower pressures the curves, starting from a considerable pressure, exhibit a portion which is virtually rectilinear, and by which we can calculate, though with less certainty, the limiting volume*.

* In this memoir I have used the term *atomic volume* to denote the value of $\alpha$ in reference to one litre of gas at zero, and under a pressure of 76 centims. I make this observation to avoid any confusion.