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R. Shida M.E.

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Does not the "rotatory coefficient" of resistance completely express the important facts discovered by Mr. Hall? Instead of expressing these facts by saying that there is a direct action of a magnetic field on a steady current as distinguished from the body conducting the current, may we not with equal convenience express them by saying that the effect of a magnetic field on a conductor is to change its coefficients of resistance in such wise that the electromotive force is no longer a *self-conjugate*-linear-vector function of the current?

L. *On the Number of Electrostatic Units in the Electromagnetic Unit.* *By* R. Shida, *M.E., Imperial College of Engineering, Tokio, Japan**.

THE object of this paper is to explain measurements made during the month of July last for an evaluation of "v," the number of electrostatic units in the electromagnetic unit —a question which has much engaged the attention of the British Association. We can evaluate "v" by determining the electrostatic and also the electromagnetic measure of any one of the following terms—Electromotive Force, Resistance, Current, Quantity, and Capacity. It is the first of these terms that I measured in the two systems of units; and the E.M.F. was that of Sir William Thomson's gravity Daniell, which is very constant. The question divides itself into two parts:—

(A) *Absolute Electrostatic Measurement of the E.M.F.*

This measurement was made by means of Sir William Thomson's absolute electrometer. It is not easy to explain shortly how the electrostatic measurement is made by this instrument; but, briefly speaking, it is as follows:—Imagine a circular disk suspended by springs in a horizontal plane inside the aperture of another larger plate in the same plane, with a continuous plate below and parallel to them. The force of electrical attraction of the continuous plate on the disk is compared with the gravitating force of a known weight. To effect this, any electrical influence having been entirely removed, a known weight is put on the disk, which is then raised by means of a micrometer-screw until it comes to its original position; and then the weight is taken away, allowing electrical force to act when the continuous plate is adjusted by the aid of another micrometer-screw, to bring the disk to the same position as before. A full account of the instrument will be found in Sir William Thomson's Report on Electro-

* Communicated by Sir William Thomson, having been read in Section A of the Meeting of the British Association at Swansea.

meters (British-Association Report, 1867), and republished along with his other papers on Electrostatics and Magnetism.

In measuring an E.M.F. by this instrument, it is important that the potential of the jar or the guard-ring or disk should be kept constant during the experiment. It was observed, however, that the jar was losing its charge, though very slowly, on account of the pieces of ebonite in the replenisher insulating imperfectly. Of course I could keep the potential of the jar the same during the experiment by means of the replenisher; but I found it very difficult to work the replenisher and to take at the same time accurate readings. For this reason I thought it better, when the experiment is conducted by one experimenter (or, I venture to think, even when there are more experimenters than one), to proceed in the following manner:—First, connect one pole (say zinc) to the continuous plate, and the other pole to the outside of the jar, and take a reading; then reverse the poles and take another reading. Repeat the same operation; that is to say, take a great number of readings by successive reversals. If the experimenter be well practised, the time each reading will take him will be very nearly the same. Let D_1, D_2, D_3, &c. be the readings corresponding to zinc, and D_1', D_2', D_3', &c. be those corresponding to copper; then the difference of the two readings of zinc and copper would be the difference between the mean of any consecutive readings of one pole and the reading of the other taken between those two consecutive readings—such, for example, as $\frac{D_1+D_2}{2}-D_2'$, or $\frac{D_1'+D_2'}{2}-D_2$, &c. Thus we get many values very nearly the same, if not exactly the same, of the true difference in question. If therefore we take the mean of all these, the error due not only to a small loss of charge, but also to a little inaccuracy in the readings, will be avoided. This is the method I used in measuring the E.M.F. of 30 Daniell cells; and the result I obtained is the mean defined as above, $=13\cdot283$ divisions of the micrometer screw-head. As regards the mathematical calculation, we have

$$V-V'=2(D-D')\sqrt{\frac{F}{R_1^2+R_2^2}};$$

where $V-V'$ is the E.M.F. of the battery, $D-D'$ the difference of the distances corresponding to the readings of the two poles, F the attracting force of the continuous plate on the disk, R_1 the radius of the disk, and R_2 that of the aperture. Since, now, one division of the micrometer-screw-head corresponds to a distance of $\frac{5\cdot08}{10,000}$ centim., we get $V-V'=\cdot904187$ (C.G.S.)

