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XXII. On the ionizing processes at a point discharging in air

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Conclusions.

When discharge occurs at a sharp point in air at atmospheric pressure, the current, dimensions, and other conditions being those considered in this paper, it is possible to calculate the strength of the field in the ionizing region at the surface of the point to within one or two per cent. for a positive and less accurately for a negative point in terms of the mechanical pull upon its surface; and this conclusion holds if the point be supplied with ions of opposite sign to itself from a second point in its neighbourhood.

XXII. *On the Ionizing Processes at a Point discharging in Air.* *By* A. P. Chattock, *Professor of Physics, and* A. M. Tyndall, *B.Sc., Lecturer in Physics, in the University of Bristol*[*].

[Plate IV.]

In explaining the phenomena of discharge at sharp points in gases under normal conditions, Sir J. J. Thomson postulates an initial ionization of a few isolated molecules in the gas as a preliminary to the process of discharge.

Suppose a point to be gradually charged with positive electricity in the presence of these isolated ions. The field near its surface is at first unable to do more than clear them away as fast as they are formed; but as soon as it is strong enough to impart to the positives among them sufficient energy to enable these to ionize fresh molecules in their turn, ordinary positive discharge sets in, and a large current may result, accompanied by glow at the point and wind.

In the case of a negative point the field has also to reach a high enough value to enable the initially formed positive ions to form fresh ions; but they now have the alternative of doing this where they bombard the surface of the metal instead of in the gas, and the field required is not necessarily so high as when gaseous molecules are to be ionized.

For both kinds of discharge the supply of positive ions is pictured as kept up by ionization due to negative ions, these having been produced by previously formed positive ions and so on. Both signs of ion have therefore to be able to ionize as each produces the other; and since positive ions require a stronger field for this than negative it is always the field required by the positive ions which has to occur at the point.

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[*] Communicated by the Authors.

In what follows $f+$ stands for the field at an electrified point in which positive ions are able to ionize, and $f-$ for the corresponding field for negative ions ; $f+$ having different values according as the positive ions produce others in the gas or at the metal surface.

Suppose now that to a charged point ions of opposite sign to itself are supplied in considerable quantities from some source in its neighbourhood. We may call such ions *external ions*.

With a positively charged point there are three distinct cases that may occur :—

(*a*) If the external ions find a field at the point which is less than $f-$ they will simply pass to the point and give rise to a current from it equal to that which they themselves carry.

(*b*) If the field lies between $f-$ and $f+$, each external ion will produce several more before reaching the point, and the current resulting may be a considerable multiple of that carried by the external ions.

(*c*) If the field exceeds $f+$ the double ionization by both positive and negative ions will accompany the ionization by the external ions, and a current due to ordinary positive point discharge will be added to that of the external ions.

With a negatively charged point these cases reduce themselves to *a* and *c*, the signs + and − being interchanged : *b* does not occur because $f+$ is greater than $f-$ and the external ions are positively charged.

If the appearance of light at the point is to be taken as indicating ionization there, it will follow that for *a* the point will be dark, while for *b* and *c* it will glow.

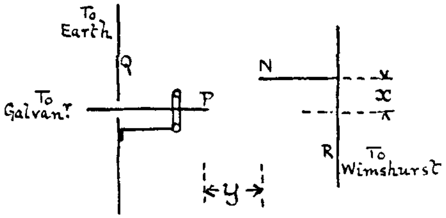
It is to be understood in the above, that the initially ionized molecules are too few in number to be taken account of in comparison with the external ions supplied.

These principles are illustrated in a general way by experiments on discharge between two points, made some years ago, which we have lately repeated and extended, and of which the following is an account :—

A horizontal platinum wire P (fig. 1) with its end rounded to a hemisphere in the blowpipe was suspended so that it protruded through a hole in a vertical metal plate Q.

P and Q were both earthed, Q directly and P through a galvanometer. Opposite P and in the same vertical plane was a sharp sewing-needle N connected to a wimshurst,

Fig. 1.



and so arranged that the vertical component x and the horizontal component y of its distance from P could be varied. The radius of P (0·031 cm.) was about 7 or 8 times greater than that of N, so that N discharged more readily than P; and the tendency of N to start first was further increased by surrounding P with a wire ring about 11 mm. in diameter, with its plane about 7 mm. behind the point P. By varying x and y it was thus possible to supply P with varying numbers of ions from N both before and after P itself began to discharge on its own account.

