

special fluids as the cerebro-spinal fluid, the aqueous humor and the fluids of the serous cavities, as well as the general fluid of the less specialized tissue-spaces.

The study of the lymphatic system throws emphasis on the importance of tissue-spaces. I am convinced that the understanding of lymphatic capillaries as definite structures, definitely placed in restricted areas, forms a secure basis from which the varied problems of absorption may be solved.

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### STATISTICAL PHYSICS<sup>1</sup>

EVERY physical measurement must be made in a region in equilibrium,<sup>2</sup> and nearly all of the correlations which have been established in physics, that is, nearly all physical laws, relate to substances in steady states or to substances in equilibrium. Furthermore, nearly all physical laws are one-to-one correspondences, and they are expressible as analytical functions. Thus the pressure of a given amount of a gas is an analytical function of the volume and temperature of the gas.

In every field of measurement, however, extreme refinement and care lead an investigator into a region of erratic action. This is evident when we consider that refined measurements are always subject to erratic error, and the atomic theory of the constitution of matter suggests that erratic action is always present everywhere, even in substances in complete thermal equilibrium.

<sup>1</sup> The substance of a lecture delivered by W. S. Franklin before the Department of Terrestrial Magnetism of the Carnegie Institution, Washington, D. C., December 20, 1915.

<sup>2</sup> Thermal equilibrium is here referred to; certain quasi states of thermal equilibrium being included. The only exception is the kind of measurement which consists of simple counting, like the counting of cattle as they pass through a gate or the counting of electrons as they enter an ionization chamber.

It has long been the custom to speak of the probable error of a precise measurement *as if perfect precision would be possible if our measuring devices were perfect and free from erratic variations*. It is important, however, to recognize two distinct types of erratic error, namely, *extrinsic error* due to uncontrollable variability of the measuring device or system, and *intrinsic error* due to inherent variability of the thing or system which is being measured. Every physical measurement involves an operation of congruence, a standard of some kind is fitted to or made congruent with successive parts (which parts are thereby judged to be equal parts) of the thing or system which is being measured; and the standard system and the measured system are both subject to erratic variations.

There is, perhaps, no case in which intrinsic error and extrinsic error can be clearly distinguished and separated from each other; but when the errors of one kind are much larger than the errors of the other kind they can, of course, be recognized. It is proper to speak of the *probable error* of a single measurement when the variations of the measuring device or system are dominant, but one should speak of the *probable departure* of the measured system from a certain mean condition at any time when the "errors" of observation are due chiefly to variability of the thing or system which is being measured. Thus in measuring the coefficient of sliding friction extrinsic error may be made negligible by making the measurements carefully, but very large "errors" persist. The thing which is being measured is inherently indefinite, and it may at any time depart widely from its average value.<sup>3</sup> In measuring the loss of

<sup>3</sup> A very brief but comprehensive statement of the proper precision method for the study of an erratic thing like friction is given by W. S. Franklin, *Transactions of American Institute of Electrical Engineers*, Vol. 20, pp. 285-286.

head in a water or gas pipe systematic errors (due to the particular details of roughness, etc., in the pipe) are not in evidence when a particular pipe is used, and extrinsic errors may be made negligible by using a precision device for measuring pressure; but the loss of head (or pressure) remains nevertheless extremely variable on account of eddy action which grows out of unstable vortex sheets; that is to say, very large "errors" persist, the thing which is being measured is inherently subject to erratic variation.

#### DESCRIPTIVE SCIENCE AND STATISTICAL SCIENCE

The greater part of physical science as applied in the arts and as used by the investigator is essentially descriptive. Thus we may wish to determine how the members of a bridge stretch or shorten as a car passes across the bridge; how electromotive force, current strength and all the changing variables play in the operation of a dynamo; how the pressure and temperature of the steam vary during the successive stages of admission, expansion and exhaust of a steam engine; and so on. But everything that takes place in this world has associated with it a substratum of complex action which baffles description. Consider, for example, a simple thing like the movement of a train of cars. The engineer is concerned only with certain broad features of what takes place, the amount of coal and water used, the draw-bar pull of the locomotive, and the forward motion of the cars as affected by steepness of grade, and the opposing force of friction. But who could describe in detail the rocking and rattling motion of the cars and the whirling and eddy motion of the surrounding air, and who could trace the motion of every particle of dust and smoke! This indescribably complex action we call by the name of *turbu-*

*lence*—it exists everywhere and in everything that goes forward in this world of ours, and it is never twice alike in detail even when the conditions are what one would consider exactly the same. All of which suggests two postulates concerning turbulence, namely (a) that it is infinitely<sup>4</sup> complicated, and (b) that it is essentially erratic in character. Let it be understood, however, that we are not speaking in terms of ordinary values in making these two statements. It is not a question, for example, as to whether a brakeman loses his hat every time he makes a trip from Albany to Buffalo, but it is a question as to whether his hat is lost every time at identically the same place because of a gust of wind of precisely the same character when he lets go of it in the same way because of a sudden jerk of the train which always occurs at the same place in exactly the same manner, and so on in endless detail of specification—if such specification were possible!

