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XXXIII. On the minimum current audible in the telephone

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alcohol an increase of ·968 dyne per centimetre. Correcting the former result for dissociation, the result is a rise corresponding to ·953 dyne for a non-dissociated substance in water; and the number ·968 for solution in alcohol would undergo but a slight diminution on account of the very small amount of dissociation in alcoholic solution.

Thus it appears that the rise, after this correction has been applied, is almost the same for these two solvents, as it ought to be in accordance with the fourth of the theoretical deductions stated above.

XXXIII. *On the Minimum Current audible in the Telephone. By* Lord RAYLEIGH, *Sec.R.S.**

THE estimates which have been put forward of the minimum current perceptible in the Bell telephone vary largely. Mr. Preece gives 6×10^{-13} ampere †; Prof. Tait, for a current reversed 500 times per second, 2×10^{-12} ampere ‡. De la Rue gives 1×10^{-8} ampere, and the same figure is recorded by Brough § as applicable to the *strongest* current with which the instrument is worked. Various methods, more or less worthy of confidence, have been employed, but the only experimenter who has described his procedure with detail sufficient to allow of criticism is Prof. Ferraris ‖, whose results may be thus expressed:—

Pitch.	Frequency.	Minimum current in amperes.
Do_3	264	23×10^{-9}
Fa_3	352	17×10^{-9}
La_3	440	10×10^{-9}
Do_4	528	7×10^{-9}
Re_4	594	5×10^{-9}

The currents were from a make-and-break apparatus, and in each case are reckoned as if only the first periodic term of

* Communicated by the Author, having been read at the Oxford Meeting of the British Association.

† Brit. Assoc. Report, Manchester, 1887, p. 611.

‡ Edin. Proc. vol. ix. p. 551 (1878). Prof. Tait speaks of a *billion* B.A. units, and, as he kindly informs me, a billion here means 10^{12}.

§ Proceedings of the Asiatic Society of Bengal, 1877, p. 255.

‖ *Atti della R. Accad. d. Sci. di Torino*, vol. xiii. p. 1024 (1877).

the Fourier series representative of the actual current were effective. On this account the quantities in the third column should probably be increased, for the presence of overtones could hardly fail to favour audibility.

Although a considerable margin must be allowed for varying pitch, varying acuteness of audition, and varying construction of the instruments, it is scarcely possible to suppose that all the results above mentioned can be correct, even in the roughest sense. The question is of considerable interest in connexion with the theory of the telephone. For it appears that *a priori* calculations of the possible efficiency of the instrument are difficult to reconcile with numbers such as those of Tait and of Preece, at least without attributing to the ear a degree of sensitiveness to aerial vibration far surpassing even the marvellous estimates that have hitherto been given *.

Under these circumstances it appeared to be desirable to undertake fresh observations, in which regard should be paid to various sources of error that may have escaped attention in the earlier days of telephony. The importance of defining the resistance of the instruments and of employing pure tones of various pitch need not be insisted upon.

As regards resistance, a low-resistance telephone, although suitable in certain cases, must not be expected to show the same sensitiveness to current as an instrument of higher resistance. If we suppose that the total space available for the windings is given, and that the proportion of it occupied by the copper is also given, a simple relation obtains between the resistance and the minimum current. For if γ be the current, n be the number of convolutions, and r the resistance, we have, as in the theory of galvanometers, $n\gamma=$const., $n^{-2}r=$const., so that $\gamma\sqrt{r}=$const., or the minimum current is inversely *as the square root of the resistance*.