The end of P was viewed through a reading microscope, and the resultant pull, P, upon its surface was measured by tilting the whole apparatus so as to keep P always upon the cross-wire.

It has been shown* that under these conditions, if the current from the point is not greater than 15 microamperes, the disturbing effect of the discharge upon the pull of the field on the point is probably negligible compared with the pull itself, and that if r is the radius of the point, and f_0 the field at the centre of its surface due to the lines of force in the ionizing layer,

$$f_0 = \frac{\sqrt{P}}{r} \times \text{constant},$$

where the constant is 2·83 for positive, and approximately 3·07 for negative discharge.

In most of the earlier observations N was about 2 cm. long and projected from a flat plate R parallel to Q; y was kept at 1·6 cm. and x was varied. In the second set which was made with entirely new apparatus, y was varied, and

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* Chattock, Phil. Mag. pp. 272-274 of present number.

the two points were kept in line with one another, the radius of the point being the same as before (0·031 cm.).

In a third set points with radii lying between 0·062 and 0·004 cm. were used.

The results of the second set for positive and negative are plotted in Curves I. and II. respectively, as they were obtained for the widest range of conditions; but all three sets agree closely in their main features.

Positive Discharge from P.

Curves I. (Pl. IV.) apparently exhibit all the three cases, *a*, *b*, *c*, described on p. 278. Each curve is made up of a steep part S, and a nearly horizontal part H, joined by a curve. Somewhere in this curve or below it P began to glow, so that along S the discharge was dark and along H luminous. The exact position of the beginning of the glow was not easy to determine, as the light nearly always grew gradually from small beginnings, and though the observations were made in the dark it was extremely difficult to tell when it first became visible. In this respect the glow differed markedly from that at a negative point which started suddenly.

S and H correspond closely with cases *a* and *b* respectively. For besides the fact that the discharge in S is dark and in H luminous, the values of f_0 for H rise as they should do when the number of external ions is reduced by increasing y; and f_0 reaches its maximum value when N is removed altogether ($y=\infty$), H then corresponding with case *c*.

When $y=0·2$ cm. f_0 has been reduced about 2·3 times, the discharge being still apparently ordinary point discharge; but if the points are put 0·15 cm. apart streams of small sparks result *.

Provided the frequency of these sparks is not too great we may write

$$f_0=\mathrm{K}t,$$

where t is the time counted from the last spark and K is a constant. If P is the average pull on the point, we have

$$\frac{3·07}{r}\sqrt{P} = \text{apparent value of } f_0$$

$$=\frac{\text{maximum value of } f_0}{\sqrt{3}}.$$

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* It is possible by getting the discharge to start with y greater than 0·15, and then decreasing it to get point discharge at this distance also. This was done in the case of the readings discussed below.

The maximum value of f_0 just before each spark passed is given by the dotted line in Curves I, for $y=0{\cdot}15$ cm.

An interesting detail was noticed in connexion with the position of the glow on P in the first set of experiments. Here N was on one side of P and the glow always appeared on the side facing N, but became symmetrical when N was removed. This is consistent with what was said above, as ionization in the reduced fields of case *b* can only occur when the external ions approach the point. A negative point, on the other hand, only glows in case *c*, and it can then discharge whether external ions are arriving or not; this agrees with the fact that the position of the negative glow was far less dependent upon the position of N than that of the positive.

Negative Discharge from P.

Here, as with positive discharge, the curves consist of two distinct portions, S and H, corresponding respectively to dark and luminous discharge. Of these S, as before, represents case *a*, and if for a moment we neglect the curve for $y=1{\cdot}0$, H in every case appears to correspond with case *c*, as it should, the ordinates of the various H curves down to $y=1{\cdot}5$ being roughly the same as those for no discharge from N ($y=\infty$).

For $y=0{\cdot}5$ and $0{\cdot}3$ the curves were cut off short by the passage of sparks between N and P, but there seems no reason to suppose that if their H portions had been obtainable they would have differed in position from the others for normal point discharge.

