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"Guns Considered as Thermodynamic Machines."

By JAMES ATKINSON LONGRIDGE, M. Inst. C.E.

IN the following Paper the Author pursues to a great extent the lines indicated by Count de St. Robert in his "Principes de Thermodynamique," Turin, 1870; and from the principles therein laid down, deduces formulas applicable to rifled guns, for determining the initial velocity of the projectile, and the velocity of recoil of the gun and carriage.

The determination of these velocities by the method usually adopted in this country is purely empirical, and it therefore appears desirable to attempt, at any rate, a more scientific method. This is the more desirable, on account of the mystification existing on the subject of so-called slow-burning powder and large charges in chambered guns. It is almost the same as if manufacturers of steam-engines were to give their chief attention to the way in which steam is generated, to the exclusion in a great measure of the consideration of how it is used.

Now a gun is just as much a machine as is a steam-engine, and in both mechanical force is obtained from a gaseous fluid. In the gun the fluid is the powder-gas which passes through a cycle, of which the initial state is the ignition of the powder, and the final state that when the projectile leaves the gun; consequently the following equation must result—

$$J \Delta H = J \Delta Q + \Delta I + \Delta W + \tfrac{1}{2} \Delta V \quad . \quad . \quad . \quad . \quad (1)$$

in which—

J is Joule's coefficient = 772 foot-lbs. to 1 unit of heat.

(1) ΔH is the heat extracted from the products of combustion in passing from the initial to the final state, *i.e.*, from the ignition of the powder to the time when the projectile leaves the muzzle.

(2) ΔQ is the quantity of heat passing from the gases into the body of the gun, and which goes to heat the gun.

(3) ΔI is the increment of internal work in the gases during the same time.

(4) Δ W is the external work done, and includes the work done in overcoming the statical resistance of the air to the projectile, but does not include the increased resistance of the air due to velocity. It also includes the work done in rotating the projectile, the friction of the same, and gas-check, the friction of the gases in the chase, and the work done in stretching the material of the gun.

(5) Δ V is the sum of the *vis viva* acquired by the system, and includes $\int R\, d\, s$, or the resistance of the air due to the velocity of the projectile.

Determination of the above.

(1) Δ H.

The powder is transformed into two portions, one of which is gaseous, the other non-gaseous, and according to Noble and Abel's researches,[1] these are by weight:—

Gaseous products . .	43 per cent.;	specific heat, 0·186
Non-gaseous products	57 „	„ 0·450

Consequently, if w be the weight of the charge, t_o and t the initial and final temperatures,

$$\Delta H = \{0 \cdot 57\, w \times 0 \cdot 45 + 0 \cdot 43\, w \times 0 \cdot 186\}\ (t_o - t) = 0 \cdot 3385\, w\, (t_o - t) \quad . \quad . \quad . \quad (2)$$

t_o is given by Noble and Abel at 2,000° to 2,100° C. or 2,274° to 2,374° C. absolute. In future calculations it is taken at 2,342° C. or 4,215° Fahr. absolute.

t is obtained from the equation (Noble and Abel)—

$$t = t_o \left(\frac{v_o\,(1 - a)}{v - a\, v_o} \right)^{\frac{C\mu - C v}{C v - a v_o}} = t_o \left(\frac{0 \cdot 43}{\frac{v}{v_o} - 0 \cdot 57} \right)^{0 \cdot 074} \quad . \quad . \quad . \quad (3)$$

v_o and v being the volumes before and after expansion.

(2) Δ Q. Heat imparted to the walls of the gun.

There is a good deal of uncertainty about this. The Author investigated it at some length in "A Treatise on the Application of Wire to the Construction of Ordnance" (Spon, London, 1884), and at page 144 of that work gave a diagram, which probably represents approximately the heat imparted to each square foot

[1] Philosophical Transactions of the Royal Society of London. For the year 1880, p. 203.

of surface of the interior of the gun. The application of this diagram will be found further on.

(3) Δ I. Internal work in gases.

The internal work in a perfect gas = *o*, and as powder gases approach very nearly to the condition of a perfect gas, we may take

$$\Delta\, 1 = 0.$$

(4) Δ W.

This comprises the following items :—

(*a*) *Resistance of Air* $= p \cdot \text{A} \cdot l$ . . . . (4)

p = atmospheric pressure.
A = area of bore. l = length of travel of shot.

(*b*) *Work done in Rotation* $= \frac{\text{W}}{2\,g} \cdot \left(\frac{0 \cdot 707\,\pi\,u}{m}\right)^2$ . . . . (5)

W = weight of shot. m = number of calibres to 1 turn of shot.
u = muzzle velocity.

(*c*) *Friction of Shot.*—Let the pitch of rifling be 1 in n, and let P be the pressure on the base of the shot, then

$$\text{Force to give rotation} = \frac{\pi\,\text{P}}{2\,n},$$

and if p be the powder pressure at any part x of the chase

$$\text{P}_1 = p\,\pi\,\rho^2. \quad \therefore \text{ force at } x = \frac{\pi^2\,\rho^2}{2\,n} \cdot p.$$

Now if P_1 be the initial pressure $p = \text{P}_1 \left(\frac{v_0\,(0 \cdot 1 - a)}{v_1 - a\,v_0}\right)^{1 \cdot 237}$

$$= \text{P}_1 \left(\frac{0 \cdot 43}{\frac{v_1}{v_0} - 0 \cdot 57}\right)^{1 \cdot 237}$$

where v_1 is the volume of gases at x,

therefore $\quad \text{Force at } x = \frac{\pi^2\,\rho^2}{2\,n} \cdot \text{P}_1 \left(\frac{0 \cdot 43}{\frac{v_1}{v_0} - 0 \cdot 57}\right)^{1 \cdot 237},$

and if the friction be taken at $\frac{1}{m}$th, and if P_1 be in tons per square inch, the total work done is found.

$$\frac{1}{m} \cdot \frac{\pi^2 \rho^2}{2 n} 2240 \, P_1 \int^{l} \left(\frac{0 \cdot 43}{\frac{v_1}{v_0} - 0 \cdot 57} \right)^{1 \cdot 237} d\, x \; . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and writing $\frac{v_1}{v_0}$ in terms of x, and integrating, the work done in foot-lbs. is ascertained.