In the motion of a simple mechanism like the sun and planets, or in the operation of a simple machine like a dynamo the accompanying erratic action is practically negligible. Thus one does not consider even the tremendous storm movements in the sun in the study of planetary motion, and one does not consider the minute details of the motion which takes place in a lubricated bearing in the study of the operation of a dynamo. In many phenomena, however, erratic action is dominant, and in the study of such phenomena the statistical method must be used. Consider, for example, the motion of the water in a brook. This motion presents a fairly definite average character at each point, and a fairly typical rhythmic variation from this average exists at each point, but there is an erratic depar-

<sup>4</sup> The idea of infinity which comes from counting, one, two, three, four and so on *ad infinitum*, is as nothing compared with the intimation of infinity that comes from things that are seen and felt!

ture from this regular motion which is by no means negligible in magnitude. So it is, in the case of the weather. There is a fairly definite average of weather conditions at a place from year to year, and a fairly typical rhythmic variation, but there is an erratic departure from average and from type, and this erratic variation of the weather can only be studied statistically.

Turbulence is characteristic of those physical and chemical changes which are called irreversible or sweeping processes.<sup>5</sup> The most familiar example of such a process is ordinary fire, and, as every one knows, a fire is not dependent upon an external driving cause, but when once started it goes forward spontaneously and with a rush. It is not, however, exactly correct to speak of a fire as *spontaneous*, because this word refers especially to the beginning of a process, whereas we are here concerned with the characteristics of a process already begun. Therefore it is better to describe a phenomenon like fire as *impetuous* because it does go forward of itself. Tyndall, in referring to the impetuous character of fire, says that it was one of the philosophical difficulties of the eighteenth century. A spark is sufficient to start a conflagration, and the effect would seem to be out of all proportion greater than the cause. Herein lay the philosophical difficulty. This difficulty may seem to be the same as that which the biologist faces in thinking of the small beginnings of such a tremendous thing as the chestnut-tree blight in the United States. The chance importation of a spore is indeed a small thing, but it is by no means an infinitesimal, whereas, under conceivable conditions a fire can be started by *a cause more minute and more nearly insignificant than anything assignable*. This possibility

<sup>5</sup> There is one type of irreversible process which is steady and amenable to measurement while under way, namely, the so-called steady sweep.

of the growth of tremendous consequences out of a cause which has the mathematical character of an infinitesimal is the remarkable thing; and this possibility is not only characteristic of fire, but it is characteristic of impetuous processes in general.

#### STATIC AND DYNAMIC INSTABILITY

Impetuous processes, such as storm movements of the atmosphere, are intimately connected with conditions of instability. Indeed, an impetuous process seems always to be the collapse of an unstable state. Let us consider, therefore, two ideal cases where the condition of instability is assumed to be completely established at the start.

(a) Imagine a warm layer of air near the ground overlaid with cold air. Such a condition of the atmosphere is unstable, and any disturbance, however minute, may conceivably start a general collapse. Thus a grasshopper in Idaho might conceivably initiate a storm movement which would sweep across the continent and destroy New York City, or a fly in Arizona might initiate a storm movement which would sweep out into the Gulf of Mexico! These results are different, surely, and the grasshopper and the fly may be of entirely unheard-of varieties, more minute and insignificant than anything assignable. Infinitesimal differences in the earlier stages of an impetuous process may, therefore, lead to finite differences in the final trend of the process. And yet it is quite generally believed that if we knew enough we could predict the weather as we predict an eclipse!

(b) Consider a smooth spherical ball traveling through still air. There certainly is no more reason to expect the ball to jump to the right than to the left. Therefore we may conclude that it will not jump either way. Similarly, a sharp pointed stick stands in a perfectly vertical position

in a perfectly quiet room, and there is no more reason to expect the stick to fall one way than another, therefore the stick will not fall at all! Every one appreciates the fallacy of this argument as applied to the stick, and the moving ball does in fact jump sidewise.

To understand the behavior of the ball let us think of the ball as standing still and of the air as blowing past in a steady stream. The air streams past the ball and slides over a body of still air behind the ball; the surface which separates the moving air and still air is called a vortex sheet, and a vortex sheet is unstable. Any cause, however minute, is sufficient to start an eddy or whirl, and once started such an eddy or whirl develops more and more. Such an eddy or whirl means that the air streaming past one side of the ball is thrown inwards or outwards, and the reaction on the ball pushes the ball sidewise. This effect can be shown by dropping a marble in a deep jar of water. Instead of moving straight downwards the marble follows an erratic zigzag path. This effect is familiar to every one in the sidewise quivering of a stick in a stream of water; and the hissing of a jet of steam is due to the rapid fluttering of the boundary between steam jet and air because of the formation of innumerable eddies.