In the case of $y=0{\cdot}3$ the curve is shown forking. The reading at the top of the lower branch was taken just before a single spark passed, while that for the upper branch corresponded with a stream of sparks. The ordinate of the latter was therefore multiplied by $\sqrt{3}$, as explained above, and this has brought it well among the rest of the H curves. The dotted lines correspond with the discharge of streams of sparks as before.

Between $y=0{\cdot}3$ and $y=0{\cdot}15$ there appears to be a fundamental change in the character of the discharge. The ordinates of $y=0{\cdot}15$ have been multiplied by $\sqrt{3}$, but this has not brought them anywhere near the top of $y=0{\cdot}3$.

It is possible that in the curve for $y=1{\cdot}0$ we have the transition stage connecting $y=0{\cdot}3$ and $y=0{\cdot}15$. In normal point discharge the glow is confined to the region near the point, but in this case ($y=1{\cdot}0$) it reached right across from N to P in the form of faintly luminous streamers, which occasionally passed into sparks for currents above about 12 microamperes. It is true that 1·0 does not lie between 0·3 and 0·15, but the current was evidently on the verge of sparking all along $y=1{\cdot}0$, and a very small change in the conditions was probably enough to cause streamers to pass into sparks or sparks into streamers for $y=1{\cdot}0$, 0·5, and 0·3. Streamers were, in fact, once or twice obtained for $y=0{\cdot}3$.

Values of Ionizing Fields.

There is no indication in Curves I., as the supply of external ions increases, of any limit to the lowering of the H portions beyond the accidental one of sparking.

This was at first sight disappointing; for according to the theory when the field at the point is below $f-$, it ought not to be possible to obtain luminous discharge, and it seemed unlikely that the positive and negative ions should require such very different fields to ionize in as corresponded with the highest and lowest H curves obtained.

Now, provided there are enough external ions present to produce a detectable amount of light, the beginnings of glow should occur when, and not before, the field reaches the value $f-$.

To test this we measured f_0 at the moment the glow first became visible for a wide range of distances between P and N. Under these conditions it was to be expected that the field in which the glow was first seen would be constant and equal to $f-$ so long as the supply of external ions was sufficient; but that when the supply fell short the glow produced in this field, though still present, would not be detectable, and the field corresponding to the first *visible* glow would therefore be higher than $f-$.

Further, the field would continue to rise as the external ions became fewer until it reached the value at which ordinary positive point discharge sets in.

The supply of external ions may be reduced by increasing either x or y. As, however, a sufficient increase of y made it impossible for our wimshurst to produce the highest field at P, we kept y constant at 1·5 cm. and varied x.

To limit the spreading of the ions from N this needle was made to project from the flat plate R (fig. 1) and x was altered by moving N and R together. In this way we were able to reach values of f_0 which were practically identical with those obtained when N was removed from R.

The resulting values of f_0 are given by the line marked Field in Curves III. (Pl. IV.), and are in good accord with this theory—rather surprisingly good accord considering the great difficulty of determining exactly when the glow started in the case of the observations at the lower fields. (In the high fields it began more suddenly and was brighter.) There is an obvious halting place at a field of 250 E.S. units in the falling of f_0 as x decreases.

The fact of the field being thus constant over a certain range of x values does not, however, prove that there is no glow below this particular field. The amount of light in the glow depends on the current arriving at the point and on the field there; and if the current happened to be constant for this range of x, a constant field might merely mean that until this field was reached the glow was too weak to be seen, and not that it was absent altogether.

To meet this objection we have plotted in Curves III. the current received by P at the moment the glow became visible. Starting with $x=0$ (N and P opposite one another), it will be seen how very far from constant this current is. It falls rapidly as x increases until the value of f_0 begins to change.

The subsequent rise and fall of the current curve at higher values of x is attributable to the fact that, when x is comparable with y, P receives most of the current on its sides. For as soon as the N ions are too few to give a detectable glow without a higher field f_0 and therefore the current from N increases, the result being that P receives a larger total current than before, though its end of course does not. At still higher values of x, f_0 becomes constant, and the same as for R without N; the current now falls off once more as it should, and the end of P presumably receives no ions at all.