(*d*) *Work done in overcoming Friction of Gas-check and Shot.*—As it is the gas-check in the Woolwich system that gives the rotation, this friction is included in the preceding items. No doubt an extra force is required at first to force it into the grooves, but as this is not continuous, and amounts probably to only a few lbs. per square inch on the area of the base of the shot at its first starting, it may be neglected.

(*e*) *Work done in overcoming Friction of the Gases in the bore.*—There must of course be some uncertainty about this, as so little is known of the laws of gaseous friction at high pressures; but as an approximation it may be assumed with Rankine[1] that the resistance $= f \rho S \frac{V^2}{2g}$.

when $f = 0 \cdot 006$.
ρ = weight in lbs. of cubic foot of gas.
S = surface of contact in square feet.
V = velocity in feet per second.

Now at the breech $V = o$, and at the muzzle it is the velocity of the shot. The weight of a cubic foot of the gas at the beginning of motion $= \frac{w}{\frac{\pi \rho_1^2 \times l}{1{,}728}} = \frac{1{,}728\, w}{\pi \rho_1^2 l}$ when l and ρ_1 are the length and radius of the powder chamber.

Therefore at any other point x the weight of a cubic foot $= \frac{1{,}728}{\pi \rho_1^2 l} \cdot \frac{l_1}{x + l_1}$ where l_1 is the equivalent length of the powder chamber, or the length which it would have, if of the same diameter as the bore. Now if it be assumed that the velocity of the gas increases uniformly from the breech to the muzzle, then

Velocity at $x = V \frac{x}{L}$, L being the travel of the shot and V the muzzle velocity.

[1] "A Manual of Applied Mechanics." By William John Macquorn Rankine, 1858, p. 584.

Consequently

$$\text{Resistance} = 0{\cdot}006 \times \frac{1{,}728\, w}{\pi\, \rho_1^2\, l} \cdot \frac{l_1}{x + l_1} \cdot \text{S} \cdot \frac{\text{V}^2\, x^2}{2\, g\, \text{L}^2},$$

and as this acts through $d\,x$—

$$\text{Work done} = \frac{0{\cdot}006 \times 1{,}728\, w \times l_1\, \text{S} \times \text{V}^2}{\pi\, \rho_1^2\, l\, 2g} \cdot \int_0^{\text{L}} \frac{x\, d\, x}{\text{L}^2} \quad . \quad (7)$$

which being integrated gives the work done in foot-lbs.

(*f*) *Work done in Stretching the Gun circumferentially.*—The powder-pressure inside the gun acts upon any elementary shell by extending it circumferentially and compressing it radially.

Let there be such a shell at radius y, and let its thickness and breadth be $d\,y$ and β respectively.

Let t_y be the tension at y per square inch of section.
f_y the radial compressive force in ditto.
x the extension under t_y.
l the length $= 2\,\pi\,y$.
E the modulus of elasticity.

Then $x = l\,\frac{t_y}{\text{E}}$.

Now for any intermediate extension z, let ϕ be the force exerted, then $z = l\,\frac{\phi}{\text{E}}$, or $\phi = \frac{\text{E}}{l}\,z$, and work done through $d\,z = \frac{\text{E}}{l}\,z\,d\,z \times \beta\,d\,y$, because $\beta\,d\,y$ is the area of section.

Integrating in respect of z, when z becomes $= x = l\,\frac{t_y}{\text{E}}$, the work done $= \frac{l\,\beta\,t_y^2}{2\,\text{E}} \cdot d\,y$, and replacing l by $2\,\pi\,y$, the work done $= \frac{\pi\,\beta\,t_y^2\,y\,d\,y}{\text{E}}$.

But t_y is a function of y, and if f_1 be the internal powder-pressure, ρ the internal radius, R the external radius, and $m = \frac{\text{R}}{\rho}$.

$$t_y = \frac{f_1}{m^2 - 1} \cdot \frac{\text{R}^2 + y^2}{y^2},$$

substituting which in the above—

$$\text{Work done} = \frac{\pi\,\beta}{\text{E}} \cdot \frac{f_1^2}{(m^2 - 1)^2} \cdot \int \left(\frac{\text{R}^2 + y^2}{y^2}\right)^2 y\,d\,y,$$

which by integration gives—

$$\text{Work done} = \frac{\pi\,\beta}{E} \cdot \frac{f_1^2}{(m^2-1)^2}\left\{\frac{m^2-1}{2}R^2 + 2R^2\log m + \frac{R^2+y^2}{2}\right\} \quad . \quad . \quad . \quad (8)$$

Proceeding in like manner for compression—

Work done

$$= \frac{\pi\,\beta}{E} \cdot \frac{f_1^2}{(m^2-1)^2}\left\{\frac{m^2-1}{2}R^2 + 2R^2\log\frac{1}{m} + \frac{R^2-y^2}{2}\right\} \quad (9)$$

therefore adding—

$$\text{Total work done} = \frac{\pi\,\beta}{E} \cdot f_1^2\,\frac{m^2+1}{m^2-1} \cdot \rho^2 \quad . \quad . \quad . \quad . \quad (10$$

and if the units be tons and inches, and $\beta = 1$, this gives inch-tons per lineal inch of bore, or foot-tons per lineal foot of bore; and since the surface of 1 lineal foot of bore $= \frac{2\,\pi\,\rho \times 12}{144} = \frac{\pi\,\rho}{6}$, finally

$$\textit{Work done per square foot of surface} = \frac{6\,f_1^2\,\rho}{E} \cdot \frac{m^2+1}{m^2-1}$$

in the chamber . . . . (11)

To find the work done in the rest of the chase.