The E.M.F. of Thomson's gravity Daniell was measured by comparing it before and after the above experiment directly with that of the above battery by means of Sir William Thomson's quadrant electrometer. The E.M.F., e, of the cell was

$$e=\frac{V-V'}{26\cdot299}=0\cdot034380 \text{ C.G.S. electrostatic units.}$$

(B) *Absolute Electromagnetic Measurement of the E.M.F.*

This measurement was made by determining the strength of the current given by the E.M.F. by means of a tangent-galvanometer, and then measuring the resistance of the circuit in the way to be described presently.

The tangent-galvanometer employed consists of a circular coil, of mean radius 18·2 centims., containing 400 turns, in 19 layers, of insulated copper wire, the breadth and the depth of the coil being 2 and 1·3 centims. respectively. The needle of the galvanometer consists of a magnet only about $\frac{1}{2}$ centim. long, made of hard-tempered steel wire, and suspended in the centre of the coil by a single silk fibre. To the needle is attached a very fine straight glass fibre, of such a length that its ends travel round a graduated dial of radius a little less than that of the coil, thus serving for taking readings.

The mathematical theory shows that, in a tangent-galvanometer,

$$c=\frac{H\sqrt{r_0^2+b^2}\tan\alpha}{2\pi n}\cdot\frac{3q^2r_0^2}{3q^2r_0^2\times d^2(q^2-1)}, \quad . \quad . \quad (1)$$

where c is the current-strength, H the horizontal component of earth-magnetism, α the angle of deflection, n the number of turns of wire in the coil, r_0 the mean radius of the coil, b half the breadth of the coil in the plane at right angles to the plane of the coil, d half the depth of the coil in its plane, and q the number of layers in the coil. If E be the E.M.F. producing the current c in a circuit of resistance R, then, by Ohm's law and from the preceding equation, we get

$$E=\frac{RH\sqrt{r_0^2+b^2}\tan\alpha}{2\pi n}\cdot\frac{3q^2r_0^2}{3q^2r_0^2+d^2(q^2-1)}. \quad . \quad . \quad (2)$$

The formula (2) shows that, in order to measure an E.M.F. in absolute electromagnetic units, we have to determine (a) the deflection α, (b) the resistance R, and (c) the horizontal component of earth-magnetism H.

(*a*) To determine α. The formula (2) also shows that, whatever be the value of R, the product R tan α is a constant quantity as long as E is kept constant; which furnishes this important suggestion—that by varying the resistance R we vary α, and thus get many values very nearly equal, if not equal, of the product R tan α, the mean of which would be the more accurate value of the product. The determination of α, therefore, was performed as follows:—The current from the gravity-cell was passed through the tangent-galvanometer g and a variable resistance r, and the deflection α was noted. The object of introducing the variable resistance is (1) to enable us to alter the resistance R, and (2) to obtain the deflection giving minimum error, which is 45°.

(*b*) To determine $R(=g+b+r)$. The resistance g of the galvanometer was measured by the Wheatstone's-bridge method, and was equal to 30·86 ohms. The resistance b of the battery was measured by measuring the deflections produced on the scale of Sir William Thomson's quadrant-electrometer by connecting the electrodes of the cell to those of the electrometer, first when the cell was unshunted, and secondly when it was shunted by a known resistance. The resistance b in this case is equal to the product of the difference of the two readings into the shunt, divided by the second reading. It was exactly equal to 2·02 ohms. The corresponding values of a, r, R, so obtained, were as follows:—

a.	r.	R.
45° 15′	80 ohms	107·88
42° 45′	100 „	112·88
51° 39′	50 „	82·88

$\therefore$ the mean value of R tan $\alpha = 104{\cdot}73 \times 10^9$.

It must, however, be remembered that in all these measurements the ohm, or B.A. unit of resistance, is assumed to be exactly 10^9 C.G.S. units; which is unfortunately doubtful, as was well remarked by Professor Adams, the President of this Section, in his address.

