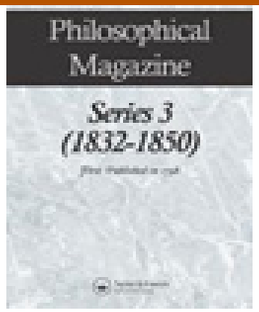

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LV. On cask-gaging

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escaping into an atmosphere of another gas, and escaping into one of its own kind. For this reason, Leslie's experiment cited in the notes of his Treatise on Heat, and Mr. Faraday's on the Escape of Gases through Capillary Tubes, (Quarterly Journal of Science, vol. iii.) present results differing from each other, from Mr. Graham's, and from the deductions of theory. It is, I think, not improbable that when gases are subjected to pass rapidly through a porous medium, as is the case when they escape into a vacuum, instead of being retarded by the opposing atmosphere of another gas, the angular irregularities of the channels of communication may present greater obstruction to those gases which are disposed to move with great velocity, than to those whose motion is not so rapid. When, on the other hand, the gases mutually diffuse into and retard each other, it becomes a question of time and not of velocity; and the supposed inequality of obstruction may greatly diminish if not entirely disappear.

This supposition has at least the advantage of reconciling, in some degree, Mr. Graham's experiments with the theoretical deductions. But it is to be hoped that future researches may throw further light on this interesting question.

Primrose, near Clitheroe, April 12, 1834.

LV. *On Cask-Gaging.* *By* J. W. Lubbock, *Esq.*, *V.P. and Treas. R.S.**

KEPLER was the first who endeavoured to reduce the art of gaging to accurate principles, in his work entitled, *Nova Stereometria Doliorum Vinariorum*, published in 1615. He gave in this work the solution of several new problems relative to the content of various solids, and he showed how the solution of others might be made to rest upon considerations more simple than those which had previously been employed. The art of gaging is one of such practical importance in all countries, but more especially in our own, in consequence of the immense duties annually levied by the Government upon various liquids, the quantity of which is ascertained by gaging separately the casks in which they are contained †, that I trust the following remarks will not be considered superfluous, although it is not in my power at the present time to do more than to show how the elementary principles of the mensuration of solids should be applied to the

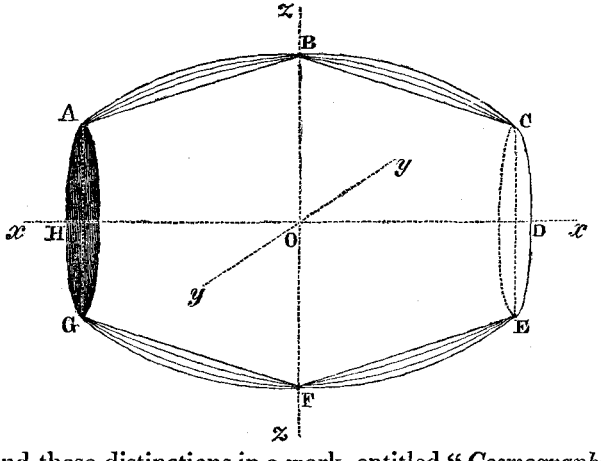
* Communicated by the Author.

† The number of casks gaged annually on the quays in this port alone, before they are deposited in store, amounts to more than 100,000. The duty on brandy is 22*s.* 6*d.* per gallon.

subject in question. I shall limit myself, in the following brief remarks, to the gaging of casks lying, not entering upon the question of the gaging of ships, or stills, or any other irregular figures, merely premising that this can only be done by quadratures, or, as it is termed in works on gaging, by the method of equidistant coordinates. A very material difference, however, exists between these problems and the one which I propose here to notice, and which consists in this circumstance, that in the one case extreme dispatch is a necessary requisite in the method employed; in the other, that is to say, in ascertaining the tonnage of ships, or the content of any fixed reservoirs, such as distillers' vats, &c., time not is so great an object; so that the peculiar difficulty of cask-gaging arises from the necessity of combining accuracy with celerity in the operation.

It has long been customary to divide casks into four varieties, which are thus defined:

1. The middle frustum of a spheroid.
2. The middle frustum of a parabolic spindle.
3. The frustums of two equal parabolic conoids.
4. The frustums of two equal cones.



I find these distinctions in a work, entitled "*Cosmographia*," by Dr. John Newton, published in 1679, and they appear to have continued ever since.

