Mathematical Association

119. [Rule for finding the number of quarts not greater than a given number N (a quart being a number which cannot be expressed a the sum of one, two, or three squares).]

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The triangles BAD, DAC are congruent, so that $BD=DC$.

Hence $2AP.AD=AB^2+AD^2-BD^2$ (II. 12 or 13)

$=AC^2+AD^2-CD^2$

$=2AQ.AD.$

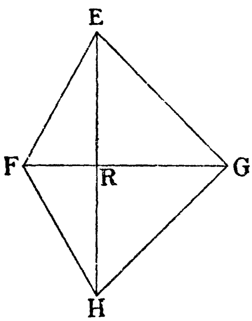
Hence $AP=AQ$ (I. 34 and 40).

Hence $BP=QC$ (I. 47).

Since $AP=AQ$, P and Q coincide, and thus the proposition is proved.

It follows that the perpendicular from A to BC bisects the angle A and the line BC, and that the median from A bisects the angle A and is perpendicular to BC.

(B) Now let EFG, HFG be two triangles on opposite sides of FG, having $EF=HF$ and the angles EFG, GFH equal. Then shall the triangles be equal in all respects.



Draw EH cutting FG, produced if need be, in R. By (A), FG bisects EH at right angles.

Hence $EG=HG$ (I. 47).

Since EGH is isosceles GR bisects the angle EGH. Hence the angles EGF, FGH are equal.

The third angles are equal (I. 32) and the areas (I. 38).

The second (symmetrical) case of I. 4, and I. 5. follow at once from this. I. 5 could also be deduced from (A) and I. 32.

For I. 8 in the symmetrical case put the bases together and use I. 5. In the congruent case compare the triangles with a symmetrical one.

In the symmetrical case of I. 26, which includes I. 6, compare the triangles with a third, made symmetrical with one of them by I. 23 and 3.

A. C. Dixon.

MATHEMATICAL NOTE.

119. [I. 13. b. α; 17. c.]

Rule for finding the number of quarts not greater than a given number N (a quart being a number which cannot be expressed as the sum of one, two, or three squares).

1. Express N in the binary scale, and let its digits, commencing with the unit-digit, be α_0, α_1, α_2, α_3, etc. (Of course each α is either 0 or 1.)

2. In this expression for N note sequences of three 1's; but only count such sequences of three as end with the unit-digit, or with a digit an *even* number of places from the unit-digit. Let β be the number of these sequences.

3. Then the number of quarts not exceeding N will be

$$\beta+(\alpha_3+\alpha_5+\alpha_7+\ldots)+2(\alpha_4+\alpha_6+\ldots)+4(\alpha_5+\alpha_7+\ldots)$$
$$+8(\alpha_6+\alpha_8+\ldots)+\text{etc.}$$

Examples of the Rule.

Ex. 1. To find the number of quarts not exceeding 125.

$125=1111101$ binary, and there are two sequences as marked. Hence the number of quarts is

$$2+(1+1)+2(1+1)+4\cdot1+8\cdot1=20.$$

Ex. 2. To find the number of quarts not exceeding 167.

$167 = 10100\widehat{111}$ binary, and there is one sequence as marked. Hence the number of quarts is

$$1+(0+1+1)+2(0+0)+4(1+1)+8\cdot 0+16\cdot 1=27.$$

N.B.—These 27 quarts are—7, 15, 23, 28, 31, 39, 47, 55, 60, 63, 71, 79, 87, 92, 95, 103, 111, 112, 119, 124, 127, 135, 143, 151, 156, 159, 167.

W. A. WHITWORTH.

REVIEWS.

Cours d'Analyse Mathématique. Tome I. By É. GOURSAT. Pp. 620. 1902. (20fr.)

This work is practically the résumé of a course of lectures given by the author to the Faculté des Sciences de Paris. The attempt has been made to give in this the first volume a general exposition of the properties of functions of real variables, the exception being made of those connected with differential equations. This part of the subject, however, will be treated in a second volume at present in the press.

As was perhaps to be expected from M. Goursat, the book is one of the most pleasing which has appeared recently on the subject in question. The matter is treated in a vigorous manner, and its arrangement leaves little to be desired. The author assumes that the student has some acquaintance with the elements of the calculus, and he also assumes some knowledge of the better known properties of irrationals. Whilst admitting that the theory of irrationals ought logically to form the ground-work of an exposition of Mathematical Analysis, appeal is made to the many well-known works on the subject.

The contents of the volume are divided primarily into four parts—(1) General theorems on differentiation, (2) Integration and properties of Definite Integrals, (3) Theory of Series, and (4) Geometrical Applications.

The first part commences with theorems on continuity and on limits. General theorems connected with differentiation are then given, some space being devoted to a consideration of the properties of Jacobians and of Hessians, and to a discussion of various transformations, such as those of contact, for example.

Taylor's Theorem, and its extension to several variables, together with properties of functions connected with it, form the next chapter, while the remainder of this part is devoted to a discussion of maxima and minima, and to problems connected therewith.

The second part commences with a close examination into the question of continuity from an analytical standpoint. General theorems on integration are then given, which are afterwards exemplified by appeal to geometrical intuition. After a discussion of ordinary methods of integration the author proceeds to consider double and multiple integrals, some space being devoted to Green's and Stokes' theorems. A few interesting examples of definite integrals are given, and this part concludes with a short account of the integration of total differentials.

The next two chapters are concerned with the theory of series. The first of these is occupied with general convergence criteria. For the purpose of obtaining many of them, use is made of Cauchy's theorem that

$$\sum_{x=0}^{\infty} \phi(a+x) \text{ and } \int_a^{\infty} \phi(x)dx$$

converge or diverge together.

There are also given in this part of the work sections on multiple series and on series with variable terms. The section on properties of power series is particularly worthy of note. The division concludes with a discussion of trigonometric series, the proof of Fourier's theorem given being Bonnet's modification of that due to Lejeune-Dirichlet.

The remainder of the volume is occupied with general properties of plane and twisted curves and of surfaces.