tively connected, and thereby submitted to the process of electric distribution.

To the theory of the sun's electric potential might still be opposed the objection that the electric attraction between the sun and the planets, and the repulsion which the latter would necessarily exert upon one another and upon their satellites, would modify the basis of the astronomical calculations, since then, besides gravitation, an additional force, the electrical, would have to be taken into account.

This objection is perfectly legitimate. But as electric force, equally with gravitation, stands in the ratio of the square of the distance of the centres, the paths of the planets would remain unaltered if a part of the gravitational were replaced by an electrical attraction. Only the calculated ratio of the masses of the sun and planets to that of the earth would be changed. These alterations would be sensible, especially in the case of the small planets and the satellites, since electric force is a function of the surface. On the other hand, however, the disturbing influences exerted by the planets and their satellites upon one another's paths must be changed if gravitation be diminished by electric repulsion.

Perhaps it is reserved for astronomy to bring out from the perturbations of the paths of Mercury, the asteroids, and the satellites the demonstration of the existence or nonexistence of an electric potential of the sun.

XXVI. *On Porous Bodies in relation to Sound.* *By* Lord Rayleigh, *D.C.L., F.R.S., Cavendish Professor of Physics in the University of Cambridge**.

IN Acoustics we have sometimes to consider the incidence of aerial waves upon porous bodies, in whose interstices some sort of aerial continuity is preserved. Tyndall has shown that in many cases sound penetrates such bodies, *e. g.* thick pieces of felt, more freely than would have been expected, though it is reflected from quite thin layers of continuous solid matter. On the other hand, a hay-stack seems to form a very perfect obstacle. It is probable that porous walls give a diminished reflection, so that within a building so bounded resonance is less prolonged than would otherwise be the case.

When we inquire into the matter mechanically, it is evident that sound is not destroyed by obstacles as such. In the absence of dissipative forces, what is not transmitted must be reflected. Destruction depends upon viscosity and upon

* Communicated by the Author.

conduction of heat; but the influence of these is enormously augmented by the contact of solid matter exposing a large surface. At such a surface the tangential as well as the normal motion is hindered, and a passage of heat to and fro takes place, as the neighbouring air is heated and cooled during its condensations and rarefactions. With such rapidity of alternations as we are concerned with in the case of audible sounds, these influences extend to only a very thin layer of the air and of the solid, and are thus greatly favoured by attenuation of the masses.

I have thought that it might be interesting to consider a little more definitely a problem sufficiently representative of that of a porous wall, in order to get a better idea of the magnitudes of the effects to be expected. We may conceive an otherwise continuous wall, presenting a flat face, to be perforated by a great number of similar small channels, uniformly distributed, and bounded by surfaces everywhere perpendicular to the face. If the channels be sufficiently numerous, the transition from simple plane waves outside to the state of aerial vibration corresponding to the interior of a channel of infinite length, occupies a space which is small relative to the wave-length of the vibration, and then the connexion between the condition of things inside and outside admits of simple expression.

Considering first the interior of one of the channels, and taking the axis of x parallel to the axis of the channel, we suppose that as functions of x the velocity-components u, v, w, and the condensation s are proportional to $e^{i\kappa x}$, while as functions of t everything is proportional to e^{int}, n being real. The relationship between κ and n depends on the nature of the gas and upon the size and form of the channel, and must be found in each case by a special investigation. Supposing it known for the present, we will go on to show how the problem of reflection is to be dealt with.

For this purpose consider the equation of continuity as integrated over the cross section of the channel σ. Since the walls are impenetrable,

$$\frac{d}{dt}\iint s\,d\sigma+\frac{d}{dx}\iint u\,d\sigma=0,$$

so that

$$n\iint s\,d\sigma+\kappa\iint u\,d\sigma=0. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This result is applicable at points distant from the open end more than several diameters of the channel.