Let p be the powder-pressure at any part x,

then $$p = P_1\left(\frac{v_0\,(1-a)}{v_1 - a\,v_0}\right)^{1\cdot237},$$

or, as was shown before,

$$= P_1\left(\frac{0\cdot43}{\frac{v_1}{v_0} - 0\cdot57}\right)^{1\cdot237},$$

which is the value of f_1 to be used in the above equation; therefore the work done on unit of length of bore

$$= \frac{\pi}{E} \cdot \frac{m^2+1}{m^2-1} \cdot \rho^2 \cdot P_1^2\left(\frac{0\cdot43}{\frac{v_1}{v_0} - 0\cdot57}\right)^{2\cdot474},$$

and work done in $d\,x$

$$= \frac{\pi}{E} \cdot \left(\frac{m^2+1}{m^2-1}\right) \cdot \rho^2 \cdot P_1^2\left(\frac{0\cdot43}{\frac{v_1}{v_0} - 0\cdot57}\right)^{2\cdot474} d\,x \quad . \quad (12)$$

and expressing $\frac{v_1}{v_0}$ in terms of x, and integrating, taking $x = L$ the length of the chase, the total work done in foot-lbs. is ascertained.

There now only remains to determine the work done in extending the gun between the breech and the trunnions. It will be assumed that the strain is uniformly distributed over the cross-section of the gun.

Then the strain per square inch is $\frac{P_1 \rho^2 \pi}{(R^2 - \rho^2)\pi} = \frac{P_1}{m^2 - 1}$ which call f, thus if l be the length from breech to trunnion, total extension $= f \frac{l}{E}$.

Now for any intermediate extension y, the force $= E \frac{y}{l}$, and work done in $d\,y = E\left(\frac{y\,d\,y}{l}\right)$. Integrating, since $y = f \frac{l}{E}$, work done $= \frac{f^2\,l}{2\,E}$ per square inch of surface, and since the area is $2\,\pi\,(R^2 - \rho^2)$, and $f = \frac{P_1}{m^2 - 1}$.

$$\text{Total work} \qquad \frac{\pi\,(R^2 - \rho^2)\,l \cdot P_1^2}{E\,(m^2 - 1)^2} \qquad \ldots\ldots \quad (13)$$

(5) Δ V.

This is made up of the following items:—

(*a*) *Vis viva* of projection $= \frac{W}{g} \cdot u^2$ . . . . . . (14)
when W = weight and u = muzzle velocity of projectile.

(*b*) *Vis viva* of gun and carriage $= \frac{W_1}{g}\,u_1^2$ . . . (15)
when W_1 is the weight of gun and recoiling part of carriage.

(*c*) *Vis viva* of the gases $= \int u_{,,}^2\,d\,\mu$, when μ is the mass or $\frac{w}{g}$, w being the weight of the charge, and $u_{,,}$ the varying velocity of the gas at varying distances from the breech at the time the shot reaches the muzzle.

This integral must be taken for the whole mass of the products of combustion from the breech to the muzzle. It depends on the state of the particles and their respective velocities at the time the shot leaves the gun.

Some uncertainty as regards this is unavoidable; but probably it will not lead to any important error if it be assumed, first, that the density of the products of combustion at any moment is uniform throughout; second, that these velocities increase uniformly from the breech to the muzzle; and lastly, that the layers or transverse slices of the products in contact with the breech and the projectile have respectively the velocities of the gun and of the projectile, viz., u_1 and u. This being so, there must be some point where the gases are at rest, and this point divides the whole length in the ratio of u and u_1.

Let l be the length of the chase, then the point of rest will be

$$\frac{u_1}{u + u_1} \cdot l \text{ from the breech,}$$

$$\frac{u}{u + u_1} \cdot l \text{ from the muzzle,}$$

and for any intermediate point x from the point of rest, on the muzzle side, the velocity will be

$$u \frac{x}{\dfrac{u\,l}{u + u_1}} = \frac{u + u_1}{l} \cdot x$$

also at y from point of rest on breech side

$$\text{velocity} \quad = \frac{u + u_1}{l} \cdot y.$$

Now let δ = density of products of combustion.
A = area of the bore.

Since the moments are equal on each side of the point of rest,

$$\text{moment on the muzzle side} = \frac{\delta\,A}{g} \cdot \frac{u + u_1}{l} \int_0^{\frac{u\,l}{u + u_1}} x\,d\,x$$

$$= \frac{\delta\,A\,l}{2\,g} \cdot \frac{u^2}{u + u_1}.$$

and on the other side the moment is

$$\frac{\delta\,A\,l}{2\,g} \cdot \frac{u_1^{\,2}}{u + u_1},$$

and as these are in opposite directions their algebraic sum is

$$\frac{\delta\,A\,l}{2\,g} \cdot \frac{u^2 - u_1^{\,2}}{u + u_1} = \frac{\delta\,A\,l}{2\,g}(u - u_1) \quad . \quad . \quad . \quad . \quad . \quad (16)$$

which is the total momentum.

Now $\delta\ A\ l$ represents the total weight of the products of combustion $= w$

$$\therefore \int u_{,,}\, d\,\mu = \frac{w}{2\,g}(u - u_1),$$

which is the total momentum of the products of combustion.

