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L. On Vaschy's or Pirani's method of comparing the self-inductance of a coil with the capacity of a condenser

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accuracy be desired, which can rarely happen owing to the uncertainty as to the value of μ. But if it be retained, the resistance becomes

$$R = \frac{2n\mu}{z^2}\left(1 - \frac{1}{12}z^4 - \frac{1}{180}z^8\right) - \frac{8na^2z^2}{\mu c^2}\left(1 - \frac{1}{24}z^4 + \frac{13}{4320}z^8\right)$$
$$+ \frac{8na^2z^2}{\mu c^2}\left(\rho - \log\frac{a}{c}\right)\left(1 - \frac{1}{3}z^4 + \frac{19}{120}z^8\right). \quad . \quad (50)$$

where z has the value in (39), and the brackets may be shortened except when z is nearly unity.

When the frequency is smaller, this makes $R = 2\sigma/\pi a^2$, the value appropriate to steady currents.

Copper Wires with Low Frequency.

The formula suited to this case is, writing $\beta^2 + \gamma^2 = \alpha$, if $(z\,\alpha\,\beta\,\gamma)$ are defined as in (39), and under the same limitations,

$$R = \frac{2n}{z} \cdot \frac{\gamma}{\alpha} - \frac{4na^2}{zc^2}(\beta - 2\alpha\rho z + 2\gamma\rho z^2)/(1 - 4\beta\rho z + 4\alpha\rho^2 z^2)$$
$$- \frac{8na^2}{c^2}\log\frac{a}{c}\left(\gamma^2 - \beta^2 - \gamma z + 2\alpha\beta\rho z\right)/(1 - 4\beta\rho z + 4\alpha\rho^2 z^2), \quad (51)$$

again reducing to $2\sigma/\pi a^2$ for steady currents.

Trinity College, Cambridge,
April 21, 1909.

L. *On Vaschy's or Pirani's Method of comparing the Self-Inductance of a Coil with the Capacity of a Condenser.* By CHARLES H. LEES, *D.Sc., F.R.S., Professor of Physics in the East London College, University of London* *.

IN a recent number of *L'Eclairage Electrique*, M. O. de A. Silva † examined in detail the validity of Vaschy's or Pirani's method of comparing the self-inductance of a coil with the capacity of a condenser for the case in which the

* Communicated by the Author.
† O. de A. Silva, *L'Eclairage Electrique*, 50. p. 113 (1907).

discharge of the condenser was non-oscillatory. In the June number of this Magazine Mr. E. C. Snow * completed the examination by dealing with the oscillatory case.

In treating the problem Messrs. Silva and Snow write down the Kirchhoff equations for the currents in each branch of the resistance-bridge, and from them deduce a differential equation of the third order for the current through the galvanometer. This equation they solve and find the quantity of electricity discharged through the galvanometer by integrating the expression for the current as a function of the time between the limits 0 and ∞. On equating this quantity to zero, there results the well-known equation for the method $L=Kr^2$. The investigations are therefore very detailed, and if the object were to determine, for example, the time which must elapse before the quantity of electricity which has passed through the galvanometer amounts to say ·999 of its final value (a question which might arise in connexion with the condition that the discharge must have passed through the galvanometer before its moving part has moved appreciably) such detail would be unavoidable. But if we start, as do Messrs. Silva and Snow, with the assumptions that the galvanometer satisfies the above condition, and that the needle starts from a symmetrical position †, the investigation may be simplified considerably.

The method of using the Electro-Kinetic Energy and Rayleigh's Dissipation Function ‡ for the treatment of problems of this kind, first introduced by Maxwell §, and extended by Fleming ‖ and Niven ¶, has proved so powerful, and it is so much in keeping with modern dynamical methods ** that a brief statement of it may not be out of place here.

If a network of conductors consist of branches having resistances R_1, R_2, R_3, &c., self-inductances L_1, L_2, L_3, &c., mutual inductances M_{12}, M_{23}, M_{34}, &c., and capacities K_1, K_2, K_3, &c., and if the quantities of electricity which have flowed through the various branches up to a given time t are

* E. C. Snow, Phil. Mag. vol. xvii. p. 849 (1909).