The telephones employed in the experiments about to be narrated were two, of which one (T_1) is a very efficient instrument of 70-ohms resistance. The other (T_2), of less finished workmanship, was rewound in the laboratory with comparatively thick wire. The interior diameter of the windings is 9 millim., and the exterior diameter is 26 millim. The width of the groove, or the axial dimension of the coil, is 8 millim., the number of windings is 160, and the resistance is 8 ohm. Since the dimensions of the coils are about the

* Proc. Roy. Soc. vol. xxvi. p. 248 (1877). Also Wien, Wied. *Ann.* vol. xxxvi. p. 834 (1889).

same in the two cases, we should expect, according to the above law, that about 10 times as much current would be required in T_2 as in T_1. Both instruments are of the Bell (unipolar) type, and comparison with other specimens shows that there is nothing exceptional in their sensibility.

In view of the immense discrepancies above recorded, it is evident that what is required is not so much accuracy of measurement as assured soundness in method. It appeared to me that electromotive forces of the necessary harmonic type would be best secured by the employment of a revolving magnet in the proximity of an inductor-coil of known construction. The electromotive force thus generated operates in a circuit of known resistance; and, if the self-induction can be neglected, the calculation of the current presents no difficulty. The sound as heard in the telephone may be reduced to the required point either by varying the distance (B) between the magnet and the inductor, or by increasing the resistance (R) of the circuit. In fact both these quantities may be varied; and the agreement of results obtained with widely different values of R constitutes an effective test of the legitimacy of neglecting self-induction. When R is too much reduced, the time-constant of the circuit becomes comparable with the period of vibration, and the current is no longer increased in proportion to the reduction of R. This complication is most likely to occur when the pitch is high.

In order to keep as clear as possible of the complication due to self-induction, I employed in the earlier experiments a resistance-coil of 100,000 ohms, constructed as usual of wire doubled upon itself. But it soon appeared that in avoiding Scylla I had fallen upon Charybdis. The first suspicion of something wrong arose from the observation that the sound was nearly as loud when the 100,000 ohms was included as when a 10,000-ohm coil was substituted for it. The first explanation that suggested itself was that the sound was being conveyed mechanically instead of electrically, as is indeed quite possible under certain conditions of experiment. But a careful observation of the effect of breaking the continuity of the leads, one at a time, proved that the propagation was really electrical. Subsequent inquiry showed that the anomaly was due to a condenser, or leyden, like action of the doubled wire of the 100,000-ohm coil. When the junction at the middle was unsoldered, so as to interrupt the metallic continuity, the sounds heard in the telephone were nearly as loud as before. In this condition the resistance should have been enormous, and was in fact about 12 megohms * as indicated by a galvanometer. It was evident that the coil was acting principally as a leyden rather than as a resistance, and that any calculation founded upon results obtained with it would be entirely fallacious.

It is easy to form an estimate of the point at which the complication due to capacity would begin to manifest itself. Consider the case of a simple resistance R in parallel with a leyden of capacity C, and let the currents in the two branches be x and y respectively. If V be the difference of potential at the common terminals, proportional to e^{ipt}, we have

$$x = \mathrm{V}/\mathrm{R}, \qquad y = \mathrm{C}\, d\mathrm{V}/dt = ip\mathrm{VC}\ ;$$

so that

$$\frac{x+y}{\mathrm{V}} = \frac{1+ip\mathrm{RC}}{\mathrm{R}}.$$

The amplitude of the total current is increased by the leyden in the ratio $\sqrt{(1+p^2\mathrm{R}^2\mathrm{C}^2)}\ :\ 1$; and the action of the leyden becomes important when $p\mathrm{RC}=1$. With a frequency of 640, $p=4020$; so that, if $\mathrm{R}=10^{14}$ C.G.S., the critical value of C is $\frac{1}{402}\times 10^{-15}$ C.G.S., or about $\frac{1}{400}$ of a microfarad.

It will be seen that even if the capacity remained unaltered, a reduction of resistance in the ratio say of 10 to 1 would greatly diminish the complication due to condenser-like action; but perhaps the best evidence that the results obtained are not prejudiced in this manner is afforded by the experiments in which the principal resistance was a column of plumbago.