For the *vis viva*, there is for those moving in the direction of the projectile

$$\frac{\delta\ A}{g}\left(\frac{u + u_1}{l}\right)^2 \int_o^{\frac{u\,l}{u + u_1}} x^2\, d\, x,$$

and for those moving in the opposite direction

$$\frac{\delta\ A}{g}\left(\frac{u + u_1}{l}\right)^2 \int_o^{\frac{u\,l}{u + u_1}} y^2\, d\, y$$

the integrals of which are

$$\frac{\delta\ A\ l}{3\ g} \cdot \frac{u^3}{u + u_1} \text{ and } \frac{\delta\ A\ l}{3\ g} \cdot \frac{u_1{}^3}{u + u_1},$$

therefore the total momentum is

$$\frac{\delta\ A\ l}{3\ g} \cdot \frac{u^3 + u_1{}^3}{u + u_1} = \frac{w}{3\ g} \cdot (u^2 + u_1{}^2 - u\ u_1),$$

which is the value of $\int u_{,,}{}^2\ \delta\ \mu$ . . . . (17)

It remains to determine the value of $\int R\ d\ s$, or the work done in overcoming the resistance of the air due to the velocity.

Taking the resistance as proportionate to the cube of the velocity and $= a\ u^3$. a is a coefficient depending on the diameter of the projectile and the form of the head.[1]

Then $R = a\ u^3$.

To determine v in terms of the space.

No exact function determining this has yet been obtained, but from an examination of the velocity curves given by the Committee on Explosives, Preliminary Report, 1870, it appears that the relation is very nearly $u = 744\ S^{\frac{1}{3}}$ for pebble powder in an 8-inch gun with a projectile of 180 lbs.

therefore $$R = a \cdot (744)^3\ S,$$

and $$\int R\ d\ s = (744)^3\ a \int_o^L s\ d\ s$$

$$= \frac{(744)^3\ a\ L^2}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

[1] "Reports on Experiments made with the Bashforth Chronograph." Part II. 1878–79. Table II., Appendix to Report VIII.

To determine u and u_1 there is the further relation from the equality of moments

$$m_1 u_1 = m u + \int u_{\prime\prime} \delta \mu,$$

or
$$\frac{W_1}{g} u_1 = \frac{W}{g} u + \frac{w}{2 g} (u - u_1),$$

or
$$W_1 u_1 = W u + \frac{w}{2 g} (u - u_1) \quad . \quad . \quad . \quad . \quad (19)$$

from which u_1 is obtained in terms of u, and substituting this in the former equation, u the muzzle velocity is found, and then from the last equation u_1 the velocity of recoil.

To proceed to an application of the preceding formula.

Take the 10 inches B.L. Woolwich gun of 27 tons.

Weight of projectile . . .	500 lbs. = W.
„ of charge	300 lbs. = w.
Length of chamber. . . .	54 inches.
Diameter of „	14 „
Length beyond chamber . .	27·0 feet.
Diameter „ „ . .	10·0 inches.
Capacity of chamber . . .	8,316 cubic inches = v_o.
Total capacity of gun . . .	29,522 „ „ = v.

Therefore $\frac{v}{v_o} = 3 \cdot 55$.

(1) Determinations of J Δ H.

As shown before, this is

$$= 0 \cdot 3585\, w\, (t_o - t),$$

now $w = 300$ lbs. $t_o = 4{,}215°$ Fahrenheit (absolute),

and
$$t = t_o \left(\frac{0 \cdot 43}{\frac{v}{v_o} - 0 \cdot 57} \right)^{0 \cdot 074}.$$

$$\text{Now } \frac{v}{v_o} = 3 \cdot 55 \therefore t = 4{,}215 \times \left(\frac{0 \cdot 43}{2 \cdot 98} \right)^{0 \cdot 074} = 3{,}652°.$$

Consequently fall of temperature = 4,215 − 3,652 = 563°,

and
$$J \Delta H = \frac{772 \times 300 \times 0 \cdot 3385 \times 563}{2{,}240} = 19{,}513 \text{ foot-tons.}$$

(2) Numerical determination of J Δ Q.

Referring to the diagram, p. 144, " A Treatise on the Application of Wire to the Construction of Ordnance," it will be found that the heat imparted to the body of the gun is as follows:—

Powder-chamber per square foot of surface			.	168 units.
In first expansion average	,,	,,	.	98 ,,
In second do. ,,	,,	,,	.	53 ,,
In 1st half of third do.	,,	,,	.	40 ,,

Therefore taking the surfaces—

Powder-chamber $\frac{54 \times 44}{144} = 16\frac{1}{2}$ sq. ft. $\times$ 168 = 2,772 units.

First expansion $\frac{107 \times 31 \cdot 41}{144} = 23\frac{1}{3}$,, $\times$ 98 = 2,307 ,,

Second ,, $\frac{107 \times 31 \cdot 41}{144} = 23\frac{1}{3}$,, $\times$ 53 = 1,237 ,,

Remainder $\frac{56 \times 31 \cdot 41}{144} = 11$,, $\times$ 40 = 440 ,,

Total . . . . 6,756 ,,

Therefore $J \Delta Q = \frac{6{,}756 \times 772}{2{,}240} = 1{,}850$ foot-tons.

(3) $\Delta I = o$.

(4) Determinations of ΔW.

(*a*) Resistance of air $p A l$

$$= \frac{14 \cdot 75 + 78 \cdot 54 + 25 \cdot 5}{2{,}240} = 13 \cdot 89 \text{ foot-tons.}$$

(*b*) For rotation

$$\text{Work done} = \frac{\frac{W}{2g}\left\{\frac{0 \cdot 707 \, \pi \, u}{m}\right\}^2}{2{,}240},$$

Now $W = 500$

m = pitch of rifling or number of calibres for 1 turn of shot, and let this be 30.