† See Russell, Phil. Mag. vol. xii. p. 202 (1906).

‡ Lord Rayleigh, Proc. Lond. Math. Soc. iv. p. 357 (1873), and Scientific Papers, i. p. 176.

§ Clerk Maxwell, 'Electricity and Magnetism,' 2nd edit. vol. ii. p. 365 (1881).

‖ J. A. Fleming, Phil. Mag. vol. xx. p. 242 (1885).

¶ J. C. Niven, Phil. Mag. vol. xxiv. p. 225 (1887).

** See E. T. Whittaker, 'Analytical Dynamics,' pp. 226, 228 (1904).

x_1, x_2, x_3, &c., respectively, then if we write down the Electrokinetic Energy

$$T = \Sigma\tfrac{1}{2}L_n\dot{x}_n^2 + \Sigma M_n M_{nn'}\dot{x}_n\dot{x}_{n'}, \quad . \quad . \quad . \quad (1)$$

the Dissipation Function

$$D = \Sigma\tfrac{1}{2}R_n\dot{x}_n^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and the Electrostatic Energy

$$V = \Sigma\tfrac{1}{2}\frac{x_n^2}{K_n}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

the equations for the flow of electricity through the various branches of the network may be written in the form:

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{x}_n}\right) + \frac{\partial D}{\partial \dot{x}_n} + \frac{\partial V}{\partial x_n} = 0. \quad . \quad . \quad . \quad . \quad (4)$$

In this equation there is no reference to electromotive forces due to cells or other causes present in the system. To extend the method to cover such cases, we make use of a device well known to readers of Heaviside *, that is we consider a constant electromotive force E as due to the presence of a condenser of large capacity K possessing an initial charge $X = EK$. The Electrostatic Energy of such a condenser when it has given up a finite quantity of electricity x is equal to $\frac{1}{2}(X-x)^2/K$, *i. e.* to $\frac{1}{2}KE^2 - Ex$ since x/X is small. As we are only concerned with changes of Energy, the term contributed to the Electrostatic Energy by the cell reduces to $-Ex$. The extended form of the Electrostatic Energy becomes therefore

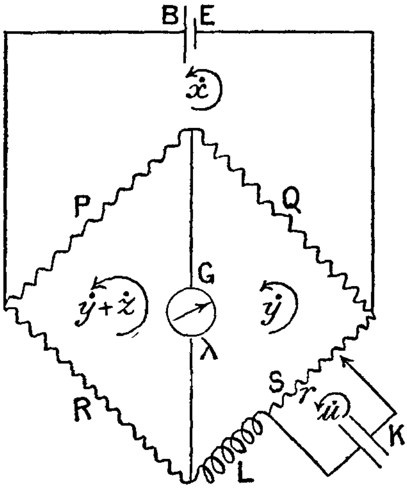
$$V = \Sigma\tfrac{1}{2}x_n^2/K_n - \Sigma E_n x_n. \quad . \quad . \quad . \quad . \quad (3')$$

Although in stating these propositions it has been convenient to take a simple symbol for the quantity of electricity which has flowed through each branch of the network, it is more convenient in applying the method to the solution of a problem to follow Maxwell's plan of assigning a simple symbol to the quantity which has flowed round a mesh of the network. Kirchhoff's first law for the distribution of currents in networks, *i. e.* that the currents leaving a node have a sum equal to zero, is then fulfilled automatically.

The following figure gives the arrangement of the circuit

* See for example, O. Heaviside, Electrical Papers, ii. p. 216 (1892).

known as Vaschy's or Pirani's method, and the arrows show the currents in the various meshes.