Although, as already explained, if we alter y instead of x, we cannot trace the field curve up to the top, it is possible to obtain the horizontal part at $f-$.

In Curves IV. are plotted the results of experimenting in this way with the same two points. The field curve becomes horizontal at about 240, which agrees with the 250 just obtained for $f-$, and the current curve also shows the same sort of behaviour as the one in Curves III.

But the most interesting feature of Curves IV. is the way in which the field drops below the horizontal when N is brought nearer to P than about a centimetre. At this point the rise of the current, when y is decreased, becomes less marked, and when N is 0·7 cm. from P the rise changes to a fall. This also implies a falling field ; and it thus appears that at a distance from their origin of less than a centimetre the N ions possess the power of ionizing air in fields which steadily decrease as this distance decreases.

This is precisely what we should expect if the N ions take time to grow to their full size, and it is interesting to consider it in connexion with other facts bearing on the growth of ions.

Franck * has shown that when discharge occurs in air from the sides of a fine wire in a strong field, it is extremely probable that the ions, whether positive or negative, do not reach their full size while travelling a distance of 7 mm. We † have shown, by a different method of experimenting, that when discharge occurs from a fine point, both the positive and the negative ions probably travel about 3 mm. before they are fully formed—a result which is consistent with Franck's, when it is remembered that the average field in the 3 mm. was probably lower than in Franck's 7 mm., and the ions consequently travelled slower. With the relatively blunt point of the present experiments the distance should be greater, and we now find that the distance of growth in the case of the negative ions seems to have increased to a centimetre or so.

All these facts thus hang well together, as far as they go, and so afford support to the view that the negative ions do really take time to grow after leaving N.

The lower limit to the size of a negative ion is the corpuscle. If the second drop in the field curve is really due to a growth of the ions, the curve ought either to become horizontal again when the still lower field is reached in which corpuscles can ionize the air, or else to cut the vertical axis at this field. We have made a number of experiments on the starting of the glow in this critical region, the mean of the results being given in Curve V. It was found impossible to bring N nearer to P than 0·14 cm. on account of sparking, but down to this distance P could be made to glow in what seemed to be the normal manner. The numbers obtained were rather irregular, and it was only by making many observations that we were able to obtain so smooth a curve. We do not therefore wish to press conclusions drawn from them until we have studied this part of the field more carefully. As the curve stands, however, it certainly does show a tendency to cut the vertical axis at a field of about 75 E.S. units.

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* Franck, *Ann. der Physik*, Vierte Folge, Bd. xxi. p. 984.

† Chattock & Tyndall, Phil. Mag. [6] vol. xix. p. 449 (1910).

Reference was made in the preceding paper to the fact that the field in which ordinary positive point discharge occurs depends upon the curvature of the point; and that in consequence of this dependence the ionizing region probably extends a sufficient distance from the metal to feel, as it were, the divergence of the lines of force.

It is interesting to find that the fields in which the glow first appears in the presence of N ions are similarly dependent upon the point. This is shown by the following Table, in which are given the results of experiments upon four points of different sizes.

r.	f_1.	$f_1\,r^{0{\cdot}45}$.	f_2.	f_1/f_2.	f_3.	f_1/f_3.
0·0619	293	84	157	1·9	30 ?	10
0·0310	410	86	245	1·7	70 ?	6
0·0105	662	85	325	2·0	130 ?	5
0·0043	975	84	433	2·2	120 ?	8

r is the radius of the point in centimetres; f_1 the field in which ordinary positive point discharge is on the verge of stopping; f_2 the lowest field in which glow is caused by fully formed N ions; and f_3 the field in which corpuscles give rise to glow, if the views expressed above are correct. The values of f_3 are queried on account of the great uncertainty attending their determination.

The third column illustrates the exactness of the empirical relation between f_1 and r, and the rough constancy of the fifth and seventh columns shows that f_2 and f_3 also depend on r in a more or less similar manner.

It must be remembered that all these fields are rapidly divergent, and that their values are given at the surface of the metal. We do not yet know the values of the *weakest* fields in which the corresponding ionizing processes can occur because we do not know how far the ionizing regions extend from the point.