Therefore

$$\text{Work done} = \frac{500 + 0 \cdot 00548}{64 \cdot 4 + 2{,}240} u^2 = 0 \cdot 00002 \, u^2.$$

(c) For friction of gas ring, and shot. The expression for this is

$$\frac{\frac{1}{m} \cdot \frac{\pi^2 \rho^2}{2 n} + 2{,}240 \text{ P}_1 \int_o^l \left(\frac{0 \cdot 43}{\frac{v_1}{v_0} - 0 \cdot 57}\right)^{1 \cdot 237} d x}{2{,}240}$$

$\frac{1}{m}$ the coefficient of friction may be taken $= \frac{1}{5}$

$$n = 30; \ \rho = 5; \ \text{P}_1 = 18; \ l = 25 \cdot 5.$$

To determine $\frac{v_1}{v_o}$ there is $v_o = 8{,}316$ cubic inches,

and $\qquad v_1 = 78 \cdot 54 \, x + 8{,}316.$

$$\therefore \frac{v_1}{v_o} = \frac{78 \cdot 54 \, x + 8{,}316}{8{,}316} = 0 \cdot 00944 \, x + 1.$$

$$\therefore \int_o^l \left(\frac{0 \cdot 43}{\frac{v_1}{v_o} - \cdot 57}\right)^{1 \cdot 237} = \int_o^l \left(\frac{0 \cdot 43}{0 \cdot 00944 \, x + 0 \cdot 43}\right)^{1 \cdot 237},$$

and the expression becomes

$$\frac{1}{5} \cdot \left(\frac{15 \cdot 7075}{60}\right)^2 \times 18 \int_o^l \left(\frac{0 \cdot 43}{0 \cdot 00944 \, n + 0 \cdot 43}\right)^{1 \cdot 237} d x$$

$$= 14 \cdot 93 \int_o^l \frac{d x}{(0 \cdot 0219 \, x + 1)^{1 \cdot 237}}.$$

Now the integral of $\frac{d x}{(a + b x)^n} = - \frac{1}{(n - 1) b (a + b x)^{n-1}}.$

Therefore the above becomes

$$\int - \frac{1}{0 \cdot 237 \times 0 \cdot 0219 \, (0 \cdot 0219 \, x + 1)^{0 \cdot 237}},$$

and this taken between $x = l$ and $x = o$ gives

$$- \frac{1}{0 \cdot 0219 \times 0 \cdot 237 \times (0 \cdot 0219 \, l + 1)^{0 \cdot 237}} - - \frac{1}{0 \cdot 237 \times 0 \cdot 0219}$$

$$= 192 \cdot 6 - \frac{192 \cdot 6}{(0 \cdot 0219 \, l + 1)^{0 \cdot 237}}$$

and making $\qquad l = 25 \cdot 5$

$$= 192 \cdot 68 - \frac{192 \cdot 68}{1 \cdot 1109} = 192 \cdot 68 - 173 \cdot 46 = 19 \cdot 22.$$

$\therefore$ Work done $= 14 \cdot 93 \times 19 \cdot 22 = 286 \cdot 9$ foot-tons.

(*d*) Friction of the gases. The weight of a cubic foot of gas at the beginning is

$$\frac{1{,}728\, w}{\pi\, \rho^2\, l} = \frac{1{,}728 \times 300}{3 \cdot 1416 \times 25 \times 54} = \frac{518{,}400}{8{,}316} = 62 \cdot 34 \text{ lbs.}$$

and since l_1 the equivalent length of the chamber $= \frac{8{,}316}{78 \cdot 52} =$ 107 inch $= 8 \cdot 93$ feet. Therefore weight of a cubic foot at $x =$ $62 \cdot 34 \times \frac{8 \cdot 93}{x + 8 \cdot 93}$, and if it be assumed that the velocity of the gas increases uniformly from the chamber to the muzzle, the velocity at $x = u \frac{x}{\text{L}}$, u being the muzzle velocity, and L the travel of the shot $= 25 \cdot 5$ feet. Hence the resistance per unit of surface at x

$$= 0 \cdot 006 \times 62 \cdot 34 \times \frac{8 \cdot 93}{x + 8 \cdot 93} \times \text{S} \cdot \frac{u^2\, x^2}{2\, g\, \text{L}^2}.$$

Now S is the unit of surface in feet

$$\therefore \text{S} = \frac{\pi\, d}{12}.$$

Therefore the work due to the resistance through $d\, x$

$$= 0 \cdot 006 \times 62 \cdot 34 \times \frac{8 \cdot 93}{x + 8 \cdot 93} \times \frac{\pi\, d}{12} \cdot \frac{u^2}{2\, g\, \text{L}^2} \cdot x^2\, d\, x$$

$$= 0 \cdot 000209\, u^2 \int_0^{\text{L}} \frac{x^2}{x + 9 \cdot 83}\, d\, x;$$

but $$\int \frac{x^2\, d\, x}{a + b\, x} = \frac{x^2}{2\, b} - \frac{a\, x}{b^2} + \frac{a^2}{b^3} \log (a + b\, x),$$

and here $\qquad a = 9 \cdot 83 \quad b = 1,$

therefore the integral is

$$\frac{x^2}{2} - 9 \cdot 83\, x + 9 \cdot 83^2 \log (9 \cdot 83 + x)$$

when $x = \mathrm{L}$ this becomes

$$\frac{\mathrm{L}^2}{2} - 9 \cdot 83\ \mathrm{L} + 9 \cdot 83^2 \log (9 \cdot 83 + \mathrm{L}),$$

and when $x = o$ it becomes $9 \cdot 83^2 \log 9 \cdot 83$.

Therefore the integral between these limits is

$$\frac{\mathrm{L}^2}{2} - 9 \cdot 83\ \mathrm{L} + 9 \cdot 83^2 (\log (9 \cdot 83 + 1) - \log 9 \cdot 83)$$

$$= \frac{\mathrm{L}^2}{2} - 9 \cdot 83\ \mathrm{L} + 9 \cdot 83^2 \left\{\log. \frac{9 \cdot 83 + \mathrm{L}}{9 \cdot 83}\right\}$$

and since $\mathrm{L} = 25 \cdot 5$.