Taking now the origin of x at the face of the wall, we have to form corresponding expressions for the waves outside; and we may here neglect the effects of friction and heat-conduction. If a be the velocity of sound in the open, and $\kappa_0=n/a$, we may write

$$s=(\;e^{i\kappa_0 x}+\mathrm{B}e^{-i\kappa_0 x})e^{int}, \quad . \quad . \quad . \quad . \quad (2)$$

$$u=a(-e^{i\kappa_0 x}+\mathrm{B}e^{-i\kappa_0 x})e^{int}; \quad . \quad . \quad . \quad . \quad (3)$$

so that the incident wave is

$$s=e^{i(nt+\kappa_0 x)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or, on throwing away the imaginary part,

$$s=\cos(nt+\kappa_0 x). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

These expressions are applicable when x exceeds a moderate multiple of the distance between the channels. Close up to the face the motion will be more complicated; but we have no need to investigate it in detail. The ratio of u and s at a place near the wall is given with sufficient accuracy by putting $x=0$ in (2) and (3),

$$\frac{u}{s}=\frac{a(-1+\mathrm{B})}{1+\mathrm{B}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

We now assume that a region about $x=0$, on one side of which (6) is applicable and on the other side of which (1) is applicable, may be taken so small relatively to the wave-length that the mean pressures are sensibly the same at the two boundaries, and that the flow into the region at the one boundary is sensibly equal to the flow out of the region at the other boundary. The equality of flow does not imply an equality of mean velocities, since the areas concerned are different. The mean velocities will be inversely proportional to the corresponding areas—that is, in the ratio $\sigma:\sigma+\sigma'$, if σ' denote the area of the unperforated part of the wall corresponding to each channel. By (1) and (6) the connexion between the inside and outside motion is expressed by

$$-\frac{n}{\kappa}\sigma=\frac{(\mathrm{B}-1)a}{\mathrm{B}+1}(\sigma+\sigma').$$

We will denote the ratio of the unperforated to the perforated parts of the wall by g, so that $g=\sigma'/\sigma$. Thus,

$$\frac{1-\mathrm{B}}{1+\mathrm{B}}=\frac{\kappa_0}{\kappa(1+g)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If $g=0$, $\kappa=\kappa_0$, there is no reflection; if there are no perforations, $g=\infty$, and then $\mathrm{B}=1$, signifying a complete reflection. In place of (7) we may write

$$\mathrm{B}=\frac{\kappa(1+g)-\kappa_0}{\kappa(1+g)+\kappa_0}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which is the solution of the problem proposed. It is understood that waves which have once entered the wall do not return. When dissipative forces act, this condition may always be satisfied by supposing the channels long enough. The necessary length of channel, or thickness of wall, will depend upon the properties of the gas and upon the size and shape of the channels.

Even in the absence of dissipative forces there must be reflection, except in the extreme case $g=0$. Putting $\kappa=\kappa_0$ in (8), we have

$$B=\frac{g}{2+g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If $g=1$ (that is, if half the wall be cut away), $B=\frac{1}{3}$, $B^2=\frac{1}{9}$, so that the reflection is but small. If the channels be circular, and arranged in square order as close as possible to each other, $g=(4-\pi)/\pi$, whence $B=\cdot121$, $B^2=\cdot015$, nearly all the motion being transmitted.

It remains to consider the value of κ. The problem of the propagation of sound in a circular tube, having regard to the influence of viscosity and heat-conduction, has been solved analytically by Kirchhoff*, on the suppositions that the tangential velocity and the temperature-variation vanish at the walls. In discussing the solution, Kirchhoff takes the case in which the dimensions of the tube are such that the immediate effects of the dissipative forces are confined to a relatively thin stratum in the neighbourhood of the walls. In the present application interest attaches rather to the opposite extreme, viz. when the diameter is so small that the frictional layer pretty well fills the tube. Nothing practically is lost by another simplification which it is convenient to make (following Kirchhoff)—that the velocity of propagation of viscous and thermal effects is negligible in comparison with that of sound.

One result of the investigation may be foreseen. When the diameter of the tube is very small, the conduction of heat from the centre to the circumference of the column of air becomes more and more free. In the limit the temperature of the solid walls controls that of the included gas, and the expansions and rarefactions take place isothermally. Under these circumstances there is no dissipation due to conduction, and everything is the same as if no heat were developed at all. Consequently the coefficient of heat-conduction will not appear in the result, which will involve, moreover, the Newtonian value of the velocity of sound (b) and not that of Laplace (a).

Starting from Kirchhoff's formulæ, we find as the value of

* Pogg. *Ann.* cxxxiv. 1868.

κ^2 applicable when the diameter $(2r)$ is very small,

$$\kappa^2 = -\frac{8in\mu'}{b^2r^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

μ' being the kinematic coefficient of viscosity. The wave propagated into the channels is thus proportional to

$$e^{px}\cos(nt+px+\epsilon), \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where

$$p = \frac{\kappa}{1-i} = \frac{2\sqrt{(n\mu')}}{br} = \frac{2\sqrt{(n\gamma\mu')}}{ar}, \quad . \quad . \quad . \quad . \quad (12)$$

γ being the ratio of the specific heats, equal to 1·41. In the derivation of (10), $nr^2/(8\nu)$, ν being the thermometric coefficient of conductivity, is assumed to be small.