The revolving magnet was of clock-spring, about $2\frac{1}{2}$ centim. long, and so bent as to be driven directly, windmill fashion, from an organ bellows. It was mounted transversely upon a portion of a sewing-needle, the terminals of which were carried in slight indentations at the ends of a U-shaped piece of brass. As fitted to the wind-trunk the axis of rotation was horizontal.

The inductor-coil, with its plane horizontal, was situated so that its centre was vertically below that of the magnet at distance B. Thus, if A be the mean radius of the coil, n the number of convolutions, the galvanometer-constant G of the coil at the place occupied by the magnet is given by

$$\mathrm{G} = \frac{2\pi n\mathrm{A}^2}{\mathrm{C}^3}, \quad \ldots\ldots \quad (1)$$

* Doubtless the insulation between the wires should have been much higher.

where $C^2=A^2+B^2$; and if m be the magnetic moment of the magnet, and ϕ the angle of rotation, the mutual potential M may be represented by *

$$M=Gm\sin\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the frequency of revolution be $p/2\pi$, $\phi=pt$; and then

$$dM/dt=Gmp\cos pt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The expression (3) represents the electromotive force operative in the circuit. If the inductance can be neglected, the corresponding current is obtained on division of (3) by R, the total resistance of the circuit.

The moment m is deduced by observation of the deflexion of a magnetometer-needle from the position which it assumes under the operation of the earth's horizontal force H. If the magnet be situated to the east at distance r, and be itself directed east and west, the angular deflexion θ from equilibrium is given by

$$\tan\theta=\frac{2m/r^3}{H}.$$

The relation between the angle θ and the double deflexion d in scale-divisions, obtained on *reversal* of m, is approximately $\theta=d/4D$, where D is the distance between mirror and scale; so that we may take

$$m=\frac{Hr^3d}{8D}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The amplitude of the oscillatory current, generated under these conditions, is accordingly

$$\frac{n\pi pHA^2r^3d}{4C^3RD}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If C.G.S. units are employed, $H=\cdot18$. A must of course be measured in centimetres; but any units that are convenient may be used for r and C, and for d and D. The current will then be given in terms of the C.G.S. unit, which is equal to 10 amperes.

The inductor-coil used in most of the experiments is wound upon an ebonite ring, and is the one that was employed as the "suspended coil" in the determination of the electro-chemical equivalent of silver †. The number of convolutions (n) is 242.

* Maxwell, 'Electricity and Magnetism,' vol. ii. § 700.
† Phil. Trans. part ii. 1884, p. 421.

The axial dimension of the section is 1·4 centim., and the radial dimension is ·97 centim. The mean radius A is 10·25 centim., and the resistance is about $10\frac{1}{2}$ ohms.

In making the observations the current from the inductor-coil was led to a distant part of the house by leads of doubled wire, and was there connected to the telephone and resistances. Among the latter was a plumbago resistance on Prof. F. J. Smith's plan * of about 84,000 ohms ; but in most of the experiments a resistance-box going up to 10,000 ohms was employed, with the advantage of allowing the adjustment of sound to be made by the observer at the telephone. The attempt to hit off the least possible sound was found to be very fatiguing and unsatisfactory ; and in all the results here recorded the sounds were adjusted so as to be *easily* audible after attention for a few seconds. Experiment showed that the resistances could then be *doubled* without losing the sound, although perhaps it would not be caught at once by an unprepared ear. But it must not be supposed that the observation admits of precision, at least without greater precautions than could well be taken. Much depends upon the state of the ear as regards fatigue, and upon freedom from external disturbance.

The pitch was determined before and after an observation by removing the added resistance and comparing the loud sound then heard with a harmonium. The *octave* thus estimated might be a little uncertain. It was verified by listening to the beats of the sound from the telephone and from a nearly unisonant tuning-fork, both sounds being nearly pure tones.