This becomes $325 \cdot 12 - 151 \cdot 17 + 96 \cdot 63 \{\log 3 \cdot 594\}$

$$= 173 \cdot 95 + 96 \cdot 63 \times 1 \cdot 2693$$

$$= 173 \cdot 95 + 122 \cdot 65 = 296 \cdot 60.$$

Therefore the work done $= 0 \cdot 000209 \times 296 \cdot 6\ u^2$ in foot-lbs.

$$= \frac{0 \cdot 000209 + 296 \cdot 6}{2{,}240} u^2 \text{ in foot-tons.}$$

$$= 0 \cdot 00002765\ u^2 \text{ in foot-tons.}$$

(*e*) Work done in stretching guns.

In the chamber it is per square foot of surface

$$\frac{6 f_1^2 \rho}{\mathrm{E}} \cdot \frac{m^2 + 1}{m^2 - 1}$$

$$f_1 = 18 \text{ tons} \quad \rho = 5 \text{ inches},$$

and if $\mathrm{R} = 20$ inches

$\rho = 7$ in the chamber

$$\frac{m^2 + 1}{m^2 - 1} = \frac{\mathrm{R}^2 + \rho^2}{\mathrm{R}^2 - \rho^2} = \frac{449}{351} = 1 \cdot 28,$$

$$\mathrm{E} = 13{,}000 \text{ tons},$$

and the surface of the chamber $= \dfrac{54 \times 43 \cdot 98}{144} = 16 \cdot 5$ square feet.

Therefore the work done

$$= \frac{6 \times 18^2 \times 5 \times 1 \cdot 28 \times 16 \cdot 5}{13{,}000} = 15 \cdot 8 \text{ foot-tons.}$$

For the rest of the chase.

To be accurate, the length of the chase should be divided into sections, but as the only term which is affected by the difference in thickness is $\frac{m^2 + 1}{m^2 - 1}$, and as this is a comparatively small factor and does not vary much, it will be sufficient to take a mean value of the outer radius, which, accordingly, will be taken at 10 inches, and $\rho = 5$,

therefore $$\frac{m^2 + 1}{m^2 - 1} = \frac{R^2 + \rho^2}{R^2 - \rho^2} = \frac{125^3}{75} = 1 \cdot 333.$$

Now the expression for the work done is

$$\frac{\pi}{E} \cdot \frac{m^2 + 1}{m^2 - 1} \rho^2 P_1^2 \int \left(\frac{0 \cdot 43}{\frac{v_1}{v_0} - 0 \cdot 57} \right)^{2 \cdot 474} d x,$$

and as was found before the part under the integral becomes $\left(\frac{1}{0 \cdot 0219 x + 1} \right)^{2 \cdot 474} d x$, of which the integral is

$$- \frac{1}{1 \cdot 474 \times 0 \cdot 0219 (0 \cdot 0219 x + 1)^{1 \cdot 474}},$$

and taking this between $x = L$ and $x = o$

$$30 \cdot 98 - \frac{30 \cdot 98}{(0 \cdot 0219 L + 1)^{1 \cdot 474}}, \text{ and since } L = 25 \cdot 5$$

$$= 30 \cdot 98 - \frac{30 \cdot 98}{1 \cdot 923} = 30 \cdot 98 - 16 \cdot 11 = 14 \cdot 87.$$

Therefore work done $$= \frac{3 \cdot 1416}{13{,}000} \times 1 \cdot 333 \times 25 \times 18^2 \times 14 \cdot 87$$

$$= 42 \cdot 04 \text{ foot-tons.}$$

Stretching the guns between breech and trunnions.

The expression for this is

$$\frac{\pi (R^2 - \rho^2) l P_1^2}{E (m^2 - 1)^2},$$

which since $$(m^2 - 1) = \frac{R^2 - \rho^2}{\rho^2} \text{ becomes}$$

$$\frac{\pi \rho^4 l P_1^2}{E (R^2 - \rho^2)}$$

$$= \frac{3 \cdot 1416 \times 625 \times 25 \cdot 5 \times 18^2}{13{,}000 \times 75} = 16 \cdot 64 \text{ foot-tons.}$$

Determination of Δ V.

This is made up of

(*a*) *Vis viva* of projectile $= \frac{W}{g} u^2$.

$$\therefore \text{Work done} = \frac{W\, u^2}{2\, g\, 2{,}240} =$$

$$= \frac{500}{64 \cdot 4 \times 2{,}240} u^2 = 0 \cdot 003465\, u^2 \text{ foot-tons.}$$

(*b*) *Vis viva* of gun and carriage. The weight of the gun is 27 tons, and if the resisting part of the carriage be taken at one-third the weight of the gun, or 9 tons, then $W_1 = 36$ tons. Therefore

$$\text{Work done} = \frac{W_1}{2\, g} \cdot u_1{}^2 = \frac{36}{64 \cdot 4} u_1{}^2 = 0 \cdot 559\, u_1{}^2 \text{ foot-tons.}$$

(*c*) *Vis viva* of the gases, or

$$\int u_{\prime\prime}{}^2\, \delta\, \mu.$$

The value of this is

$$\frac{W}{3\, g} (u^2 + u_1{}^2 - u\, u_1).$$

Therefore work done in foot-tons

$$\frac{300}{6\, g \times 2{,}240} (u^2 + u_1{}^2 - u\, u_1) = 0 \cdot 0006931\, (u^2 + u_1{}^2 - u\, u_1) \text{ foot-tons.}$$

Determination of $\int R\, d\, x$ the resistance of the air due to the velocity.