I place the origin of rectangular coordinates at the centre of the cask, and I suppose the cask to be a figure of revolution about the horizontal axis Ox, as in the diagram annexed. According to the preceding definitions, in the first variety the arc ABC is considered to be a portion of an ellipse of

which BO is the semiaxis minor, the axis major coinciding in direction with Ox.

In the second variety the arc ABC is considered to be a portion of a parabola whose vertex is B, and whose axis coincides in direction with BO.

In the third variety the arc BC is considered to be a portion of a parabola whose vertex is in the line Ox beyond D.

In the fourth variety BC is a straight line.

In works on gaging, the second and third varieties are supposed to be intermediate between the first and fourth, as in the figure, which is copied from Symons's Practical Gager, p. 187; but this is incorrect, for the second variety, *as thus defined*, coincides very nearly with the first, and the third with the fourth.

Let BF (the internal bung diameter) be called b.
CE (the internal head diameter) be called h.
HD (the internal length) be called l.

In the first variety the equation to the curves ABC, GFE, in the plane xz, is

$$z^2 = 4b^2 - \frac{b^2-h^2}{l^2}x^2.$$

In the second variety the equation to the curve ABC is

$$z = 2b - \frac{b-h}{2l^2}x^2.$$

In the third variety the equation to the curves BC, FE, is

$$z^2 = 4b^2 - \frac{2(b^2-h^2)}{l}x.$$

In the fourth variety the equation to the curve BC is

$$z = 2b - \frac{b-h}{l}x.$$

In the first variety the content of the cask is easily found by means of the triple integral $\iiint dx, dy, dz$ taken between the proper limits,

$$= \frac{\pi l}{12}\left\{2b^2 + h^2\right\} \qquad (\pi = 3{\cdot}14159).$$

In the second variety the content

$$= \frac{\pi l}{20}\left\{\frac{8}{3}b^2 + \frac{4}{3}bh + h^2\right\}.$$

In the third variety,

$$= \frac{\pi}{8}l\left\{b^2 + h^2\right\}.$$

In the fourth variety,

$$= \frac{\pi}{12}l\left\{b^2 + bh + h^2\right\}.$$

These are the rigorous expressions for the content of the cask, considered of either variety, and rules founded upon them are given in Symons's Practical Gager, p. 193.

The preceding expressions give the result in cubic inches, cubic feet, &c., accordingly as the linear dimensions of the cask b, h, l are expressed in one or the other denomination. The Imperial gallon contains 277·273 cubic inches of water at 62°; and *if we consider a gallon of any other fluid to have the same volume at the same temperature*, and not as a measure of weight, when the dimensions of the cask, as is usual, are expressed in inches, the expression for the content must be divided by 277·273, in order to have the content in Imperial gallons.

The readiest mode of obtaining *accurately* the content of the cask, considered either of the first or of the third variety, appears to me to be by employing the Tables* given by Symons, p. 210 and following, which consist, the first of values of $\frac{\pi h^3}{12 \times 277{\cdot}273}$ and the second of $\frac{\pi b^2}{6 \times 277{\cdot}273}$ for different values of h and b. The second and fourth varieties may be rejected altogether, as coinciding with the first and third. Thus: required the content of a cask of the first variety when $b = 31{\cdot}2$, $h = 26{\cdot}1$, and $l = 46{\cdot}9$. I find by the Tables

$$\frac{\pi h^3}{12 \times 277{\cdot}273} = {\cdot}6432$$

$$\frac{\pi b^3}{6 \times 277{\cdot}273} = 1{\cdot}8382$$

$$\text{sum} = 2{\cdot}4814$$

$2{\cdot}4814 \times 46{\cdot}9 = 116{\cdot}37$ = content of cask in Imperial gallons *considered as of the first variety.*

The content of the third variety may be found in the same way.

$$\left.\begin{aligned} \frac{\pi h^3}{12 \times 277{\cdot}273} &= {\cdot}6432 \\ \frac{\pi b^3}{12 \times 277{\cdot}273} &= {\cdot}9191 \end{aligned}\right\} \text{By the Table entitled "} \textit{Areas of the Head Diameter.}\text{"}$$

$$\text{sum} = 1{\cdot}5623$$

$$\frac{3l}{2} = 70{\cdot}35$$

$70{\cdot}35 \times 1{\cdot}5623 = 109{\cdot}91$ = content of the cask in Imperial gallons *considered as of the third variety.* The extreme difference in the content, due to the variety, obtained by con-

* These tables are *headed* inaccurately.

sidering the cask, with the same length, bung and head diameter, is thus in this instance 6·46 gallons, or about 5·5 per cent.