When the magnet was driven at full speed the frequency was found to be 307, and at this pitch a series of observations was made with various values of C and of R. Thus when B=7·75 inches, or C=8·7 inches, the resistance from the box required to produce the standard sound in telephone T_1 was 8000 ohms, so that $R=8100\times10^9$. The quantities required for the calculation of (5) are as follows :—

$$n=242, \qquad p=2\pi\times307, \qquad H=\cdot18,$$
$$A=10\cdot25, \qquad r=8\cdot25, \qquad d=140,$$
$$C=8\cdot7, \qquad R=81\times10^{11}, \qquad D=1370,$$

r and C being reckoned in inches, d and D in scale-divisions of about $\frac{1}{40}$ inch. From these data the current required to

* Phil. Mag. vol. xxxv. p. 210 (1893).

produce the standard sound is found to be $7{\cdot}4\times10^{-8}$ C.G.S., or $7{\cdot}4\times10^{-7}$ amperes, for telephone T_1.

The results obtained by the method of the revolving magnet are collected into the accompanying table. The "wooden coil" is of smaller dimensions than the "ebonite coil," the mean radius being only 3·5 centim. The number of convolutions is 370.

Frequency = 307. Ebonite coil.

Telephone.	R in ohms.	Current in amperes.	Sound.
T_1	84100 Plumbago.	$3{\cdot}8\times10^{-7}$	Below standard.
T_1 ...	8100 Box.	$7{\cdot}4\times10^{-7}$	Standard.
T_1	4100 Box.	$5{\cdot}2\times10^{-7}$	......
T_2	500 Box.	$1{\cdot}2\times10^{-5}$	......
T_2	200 Box.	$1{\cdot}0\times10^{-5}$	......

Frequency = 307. Wooden coil.

T_1	84100 Plumbago.	$3{\cdot}6\times10^{-7}$	Standard.
T_1	10100 Box.	$3{\cdot}7\times10^{-7}$	......
T_1	1600 Box.	$5{\cdot}4\times10^{-7}$	......
T_2	350 Box.	$1{\cdot}1\times10^{-5}$	......

Frequency = 192. Ebonite coil.

T_1	3100 Box.	$2{\cdot}5\times10^{-6}$	Standard.

The method of the revolving magnet seemed to be quite satisfactory so far as it went, but it was desirable to extend the determinations to frequencies higher than could well be reached in this manner. For this purpose recourse was had to magnetized tuning-forks, vibrating with known amplitudes. If, for the moment, we suppose the magnetic poles to be concentrated at the extremities of the prongs, a vibrating-fork may be regarded as a simple magnet, fixed in position and direction, but of moment proportional to the instantaneous distance between the poles. Thus, if the magnetic axis pass perpendicularly through the centre of the mean plane of the inductor-coil, the situation is very similar to that obtaining in the case of the revolving magnet. The angle ϕ in (2) is no longer variable, but such that $\sin\phi=1$ throughout. On the other hand m varies harmonically. If l be the mean distance between the poles, 2β the extreme arc from rest to rest traversed by each pole during the vibration, m_0 the mean magnetic moment,

$$m/m_0=1+2\beta/l\,.\sin pt,$$

and

$$dM/dt = Gm_0p \,.\, 2\beta/l \,.\, \cos pt. \quad . \quad . \quad . \quad (6)$$

The formula corresponding to (5) is thus derived from it by simple introduction of the factor $2\beta/l$.

The forks were excited by bowing, and the observation of amplitude was effected by comparison with a finely divided scale under a magnifying-glass. It was convenient to observe the extreme end of a prong where the motion is greatest, but the double amplitude thus measured must be distinguished from 2β. In order to allow for the distance between the resultant poles and the extremities of the prongs, the measured amplitude was reduced in the ratio of 2 to 3. The observation of the magnetic moment at the magnetometer is not embarrassed by the diffusion of the free polarity.