Now $d = 10$ inches, $W = 500$ lbs.

To find a from Table II. of the Reports on Experiments made with the Bashforth Chronograph, above referred to, it will be seen that the resistance to a 10-inch ogival-headed projectile at 1,000 feet per second is 233 lbs., and as the resistance is as the cube of the velocity the resistance at velocity $V = 233 \left(\frac{V}{1{,}000}\right)^3$

$$= 0 \cdot 000000233\, V^3 \qquad \therefore a = 0 \cdot 000000233,$$

but the velocity is approximately $\doteq 744\ S^{\frac{1}{3}}$ = therefore the resistance $= (744)^3 \times 0 \cdot 000000233 \int S\, \delta\, s = 95 \cdot 95 \int S\, \delta\, s$, which when $S = L = 47 \cdot 97\, L^2$, and when $L = 25 \cdot 5$ the above gives $13 \cdot 89$ foot-tons.

Therefore

$J \Delta Q$ or equivalent of heat absorbed . .		1,850	foot-tons.
ΔI „ internal work = 0.			
$A W$ (*a*) Expulsion of air		13·89	„
(*b*) Rotation		$0 \cdot 00002\, u^2$	„
(*c*) Friction of gas-check, &c.. . . .		286·9	„
(*d*) „ of gases . . .		$0 \cdot 00002765\, u^2$	„
(*e*) Stretching guns—			
In chamber . . .	15·80		
Chase	42·04		
Longitudinally . .	16·04		
		73·84	„
ΔV on projectile		$0 \cdot 003465\, u^2$	„
„ „ gun and carriage		$0 \cdot 559\, u_1^2$	„
„ „ gases . .		$0 \cdot 0006931\, (u^2 + u_1^2 - u\, u_1)$	„
$\int R\, d\, s$ Resistance of air		13·89	„

Now all this work is done by the gases, and should be equivalent to $J \Delta H$, the equivalent of the heat expended, and which has been already found = 19,513 foot-tons.
Therefore

$$19{,}513 = 1{,}850 + 13 \cdot 89 + 0 \cdot 00002\, u^2 + 286 \cdot 9 + 0 \cdot 00002765\, u^2 + 73 \cdot 84 + 0 \cdot 003465\, u^2 + 0 \cdot 559\, u_1^2 + 0 \cdot 000693\, (u^2 + u_1^2 - u\, u) + 13 \cdot 89\,;$$

or

$$17{,}275 = 0 \cdot 0042056\, u^2 + 0 \cdot 559693\, u_1^2 - 0 \cdot 000693\, u\, u_1,$$

but as shown above

$$u_1\, u_1 = W\, u + \frac{w}{2}\, (u - u_1),$$

from which

$$u_1 = \frac{W + \frac{w}{2}}{W_1 + \frac{w}{2}} \cdot u$$

and

$$W = 500, \qquad w = 300$$

$$W_1 = 36 \times 2{,}240, \text{ therefore}$$

$$u_1 = \frac{650}{80{,}790} \cdot u = 0 \cdot 008046\, u$$

substituting this value in the above equation

$$17{,}275 = 0 \cdot 0042056\, u^2 + 0 \cdot 0000362\, u^2 - 0 \cdot 000005576\, u^2$$
$$= 0 \cdot 0042362\, u^2$$
$$\therefore u = 2{,}020 \text{ feet per second,}$$

which is the muzzle velocity,

and u_1 = velocity of recoil $= 0 \cdot 008026 \times 2{,}020 = 16 \cdot 25 f s.$

and

Work done for rotation	$= 0 \cdot 00002\, u^2$	$= 81 \cdot 62$	foot-tons;
„ friction of gases	$= 0 \cdot 00002765\, u^2$	$= 112 \cdot 78$	„
„ on projectile	$= 0 \cdot 003465\, u^2$	$= 14{,}107$	„
„ on gun and carriage	$= 0 \cdot 559\, u_1^2$	$= 147 \cdot 61$	„
„ on gases	$= (0 \cdot 0006931)\,(u^2 + u_1^2 - u\, u_1)$	$= 2{,}828$	„

Summary of work done—

On projectile	14,107	foot-tons;
On gases	2,828	„
On gun and carriage	148	„
On rotation	82	„
Friction of gas-check	287	„
Expulsion of air	14	„
Friction of gases	113	„
Resistance of air to shot	14	„
Stretching of gun	74	„
Equivalent of heating gun	1,850	„
Total	19,517	„
Total J Δ H =	19,513	„
Difference	4	„

The total heat developed per kilogram of powder, according to Noble and Abel is

$$721 \cdot 400 \text{ French units}$$
$$= 1{,}298 \cdot 4 \text{ English units per lb.}$$

Therefore the equivalent work of 300 lbs.

$$= \frac{1{,}298 \cdot 4 \times 300 \times 772}{2{,}240} = 134{,}260 \text{ foot-tons}$$

of which there is expended

In the gun	19,517	foot-tons.
Loss	114,743	„

The amount actually utilized in the shot is 14,107 foot-tons.

The whole of the above loss is in the residual power of the gases as they escape into the atmosphere at an absolute temperature of 3,652° Fahrenheit.

The loss to be accounted for is 114,743 foot-tons. Now the equivalent work remaining in the gases if reduced down to 542° Fahrenheit (absolute) = 51° of Fahrenheit

$$= \frac{300 \times 0\cdot3381 \times 772 \times (3,652^\circ - 542^\circ)}{2,240} = 108,850 \text{ foot-tons,}$$

leaving a remainder of 5,893 foot-tons still in the gases.

The velocity of the projectile, as determined by the above calculation, is 2,020 *f.s*, and the energy 14,107 foot-tons.