We found that there was a certain hysteresis in the appearance and disappearance of the glow, especially for small values of y, the current having to be raised considerably before the glow would start, after which it slowly worked back to a minimum. At this minimum the glow could be made to appear and disappear by slightly increasing or diminishing the current strength, and it was there that the field was measured in each case.

The hysteresis is perhaps connected with the fact that when P begins to discharge it sends + ions to N. These, by rendering the escape of corpuscles easier (see below), may increase the average ionizing power of the negative ions sent to P and so diminish the field necessary for glow, and therefore indirectly the current.

Ageing of the Point.

While external ions appear to exercise little influence upon negative discharge from a new point the case is different for an old one. It is well known that when a point has been used a good deal it "ages" for negative discharge by requiring, not only a higher field to keep a given current flowing from it, but a field which fluctuates widely; the ageing having apparently no effect upon positive discharge from the same point.

In the first set of experiments thirty curves were obtained altogether for positive and negative discharge with and without N, and by the end of the nineteenth the point showed signs of ageing. This appears from Curves VI. (Pl. IV.), where the unconnected dots and circles represent discharges from a negative point against a plate without N.

Those observations made before the nineteenth curve are marked by the dots, and if joined up by lines give curves that are more or less smooth; but the circles which mark the later observations give curves which zigzag up and down in the most irregular way if treated similarly. Instances of this irregularity are shown in Curves VII., where are plotted the twenty-first and twenty-eighth curves taken for negative discharge without N.

If, however, external ions are supplied to the point the irregularity vanishes. This is illustrated by the twenty-fourth curve, also plotted in Curves VII., which was taken with N at distances $x=1{\cdot}5$, $y=1{\cdot}6$ cm.

In these earlier observations, as in the later ones of Curves II., those for which external ions were supplied give curves which are in close agreement with one another. The mean position of the earlier of these curves is shown by the line A A in Curves VI., and may be said to follow approximately the line of dots. It is true that at small currents A A is appreciably above the dots, as it is above the lowest dots throughout its length, but if we allow for the fact that the abscissæ of A A are all too large by the currents carried on the N ions and shift A A to the left the discrepancy becomes less marked.

Roughly, then, it may be said that the effect of external ions upon negative discharge is to remove temporarily the two signs of ageing—high field and fluctuating field at the point; in other words, to render the old point new for the time being.

Ageing has been attributed to some change in the surface of the point, which makes it difficult for positive ions to knock corpuscles out of the metal.

Considering that a point discharging negative electricity produces quantities of positive ions in its neighbourhood, it is not easy at first sight to see why the arrival of a relatively small number of external positives should facilitate the escape of the corpuscles so much. For the only obvious difference between these two sets of ions is that those produced at the point are newly formed, while the externals are old —and this ought to render the externals less able to set free corpuscles instead of more.

An explanation is perhaps to be found in the following theory.

The negative discharge starts in a very small spot upon the point surface, the glow standing out in the form of a luminous trumpet to a distance comparable with the diameter of the point, in a manner suggestive of a rush of corpuscles escaping through some weak spot in the surface of the metal.

When the point ages a very characteristic feature of the discharge is observable. It is often impossible to get small currents to flow steadily. With a new point the current can be made to sink gradually to nothing as the wimshurst is slowed down; but with an aged point it sinks gradually to some low finite value and then stops dead, just as if it had been suddenly switched off.

In the light of this fact let us test the following hypothesis; whatever the nature of the ageing change may be, let its effect be such that the metal refuses to yield up corpuscles under bombardment by positive ions *unless the number of these ions is considerable.*

The hypothesis is consistent with the switching off effect just referred to.

It explains, what appears to be the case from Curves VI. and VII., that ageing has less effect on the field at large currents than at small ones.

It explains the fact that discharge will not start as a rule from an aged point until its electrification is far in excess of what is required when the point is new. For this starting of discharge depends on the presence of initially formed positive ions and if, as these are very few in number, they fail to obtain corpuscles from the metal, their only alternative will be to ionize the gas, which of course means a higher field.