The term *ullage* appears to be used indifferently to denote either the liquor contained in a cask, or the vacuity above the liquor. It has been seen that accurate, that is to say, *rigorous* and extremely simple expressions can be obtained for the entire content of a cask of either variety; but it seems to me impossible to find expressions sufficiently simple to be used in practice for the ullage of a cask *lying upon its bulge*, except in the case of the first variety, and when the *dry inches* do not exceed the half difference of the bung and head diameters. This difficulty arises from the limits of the integral which has to be employed.

The cask considered as of the first variety is a portion of the spheroid, of which the equation is

$$l^2(z^2+y^2)+(b^2-h^2)x^2=4b^2l^2.$$

If this spheroid be intersected by a horizontal plane, the curve of intersection will be an ellipse, of which the semiaxes are $\frac{l\sqrt{4b^2-z^2}}{\sqrt{b^2-h^2}}$ and $\sqrt{4b^2-z^2}$:

the area of this ellipse $=\frac{\pi l(4b^2-z^2)}{\sqrt{b^2-h^2}}$.

This quantity multiplied by d z and integrated between the proper limits will give the ullage in the first variety; and I find that if d represent the *dry inches*, or the distance from the bung-hole at B to the surface of the liquor,

the ullage or vacuity $=\frac{\pi l d^2}{3\sqrt{b^2-h^2}}\left(\frac{3}{2}b-d\right)$,

when the dry inches do not exceed the half-difference of the bung and head diameters, that is, when the surface of the liquor is not beneath a straight line joining A and C. I understand that in practice this condition generally obtains, except where a cask has leaked, or where the liquor has made a long voyage, as in the case of *rums*. This formula might be useful if casks could be considered as belonging to this variety, but of course the difference of figure has a sensible effect upon the ullage.

In the other varieties, and in the first variety except in the particular case I have mentioned, the integral which has to be taken in order to obtain the ullage is extremely complicated; and I apprehend the simplest way to arrive at the ullage in all these cases is to construct Tables for various casks by quadratures, that is, to divide the liquor into parallel and thin slices, vertical and perpendicular to the axis of revolution, each of which may be considered as a small cylinder of which the base is a circular segment.

The rule which is given by Symons, p. 217, for finding the ullage of a cask *lying*, by the pen, is erroneous, and I apprehend of no utility.

In practice, the sliding-rule (or *head-rod*) is invariably used in gaging casks, and the content of the cask, considered generally of the first variety, is sought by adding to the head diameter a certain quantity, depending only on the difference of the bung and head diameter, and considering the sum so obtained as a mean diameter, and the content of the cask to be the same as that of a cylinder of the same altitude or length, with that quantity as the diameter of the base.

In the first variety,

$$\text{content} = \frac{\pi l}{12}(2b^2+h^2),$$

$$\text{if } 2b^2+h^2 = 3(h+q)^2$$

$$h+q = \sqrt{\frac{2b^2+h^2}{3}}; \quad \text{and since } b = h+b-h$$

$$= \sqrt{h^2+\frac{4h}{3}(b-h)+\frac{2(b-h)^2}{3}}$$

$$= h + \tfrac{2}{3}(b-h) + \frac{(b-h)^2}{9h} + \&c.$$

$$q = \tfrac{2}{3}(b-h) \text{ nearly.}$$

The error of the content obtained by this approximate method $= -\frac{\pi l}{18}\{b-h\}^2$.

This, however, is only to be considered as a rough approximation, and the quantity q cannot be made strictly to depend only upon $b-h$. If we take a cask of which the dimensions are $l = 46{\cdot}9$, $b = 31{\cdot}2$, $h = 26{\cdot}1$ (which are those of an average brandy piece,) the content of which is accurately 116·37 gallons, we find $q = \frac{2}{3}(5{\cdot}1) = 3{\cdot}4$, $h+q = 29{\cdot}5$, and the content is found by the approximate method above equal to 115·61 gallons. It is obvious that the error will be nearly constant for *the same kind of cask*, and hence either a correction may be obtained experimentally or by calculation to be added in all cases for each kind of cask, or this correction may be included in that due to the variety. The direction given by Symons is, "Look for the difference of the bung and head diameters on the line of inches on the edge of the rule, and whatsoever number stands opposite to it on the respective lines of varieties, add to the head diameter, and the respective sums will be mean diameters, or diameters of cylinders equal to the solidities of the cask when the altitudes of those cylinders and the lengths of the casks correspond." When this was written the four varieties were introduced on the sliding-rule; now only the *spheroid* is placed there, and I find opposite 5·1, 3·5 instead of 3·4. How this difference arises is difficult

to ascertain: the line has most likely been laid down for a particular value of h.