(*c*) To determine H. The method of determining this element consisted in (1) observing the period of vibration of a magnet under H, and (2) observing the deflection of a magnetometer placed in the magnetic meridian by the action of the magnet placed at a fixed distance in a line at right angles to the magnetic meridian and passing through the centre of the magnetometer. I made the experiment with two different magnets made out of very hard-tempered steel wire, about 0·97 centim. in diameter, and also experimented with each magnet by varying the distance of the magnet, and found the results to agree very closely with one another. The mean value of H obtained with one magnet is ·15955; and the mean value obtained with the other is ·15937; so that the mean of these two is

$$H = \cdot 15947.$$

The formula used in the calculation of H is

$$H = \frac{2\pi}{t(k-l)(k+l)}\sqrt{\frac{2ki}{\tan\phi}},$$

where t is the period of vibration of magnet under H, k the distance of the centre of the magnet from the magnetometer, l half the length of magnet, i the moment of inertia of the magnet, and ϕ the angle of deflection of the magnetometer.

We have now come to the evaluation of "v." The formula (2) gives

$$\epsilon = 1\cdot 01172 \times 10^8 \text{ (C.G.S.) electromagnetic units.}$$

Hence

$$v = 294\cdot 4 \times 10^8 \text{ centims. per second,}$$

which agrees well with the latest value obtained by Sir William Thomson, namely 293×10^8.

Although I took as much care as possible in making all the above measurements leading to this evaluation of "v," yet since, from want of time, it was only on one occasion that I was able to make the complete measurements, there may have been some cause or causes of error unnoticed. I intend therefore to repeat the whole experiment, and hope to be able to make a further communication.

In conclusion, I must say (and I say with extreme gratitude) that if the experiment be in any way satisfactory, it is chiefly due to the very able and kind instructions given me by Sir William Thomson and his assistants in carrying it out.

Addition, Nov. 18, 1880.—These experiments have, since the communication of the above paper to the British Association, been several times very carefully repeated, with in every case a confirmation of the close accuracy of the determination of the electrostatic value of electromotive force. In the electromagnetic determination, however, a correction has been made for the torsion of the single silk fibre by which the needle of the tangent-galvanometer was suspended. As it was supposed that the torsion of a single fibre of silk might be neglected, no correction was made in the first results; but the error due to this cause has in these later experiments been determined and allowed for. Five experiments were made, and the corresponding valnes of "*v*" calculated from the results. These values are as follows:—

v.
$299{\cdot}9 \times 10^8$
$300{\cdot}3 \times 10^8$
$299{\cdot}4 \times 10^8$
$298{\cdot}0 \times 10^8$
$299{\cdot}9 \times 10^8$

Mean value . . . $299{\cdot}5 \times 10^8$.

LI. *Note on the Relation between the Mechanical Equivalent of Heat and the Ohm.* *By* L. B. Fletcher, *Student in Physics, Johns Hopkins University**.

A SINGULAR error occurs in a paper, published in the Philosophical Magazine (April and May 1880), by Dr. C. R. A. Wright. After remarking that Joule's value of the mechanical equivalent of heat, derived from experiments on the heat generated by a measured current in a wire of known resistance, is probably, when corrected for the error in the resistance-estimation due to superheating of the wire, from 1·5 to 2 per cent. higher than Joule's water-friction value, Dr. Wright goes on to say (p. 264) that:—"This difference between the two values is precisely that which would subsist did an error to an equal amount exist in the B.A. resistance-unit valuation: *i. e.* if the B.A. unit were 1·015 to 1·020 earth-quadrant per second instead of being exactly 1 earth-quadrant per second, the value of J deduced from Joule's 1867 experiments would be 1·015 to 1·020 times the true value; for it is calculated by the formula

$$J = \frac{C^2Rt}{H},$$

where C is the current, R the resistance, *t* the time, and H the heat evolved." This statement is evidently incorrect; for if the ohm is really 1·02 earth-quadrant per second, and was assumed by Joule to be exactly 1·00 earth-quadrant per second, Joule's value for the resistance of his wire, and consequently his value for J obtained by this method, must be 2 per cent. too *small*.

* Communicated by the Author.