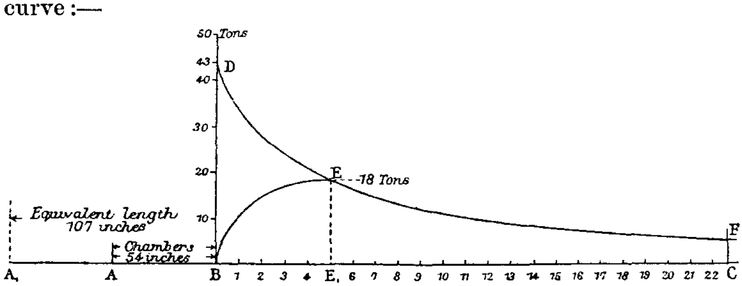
It is stated by Mr. Anderson in a Lecture delivered before the Society of Arts,[1] that the velocity was 2,100 *f.s*, but it is not said whether this was the observed or only the estimated velocity.[2]

Supposing it to be the former, it gives 15,280 foot-tons for the energy of the projectile, which exceeds the calculated energy by 1,173 foot-tons.

It is very possible that a good deal of this difference may be attributable to an overestimate of the amount of heat communicated to the gun, and which has been estimated above at the equivalent of 1,850 foot-tons.

A few carefully-conducted experiments would throw much light on this subject. Enough, however, has been done in this Paper to show that the actual muzzle-velocity may be very approximately estimated without reference to the actual pressure of the pressure-curves.

The following figure illustrates the method usually adopted by the Author, for estimating the muzzle-velocity from the pressure-curve :—



[1] "Journal of the Society of Arts." Vol. xxxiii., p. 727.

[2] The observed velocity is somewhat greater than the real muzzle velocity, because the gases do not cease their action on the projectile immediately on its quitting the muzzle.

A B represents the length of the powder-chamber.

A_1 B its equivalent length, if of the same diameter as the bore.

B C represents the chase outside the chamber.

In the present case the charge of 300 lbs. at 27·7 cubic inches to the lb. would just fill the chamber, or its equivalent length.

D E F represents Noble and Abel's curve calculated from the formula

$$p = p_o \left(\frac{v_0 (\iota - a)}{v - a v_0} \right)^{\frac{C p + \beta a}{C p_1 + \beta a}}$$

which is numerically

$$p = p_o \left(\frac{0 \cdot 43}{\frac{v}{v_0} - 0 \cdot 57} \right)^{1 \cdot 0748}.$$

E is the point in this curve where the pressure is 18 tons per square inch, the observed maximum pressure which is attained when the projectile has reached the corresponding point E.

After this point the work on the projectile is represented by the area E_1 E F E_1.

Previous to this the projectile has been acted on by an increasing pressure, which would be represented by a curve rising vertically from B and terminating horizontally at E. The exact form of this curve is at present unknown, but the Author has assumed it to be elliptical, and he therefore adds the area of the quarter ellipse B E E to the area E_1 E F E, and this sum he takes to represent the work done on the shot and the gases per square inch of the bore.

In the present instance this area is 231, which, multiplied by 78·54, the area of the other gives 18,142 foot-tons.

Now, by the preceding calculation, the energy accounted for was found to be:—

	Foot-Tons.
On the projectile for velocity	14,107
,, for rotation	82
,, friction	287
	14,476
Expulsion of air	14
Resistance of air	14
Giving velocity to the products	2,828
Friction of ditto	113
Recoil velocity	148
Total	17,693
Energy from curve	18,143
Difference	450

Which may probably be due to a slight escape of the gases before the gas-check comes fully into action, or to a slight amount of windage from its not exactly sealing the bore.

The difference is, however, so small that it confirms the Author in his method of estimating the velocity from the pressure-curve, deducting from the area of this curve about 22 per cent. for work done in giving rotation, overcoming friction, giving velocity to the gases, &c., &c.

The difficulty, however, in applying this method is, that it involves the prior knowledge of the maximum powder-pressure, which at present there is no *à priori* method of determining.

One advantage is claimed for the method in the preceding calculation by Count de St. Robert, who observes that it seems to eliminate all consideration of the mode of combustion of the charge.

He says, "Principes de Thermodynamique," p. 252—Whatever be the mode of combustion in the guns, whether it burns instantaneously or successively, the two temperatures t_o and t are always the same. The first depends on the composition of the powder, and is determined by the chemical reaction which takes place whilst it passes to the gaseous state. The second depends only on the ratio of the space occupied by the gases whilst they have the temperature t_o to the space they occupy when they are expanded in the chase, when the projectile leaves the muzzle, and on the atmospheric pressure—quantities which remain invariable.

The Author is not prepared to admit the correctness of this remark without limitation, as it seems contrary to the thermodynamic law that "any thermal machine which works between given limits of temperature gives the maximum useful effect when all the heat is received at the highest temperature and rejected at the lowest."[1]

It is evident from an inspection of the curve given above that the effect of the powder must increase as the point E_1 approaches to B—that is to say, as pressure accumulates more rapidly behind the projectile, or as the powder burns quicker.

The Author, therefore, sees no reason to alter the views he has so often expressed about slow-burning powder.

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[1] The error in Count de St. Robert's remark seems to be that he makes t_o and t both invariable. But this is not consistent with the fact that $t = t_o \left(\dfrac{0 \cdot 43}{\dfrac{v}{v_o} - 0 \cdot 57}\right)^{0 \cdot 074}$, because v_o is the volume when the temperature is t_o, and this volume is less as the rate of burning of the powder is greater.

He proposes on a future occasion to treat at some length on this subject, it being foreign to the purpose of the present Paper.

What he thinks he has established in this communication is that it is quite possible to estimate very approximately the ballistic effect of a gun from purely thermodynamic principles.

The Paper is accompanied by a diagram, from which the figure in the text has been engraved.