To take a numerical example, suppose that the pitch is 256 (middle *c* of the scale), so that $n=2\pi\times 256$. The value of μ' for air is ·16 C.G.S. (Maxwell), and that of ν is ·256. If we take $r=\frac{1}{1000}$ centim., we find $nr^2/8\nu$ equal to about $\frac{1}{1000}$. If r were 10 times as great, the approximation would perhaps still be sufficient.

From (12), if $n=2\pi\times 256$,

$$p = \frac{1{\cdot}15\times 10^{-3}}{r}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

so that if $r=\frac{1}{1000}$, $p=1{\cdot}15$. In this case the amplitude is reduced in ratio $e:1$ in passing over the distance p^{-1}—that is, about one centimetre. The distance penetrated is proportional to the radius of the channel.

The amplitude of the reflected wave is, by (8),

$$\mathrm{B} = \frac{p(1+g)(1-i)-\kappa_0}{p(1+g)(1-i)+\kappa_0},$$

or, as we may write it,

$$\mathrm{B} = \frac{p'(1-i)-1}{p'(1-i)+1} = \frac{p'-1-ip'}{p'+1-ip'}, \quad . \quad . \quad (14)$$

where

$$p' = (1+g)p/\kappa_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

If I be the intensity of the reflected sound, that of the incident sound being unity,

$$\mathrm{I} = \frac{2p'^2-2p'+1}{2p'^2+2p'+1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The intensity of the intromitted sound is given by

$$\mathrm{I}' = 1-\mathrm{I} = \frac{4p'}{2p'^2+2p'+1}. \quad . \quad . \quad . \quad . \quad . \quad (17)$$

By (12), (15),

$$p' = \frac{2(1+g)\sqrt{(\mu'\gamma)}}{r\sqrt{n}}. \quad . \quad . \quad . \quad (18)$$

If we suppose $r = \frac{1}{1000}$ centim., and $g = 1$, we shall have a wall of pretty close texture. In this case, by (18), $p' = 47{\cdot}4$, and $I' = {\cdot}0412$. A four-per-cent. loss may not appear to be much; but we must remember that in prolonged resonance we are concerned with the accumulated effects of a large number of reflections, so that rather a small loss in a single reflection may well be material. The thickness of the porous layer necessary to produce this effect is less than one centimetre.

Again, suppose $r = \frac{1}{100}$ centim., $g = 1$. We find $p' = 4{\cdot}74$, $I' = {\cdot}342$, and the necessary thickness would be less than 10 centimetres.

If r be much greater than $\frac{1}{100}$ centim., the exchange of heat between the air and the walls of the channels is no longer sufficiently free for the expansions to be treated as isothermal. When r is so great that the thermal and viscous effects extend only through a small fraction of it, we have the case discussed by Kirchhoff. If we suppose for simplicity $g = 0$ (a state of things, it is true, not strictly consistent with channels of circular section*), we have

$$I = \frac{\gamma'^2}{4nr^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

in which

$$\gamma' = \sqrt{\mu'} + \left(\frac{a}{b} - \frac{b}{a}\right)\sqrt{\nu}. \quad . \quad . \quad . \quad . \quad (20)$$

The incident sound is absorbed more and more completely as the diameter of the channels increases; but at the same time a greater thickness becomes necessary in order to prevent a return from the further side. If $g = 0$, there is no theoretical limit to the absorption; and, as we have seen, a moderate value of g does not by itself entail more than a comparatively small reflection. A loosely compacted hay- or straw-stack would seem to be as effective an absorbent of sound as anything likely to be met with.

In large spaces bounded by non-porous walls, roof, and floor, and with few windows, a prolonged resonance seems inevitable. The mitigating influence of thick carpets in such cases is well known. The application of similar material to the walls, or to the roof, appears to offer the best chance of further improvement.

* The problem in two dimensions is somewhat simpler than that treated by Kirchhoff. Although it would allow us without violence to suppose $g = 0$, it seems scarcely worth while to enter upon it here, as the results are of precisely the same character. The principal difference is that the hyperbolic functions cosh &c. replace that of Bessel.