The value of q, for the third variety, is found in the same way to be approximately $\frac{1}{2}(b-h)$, and the error of the content would be $= -\frac{\pi l}{16}\{b-h\}^3$.

The sliding-rule or head-rod used in gaging affords the means of multiplying the square of the *mean diameter* by

$$\frac{\pi l}{4 \times 255{\cdot}273}.$$

For gaging *by the pen*, it would be convenient to have a table of the logarithms of the numbers in Mr. Symons's Table, entitled "Areas of the Head Diameters," instead of the numbers themselves. With these, and a table of the logarithms of numbers from 1 to 1000, to four decimal places (both of which might be contained on a single quarto page,) the content of casks might probably be calculated as readily as by means of the sliding-rule. It must be recollected that the principle upon which the line on the head-rod gives the quantity q to be added to the head-diameter is only a rough approximation, which may be out nearly half a gallon, and which stands in need of a subsequent correction.

The method of determining the ullage of a cask by the line on the head-rod marked S L y (Symons, p. 214,) is not strictly accurate *in principle*; but how far the error which is so introduced is sensible in practice, depends upon how far it is desirable that the approximation should be carried, and how far the principle itself is modified afterwards by any subsequent correction. The ullage cannot be considered strictly as dependent on (or as a function of) the *dry inches* and the content of the cask as it is by the method alluded to.

In practice an error in determining the content of a cask may arise either from the errors above noticed, which are introduced by calculating the content approximately by the sliding-rule, having given the dimensions and the *variety*, or from an error in ascertaining the dimensions, or from an erroneous judgement in deciding upon the *variety* to which the cask is to be referred.

An error of a tenth of an inch in all the linear dimensions of a cask will make a difference in a cask holding 116 gallons, of about a gallon, which is, perhaps, the maximum of this error. It must be recollected also, that the instruments used in ascertaining the external dimensions are of a nature to stand in need of occasional adjustment, and that the difficulty of ascertaining the *internal* dimensions is considerable, from the irregularity of the thickness of the staves. The rules and callipers used in gaging are liable to slight but not insensible

variations from heat and moisture, and should therefore be constantly compared with standard metallic scales provided for the purpose and kept in protected situations.

The extreme limits of the error due to the determination of the variety may of course be ascertained by calculating the content upon either extreme hypothesis. I find that the content of the cask for which $l = 46{\cdot}9$, $b = 31{\cdot}2$, $h = 26{\cdot}1$,

by the formula $\frac{\pi}{12} l(2b^2+h^2)$ is 116·37 gallons,

——————— $\frac{\pi}{8} l(b^2+h^2)$ is 109·91 gallons,

showing 6·46 gallons as the difference between the content obtained, according to one or the other hypothesis. This error may be greatly diminished by ascertaining *experimentally* the correction required for any particular cask, when one or the other hypothesis has been used, which proceeding is facilitated by the circumstance that casks coming from the same country generally maintain a separate and peculiar character, so that a particular correction might be easily deduced for a sherry butt, a port pipe, &c., without any reference to mathematical considerations. This would amount to the same as to deduct a certain quantity from the length of the cask, calculating the content as though it were accurately of the first variety. So if the brandy piece be, in fact, of the variety which I call the fourth, namely, that of which the content is given by the expression

$$\frac{\pi}{8} l(b^2+h^2),$$

it is sufficient to take $l = 44{\cdot}293$, that is, to deduct 2·607 from the length, and calculate the content by the expression

$$\frac{\pi}{12} l(2b^2+h^2),$$

which gives the same result.

