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XXXVI. Graphic representation of currents in a primary and a secondary coil

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frequency employed; but, as is seen from the previous theoretical investigation, by employing a primary and a secondary having equal time-constants suitably related to the frequency, and a pure sine function P.D., an increase of impedance of from 10 to 12 per cent. ought to be obtained; $15\frac{1}{2}$ per cent. increase could never be obtained in practice, as there must always be some magnetic leakage.

In transformers with iron cores this effect would never be likely to escape notice as the values of $\frac{L}{R}$ would be so large that the critical frequency would be very small, so that for all frequencies employed in practice the impedance of the primary would diminish on closing the secondary. The iron core would also distort the current from a pure sine function.

XXXVI. *Graphic Representation of Currents in a Primary and a Secondary Coil.* *By* Prof. G. M. Minchin, *M.A.**

In this short paper is contained a solution of the following problem:—*A primary and a secondary coil occupy given positions; an alternating E.M.F., expressed by a sine function of the time, being applied to the primary, it is required to represent graphically the impedances and phases of the primary and secondary currents for all speeds of alternation.* (No iron cores employed.)

The occasion of this communication was a paper read to the Physical Society on the 27th of October, 1893, by Mr. Rimington, in which the subject was presented in a different manner.

Adopting the notation of Mr. Rimington's paper, let L, M, N, r_1, r_2 be, respectively, the coefficient of self-induction of the primary coil, the coefficient of mutual induction, that of self-induction of the secondary, and the resistances of the primary and secondary. Also let n be the frequency of the alternation, *i. e.* the number of alternations per second; $p=2\pi n$; E = maximum value of the impressed E.M.F. Then, if the secondary coil is open (or $r_2=\infty$), the impedance, I, of the primary is given by the expression

$$I_1=\sqrt{r_1^2+L^2p^2}.$$

The impedance, I_2, of the secondary, if the primary were absent and the secondary plied by an alternating E.M.F.,

* Communicated by the Author.

would be similarly given by the expression

$$I_2 = \sqrt{r_2^2 + N^2 p^2}.$$

Let p^2 be denoted by x, and, for shortness, let

$$a = r_1 r_2 - (LN - M^2)x,$$

$$b = (Nr_1 + Lr_2)\sqrt{x}.$$

Then, assuming that the E.M.F., namely e, applied to the primary is at any time, t, given by the equation

$$e = E \sin(pt + \phi),$$

the periodic parts, c_1 and c_2, of the currents at this time in the primary and secondary, respectively, are given by

$$c_1 = \frac{EI_2}{\sqrt{a^2+b^2}} \sin pt,$$

$$c_2 = \frac{EM\sqrt{x}}{\sqrt{a^2+b^2}} \sin(pt+\theta),$$

and we have

$$\theta = \tfrac{3}{2}\pi - \chi, \text{ where } \tan\chi = \frac{N\sqrt{x}}{r_2},$$

$$\phi = \psi - \chi, \text{ where } \tan\psi = \frac{b}{a}.$$

It thus follows that the actual impedances, I, I′, of the primary and the secondary coil during the working of both are given by

$$I^2 = \frac{a^2+b^2}{I_2^2}; \; I'^2 = \frac{a^2+b^2}{M^2 x}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

To represent I, I′, and the phase-angles θ and ϕ graphically is the problem in hand. Take two rectangular axes, Ox, Oy, and along the first lay off the numerical values of p^2; then, taking $k^2 y$ to represent the value of I^2 corresponding to any value of p (or x), where k^2 is any constant which (according to the numerical values of L, M, N, &c.) may be required to confine the figure to any convenient size, we have

$$k^2 y = \frac{a^2+b^2}{r_2^2 + N^2 x}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

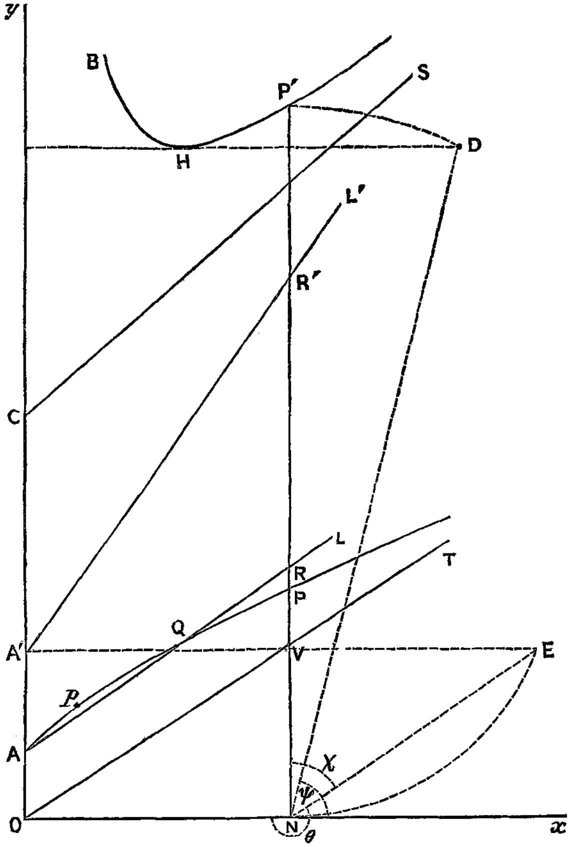
or if for shortness we put $A^2 = LN - M^2$, and $B^2 = Nr_1 + Lr_2$,

$$k^2 y (N^2 x + r_2^2) = (r_1 r_2 - A^2 x)^2 + B^4 x; \quad . \quad . \quad . \quad (3)$$

which equation denotes an hyperbola, $\mathrm{A}p\mathrm{QP}$, making an intercept, OA, on the axis of y such that

$$\mathrm{OA} = \frac{r_1^2}{k^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Call this curve the *primary hyperbola*.