Lastly it explains the effect of external ions on an aged point. When discharge starts in the manner just described by ionizing the gas, the region on the point at which it takes place will be determined by geometrical conditions alone, and will therefore have no particular connexion with the place where corpuscles come out most easily. It will, in fact, tend to be the place where the point has been most used, and therefore where they come out with greatest difficulty, so that even when the current is well started and the supply of positive ions sufficient to obtain corpuscles from the point we may still expect an abnormally high field there.

Now allow the initially formed ions to be reinforced by supplies of external ions sufficiently large to knock out the corpuscles freely. It will no longer be necessary to raise the field to that required for ionizing the gas before discharge will start, as the conditions for ordinary negative discharge will obtain. But whereas when discharge was started by initially formed ions alone it tended to occur at an aged place on the point, it now starts at the place where corpuscles come out easiest, since a large area of the point surface is bombarded by the external ions, and the unaged spots upon it are therefore sure to be discovered. The point should consequently behave like a new one, and this, as the experiments show, is precisely what happens.

Our somewhat arbitrary assumption, that a small supply of positive ions is prevented by the ageing change at a point from bombarding corpuscles out of it, while a large supply is not, thus seems to fit the facts fairly well.

In time of course a point ought to become aged all over if persistently supplied with external ions. We do not know whether this happens or not, but it is possible that the beginnings of the process are to be seen in those curves of the first set which were taken for negative discharge with

N present. Nos. 9, 10, 13 and 14 agree with one another to within about 0·5 per cent., and nos. 24 and 25, the only others available, are practically coincident with one another, but 24 and 25 are about 3 per cent. higher than the four earlier ones.

Relation between the Fields for Positive and Negative Discharge.

The field at the outer surface of the ionizing layer at a positive point is the minimum in which positive ions can ionize. At a negative point the field at the surface of the metal is that required by positive ions to knock out corpuscles; and if from any cause they are unable to do this there is still the ionizing of the gas itself open to them. It follows that the ionizing field at the surface of a negative point can never be quite as great as that at the surface of a positive point if the positive ions produced at each are the same.

In Curves VI., the line B B represents the field-current curve for positive discharge against a plate only, and it will be seen that the majority of the negative points are well above it.

As explained in the preceding paper, the absolute values of the negative fields are not so accurately known as those of the positives, but it is unlikely that this will account for so large a discrepancy as the one in question.

We are inclined to explain it as follows:—The negative glow stands out a long way from the point. A considerable proportion of the positive ions formed in it have consequently some distance to travel before reaching the point, and will have grown beyond their initial size when they arrive. We shall thus probably be dealing with older positive ions on the average in negative than in positive point discharge, and the occurrence of the stronger fields at the negative than at the positive point is thus reasonable.

It is consistent with this that the field at a negative point becomes less, relatively to that at the same point positive, as the sharpness of the point increases*; for at sharp points the glow does not stand out so far, and as the ions thus have a shorter distance to go, and also move faster in approaching the point, they will be newer when they get there.

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* Chattock, Phil. Mag. [5] vol. xxxii. p. 285 (1891).

Summary.

1. A supply of negative ions from without to a positively electrified point lowers the ionizing field at its surface.

2. Positive ions supplied to a negative point are without effect when the point is new.

These two facts are shown to be consistent with accepted theory.

3. A negative point may become aged with use, but temporarily acquires the properties of a new one when bombarded with positive ions.

4. The minimum ionizing field for fully formed negative ions is about half, and that for corpuscles about one-seventh of the field in which ordinary positive point discharge takes place. In each case the field is measured at the surface of the metal.

XXIII. *The Flow of Energy in an Interference Field. By* MAX MASON, *Ph.D., Professor of Mathematical Physics, University of Wisconsin*[*].

THE following investigation may answer some of the questions recently raised by Professor R. W. Wood[†], regarding the lines of energy flow in a field produced by two similar light sources.

The discussion will be limited, for simplicity, to the following case: Two points A_1, A_2 are centres of electromagnetic radiation, produced by the isochronous vibration of equal point charges. The direction of vibration will be taken at right angles to the line A_1A_2. Those lines of energy flow will be studied which lie in the plane containing the line A_1A_2 and perpendicular to the direction of vibration

§ 1. *The differential equation of the lines of mean energy flow.*

Let r_1 and r_2 be the distances from A_1 and A_2 to the point P (fig. 1). The electric and magnetic vectors at P due to the radiation from A_1 and from A_2 will be denoted by $\mathbf{E}_1$, $\mathbf{H}_1$; $\mathbf{E}_2$, $\mathbf{H}_2$. The vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are of unit length and have the directions from A_1 to P and from A_2 to P respectively; $\mathbf{j}$ is a unit vector in the direction of vibration.