It is in determining the *variety* of a cask, or the allowance to be deducted from the content previously obtained by considering the cask as of the first variety, that the skill and experience of the gager are principally required. Difference of opinion appears to exist as to the limits of the error which obtains in practice when a single cask is gaged. Mr. Archer, a gentleman particularly qualified by long experience to form an accurate opinion upon this point, has assured me that it generally amounts to 2 per cent., but that by extreme care it may be reduced to a gallon in a single cask, or to 1 per cent. nearly; on the other hand Mr. Nairn, the principal gager of the St. Katherine Docks, in a communication with which he has favoured me, considers the usual error at no more than the eighth of a gallon.

Were the practice of ascertaining the content by weighing introduced, of course all the sources of error which I have described would be got rid of, but it would be difficult to ascertain the *tare*, which might vary, probably, between 5 and 8 per cent.

In gaging wines it is not usual to make any allowance for temperature. The following Table, which was obtained by means of experiments made under my direction by Mr. Ladd, a workman in the employ of Mr. Bate, shows the specific gravity of each liquid, that of the liquid at 62° Fahrenheit being unity.

	Brandy.	Sherry.	Port.	
100	·9813	·9900	·9913	100
95	·9838	·9913	·9920	95
90	·9863	·9928	·9933	90
85	·9888	·9943	·9966	85
80	·9912	·9957	·9958	80
75	·9939	·9970	·9972	75
70	·9965	·9982	·9984	70
65	·9986	·9993	·9994	65
62	1·0000	1·0000	1·0000	62
60	1·0009	1·0005	1·0003	60
55	1·0034	1·0017	1·0012	55
50	1·0058	1·0028	1·0022	50
45	1·0078	1·0037	1·0031	45
40	1·0102	1·0046	1·0041	40

This Table gives the means of ascertaining the correction due to temperature, and which in extreme cases ought not to be neglected.

As regards the practice of gaging, I will venture to observe, that the methods employed by the officers of the Customs in ascertaining the content of casks should be definite, and not left, as at present, entirely to the arbitrary opinion of different individuals. It is usual for the Dock-gagers to gage the casks also, and their measures serve for the benefit of the merchant to check the measures or gages obtained by the officers of the Customs; but it is impossible for this check to operate effectually, unless the officers and the Dock-gagers proceed independently, and both upon recognised principles. It is difficult now, even to arrive at the method by which the usual allowances are made; nor are they, I believe, sufficiently described in any printed work. This arises, perhaps, from an unwillingness to make public the methods employed, for fear casks should, in consequence, be made purposely to defraud the revenue, called technically *game-casks*.

It seems to me, nevertheless, that uniform instructions, as far as they can be generally applicable, and leaving as little as possible to the judgement, that is, to the guess of the gager, should be carefully drawn up and published for general use, so as to be open to the examination of any one who may question their accuracy. Moreover, if the use of the sliding-rule be permitted, whether in the determination of the content or of the ullage of casks, it should be considered merely as a readier method of *working*; and it ought not to supersede the construction of accurate tables, approved by the Customs and Excise, of which the rule itself would contain the representation, and which might therefore be verified by any one without difficulty. Perhaps the best method would be to calculate the content as in p. 331, employing a mean diameter $= h + \frac{2}{3}(b-h)$, by means of a table of the logarithms of $\frac{\pi h^2}{4 \times 255{\cdot}273}$, or by the lines C and D of the sliding-rule, (See Symons, p. 192,) having previously determined experimentally for various kinds of cask a correction to be applied, in order to improve the content so obtained. Accurate tables should also be formed of the ullages of various casks, and in cases of difficulty the ullage might be gaged, the cask *standing*, which is obviously a much simpler problem.

At present it is not even known how some of the lines on the sliding-rule were originally laid down, or, I believe, for what cask they are intended. Lately an alteration was made in the line marked S L y, by which an obvious inaccuracy was removed; but this alteration, by which the ullage (vacuity) is diminished, operates disadvantageously to the merchant who pays duty on the liquor. In consequence, the *Wine and Spirit Committee* requested me to offer them an opinion as to the propriety of the change and the accuracy of the method used in determining the ullage. It is obviously impossible to give a complete and satisfactory reply to the latter question, except by means of direct experiments, while we are ignorant what function of the dimensions of the cask the line S L y is intended to represent, and without tables of the ullages of various casks with which to compare it.

No doubt when a general system of public instruction is introduced, with a greater tendency than exists at present to the cultivation of drawing, and of those arts which are most generally useful, the elements of the art of gaging will not be altogether overlooked.