In order to explain the determination more completely, I will give full details of an observation with a fork c' of frequency 256. The distance l between the middles of the prongs was ·875 inch, and the double amplitude of the vibration at the end of one of the prongs was ·09 inch. Thus 2β is reckoned as ·06 inch. The inductor-coil was the ebonite coil already described, and the sound was judged to be of the standard distinctness when, for example, B=15 inches, or C=15·5 inches, and the added resistance was 1000 ohms, so that $R=1100\times10^9$. The quantities required for the computation of (5) as extended are

$$n=242, \qquad p=2\pi\times256, \qquad H=\cdot18,$$
$$A=10\cdot25, \qquad r=15, \qquad d=410,$$
$$C=15\cdot5, \qquad R=11\times10^{11}, \qquad D=1370,$$
$$2\beta=\cdot06, \qquad l=\cdot875\,;$$

and they give for the current corresponding to the standard sound $9\cdot8\times10^{-8}$ C.G.S., or $9\cdot8\times10^{-7}$ amperes.

A summary of the results obtained with forks of pitch c, c', e', g', c'', e'', g'' is annexed. As the pitch rose, the difficulties of observation increased, both on account of the less duration of the sound and of the smaller amplitudes available for measurement. In one observation with telephone T_2 at pitch c'', the resistance, estimated at 11 ohms, was that of the coil, telephone, and leads only. No trustworthy result was to be expected under such conditions, but the number is included in order to show how small was the influence of self-induction, even where it had every opportunity

Telephone.	R in ohms.		Current in amperes.	
		$c = 128.$		
T_1	1100	....	$2{\cdot}8 \times 10^{-5}$	
		$c' = 256.$		
T_1	8100	Box.	$6{\cdot}8 \times 10^{-7}$	
T_1	1100	....	$9{\cdot}8 \times 10^{-7}$	
T_2	500	....	......	$1{\cdot}1 \times 10^{-5}$
		$e' = 320.$		
T_1	84000	Plumb.	$3{\cdot}8 \times 10^{-7}$	
T_1	6100	Box.	$2{\cdot}6 \times 10^{-7}$	
T_1	1600	....	$3{\cdot}1 \times 10^{-7}$	
		$g' = 384.$		
T_1	84000	Plumb.	$1{\cdot}4 \times 10^{-7}$	
T_1	9500	Box.	$1{\cdot}6 \times 10^{-7}$	
T_1	2100	....	$1{\cdot}4 \times 10^{-7}$	
T_1	900	....	$1{\cdot}7 \times 10^{-7}$	
T_2	600	....	......	$1{\cdot}9 \times 10^{-6}$
T_2	300	....	......	$2{\cdot}2 \times 10^{-6}$
		$c'' = 512.$		
T_1	84000	Plumb.	$8{\cdot}9 \times 10^{-8}$	
T_1	9000	Box.	$4{\cdot}8 \times 10^{-8}$	
T_1	3600	....	$5{\cdot}2 \times 10^{-8}$	
T_1	700	....	$8{\cdot}2 \times 10^{-8}$	
T_2	11?		......	$5{\cdot}2 \times 10^{-6}$?
T_2	100	Box.	......	$1{\cdot}9 \times 10^{-6}$
T_2	300	....	......	$1{\cdot}4 \times 10^{-6}$
T_2	500	....	......	$2{\cdot}5 \times 10^{-6}$
T_2	900	....	......	$2{\cdot}4 \times 10^{-6}$
		$e'' = 640.$		
T_1	84000	Plumb.	$3{\cdot}8 \times 10^{-8}$	
T_1	5100	Box.	$3{\cdot}8 \times 10^{-8}$	
T_1	1100	....	$5{\cdot}5 \times 10^{-8}$	
		$g'' = 768.$		
T_1	84000	Plumb.	$1{\cdot}1 \times 10^{-7}$	
T_1	7100	Box.	${\cdot}9 \times 10^{-7}$	
T_1	2100	....	$1{\cdot}1 \times 10^{-7}$	

of manifesting itself. If we bring together the numbers* derived with the revolving magnet and with the forks, we obtain in the case of T_1:—