#### METEOROLOGY<sup>6</sup>

There are three fairly distinct objects to be attained in the analysis of weather observations, namely, (a) the determination of systematic variations in time and place; (b) the elaborate classification of individual storm movements with respect to a great number of measurable characteristics, and the establishment of coefficients

<sup>6</sup> The proposal here set forth was mentioned in a semi-humorous way in a very short article by W. S. Franklin in *SCIENCE*, Vol. 14, pages 496-497, September 27, 1901.

of correlation (statistical) between the measurable characteristics of a given type or class of storm on successive days so that weather predictions can be made rationally, that is, definite predictions qualified by probable departures; and (c) the recognition of critical states in an individual storm movement (conditions of static or dynamic instability) with the hope of devising means for controlling the storm by the suitable expenditure of a very small amount of energy at the critical time and place. If we are ever to control the weather we must, as it seems, do it in this way, and this would be singing Dan Tucker to a hurricane *not* in accordance with Uncle Remus's idea.

The above-mentioned objects are now kept in view by meteorologists, but the study of classifications and departures should be increased a thousand-fold. The point of view of the meteorologist has in the past been the point of view of the classicist in physics with his preconception of a universe of one-to-one correspondences; but statistical studies are the thing.

#### STATISTICAL PHYSICS AND THE POSTULATE OF INDETERMINATION

Whenever the postulate of erratic action is set forth, and the probable departure of a natural phenomenon from the most carefully considered prediction is urged as in the nature of things inevitable, we meet objections from two classes of men, namely, the average man who thinks frankly in terms of human values and the classicist in science who idealizes nature in one-to-one correspondences. Surely, the classicist says, "if we knew all" the data we could make an unqualified prediction in any case. But, ignoring the hopelessly unscientific attitude of mind of one who can postulate infinite knowledge, let it be understood that to speak of data in physics is to speak of a very narrow and limited

kind of thing, for data are only conceivable where measurements can be made or where we have, contrary to Bacon's exhortation, accepted a dream of fancy for a model of the world.

In that branch of mathematical physics which is called statistical mechanics and which includes the atomic theory, we speak of the *completion* of a system when we wish to refer to the positions and velocities of all the elements or particles of the system; let us use this word in the statement of the postulate of indetermination. *The completion of the world to-morrow is not determinate, that is to say, it does not grow out of the completion of the world to-day as a single-valued determinate thing.* This is a postulate which, as it seems, must be accepted as a working hypothesis in the "extra-equilibrium" world, the world of actual happenings, where things never do stand still but go forward by fits and starts impetuously and beyond all control.

#### LITTLE PHYSICS AND BIG PHYSICS

The most fertile source of ideas in physics is the atomic theory which now runs through the whole of physics. Indeed we now have our atomic theory of elasticity, our atomic theory of crystal structure, our atomic theory of gases, our atomic theory of heat (including the whole of chemistry), our atomic theories in nearly every branch of electricity and magnetism, and our quasi-atomic theories of radiation; and the atomic theory suggests that erratic action is universally dominant in the physics of the very small. Therefore the term micro-physics, or little physics, is frequently used to designate what we have called statistical physics, and the term macro-physics, or big physics, is frequently used to designate the classical physics where nature is idealized more or less and one-to-one correspondences rule.

W. S. FRANKLIN

#### THE MINING INDUSTRY

THE accomplishment of the mining industry in the six-month period just completed warrants the forecast that 1916 is to be a record-breaking year, according to the director of the United States Geological Survey. Active demands and good prices have furnished the mine operators with full opportunity for success in working developed properties, and this in turn has given added incentive and available funds for exploration, prospecting and experimentation with new processes.

Summarizing the special reports which are now being made public, Director Smith continues his review:

The returns for six months furnish a basis for the belief that 1916 will set up a new record for the soft-coal mines. Every coal-mining state is sharing in this prosperity and of course this demand for coal is to be traced back to the increased business of the railroads and of the steel and other large industries.

Drilling activity throughout the oil-producing states has brought about a gratifying increase in production of crude oil that promises to make 1916 a record year for marketed petroleum. Already production and consumption are reported by the surveys specialist as essentially in balance east of the Rocky Mountains, with a tendency to lower prices.

The Portland cement industry has had a busy six months and the manufacturers are optimistic. It is predicted that in both production and shipments of cement this year will show a gain over last year, if indeed it does not establish a new record for cement.

Among the metals copper is continuing the steady increase in production which began early last year, and the forecast for 1916 indicates not only the largest output ever known but also the largest profits.

Shipments of iron ore from Lake Superior points for five months of 1916 exceeded by more than 80 per cent. those for the same months in 1915, and the indications for the year are favorable for a new high record on iron-ore production, and of pig iron as well. Higher prices with a steady demand are stimulating the mining of manganese, with the result that