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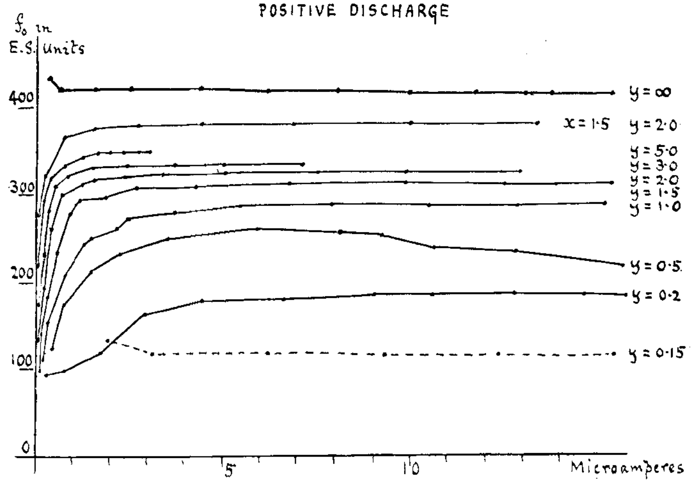
[*] Communicated by the Author.

[†] Phil. Mag. 1909, xviii. p. 250.

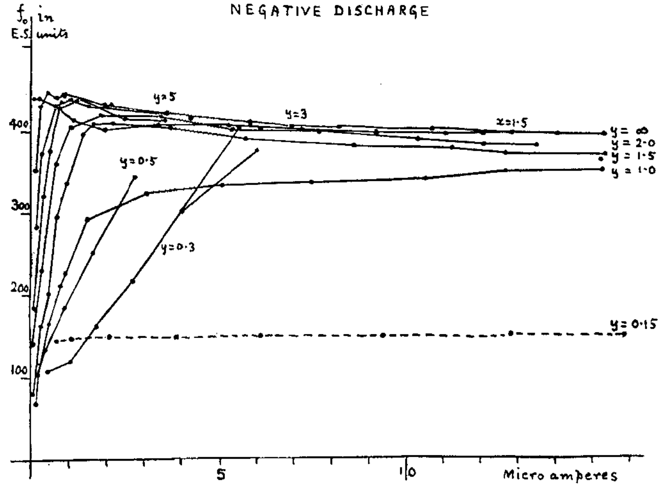
Curves I.

POSITIVE DISCHARGE

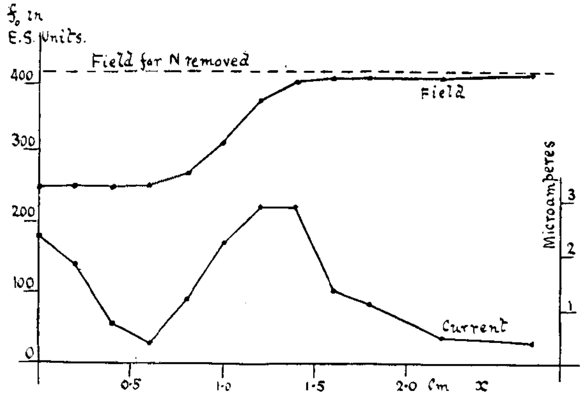


Curves II.