The following are the expressions for the Electrokinetic Energy, the Dissipation Function, and the Electrostatic Energy respectively, the mutual inductances being taken zero:—

$$T = \tfrac{1}{2}L\dot{y}^2 + \tfrac{1}{2}\lambda\dot{z}^2, \quad \ldots \quad (5)$$

$$D = \tfrac{1}{2}B\dot{x}^2 + \tfrac{1}{2}P(\dot{x}-\dot{y}-\dot{z})^2 + \tfrac{1}{2}Q(\dot{x}-\dot{y})^2 + \tfrac{1}{2}R(\dot{y}+\dot{z})^2 + \tfrac{1}{2}(S-r)\dot{y}^2 + \tfrac{1}{2}r(\dot{y}-\dot{u})^2 + \tfrac{1}{2}G\dot{z}^2, \quad \ldots \quad (6)$$

$$V = \tfrac{1}{2}u^2/K - Ex. \quad \ldots \quad (7)$$

Differentiating these expressions to find the terms of the equations of type (4), or applying Kirchhoff's second law directly to each mesh of the network, we have

$$B\dot{x} + P(\dot{x}-\dot{y}-\dot{z}) + Q(\dot{x}-\dot{y}) = E, \quad \ldots \quad (8)$$

$$L\ddot{y} - P(\dot{x}-\dot{y}-\dot{z}) - Q(\dot{x}-\dot{y}) + R(\dot{y}+\dot{z}) + S\dot{y} - r\dot{u} = 0\,. \quad (9)$$

$$\lambda\ddot{z} - P(\dot{x}-\dot{y}-\dot{z}) + R(\dot{y}+\dot{z}) + G\dot{z} = 0, \quad \ldots \quad (10)$$

$$-r(\dot{y}-\dot{u}) + u/K = 0. \quad \ldots \quad (11)$$

Rearranging equations (8), (9), and (10) we have

$$(B+P+Q)\dot{x}-(P+Q)\dot{y}-P\dot{z} \qquad = E, \quad . \quad . \quad . \quad (8')$$

$$-(P+Q)\dot{x}+(P+Q+R+S)\dot{y}+(P+R)\dot{z} = r\dot{u}-L\ddot{y}, \quad . \quad (9')$$

$$-P\dot{x}+(P+R)\dot{y}+(R+G)\dot{z} \qquad = -\lambda\ddot{z}. \quad . \quad . \quad (10')$$

In the steady state when $\dot{u}$, $\ddot{y}$, and $\ddot{z}$ are each $=0$, the current $\dot{z}$ will be $=0$ if the minor of E in the determinant for z is zero, that is if $P/Q=R/S$.

Integrating the above equations with respect to the time between the limit 0, at which the currents and quantities are zero, and the time t_1, at which the currents are steady and the quantities of electricity which have passed have attained the values x_1, y_1, z_1, we have :—

$$(B+P+Q)x_1-(P+Q)y_1-Pz_1 \qquad = \int_0^{t_1} E\,dt, \quad . \quad (12)$$

$$-(P+Q)x_1+(P+Q+R+S)y_1+(P+R)z_1 = (Kr^2-L)\dot{y}_1, \quad (13)$$

$$-Px_1+(P+R)y_1+(R+G)z_1 \qquad = 0. \quad . \quad . \quad . \quad (14)$$

From the second of which the relation $u_1=Kr\dot{y}_1$ given by (11) has been used to eliminate u_1.

From these equations it is seen by inspection that $z_1=0$ if $Kr^2-L=0$, and the minor of $\int_0^{t_1} E\,dt$ in the determinant for z_1 is zero, that is if $P/Q=R/S$, *i. e.* the condition for a steady balance previously obtained.

Since the only conditions assumed to hold in the above proof are that the current in each mesh of the network is initially zero and finally steady, the question whether oscillations take place in the interval or not, does not influence the result.

Whether t_1 can be so chosen that the currents throughout the network have become sufficiently steady, without the condition that there has been no motion of the galvanometer-needle or coil during that time being violated, is quite another question. With a modern ballistic galvanometer of the type recently constructed by Prof. B. O. Peirce *, of Harvard, having a period of 10 minutes, there will be very few cases in which there is any doubt that both conditions are satisfied.

It is well to remember that even then, the needle or coil should start from a symmetrical position if the absence of reflexion is to be taken as a proof that the time integral of the current throughout the instrument is zero †.

* B. O. Peirce, Proc. Amer. Acad. xliv. p. 283 (1909).
† A. Russell, Phil. Mag. xii. p. 202 (1906).