Pitch.	Source.	Current in 10^{-8} amperes.
128	Fork..............	2800
192	Revolving magnet ..	250
256	Fork..............	83
307	Revolving magnet ..	49
320	Fork..............	32
384	....	15
512	....	7
640	....	4·4
768	....	10

It would appear that the maximum sensitiveness to current occurs in the region of frequency 640; but observations at still higher frequencies would be needed to establish this conclusion beyond doubt. Attention must be paid to the fact that the sounds were not the least that could be heard, and that before a comparison is made with the numbers given by other experimenters there should be a division by 2, if not by 3. But this consideration does not fully explain the difference between the above table and that of Ferraris already quoted, from which it appears that in his experiments a current of 5×10^{-9} amperes was audible.

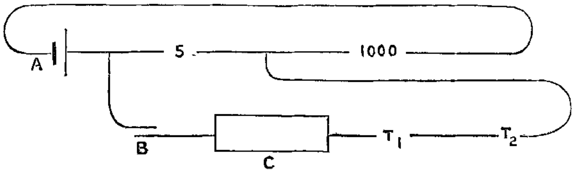
It is interesting to note that the sensitiveness of the telephone to periodic currents is of the same order as that of the galvanometer of equal resistance to steady currents †, viz. that the currents (at pitch 512) just audible in the telephone would, on commutation, be just easily visible by a deflexion in the latter instrument. But there is probably more room for further refinements in the galvanometer than in the telephone.

If we compare the performances of the two telephones T_1 and T_2, we find ratios of sensitiveness to current ranging from 13 to 30; so that T_2 shows itself inferior in a degree beyond what may be accounted for by the resistances. It is singular that an experiment of another kind led to the opposite conclusion. The circuit of a Daniell cell A was permanently closed through resistance-coils of 5 ohms and of 1000 ohms. The two telephones in series with one another and with a resistance-box C were placed in a derived circuit where was

* The observations recorded were made with my own ears. Mr. Gordon obtained very similar numbers when he took my place.

† See, for example, Ayrton, Mather, and Sumpner, Phil. Mag. vol. xxx. p. 90, 1890, "On Galvanometers."

also a scraping contact-apparatus B, as indicated in the figure. The adjustment was made by varying the resistance in C until the sound was just easily audible in the telephone under trial. Experiments conducted upon this plan showed that T_1 was only about five times as sensitive to current as T_2. It was



noticed, however, that the sounds, though as equal as could be estimated, were not of the same quality, and in this probably lies the explanation of the discrepancy between the two methods of experimenting. In the latter the original sound is composite, and the telephone selects the most favourable elements—that is, those nearly in agreement with the natural pitch of its own plate. In this way the loudness of the selected sound becomes a question of the freedom of vibration of the plate, an element which is almost without influence when the sound is of pitch far removed from that of the proper tone of the telephone. There was independent reason for the suspicion that T_1 had not so well defined a proper pitch as was met with in the case of some other telephones.

P.S.—Measurements with the electro-dynamometer have been made by Cross and Page* of the currents used in practical telephony. The experiments were varied by the employment of several transmitters, and various vowel sounds were investigated. The currents found were of the order 2×10^{-4} amperes.

XXXIV. *An Attempt at a Quantitative Theory of the Telephone.* *By* Lord Rayleigh, *Sec. R.S.*†

THE theory of the telephone cannot be said to be understood, in any but the most general manner, until it is possible to estimate from the data of construction what its sensitiveness should be, at least so far as to connect the magnitude of the vibratory current with the resulting condensations and rarefactions in the external ear-passage.

* Electrical Review, Nov. 14, 1885. I owe this reference to Mr. Swinburne.

† Communicated by the Author, having been read at the Oxford Meeting of the British Association.