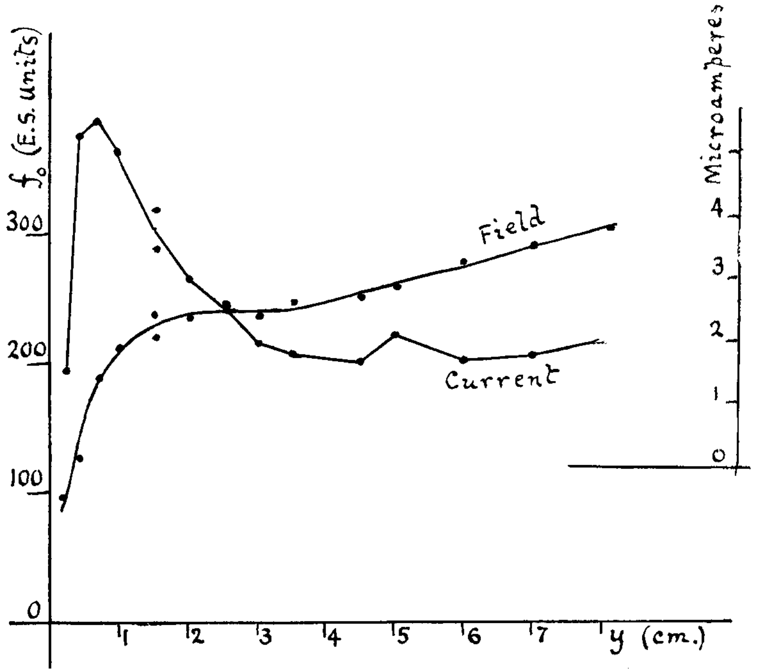
NEGATIVE DISCHARGE



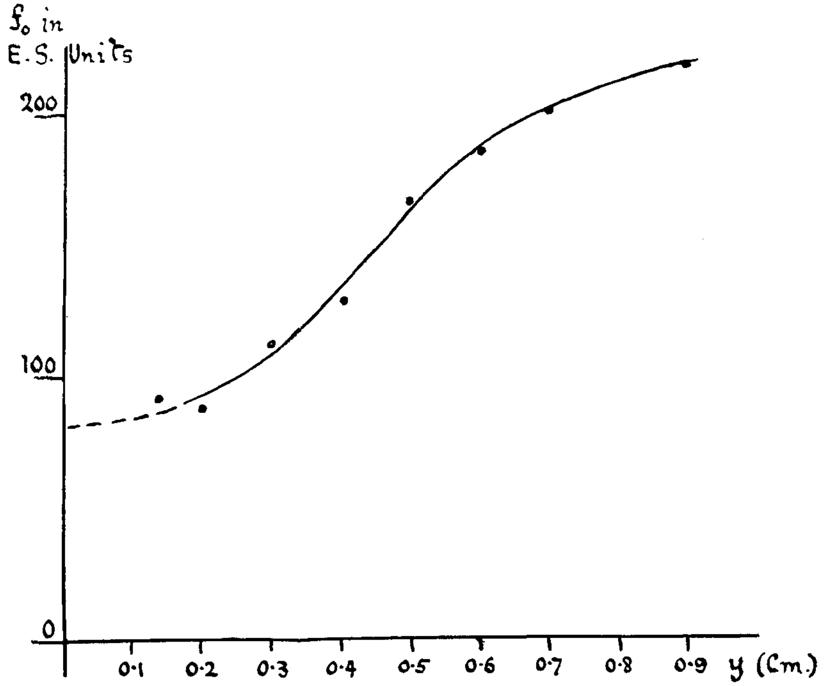
Curves III.



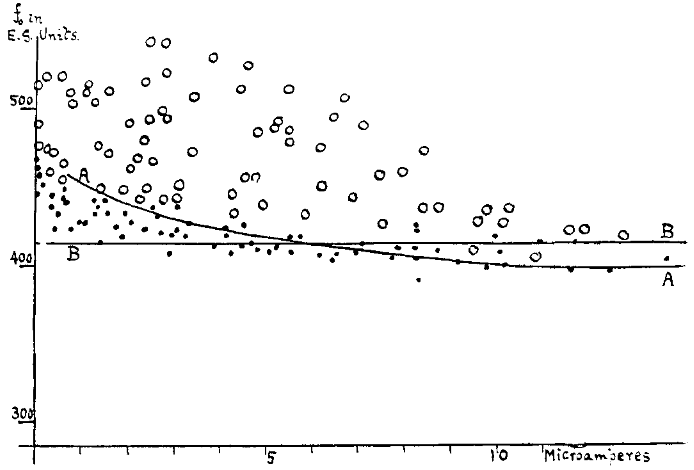
Curves IV.



Curves V.



Curves VI.



Curves VII.