By successive differentiation, we have

$$\begin{aligned} U &= x^3-2x-5, \\ U' &= 3x^2-2, \\ U'' &= 6x, \\ U''' &= 6. \end{aligned}$$

A real root of the equation evidently lies near to the number 2; and for $x=2$ we have

$$-U=1, \quad U'=10, \quad U''=12, \quad U'''=6;$$

$$\therefore \quad z=\frac{1}{10}, \quad c_2=\frac{3}{5}, \quad c_3=\frac{1}{10};$$

$$v=\frac{3}{50}, \quad q=\frac{\sqrt{31}}{5}, \quad p=\frac{\sqrt{31}+5}{10},$$

$$\omega=\frac{\sqrt{31}-5}{6}, \qquad h_3=\frac{1}{2\sqrt{31}};$$

$$\begin{aligned} \therefore \quad x &= 2+\omega-h_3\omega^3 \\ &= 2+\frac{\sqrt{31}-5}{6}-\frac{1}{2\sqrt{31}}\left(\frac{\sqrt{31}-5}{6}\right)^3 \\ &= \frac{1}{216}\left(\frac{1411}{31}\sqrt{31}+199\right) \\ &= 2\cdot0945513, \text{ which is true to the last figure.} \end{aligned}$$

These examples will serve to show the practical utility of the formula, which may with unfailing efficacy be similarly applied to all kinds of problems. It is, however, in the numerical solution of complicated and otherwise unmanageable equations that the value of the method will be most conspicuous. All complication is entirely got rid of so soon as the successive numerical values are obtained by direct computation.

Remark on the preceding Paper. *By* A. DE MORGAN, ESQ., F.R.A.S.

Having seen Mr. Woolhouse's Paper before it was publicly read, I arrived at an independent establishment of the development which is, in the Paper, deduced from the known series for reversion. This mode of establishment is also an extension; though it is hardly to be expected that any extended application will be found desirable. At the request of Mr. Woolhouse, I subjoin a short notice.

Let $\phi\omega=\phi x+\psi x$, where ψx will be small; and write fx, when convenient, for $\phi x+\psi x$. Assume $x=\omega+t$, which gives

$$-\psi\omega=f'\omega\,.\,t+f''\omega\,.\,\frac{t^2}{2}+\ldots.$$

Find t in powers of $-\psi\omega$ by common inversion, omitting ω after f for abbreviation, and writing f_n for $f^{(n)} : 2.3\ldots n$,

$$x = \omega - \frac{1}{f_{,}} . \psi\omega - \frac{f_{,,}}{f_{,}^{3}} . (\psi\omega)^2 - \frac{2f_{,,}^{2} - f_{,} f_{,,,}}{f_{,}^{5}} . (\psi\omega)^3$$
$$- \frac{5f_{,,}^{3} - 5f_{,} f_{,,} f_{,,,} + f_{,}^{2} f_{,,}}{f_{,}^{7}} . (\psi\omega)^4 - \ldots\ldots,$$

a representation of $f^{-1}\phi\omega$, or of $f^{-1}(f\omega - \psi\omega)$. Three terms will be more than sufficient.

Let $\phi x + \psi x$ be $\quad ax + bx^2 + \ldots + kx^m + (p + qx + \ldots + sx^n)\, x^{n+1}$;
let $\alpha = \phi x + \psi x$ be the equation to be solved, and let $\alpha = \phi x$ give $x = \omega$. The root of $\alpha = \phi\omega$ being known, $\phi'\omega$ is known; call it l. Expanding the above in powers of ω, it is clear that the first three terms will give all short of ω^{3m+3}; Mr. Woolhouse, by an entirely different method, stops at ω^8 when $m = 2$. Taking this case, we find $\alpha = a\omega + b\omega^2$, $a + 2b\omega = \sqrt{(a^2 + 4b\alpha)} = l$. And, writing down no more than necessary for ω^8 inclusive,

$$x = \omega - \frac{c + e\omega + f\omega^2 + g\omega^3 + h\omega^4 + k\omega^5}{l + 3c\omega^2 + 4e\omega^3 + 5f\omega^4 + 6g\omega^5 + 7h\omega^6 + 8k\omega^7}\,\omega^3$$
$$- \frac{(b + 3c\omega + 6e\omega^2 + \ldots + 28k\omega^6)\,(c + e\omega + f\omega^2)^2}{(l + 3c\omega^2 + \ldots + 8k\omega^7)^3}\,\omega^6 - \ldots.$$

For l write 1; and, remembering that in this result al^{-1}, bl^{-1}, &c. must be written for a, b, &c., we have

$$x = \omega - c\omega^3 - e\omega^4 - (f - 3c^2)\,\omega^5$$
$$-(g - 7ce + bc^2)\,\omega^6 - (h - 8cf - 4e^2 + 12c^3 + 2bce)\,\omega^7$$
$$-(k - 9cg - 9ef + 45c^2e - 9bc^3 + 2bcf + be^2)\,\omega^8 - \ldots.$$

This result agrees entirely with that of Mr. Woolhouse.

The following Paper, read May 28th, 1868, could not be inserted in the account of the Proceedings of that day :—

On some Geometrical Constructions.

By H. J. Stephen Smith, F.R.S., Savilian Professor of Geometry in the University of Oxford.

Art. 1. A conic A is said to *circumscribe harmonically* a conic B, when A circumscribes a triangle which is self-conjugate with regard to B. Similarly, A is said to be *inscribed harmonically in* B, when A is