In the same way, let I'^2 be denoted by k^2y'; then

$$k^2\mathrm{M}^2xy' = (r_1r_2 - \mathrm{A}^2x)^2 + \mathrm{B}^4x, \quad . \quad . \quad . \quad . \quad (5)$$

showing that the values of I'^2 are also represented by the ordinates of an hyperbola, B H P′. We shall discuss this hyperbola more particularly, and show that it may be easily and rapidly drawn. Call it the *secondary hyperbola*. In the first place, it passes through the point $x = \frac{r_1r_2}{\mathrm{A}_2}$, $y' = \frac{\mathrm{B}^4}{k^2\mathrm{M}^2}$; and the tangent at this point is parallel to $\mathrm{O}x$. The point is H.

whose ordinate we shall denote by h, so that

$$h=\frac{B^4}{k^2M^2}. \qquad (6)$$

It is easily seen that both hyperbolas cut the axis of x in the same two points. These points are at the left of O and not shown, since negative values of x (i. e. p^2) do not belong to the physical problem. Moreover, the x of H is the geometric mean between the intercepts of the hyperbola on Ox.

Again, the centre of the hyperbola (6) is at the point C whose coordinates are

$$0 \text{ and } \frac{N^2r_1{}^2+L^2r_2{}^2+2M^2r_1r_2}{k^2M^2};$$

while one asymptote is Oy and the other is CS whose direction is easily known, since the tangent of its inclination to Ox is $\frac{A^4}{k^2M^2}$.

Hence we have at once the asymptotes, Cy, CS and one point, H, of the hyperbola, from which the curve is rapidly drawn by the well-known rule that the parts intercepted between the curve and its asymptotes on every line drawn through H are equal. The other branch of this hyperbola is not represented, as it is irrelevant.

We shall now show that the primary hyperbola can be drawn from the secondary. Representing the values of $I_1{}^2$ and $I_2{}^2$ for all values of x by ordinates, so that $k^2y_1=I_1{}^2$, $k^2y_2=I_2{}^2$, we see that

$$k^2y_1=L^2x+r_1{}^2, \qquad (7)$$

$$k^2y_2=N^2x+r_2{}^2; \qquad (8)$$

so that the impedances are now represented by two right lines, AL and A′L′. (These are the impedances of the coils, each treated separately, as before explained.)

The primary line (7) passes, of course, through A, and always intersects the primary hyperbola in a point, Q, having a positive abscissa, viz.,

$$\frac{2r_1r_2}{2LN-M^2},$$

which is < twice abscissa of H. Hence for some speed less than that represented by the abscissa of Q the ratio $\frac{I}{I_1}$ attains a maximum value. The point, p, representing this maximum value is easily found; for, no matter what curve AQP may be, if y is the ordinate of a point on it, and y_1 the corresponding ordinate of the right line AQ, the ratio $\frac{y}{y_1}$ is a maximum at the point, p, of contact of a tangent to the curve drawn from the point where the right line AQ meets Ox.

Construct also the right line OT whose equation is

$$k^2\eta = M^2 x; \quad \ldots \ldots \quad (9)$$

then, taking any value, ON, of x, draw the ordinate NP, and we have

$$y' = P'N; \quad y_2 = R'N; \quad y = PN; \quad \eta = VN.$$

Moreover, it is obvious from the previous values that

$$\frac{y'}{y_2} = \frac{y}{\eta}, \quad \therefore y = \frac{y'}{y_2}\eta, \quad \ldots \ldots \quad (10)$$

which shows that the point P on the primary hyperbola is deduced from the point P′ on the secondary by the simple construction or calculation of a fourth proportional. (Though not belonging to the physical problem, it may be noted that one asymptote of the primary hyperbola is the parallel to Oy at the point where the secondary line, A′L′, cuts Ox, the other asymptote making with Ox the angle whose tangent is $\frac{A^4}{k^2N^2}$.)

Finally, as regards the phase-angles, take χ first. We have $\tan\chi = \frac{N\sqrt{x}}{r_2}$;

$$\therefore \sec^2\chi = \frac{I_2^2}{r_2^2} = \frac{k^2 y_2}{r_2^2} = \frac{y_2}{OA'},$$

$$\therefore \cos\chi = \sqrt{\frac{OA'}{NR'}}. \quad \ldots \ldots \quad (11)$$

Hence, describing a circle on NR′ as diameter, and drawing A′E parallel to Ox, meeting the circle in E, we have

$$\chi = ENR';$$

$$\therefore \theta = \text{re-entrant angle ONE}.$$

Again, we have

$$\sin^2\psi = \frac{b^2}{a^2+b^2} = \frac{b^2}{k^2M^2y'x} = \frac{b^2}{k^4\eta y'} = \frac{B^4 x}{k^4\eta y'}.$$

But from (6), $h = \frac{B^4 x}{k^4 \eta}$;

$$\therefore \sin^2\psi = \frac{h}{y'}, \quad \ldots \ldots \quad (12)$$

which shows that if we construct a circle on NP′ as diameter, and take the point, D, in which this circle is cut by the tangent HD at H to the secondary hyperbola, we shall have

$$\psi = DNx. \quad \ldots \ldots \quad (13)$$